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Alternating Offer Bargaining with Endogenous Information*

Tri Vi Dang

Department of Economics
University of Mannheim
L7, 3-5
68131 Mannheim, Germany
Email: dang@pool.uni-mannheim.de

Abstract

Two ex ante identically informed agents play an alternating offer bargaining game with endogenous information acquisition and common values. This paper shows that perfect Bayesian equilibria *may* have the following properties. (i) No agreement exists in ultimatum bargaining although both agents maintain symmetric information in equilibrium and the gain from trade is common knowledge. (ii) The agent responding to a take-it-or-leave-it offer captures some or even the full trading surplus. (iii) In the two period case the equilibrium payoffs of the agents are non-monotonic in the discounting of the trading surplus. Further implications for the dynamics in alternating offer bargaining are derived.

Key words: bargaining, endogenous lemons problem, endogenous bargaining position, information acquisition

JEL Classification Numbers: C78, D82, D83

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1. Introduction

An interesting question in economics is why rational agents may have difficulties in reaching mutually beneficial agreements. Inefficient outcomes may take on different forms such as the failure to reach an agreement when gains from trade exist, costly delay in reaching an agreement, or settling on contractual terms that fail to fully realize all gains from trade. The bargaining literature provides asymmetric information as a dominant reason for these inefficiencies. See the survey in Ausubel et al. (2002).

This paper relaxes the assumption of actual asymmetric information and assumes that the agents start with symmetric information about all relevant aspects of trade but information is endogenous. In particular, this paper analyses information acquisition in alternating offer bargaining. A risk neutral buyer and a risk neutral seller seek to agree on a price at which to trade an asset with a common value component. It is common knowledge that a trading surplus exists and both agents have symmetric information about the uncertain common value of the asset. However, in each bargaining period prior to making an offer or a response an agent can acquire information about the true value of the asset.

Common values uncertainty typically plays a role in real restate and financial transactions. If the asset is a piece of land, then the agents may have an incentive to obtain information about the intrinsic value of the piece of land by studying real estate reports or contacting a real estate agency. Agents trading financial assets also face common values uncertainty because of the underlying risky cash flow stream. In particular, in secondary markets the seller of a financial asset does not necessarily possess better information about the value of the asset than a potential buyer. Irrespective of asset ownership, an agent can spent resources to obtain information about the asset before they trade. For example, the trading of stocks on upstairs markets are conducted on a bilateral basis where the market makers can identify the counter parties.[1]

This paper analyses information acquisition in one period and two period bargaining and shows that perfect Bayesian equilibria *may* have the following properties. (i) In ultimatum bargaining no agreement exists although both agents maintain symmetric information in equilibrium and the gain from trade is common knowledge. (ii) The agent responding to a take-it-or-leave-it-offer captures some or even the full trading surplus.

[1] Smith et al. (2001) document that the upstairs market facilitates the execution of very large liquidity motivated trades. They show that these block trades have no information content, whereas information-motivated orders are sent downstairs. Therefore, bilateral and non-anonymous trading can realize lower order execution cost. Also, bonds are traded on decentralized dealership markets.

Whether there is a first mover or second mover advantage in ultimatum bargaining depends on the information cost. (iii) The proposer acquires information and disagreement arises with positive probability. (iv) In two period alternating offer bargaining the equilibrium payoffs of the agents are non-monotonic in the discount factor δ of the trading surplus. In particular, the equilibrium payoff of the first period proposer can increase in δ. (v) If the discounting of the trading surplus is lower than the discounting of the information cost, the delay of agreement arises and is (constraint) efficient. (vi) On the other hand two period bargaining may also cause a more severe inefficiency than one period bargaining.

The driving force for these results is that endogenous information acquisition in a common values environment exerts two effects. It can cause an endogenous lemons problem and it implies that the bargaining positions of the agents are endogenous. The intuition why perfect equilibria exist in which the agent responding to a take-it-or-leave-it-offer captures some or even the full trading surplus is the following. Suppose the asset is worth $v+\Delta$ to the buyer and $v-\Delta$ to the seller where Δ is a constant and v the uncertain common value component which is either high or low. Suppose that the buyer makes the offer and the seller is the responder.

If the information cost c is larger than the total surplus (i.e. $c>2\Delta$), then the buyer does not acquire information in an equilibrium. Suppose the buyer is willing to give the full surplus 2Δ to the seller and proposes $E[v]+\Delta$. If the seller accepts the offer, he gets the expected payoff 2Δ. Alternatively, the seller acquires information and tries to exploit the buyer. If he sees that the value of the asset is high, he rejects the offer. If he sees that the value is low, he accepts the offer. In this state the uninformed buyer suffers an endogenous lemons problem while the informed seller realizes the trading surplus as well as makes some speculative profits. However, ex ante speculation causes an opportunity cost in the sense that the seller forgoes some surplus because there is no trade in the high state.

So if the uninformed buyer offers the seller the full trading surplus and if the information cost c is larger than the expected speculative profit π net the expected opportunity cost of speculation c^{Spec}, then the seller accepts the offer without information acquisition and gets 2Δ. On the other hand if the buyer wants to capture the full trading surplus and proposes $E[v]-\Delta$, then the seller faces no opportunity cost of speculation. If $c<\pi$, then the seller acquires information and speculates instead of just getting zero payoff. Therefore, if $\pi-c^{Spec}<c<\pi$, then there exists a critical offer which the seller accepts without information acquisition. This offer must give the seller some trading surplus.

The possibility to acquire information may endow the responder with a credible speculative threat in the sense of saying, that if he does no get enough trading surplus, he acquires information and exploits the uninformed proposer. In particular, for $2\Delta \leq c = \pi - c^{Spec}$, in any perfect Bayesian equilibrium the proposer gets zero payoff and a perfect Bayesian equilibrium exists in which the responder to a take-it-or-leave-it-offer captures the full trading surplus. Whether there is a first-mover or second-mover advantage in ultimatum bargaining in this type of environment depends on the information cost.

If the information cost is smaller than the speculative profit net the maximal opportunity cost of speculation (i.e. $c < \pi - c^{Spec} = \pi - \Delta$), then the buyer will not propose an offer which reflects the average quality of the object. Although the seller is offered the full surplus he speculates. Anticipating the endogenous lemons problem an uninformed buyer proposes a defensive offer which an uninformed seller does not accept. So if $2\Delta < c < \pi - \Delta$, then no equilibrium with agreement exists although the agents maintain symmetric information.[2] If the information cost is low, the proposer acquires information and only mixed strategy equilibria exist. Because of the lemons problem the proposer endogenously creates, disagreement arises with positive probability.

This paper also shows that the equilibrium payoffs of the agents in two period alternating offer bargaining may be non-monotonic in the discount factor of the trading surplus. The intuition is as follows. There exist parameter constellations (i.e. $2\delta\Delta < \beta c < \pi - 2\delta\Delta$ where δ and β denote discount factors) such that when the agents reach the second period without information acquisition then no trade occurs in the second period, too. In such a case the continuation payoff of the seller is zero. In the first period the buyer faces a trade-off when comparing the following two alternatives. (i) If the buyer acquires information, the continuation payoff of the seller is positive. Since the buyer is informed trade occurs with positive probability in the second period and the seller can capture the expected surplus $\delta\Delta$. Information acquisition exerts a positive externality. (ii) If the buyer does not acquire information but induces the seller to acquire information in the first period by just compensating him for the information cost, the buyer can keep the continuation payoff of the seller at zero. The uninformed buyer accounts for the lemons problem. In the first period trade only occurs in the low state. If there is no trade, then in the second period the seller is

[2] This no efficient trade result is neither driven by asymmetric information about the common valuation as in Akerlof (1970), Samuelson (1984), or Gresik (1991) nor by asymmetric information about the private valuation as in Myerson and Satterthwhaite (1983) but by potential asymmetric information about the common valuation due to the mere possibility of information acquisition.

informed and the surplus $2\delta\Delta$ can be realized with positive probability. With an appropriate offer, the buyer can capture this additional surplus in the first period. The seller may accept the offer because his continuation payoff is anyway zero. Therefore, the equilibrium payoff of the buyer may increase in δ.

The remainder of the paper is organized as follows. The next section relates this paper to the literature. Section 3 introduces the model. Section 4 analyzes ultimatum bargaining. Section 5 analyses two period alternating offer bargaining. Section 6 discusses the results and some implications for the dynamics in T period alternating offer bargaining. Section 7 concludes. The Appendix contains the proofs of all Propositions.

2. Relation to the Literature

The aim of the paper is to study the implication for alternating offer bargaining if the agents can acquire information for strategic reasons, i.e. an agent may want to acquire information to exploit the other agent or to avoid being exploited. In order to focus on this question, this paper assumes that the agents are risk neutral and the trading gain is fixed so that information has no social value. This paper is most closely related to Shavell (1994) who analyses one-sided information acquisition and the disclosure of information prior to the sale of an object through a take-it-or-leave-it-offer. In his setting there is a continuum of sellers and buyers who face different information cost. He compares the equilibrium information acquisition with socially efficient information acquisition for four constellations. (i) Information has social value versus no social value. (ii) Disclosure is mandatory versus voluntary. In his model if information is disclosed it is credible.

Shavell (1994) shows that for the case where information has no social value and disclosure is voluntary, if the cost of information is low, then socially useless information is acquired in equilibrium.[3] If disclosure is mandatory, no wasteful information is acquired. For the case where information has social value the identity of the proposer has different implications for efficient information acquisition. The key difference to Shavell (1994) is that in this model both agents are allowed to acquire information and this can be done during the bargaining process. Under these assumptions the equilibrium payoffs of the agents are

[3] Proposition 5 of this paper derives the same result. The observation that risk neutral agents may overinvest in information is also stated by Matthews (1984) and Hausch and Li (1993) who show that bidders acquire excessive information in pure common values auctions. Bergemann and Valimäki (2002) employs a mechanism design approach and a *local* efficiency concept. They show that any ex post efficient allocation mechanism causes an ex ante information acquisition inefficiency.

non-monotonic in the information cost. In particular, the responder may capture some or even the full surplus in a perfect Bayesian equilibrium. In Shavell (1994) the responder always receives zero payoff in equilibrium.

In addition, the no trade result does not occur in Shavell (1994) since he assumes that the seller always wants to sell. In the terminology of this paper where $u^B=v+\Delta$ and $u^S=v-\Delta$, Δ is assumed to be large so that the lemons problem is not severe relative to the trading gain even though the other agent is better informed.[4] In this paper if Δ is large, then the lemons problem does not arise as an equilibrium outcome. The responder has no incentive to speculate, if $c<\pi-c^{Spec}=\pi-\Delta$ and this cannot arise in equilibrium since $\pi-\Delta<0$.

Shavell (1994) assumes that information can only be acquired prior to the bargaining stage. Section 6 argues that if this assumption is imposed, a hold-up problem would arise in this model and the responder does not acquire information in equilibrium. Therefore, the one-sided information acquisition assumption is immaterial in his model. In contrast, this paper assumes that the responder can acquire information after seeing an offer. It is this assumption which endows the responder with a credible speculative threat so that he may obtain a share of the surplus.[5]

In a series of paper Cremer et al. analyze information acquisition in a principal agent framework. A principal contracts with an agent for the production of a good. In Cremer et al. (1998b) (Cremer and Khalil (1992)) the agent can acquire information about the production cost before (after) the principal offers him a contract. Both models assumes that information acquisition is socially wasteful. The information can be obtained for free when the good is produced. In the first case they show that the optimal contract has the following properties. If the information cost is low (high), then the agent acquires (does not acquire) information in the contracting equilibrium. If the information cost is in an intermediate range, the agent randomizes his information acquisition decision so that the principal faces a maybe informed agent. In the second case no information is acquired even for low information cost.[6]

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[4] Shavell (1994, p.25) states "*In the absence of such an assumption, the complicating issue would arise that the seller without information might not sell, a problem similar to the "lemons" problem in Akerlof (1970)*".

[5] For example, there are circumstances where the responder has some time to react to an offer and in the meanwhile he can acquire information.

[6] Although the sequence of moves in the present model is similar to Cremer and Khalil (1992) the equilibrium outcome is similar to Cremer et al. (1998b). The "interesting" equilibra arise if the information cost c is in an intermediate range. (In this model mixed strategy equilibria only exist if c is low.) Cremer et al. (1998a) analyses contract design in which the agent can acquire socially useful information after seeing the contract offer. For this setting, they characterize how the optimal contract varies with the information cost.

Both papers show that although the agent is or maybe uninformed in equilibrium, it has implications for the trade-off between efficiency and rent extraction in the optimal contract. The agent may obtain some rents because of the ability to acquire information although he may not make use of it. This paper derives a similar result in a bilateral exchange setting. The ability to acquire socially useless information solely for strategic reason can enhance the bargaining position of the agent responding to a take-it-or-leave-offer. In contrast to the standard result, the responder may capture some or even the full trading surplus in a perfect Bayesian equilibrium.

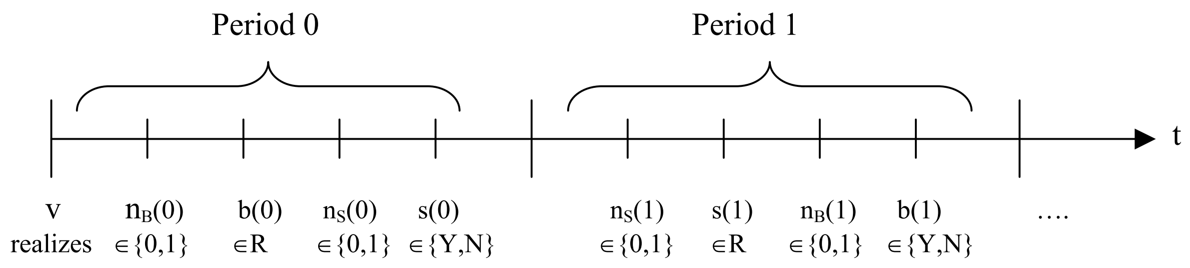
In addition, this paper shows that the mere possibility to acquire information may already render efficient trade unattractive although in the no trade equilibrium the agents maintain symmetric information and the gain from trade is common knowledge. Most bargaining papers assume exogenous asymmetric information and show that private information is a source of inefficiencies in bargaining. See Ausubel et al. (2002). The main contribution of this paper is to show that endogenous information affects both efficiency in bargaining and the bargaining positions of the agents.

3. The Model

Two risk neutral agents play a T period alternating offer bargaining game and seek to agree on a price p_t at which to trade an asset. It is common knowledge that in period t the asset is worth $v+\Delta_t$ to the buyer and $v-\Delta_t$ to the seller. $2\Delta_t$ captures the difference in the valuation between the buyer and the seller where $\Delta_t=\delta^t\cdot\Delta$ with $\delta\in[0,1]$ and $\Delta>0$. v is the uncertain quality which is either v_L or v_H with equal probability where $v_H>v_L>\Delta$. If trade occurs in period t, the surplus $2\Delta_t$ is realized and $U^B=(v+\Delta_t)-p_t$ and $U^S=p_t-(v-\Delta_t)$. If no agreement is reached until period T, the payoffs of the agents are normalized to zero.

In period t=0,2,4,.. the buyer's action is to acquire $n_B(t)\in\{0,1\}$ unit of information and then to choose an offer $b(t)\in R$. The seller's action is to acquire $n_S(t)\in\{0,1\}$ unit of information and to choose a response $s(t)\in\{Y,N\}$ to b(t). If s(t)=Y, trade occurs at the price b(t) and the game ends. Otherwise, the next bargaining period begins. In period t=1,3,... the seller chooses $n_S(t)\in\{0,1\}$and $s(t)\in R$ and the buyer chooses $n_B(t)\in\{0,1\}$ and $b(t)\in\{Y,N\}$. If an agent acquires information, he knows the true value v. The information cost is $c_t=\beta^t\cdot c$ with $\beta\in[0,1]$ and c>0. Information acquisition is observable but the information acquired cannot be disclosed credibly.[7] The solution concept is perfect Bayesian equilibrium (PBE). The sequence of actions is depicted in Figure 1.

Figure 1



4. Ultimatum Bargaining

Given that there is a gain from trade and information has no social value, the efficient outcome is trade without costly information acquisition irrespective of how the surplus is divided. This section shows that the set of perfect Bayesian Equilibria (PBE) depends on the information cost and has the following properties. (i) If the information cost c is low, in a PBE in mixed strategies the buyer acquires information and only he captures some surplus but disagreement arises with positive probability. If c is in an intermediate range, then three cases can arise. (ii) No PBE with trade exists. (iii) In the unique PBE no agent acquires information and both agents capture some surplus. (iv) There exist a PBE in which no agent acquires information and the seller captures the full surplus. (v) If c is high, then the standard case in ultimatum bargaining arises. In the unique PBE no agent acquires information and the buyer captures the full surplus. Whether there is a first or second mover advantage in take-it-or-leave-it offer bargaining depends on the information cost.

To get started, suppose that the buyer does not acquire information and is willing to offer the seller the full surplus 2Δ by proposing the price $b=E[v]+\Delta$. The seller has two potentially profitable responses. (i) The seller accepts this offer and chooses s=Y. He gets $EU^S=2\Delta$ and the buyer obtains $EU^B=0$. (ii) The seller acquires information and chooses s=Y if he sees that v_L; and s=N if he sees v_H. An agreement is only reached at v_L and trade occurs

7 Following most bargaining models, the main analysis is based on the assumption that information cannot be disclosed credibly. If private information is easily verifiable, a complete and fully state contingent contract can resolve the problem of asymmetric information. Section 6 argues that the observability assumption is not crucial for the results but simplifies the analysis. If information acquisition is not observable, then no Perfect Bayesian equilibria in pure strategies may exist.

at the price $p=E[v]+\Delta=\frac{1}{2}(v_H+v_L)+\Delta$. The seller's payoff is $EU^S=\frac{1}{2}[p-(v_L-\Delta)]-c=\frac{1}{4}(v_H-v_L)+\Delta-c$.

The second strategy dominates the first one if $\frac{1}{4}(v_H-v_L)+\Delta-c>2\Delta$, which is the case for $c<\frac{1}{4}(v_H-v_L)-\Delta$. While $\frac{1}{4}(v_H-v_L)$ is the expected speculative profit the informed seller makes, Δ can be interpreted as the expected opportunity cost of speculation. If the seller speculates, no trade occurs in the state v_H and ex ante he forgoes with probability 0.5 the surplus 2Δ. So if $c<\frac{1}{4}(v_H-v_L)-\Delta$, the seller wants to acquire information and exploit the uninformed buyer *although* he is offered the full surplus. Given response (ii), the payoff of the buyer is $EU^B=\frac{1}{2}[(v_L+\Delta)-p]=-\frac{1}{4}(v_H-v_L)$. Therefore, the buyer does not propose an offer which reflects the average quality of the asset. He has to submit a defensive offer so as to account for the endogenous lemons problem.

Proposition 1 shows that if the information cost c is higher than the maximum surplus the *informed* buyer can capture (in any mixed strategy trading equilibrium), the buyer does not acquire information. Secondly, if c is also higher than the trading surplus which the uninformed buyer can capture when providing the seller an incentive to acquire information by just compensating him for c, the buyer does not do this either.[8] The maximum price the uninformed buyer is willing to propose is his expected valuation $E[v]+\Delta$. The minimum price the uninformed seller is willing to accept is his expected valuation $E[v]-\Delta$. However, since $c<\frac{1}{4}(v_H-v_L)-\Delta$ the buyer does not offer any price within this interval because of the endogenous lemons concern. He proposes a lower price which the uninformed seller does not accept. Therefore, no agreement exists although in the no trade equilibrium the buyer and seller maintain symmetric information.[9]

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[8] The Appendix shows that the first alternative yields a higher surplus for the buyer. If the buyer acquires information, he randomizes his offer and the seller randomizes his response and trade occurs with probability $k>0.5$. If the buyer induces the seller to acquire information the buyer accounts for the lemons problem and trade only occurs in the low state and $k=0.5$.

[9] This no efficient trade result is different from Akerlof (1970), Samuelson (1984) and Gresik (1991) as well as Myerson and Satterthwhaite (1983) since no agent has private information. Dang (2005) shows that the no-trade result also holds (i) in double-auction bargaining and (ii) for the case where the quality is a continuous random variable and the agents can acquire $n\in N$ units of information.

Proposition 1

If $2k\Delta<c<\frac{1}{4}(v_H-v_L)-\Delta$ where $k\in(\frac{1}{2},1)$, the set of PBE is given as follows. The buyer chooses $n_B=0$ and $b<v_L-\Delta+2c$ and the seller chooses $n_S=0$ and s=N. No PBE with trade exists.

For a formal statement of k see Step 3d in the Appendix which shows that k>0.5. (Since the total surplus is 2Δ, this implies that $k\leq1$.) The argument above also shows that if the buyer offers $E[v]+\Delta$ and the information cost is larger than the expected speculative profit net the expected opportunity cost of speculation, i.e. $c\geq\frac{1}{4}(v_H-v_L)-\Delta$, then the seller accepts the offer and gets $EU^S=2\Delta$. On the other hand if the buyer offers $E[v]-\Delta$ and $c<\frac{1}{4}(v_H-v_L)$, the seller acquires information and speculates instead of just getting $EU^S=0$. Therefore, if $\frac{1}{4}(v_H-v_L)-\Delta<c<\frac{1}{4}(v_H-v_L)$, there exists a critical offer which the seller accepts without information acquisition. This offer must give the seller some trading surplus.

Proposition 2

If $\max\{\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L),\ \frac{1}{4}(v_H-v_L)-\Delta\}<c<\frac{1}{4}(v_H-v_L)$, then in the unique PBE the buyer chooses $n_B=0$ and $b=v_H-\Delta-2c$ and the seller chooses $n_S=0$ and s=Y. Trade occurs with probability 1 and $EU^B=2\Delta+2c-\frac{1}{2}(v_H-v_L)$ and $EU^S=\frac{1}{2}(v_H-v_L)-2c$.

The possibility to acquire information endows the responder to a take-it-or-leave-offer with a credible speculative threat in the sense of saying, that if he does no get enough trading surplus, he acquires information and exploits the proposer. The next proposition shows that a PBE exists in which the responder captures the full trading surplus.

Proposition 3

If $\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)\leq c=\frac{1}{4}(v_H-v_L)-\Delta$, then the set of PBE has the following properties. In any PBE the buyer gets $EU^B=0$. There exists a PBE in which the buyer chooses $n_B=0$ and $b=\frac{1}{2}(v_H+v_L)+\Delta$ and the seller chooses $n_S=0$ and s=Y. Trade occurs with probability 1 and $EU^S=2\Delta$.

If the information cost is higher than the speculative profit, then the buyer is not concerned about the endogenous lemons problem. In the unique PBE no agent acquires information, trade occurs with probability 1 and the buyer captures the full surplus. This corresponds to the standard take-it-or-leave-it-offer setting.

Proposition 4

If $c \geq \frac{1}{4}(v_H - v_L)$, then in the unique PBE the buyer chooses $n_B=0$ and $b=\frac{1}{2}(v_H+v_L)-\Delta$ and the seller chooses $n_B=0$ and $s=Y$. Trade occurs with probability 1 and $EU^B=2\Delta$ and $EU^S=0$.

The last case which has not been addressed so far is the low information cost case. The next proposition shows that in a PBE the buyer acquires information and a signaling game arises in which the seller also has the option to acquire information. For a formal statement of α_c, β_c, γ_c, k and t in Proposition 5 see Step 3 in the Appendix.

Proposition 5

If $c<\min\{2k\Delta, \frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)\}$, then a PBE in mixed strategies has the following properties. The buyer chooses $n_B=1$. If $v=v_L$ the buyer chooses $b_L=v_L-\Delta+t$. If $v=v_H$ the buyer chooses $b_H=v_H-\Delta$ with probability $1-\alpha_c$ and $b_L=v_L-\Delta+t$ with probability α_c. The seller chooses the following response: If he sees $b=v_H-\Delta$, he chooses=Y. If the seller sees $b=v_L-\Delta+t$, he chooses $n_S=1$ with probability $1-\beta_c$ and $n_S=0$ with probability β_c. If he is supposed to choose $n_S=1$, then seeing $v=v_L$ the seller chooses s=Y. Otherwise he chooses s=N. If the seller is supposed to choose $n_S=0$, then he chooses s=Y with probability γ_c and s=N with probability $1-\gamma_c$. Trade occurs with probability $k>0.5$ and $EU^B=2k\Delta-c$ and $EU^S=0$.

Corollary

If c=0, then two PBE exist. The buyer chooses $n_B=1$ and $b_L=v_L-\Delta$ at v_L and $b_H=v_H-\Delta$ at v_H. If the seller sees b_H, he chooses $n_S=0$ *or* $n_S=1$ and s=Y. If the seller sees b_L he chooses $n_S=1$ and s=Y at $v=v_L$ and s=N at $v=v_H$. In both PBE $U^B=2\Delta$ and $U^S=0$.

Figure 2 (a) plots the equilibrium payoffs of the agents as a function of the information cost for the parameter values $\Delta=\frac{1}{20}$ and $v_H-v_L=1$ and shows that the Propositions 5 (with $k\approx0.52$), 1, 3, 2 and 4 arise consecutively. In Figure 2 (b) where $\Delta=\frac{1}{8}$ and $v_H-v_L=1$, the Propositions 5 (with $k\approx0.56$), 2 and 4 arise consecutively. There is a discrete jump in the payoff of the buyer at c=0 from 2Δ to $2k\Delta$, since k<1 for c>0. (In a mixed strategy equilibrium the probability k of trade is strictly smaller than one.)

Figure 2

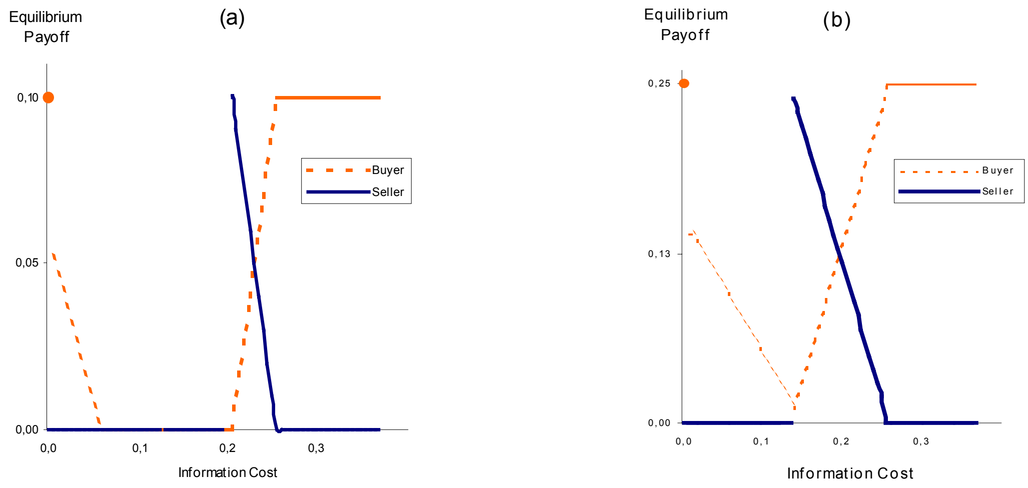

Figure 2. The equilibrium payoff of the buyer (proposer) and the seller (responder) in take-it-or-leave-it-offer bargaining is plotted as a function of the information cost for the parameter values in (a) $2\Delta=1/10$, $v_H-v_L=1$ and (b) $2\Delta=1/4$, $v_H-v_L=1$.

5. Two Period Alternating Offer Bargaining

This section extends the one-period case to two period alternating offer bargaining and analyses the impact of the discounting of the trading surplus and the discounting of the information cost on the timing of information acquisition and trade as well as the terms-of-trade. To focus on the interesting case, the information cost is assumed to be low such that there is an endogenous lemons concern in any of the two periods. Proposition 5 shows that in the one-period setting only mixed strategy equilibria exist. Therefore, one has to deal with a two-sided signaling game with endogenous information acquisition. In order to highlight the main results, the following simplifying assumption is employed.

<u>Assumption T</u>

(1) If the informed agent discloses information, he can disclose it credibly.

(2) $c<\min\{2\Delta, \frac{1}{4}(v_H-v_L)-\Delta\}$.

Assumption T1 implies that k=1 in Propositions 1 to 5. (In equilibrium, an informed agent discloses information since it increases the probability of trade.) Assumption T2 makes the problem interesting and implies that if trade occurs, then at least one agent acquires information because of the endogenous lemons threat. Appendix C analyses the full set of

PBE without Assumption T1 and shows that the qualitative implications hold. The next lemma describes the continuation payoff or default option of the seller in period 1 if he does not acquire information and rejects any offer in period 0.

Lemma 1

Assumptions T1 and T2 hold. The default option D of the seller is given as follows.

(i) For $\delta \leq \frac{c_1}{2\Delta}$, D=0 if $n_B(0)=0$ and $D=2\delta\Delta$ if $n_B(0)=1$.

(iii) For $\delta \geq \frac{c_1}{2\Delta}$, $D=2\delta\Delta-c_1$ if $n_B(0)=0$ and $D=2\delta\Delta$ if $n_B(0)=1$.

Lemma 1 shows that if the buyer acquires information in period 0, he increases the default option of the seller. The intuition is as follows. Given the endogenous lemons problem, if both agents are uninformed, then no trade occurs at all. If one agent is informed trade occurs with positive probability in any of the two periods.[10] Therefore, the information acquisition of the buyer exerts a positive externality on the seller since trade occurs with positive probability in period 1. So the default option of the seller is endogenous and depends on both the discounting and the information acquisition decision of the buyer. The next proposition shows that given the endogenous lemons constraint, the delay of information acquisition and agreement arise in equilibrium and is not caused by signaling but by a kind of optimal timing consideration subject to an endogenous lemons constraint.[11]

Proposition 6

Assumptions T1 and T2 hold. If $\delta>\max\{\frac{c_1}{2\Delta}, 1-\frac{c_0+c_1}{\Delta}, 1-\frac{c_0}{\Delta}\}$, then the set of PBE has the following properties. No agent acquires information and no trade occurs in period 0. In period 1 the seller acquires information, trade occurs with probability 1 and $EU^B=0$ and $EU^S=2\delta\Delta-c_1$.

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[10] In some sense actual asymmetric information is "better" than potential asymmetric information in this environment.

[11] The bargaining literature provides as a dominant reason for delay a signaling or screening story due to (actual) asymmetric information. Admati and Perry (1987) and Cramton (1992) show that asymmetric information about the private valuation of the asset can cause delay. Evans (1989) and Vincent (1989) show that asymmetric information about the common valuation can lead to delay, too. See also Cho (1990), Watson (1998), Feinberg and Skrzypacz (2005), and the survey in Ausubel et al. (2002).

Proposition 6 shows that the agents have symmetric information during the period of disagreement. Rewriting the condition in Proposition 6 shows that delay occurs if $\Delta(1-\delta)<c_0-c_1$, i.e. if the drop in the trading surplus is smaller than the drop in the cost of information acquisition (assuming that one agent acquires information at some time). Therefore, if information is acquired in equilibrium it is socially optimal to delay information acquisition and trade. As an extreme case, if $\delta=1$ and $\beta=0$ (i.e. the information can be obtained for free in period 1), then it is efficient to acquire information in period 1. This results is similar in flavor to Ingersoll and Ross (1992) who derive an optimal waiting time argument to invest under uncertainty without strategic interactions. Proposition 6 proposes a kind of waiting-to-agree-result or optimal timing argument to invest in information subject to an endogenous lemons constraint.

Proposition 7

Assumptions T1 and T2 hold. If $\max\{\frac{1}{3},\frac{c_0}{\Delta}-1\}<\delta<\frac{c_1}{2\Delta}$, then the PBE has the following properties. The seller acquires information in period 0. If $v=v_L$ trade occurs. If $v=v_H$, there is disagreement. In period 1 trade occurs with probability 1 and $EU^B=\Delta+\delta\Delta-c_0$ and $EU^S=0$.

Proposition 7 contains two observations. (1) The buyer does not acquire information but provides the seller an incentive to acquire information in period 0. The reason is the positive externality of information acquisition. Lemma 1 shows that for $\delta<\frac{c_1}{2\Delta}$ the continuation payoff of the seller is zero, if the buyer does not acquire information in period 0. If the buyer acquires information he increases the seller's continuation payoff to $2\delta\Delta$. In period 0 the buyer faces a trade-off when comparing the two alternatives. (i) If the buyer does not acquire information but provides the seller an incentive to do so, the buyer is able to keep the default option of the seller at zero but trade only occurs with probability 0.5 in period 0. His payoff is $EU^B=\Delta+\delta\Delta-c_0$. (ii) If the buyer acquires information the default option of the seller increases to $2\delta\Delta$ but trade occurs with probability 1. His payoff is $EU^B=2\Delta-\delta\Delta-c_0$. Proposition 7 gives conditions such that alternative (i) dominates (ii) and alternative (i) yields $EU^B>0$.

(2) The equilibrium payoff of the buyer increases in the discount factor δ of the trading surplus. The intuition is as follows. In period 0 the buyer does not acquire information but provides the seller an incentive to do so. The buyer accounts for the lemons problem and trade only occurs in the low state. If there is no trade in period 0, in period 1 the

seller is informed and trade occurs with probability 1 and the seller gets $2\delta\Delta$. In period 0 the buyer can capture this additional surplus by proposing an appropriate offer and his expected payoff increases in δ. The seller may accept this offer since his continuation payoff is anyway zero.

Proposition 8

Assumptions T1 and T2 hold. If $1-\frac{c_0}{2\Delta}<\delta<\min\{\frac{c_1}{2\Delta},\frac{1}{3}\}$ or $\frac{1}{3}<\delta<\min\{\frac{c_1}{2\Delta}, \frac{c_0}{\Delta}-1\}$, then no PBE with trade exists.

Proposition 8 can be interpreted as saying that there exist parameter constellations such that neither the first period nor the second period proposer is able to capture enough trading surplus so that nobody acquires costly information. Because of the endogenous lemons problem no trade occurs at all. If the agents are only allowed to bargain for one period, then the buyer acquires information and trade occurs with positive probability. This result is similar to Deneckere and Liang (2006) who show that the outcome in infinite horizon bargaining may perform worse than the outcome in one period bargaining. Figure 3 plots the equilibrium payoffs of the agents as a function of the discount factor while keeping all other parameters fixed.[12]

Figure 3

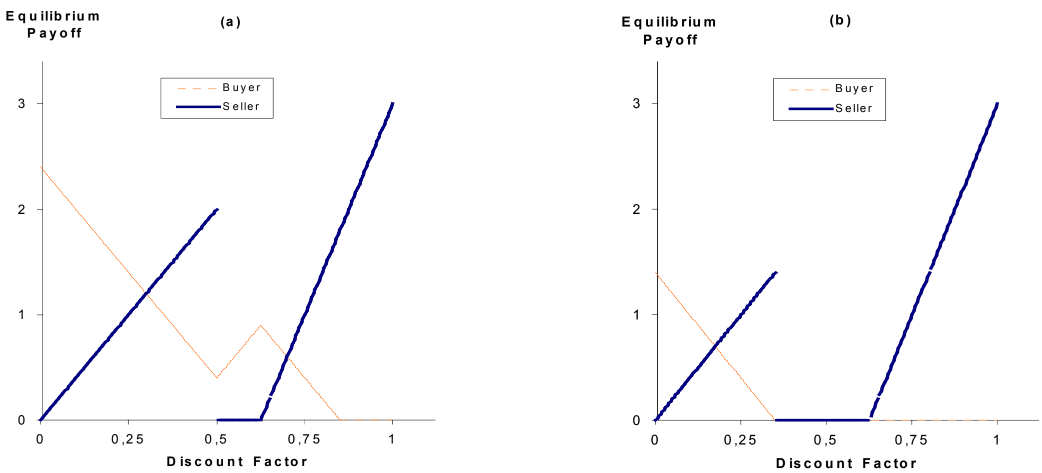

Figure 3. The equilibrium payoff of the buyer (first-period proposer) and the seller (second-period proposer) in two-period alternating-offer bargaining is plotted as a function of the discount factor of the trading surplus for the parameter values in (a) c_0=5.6, c_1=5, 2Δ=8, and v_H-v_L=60 and (b) c_0=6.6, c_1=5, 2Δ=8, and v_H-v_L=60.

[12] Proposition 7 also holds for $c_1=c_0$ and Proposition 6 and 8 also hold for $\beta=\delta$.

6. Discussion

A) On the Assumptions

This section discusses the assumptions. (1) Given that the agents are risk neutral, if the signal the agents acquire is not perfect, this only changes the expected speculative profits an informed agents might make and the critical values of the information cost for the different types for equilibria to arise. (2) If the agents can acquire multiple units of information, then in a PBE in which the proposer acquires information, he endogenously acquires n* units of information such that the responder anticipates that when he accepts an offer he does not suffer a lemons problem. Given the defensive offer, the proposer acquires n* units of information as a self-fulfilling equilibrium. Dang (2005) analyses the case where the quality of the asset is a continuous random variable and the agents can acquire $n \in N$ units of information. The qualitative implication remains.

(3) This paper assumes that information acquisition is observable. If information acquisition is not observable, then no equilibrium in pure strategies exists if $c<\frac{1}{4}(v_H-v_L)$ and $\Delta<\frac{1}{8}(v_H-v_L)$. For example, if the seller does not know whether the buyer is informed and is to accept the offer $b=E[v]+\Delta$ without information acquisition, the buyer may have an incentive to acquire information and speculate. The informed buyer only proposes this offer if he sees v_L. The expected payoff of the buyer is $EU^B=\frac{1}{4}(v_H-v_L)-c$ while the seller obtains $EU^S=2\Delta-\frac{1}{4}(v_H-v_L)$ and suffers a lemons problem. Although no pure strategy equilibria may exist if information acquisition is not observable, yet all qualitative results may hold since a potential second mover advantage carries over to this case. If $c<\frac{1}{4}(v_H-v_L)$, on average the proposer receives a share of the surplus even in mixed strategy equilibrium.

(4) Suppose that both agents can only acquire information prior to the bargaining stage. (i) If information acquisition is observable, then the efficient outcome is achievable. Both agents are not concerned about being exploited since they know for sure whether the opponent is informed or not. In equilibrium no agent acquires information and the responder captures no surplus in a perfect equilibrium. (ii) If information acquisition is not observable and the information cost is low, then the proposer acquires information as a self-fulfilling equilibrium. The responder does not acquire information prior to the bargaining stage because he faces an hold-up problem since the information cost is sunk.

B) T-Period Bargaining

A potential difficulty in analyzing a T-period or an infinite horizon version is that the continuation payoffs of the agents depend on the information acquisition as well as the discounting processes $\{\Delta_t\}$and $\{c_t\}$ in a complex fashion. The reason is that information acquisition may exert a positive externality by changing the probability of trade and therefore the continuation payoff of the counter party. This section discusses some special cases and shows that the delay or waiting-to-agree-result holds for T-period and infinite horizon bargaining as well as for the case where the time interval between offers converges to zero. This section also shows that the *analysis of information acquisition* in infinite horizon bargaining reduces to a finite consideration if the discounting of the trading surplus is larger than the discounting of the information cost.

(a) Suppose that $t\in[0,1]$ is real time, and each bargaining round has a length of γ. If there is T bargaining rounds within the time interval 0 and 1, then $\gamma=1/T$. Furthermore, suppose that $\Delta_t=\delta^{\gamma\cdot t}\Delta$ for $t\in\{0,\frac{1}{T},\frac{2}{T},..,1\}$ and $c_t=\beta^{\gamma\cdot t}c$ for $t\in\{0,\frac{1}{T},\frac{2}{T},..,\frac{T-1}{T}\}$ and $c_1=\varepsilon$ where $2\Delta<c<\frac{1}{4}(v_H-v_L)-\Delta$, $\beta\geq\delta$, and ε small. It is straightforward to show that in no perfect Bayesian equilibrium does any agent acquire information at time $t<1$ or in any of the bargaining rounds 0 to $T-1$ because $c_t>2\Delta_t$. Since $c_t<\frac{1}{4}(v_H-v_L)-\Delta_t$, the uninformed proposer submits a defensive offer which the uninformed responder does not accept. So no trade occurs at time $t<1$. If $T\rightarrow\infty$, then the time interval between offers converges to zero. This argument shows that the specification of the length of single bargaining rounds is not crucial for the delay of agreement in this model. At time $t=1$ or in the final bargaining round T, information is acquired by at least one agent and trade occurs with positive probability.

(b) Suppose that time is discrete, the bargaining can last forever, $\Delta_t=\delta^t\Delta$, $c_t=\beta^t c$, and $c<\frac{1}{4}(v_H-v_L)-\Delta$ (for t=0,1,2,..). (i) If $2\Delta<c$ and $\delta\leq\beta$ then the agents never reach an agreement. (ii) If $c<2\Delta$ and $\delta<\beta$ then the *analysis of information acquisition* in infinite horizon bargaining reduces to a finite consideration. Since the trading surplus is discounted more than the information cost, there exists a t^* such that $2\Delta_t<c_t$ for $t>t^*$. If no agent acquires information in any period $t<t^*$, no agreement will be reached anymore. So one can start with the backward induction argument in period t^* to determine information acquisition. However, once information is acquired, the game switches back to the infinite horizon version. Therefore, standard stationarity arguments may not apply. Further research on this case as well as the case where $c<2\Delta$ and $\delta=\beta$ might be of interest.

7. Conclusion

This paper analyses information acquisition in alternating offer bargaining with common values uncertainties. This paper shows that information acquisition in such an environment can cause an endogenous lemons problem and implies that the bargaining positions of the agents are endogenous. Depending on the underlying parameters of the bargaining environment perfect Bayesian equilibria may have very different properties. For example, in ultimatum bargaining the agent responding to a take-it-or-leave-it-offer may capture some or even the full trading surplus. The equilibrium payoffs of the agents in two period alternating offer bargaining may be non-monotonic in the discount factor of the trading surplus. In particular, this paper shows that the mere possibility to acquire information may already render efficient trade unattractive although the agents maintain symmetric information in equilibrium and the gain from trade is common knowledge.

Appendix A

This Appendix proves Propositions 1 to 5 together. The proof proceeds as follows. Step 1 analyses the best response correspondence of the seller to $n_B=0$ and b. Step 2 analyzes the buyer's payoff expectations at $n_B=0$ and different b, ensuring best responses of the seller. Step 3 analyses the best response correspondences for the case where $n_B=1$. Step 4 characterizes the decision of the buyer and step 5 summarizes the PBE paths.

Step 1

This step analyzes the best response correspondence of the seller to $n_B=0$ and b which is denoted with (0,b). If the seller does not acquire information his strategy is denoted with $(n_S,s)=(0,s)$. If the seller acquires information his strategy is denoted with $(n_S,s_L,s_H)=(1,s_L,s_H)$ where s_L and s_H describe his responses when seeing $v=v_L$ and $v=v_H$, respectively.

Step 1a

Case 1: If $b \le v_L-\Delta$, then the seller never wants to sell, so he has nothing to gain from buying information. The best response to (0,b) with $b \le v_L-\Delta$ is given by (0,s) where s=N.

Case 2: Suppose $v_L-\Delta<b<\frac{1}{2}(v_H+v_L)-\Delta$. (a) If the seller acquires no information, he can only loose from trading, so (0,s) with s=Y is a dominated choice. His maximal payoff without information acquisition is therefore $EU^S=0$. (b) If the seller buys information, then he will choose $s_H=N$ and $s_L=Y$. His maximal payoff with information acquisition is $EU^S=\frac{1}{2}[b-(v_L-\Delta)]-c$.

Consequently, if $\frac{1}{2}(b-v_L+\Delta)-c<0$, the best response of the seller to (0,b) where $v_L-\Delta<b<\frac{1}{2}(v_H+v_L)-\Delta$ is given by (0,s) with s=N. If $\frac{1}{2}(b-v_L+\Delta)-c>0$, the best response of the seller to (0,b) where $v_L-\Delta<b<\frac{1}{2}(v_H+v_L)-\Delta$ is given by $(1,s_L,s_H)$ with $s_L=Y$ and $s_H=N$. If $\frac{1}{2}(b-v_L+\Delta)-c=0$, the set of best responses of the seller is given by (0,s) with s=N and $(1,s_L,s_H)$ with $s_L=Y$ and $s_H=N$.

Case 3: Suppose $b=\frac{1}{2}(v_H+v_L)-\Delta$. The same argument shows that if $\frac{1}{2}(b-v_L+\Delta)-c<0$ the set of best response of the seller is given by (0,s) with s=N and (0,s) with s=Y. If $\frac{1}{2}(b-v_L+\Delta)-c>0$, then as before, the best response of the seller is given by $(1,s_L,s_H)$ with $s_L=Y$ and $s_H=N$. If $\frac{1}{2}(b-v_L+\Delta)-c=0$, then the set of best responses of the seller is given by (0,s) with s=Y, (0,s) with s=N, and $(1,s_L,s_H)$ with $s_L=Y$ and $s_H=N$.

Case 4: Suppose $\frac{1}{2}(v_H+v_L)-\Delta<b<v_H-\Delta$. (a) If the seller acquires no information, he chooses s=Y. His payoff is $EU^S=b-\frac{1}{2}(v_H+v_L)+\Delta$. (b) If the seller buys information, then he chooses s_L=Y and s_H=N. His maximal payoff with information acquisition is $EU^S=\frac{1}{2}[b-(v_L-\Delta)]-c$, as before.

It follows that if $\frac{1}{2}(b-v_L+\Delta)-c<b-\frac{1}{2}(v_H+v_L)+\Delta$, the best response of the seller to (0,b) with $\frac{1}{2}(v_H+v_L)-\Delta<b<v_H-\Delta$ is given by (0,s) with s=Y. If $\frac{1}{2}(b-v_L+\Delta)-c>b-\frac{1}{2}(v_H+v_L)+\Delta$, the best response of the seller to (0,b) with $\frac{1}{2}(v_H+v_L)-\Delta<b<v_H-\Delta$ is given by $(1,s_L,s_H)$ with s_L=Y and s_H=N. Otherwise the seller is indifferent between the two responses.

Case 5: Suppose $b=v_H-\Delta$. (a) If the seller does buy information, he chooses s_L=Y and he is willing to set s_H=Y, allowing a trade to occur albeit without any net gain to himself. His payoff is $EU^S=\frac{1}{2}(b-v_L+\Delta)+\frac{1}{2}(b-v_H+\Delta)-c= b-\frac{1}{2}(v_L+v_H)+\Delta-c$. (b) If the seller does not acquire information, then he chooses s=Y and $EU^S=b-\frac{1}{2}(v_L+v_H)+\Delta$. Consequently, buying information is dominated by not buying information. The seller's best response to (0,b) with $b=v_H-\Delta$ is to choose (0,s) with s=Y.

Case 6: Suppose $b>v_H-\Delta$. The same argument as in case 5 shows that the seller's best response to (0,b) with $b>v_H-\Delta$ is to choose (0,s) with s=Y.

Step 1b

The preceding discussion has not yet gone into much detail about the seller's information acquisition decision. In case 1, 5, and 6 the information acquisition best response of the seller is not to acquire information. Only if $v_L-\Delta<b<v_H-\Delta$, it is potentially worthwhile for the seller to acquire information. In case 2 and 3, the information acquisition decision turns on whether

(1) $\quad \frac{1}{2}(b-v_L+\Delta)-c <=> 0,$

in case 4 on whether

(2) $\quad \frac{1}{2}(b-v_L+\Delta)-c <=> b-\frac{1}{2}(v_H+v_L)+\Delta.$

Given that the left-hand side of (1) is increasing in b and the difference between the left-hand side and the right-hand side of (2) is decreasing in b, information acquisition is not attractive at any price b if it is not attractive at $b=\frac{1}{2}(v_H+v_L)-\Delta$, the upper bound of the interval defining Cases 2 and 3 and the lower bound of the interval defining Case 4. Substituting $b=\frac{1}{2}(v_H+v_L)-\Delta$ into the left-hand side of (2) yield $\frac{1}{4}(v_H-v_L)-c$. Thus there are three possibilities.

<u>Alternative I</u>: $c>\frac{1}{4}(v_H-v_L)$.

In this case, at $b=\frac{1}{2}(v_H+v_L)-\Delta$, information acquisition is not worthwhile, i.e.

$$\tfrac{1}{2}[\tfrac{1}{2}(v_H+v_L)-\Delta-v_L+\Delta]-c=\tfrac{1}{4}(v_H-v_L)-c<0$$

and

$$\tfrac{1}{4}(v_H+v_L)-c<\tfrac{1}{2}(v_H+v_L)-\Delta-\tfrac{1}{2}(v_H+v_L)+\Delta.$$

So if $c>\frac{1}{4}(v_H-v_L)$, then information acquisition is not worthwhile to the seller regardless of what price he expects the uninformed buyer to set. The seller's best response to (0,b) is to choose (0,s) where (i) s=N if $b<\frac{1}{2}(v_H+v_L)-\Delta$, (ii) s=Y or s=N if $b=\frac{1}{2}(v_H+v_L)-\Delta$, and (iii) s=Y if $b>\frac{1}{2}(v_H+v_L)-\Delta$.

<u>Alternative II</u>: $c<\frac{1}{4}(v_H-v_L)$.

In this case, at $b=\frac{1}{2}(v_H+v_L)-\Delta$, information acquisition is worthwhile, i.e.

$$\tfrac{1}{2}[\tfrac{1}{2}(v_H+v_L)-\Delta-v_L+\Delta]-c=\tfrac{1}{4}(v_H-v_L)-c>0$$

and

$$\tfrac{1}{4}(v_H+v_L)-c>\tfrac{1}{2}(v_H+v_L)-\Delta-\tfrac{1}{2}(v_H+v_L)+\Delta.$$

Denote $\underline{b}$ as the price where the left-hand side of (1) is just zero and $\overline{b}$ where the left-hand side equals the right-hand side of (2). There exist critical prices

$$\underline{b}=v_L-\Delta+2c<\tfrac{1}{2}(v_H+v_L)-\Delta,$$

and

$$\overline{b}=v_H-\Delta-2c>\tfrac{1}{2}(v_H+v_L)-\Delta,$$

such that information acquisition is not worthwhile to the seller if the buyer sets $b<\underline{b}$ or $b>\overline{b}$. If the buyer sets $b\in(\underline{b},\overline{b})$, then it is worthwhile to the seller to acquire information. (At $\underline{b}$ and $\overline{b}$, the seller is indifferent.)

(i) The seller's best response to (0,b) with $b<\underline{b}$ or $b>\overline{b}$ is to choose (0,s) where s=N if $b<\underline{b}$ and s=Y if $b>\overline{b}$. (ii) The seller's best response to (0,b) with $b\in(\underline{b},\overline{b})$, is to choose $(1,s_L,s_H)$ with s_L=Y and s_H=N. (iii) For $b=\underline{b}$, the seller is indifferent between (0,s) with s=N and $(1,s_L,s_H)$ with s_L=Y and s_H=N. (iv) For $b=\overline{b}$, the seller is indifferent between (0,s) with s=N and $(1,s_L,s_H)$ with s_L=Y and s_H=N.

<u>Alternative III</u>: $c=\frac{1}{4}(v_H-v_L)$.

This is the boundary between Alternatives I and II. For $b=\frac{1}{2}(v_H+v_L)-\Delta$, the seller is indifferent between (0,s) with s=Y, (0,s) with s=Y, and $(1,s_L,s_H)$ with s_L=Y and s_H=N. For $b\neq\frac{1}{2}(v_H+v_L)-\Delta$, the best response of the seller is to choose (0,s) where s=N if $b<\frac{1}{2}(v_H+v_L)-\Delta$, and s=Y if $b>\frac{1}{2}(v_H+v_L)-\Delta$.

<u>Step 2</u>

This step analyses the buyer's payoff expectations at $n_B=0$ and b, ensuring best responses of the seller.

<u>Alternative I</u>: $c>\frac{1}{4}(v_H-v_L)$.

As mentioned, the seller's best response correspondence to (0,b) is to choose (0,s) where (i) s=N if $b<\frac{1}{2}(v_H+v_L)-\Delta$, (ii) s=Y or s=N if $b=\frac{1}{2}(v_H+v_L)-\Delta$, and (iii) s=Y if $b>\frac{1}{2}(v_H+v_L)-\Delta$. The buyers' payoff is zero if $b<\frac{1}{2}(v_H+v_L)-\Delta$ or if $b=\frac{1}{2}(v_H+v_L)-\Delta$ and s=N. The buyer's payoff is $EU^B=\frac{1}{2}(v_L+v_H)+\Delta-b$ if $b=\frac{1}{2}(v_H+v_L)-\Delta$ and s=Y or $b>\frac{1}{2}(v_H+v_L)-\Delta$.

Thus, by setting $b=\frac{1}{2}(v_L+v_H)$, the buyer can ensure himself the payoff Δ. All (0,b) with $b<\frac{1}{2}(v_H+v_L)-\Delta$ provides the buyer with a lower payoff than (0,b) with $b=\frac{1}{2}(v_H+v_L)$. Similarly, all (0,b) with $b>\frac{1}{2}(v_H+v_L)-\Delta$ provides the buyer with a worse payoff than (0,b) where $\frac{1}{2}(v_H+v_L)-\Delta<b'<b$.

The only strategy without information acquisition of the buyer which is a candidate for being best response to a subform perfect strategy of the seller is thus given by (0,b) with $b=\frac{1}{2}(v_H+v_L)-\Delta$. However, if this is to be best response of the buyer, it must be the case, that the seller's response to this choice is to set (0,s) with s=Y, i.e. the seller must resolve his indifference by opting for trade. In this case $EU^B=2\Delta$.

Remark A1

This line of arguments will be used repeatedly to establish (the existence of) best responses of the agents. Otherwise the proposer has no best responses. (See Fudenberg and Tirole (1998, p.116).) Therefore, the subsequent steps assume that the indifferent responder chooses a response from his set of best responses which the proposer prefers most.

Alternative II: $c<\frac{1}{4}(v_H-v_L)$.

Case 1 and 2a: (i) If the buyer chooses (0,b) with $b<\underline{b}=v_L-\Delta+2c$, the seller chooses (0,s) with s=N. (ii) If the buyer chooses (0,b) with $b=\underline{b}$, the seller is indifferent between (0,s) and s=N and $(1,s_L,s_H)$ with s_L=Y and s_H=N. Depending on which alternative the seller chooses, the buyer's payoff is $EU^B=0$ or $EU^B=\frac{1}{2}(v_L+\Delta-\underline{b})=\Delta-c$.

Case 2b, 3, 4a: (i) If the buyer chooses (0,b) with $\underline{b}=v_L-\Delta+2c<b<\overline{b}=v_H-\Delta-2c$, the seller chooses $(1,s_L,s_H)$ with s_L=Y and s_H=N and $EU^B=\frac{1}{2}(v_L+\Delta-b)$. (ii) If the buyer chooses (0,b) with $b=\overline{b}$, the seller is indifferent between choosing (0,s) with s=Y and $(1,s_L,s_H)$ with s_L=Y and s_H=N. If the seller chooses the first alternative, then $EU^B=\frac{1}{2}(v_L+v_H)+\Delta-\overline{b}=\frac{1}{2}(v_L+v_H)+\Delta-(v_H-\Delta-2c)=2\Delta+2c-\frac{1}{2}(v_H-v_L)$. If the seller chooses the second alternative, then $EU^B=\frac{1}{2}(v_L+\Delta-\overline{b})=\Delta+c-\frac{1}{2}(v_H-v_L)<2\Delta+2c-\frac{1}{2}(v_H-v_L)$. So the buyer has a strict preference to have the seller resolve his indifference by not acquiring information.

Case 4b, 5, 6: If the buyer chooses (0,b) with $b>\overline{b}=v_H-\Delta-2c$, the seller chooses (0,s) where s=Y and $EU^B=\frac{1}{2}(v_L+v_H)+\Delta-b<2\Delta+2c-\frac{1}{2}(v_H-v_L)$.

Given these observations, any choice (0,b) with $b>\overline{b}$ is obviously worse for the buyer than the choice (0,b) with $b=\frac{1}{2}(b+\overline{b})$. Similarly, any choice (0,b) with $\underline{b}<b<\overline{b}$ is worse for the buyer than (0,b) with $b=\frac{1}{2}(b+\underline{b})$; as is the choice (0,b) with $b=\overline{b}$ followed by information acquisition of the seller, i.e. $(1,s_L,s_H)$ with s_L=Y and s_H=N.

The only strategies without information acquisition of the buyer which remain as possible candidates for being best responses to a subform perfect strategy of the seller are the following: (i) (0,b) with $b=\overline{b}$, assuming that this is followed by the seller choosing (0,s) with s=Y, (ii) (0,b) with $b=\underline{b}$, assuming that this is followed by $(1,s_L,s_H)$ with s_L=Y and s_H=N, (iii) (0,b) with $b<\underline{b}$, followed by (0,s) where s=N. Path (i) implies $EU^B=2\Delta+2c-\frac{1}{2}(v_H-v_L)$, path (ii) implies $EU^B=\Delta-c$, and path (iii) implies no trade and $EU^B=0$.

Alternative III: $c=\frac{1}{4}(v_H-v_L)$.

Based on an analogous argument as above, the only strategy without information acquisition of the buyer which is a candidate for being best response to a subform perfect strategy of the seller is given by (0,b) with $b=\frac{1}{2}(v_H+v_L)-\Delta$, assuming that the indifferent seller chooses (0,s) with s=Y. Then $EU^B=2\Delta$ and $EU^S=0$.

Step 3

This step analyses best responses for the case where the buyer chooses $n_B=1$.

Remark A2

(a) The following arguments show that no best responses in pure strategies exist once the buyer acquires information. Suppose the informed buyer is honest and chooses $b=v_L-\Delta$ if $v=v_L$ and $b=v_H-\Delta$ if $v=v_H$. In this case the seller is willing to choose s=Y. However, if the seller always chooses s=Y, the buyer has an incentive always to choose $b=v_L-\Delta$. (If the seller always chooses s=N when seeing $b<v_H-\Delta$ then the buyer always chooses $b=v_H-\Delta$ if $v=v_H$. In this case seeing $b=v_L-\Delta$ is fully revealing of v_L and the seller may choose s=Y). So no best responses in pure strategies exists.

(b) It is easy to see that choosing s=Y when seeing $b=v_L-\Delta$ is a weakly dominated strategy. The seller never gets some surplus but may suffer a lemons problem.

Step 3a (Mixed strategies)

Define $b_L\equiv v_L-\Delta+t$ for $0<t\leq 2\Delta$ and $b_H\equiv v_H-\Delta$. (Note, the informed buyer would not choose $b>v_L+\Delta$ at $v=v_L$. So any $b>v_L+\Delta$ reveals that $v\neq v_L$.)

(1) Suppose the buyer considers the following strategies. If the buyer sees $v=v_H$, then he chooses $b=b_H$ with probability $1-\alpha$ and $b=b_L$ with probability α (where b_L and b_H are as defined above). If he sees $v=v_L$, then he chooses $b=b_L$.

(2) Suppose the seller considers the following strategies. If the seller sees $b=b_H$, he chooses s=Y. If he sees $b=b_L=v_L-\Delta+t$, two cases arises. (a) If $t\geq 2c$, he may choose $n_S=1$ with probability $1-\beta$, and $n_S=0$ with probability β. If he is supposed to choose $n_S=1$, then seeing $v=v_L$ he chooses s=Y; and seeing $v=v_H$ he chooses s=N. If he is supposed to choose $n_S=0$, then he chooses s=Y with probability γ_1 and s=N with probability $1-\gamma_1$. (b) If $t<2c$, he chooses $n_S=0$ and s=Y with probability γ_0 and s=N with probability $1-\gamma_0$.

Step 3b (Making the buyer indifferent at $v=v_H$)

(1) Suppose the seller chooses $n_S=0$ and randomizes his yes/no-decision as described above. At $v=v_H$, if the buyer chooses $b=b_H$, then his payoff is $U^B=2\Delta$. If the buyer chooses $b=b_L$, then $EU^B=\gamma_0\cdot[v_H+\Delta-(v_L-\Delta+t)]$. The buyer is indifferent between choosing $b=b_L$ and $b=b_H$ at $v=v_H$ if $\gamma_0\cdot[v_H+\Delta-(v_L-\Delta+t)]=2\Delta$. (Note, c is sunk at the offer stage.) In order to make the buyer indifferent the seller chooses $\gamma_0=2\Delta/(v_H-v_L+2\Delta-t)$.

(2) Suppose the seller chooses $n_S=1$ with probability $1-\beta$; and $n_S=0$ with probability β and randomizes his yes/no-decision as describe above. In this case, at $v=v_H$ if the buyer chooses $b=b_L$ then $EU^B=(1-\beta)\cdot 0+\beta\cdot\gamma_1\cdot[v_H+\Delta-(v_L-\Delta+t)]$. The buyer is indifferent between choosing $b=b_L$ and $b=b_H$ at $v=v_H$ if $\beta\cdot\gamma_1(v_H-v_L+2\Delta-t)=2\Delta$. In order to make the buyer indifferent the seller chooses $\beta\cdot\gamma_1=2\Delta/(v_H-v_L+2\Delta-t)$.[13]

Step 3c (Making the seller indifferent when seeing $b=b_L$)

Case 1: $t<2c$.

The seller never chooses $n_S=1$; see Case 2 below. If the uninformed seller sees $b=b_L$ and if he chooses $s=Y$, then $EU^S=\frac{1}{2}[(v_L-\Delta+t-(v_L-\Delta)]+\frac{1}{2}\alpha_0[(v_L-\Delta+t-(v_H-\Delta)]=\frac{1}{2}t+\frac{1}{2}\alpha_0(v_L-v_H+t)$. If the seller chooses $s=N$, then $U^S=0$. In order to make the seller indifferent the buyer chooses $\alpha_0=t/(v_H-v_L-t)$.

Case 2: $t\geq 2c$.

The seller may choose $n_S=1$. If the seller chooses $n_S=1$ and sees $v=v_L$, then he chooses $s=Y$. Otherwise he chooses $s=N$. $EU^S=\frac{1}{2}[v_L-\Delta+t-(v_L-\Delta)]-c=\frac{1}{2}t-c$. If the seller chooses $n_S=0$ then $EU^S=\frac{1}{2}t+\frac{1}{2}\alpha_1(v_L-v_H+t)$. The seller is indifferent between $n_S=0$ and $n_S=1$ if $\frac{1}{2}t-c=\frac{1}{2}t+\frac{1}{2}\alpha_1(v_L-v_H+t)$. In order to make the seller indifferent the buyer chooses $\alpha_1=2c/(v_H-v_L-t)$.

Step 3d (Choosing the optimal t as a pre-game decision)

Case 1: $t<2c$.

The expected payoff of the buyer (before information acquisition) is

$$EU^B=\frac{1}{2}\gamma_0[(v_L+\Delta-(v_L-\Delta+t)]+\frac{1}{2}[(1-\alpha_0)(v_H+\Delta-(v_H-\Delta))+\alpha_0\gamma_0(v_H+\Delta-(v_L-\Delta+t))]-c.$$

$$EU^B=\Delta+\Delta(2\Delta-t)/(v_H-v_L+2\Delta-t)-c\in(\Delta-c,2\Delta-c).$$

[13] In this case, the seller has some degree of freedom so as to make the buyer indifferent between lying and telling the truth at $v=v_H$.

Case 2: $t \geq 2c$.

The expected payoff of the buyer (before information acquisition) is

$$EU^B = \tfrac{1}{2}[(\beta\gamma_1[(v_L+\Delta-(v_L-\Delta+t)] + \tfrac{1}{2}[(1-\alpha_1)(v_H+\Delta-(v_H-\Delta)) + \alpha_1\beta\gamma_1(v_H+\Delta-(v_L-\Delta+t))]-c$$

$$EU^B = \Delta(2\Delta-t)/(v_H-v_L+2\Delta-t)+\Delta-c$$

Both responses of the seller yields the same payoff to the buyer. The buyer chooses $t \in (0,2\Delta)$ to maximizes his payoff and therefore,

$$t = \tfrac{1}{2}(-v_H + v_L - \Delta) + \sqrt{\tfrac{1}{4}(-v_H + v_L - \Delta)^2 + \Delta(v_H - v_L) + \Delta^2} > 0 .$$

Define k such that $2k\Delta = \Delta + \Delta(2\Delta-t)/(v_H-v_L+2\Delta-t)$ then

$$k = \tfrac{1}{2} + \tfrac{1}{2}(2\Delta - t)/(v_H - v_L + 2\Delta - t) \in (\tfrac{1}{2}, 1).$$

So the payoff of the buyer is $EU^B = 2k\Delta - c$.

Remark A3

(i) k is also the probability of trade $\tfrac{1}{2} + \tfrac{1}{2}(\gamma_0 - \alpha_0 + \gamma_0\alpha_0)$.

(ii) If v_H-v_L is large, then the optimal $t>2\Delta$. In this case the buyer wants to choose a t which is as close as 2Δ. So no best response exists.

Step 4 (The buyer's decision)

Alternative I: $c > \tfrac{1}{4}(v_H-v_L)$.

It is easy to see that the best response of the buyer is to choose $n_B=0$ and $b=\tfrac{1}{2}(v_H+v_L)-\Delta$, assuming that this is followed by (the indifferent seller choosing) $n_S=0$ and s=Y. Trade occurs with probability 1 and $EU^B=2\Delta$ and $EU^S=0$.

Alternative II: $c = \tfrac{1}{4}(v_H-v_L)$.

As above, the best response of the buyer is to choose $n_B=0$ and $b=\tfrac{1}{2}(v_H+v_L)-\Delta$, assuming that this is followed by $n_S=0$ and s=Y. Then $EU^B=2\Delta$ and $EU^S=0$.

Alternative III: $c < \tfrac{1}{4}(v_H-v_L)$.

The set of candidates without information acquisition for being best responses is the following: (a) $n_B=0$ and $b=v_H-\Delta-2c$ assuming it is followed by $n_S=0$ and s=Y. (b) $n_B=0$ and $b=v_L-\Delta+2c$ assuming it is followed by $n_S=1$ and s_L=Y and s_H=N. (c) $n_B=0$ and $b<v_L-\Delta+2c$, assuming it is followed by $n_S=0$ and s=N. (d) A candidate with information acquisition for

being best responses is described in Step 3. The buyer's expected payoff of the various strategies are given as follows: (a) $2\Delta+2c-\frac{1}{2}(v_H-v_L)$, (b) $\Delta-c$, (c) 0, and (d) $2k\Delta-c$. (Since $k>\frac{1}{2}$, strategy (d) dominates strategy (b)).

Case 1: $c>\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)$.

Strategy (a) dominates strategy (d). So the buyer compares strategy (a) with (c). If $c<\frac{1}{4}(v_H-v_L)-\Delta$, the buyer chooses strategy (c). If $c>\frac{1}{4}(v_H-v_L)-\Delta$, the buyer chooses strategy (a). If $c=\frac{1}{4}(v_H-v_L)-\Delta$, the buyer is indifferent between the two choices.

Case 2: $c<\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)$.

Strategy (d) dominates (a). The buyer compares strategy (d) with (c). If $c>2k\Delta$, the buyer chooses strategy (c). If $c<2k\Delta$, the buyer chooses (d). If $c=2k\Delta$ the buyer the buyer is indifferent between the two strategies.

Case 3: $c=\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)$.

The buyer is indifferent between strategy (a) and (d). So the buyer compares (a,d) with (c). If $c>2k\Delta$ the buyer chooses (c). If $c<2k\Delta$, the buyer is indifferent between the alternatives (a) and (d). If $c=2k\Delta$ the buyer the buyer is indifferent between the three strategies.

Step 5 (Equilibrium paths)

If $c\geq\frac{1}{4}(v_H-v_L)$, then in the unique PBE the buyer chooses $n_B=0$ and $b=\frac{1}{2}(v_H+v_L)-\Delta$ and the seller chooses $n_S=0$ and s=Y. Trade occurs with probability 1 and $EU^B=2\Delta$ and $EU^S=0$. (*Proposition 4*)

If $\max\{\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L),\ \frac{1}{4}(v_H-v_L)-\Delta\}<c<\frac{1}{4}(v_H-v_L)$, then in the unique PBE the buyer chooses $n_B=0$ and $b=v_H-\Delta-2c$ and the seller chooses $n_S=0$ and s=Y. Trade occurs with probability 1 and $EU^B=2\Delta+2c-\frac{1}{2}(v_H-v_L)$ and $EU^S=\frac{1}{2}(v_H-v_L)-2c$. (Note, the buyer chooses this strategy if $\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)<c<\min\{2k\Delta,\frac{1}{4}(v_H-v_L)\}$ or $\max\{2k\Delta,\frac{1}{4}(v_H-v_L)-\Delta\}<c<\frac{1}{4}(v_H-v_L)$.) (*Proposition 2*)

If $\frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)\leq c=\frac{1}{4}(v_H-v_L)-\Delta$, then two types of PBE exist. (i) The buyer chooses $n_B=0$ and $b<v_L-\Delta+2c$ and the seller chooses $n_S=0$ and s=N. No trade occurs. (ii) The buyer chooses $n_B=0$ and $b=v_H-\Delta-2c=\frac{1}{2}(v_H+v_L)+\Delta$ and the seller chooses $n_S=0$ and s=Y. Trade occurs with probability 1 and $EU^B=0$ and $EU^S=\frac{1}{2}(v_H-v_L)-2c=2\Delta$. (*Proposition 3*)

If $2k\Delta<c<\frac{1}{4}(v_H-v_L)-\Delta$, then the set of PBE is given as follows. The buyer chooses $n_B=0$ and $b<v_L-\Delta+2c$ and the seller chooses $n_S=0$ and s=N. There is no PE with trade. (*Proposition 1*)

If $c<\min\{2k\Delta, \frac{2}{3}\Delta(k-1)+\frac{1}{6}(v_H-v_L)\}$, then a PBE in mixed strategies has the following properties. The buyer chooses $n_B=1$. If $v=v_L$ the buyer chooses $b_L=v_L-\Delta+t$. If $v=v_H$ the buyer chooses $b_H=v_H-\Delta$ with probability $1-\alpha_c$ and $b_L=v_L-\Delta+t$ with probability α_c. The seller chooses the following response: If he sees $b=v_H-\Delta$ he chooses=Y. If the seller sees $b=v_L-\Delta+t$ he chooses $n_S=1$ with probability $1-\beta_c$ and $n_S=0$ with probability β_c. If he is supposed to choose $n_S=1$, then seeing $v=v_L$ the seller chooses s=Y. Otherwise he chooses s=N. If the seller is supposed to choose $n_S=0$ then he chooses s=Y with probability γ_c and s=N with probability $1-\gamma_c$. Trade occurs with probability k>0.5 and $EU^B=2k\Delta-c$ and $EU^S=0$. (*Proposition 5*) **QED**

Appendix B

This Appendix proves the results in section 5. It is assumed that $c_0<\min\{2\Delta,\frac{1}{4}(v_H-v_L)-\Delta\}$. $b_L(t)$ and $b_H(t)$ denote the choice of an informed buyer in the state L and H, respectively. Analogously for $s_L(t)$ and $s_H(t)$. To save on notation, $c_1=\beta c$.

<u>Proof of Lemma 1</u>

<u>Case 1</u> : $n_B(0)=0$.

The situation is analogous to the one-period case. In period 1 the seller is the proposer and the buyer is the responder. (Note, $c_1\leq c_0$ and $\Delta_1=\delta\Delta\leq\Delta_0$.)

If $\delta>\frac{c_1}{2\Delta}$, then $c_1<\min\{2\Delta_1, \frac{1}{4}(v_H-v_L)-\Delta_1\}$. Given Assumption T1, the seller chooses $n_S(1)=1$ and trade occurs with probability 1. The continuation payoff of the seller is $EU^S=2\delta\Delta-c_1$.

If $\delta<\frac{c_1}{2\Delta}$, then $2\Delta_1<c_1<\frac{1}{4}(v_H-v_L)-\Delta_1$. Analogous to Proposition 1, the best responses in period 1 imply no trade. The continuation payoff is $U^S=0$.

If $\delta=\frac{c_1}{2k_1\Delta}$, then the seller's continuation payoff is $U^S=0$.

<u>Case 2</u> : Suppose $n_B(0)=1$.

The buyer discloses information. Therefore, the seller knows the true value. The seller chooses $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$. Assuming that the indifferent informed buyer opts for trade and chooses $b_L(1)=Y$ and $b_H(1)=Y$, the seller gets $U^S=2\delta\Delta$.

Proof of Propositions 6-8

Step 1 : $\delta \leq \frac{c_1}{2\Delta}$.

From Lemma 1, the default option D of the seller is D=0 if $n_B(0)=0$ and $D=2\delta\Delta$ if $n_B(0)=1$.

Step 1.1 (Seller's best response correspondence in period 0)

Case 1: $n_B(0)=0$. Define $\hat{b} \equiv v_L-\Delta-2\delta\Delta+2c_0$.

(i) : If $b=\hat{b}$ then the seller is indifferent between choosing $n_S(0)=0$ and $s(0)=N$ and choosing $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$.

Proof : If the seller chooses the first response, no trade occurs in period 0. In period 1 there is also no trade and $U^S=0$. If the seller chooses the second response, then trade occurs in state L and $U^S=(v_L-\Delta-2\delta\Delta+2c_0-(\Delta-v_L))-c_0=-2\delta\Delta+c_0\geq 0$ (since $\delta \leq \frac{c_1}{2\Delta}$). If the true state is H, then no trade occurs in period 0. In period 1 the informed seller gets $EU^S=2\delta\Delta-c_0\leq 0$. So his expected payoff $EU^S=0$.

(ii) If $b<\hat{b}$, then the best response of the seller is to choose $n_S(0)=0$ and $s(0)=N$ and $U^S=0$.

(iii) If $b>\hat{b}$ then the seller chooses $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$ and $EU^S>0$.

Case 2: $n_B(0)=1$. Define $\hat{b}_L \equiv v_L-\Delta+2\delta\Delta$ and $\hat{b}_H \equiv v_H-\Delta+2\delta\Delta$.

The buyer discloses the information credibly. The seller chooses $s(0)=Y$ and trade occurs with probability 1 in period 0 and $U^S=2\delta\Delta$.

Step 1.2 (Buyer's decision in period 0)

(a) Suppose $n_B(0)=0$. If the buyer chooses alternative (i) then $EU^B=0.5[v_L+\Delta-(v_L-\Delta-2\delta\Delta+2c_0)]=\Delta-c_0+\delta\Delta$. If the buyer chooses alternative (ii) then $EU^B=0$. If he chooses alternative (iii) then $EU^B<\Delta-c_0+\delta\Delta$. (b) Suppose $n_B(0)=1$, then $EU^B=2\Delta-2\delta\Delta-c_0$. Consequently, the buyer compares the alternatives (ai), (aii) and (b).

Case 1: If $\delta<\frac{1}{3}$ then alternative (b) dominates (ai). The buyer compares (b) with (aii). If $\delta<1-\frac{c_0}{\Delta}$, the buyer chooses (b) and $EU^B=2\Delta-2\delta\Delta-c_0$. If $\delta>1-\frac{c_0}{\Delta}$, the buyer chooses (aii) and $EU^B=0$. If $\delta=1-\frac{c_0}{\Delta}$, the buyer is indifferent between the two alternatives.

Case 2: If $\delta>\frac{1}{3}$, then alternative (ai) dominates (b). The buyer compares (ai) with (aii). If $\delta>\frac{c_0}{\Delta}-1$, the buyer chooses alternative (ai) and $EU^B=\Delta-c_0+\delta\Delta$. If $\delta<\frac{c_0}{\Delta}-1$, the buyer chooses alternative (aii) and $EU^B=0$. If $\delta=\frac{c_0}{\Delta}-1$, the buyer is indifferent between the two alternatives.

Step 2: $\delta>\frac{c_1}{2\Delta}$.

The seller's default option is $D=2\delta\Delta-c_1$ if $n_B(0)=0$ and $D=2\delta\Delta$ if $n_B(0)=1$.

Step 2.1 (Seller's best response correspondence in period 0)

Case 1: $n_B(0)=0$. Define $\hat{b}\equiv v_L-\Delta+2\delta\Delta-2c_1+2c_0$.

(i) If $b=\hat{b}$ then the seller is indifferent between choosing $n_S(0)=0$ and $s(0)=N$ and choosing $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$. If the seller chooses the first response, no trade occurs in period 0. In period 1 the seller gets $EU^S=2\delta\Delta-c_1$. If the seller chooses the second response then trade occurs in state L and $U^S=(v_L-\Delta+2\delta\Delta-2c_1+2c_0)-(v_L-\Delta)-c_0=2\delta\Delta-2c_1+c_0>2\delta\Delta-c_1$. If $v=v_H$, no trade occurs in period 0. In period 1 trade occurs with probability $p_1>0.5$ and the seller gets $EU^S=2\delta\Delta-c_0$. So his expected payoff is $EU^S=2\delta\Delta-c_1$. (ii) If $b<\hat{b}$, then the best response of the seller is to choose $n_S(0)=0$ and $s(0)=N$ and $EU^S=2\delta\Delta-c_1$. (iii) If $b>\hat{b}$ then the seller chooses $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$ and $EU^S>2\delta\Delta-c_1$.

Case 2: $n_B(0)=1$. See Step 1.1 Case 2.

Step 2.2 (Buyer's decision in period 0)

(a) Suppose $n_B(0)=0$. If the buyer chooses alternative (i) then $EU^B=0.5(v_L+\Delta-(v_L-\Delta+2\delta\Delta-2c_1+2c_0))=\Delta-c_0-\delta\Delta+c_1$. If the buyer chooses alternative (ii) then $EU^B=0$. If the buyer chooses alternative (iii) then $EU^B<\Delta-c_0-\delta\Delta+c_1$. (b) Suppose $n_B(0)=1$. Trade occurs with probability 1 and $EU^B=2\Delta-2\delta\Delta-c_0$. Consequently, the buyer compares the alternatives (ai), (aii) and (b).

Case 1: If $\delta>\frac{\Delta-c_0+c_1}{\Delta}$, then (aii) dominates (ai). So alternatives (b) and (aii) remain. If $\delta>1-\frac{c_0}{2\Delta}$, the buyer chooses alternative (aii) and $EU^B=0$. If $\delta<1-\frac{c_0}{2\Delta}$, the buyer chooses alternative (b) and $EU^B=2\Delta-2\delta\Delta-c_0$. If $\delta=1-\frac{c_0}{2\Delta}$, the buyer is indifferent between the two alternatives.

Case 2: If $\delta<\frac{\Delta-c_0+c_1}{\Delta}$, then (ai) dominates (aii). So alternatives (b) and (ai) remain. If $\delta>1-\frac{c_0}{\Delta}$, then the buyer chooses (ai) and $EU^B=\Delta-c_0-\delta\Delta+c_1$. If $\delta<1-\frac{c_0}{\Delta}$, then the buyer chooses (b) and $EU^B=2\Delta-2\delta\Delta-c_0$. If $\delta=1-\frac{c_0}{\Delta}$, then the buyer is indifferent between the two alternatives.

Case 3: If $\delta=\frac{\Delta-c_0+c_1}{\Delta}$, then the buyer is indifferent between (ai) and (aii). If $\delta>1-\frac{c_0}{\Delta}$, then the buyer chooses (ai) or (aii) and $EU^B=\Delta-c_0-\delta\Delta+c_1$. If $\delta<1-\frac{c_0}{\Delta}$, then the buyer chooses (b) and $EU^B=2\Delta-\delta\Delta-c_0$. If $\delta=1-\frac{c_0}{\Delta}$, the buyer is indifferent between three alternatives.

Step 3 (Equilibrium paths)

If $\delta<\min\{\frac{c_1}{2\Delta},\frac{1}{3},1-\frac{c_0}{2\Delta}\}$, $\frac{c_1}{2\Delta}<\delta<\min\{1-\frac{c_0+c_1}{\Delta}, 1-\frac{c_0}{\Delta}\}$ or $\max\{\frac{c_1}{2\Delta},1-\frac{c_0+c_1}{\Delta}\}<\delta<1-\frac{c_0}{\Delta}$, then the PBE is given as follows. In period 0 the buyer chooses $n_B(0)=1$, $b_L(0)=v_L-\Delta+2\delta\Delta$ and $b_H(0)=v_H-\Delta+2\delta\Delta$. The seller chooses $n_S(0)=0$ and $s(0)=Y$. Trade occurs with probability 1 in period 0 and $EU^B=2\Delta-2\delta\Delta-c_0$ and $EU^S=2\delta\Delta$.

If $1-\frac{c_0}{2\Delta}<\delta<\min\{\frac{c_1}{2\Delta},\frac{1}{3}\}$ or $\frac{1}{3}<\delta<\min\{\frac{c_1}{2\Delta}, \frac{c_0}{\Delta}-1\}$, then the set of PBE is given as follows. In period 0 the buyer chooses $n_B(0)=0$ and $b(0)<v_L-\Delta-2\delta\Delta+2c_0$ and the seller chooses $n_S(0)=0$ and $s(0)=N$. In period 1 the seller chooses $n_S(0)=0$ and $s(1)>v_{1H}+\delta\Delta-2c_1$ and the buyer chooses $n_B(1)=0$ and $b(1)=N$. No PE with trade exists. (*Proposition 8*)

If $\max\{\frac{1}{3},\frac{c_0}{\Delta}-1\}<\delta<\frac{c_1}{2\Delta}$, then the PE is given as follows. In period 0 the buyer chooses $n_B(0)=0$ and $b(0)=v_L-\Delta+2c_0-\delta\Delta$ and the seller chooses $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$. If $v=v_L$ trade occurs. If $v=v_H$, there is disagreement. In period 1 the informed seller chooses $n_S(1)=0$, $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$. The buyer chooses $n_B(1)=0$ and $b(1)=Y$ Trade occurs with probability 1 in period 1 and $EU^B=\Delta+\delta\Delta-c_0$ and $EU^S=0$. (*Proposition 7*)

If $\max\{\frac{c_1}{2\Delta},1-\frac{c_0}{\Delta}\}<\delta<1-\frac{c_0+c_1}{\Delta}$, the PBE is given as follows. In period 0 the buyer chooses $n_B(0)=0$, and $b(0)=v_L-\Delta+2\delta\Delta-2c_1+2c_0$. The seller chooses $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$. If $v=v_L$ trade occurs. If $v=v_H$, there is disagreement. In period 1 the informed seller chooses $n_S(1)=0$, $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$. The buyer chooses $n_B(1)=0$ and $b(1)=Y$ Trade occurs with probability 1 in period 1 and $EU^B=\Delta-c_0-\delta\Delta+c_1$ and $EU^S=2\delta\Delta-c_1$.

If $\delta>\max\{\frac{c_1}{2\Delta},1-\frac{c_0+c_1}{\Delta},1-\frac{c_0}{\Delta}\}$, the PBE is given as follows. In period 0 the buyer chooses $n_B(0)=0$, and $b(0)<v_L-\Delta+2\delta\Delta-2c_1+2c_0$. The seller chooses $n_S(0)=0$ and $s(0)=N$. In period 1 In period 1 the seller chooses $n_S(1)=1$, $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$. The buyer chooses $n_B(1)=0$ and $b(1)=Y$ Trade occurs with probability 1 and $EU^B=0$ and $EU^S=2\delta\Delta-c_1$. (*Proposition 6*)

Appendix C

This Appendix analyses two period alternating offer bargaining without imposing the simplifying Assumption T1.

Lemma 1

Assumptions T1 and T2 hold. The default option D of the seller is given as follows.

(i) For $\delta \leq \frac{c_1}{2k_1\Delta}$, D=0 if $n_B(0)=0$ and $D=\delta\Delta$ if $n_B(0)=1$.

(ii) For $\frac{c_1}{2k_1\Delta} \leq \delta \leq \frac{c_1}{\Delta}$, $D=2\delta k_1\Delta-c_1$ if $n_B(0)=0$ and $D=\delta\Delta$ if $n_B(0)=1$.

(iii) For $\delta \geq \frac{c_1}{\Delta}$, $D=2\delta k_1\Delta-c_1$ if $n_B(0)=0$ and $D=2\delta\Delta-c_1$ if $n_B(0)=1$

where $k_1 \in (0.5,1)$.

<u>**Proof of Lemma 1**</u>

<u>Case 1</u> : $n_B(0)=0$.

If $\delta > \frac{c_1}{2k_1\Delta}$, then $c_1<\min\{2\Delta_1, \frac{1}{4}(v_H-v_L)-\Delta_1\}$. Analogous to Proposition 5, a mixed strategy equilibrium in period 1 has the following properties. The seller chooses $n_S(1)=1$, and randomizes his offer. Trade occurs with probability $k_1>0.5$. The continuation payoff of the seller is $EU^S=2\delta k_1\Delta-c_1$.

If $\delta < \frac{c_1}{2k_1\Delta}$, then $2\Delta_1<c_1<\frac{1}{4}(v_H-v_L)-\Delta_1$. Analogous to Proposition 1, the best responses in period 1 imply no trade. The continuation payoff is $U^S=0$.

If $\delta = \frac{c_1}{2k_1\Delta}$, then the seller's continuation payoff is also $U^S=0$.

<u>Case 2</u> : Suppose $n_B(0)=1$.

The seller may learn something about v from observing b(0). The seller compares the following alternatives.

(i) If $n_S(1)=0$, then the uninformed seller accounts for the lemons problem and chooses $s(1)=v_H+\delta\Delta$. Trade occurs with probability 0.5, assuming that the indifferent buyer opts for partial trade and chooses $b_L(1)=N$ and $b_H(1)=Y$. The seller's continuation payoff is $EU^S=\delta\Delta$. (Note, if $s(1)=v_{1L}+\delta\Delta$ trade may occur with probability 1 but the seller obtains $EU^S=2\delta\Delta-(v_H-v_L)/2<0$ since $\Delta<(v_H-v_L)/4$.)

(ii) If $n_S(1)=1$, then the seller chooses $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$. Assuming that the indifferent informed buyer opts for trade and chooses $b_L(1)=Y$ and $b_H(1)=Y$, the seller gets $U^S=2\delta\Delta-c_1$.

So if $2\delta\Delta - c_1 > \delta\Delta$, i.e. $\delta > \frac{c_1}{\Delta}$, then the seller acquires information and his continuation payoff is $U^S = 2\delta\Delta - c_1$. If $\delta < \frac{c_1}{\Delta}$, then the seller acquires no information and his continuation payoff is $EU^S = \delta\Delta$. If $\delta = \frac{c_1}{\Delta}$, then the seller is indifferent between the two alternatives.

Step 1 : $\delta \le \frac{c_1}{2k_1\Delta}$.

From Lemma 1, the default option D of the seller is D=0 if $n_B(0)=0$ and $D=\delta\Delta$ if $n_B(0)=1$.

Step 1.1 (Seller's best response correspondence in period 0)

Case 1: $n_B(0)=0$. Define $\hat{b} \equiv v_L - \Delta - 2\delta k_1\Delta + 2c_0$.

(i) : If $b=\hat{b}$ then the seller is indifferent between choosing $n_S(0)=0$ and $s(0)=N$ and choosing $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$.

Proof : If the seller chooses the first response, no trade occurs in period 0. In period 1 there is also no trade and $U^S=0$. If the seller chooses the second response, then trade occurs in state L and $U^S=(v_L-\Delta-2\delta k_1\Delta+2c_0-(\Delta-v_L))-c_0=-2\delta k_1\Delta+c_0 \ge 0$ (since $\delta \le \frac{c_1}{2k_1\Delta}$). If the true state is H, then no trade occurs in period 0. In period 1 the informed seller gets $EU^S=2\delta k_1\Delta - c_0 \le 0$. So his expected payoff $EU^S=0$.

(ii) If $b<\hat{b}$, then the best response of the seller is to choose $n_S(0)=0$ and $s(0)=N$ and $U^S=0$.

(iii) If $b>\hat{b}$ then the seller chooses $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$ and $EU^S>0$.

Case 2: $n_B(0)=1$. Define $\hat{b}_L \equiv v_L-\Delta+\delta\Delta+t$ and $\hat{b}_H \equiv v_H-\Delta+\delta\Delta$.

In the signaling game, the buyer randomizes over $\hat{b}_L$ and $\hat{b}_H$ such that the seller is indifferent between $s(0)=Y$ and $s(0)=N$. Trade occurs with probability $k_0>0.5$ in period 0.

Step 1.2 (Buyer's decision in period 0)

(a) Suppose $n_B(0)=0$. If the buyer chooses alternative (i) then $EU^B=0.5[v_L+\Delta-(v_L-\Delta-2\delta k_1\Delta+2c_0)]=\Delta-c_0+\delta k_1\Delta$. If the buyer chooses alternative (ii) then $EU^B=0$. If he chooses alternative (iii) then $EU^B<\Delta-c_0+\delta k_1\Delta$. (b) Suppose $n_B(0)=1$. Then the mixed strategy payoff is $EU^B= k_0(2\Delta-\delta\Delta)-c_0$. Consequently, the buyer compares the alternatives (ai), (aii) and (b).

Case 1: If $\delta < \frac{2k_0-1}{k_0+k_1}$ then alternative (b) dominates (ai). The buyer compares (b) with (aii). If $\delta < 2-\frac{c_0}{k_0\Delta}$, the buyer chooses (b) and $EU^B=k_0(2\Delta-\delta\Delta)-c_0$. If $\delta > 2-\frac{c_0}{k_0\Delta}$, the buyer chooses (aii) and $EU^B=0$. If $\delta = 2-\frac{c_0}{k_0\Delta}$, the buyer is indifferent between the two alternatives.

Case 2: If $\delta > \frac{2k_0-1}{k_0+k_1}$ then alternative (ai) dominates (b). The buyer compares (ai) with (aii). If $\delta > \frac{c_0-\Delta}{k_1\Delta}$, the buyer chooses alternative (ai) and $EU^B=\Delta-c_0+\delta k_1\Delta$. If $\delta < \frac{c_0-\Delta}{k_1\Delta}$, the buyer chooses alternative (aii) and $EU^B=0$. If $\delta = \frac{c_0-\Delta}{k_1\Delta}$, the buyer is indifferent between the two alternatives.

Case 3: If $\delta = \frac{2k_0-1}{k_0+k_1}$ then the buyer compares (ai, b) with (aii). If $\delta > \frac{c_0-\Delta}{k_1\Delta}$, the buyer is indifferent between (ai) and (b) and $EU^B=\Delta-c_0+\delta\ k_1\Delta$. If $\delta < \frac{c_0-\Delta}{k_1\Delta}$, the buyer chooses alternative (aii) and $EU^B=0$. If $\delta = \frac{c_0-\Delta}{k_1\Delta}$, the buyer is indifferent between the three alternatives.

Step 2: $\frac{c_1}{2k_1\Delta} < \delta \le \frac{c_1}{\Delta}$.

The seller's default option is $D=2k_1\delta\Delta-c_1$ if $n_B(0)=0$ and $D=\delta\Delta$ if $n_B(0)=1$.

Step 2.1 (Seller's best response correspondence in period 0)

Case 1: $n_B(0)=0$. Define $\hat{b} \equiv v_L-\Delta+2\delta k_1\Delta-2c_1+2c_0$.

(i) If $b=\hat{b}$ then the seller is indifferent between choosing $n_S(0)=0$ and $s(0)=N$ and choosing $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$. If the seller chooses the first response no trade occurs in period 0. In period 1 the seller gets $EU^S=2\delta k_1\Delta-c_1$. If the seller chooses the second response then trade occurs in state L and $U^S=(v_L-\Delta+2\delta k_1\Delta-2c_1+2c_0)-(v_L-\Delta)-c_0=2\delta k_1\Delta-2c_1+c_0>2\delta k_1\Delta-c_1$. If $v=v_H$, no trade occurs in period 0. In period 1 trade occurs with probability $p_1>0.5$ and the seller gets $EU^S=2\delta k_1\Delta-c_0$. So his expected payoff is $EU^S=2\delta k_1\Delta-c_1$. (ii) If $b<\hat{b}$, then the best response of the seller is to choose $n_S(0)=0$ and $s(0)=N$ and $EU^S=2\delta k_1\Delta-c_1$. (iii) If $b>\hat{b}$ then the seller chooses $n_S(0)=1$ and $s_L(0)=Y$ and $s_H(0)=N$ and $EU^S>2\delta k_1\Delta-c_1$.

Case 2: $n_B(0)=1$. See Step 1.1 Case 2.

Step 2.2 (Buyer's decision in period 0)

(a) Suppose $n_B(0)=0$. If the buyer chooses alternative (i) then $EU^B=0.5(v_L+\Delta-(v_L-\Delta+2\delta k_1\Delta-2c_1+2c_0))=\Delta-c_0-\delta k_1\Delta+c_1$. If the buyer chooses alternative (ii) then $EU^B=0$. If the buyer chooses alternative (iii) then $EU^B<\Delta-c_0-\delta k_1\Delta+c_1$. (b) Suppose $n_B(0)=1$. Then the mixed strategy payoff is $EU^B=k_0(2\Delta-\delta\Delta)-c_0$. Consequently, the buyer compares the alternatives (ai), (aii) and (b).

Case 1: If $\delta>\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then (aii) dominates (ai). So alternatives (b) and (aii) remain. If $\delta>2-\frac{c_0}{k_0\Delta}$, the buyer chooses alternative (aii) and $EU^B=0$. If $\delta<2-\frac{c_0}{k_0\Delta}$, the buyer chooses alternative (b) and $EU^B=k_0(2\Delta-\delta\Delta)-c_0$. If $\delta=2-\frac{c_0}{k_0\Delta}$, the buyer is indifferent between the two alternatives.

Case 2: If $\delta<\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then (ai) dominates (aii). So alternatives (b) and (ai) remain. If $\delta>\frac{(2k_0-1)\Delta-c_1}{(k_0-k_1)\cdot\Delta}$ then the buyer chooses (ai) and $EU^B=\Delta-c_0-\delta k_1\Delta+c_1$. If $\delta<\frac{(2k_0-1)\Delta-c_1}{(k_0-k_1)\cdot\Delta}$ then the buyer chooses (b) and $EU^B=k_0(2\Delta-\delta\Delta)-c_0$. If $\delta=\frac{(2k_0-1)\Delta-c_1}{(k_0-k_1)\cdot\Delta}$ then the buyer is indifferent between the two alternatives.

Case 3: If $\delta=\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then the buyer is indifferent between (ai) and (aii). If $\delta>\frac{(2k_0-1)\Delta-c_1}{(k_0-k_1)\cdot\Delta}$ then the buyer chooses (ai) or (aii) and $EU^B=\Delta-c_0-\delta k_1\Delta+c_1$. If $\delta<\frac{(2k_0-1)\Delta-c_1}{(k_0-k_1)\cdot\Delta}$ then the buyer chooses (b) and $EU^B=k_0(2\Delta-\delta\Delta)-c_0$. If $\delta=\frac{(2k_0-1)\Delta-c_1}{(k_0-k_1)\cdot\Delta}$ the buyer is indifferent between three alternatives.

Step 3: $\delta>\frac{c_1}{\Delta}$.

The seller's default option is $D=2\delta k_1\Delta-c_1$ if $n_B(0)=0$ and $D=2\delta\Delta-c_1$ $n_B(0)=1$.

Step 3.1 (Seller's best response correspondence in period 0)

Case 1: $n_B(0)=0$. See Step 2.1, Case 1.

Case 2: $n_B(0)=1$. Define $\hat{b}_L\equiv v_L-\Delta+2\delta\Delta-c_1+t$ and $\hat{b}_H\equiv v_H-\Delta+2\delta\Delta-c_1$.

In the signaling game, the buyer randomizes over $\hat{b}_L$ and $\hat{b}_H$ such that the seller chooses $n_S(0)=0$ and is indifferent between $s(0)=Y$ and $s(0)=N$. Trade occurs with probability $k_0>0.5$. If there is disagreement, the seller chooses $n_S(1)=1$, $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$..

Step 3.2 (Buyer's decision in period 0)

(a) Suppose $n_B(0)=0$. If the buyer chooses alternative (i) then $EU^B=\Delta-c_0-\delta k_1\Delta+c_1$. If the buyer chooses the alternative (ii) then $EU^B=0$. If the buyer chooses alternative (iii) then $EU^B<\Delta-c_0-\delta k_1\Delta+c_1$. (b) Suppose $n_B(0)=1$. Then the mixed strategy payoff is $EU^B=k_0(2\Delta-2\delta\Delta+c_1)-c_0$. Consequently, the buyer compares the alternatives (ai), (aii) and (b).

Case 1: If $\delta>\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then (aii) dominates (ai). So alternatives (b) and (aii) remain. If $\delta>1-\frac{c_0-k_0c_1}{2k_0\Delta}$, the buyer chooses alternative (aii) and $EU^B=0$. If $\delta<1-\frac{c_0-k_0c_1}{2k_0\Delta}$, the buyer chooses alternative (b) and $EU^B=k_0(2\Delta-2\delta\Delta+c_1)-c_0$. If $\delta=1-\frac{c_0-k_0c_1}{2k_0\Delta}$, the buyer is indifferent between the two alternatives.

Case 2: If $\delta<\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then (ai) dominates (aii). So alternatives (b) and (ai) remain. If $\delta>\frac{(2k_0-1)\Delta-c_1(1-k_0)}{(2k_0-k_1)\cdot\Delta}$, the buyer chooses alternative (ai) and $EU^B=\Delta-c_0-\delta k_1\Delta+c_1$. If $\delta<\frac{(2k_0-1)\Delta-c_1(1-k_0)}{(2k_0-k_1)\cdot\Delta}$, the buyer chooses alternative (b) and $EU^B=k_0(2\Delta-2\delta\Delta+c_1)-c_0$. If $\delta=\frac{(2k_0-1)\Delta-c_1(1-k_0)}{(2k_0-k_1)\cdot\Delta}$, the buyer is indifferent between the two alternatives.

Case 3: If $\delta=\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then the buyer is indifferent between (ai) and (aii). If $\delta>\frac{\Delta-c_0+k_1c_1}{k_1\cdot\Delta}$, the buyer chooses alternative (ai) or (aii) and $EU^B=\Delta-c_0-\delta k_1\Delta+c_1$. If $\delta<\frac{\Delta-c_0+k_1c_1}{k_1\cdot\Delta}$, the buyer chooses alternative (b) and $EU^B=2k_0\Delta-c_0-2\delta\Delta+c_1$. If $\delta=\frac{\Delta-c_0+k_1c_1}{k_1\cdot\Delta}$, the buyer is indifferent between the three alternatives.

Step 4 (Equilibrium paths)

If $\delta<\min\{\frac{c_1}{2k_1\Delta},\frac{2k_0-1}{k_0+k_1},2-\frac{c_0}{k_0\Delta}\}$, $\max\{\frac{c_1}{2k_1\Delta},\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}\}<\delta<\min\{\frac{c_1}{\Delta},2-\frac{c_0}{k_0\Delta}\}$ or $\frac{c_1}{2k_1\Delta}<\delta<\min\{\frac{c_1}{\Delta},\frac{\Delta-c_0+c_1}{k_1\cdot\Delta},\frac{(2k_0-1)\Delta-c_1}{(1-k_1)\cdot\Delta}\}$, then a of PE has the following properties. In period 0 the buyer chooses $n_B(0)=1$. At v_{0L} the buyer chooses $b_L(0)=v_L-\Delta+\delta\Delta+t$. At v_{0H} the buyer randomizes over $b_H(0)=v_H-\Delta+\delta\Delta$ and $b_L(0)$. The seller chooses $n_S(0)=0$. Seeing $b_H(0)$, he chooses $s(0)=Y$. Seeing $b_L(0)$, he randomizes over Y and N. Trade occurs with probability $k_0>0.5$ in period 0. If there is disagreement, the seller chooses $n_S(1)=0$ and $s(1)=v_H+\delta\Delta$. The informed buyer chooses $n_B(1)=0$, $b_N(1)=N$ and $b_H(1)=Y$. Trade occurs with probability 0.5 in period 1. $EU^B=k_0(2\Delta-\delta\Delta)-c_0$ and $EU^S=\delta\Delta$.

If $2-\frac{c_0}{k_0\Delta}<\delta<\min\{\frac{c_1}{2k_1\Delta},\frac{2k_0-1}{k_0+k_1}\}$ or $\frac{2k_0-1}{k_0+k_1}<\delta<\min\{\frac{c_0-\Delta}{k_1\Delta},\frac{c_1}{2k_1\Delta}\}$, then the set of PE is given as follows. In period 0 the buyer chooses $n_B(0)=0$ and $b(0)<v_L-\Delta-2\delta\Delta+2c_0$ and the seller chooses $n_S(0)=0$ and $s(0)=N$. In period 1 the seller chooses $n_S(0)=0$ and $s(1)>v_H+\delta\Delta-2c_1$ and the buyer chooses $n_B(1)=0$ and $b(1)=N$. No PE with trade exists.

If $\max\{\frac{c_0-\Delta}{k_1\Delta},\frac{2k_0-1}{k_0+k_1}\}<\delta<\frac{c_1}{2k_1\Delta}$, then the set of PE is given as follows. In period 0 the buyer chooses $n_B(0)=0$ and $b(0)=v_L-\Delta+2c_0-\delta k_1\Delta$ and the seller chooses $n_S(0)=1$ and $s_L(0)=Y$

and $s_H(0)$=N. If $v=v_L$ trade occurs. If $v=v_H$, there is disagreement. In period 1 the informed seller chooses $n_S(1)=0$. At v_{1H} he chooses $s_H(1)=v_H+\delta\Delta-t$. At v_L he randomizes over and $s_H(1)$ and $s_L(1)=v_L+\delta\Delta$. The buyer chooses $n_B(1)=0$. Seeing $s_L(1)$, the buyer chooses b(1)=Y. Seeing $s_H(1)$, the buyer randomizes over Y and N. Trade occurs with probability $k_1>0.5$ in period 1 and $EU^B=\Delta+\delta k_1\Delta-c_0$ and $EU^S=0$.

If $\max\{\frac{c_1}{2k_1\Delta},\frac{(2k_0-1)\Delta-c_1}{(1-k_1)\cdot\Delta}\}<\delta<\min\{\frac{c_1}{\Delta},\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}\}$ or $\max\{\frac{c_1}{\Delta},\frac{(2k_0-1)\Delta-c_1(1-k_0)}{(2k_0-k_1)\cdot\Delta}\}<\delta<\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}$, then a PE has the following properties. In period 0 the buyer chooses $n_B(0)=0$ and $b(0)=v_L-\Delta+2\delta k_1\Delta-2c_1+2c_0$. The seller chooses $n_S(0)=1$ and $s_L(0)$=Y and $s_H(0)$=N. If $v=v_H$, there is disagreement. In period 1 the informed seller chooses $n_S(1)=0$. At v_H the informed seller chooses $s_H(1)=v_L+\delta\Delta-t$. At v_L the seller randomizes over $s_L(1)=v_L+\delta\Delta$ and $s_H(1)$. The buyer chooses $n_B(1)=0$. Seeing $s_L(1)$ the buyer chooses b(1)=Y. Seeing $s_H(1)$ the buyer randomizes over Y and N. Trade occurs with probability 0.5 in period 0. If there is disagreement trade occurs with probability $k_1>0.5$ in period 1. $EU^B=\Delta-c_0-\delta k_1\Delta+c_1$ and $EU^S=2\delta k_1\Delta-c_1$.

If $\frac{c_1}{\Delta}<\delta<\min\{\frac{\Delta-c_0+c_1}{k_1\cdot\Delta},\frac{(2k_0-1)\Delta-c_1(1-k_0)}{(2k_0-k_1)\cdot\Delta}\}$ or $\max\{\frac{c_1}{\Delta},\frac{\Delta-c_0+c_1}{k_1\cdot\Delta}\}<\delta<1-\frac{c_0-k_0c_1}{2k_0\Delta}$, then a PE has the following properties. In period 0 the buyer chooses $n_B(0)=1$. At v_L the buyer chooses $b_L(0)=v_L-\Delta+2\delta\Delta-c_1+t$. At v_H the buyer randomizes over $b_H(0)=v_H-\Delta+2\delta k_1\Delta-c_1$ and $b_L(0)$. The seller chooses $n_S(0)=0$. Seeing $b_H(0)$, the seller chooses s(0)=Y. Seeing $b_L(0)$ the seller randomizes over Y and N. Trade occurs with probability $p_0>0.5$ in period 0. If there is disagreement, the seller chooses $n_S(1)=1$, $s_L(1)=v_L+\delta\Delta$ and $s_H(1)=v_H+\delta\Delta$. The informed buyer chooses $b_L(1)$=Y and $b_H(1)$=Y. $EU^B=k_0(2\Delta-2\delta\Delta+c_1)-c_0$ and $EU^S=2\delta\Delta-c_1$.

If $\delta>\max\{\frac{\Delta-c_0+c_1}{k_1\cdot\Delta},\frac{c_1}{\Delta},1-\frac{c_0-k_0c_1}{2k_0\Delta}\}$ or $\max\{\frac{c_1}{2k_1\Delta},\frac{\Delta-c_0+c_1}{k_1\cdot\Delta},2-\frac{c_0}{k_0\Delta}\}<\delta<\frac{c_1}{\Delta}$, then the set of PE is given as follows. In period 0 the buyer chooses $n_B(0)=0$ and $b(0)<v_L-\Delta+2\delta\Delta-c_1+2c_0$ and the seller chooses $n_S(0)=0$ and s(0)=N. In period 1, the seller chooses $n_S(1)=1$. At v_H the seller chooses $s_H(1)=v_H+\delta\Delta-t$. At v_{1L} the seller randomizes over $s_L(1)=v_L+\delta\Delta$ and $s_H(1)$. The buyer chooses $n_B(1)=0$. Seeing $s_L(1)$ the buyer chooses b(1)=Y. Seeing $s_H(1)$ the buyer randomizes over Y and N. No trade occurs in period 0. Trade occurs with probability $k_1>0.5$ in period 1. $EU^B=0$ and $EU^S=2\delta k_1\Delta-c_1$.

References

Admati, A.R. and Perry, M. (1987): "Strategic Delay in Bargaining", *Review of Economic Studies 54*, 345-364

Akerlof, G. (1970) : "The Market for Lemons : Qualitative Uncertainty and the Market Mechanism", *Quarterly Journal of Economics 84*, 488-500

Ausubel, L.M., Cramton, P. and Deneckere R.J. (2002): "Bargaining with Incomplete Information", in Aumann, R. and Hart, S.: *Handbook of Game Theory*, Vol.3, Ch. 50

Bergemann, D. and Välimäki, J. (2002): Information Acquisition and Efficient Mechanism Design, *Econometrica 70*, 1007-1033

Cho, I.K. (1990): "Uncertainty and Delay in Bargaining", *Review of Economic Studies 57*, 575-595

Cramton, P.C. (1992): "Strategic Delay in Bargaining with Two-Sided Uncertainty", *Review of Economic Studies 59*, 205-225

Cremer, J. and Khalil, F. (1992): "Gathering Information Before Signing a Contract", *American Economic Review 82*, 566-578

Cremer, J., Khalil, F. and Rochet, J.C. (1998a): "Contracts and Productive Information Gathering", *Games and Economic Behavior 25*, 174-193

Cremer, J., Khalil, F. and Rochet, J.C. (1998b): "Strategic Information Gathering before a Contract is Offered", *Journal of Economic Theory 81*, 163-200

Dang, T.V. (2005): "Essays on Information Acquisition", Dissertation Thesis, University of Mannheim

Deneckere, P. and Liang, M.Y. (2006): " Bargaining with Interdependent Values", *Econometrica 74*, 1309-1364

Evans, R. (1989): "Sequential Bargaining with Correlated Values", *Review of Economic Studies 56*, 499-510

Feinberg, Y. and Skrzypacz, A. (2005): "Uncertainty about Uncertainty and Delay in Bargaining", *Econometrica 73*, 69-91

Gresik, T.A. (1991): "Ex Ante Incentive Efficient Trading Mechanism without the Private Valuation Restriction", *Journal of Economic Theory 55*, 41-63

Hausch, D.B. and Li, L. (1993): "A Common Value Model with Endogenous Entry and Information Acquisition", *Economic Theory 3*, 315-334

Ingersoll, J.E. and Ross, S.A. (1992): "Waiting to Invest: Investment under Uncertainty", *Journal of Business 65*, 1-29

Matthews, S. (1984): "Information Acquisition in Discriminatory Auctions", in: Boyer, M. and Kihlstrom, R.E. (Eds), *Bayesian Models in Economic Theory*, 181-207

Myerson, R. and Satterthwaite, M. (1983): "Efficient Mechanism for Bilateral Trading", *Journal of Economic Theory 29*, 256-281

Samuelson, W. (1984): "Bargaining under Asymmetric Information", *Econometrica 52*, 995-1005

Shavell, S. (1994): "Acquisition and Disclosure of Information Prior to Sale", *Rand Journal of Economics 25*, 20.36

Smith, B.F., Turnbull, A.S. and White, R.W. (2001): "Upstairs Markets for Principal and Agency Trades: Analysis of Adverse Information and Price Effects", *Journal of Finance 56*, 1723-1746

Vincent, D.R. (1989): "Bargaining with Common Values", *Journal of Economic Theory 48*, 47-62

Watson, J. (1998): "Alternating-Offer Bargaining with Two-Sided Incomplete Information", *Review of Economic Studies 65*, 573-594