Forward Contracts and Collusion in the Electricity Markets

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October 2006

Abstract

European competition authorities intend to mitigate market power in electricity markets by compelling merging firms to sell virtual capacity to prospective competitors. In the economic literature, Allaz and Vila (1993) argue that trading forward contracts enhances competition, which is prevalently accepted by many articles on electricity market competition. However, this result does not hold with the framework of an infinitely repeated game. We find that introducing forward trading allows firms to sustain collusive profits forever. The obligation to sell more contracts imposed by the regulator makes it more difficult for the incumbent firm to collude tacitly with its competitor. But this conclusion only holds when the competitor of the incumbent is a non-fringe firm and when the profit sharing rule on the collusive path is specific. Otherwise, trading forward contract either has no effect or possibly facilitates tacit collusion if the competitor is a fringe firm. The analysis suggests that competition authorities should not only worry about the frequency of trading forward contracts, but also be careful with the regulation of contract quantity.

Keywords: Contract market, Electricity, Spot Market, Forward, Tacit collusion.

JEL Classification: C72 D43 G13 L13 L94

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*Support from the European Association for Research in Industrial Economics (E.A.R.I.E.), through the award of Paul Geroski Prize (Young Economists' Essay Award) is gratefully acknowledged. I am grateful to Claude Crampes for many helpful comments and encouragement. I would also thank the sector *competition and regulation* at CPB (Netherlands Bureau for Economic Policy Analysis) for its hospitality and, in particular, Machiel Mulder and Gijsbert Zwart for their help in the early stage of this research. I thank all the seminar and conference participants at the University of Paris IX-Dauphine, IAEE2006 (Potsdam) and EARIE2006 (Amsterdam) for their valuable comments. Any errors are of course my own.

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1 Introduction

There is a widespread presumption among economists that forward trading is socially beneficial. Allaz and Vila (1993) argue that forward trading raises welfare even in the absence of any risk. They investigate the case of Cournot duopolists and characterize an equilibrium outcome with larger outputs, hence a lower spot price, compared to the case without forward trading. The point is that selling forward allows each duopolist to commit to a Stackelberg level of output in the spot market but, since both do so in equilibrium, no one succeeds in acquiring a leader's advantage. As a result, competition is tougher in the spot market and welfare is improved, compared to a situation without forward trading. Allaz and Vila also extend this result to the case where there is more than one time period for forward trading. Without uncertainty, they establish that as the number of periods of forward trading tends to infinity, producers lose their ability to raise market prices above marginal cost and the outcome tends to the competitive solution. But it is still a hot topic to verify the robustness of Allaz and Villa's result.

In the real world, there are roughly two kinds of forward contracts: physical contract and financial contract. In several merger cases in European electricity markets, competition authorities have ordered dominant firms to sell special forward contracts, named virtual power plants (VPP), in order to reduce the market power of large electricity companies.

The earliest VPP capacity auctions were launched by EDF in France in 2001. As a national dominant power provider, EDF set out to acquire a substantial interest in the Germany-based vertical utility and electricity trader EnBW in 2000. Being judged to enjoy a dominant position by the European Commission, EDF had to commit in an agreement with the European Commission to begin the virtual divestment of 6,000 megawatts of generation capacity in 2001, which represented approximately 10% of France's electricity supply. VPPs have also been used in Belgium for the acquisition by Electrabel of the supply activities of IVEKA in 2003, one of the former associations of local-government-owned utilities active in local distribution grid operation and supply of both electricity and gas. Now it is also discussed in the Netherlands, Denmark and Czech Republic.

Those VPP contracts are actually financial contracts, specifically call options. In this paper, we will focus on physical forward contracts instead of financial ones. We would like to draw out the following practical questions: does the introduction of the opportunity of forward trading facilitate collusion and to what extent can forward trading mitigate market power? One intuition is that contract length does influence the effectiveness of the market power mitigation that results from the contract position. Clearly, an obligation to sell ten years contracts seems to have a more robust effect on the company's incentives to raise prices than the obligation to sell daily contracts each day during ten years. Observe that the benefits of exercising market power may be reaped both in the spot market, where the actual strategic behavior may take place, and in the regulated contract market. In this contract market (which would typically take the form of an auction market) potential buyers would base their willingness to pay on the expected behavior of the incumbent, which might be cooperation on the price or quantity, or no cooperation at all along the commercial trading periods.

Allaz and Villa (1993) is the first theoretical analysis of the competition enhancing effect of forward trading. Based on this idea, Green (1999) analyzes the case of two dominant producers of electricity who can raise spot prices well above marginal costs, which is profitable for them in the absence of contracts. If fully hedged, however, the generators lose their incentive to raise prices above marginal costs. In addition, competition in the contract market can lead the generators to sell contracts for the greater part of their output. Empirically, Green finds that since privatization British generators have indeed sold most of their energy in the contract market.

Brandts, Pezanis-Christou and Schram (2003) use experiments to study the efficiency effects of adding the possibility of forward contracting to a pre-existing spot market. They deal separately with the cases where spot market competition is in quantities and where it is in supply functions. In both cases they compare the effect of adding a contract market with the introduction of an additional competitor, changing the market structure from a triopoly to a quadropoly. They find that, as theory suggests, for both types of competition the introduction of a forward market significantly lowers prices. The combination of supply function competition with a forward market leads to high efficiency levels. Meanwhile, Le Coq and Orzen (2004) also examine the A&V's prediction in a controlled laboratory environment. They investigate how and to what extent the market institution and the number of firms affect competition, in theory and in their experimental markets. Their findings support the main comparative-static predictions of the model but also suggest that the competition-enhancing effect of a forward market is weaker than predicted. In contrast, entry has a stronger competition-enhancing effect.

Nevertheless, there is some controversy about A&V's result. Hughes and Kao (1997) find that if the contract position is not perfectly observed by the other players, i.e., one's contract choice has no influence on the others' strategy, then the firm has no more incentive to sell forward contracts. Harvey and Hogan (2000) and Kamat and Oren (2004) doubt that the competition-enhancing effect holds if firms play the game repeatedly, as is undeniably the case in most real markets. They argue that a dynamic setting may enable firms to commit to keeping their forward positions to a minimum. Moreover, Liski and Montero (2005) consider an infinitely-repeated oligopoly in which at each period firms not only serve the spot market by either competing in prices or quantities but also have the opportunity to trade forward contracts. Contrary to the pro-competitive results of finite-horizon models, they find that the possibility of forward trading allows firms to sustain collusive profits that otherwise would not be possible. The result holds both for price and quantity competition and follows because (collusive) contracting of future sales is more effective in deterring deviations from the collusive plan than in inducing the previously identified pro-competitive effects. Ferreira (2003) studies an oligopolistic industry where firms are able to sell in a futures market at infinitely many moments prior to the spot market. A kind of Folk theorem is established: any outcome between perfect competition and Cournot can be sustained in equilibrium. However, the competitive outcome is not renegotiation-proof and only the monopolistic outcome is renegotiation-proof if firms can buy and sell in the futures market. These results suggest, contrary to existing literature, that the introduction of futures markets may have an anti-competitive effect. Mahenc and Salanie (2004) show that, buying forward (rather than selling) commits a producer to set a higher spot price in order to increase the value of his position. Due to Bertrand competition in the spot market, the other producer reacts by raising his price, which increases the profit of the first producer. In equilibrium, both producers buy forward and spot prices are raised above the levels reached in the absence of forward trading. Therefore, duopolists soften competition through forward trading.

Clearly, whether forward contracts can enhance competition depends strongly on the setup of the game. Changing the assumptions of the game towards a slightly different direction might dramatically change the conclusion. As we have noticed above, the Allaz and Villa's market power mitigating effect of forward contracts is based on several critical assumptions: perfect observation of forward contracts, selling instead of buying contracts, unrepeated game setting, rational expectation, Cournot competition and prisoner's dilemma. Actually Allaz even indicated in his dissertation that changing the response assumption from fixed quantity to fixed market share would reverse the conclusion. This result could also be found in Green (2002) with supply function equilibrium. Meanwhile, it is well-known that the prisoner's dilemma may not hold if the game is played repeatedly. The repetition of interactions will help, somehow, players to obtain collusive outcomes. Therefore, it is still ambiguous how forward contracts influence the players to exercise market power.

The main result of the paper is the following: given that introducing forward trading allows firms to sustain collusive profits for an infinite number of periods, we show that selling more contracts, as imposed by the regulator, makes it more difficult for the incumbent firm to collude tacitly with its competitor. If we use the threshold of the discount factor in collusion-sustainable conditions to measure the difficulty of collusion, augmenting contract quantity will increase this threshold, which means, harming tacit collusion between firms. But this conclusion only holds when the competitor of the incumbent is a non-fringe firm and when the profit sharing rule on the collusive path is specific. Otherwise, trading forward contract either has no effect or possibly facilitates tacit collusion if the competitor is a fringe firm. The analysis suggests that competition authorities should worry about the frequency of trading forward contract and the regulation of contract quantity.

Here we analyze two kinds of competition patterns: Cournot and Bertrand. There are two separating conditions in the model: whether the entrant firm is a fringe firm and whether the entrant firm has its own production facility besides the forward contract it buys. On the one hand, forward trading makes it indeed more difficult for firms to sustain collusion because it reduces the remaining non-contracted sales along the collusive plan. This is the pro-competitive effect of repeated forward trading like in the static setting. On the other hand, it becomes less attractive for firms to deviate from the collusive plan, since forward contracts reduce the market share that a deviating firm can capture in the deviation period but the punishment is not milder than in the repeated single spot market. However, things are more complicated when we consider the cases where the entrant firm is a fringe firm and it has its own production facility.

There are three papers which are most related to our work. Liski and Montero (2005) first studied forward trading's effect on tacit collusion. In their model, duopolist firms are selling forward contract simultaneously on the market and there is no strategic and timing advantages for either firms. However, we let the incumbent firm sell forward contracts to entrant firms and both compete in the spot market afterwards. The strategies of sustaining tacit collusion are different from Liski and Montero (2005). Schultz (2005) and Zhang and Zwart (2006) both study the reputation effect of virtual power plant contracts and recommend shorter trading frequency, i.e. longer contract durations, for European competition authorities. However, Schultz (2005) analyzes virtual power plant contracts based on the physical contract form. The reputation effect built through trading forward contracts along infinite periods makes it easier for both firms to capture monopoly profits under certain circumstances. In our work, we focus on the physical forward contract, and find that forward trading makes it more difficult for firms to sustain tacit collusion under both modes of market competition other than the reputation effect. But this result indeed depends not only on the sharing rule of monopoly profit along the collusive path, but also on whether the entrant owns its production facilities and whether it is capacity constrained. Under certain conditions it might have no influence on tacit collusion. Zhang and Zwart (2006) use a signaling game approach to analyze the reputation effect. The incumbent firm's choice on forward contract signals its private information on cost, which might influence the entrant firm's belief when it bids in the forward market. Their research focuses on finite time periods and the strategy choice is different from our results.

The remainder of this paper is organized as follows: In section 2, we set up the model. Section 3 and 4 presents our results for repeated forward markets, in which we separate the cases where the entrant is a fringe firm or a non-fringe firm, with or without a production facility, being capacity constrained or without any production limit. We also distinguish the cases of Cournot competition and Bertrand competition. Section 5 summarizes and concludes.

2 The setup

2.1 The static model

We start with a static model where the firms can only play the game once. Firm 1, the incumbent firm, is forced by the regulator to sell forward contract x. In the first stage of the game, firm 2 bids the unit price p_f for the regulated quantity x. We first assume that the forward market is conducted through a competitive auction and there are sufficiently many potential bidders so that firm 2 is only a fringe firm and a short term player. In the next section, we will relax this assumption and analyze the situation where firm 2 is no longer a fringe firm and becomes a long-term player.

The timing is as follows: at stage -1 the contract quantity x, which is going to be traded in the forward market, is fixed by the regulator. At stage 0 firm 2 chooses to bid on price. At stage 1 firm 1 and firm 2 compete in the spot market, either *à la Cournot* or *à la Bertrand*. In this static model, we study the simplest case where firm 2 has no production facility so that it cannot sell more than x in the spot market. In addition, this forward quantity is relatively small compared to the total demand and firm 2 is constrained by x.

By backward induction we first consider the situation in stage 1. After the auction of forward contract x, firm 1 and firm 2 compete in the spot market. The quantity sold by firm 1 in the spot market is denoted as q_1. The inverse demand function of the spot market is given by $p(Q) = a - Q$, where Q is the total production. Given that firm 2 has no production facility and sell all its capacity x [1], the optimization problem of firm 1 in quantity competition is

$$\max_{q_1}(a - q_1 - x)q_1 - cq_1 + (p_f - c)x \tag{1}$$

The price of forward contracts, p_f, is determined through the auction in the forward market and has no influence in the spot market. Firm 1 maximizes the profit from the residual demand with respect to its own quantity q_1 in the spot market. The first order condition gives the best choice of firm 1 conditional on x :

$$q_1^{mf} = \frac{a-c}{2} - \frac{x}{2}$$

Here superscript mf stands for monopoly with a fringe firm. Therefore, the total production in the spot market is

$$q_1^{mf} + x = \frac{a-c}{2} + \frac{x}{2} \tag{2}$$

The price in the spot market is

$$p^{mf} = \frac{a+c}{2} - \frac{x}{2} \tag{3}$$

Given the fact that firm 2 has no own production facility and is constrained by this small contract capacity x, firm 1 acts as a monopolist. The result would be the same if firm 1 could determine its optimal spot price facing the residual demand. Indeed,

$$\max_{p_s}(a - p_s - x)(p_s - c) + (p_f - c)x \tag{4}$$

gives

$$p_s^* = \frac{a+c}{2} - \frac{x}{2} = p^{mf}$$

Now we consider the forward market. The prospective bidders are rational[2] and

[1] We will check that it is its best choice

[2] This rational expectation is prevailing among the bidders. Suppose that we have at least two bidders

forecast the competitive equilibrium spot price of the next period, consequently, firm 2 should get no profit by buying and selling forward contracts. Comparison of profits is shown in the following matrix [3], where p^M represents the monopoly price $p^M = \frac{a+c}{2}$. At equilibrium, firm 2 bids the forward price p_f equal to the expected spot price p^{mf}.

		Firm 1	
		p^{mf}	p^M
Firm 2	p^{mf}	0 ; $\frac{(a-c)^2}{4} - \frac{x^2}{4}$	$\frac{x^2}{2}$; $\frac{(a-c)^2}{4} - \frac{x^2}{2}$
	p^M	$-\frac{x^2}{2}$; $\frac{(a-c)^2}{4} + \frac{x^2}{4}$	0 ; $\frac{(a-c)^2}{4}$

Diagram 3-1: Profit Matrix

Lemma 1 *When the fringe bidder (firm 2) has no production facility, there exists a unique regulated-monopoly perfect equilibrium, where firm 2 bids p^{mf} and the incumbent firm (firm 1) plays the dominant strategy p^{mf}.*

Proof. Suppose firm 2 has chosen p^{mf}. Then the best strategy for firm 1 is to choose p^{mf} because it always earns more by choosing p^{mf} (in this case firm 1's profit is $\frac{(a-c)^2}{4} - \frac{x^2}{4}$) than by choosing p^M (in this case firm 1's profit is $\frac{(a-c)^2}{4} - \frac{x^2}{2}$). Then let us suppose firm 2 has choosen p^M. Firm 1's best strategy is to choose p^{mf} because it always earns more by choosing p^{mf} (in this case firm 1's profit is $\frac{(a-c)^2}{4} + \frac{x^2}{4}$) than by choosing p^M (in this case firm 1's profit is $\frac{(a-c)^2}{4}$). Therefore in this two-stage static model, no matter what firm 2 chooses, firm 1 prefers p^{mf}.

in the contract market and the one who wins the contract is rational. It will also expect the other one, or the rest of the contract bidders, is as rational as itself. None of them would like to bid a price higher than the spot market price and to lose money (get negative profit) in order to favor the incumbent firm.

[3] We leave the calculation to the readers for the other profits in the matrix: firm 2's profit can be obtained by the following formula $\pi_2 = (p_f - p_s)x$, where p_f is the choice of firm 2 and p_s is the choice of firm 1. π_2 is presented in the south-western corner of each cell. Firm 1's profit comes from (4) and π_2 is presented in the north-eastern corner of each cell.

Now let us look at firm 2's choice. Knowing that firm 1 always prefers p^{mf} in the second stage, firm 2 will be happier if it chooses p^{mf}, in which case firm 2 earns zero, than choosing p^M, in which case firm 2 gets negative profits $-\frac{x^2}{2}$.

In summary, when the fringe bidder has no production facility, a unique regulated-monopoly perfect equilibrium exists in this two-stage static game, where both firms bid p^{mf}. ■

The total profit that firm 1 can gain through selling its own quantity in the spot market and through selling contracts in the forward market under the static setting is

$$\pi = (a - q_1^{mf} - x - c)q_1^{mf} + (p^{mf} - c)x = \frac{(a-c)^2 - x^2}{4}$$

It is obvious that selling forward contracts reduces the profit of the monopoly incumbent and mitigates its market power. This competition enhancing effect in the static model supports the regulator's policy of compulsory sale of forward contract by the incumbent monopoly generator.

As we mentioned above, the contract quantity x is relatively small and firm 2 is constrained by this capacity. We must check whether firm 2 is not better off when buying only a fraction of x. Bidding for the whole x is profitable only if x is less than the firm 2's best response to firm 1's quantity in a Cournot game. In other words, x must be less than the quantity

$$\frac{a - q_1(x)}{2} = \frac{a - \frac{a-c-x}{2}}{2}$$

which implies

$$0 \leq x < \frac{a+c}{3}$$

Recall that the Cournot price in this asymmetric duopoly case, where one has marginal production cost c but the other has no production cost, is $p^C = \frac{a+c}{3}$. From (3) we see that the monopoly price with a fringe firm p^{mf} belongs to the interval $(p^C, p^M]$.

3 The repeated game with a fringe firm

In this section we have several bidding candidates in the forward market auction. We call them fringe firms because they are all small and short-term players, and we study the case where the incumbent auctions off its forward contracts repeatedly for an infinite number of periods $t = 1, 2, ..., \infty$. We adopt the timing pattern used in Liski and Montero (2005): the spot market opens at odd periods ($t = 1, 3, ...$) and the auction market opens at even periods ($t = 0, 2, ...$) and . This timing assumption implies that all spot markets are preceded by a forward market where the incumbent firm has the opportunity to sell forward contracts. A fringe firm cannot secure whether it will be the winner of the next period auction. Its objective is to gain as much as possible in each individual period. It is not in its interest to withdraw capacity in order to sustain a higher price in the spot market. Therefore, the fringe firm sells all the acquired capacity x in the spot market and by our competitive auction assumption it always earns zero profit (see Diagram 3-1: the profit matrix).

However, the incumbent has the interest to raise the price to a monopoly level since it is a long-term player in this repeated game. Given that the auction is conducted in a competitive way, firm 1 would like to give the bidders an impression that it will carry out the monopoly result and it would like to sustain this reputation across periods. We start with the case where firm 2 has no production facility and can sell at most x. We will loose this assumption in the next sections.

We first look for the condition under which an equilibrium of the repeated game exists, where firm 1 earns monopoly profits in the market. On the collusive path, firm 1 always sells $(q^M - x)$ in the spot market and x in the forward market at price $p^M = \frac{a+c}{2}$. Therefore, it earns the monopoly profits through selling the contracts along the infinite number of periods. We also assume that firm 1 cares about the sum of the discounted future profits. The per-period discount factor is denoted by $\sqrt{\delta}$, where $0 < \delta < 1$, so the discount factor between two consecutive spot market openings is δ, which facilitates the comparison with pure-spot repeated games.

3.1 Firm 2 has no production facility

3.1.1 Cournot competition

When firm 2 has no production facility and competes with firm 1 in quantity on the spot market, if firm 1 deviates from the collusive path at period t, this deviation always happens in the spot market. The reasons are the following: firm 1 can deviate by either undercutting its spot price through increasing its own spot production, or increasing its forward sale. Here the forward sale is fixed by the regulator and the incumbent firm cannot change it easily. Even if the incumbent firm could decide the forward sale, it would never be profitable because any deviation in the forward market would instantly be detected by bidding firms who would pay no more than the next period spot market price. If firm 1 deviates from the collusive path and maximizes its profit with respect to quantity, it is more profitable for firm 2 to detect it on the forward market of period $t+1$. Therefore, firm 1 will maximize the profit from the residual demand

$$\max_{q_1}(a - q_1 - x)q_1 - cq_1 + (p_f - c)x$$

and we are back to the static equilibrium

$$\begin{aligned} q_1^{mf} &= \frac{a-c}{2} - \frac{x}{2} \\ p^{mf} &= \frac{a+c}{2} - \frac{x}{2} \end{aligned}$$

The deviation profit earned by firm 1 is

$$\pi_1^d = (a - q_1^{mf} - x)q_1^{mf} - cq_1^{mf} = \frac{(a-c-x)^2}{4}$$

In period $t+1$, auction participants will only bid at price p^{mf} to change their expectation that firm 1 always chooses the highest price p^M, after detecting firm 1's deviation in the spot market of period t. This reaction is equivalent to punishment. To summarize, the auction participants have the following expectations function (implicitly, they also expect that the winner of the auction will sell x units in the spot market)

$$p_t^e = \begin{cases} p^M & if \quad p_{t'} = p^M \quad \forall\, t' < t \text{ or } t = 0 \\ p^{mf} & \qquad otherwise. \end{cases}$$

On the punishment path, there are trigger strategy expectations, which means, if firm 1 deviates from the monopoly path and floods the market with price lower than p^M, it will get punished as under the static model given that the fringe firm has not its own production facility. We exclude the cases where the participants in the auction collude to bid prices varying from zero to $(p^{mf} - \varepsilon)$, where ε is extremely small. Even though those punishments are much harsher than in the former case we talked above, we assume that the number of participants in the auction is sufficiently large for such collusion on bidding prices lower than p^{mf} to be impossible. However, notice that if such a collusion among auction participants is possible, the incumbent will be punished much harder than what we assume, and it will be even easier for firm 1 to sustain the monopoly outcome. Therefore, the monopoly result is sustainable if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\frac{(a-c)^2}{4} \geq \frac{(a-c-x)^2}{4} + \frac{(a-c)}{2}x + \frac{\delta}{1-\delta}\left[\frac{(a-c-x)^2}{4} + \frac{(a-c-x)}{2}x\right]$$

The left hand side of the inequality represents the monopoly collusive profit which the incumbent firm captures. The first two terms on the right hand side of the inequality stand for the deviation profit which firm 1 can earn when it gets off the collusive path, and the last term on the right hand side of the inequality is the discounted profit of firm 1 when firm 2 retaliates under trigger strategy. That inequality implies

$$\delta \geq \underline{\delta}^C = \frac{1}{2} \tag{5}$$

where $\underline{\delta}^C$ is defined as the threshold of the discount factor above which collusion is sustainable in Cournot competition.

In this linear demand model, the competition enhancing effect of forward trading that existed in the static model vanishes since it makes it easier to sustain collusion than in the case where there is no forward trading possibility[4]. However, this results is special because it strongly depends on the linearity of the demand function. Results for a more general demand function are shown in Appendix A.1.

3.1.2 Bertrand competition

Consider the case where firms play price competition in the spot market. Given that firm 2 has no production facility, it can only compete against firm 1 under its capacity constraint x, where $x < \frac{a+c}{3}$. Like in the former subsection, if firm 1 deviates from the collusive path in the spot market of period t by charging

$$p_1^d = \arg\max \{(p_1 - c)(a - x - p_1)\} = \frac{a+c}{2} - \frac{x}{2} = p^{mf}$$

in the following periods $t+1, ..., \infty$, auction participants pays p^{mf} instead of p^M.

The deviation profit earned by firm 1 in the spot market is

$$\pi_1^d = (a - q_1^{mf} - x)q_1^{mf} - cq_1^{mf} = \frac{(a-c-x)^2}{4}$$

The monopoly result is sustainable if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\frac{(a-c)^2}{4} \geq \frac{(a-c-x)^2}{4} + \frac{(a-c)}{2}x + \frac{\delta}{1-\delta}\left[\frac{(a-c-x)^2}{4} + \frac{(a-c-x)}{2}x\right]$$

which implies

$$\delta \geq \underline{\delta}^B = \frac{1}{2} \tag{6}$$

where $\underline{\delta}^B$ is defined as the threshold of the discount factor above which collusion is sustainable in price competition with one agent capacity constrained.

We retrieve the same results as those of quantity competition. It is not surprising since the fringe firm, i.e. firm 2, has no production facility and cannot produce more than the contract quantity x, which constrains price competition. Therefore, on the

[4] The threshold of the discount factor in the standard Cournot competiton is $\frac{9}{17}$, which will be proved later in the proof of Proposition 1

punishment path, firm 2 cannot punish firm 1 for deviating as harshly as under Bertrand competition without capacity constraint, where the price is pushed down to the marginal production cost and none of the firms make profits along the post-deviation periods. In the meantime, the introduction of forward trading does not change the difficulty of collusion since the threshold of the discount factor is identical to the one in standard Bertrand competition[5].

Lemma 2 *When the fringe firm has no production facility and when it is a short run player, there exists a subgame perfect equilibrium, where the incumbent firm earns monopoly profits from both the contract and spot markets in each period, if the discount factor* δ *exceeds the threshold* $\underline{\delta}^C = \frac{1}{2}$ *(resp.* $\underline{\delta}^B = \frac{1}{2}$*) which is derived from (5) (resp. (6)).*

Proof. It is clear that when the fringe firm has no production facility and when it is a short run player, we retrieve the standard result that Cournot competition is equivalent to Bertrand competition since firm 2 has capacity constraint and firm 1 maximizes its profit from the residual demand. ■

Proposition 1 *In this specific repeated game, when the contract quantity* x *is relatively small, introducing forward trading opportunity does not change the level of difficulty to sustain collusion if both firms compete à la Bertrand in the spot market. However, introducing forward contract facilitates collusion if both firms compete à la Cournot in the spot market.*

Proof. This threshold $\underline{\delta}^B$ coincides to the standard collusion-sustainable threshold of Bertrand competition, which is $\underline{\delta}^{SB} = \frac{1}{2}$ (Tirole (1988)).

$$\frac{1}{1-\delta}\frac{(a-c)^2}{8} \geq \frac{(a-c)^2}{4} + \frac{\delta}{1-\delta} * 0$$

The left hand side of the inequality is the equally shared monopoly collusive profit which one firm captures. The first term on the right hand side of the inequality stands for the deviation profit which one firm can earn when it gets off the collusive path, and the

[5] The threshold of the discount factor in the standard Bertrand competiton is $\frac{1}{2}$, which will be proved later in the proof of Proposition 1

second term on the right hand side of the inequality represents the discounted profit of one firm when its rival retaliates under trigger strategy. The inequality which implies

$$\delta \geq \underline{\delta}^{SB} = \frac{1}{2}$$

where $\underline{\delta}^{SB}$ is defined as the threshold of the discount factor above which collusion is sustainable in the standard Bertrand competition without capacity constraint. The introduction of forward contract actually makes no difference from the situation where there is no forward trading opportunity if both firms compete *à la Bertrand*.

The result for quantity competition can be easily verified by comparing $\underline{\delta}^{C} = \frac{1}{2}$ with the standard collusion-sustainable threshold of Cournot competition $\underline{\delta}^{SC} = \frac{9}{17}$. In fact,

$$\frac{1}{1-\delta}\frac{(a-c)^2}{8} \geq \frac{(a-c)^2}{2} + \frac{\delta}{1-\delta}\left[\frac{(a-c)^2}{9}\right]$$

The equally shared monopoly collusive profit stays on the left hand side of the inequality. One firm's deviation profit is presented through the first term on the right hand side of the inequality and the discounted profit under retalization is the second term. The inequality implies

$$\delta \geq \underline{\delta}^{SC} = \frac{9}{17}$$

where $\underline{\delta}^{SC}$ is defined as the threshold of the discount factor above which collusion is sustainable in the standard Cournot competition ■

3.2 Firm 2 has production facility

When firm 2 has its own production facility, it will not be constrained by the contract quantity x. We assume that firm 2 has the same marginal production cost as firm 1, i.e. c, and a large production capacity. If firm 2 is a short run player and it cannot secure that it will win the auction in the following successive periods, firm 2 has only the interest to maximize its short run profit and will not coordinate with the incumbent firm to restrict its production in order to sustain the monopoly profit. In other words, there is no monopoly subgame perfect equilibrium and the short run fringe firm will compete against the incumbent, in the same manner as in the static model.

Proposition 2 *When the fringe firm has its own production facility and behaves as a short run player, it is not possible to sustain collusion in the forward market and in the spot market.*

Proof. In the spot market if both firms compete in price without any capacity constraint, we will retrieve static Bertrand results and the equilibrium spot price is equal to the marginal production cost c. Anticipating this equilibrium price, none of the auction participants would like to bid more than c in the forward market. On the other hand, if both firms compete in quantity without any capacity constraint in the spot market, we will retrieve static Cournot results and the equilibrium price is $\frac{a+2c}{3}$, none of the auction participants would like to bid more than this Cournot price in the forward market. ■

4 The repeated game with a non-fringe firm

In this section, we successively examine the case where firm 2 has production facility and is not capacity constrained, and the case where firm 2 has capacity constraint. In each case, we will consider different profit sharing rules on the collusive path because of this change of bargaining power for firm 2 to get its share in this collusion

4.1 Firm 2 is not capacity constrained

4.1.1 Cournot competition

Suppose that firm 2 is not a price-taking fringe firm but can play strategically against firm 1. However, only firm 1 has the obligation to sell forward contracts and firm 2 only has the choice to buy forward contracts. In the second stage, the two firms compete against each other in the spot market *à la Cournot.* When firm 2 owns production facility and faces no capacity constraint, the next proposition shows that forward trading opportunity may make collusion difficult to sustain.

Lemma 3 *The threshold of the discount factor that makes firm 1 incline to sustain collusion is* $\underline{\delta}_1^C(x) = \frac{9(a-c)}{17(a-c)+96x}$.

Proof. see Appendix A.2 ■

Lemma 4 *The threshold of the discount factor that makes firm 2 incline to sustain collusion is* $\underline{\delta}_2^C(x) = \frac{9(a-c)}{17(a-c)-96x}$.

Proof. see Appendix A.2 ■

Proposition 3 *When firms compete in quantity in the spot market and firm 2 has no production capacity constraint, selling forward contract makes it more difficult for both firms to collude than in the case where there is no forward trading opportunity.*

Proof. The thresholds which are determined in Lemma 3 and Lemma 4 imply $\underline{\delta}_2^C > \underline{\delta}_1^C$. Therefore, the relevant threshold is $\underline{\delta}_2^C$ and collusion only exists when $\delta \geq \underline{\delta}_2^C(x)$, which is regime C in figure 1. Recall from the proof of proposition (1) that, without contract market, the standard threshold to sustain collusion in Cournot competition is $\delta \geq \frac{9}{17}$ We can compare it with the relevant threshold and get the result that $\underline{\delta}_2^C(x) > \frac{9}{17}$ when $0 < x < \frac{a-c}{12}$. Therefore, selling forward contract makes it more difficult for both firms to collude than in the case where there is no forward trading opportunity. ■

Remark 1 *If the forward contract quantity is sufficiently small* $\left(0 < x < \frac{a-c}{12}\right)$, *introducing the forward trading opportunity increases the difficulty for the firms to sustain tacit collusion and this argument can be utilized by regulatory authorities to mitigate market power. However, if the forward contract quantity is relatively large, i.e.* $x \geq \frac{a-c}{12}$, *the condition of sustaining tacit collusion does not hold any more and it is not possible for firm 2 to collude. In other words, when* $x \geq \frac{a-c}{12}$, *any increase of the contract quantity has no effect on the difficulty of collusion.*

The relationship between the volume of forward contract x and the discount factor δ, which describes the thresholds between collusion incentive and collusion deterrence, are presented in figure 1. Regime A stands for the regime where none of the firms would like

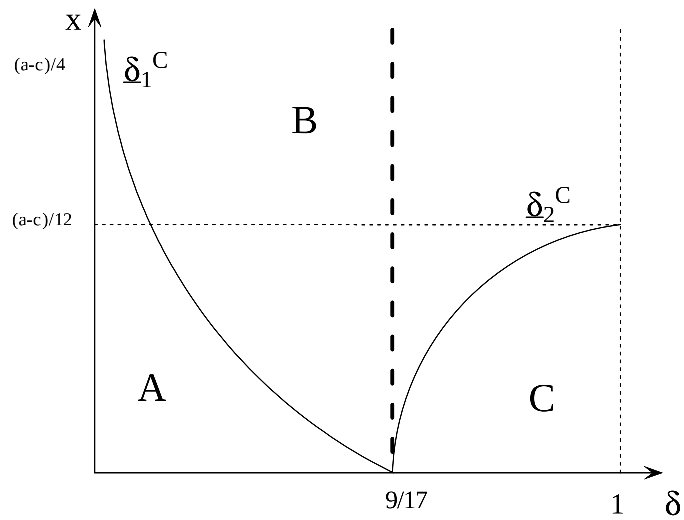

Figure 1: Collusion Regimes-Cournot Competition

to participate into collusion. Regime B stands for the regime where firm 1 is inclined to collude but firm 2 is not. Regime C stands for the regime where both firms would like to sustain collusion.

4.1.2 Bertrand competition

We still stick to the case where only firm 1 has the obligation to sell forward contracts and firm 2 only has the choice to buy the forward contracts and then compete against firm 1 in the spot market. Now the second stage competition is *à la Bertrand.* When firm 2 has a production facility and faces no capacity constraint, proposition 4 shows that forward trading opportunity may make collusion more difficult to sustain.

Lemma 5 *The threshold of the discount factor that makes firm 1 inclined to sustain collusion is* $\underline{\delta}_1^B(x) = \frac{1}{2} - \frac{4(a-c)x-x^2}{2[(a-c)^2+x^2]}$.

Proof. see Appendix A.3 ■

Lemma 6 *The threshold of the discount factor that makes firm 2 inclined to sustain collusion is* $\underline{\delta}_2^B(x) = \frac{1}{2} + \frac{x}{(a-c)-2x}$.

Proof. see Appendix A.3 ■

Proposition 4 *When firms compete in price in the spot market and when firm 2 faces no production capacity constraint, selling forward contract makes it more difficult for both firms to collude than in the case where there is no forward trading opportunity.*

Proof. The thresholds which are determined in Lemma 5 and Lemma 6 imply $\underline{\delta}_2^B > \underline{\delta}_1^B$. Therefore, the relevant threshold is $\underline{\delta}_2^B$ and collusion only is feasible when $\delta \geq \underline{\delta}_2^B(x)$, which is regime C in figure 2. Recall that, without forward, the standard threshold to sustain collusion in Bertrand competition is $\frac{1}{2}$. Comparing it with the relevant threshold, we get the result that $\underline{\delta}_2^B(x) > \frac{1}{2}$ when $0 < x < \frac{a-c}{4}$. Therefore, selling forward contract makes it more difficult for both firms to collude than in the case where there is no forward trading opportunity. ■

Remark 2 *If the forward contract quantity is sufficiently small* $0 < x < \frac{a-c}{4}$*, introducing the forward trading opportunity will increase the difficulty for both firms to sustain tacit collusion and this argument can be utilized by regulatory authorities to mitigate market power. However, if the forward contract quantity is relatively large, i.e.* $x \geq \frac{a-c}{4}$*, forward trading opportunity has reached its maximal effect on tacit collusion since it is not possible to collude for firm 2's interest and this coalition relationship cannot be built up in any case. Increasing the quantity of forward contract has no effect on the difficulty to collude and cannot be utilized by regulatory authorities as the argument of market-power mitigation.*

The relationship between the forward contract quantity x and the discount factor δ can be presented by figure 2. Regime A stands for the regime where none of the firms would like to participate into collusion. Regime B stands for the regime where firm 1 is inclined to collude but firm 2 is not. Regime C stands for the regime where both firms would like to sustain collusion.

4.2 Firm 2 is capacity constrained

It is not difficult to predict that price competition corresponds to quantity competition when firm 2 faces production limitation. A short-run player which has its own production

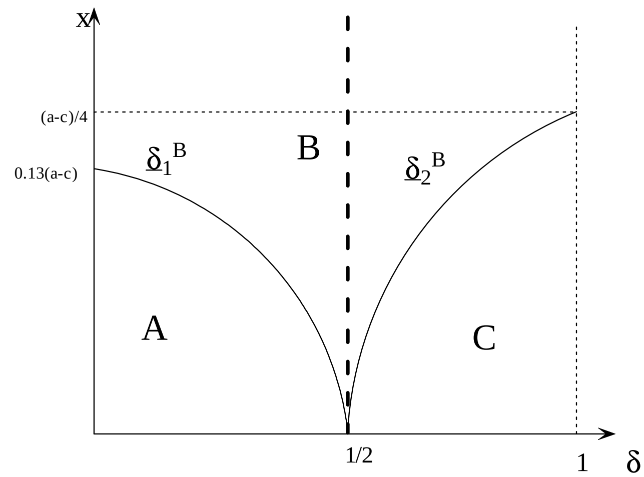

Figure 2: Collusion regimes -Bertrand Competition

capacity will not participate into collusion and will only maximize its short run profit. Only the long-run player would like to participate into collusion since it could get a share of monopoly profit in future periods. Now it becomes crucial how to share the monopoly profit between the firms on the collusive path. If both firms are symmetric, a natural assumption is to share the monopoly profit equally. Then firm 2 who has no right to sell forward contract would like to buy zero forward contract. The reason is as follows: the only profit earned by firm 2 comes from $(q_2 - x)p^M$, where superscript M stands for *monopoly* on the collusive path. Therefore, firm 2 has an incentive to buy as few forward contracts as possible in order to get most monopoly profit on the collusive path if firm 2 buys forward contracts at price p^M. If both firms are asymmetric and firm 2 is smaller than firm 1, firm 2 will be constrained by its capacity $q_2 \leq k_2$ off the collusive phase. Another sharing rule is to split the monopoly profit according to the same ratio as on the punishment path. If firm 2 is relatively smaller and is constrained by its capacity on the punishment path, its best response to firm 1's deviation is to produce at full capacity. Therefore, the market share on the punishment path depends on firm 2's capacity k_2, as well as the forward contract x. We denote the market share of firm 1 (resp. firm 2) by s_1 (resp. s_2): $s_1(k_2, x), s_2(k_2, x)$ where $s_1(k_2, x) + s_2(k_2, x) = 1$.

On the collusive path, both firm produce the collusive monopoly quantities and capture the collusive monopoly profits according to their market shares

$$q_1^M = s_1 Q^M; q_2^M = s_2 Q^M \quad (7)$$

$$\pi_1^M = s_1 \pi^M; \pi_2^M = s_2 \pi^M \quad (8)$$

where the monopoly quantity is $Q^M = \frac{a-c}{2}$, and monopoly profit is $\pi^M = \frac{(a-c)^2}{4}$.

On the punishment path, the best response of firm 1 after detecting firm 2's deviation is

$$q_1^R(k_2) = \arg\max_{q_1}(a - q_1 - k_2 - x - c)q_1 + (p^f - c)x = \frac{a - c - k_2 - x}{2} \quad (9)$$

where superscript R stands for *the best response*. From the firm 1's objective function we can find that firm 1 produces $(q_1 + x)$ but only sells q_1 in the spot market. However, firm 1 also captures its profit by selling x contract in the forward market. Even though officially firm 2 sells quantity x in the spot market, this contract quantity is actually sold by firm 1 through forward trading. Therefore, we should count $q_1^R(k_2) + x$ for firm 1's quantity and k_2 for firm 2's quantity in the calculation of their market shares

$$s_1 = \frac{q_1^R(k_2) + x}{q_1^R(k_2) + k_2 + x}$$

$$s_2 = \frac{k_2}{q_1^R(k_2) + k_2 + x}.$$

The market shares become

$$s_1 = \frac{a - c - k_2 + x}{a - c + k_2 + x}, \quad s_2 = \frac{2k_2}{a - c + k_2 + x}$$

and the punishment profits are

$$\pi_1^P = \frac{(a - c - k_2)^2}{4} - \frac{x^2}{4}; \quad \pi_2^P = \frac{(a - c - k_2 - x)k_2}{2}$$

where superscript P stands for *punishment*.

The total profit is

$$\pi^P = \pi_1^P + \pi_2^P = \frac{(a-c+k_2+x)(a-c-k_2-x)}{4}$$

$$\text{and } \pi_1^P = s_1\pi^P; \;\; \pi_2^P = s_2\pi^P \tag{10}$$

On the collusive path, according to (8) firms' production are

$$\begin{aligned} q_1^M &= \frac{a-c-k_2+x}{a-c+k_2+x}\left(\frac{a-c}{2}\right) \\ q_2^M &= \frac{2k_2}{a-c+k_2+x}\left(\frac{a-c}{2}\right) \end{aligned} \tag{11}$$

On the deviation path, the best response can be calculated in the same way as (9) if firm 1 deviates

$$\begin{aligned} q_1^D &= q_1(q_2^M) = \frac{a-c-q_2^M}{2} = \frac{(a-c)(a-c+x)}{2(a-c+k_2+x)} \\ q_2 &= q_2^M \end{aligned}$$

The deviation profit of firm 1 is $\pi_1^D = \left(a - q_1^M - q_1(q_2^M)\right) q_1(q_2^M) - cq_1(q_2^M) - \left(p^M - c\right) x$

$$\pi_1^D = \left[\frac{(a-c)}{2}\left(\frac{a-c+x}{a-c+k_2+x}\right)\right]^2 + \frac{(a-c)}{2}x \tag{12}$$

where superscript D stands for *deviation.* If the long-term player 2 owns its production facility, the non-deviation condition of firm 1 is

$$\frac{1}{1-\delta}s_1\pi^M \geq \pi_1^D + \frac{\delta}{1-\delta}\pi_1^P,$$

which can be simplified by using (10):

$$\frac{1}{1-\delta}\pi^M \geq \frac{\pi_1^D}{s_1} + \frac{\delta}{1-\delta}\pi^P \tag{13}$$

If firm 2 deviates from the collusion path, the best strategy it may apply is

$$q_2 = k_2 + x \tag{14}$$

At this time, firm 1 still produces $q_1 = q_1^M$. Therefore, the price on the deviation path is

$$p^D = a - q_1^M - q_2$$

where q_1^M and q_2 are given by (11) and (14). The corresponding deviation profit for firm 2 is $\pi_2^D = \left(a - q_1^M - k_2 - x\right)(k_2 + x) - ck_2 - p^M x$

$$\pi_2^D = \left[\frac{(a-c)}{2}s_2 - (k_2 + x)\right](k_2 + x) + \frac{(a-c)}{2}k_2 \tag{15}$$

The non-deviation condition of firm 2 is

$$\frac{1}{1-\delta}s_2\pi^M \geq \pi_2^D + \frac{\delta}{1-\delta}\pi_2^c$$

which can be simplified by using (10):

$$\frac{1}{1-\delta}\pi^M \geq \frac{\pi_2^D}{s_2} + \frac{\delta}{1-\delta}\pi^P \tag{16}$$

All those results above can be summarized in the profit matrix of Diagram 3-2[6]

[6] In the profit cell (q_1^M, q_2^D), the southwestern part π_2^D is the deviation profit that firm 2 earns when it cheats and it is shown by (15). The northeastern part π_1 reprensents the profit that firm 1 earns when firm 2 deviates but firm 1 still produces collusive quantity:

$$\pi_1 = \left[\left(\frac{a-c}{2}\right)(1-s_1) - (k_2 + x)\right]\left[\left(\frac{a-c}{2}\right)s_1 - x\right] + \left(\frac{a-c}{2}\right)^2 s_1$$

Moreover, in the profit cell (q_1^D, q_2^M), the northeastern part π_1^D is the deviation profit that firm 1 earns when it cheats and it is shown by (12). The southwestern part π_2 reprensents the profit that firm 2 earns when firm 1 deviates but firm 2 still produces collusive quantity:

$$\pi_2 = \left(\frac{a-c}{2}\right)\left[s_2\left(\frac{a-c}{2}\right)\left(1 - \frac{3}{2}s_2\right) - s_2 c - x\right]$$

		Firm 1	
		q_1^P or q_1^D (if firm 2 takes q_2^M)	q_1^M
Firm 2	q_2^P or q_2^D (if firm 1 takes q_1^M)	π_1^P, π_2^P	π_1, π_2^D
	q_2^M	π_1^D, π_2	π_1^M, π_2^M

Diagram 3-2: the profit matrix 2

It is easy to verify that $\frac{\pi_1^D}{s_1} > \frac{\pi_2^D}{s_2}$ when $x < \frac{(a-c)}{4}$ and $k_2 < \frac{3(a-c)}{8} - x$. Therefore, the relevant inequality to determine the threshold of collusion-sustainable discount factor is (13), which we can rewrite as follows:

$$\frac{1}{1-\delta}\frac{(a-c)^2}{4}s_1 \geq$$

$$\left[\frac{(a-c)}{2}\left(\frac{a-c+x}{a-c+k_2+x}\right)\right]^2 + \frac{(a-c)}{2}x + \frac{\delta}{1-\delta}\frac{(a-c+k_2+x)(a-c-k_2-x)}{4}s_1$$

From this inequality, we derive the threshold value of δ, denoted by $\underline{\delta}^K$. Collusion is sustainable if and only if

$$\delta \geq \underline{\delta}^K = \frac{\frac{(a-c)}{2}\left(\frac{k_2}{a-c+k_2+x}\right)^2 + x}{\frac{(a-c)}{2}\left(\frac{a-c+x}{a-c+k_2+x}\right) - \frac{(a-c-k_2)^2-x^2}{2(a-c)} + x} \tag{17}$$

As the formula becomes heavy-handed, we focus on the special case where $a = 1$ and $c = 0$. (17) becomes

$$\delta \geq \underline{\delta}^K = \frac{\left(\frac{k_2}{1+k_2+x}\right)^2 + 2x}{\left(\frac{1+x}{1+k_2+x}\right) - (1-k_2)^2 + x^2 + 2x}$$

Taking $x = 0.1$ and plotting $\underline{\delta}^K$ on the vertical axis, we have the impact of k_2 on the

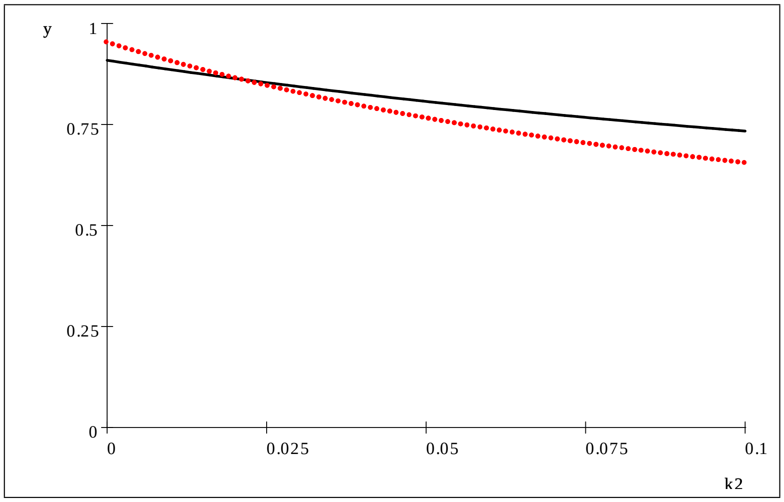

Figure 3: The role of k_2 when $x = 0.1$ (dot) and when $x = 0.2$ (solid)

discount factor $\underline{\delta}^K$, which is shown by the dot line in figure 3. The solid line in figure 3 shows the impact of k_2 on the discount factor $\underline{\delta}^K$ when $x = 0.2$. Clearly it is much easier to sustain tacit collusion between firm 1 and firm 2 when firm 2's capacity constraint k_2 increases. The intuition is the following: when firm 2's capacity constraint k_2 increases, both firms become more and more symmetric, since it is easier to sustain tacit collusion when both firm's sizes are similar, we converge to the standard tacit collusion results.

Taking $k_2 = 0.1$ and plotting $\underline{\delta}^K$ on the vertical axis (named axis y), we also get the impact of x on the discount factor $\underline{\delta}^K$, which is shown by the dot line in figure 4. The solid line in figure 4 shows the impact of x on the discount factor $\underline{\delta}^K$ when $k_2 = 0.2$. Clearly it is more difficult to sustain tacit collusion between firm 1 and firm 2 when forward contract quantity x increases. The intuition is the following: when firm 2's has capacity constraint k_2 in the spot market, selling more forward contract will make the incumbent firm occupy more market share, which increases asymmetry between the incumbent and the entrant. Therefore it is more difficult to sustain tacit collusion when we have more asymmetric firms, as established by standard tacit collusion results.

Conjecture 1 *When firms have opportunity to trade forward contracts, i.e. when x is different from 0, it is easier to sustain tacit collusion when firm 2's capacity increases. Moreover, when firm 2 has a limited production capacity, it is more difficult to sustain*

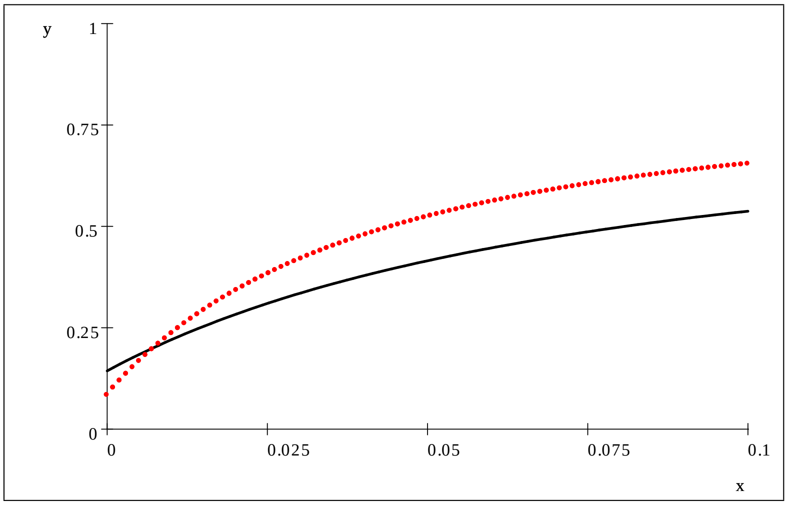

Figure 4: The role of x when $k_2 = 0.1$ (dot) and when $k_2 = 0.2$ (solid)

tacit collusion when forward contract quantity is larger.

Remark 3 *More examples can be analyzed by denoting* $x = \alpha k_2$*. Figure 5 shows the impact of* α *on the discount factor* $\underline{\delta}^K$ *when firm 2's capacity is either* $k_2 = 0.03$ *or* $k_2 = 0.1$*. Those numerical results show that, when firm 2 faces capacity constraints, regulatory authorities may prefer selling more forward contracts in order to increase the difficulty for the two firms to sustain tacit collusion.*

4.3 Two entrant firms bid forward contracts

Let us extend the model to two entrant firms, denoted as firm i and firm j, competing to obtain the forward contract offered by the incumbent firm. We assume that communication is forbidden between firm i and firm j, and they cannot collude either tacitly or publicly. In the forward market, each bidding firm would like to submit the highest bid that it believes will secure the contract, taking into account the likely bid of the rival. Any bid lower than the rival's bid will make the bidding firm lose the auction. Therefore, competition for forward contracts does not necessarily increase as the number of entrant firms increases. As long as there are at least two firms capable of making credible bids, competition can be as vigorous with two firms as with three or more. This is the

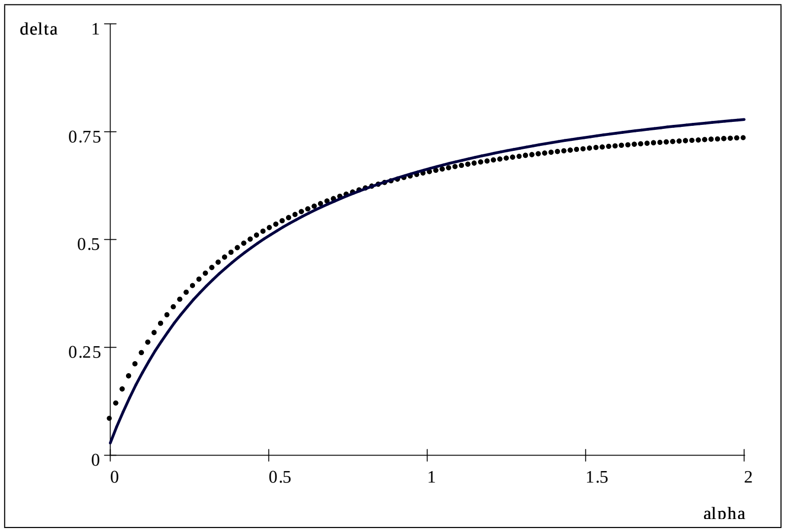

Figure 5: The impact of α on the discount factor $\underline{\delta}_3$ when $k_2 = 0.03$ (solid); $k_2 = 0.1$ (dot)

famous argument "*Two Is Enough*" concerning competition in bidding markets. Then we are back to the competitive auction case and will retrieve the same results which are analyzed in section 3 – the repeated game with a fringe firm.

5 Conclusion

We have studied the effect of trading forward contracts on tacit collusion. Specially we have analyzed forward contract as a novel and uninvestigated feature in major merger cases. European competition authorities intend to mitigate market power by compelling the merging firm to sell forward contracts to prospective competitors in the spot market. We have shown that under certain conditions selling more contracts, which are imposed by the regulator, makes it more difficult for the incumbent firm to collude tacitly with its competitor. But this conclusion only holds when the competitor of the incumbent is a non-fringe firm and when the profit sharing rule on the collusive path is specific. Otherwise, trading forward contract either has no effect or possibly facilitates tacit collusion if the competitor is a fringe firm. The analysis suggests that competition authorities should worry about both the frequency of contract trading and the regulation of contract quantity.

A Appendix

A.1 Solutions of the general reverse demand function

We still keep the same timing pattern for the repeated game as what is discussed in section 3.1. Assume the inverse demand function is $p = D(q)$ and the cost function is $C(q)$, where $C'(q) > 0$ and $C"(q) > 0$. The simplest case will start with the one where firm 2 has not its own production facility and can only sell up to the contract quantity x. On the collusive path, firm 1 will capture all the monopoly profit not only by selling monopoly price p^M in the spot market, but also by selling monopoly price p^M in the forward market. Firm 1 maximizes the profit facing the total demand

$$\max_{q_1} D(q_1)q_1 - C(q_1) \tag{18}$$

The first order condition gives

$$D'(q_1)q_1 + D(q_1) = C'(q_1) \tag{19}$$

q_1^M can be driven out from (19) and the monopoly price p^M is solved by the the inverse demand function $D(q_1^M)$. Firm 1 obtains the profit

$$\pi^M = D(q_1^M)q_1^M - C(q_1^M)$$

As the same reason as before, if firm 1 deviates from the collusive path at period t, it always happens in the spot market, denoted $t - spot$. Therefore, firm 1 will maximize the profit facing the residual demand

$$\max_{q_1} D(q_1 + x)q_1 - C(q_1 + x) + xp_f \tag{20}$$

The first order condition gives

$$D'(q_1^d + x)q_1 + D(q_1^d + x) = C'(q_1^d + x) \tag{21}$$

The deviation profit earned by firm 1 is

$$\pi_1^d = D(q_1^d + x)q_1^d + xD(q_1^M) - C(q_1^d + x)$$

In period $t+1$, auction participants will only bid at price p^S to punish firm 1, where p^S is solved by using (21), after detecting firm 1's deviation in period $t - spot$. Firm 1 only gets π_1^P on the successive punishment path

$$\pi_1^P = D(q_1^d + x)(q_1^d + x) - C(q_1^d + x)$$

To summarize, the auction participants have the following expectations function (implicitly, they also expect that the winner of the auction will sell x units in the spot market)

$$p_t^e = \begin{cases} p^M & if \quad p_{t'} = p^M \quad \forall\, t' < t \text{ or } t = 0 \\ p^{mf} & otherwise. \end{cases}$$

On the punishment path, there are trigger strategy expectations, which means, if firm 1 deviates from the monopoly path and floods the market with price lower than p^M, it will get punished as under the static model given the fringe firm has not its own production capacity. We also exclude the cases that the participants in the auction collude to bid the prices varying from zero to $(p^S - \varepsilon)$, where ε is extremely small. Even though those punishments are much more harsh than the case we talked above, we assume that the number of participants in the auction is sufficiently large that such collusion on bidding lower prices than p^S is not possible. Notice that, however, if such kind of collusion among auction participants is possible and the incumbent firm, i.e. firm 1, thus will be punished much harder than what we assume, it will be even easier for firm 1 to sustain the monopoly result. Therefore, the monopoly result is sustainable if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\pi^M \geq \pi_1^d + \frac{\delta}{1-\delta}\pi_1^P$$

which implies

$$\delta \geq \underline{\delta}_1 = \frac{\pi_1^d - \pi^M}{\pi_1^d - \pi_1^P} \tag{22}$$

(22) can be further simplified as the following inequality

$$\delta \geq \underline{\delta}_1 = \frac{D(q_1^M)\left(q_1^d + x - q_1^M\right) - \left[C(q_1^d + x) - C(q_1^M)\right] - \left[D(q_1^M) - D(q_1^d + x)\right] q_1^d}{x\left[D(q_1^M) - D(q_1^d + x)\right]}$$

A.2 Proof of Lemma 3 and Lemma 4

Proof. As we have analyzed above, a short-run player which has its own production capacity will not participate into collusion and will only maximize its short run profit. Only the long-run player would like to participate into collusion since it can get the monopoly profit along the following successive periods. Now it becomes crucial how to share the monopoly profit between the firms on the collusive path. If both firms are symmetric, a natural assumption is to share the monopoly profit equally. But here firm 1 is the incumbent and is forced to sell quantity x . Even though firm 2 is big enough to compete against firm 1 in the spot market without any capacity constraint, it is always an entrant in the market and has less bargaining power in the carve-up of monopoly profits.

First, we discuss the case where firm 2 has no capacity limit and it can produce whatever it wants. When x is small enough, i.e. $x < \frac{(a-c)}{4}$, firm 1 and firm 2 will sell a total of monopoly quantity $Q^M = q_1^C + q_2^C = \frac{a-c}{2}$ in the spot market on the collusive path. Firm 1 has to sell quantity x to firm 2 in the forward market and realizes part of its monopoly profit through forward contracting. Therefore, in the carve-up of the whole monopoly profit, firm 1 produces $q_1^C = (Q^M/2 + x)$ and firm 2 produces $q_2^C = (Q^M/2 - x)$. In each period, firm 1's profit on the collusive path is

$$(p^M - c)(Q^M/2 + x) = (\frac{a-c}{2})\left(\frac{a-c}{4} + x\right)$$

When firm 1 deviates in the spot market–t, firm 2 still produces q_2^C and sells $(q_2^C + x)$.

Firm 1's maximization problem is

$$\max_{q_1}(a - q_1 - q_2^C - x)q_1 - cq_1 + (p_f - c)x$$

The first order condition gives

$$q_1 = \frac{a-c}{2} - \frac{q_2^C + x}{2}$$

recall that $q_2^C + x = Q^M/2 = \frac{a-c}{4}$. The optimal deviation for firm 1 is

$$\begin{aligned} q_1^D &= \frac{3(a-c)}{8} \\ p^D &= \frac{3a+5c}{8} \\ \pi_1^D &= \left[\frac{3(a-c)}{8}\right]^2 + \left(\frac{a-c}{2}\right)x \end{aligned}$$

On the punishment path, there are trigger strategy expectations, which means, if firm 1 deviates from the monopoly path and floods the market with price lower than p^M, it will get punished as under the static quantity competition model given firm 2 has its own production facility. Firm 2 would like to produce q_2 and sell $(x + q_2)$ in order to punish firm 1's deviation as harshly as possible, since x is sufficiently small ($x < \frac{(a-c)}{4}$) and it is not big enough to punish firm 1 just with contract quantity x. In the forward market firm 2 is opt to pay only the expected punishment price p^P. The optimization problem of firm 2 in the spot market along the punishment periods is

$$\max_{q_2}(a - q_1 - q_2 - x)(q_2 + x) - cq_2 - p^P x$$

The first order condition gives

$$q_2 = \frac{a - c - q_1}{2} - x$$

The optimization problem of firm 1 in the spot market along the punishment periods is

$$\max_{q_1}(a - q_1 - q_2 - x - c)q_1 + (p^P - c)x$$

The first order condition gives

$$q_1 = \frac{a - c - q_2 - x}{2}$$

The equilibrium in quantity competition is

$$\begin{aligned} q_1^P &= \frac{a-c}{3} \\ q_2^P &= \frac{a-c}{3} - x \\ p^P &= \frac{a+2c}{3} \end{aligned}$$

In period $t+1$, firm 2 has the expectation of punishment price and will only pay price p^P for the forward contract. The profit earned by firm 1 on the punishment path is

$$\pi_1^P = (a - q_1^P - q_2^P - x - c)q_1^P + (p^P - c)x = \left(\frac{a-c}{3} + x\right)\frac{(a-c)}{3}$$

Therefore, the monopoly collusive result is sustainable for firm 1 if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\frac{(a-c)}{2}\left(\frac{a-c}{4} + x\right) \geq \left[\frac{3(a-c)}{8}\right]^2 + \left(\frac{a-c}{2}\right)x + \frac{\delta}{1-\delta}\left(\frac{a-c}{3} + x\right)\frac{(a-c)}{3}$$

which implies

$$\delta \geq \underline{\delta}_1^C(x) = \frac{9(a-c)}{17(a-c) + 96x} \tag{23}$$

Now let's look at firm 2's problem and the condition of sustaining tacit collusion. In each period, firm 2's profit on the collusive path is

$$(p^M - c)(Q^M/2 - x) = (\frac{a-c}{2})\left(\frac{a-c}{4} - x\right)$$

When firm 2 deviates, its maximization problem is

$$\max_{q_2} \left[a - (q_1 - x) - (q_2 + x)\right](q_2 + x) - cq_2 - p^M x$$

The first order condition gives

$$q_2 = \frac{a - c - q_1 - x}{2}$$

Recall that in the spot market firm 1 produces $q_1 = Q^M/2 + x = \frac{a-c}{4} + x$, but sells $(q_1 - x)$ since x has been sold through the forward market. The optimal deviation for firm 2 is

$$\begin{aligned} q_2^D &= \frac{3(a-c)}{8} - x \\ p^D &= \frac{3a + 5c}{8} \\ \pi_2^D &= \left[\frac{3(a-c)}{8}\right]^2 - \left(\frac{a-c}{2}\right)x \end{aligned}$$

On the punishment path, there are trigger strategy expectations, which means, if firm 2 deviates from the monopoly path and floods the market with price lower than p^M, it will get punished as under the static quantity competition model. Firm 1 sells q_1 in the spot market. In the forward market firm 2 would like to pay the expected punishment price p^P. The optimization problem of firm 2 in the spot market along the punishment periods is

$$\max_{q_2}(a - q_1 - q_2 - x)(q_2 + x) - cq_2 - p^P x$$

The first order condition gives

$$q_2 = \frac{a - q_1}{2} - x$$

The optimization problem of firm 1 in the spot market along the punishment periods is

$$\max_{q_1}(a - q_1 - q_2 - x - c)q_1 + (p^P - c)x$$

The first order condition gives

$$q_1 = \frac{a - c - q_2 - x}{2}$$

The equilibrium under quantity competition is

$$\begin{aligned} q_1^P &= \frac{a-c}{3} \\ q_2^P &= \frac{a-c}{3} - x \\ p^P &= \frac{a+2c}{3} \end{aligned}$$

In period $t+1$, firm 2 has the expectation of punishment price and will only pay for the forward contract at price p^P. The profit earned by firm 2 on the punishment path is

$$\pi_2^P = (a - q_1 - q_2 - x)(q_2 + x) - cq_2 - p^P x = \frac{(a-c)}{3}\left(\frac{a-c}{3} - x\right)$$

Therefore, the monopoly collusion result is sustainable for firm 2 if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\frac{(a-c)}{2}\left(\frac{a-c}{4} - x\right) \geq \left[\frac{3(a-c)}{8}\right]^2 - \left(\frac{a-c}{2}\right)x + \frac{\delta}{1-\delta}\left(\frac{a-c}{3} - x\right)\frac{(a-c)}{3}$$

which implies

$$\delta \geq \underline{\delta}_2^C(x) = \frac{9(a-c)}{17(a-c) - 96x} \tag{24}$$

As a result, if both firms would like to sustain collusion, the relevant constraint is (24). When $0 < x < \frac{a-c}{12}$, forward trading opportunity makes it more difficult for both firms to participate and sustain tacit collusion under this special division rule of monopoly profit, since $1 > \underline{\delta}_2^C > \frac{9}{17}$, if the forward contract quantity x is sufficiently small $0 < x < \frac{a-c}{12}$. When the forward contract quantity x is relatively large, i.e. $x \geq \frac{a-c}{12}$, it is not possible for firm 2 to participate into collusion and there is no monopoly subgame perfect equilibrium existing.

Q.E.D. ■

A.3 Proof of Lemma 5 and Lemma 6

Proof. When firm 2 can produce whatever it wants and when x is small enough, i.e. $x < \frac{(a-c)}{4}$, firm 1 and firm 2 will sell a total of monopoly quantity $Q^M = q_1^C + q_2^C = \frac{a-c}{2}$ in the spot market on the collusive path. Firm 1 has to sell quantity x to firm 2 in the forward market and realize part of its monopoly profit through forward contracting. Therefore, in the carve-up of the whole monopoly profit, firm 1 produces $q_1^C = (Q^M/2 + x)$ and firm 2 produces $q_2^C = (Q^M/2 - x)$ in order to keep the market price to sustain the monopoly level: $p^M = \frac{a+c}{2}$. In each period, firm 1's profit on the collusive path is

$$(p^M - c)(Q^M/2 + x) = (\frac{a-c}{2})\left(\frac{a-c}{4} + x\right)$$

When firm 1 deviates in the spot market $t-spot$, firm 1 can decreases the price, which is slighter lower than the monopoly one, and captures all the residual market. Actually when firm 1 deviates, it becomes the monopolist and maximizes its profits under the residual demand in the spot market. Firm 1's maximization problem is

$$\max_{p_1}(a - p_1 - x)(p_1 - c) + (p_f - c)x$$

The first order condition gives

$$p_1^d = \frac{a + c - x}{2}$$

The optimal deviation for firm 1 is

$$\pi_1^D = \frac{(a - c - x)^2}{4} + \left(\frac{a-c}{2}\right) x$$

On the punishment path from period $t+1$ and forth on, firm 2 will apply the trigger strategy and revenge most harshly to push the price to competitive price c. Therefore, the expected price in the forward market is also driven to the competitive price c and Firm 1 earns no profits at all.

$$\pi_1^P = 0$$

Therefore, the monopoly collusion result is sustainable for firm 1 if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\frac{(a-c)}{2}\left(\frac{a-c}{4}+x\right)\geq\frac{(a-c-x)^2}{4}+\left(\frac{a-c}{2}\right)x+\frac{\delta}{1-\delta}*0$$

This is equivalent to

$$\delta\geq\underline{\delta}_1^B(x)=\frac{1}{2}-\frac{4\,(a-c)\,x-x^2}{2\left[(a-c)^2+x^2\right]}$$

Recall that $0\leq x<\frac{(a-c)}{4}$, we can verify that $\underline{\delta}_1^B<\frac{1}{2}$ when $x>0$ and $\underline{\delta}_1^B=\frac{1}{2}$ when $x=0$. The opportunity to trade forward contracts makes it easier for firm 1 to collude than the situation where there is no forward trading possibility.

Now let's look at the case when firm 2 deviates. As the same reasoning, firm 2 can compete in price and capture all the residual market. Actually when firm 2 deviates, it becomes the monopolist and maximizes its profits from the residual demand in the spot market if firm 2 has sufficient production facility. Firm 2's maximization problem is

$$\max_{p_2}(a-p_2-x)(p_2-c)+x(p_2-p_f)$$

The first order condition gives

$$p_2^d=\frac{a+c}{2}$$

The optimal deviation for firm 1 is

$$\pi_2^D=\frac{(a-c)^2}{4}-\left(\frac{a-c}{2}\right)x$$

On the punishment path – from period $t+1$ and forth on – firm 2 will apply the trigger strategy and revenge most harshly to push the price down to the competitive price c. Therefore, the expected price in the forward market is also driven to the competitive price c and firm 1 earns no profits at all.

$$\pi_2^P=0$$

Therefore, the monopoly collusive result is sustainable for firm 1 if and only if the following condition is satisfied

$$\frac{1}{1-\delta}\frac{(a-c)}{2}\left(\frac{a-c}{4}-x\right) \geq \frac{(a-c)^2}{4}-\left(\frac{a-c}{2}\right)x+\frac{\delta}{1-\delta}*0$$

This is equivalent to

$$\delta \geq \underline{\delta}_2^B(x)=\frac{1}{2}+\frac{x}{(a-c)-2x} \tag{25}$$

Recall that $x < \frac{(a-c)}{4}$, we can verify that $\underline{\delta}_2^B > \frac{1}{2}$. The opportunity to trade forward contracts makes it easier for firm 1 to collude than the situation where there is no forward trading possibility.

Therefore, if both firms would like to sustain collusion, the relevant constraint is (25). If the forward contract quantity x is sufficiently small, i.e. $0 < x < \frac{a-c}{4}$, introducing forward trading opportunity makes it more difficult for both firms to participate and sustain tacit collusion under this special division rule of monopoly profit, since $1 > \underline{\delta}_2^B > \frac{1}{2}$. However, if the forward contract quantity x is relatively large, i.e. $x \geq \frac{a-c}{4}$, it is not possible for firm 2 to participate into collusion and there is no monopoly subgame perfect equilibrium existing.

Q.E.D. ■

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