Signal Effect, Service Rule on an Equal Basis, and Bank Runs*

Wan-Ru Yang**

A number of related literatures about bank runs recommend policies such as depositor insurance and suspension of convertibility to prevent bank runs. However, depositor's insurance may contribute to banks' hasty lending and suspension of convertibility will make some depositors' welfare be sacrificed. In my paper, when the bank has a financial problem at that time, I study how a bank, an entrepreneur, and depositors renegotiate without the government's intervention to avoid an inefficient bank run and the problem of the bank's moral hazard. This paper shows a perfect Bayesian semi-separating equilibrium without bank runs and a pooling equilibrium with bank runs. The renegotiation does not succeed if the bank is more likely to have bad investment; thus, the problem of moral hazard is eliminated. Depositors have strategic behavior so they are able to obtain their best outcome even during the state of bank runs. In this model, the bank serves depositors on an equal basis. This paper demonstrates that when the market has numerous patient depositors, service on an equal basis will increase the probability of bank runs.

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**Author contact information: Dept. of Economics, University of California, Santa Brbara, CA 93106-9210, U.S.A. Email:yang@econ.ucsb.edu

I. Introduction

The threat of bank runs from depositors is able to prevent a bank's hasty financial investment. But a bank run may be inefficient to the whole financial system if a few bank runs due to financial crises trigger runs in the healthy banks. Therefore, the numerous literatures study related policy issues to prevent bank runs. For example, Diamond and Dybvig (1983) demonstrate how deposit insurance and suspension convertibility prevent bank runs. However, deposit insurance may result in the problem of a bank's moral hazard and suspension convertibility will sacrifice some depositors' welfare. This paper proposes a coordination scheme among the bank, depositors, and entrepreneur to prevent the panic-based bank runs, a scheme in which flaws raised by government interventions do not exist. The feature of this paper is that each agent's action depends not only on his own information about the type of the bank's investment but also on how other agents respond to his decision. Therefore, in a banking system without any insurance coverage, the uninformed depositors who do not know the quality of the bank's investment still have the incentive to be patient and await the bank-specific information revealed from the informed depositors. This paper studies how the bank uses strategies to coordinate with the informed depositors and entrepreneur to stop a financial crisis due to the signal effect between informed and uninformed depositors.

The first-come, first-served rule imposed in the deposit contract can cause the negative payoff externality to the depositors who withdraw late, so the probability of bank runs is increased. This paper studies whether the bank's service to depositors on an equal basis increases the depositors' incentive to withdraw early. The model in this paper shows that because of the service rule on an equal basis, the more depositors who choose to withdraw late, the fewer payoffs they will obtain in the future. Therefore, this crowding effect makes the uninformed depositors have more incentive to trigger panic-based bank runs even in a healthy bank.

My paper is related to a number of papers in the literature. Diamond and Dybvig (1983) propose that banks use deposit insurance to avoid the inefficient bank runs. The technology in Diamond and Dybvig's model is riskless, so the bank can operate smoothly with a contract that is dominated by the deposit insurance. However, if the bank's portfolios are risky and the bank's selection is unobserved by depositors, the deposit insurance generates a moral hazard problem and increases the probability of bank runs. The reason is that, with the deposit insurance, the bank does not suffer the total loss from bank runs so it has strong incentive to promise the depositors an extremely high payment and invests deposits in the high-risk portfolios. My paper demonstrates that, in order to eliminate the bank's moral hazard problem and bank runs simultaneously, in a system without any deposit insurance, the bank renegotiates with the entrepreneur and depositors to prevent bank runs.

Chen (1999) proposes a deposit insurance system to avoid moral hazard in the banking system and to make depositors respond to more bank-specific information. In Chen's model, the informed depositors do not obtain any insurance; therefore, they have the incentive to monitor banks. But uninformed depositors have full insurance coverage since they have strong incentive to start a bank run. In contrast to Chen's model, my model shows that in a coordination scheme between the informed and uninformed depositors, uninformed depositors without any insurance have less inventive to trigger panic runs.

In my model, depositors have strategic behavior. Therefore, the depositors who have to withdraw early are able to obtain their payment from the renegotiation with the bank. This result is different from suspension convertibility in Diamond and Dybvig's paper. Suspension convertibility allows the banks to survive when bank runs happen because the banks are able to restrict the number of depositors who withdraw money early. However, this restriction creates a loss for the depositors who have needs to withdraw early but are unable to obtain the payments.

Diamond and Dybvig suppose that a deposit contract implies the first-come, first-served rule. Any depositor who withdraws early is able to receive the bank's payment before the bank is placed into receivership. But the negative payoff externality that results from the first-come, first-served rule makes bank runs happen when all depositors expect a bankruptcy. However, there are some debates about this assumption as well. Wallace (1988) demonstrates that this assumption is crucial in Diamond and Dybvig's model because, without this assumption, the model does not give an explanation of the banking system illiquidity. But Diamond and Dybvig does not clearly indicate in which environment the banks pay their depositors' withdrawals sequentially and why they do not use other payment rules. Calomiris and Kahn (1991) question, "Why in the case of banking (bankruptcy) should those who run the bank receive preferential treatment in liquidation states?"

The bank's payment rule in my model is the same as that in Allen and Gale's (1998) paper. They suppose that the bank's available liquidity is divided equally among the depositors who withdraw early. Similarly, the depositors who do not withdraw early share equally the bank's remaining assets that are available after those early withdrawals. In their model, this arrangement has the optimal risk-sharing allocation between the early and late consumers even in bank runs.

The rest of my paper is organized as follows. Section II introduces the model. Section III presents an optimal deposit contract when the quality of the bank's investment and the type of a depositor are unrealized. Section IV shows if the entrepreneur fails to pay the promised payment to the bank, how the bank renegotiates with the entrepreneur and depositors and when a bank run equilibrium occurs. Section V contains the conclusions.

II. Model

A. The project

The economy for this model has three dates (dates 0, 1, and 2). There is one bank, one entrepreneur and n depositors in the market. The entrepreneur needs the bank to make the investment with one dollar in his project at date 0. At date 1, the type of the project is realized and it is either a good or bad type. Suppose that θ is the type of the project and $\theta \in \Theta = \{g, b\}$. I assume that the probability of the project being good, p (g), is m.

Working the project, the entrepreneur earns either a high C_t^H or low cash flow C_t^L at date t, where t = 1, 2. For a good project, it is more likely to get a high cash flow relative to a bad project. I make the following assumptions about the probability distributions of the cash flow levels in different types of the project.

Assumption 1. $p(C_1^H \mid g) = \alpha_g$, $p(C_1^H \mid b) = \alpha_b$, and $\alpha_g > \alpha_b$.

Assumption 2. $p(C_2^H \mid C_1^H \text{and } g) = \beta_1$, $p(C_2^H \mid C_1^L \text{and } g) = \beta_2$,

$p(C_2^H \mid C_1^H \text{and } b) = \beta_3$, $p(C_2^H \mid C_1^L \text{and } b) = \beta_4$, and

$\beta_1 > \beta_2 > \beta_3 > \beta_4$.

Assumption 1 states that the conditional probabilities of a high cash flow of date 1 given a good or bad project is α_g or α_b. A good project has a higher probability to produce a high cash flow. Assumption 2 shows the conditional probabilities of a high cash flow of date 2 given the cash flow level at date 1 and project's type. If the project is a good type and its cash flow at date 1 is high, it has the highest probability to get a high cash flow at date 2. The probability of getting a high cash flow at date 2 is the lowest when the project is a bad type and its cash flow at date 1 is low.

At date 0, the type of the project is unknown to all agents, including the entrepreneur, and the public information is the probability distribution about the project's type and the conditional probability distributions about the cash flows. If the bank is willing to finance the project, it offers the entrepreneur a loan contract that specifies the entrepreneur's required payments of dates 1 and 2 to the bank. The required payment at date t, P_t (m, C_t^H, C_t^L,…), is contingent on m and other model parameters. I suppose that, at date 1, the level of cash flow is observable to the public, but the type of the project is the entrepreneur's private information.

B. The depositors

Each depositor has an endowment of $\frac{1}{n}$ dollar at date 0. Because the bank does not have any capital, in order to invest the entrepreneur's project, it issues a deposit contract to each depositor at date 0. In this deposit contract, if a depositor deposits $\frac{1}{n}$ dollar at date 0, he can withdraw the payment D_1 at date 1 or D_2 at date 2.

Suppose that a proportion $(1 - \lambda)$ of depositors face liquidity shocks at date 1. The depositors who suffer a liquidity shock at date 1 are called early consumers and they have to make consumption and withdraw money at date 1. The depositors who don't experience the liquidity shock at date 1 are called late consumers and they can wait to withdraw at date 2. At date 0, the type of each depositor is unknown to all agents and, at date 1, it is the depositors' private information. However, each depositor knows only his own consumer type but not that of other depositors.

C. Information revelation in renegotiation

When the entrepreneur defaults on the required payment P_1 at date 1, the bank offers a menu of collateral contracts, $L = \{L_1, \ldots, L_k\}$, to the entrepreneur, where $L_1 > 0$ and L_k is the highest collateral value that an entrepreneur with a good project is able to offer. Each collateral contract specifies a collateral value, L_s, and $L_s \in L = \{L_1, \ldots, L_k\}$. The bank infers the type of the project by observing how much of the collateral value the entrepreneur is willing to accept. This is a screening game.

To make the entrepreneur have the incentive to offer a collateral, the bank proposes any L_s less than P_t, $L_s \in L$. If the entrepreneur rejects any collateral contract, the bank liquidates the project at date 1. The liquidation value of a good project is X_g and that of a bad project is X_b. I suppose that a good project is liquidated with a higher value so $X_g > X_b$ and the liquidation value is collapsed to zero when the entrepreneur finishes the project at date 2. If the bank is willing to loan the entrepreneur P_1 + P_2 should be less than X_θ to make the entrepreneur accept the loan contract, where $\theta \in \Theta = \{g, b\}$.

When the entrepreneur defaults on P_1 and offers his collateral to the bank, the bank's available asset at date 1 is $C_1^L + L$. If $C_1^L + L > (1-\lambda)nD_1$, the bank is able to pay each early consumer's withdrawal so the late consumers will not start a bank run. If $C_1^L + L < (1-\lambda)nD_1$, the early consumers can not obtain the payment from the bank, the late consumers have strong incentive to withdraw early because of the fear of the bank's bankruptcy. From the following analysis, this model studies the case of $C_1^L + L < (1-\lambda)nD_1$.

In this model, the deposit contract is renegotiable at date 1. When the bank fails to pay D_1, in order to avoid the bankruptcy, the bank has to renegotiate a lower promised payment of date 1 with the depositors who claim to withdraw early. If the depositors reject the new offer, the bank has to liquidate the project. Therefore, the bank is bankrupt at date 1 and the payoff to the depositor who waits to withdraw at date 2 is 0. The depositors who claim to withdraw early are informed about the project's type by renegotiating with the bank. Thus, by observing the informed depositors' decision, the uninformed depositors infer the project's type and decide to withdraw at date 1 or date 2.

To analyze the model, I make Assumption 3.

Assumption 3. $X_b > \frac{1-\lambda}{\lambda} C_1^L$.

The time line of events in this model is in Table 1.

III. The Optimal Loan and Deposit Contracts

This section studies the optimal loan and deposit contracts to the bank, entrepreneur, and depositors when they have the symmetric information at date 0. The banking industry in this model is a monopolistic market; therefore, the bank is able to make a positive profit dealing with the entrepreneur and depositors.

Date 0
1. The bank decides whether to invest an entrepreneur's three-date project.
2. Depositors decide whether to deposit. If they deposit, the bank begins to invest the entrepreneur's project.
3. The quality of the project and types of depositors are unknown to all agents.
Date 1
1. A cash flow C_1 is realized and observable to all agents.
2. The entrepreneur pays P_1 to the bank. When the entrepreneur defaults on P_1, he has to offer a collateral to the bank or the project is liquidated.
3. The bank infers the project's type from the entrepreneur's choice about the collateral value.
4. The bank pays D_1 to the early consumers. When the bank defaults on D_1, it has to renegotiate with the depositors who claim an early withdrawal.
5. The depositors infer the type of the project by renegotiating with the bank about a revised payment.
6. The uninformed depositors infer the project's type from the informed depositor's actions.
7. Bank runs occur if all depositors withdraw.
Date 2
1. If the project is not liquidated at date 1, a cash flow C_2 is realized and the entrepreneur pays $P_{2.}$
2. If the entrepreneur fails to pay P_2, the project is liquidated.
3. If the bank is not bankrupt at date 1, the late consumers withdraw.
4. The project matures.

Table 1. Time Events

At date 0, before making offers to depositors, the bank negotiates with the entrepreneur about the entrepreneur's required payment based on the expected cash flow at each date. The required payment profile P = {P_1(m, α_θ, C_1^H, C_1^L), P_2(m, α_θ, β_j, C_2^H, C_2^L)} is the functions of m, C_t^H, C_t^L, α_θ, and β_j, where $\theta \in \Theta$ = {g, b} and j $\in$ {1, 2, 3, 4}. To the bank, the optimal entrepreneur's required payment of each date is that P_t equals the entrepreneur's expected cash flow at date t.[1] If the entrepreneur's cash flow at date t is low, the entrepreneur will default on P_t, which is shown in Lemma 1.

Lemma 1. *$P_t(m,..) < C_t^H$ and $P_t(m,..) > C_t^L$ if the following conditions hold.*

$$P_1 = p(g)E(C_1 \mid g) + p(b)E(C_1 \mid b) \tag{1}$$

$$P_2 = p(g)[\, p(C_1^H \mid g)\, E(C_2 \mid C_1^H \text{ and } g) + p(C_1^L \mid g)\, E(C_2 \mid C_1^L \text{ and } g)] + p(b)[\, p(C_1^H \mid b)\, E(C_2 \mid C_1^H \text{ and } b) + p(C_1^L \mid b)\, E(C_2 \mid C_1^L \text{ and } b)] \tag{2}$$

Proof: In the Appendix.

The bank will pay the fixed payments D_1 to each early consumer and D_2 to each late consumer. Given P_t, the bank chooses the optimal withdrawal D_t^* satisfying the following constraints:

$$(1-\lambda)nD_1 + \lambda nD_2 \le P_1 + P_2 \tag{3}$$

$$(1-\lambda)D_1 + \lambda D_2 \ge \frac{1}{n} \tag{4}$$

$$D_1 \le D_2 \tag{5}$$

[1] Consider a cash flow dependent contract { $P_t(C_t^H)$, $P_t(C_t^L)$ }. The loan contract in my model yields a (weak) higher bank's payoff than any cash flow dependant contract. The proof is in the Appendix.

The constraint (3) insures that D_t^* is feasible to the bank at each date. The constraint (4) is a participation constraint. It insures that each depositor's expected payoff is not less than his initial deposits. The constraint (5) is an incentive compatibility constraint which means that the late consumers have no incentive to withdraw early. In this model, the bank maximizes its profit so if D_1^* and D_2^* are the optimal payments, the constraint (4) is binding and the constraint (3) is not.

IV. The Renegotiation between the Bank and Entrepreneur

I assume that the entrepreneur obtains a low cash flow at date 1 and it is public information. Lemma 1 shows that the entrepreneur with the cash flow C_1^L will fail to pay the required payment of date 1, P_1^*. This section studies renegotiation with the entrepreneur defaulting on P_1^*; the bank will then be able to discern the quality of its investment.

Based on a low cash flow C_1^L, the bank and all depositors modify their belief about the quality of the bank's investment. By Baye's rule, the posterior probability is:

$$P(g \mid C_1^L) = \frac{(1-\alpha_g)m}{(1-\alpha_g)m + (1-\alpha_b)(1-m)} \tag{6}$$

Corollary 1 implies that if the project produces a low cash flow, the bank and depositors believe that this project is more likely to be a bad type.

COROLLARY 1. *The posterior probability, $p(g \mid C_1^L)$, is smaller than the prior probability, $p(g)$.*

Proof: In the Appendix.

Suppose the renegotiation between the bank and entrepreneur with a low cash flow is a screening game. After the level of a cash flow is realized at date 1, the bank offers a menu of collateral contracts, L = {L_1,…, L_k}, to the entrepreneur obtaining a low cash flow C_1^L. Regarding the bank's offer, the entrepreneur's strategy, $L_s(\theta)$, is to choose a collateral value $L_s \in L$ = {L_1,…, L_k} dependent on his project's type. I assume that the bank splits its available assets at date 1, C_1^L and L_s, to each early consumer equally so the collateral value influences the early consumer's payoff as well. Thus, regarding the renegotiation of the bank's alternative offer, an early consumer's strategy is $a_e(L_s) \in A_e$ = {rejects a lower payment, accepts a lower payment}.[2] Then, a late consumer observes the early consumer's action and determines whether to withdraw at date 1 or not. Let $a_l(a_e)$ denote a late consumer's strategy and $a_l(a_e) \in A_l$ = {withdraws deposits at date 1, withdraws deposits at date 2}. The entrepreneur's payoff of date 2 is $u(L_s, a_e, a_l, \theta)$, which is affected by depositors' decisions. $u(L_s, a_e, a_l, \theta)$ = 0 if a_e = rejects a lower payment or a_l = withdraws at date 1. $u(L_s, a_e, a_l, \theta)$ = $E(C_2 \mid \theta) - P_2$ if a_e = accepts a lower payment and a_l = withdraws at date 2.

[2] In this section, the early consumer's strategy, $a_e(L_s)$, depends on the collateral value. The collateral value will fully reveal the information about the project's type. Therefore, in the next section, $a_e(\theta)$ denotes the early consumer's strategy, where $\theta \in \Theta$ = {g, b}.

Therefore, the collateral value that the entrepreneur is willing to offer is determined by the type of the project and how depositors will respond to the bank's alternative offer. Definition 1 is an equilibrium in this screening game.

Definition 1. *An equilibrium of this screening game is that a menu of collateral contract,* $L = [L_1,\dots, L_k]$*, exists such that*

(i) the entrepreneur chooses his optimal collateral value, $L_s^*(\theta)$*, from L. To* $L_s^*(\theta)$*,*

$u(L_s^*, a_e(L_s^*), a_l(a_e), \theta) \geq u(\hat{L}_s, a_e(\hat{L}_s), a_l(a_e), \theta)$, $\hat{L}_s \in L$.

(ii) A new collateral contract does not exist that makes the bank obtain a higher profit .

The equilibrium is solved backward. Throughout this paper, it is given that the bank renegotiates with each early consumer about a lower promised payment $\frac{C_1^L + L_s}{(1-\lambda)n}$ compared with the original payment. If the early consumers reject this new payment, the bank has to liquidate the project. Because the liquidation makes the late consumers run to the bank, and the bank's available assets are split equally among all depositors, each depositor obtains $\frac{C_1^L + X_\theta}{n}$. Proposition 1 shows the properties of a separating equilibrium in this screening game.

Proposition 1. *Given that* L_b *is the highest collateral value that the entrepreneur with a bad project can offer,* $L_b \in L$*, and* $L_b = (1-\lambda)X_b - \lambda C_1^L$*. The separating equilibrium (* $L_s^*(b), L_s^*(g)$ *) is (* L_b, L_g*), where* $L_g \in L$*, and* L_g *is the lower bound of* *the values that are greater than* $(1-\lambda)X_g - \lambda C_1^{L}$[3].

Proof: In the Appendix.

The intuition of Proposition 1 is : the entrepreneur will suffer from a loss made by bank runs if his collateral value is less than $(1-\lambda)X_b - \lambda C_1^{L}$. It is indifferent to an early consumer to accept or reject the alternative payment if the collateral value equals $(1-\lambda)X_b - \lambda C_1^{L}$ so bank runs occur randomly. Therefore, the entrepreneur with a good project is willing to offer a higher collateral value because the early consumers have the strong incentive to accept the bank's new repayment. Because of the threat of bank runs, a pooling equilibrium does not exist in this model. The bank can be informed about the project's type from renegotiating with the entrepreneur about the collateral value.

V. The Renegotiation between the Bank and Depositors

Given collateral values in Proposition 1, the bank renegotiates with the early consumers about a lower repayment of date 1. Because the depositor is unable to recognize each other depositor's type, the question is : do some late consumers mimic the early consumers and withdraw immediately when information that the bank cannot pay D_1^* is revealed? In this model, the proportion of early consumers, $(1-\lambda)$, is public information such that the aggregate withdrawals at each date is observable. If

[3] By Assumption 3, L_g and L_b are positive.

some late consumers withdraw at date 1, the total number of the depositors who claim to withdraw early is greater than (1 − λ)n. Therefore, this aggregate withdrawal reveals information that some of the late consumers have run to the bank[4]. Then, because of a negative payoff externality[5], the others join the run as well. However, bank runs may create a loss to the late consumers if the project is good. So each late consumer has the incentive to wait until the early consumers' signal is revealed before making their withdrawal decision. Based on the above induction, the depositors renegotiating with the bank are all early consumers.

From Proposition 1, the bank with a good project is able to offer the early consumer the revised payment up to $\frac{C_1^L + L_g}{(1-\lambda)n}$. This bank has no incentive to make a repayment of date 1 less than $\frac{C_1^L + L_g}{(1-\lambda)n}$. If the bank owns a good project but offers a less payment $\frac{C_1^L + L_b}{(1-\lambda)n}$, the early consumers may reject the renegotiation. If the bank promises the payment between $\frac{C_1^L + L_b}{(1-\lambda)n}$ and $\frac{C_1^L + L_b}{(1-\lambda)n}$, the information about a good project is revealed to the early consumers but the early consumers prefer to liquidate the project since $\frac{C_1^L + X_g}{n}$ is larger than any value among $\frac{C_1^L + L_b}{(1-\lambda)n}$ and

4 Because the aggregate withdrawals of the early consumers is able to be computed in this model, there is no signal-extract problem as in Chari and Jagannathan's (1988) model even though each depositor does not know other depositors' types.

5 This model does not assume first-come, first-served rule as in Diamond and Dybvig's model. Thus, the negative payoff externality is caused by the liquidation of the project.

$\frac{C_1^L + L_g}{(1-\lambda)n}$. Because of the threat of bank runs, the bank does not lie about its investment type and the early consumers are able to learn the full information from renegotiating with the bank.

Regarding the bank's renegotiation, the early consumer's strategy is $a_e(\theta) \in$ A_e = {accepts a lower payment, rejects a lower payment} for each type θ. The late consumer has updated his belief about the project's type when the low cash flow C_1^L is realized. Therefore, before receiving the early consumer's signal, the late consumer's prior belief, $\tilde{p}$(g), is the same as (6). The late consumer determines his best withdrawal date by observing the early consumer's actions then updating his belief. A strategy for the late consumer is $a_l(a_e) \in$ A_l = {withdraws deposits at date 1, withdraws deposits at date 2} for the early consumer's each action. Suppose that $u_e(a_e, a_l, \theta)$ and $u_l(a_e, a_l, \theta)$are the early and late consumers' payoffs. Definition 2 states a perfect Bayesian equilibrium in this signaling game.

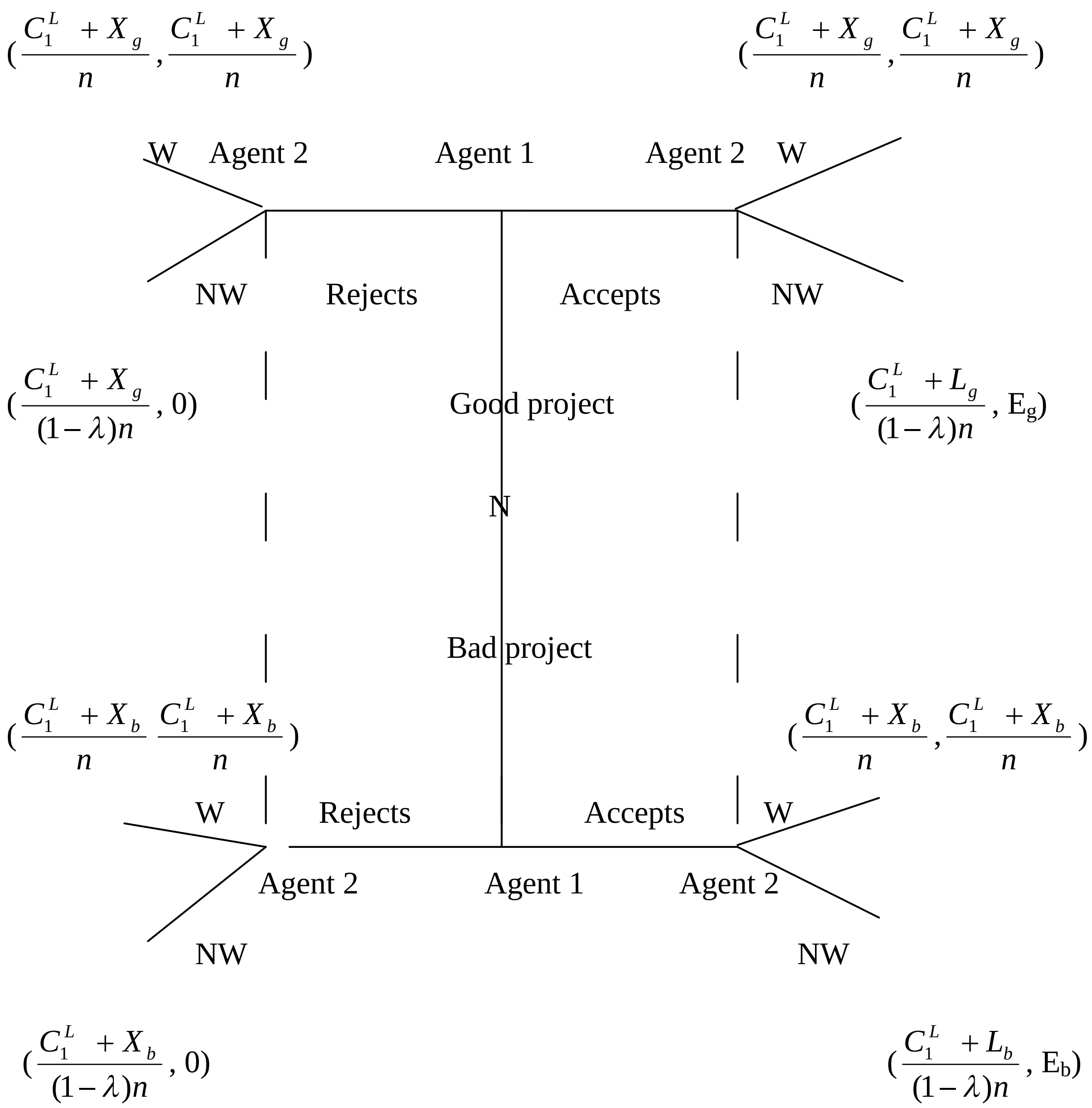
Definition 2. *A perfect Bayesian equilibrium of this signaling game is a strategy profile* (a_e^*, a_l^*) *and the late consumer's posterior beliefs* $\mu(\cdot \mid a_e)$ *such that:*

(i) $\forall\ \theta,\ a_e^*(\theta) \in \arg\max_{\tilde{a}_e} u_e(a_e, a_l^*, \theta)$,

(ii) $\forall\ a_e,\ a_l^*(a_e) \in \arg\max_{a_l} \sum_{\theta} \mu(\theta \mid a_e)\ u_l(a_e, a_l, \theta)$, *and*

(iii) $\mu(\theta \mid a_e) = \tilde{p}(\theta) a_e(\theta) \Big/ \sum_{\theta' \in \Theta} \tilde{p}(\theta')\ a_e(\theta')$.

Figure 1 illustrates the payoffs to the early and late consumer at each strategy profile. The dashed lines in Figure 1 represent that the late consumer can observe the early consumer's action but he is unable to distinguish the type of the project if the early consumer takes the same strategies regardless of the project's type.



N: Nature.

Agent 1: the early consumer. Agent 2: the late consumer.

W: withdraws at date 1. NW: withdraws at date 2.

$\mathrm{E_g} = \beta_2(\frac{P_2 - \varepsilon}{\lambda n}) + (1 - \beta_2)(\frac{C_2^L}{\lambda n}).$

$\mathrm{E_b} = \beta_4(\frac{P_2 - \varepsilon}{\lambda n}) + (1 - \beta_4)(\frac{C_2^L}{\lambda n})$

Figure 1

If the bank is not bankrupt at date 1, its promised payment at date 2 is $\frac{P_2 - \varepsilon}{\lambda n}$ or $\frac{C_2^L}{\lambda n}$, depending on the state of the cash flow at date 2. The reason that the bank has the incentive to renegotiate with the early consumers is that the bank is still able to make a positive profit ε. Thus, each late consumer is paid $\frac{P_2 - \varepsilon}{\lambda n}$, not $\frac{P_2}{\lambda n}$, at the state of a high cash flow. To avoid bank runs, $\frac{P_2 - \varepsilon}{\lambda n}$ should be greater than the payment from running to the bank, $\frac{C_1^L + X_\theta}{n}$. However, with Assumption 3, if the entrepreneur defaults again at date 2, the late consumer obtains $\frac{C_2^L}{\lambda n}$ that is less than $\frac{C_1^L + X_\theta}{n}$. Assume that $\hat{\beta}_2$ and $\hat{\beta}_4$ exist. $\forall \beta_2 > \hat{\beta}_2$, $E_g > \frac{C_1^L + X_g}{n}$ and $\forall \beta_4 < \hat{\beta}_4$, $E_b < \frac{C_1^L + X_b}{n}$. Thus, the late consumer will receive a loss if he withdraws at date 2 but the project is bad.

This model has perfect Bayesian semi-separating and pooling equilibria. Proposition 2 states the characteristics of a perfect Bayesian semi-separating equilibrium.

Proposition 2. *By an equilibrium (L_b, L_g) of Proposition 1, a perfect Bayesian semi-separating equilibrium is:*

(i) $a_e^*(g)$ *= accepts,* $\sigma_e^*(rejects \mid b) = \delta$*, and* $\sigma_e^*(accepts \mid b) = 1 - \delta$*, where* σ_e^* *is the early consumer's mixed strategy and* $0 < \delta < 1$*.*

(ii) $a_l^*(rejects)$ *= withdraws at date 1, and* $a_l^*(accepts)$ *= withdraws at date 2.*

(iii) $\mu(g \mid rejects) = 0$*,* $\mu(g \mid accepts) = \dfrac{(1-\alpha_g)m}{(1-\alpha_g)m+(1-\delta)(1-\alpha_b)(1-m)}$,

and

$$\mu(g \mid accepts)/\mu(b \mid accepts) > \frac{\lambda(C_1^L + X_b) - [\beta_4(P_2 - \varepsilon) + (1-\beta_4)C_2^L]}{[\beta_2(P_2 - \varepsilon) + (1-\beta_2)C_2^L] - \lambda(C_1^L + X_g)}.$$

.

Proof: In the Appendix.

In the semi-separating equilibrium, the early consumer's actions reveal more information about the project's quality. If the early consumer accepts the renegotiation, the late consumer is not fully informed about the project's type, but he infers that the project is more likely to be a good type. If the early consumer rejects the renegotiation, the late consumer knows that the project is a bad type. Therefore, the probability of the occurrence of a bank run is 1 as the early consumer rejects the bank's revised offer. When the early consumer accepts the offer, the condition of bank runs is

$$\frac{m(1-\alpha_g)}{(1-m)(1-\alpha_b)} < \frac{(1-\delta)\{\lambda(C_1^L + X_b) - [\beta_4(P_2 - \varepsilon) + (1-\beta_4)C_2^L]}{[\beta_2(P_2 - \varepsilon) + (1-\beta_2)C_2^L] - \lambda(C_1^L + X_g)} \qquad (7)$$

(7) is induced from Proposition 2(iii) and implies that for a late consumer, the ratio of his beliefs about a good project to that about a bad project is below a threshold value. Thus, the late consumer expects that the project is more likely to be bad.

Now I consider a perfect Bayesian pooling equilibrium in this model, which is illustrated in Proposition 3.

Proposition 3. *By an equilibrium (L_b, L_g) of Proposition 1*[6]*, a perfect Bayesian pooling equilibrium is:*

(i) $a_e^*(g)$ *= rejects, and* $a_e^*(b)$ *= rejects.*

---

[6] If the equilibrium value of the collateral is less than $(1-\lambda)X_b - \lambda C_1^L$, this model has perfect Bayesian separating and pooling equilibria. Moreover, in these two equilibria, the bank with a bad project will eventually be bankrupt. Therefore, the entrepreneur with a bad project has no incentive to renegotiate with the bank and the depositors have full information about the bank's investment.

(ii) a_l^* *(rejects) = withdraws at date 1, and* a_l^* *(accepts) = withdraws at date 1.*

(iii) $\mu(g \mid rejects) \in [0, 1]$, $\mu(g \mid accepts) = \dfrac{(1-\alpha_g)m}{(1-\alpha_g)m + (1-\alpha_b)(1-m)}$,

and

$$\mu(g \mid accepts) / \mu(b \mid accepts) < \frac{\lambda(C_1^L + X_b) - [\beta_4(P_2 - \varepsilon) + (1-\beta_4)C_2^L]}{[\beta_2(P_2 - \varepsilon) + (1-\beta_2)C_2^L] - \lambda(C_1^L + X_g)}.$$

.

Proof: In the Appendix.

Relative to the semi-separating equilibrium, in the pooling equilibrium, the late consumer does not obtain new information from the early consumer's signals. Suppose that the early consumer rejects the bank's revised payment regardless of the project type. Because the project has to be liquidated and the bank is bankrupt at date 1, the late consumer rushes to the bank rather than waiting to withdraw at date 2. Consider that the early consumer has the incentive to accept the new offer. From Proposition 3(iii), the condition of bank runs is

$$\frac{m(1-\alpha_g)}{(1-m)(1-\alpha_b)} < \frac{\lambda(C_1^L + X_b) - [\beta_4(P_2 - \varepsilon) + (1-\beta_4)C_2^L]}{[\beta_2(P_2 - \varepsilon) + (1-\beta_2)C_2^L] - \lambda(C_1^L + X_g)} \tag{8}$$

By (8), the late consumer gets a higher expected payoff from withdrawing early rather than waiting until date 2. Because of bank runs, accepting the renegotiation is not an optimal strategy to the early consumer. Thus, the early consumer prefers that the project is liquidated at date 1 even if the bank invests a good project.

Next, I will study how the probability of being a good project, m, and the proportion of the late consumers in the market, λ, affect the probability of bank runs. From Lemma 1, P_2 is a function of m. Thus, I can substitute P_2 in (8) to be the expected cash flow of date 2 and get $Wm^2 + Ym + Z < 0$. Based on Assumption 1, 2, and $E_b < \frac{C_1^L + X_b}{n}$, $W > 0$ and $Z < 0$ such that $\frac{-Y-\sqrt{Y^2-4WZ}}{2W} < m < \frac{-Y+\sqrt{Y^2-4WZ}}{2W}$. In the model, m is between 0 and 1. It is sure that $\frac{-Y-\sqrt{Y^2-4WZ}}{2W} < 0$. If $-Z-W < Y$, $\frac{-Y+\sqrt{Y^2-4WZ}}{2W} \leq 1$. The condition $-Z-W < Y$ implies $\lambda < \omega$. Proposition 4 states how λ influences the solutions of m. The values of W, Y, Z, and ω are shown in the Appendix.

Proposition 4. *For the solution of $Wm^2 + Ym + Z < 0$,*

(i) if $0 < \lambda < \omega$, $0 \leq m < \frac{-Y+\sqrt{Y^2-4WZ}}{2W}$.

(ii) if $\lambda \geq \omega$, $0 \leq m \leq 1$.

Proof: In the Appendix.

Proposition 4 implies that if the proportion of late consumers in the market is less than ω, a bank run will happen when the probability of being a good project is between 0 and $\frac{-Y+\sqrt{Y^2-4WZ}}{2W}$. If the proportion of late consumers in the market is increased and greater than ω, a bank run occurs in the bank even with a good project. The amount of the payment that each late consumer can withdraw at date 2 depends on the number of late consumers in the market. The promised payment at date 2 is decreased when many depositors are late consumers. Thus, the late consumer has more incentive to withdraw at date 1, which increases the probability of bank runs.

VI. Conclusion

This paper purposes a renegotiation among the bank, entrepreneur, and depositors to prevent bank runs. This mechanism does not induce the problem of the bank's moral hazard occurring with deposit insurance. The deposit contract is renegotiable when the bank has the financial problem of bankruptcy. However, the renegotiation between the bank and the informed depositors does not succeed and then bank runs happen if the bank is more likely to have bad investment. Therefore, the bank has more incentive to evaluate the quality of loans before it makes investment.

In this model, the decisions of the informed depositors having early withdrawal needs rely on how the uninformed depositors respond to them. Because the informed depositors have strategic behavior, they make their best decision and do not receive loss even if bank runs happen. Thus, the problem that some early consumers cannot withdraw any deposit does not exist in my model.

The results of this model can be applied in policy analysis. First, in this model, the depositors are also the investors of the entrepreneur's project. If the depositors withdraw their money from the bank before the project is matured, the entrepreneur will have a problem to obtain financial support. Therefore, a bank run is a threat to the entrepreneur as well. When the entrepreneur is unable to pay the required payment to the bank, the bank can acquire a higher penalty or collateral value from the entrepreneur and obtain more information about the quality of its investment from the entrepreneur's offer.

Second, the condition of bank runs in this paper makes the bank realizes how to revise their payments to depositors based on the market's variables when it has the financial problem of bankruptcy. The informed depositors know that their actions have the "signaling effect" to uninformed depositors' decisions. Because of this coordination scheme between the informed and uninformed depositors, using the excuse of bank runs, the bank is able to threaten the informed depositors to accept a lower payment if the bank defaults on the original promised payment. However, this threat is not successful if all the informed and uninformed depositors believe that the bank is more likely to loan a bad investment. The reason is that the uninformed depositors have a strong incentive to withdraw money early even though the informed depositors accept the bank's lower payment; therefore, the informed depositors tend to reject the revised offer.

Finally, in this model, the assumption about the rule of the banking payment to the depositors is different with the first-come, first-served rule in the Diamond and Dybvig model. In the literature, the first-come, first-served rule is one of the issues that causes bank runs and is also the subject of debate as to whether or not it is an optimal arrangement. This model assumes a service rule on an equal basis. Because of this assumption, the negative payoff externality that reults from the first-come, first-served rule and inducing bank runs does not exist and the depositors withdrawing early do not receive privilege when the bank has a financial crisis. However, the result of this model shows that if numerous depositors plan to withdraw late, the payment on an equal basis may lead to bank runs as well. With serving equally, the promised payment in a late withdrawal is close to that in an early withdrawal so the depositors have the strong incentive to withdraw early. Therefore, determining which rule of promised payment in the deposit contract is appropriate in which circumstance is worth more analysis in future research.

Appendix

Proof of Note 1

The loan contract of date 1 in my model is (9).

$$P_1(m, \alpha_g,..) = m\alpha_g C_1^H + m(1-\alpha_g)C_1^L + (1-m)\alpha_b C_1^H + (1-m)(1-\alpha_b)C_1^L \qquad (9)$$

where p(good type) = m, p(C_1^H | good type) = α_g, and p(C_1^H | bad type) = α_b.

The cash flow dependent contract is { $P_1(C_1^H)$, $P_1(C_1^L)$ }.

Case 1: If $P_1(C_1^H) \leq C_1^H$ and $P_1(C_1^L) \leq C_1^L$, the contract $P_1(m, \alpha_g,..)$ defined in (9) yields a weak higher payoff than the contract { $P_1(C_1^H)$, $P_1(C_1^L)$ }.

Proof: Bank's expected payoff by offering { $P_1(C_1^H)$, $P_1(C_1^L)$ } is :

$$\{m\alpha_g+(1-m)\alpha_b\} P_1(C_1^H) + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\} P_1(C_1^L). \qquad (10)$$

Because $P_1(C_1^H) \leq C_1^H$ and $P_1(C_1^L) \leq C_1^L$,

$$\{m\alpha_g+(1-m)\alpha_b\} P_1(C_1^H) + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\} P_1(C_1^L)$$

$$\leq \{m\alpha_g+(1-m)\alpha_b\} C_1^H + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\} C_1^L. \qquad (11)$$

Therefore, the contract $P_1(m, \alpha_g,..)$ yields a weak higher payoff than the contract { $P_1(C_1^H)$, $P_1(C_1^L)$ }

Case 2: If $P_1(C_1^H) \leq C_1^H$, $P_1(C_1^L) > C_1^L$ and the bank obtains C_1^L when the entrepreneur defaults, the contract $P_1(m, \alpha_g,..)$ defined in (9) yields a weak higher payoff than the contract { $P_1(C_1^H)$, $P_1(C_1^L)$ }.

Proof: Bank's expected payoff by offering { $P_1(C_1^H)$, $P_1(C_1^L)$ } is :

$$\{m\alpha_g+(1-m)\alpha_b\} P_1(C_1^H) + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\} C_1^L. \qquad (12)$$

Therefore,

$$\{m\alpha_g+(1-m)\alpha_b\}\,P_1(C_1^H) + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\}\,C_1^L$$

$$\leq \{m\alpha_g+(1-m)\alpha_b\}\,C_1^H + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\}\,C_1^L. \qquad (13)$$

The contract $P_1(m, \alpha_g,..)$ yields a weak higher payoff than the contract $\{P_1(C_1^H), P_1(C_1^L)\}$.

Case 3: Suppose that the conditions (14) and (15) hold. If $P_1(C_1^H) \leq C_1^H$, $P_1(C_1^L) > C_1^L$, and the bank obtains the liquidation value X_g or X_b when the entrepreneur defaults, the contract $\{P_1(C_1^H), P_1(C_1^L)\}$ yields a weak higher payoff than the contract $P_1(m, \alpha_g,..)$.

$$\alpha_g < \frac{X_g - C_1^L}{X_g - C_1^L + C_1^H - P_1(C_1^H)} \qquad (14)$$

$$m \geq \frac{\alpha_b(C_1^H - P(C_1^H) - (1-\alpha_b)(X_b - C_1^L)}{(1-\alpha_g)(X_g - C_1^L) - (1-\alpha_b)(X_b - C_1^L) - (\alpha_g - \alpha_b)(C_1^H - P(C_1^H)} \qquad (15)$$

Proof: Bank's expected payoff by offering $\{P_1(C_1^H), P_1(C_1^L)\}$ is :

$$\{m\alpha_g+(1-m)\alpha_b\}\,P_1(C_1^H) + m(1-\alpha_g)\,X_g + (1-m)(1-\alpha_b)\}\,X_b.$$

Given the conditions (14) and (15),

$$\{m\alpha_g+(1-m)\alpha_b\}\,P_1(C_1^H) + m(1-\alpha_g)\,X_g + (1-m)(1-\alpha_b)\}\,X_b$$

$$\geq m\alpha_g+(1-m)\alpha_b\}\,C_1^H + \{m(1-\alpha_g)+(1-m)(1-\alpha_b)\}\,C_1^L.$$

The contract $\{P_1(C_1^H), P_1(C_1^L)\}$ yields a weak higher payoff than the contract $P_1(m, \alpha_g,..)$.

In the case 1 and 2, the bank gets a weak higher payoff by offering the contract P_1(m , α_g,..). In the case 3, the bank may get a weak higher payoff by offering the { $P_1(C_1^H)$, $P_1(C_1^L)$ } if the bank is more likely to liquidate a good project. However, the loan market may become a "lemon market" where most loan applicants have bad projects. Therefore, I do not adopt a cash flow dependent contract, { $P_1(C_1^H)$, $P_1(C_1^L)$ }, in the model. ■

Proof of Lemma 1

In the model, $P_1^* = p(g)E(C_1 \mid g) + p(b)E(C_1 \mid b)$. By Assumption 1,

$$P_1^* = m[\alpha_g C_1^H + (1-\alpha_g)C_1^L] + (1-m)[\alpha_b C_1^H + (1-\alpha_b)C_1^L]. \qquad (16)$$

Because $C_1^H > C_1^L$,

$$P_1^* < m[\alpha_g C_1^H + (1-\alpha_g)C_1^H] + (1-m)[\alpha_b C_1^H + (1-\alpha_b)C_1^H]$$

$$= C_1^H, \qquad (17)$$

and $$P_1^* > m[\alpha_g C_1^L + (1-\alpha_g)C_1^L] + (1-m)[\alpha_b C_1^L + (1-\alpha_b)C_1^L]$$

$$= C_1^L. \qquad (18)$$

The proof for $P_2^* < C_2^H$ and $P_2^* > C_2^L$ is similarly. ■

Proof of Corollary 1

From Assumption 1, $\alpha_g > \alpha_b$ and $\frac{(1-\alpha_b)}{(1-\alpha_g)} > 1$; therefore,

$$p(g \mid C_1^*) = \frac{(1-\alpha_g)m}{(1-\alpha_g)m + (1-\alpha_b)(1-m)}$$

$$= \frac{1}{1+(\frac{1-\alpha_b}{1-\alpha_g})(\frac{1-m}{m})}$$

$$< \frac{1}{1+(\frac{1-m}{m})} = \text{m} = \text{p (g)}. \ \blacksquare \qquad (19)$$

Proof of Proposition 1

Suppose $L_s^*(b) = \tilde{L}_b < \tilde{L}_b$ is less than L_b. Then the early consumer obtains a revised payment $\frac{C_1^L + \tilde{L}_b}{(1-\lambda)n}$. Because $\frac{C_1^L + \tilde{L}_b}{(1-\lambda)n} < \frac{C_1^L + L_b}{(1-\lambda)n} = \frac{C_1^L + X_b}{n}$, the early consumer will reject the revised payment and then bank runs occur. Thus, offering $\tilde{L}_b$ is not an optimal strategy for the entrepreneur with a bad project.

Suppose that $L_s^*(g) = \tilde{L}_g$ and $L_b < \tilde{L}_g < L_g$. Then the early consumer obtains a revised payment $\frac{C_1^L + \tilde{L}_g}{(1-\lambda)n}$ and realizes the project's type because of $L_b < \tilde{L}_g$. L_g is the lower bound of the collateral values greater than $(1-\lambda)X_g - \lambda C_1^L$ so $\frac{C_1^L + \tilde{L}_g}{(1-\lambda)n}$ $< \frac{C_1^L + X_g}{n}$. The early consumer prefers to liquidate the project; therefore, offering $\tilde{L}_g$ is not an optimal strategy for the entrepreneur with a good project. $\blacksquare$

Proof of Proposition 2

If the early consumer's strategy in the equilibrium is $\{ a_e^*$ (g), (σ_e^* (rejects | b), σ_e^* (accepts | b))} = { accepts, (δ, $1-\delta$)}, by Bayes' Rule, the late consumer's beliefs are

μ(b | rejects) = 1, and

$$\mu\text{(g | accepts)} = \frac{(1-\alpha_g)m}{(1-\alpha_g)m+(1-\delta)(1-\alpha_b)(1-m)}. \quad (20)$$

With the belief μ(b | rejects) = 1 and a rejection from the early consumer, the late consumer's best response is withdrawing at date 1. Given the ratio of the belief μ (g | accepts) to μ (b | accepts) shown in Proposition 2(iii),

$$\mu\text{(g | accepts) } E_g + \mu\text{(b | accepts) } E_b$$

$$> \mu\text{(g | accepts)} \frac{C_1^L + X_g}{n} + \mu\text{(g | accepts)} \frac{C_1^L + X_b}{n} \quad (21)$$

(21) means that the late consumer's expected payoff from withdrawing at date 2 is greater than that from bank runs, Thus, if the early consumer accepts a revised payment, the late consumer's best response is withdrawing at date 2.

To determine whether a_e(g) = accepts is the best action to the early consumer, I suppose a_e(g) = rejects. The late consumer will run to the bank if the early consumer rejects the revised payment. Then, the early consumer can withdraw $\frac{C_1^L + X_g}{n}$, which is less than $\frac{C_1^L + L_g}{(1-\lambda)n}$ from accepting the renegotiation. Therefore, the strategy profile (a_e^*, a_l^*) and posterior beliefs $\mu(\cdot \mid a_e)$ shown in Proposition 2 is a perfect Bayesian semi-separating equilibrium. ■

Proof of Proposition 3

Suppose that the early consumer's strategy in the equilibrium is (a_e^*(g), a_e^*(b)) =

(rejects, rejects). For any value of the posterior belief μ(g | rejects), the late consumer's best response to the rejection from the early consumer is to withdraw at date 1 so the early consumer obtains $\frac{C_1^L + X_g}{n}$ from a good project. If the early consumer accepts the revised payment, given the ratio of the belief μ(g | accepts) to μ(b | accepts) shown in Proposition 3(iii), the late consumer obtains a higher expected payoff by withdrawing at date 1, which is (22). Thus, the late consumer runs to the bank when the early consumer accepts the renegotiation.

$$\mu(\text{g} \mid \text{accepts})\, E_g + \mu(\text{b} \mid \text{accepts})\, E_b$$

$$< \mu(\text{g} \mid \text{accepts}) \frac{C_1^L + X_g}{n} + \mu(\text{g} \mid \text{accepts}) \frac{C_1^L + X_b}{n} \tag{22}$$

Given the late consumer's response and a good project, the early consumer still obtains $\frac{C_1^L + X_g}{n}$. Therefore, the strategy profile (a_e^*, a_l^*) and posterior beliefs μ (· | a_e) shown in Proposition 3 is a perfect Bayesian pooling equilibrium. ■

Proof of Proposition 4

$$P_2 = m\{\alpha_g[\beta_1 C_2^H + (1-\beta_1)C_2^L] + (1-\alpha_g)[\beta_2 C_2^H + (1-\beta_2)C_2^L]\} + (1-m)\{\alpha_b[\beta_3 C_2^H + (1-\beta_3)C_2^L] + (1-\alpha_b)[\beta_4 C_2^H + (1-\beta_4)C_2^L]\}. \tag{23}$$

Substitute Equation (23) in (8) and I can get Inequality (24).

$$Wm^2 + Ym + Z < 0 \tag{24}$$

where

$W := (C_2^H - C_2^L)[\alpha_g\beta_1 + (1-\alpha_g)\beta_2 - \alpha_b\beta_3 - (1-\alpha_b)\beta_4][(1-\alpha_g)\beta_2$

$$-\ (1-\alpha_b)\beta_4] \tag{25}$$

$$Y := (1-\alpha_g)\{\beta_2[\alpha_b(\beta_3 C_2^H + (1-\beta_3)C_2^L) + (1-\alpha_b)(\beta_4 C_2^H + (1-\beta_4)C_2^L) - \varepsilon] + (1-\beta_2)C_2^L - \lambda(C_1^L + X_g)\} + (1-\alpha_b)\{\lambda(C_1^L + X_b) + \beta_4[(C_2^H - C_2^L)(\alpha_g\beta_1 + (1-\alpha_g)\beta_2 - \alpha_b\beta_3 - (1-\alpha_b)\beta_4) - \alpha_b(\beta_3 C_2^H + (1-\beta_3)C_2^L) - (1-\alpha_b)(\beta_4 C_2^H + (1-\beta_4)C_2^L) - \varepsilon] - (1-\beta_4)C_2^L\} \tag{26}$$

$$Z := \beta_4\{\alpha_b(\beta_3 C_2^H + (1-\beta_3)C_2^L) + (1-\alpha_b)(\beta_4 C_2^H + (1-\beta_4)C_2^L) - \varepsilon\} + (1-\beta_4)C_2^L - \lambda(C_1^L + X_b) \tag{27}$$

In (25), $C_2^H > C_2^L$,

$$\begin{aligned}
&\alpha_g\beta_1 + (1-\alpha_g)\beta_2 - \alpha_b\beta_3 - (1-\alpha_b)\beta_4 \\
&= \alpha_g(\beta_1 - \beta_2) - \alpha_b(\beta_3 - \beta_4) + (\beta_2 - \beta_4) \\
&> \alpha_g(\beta_1 - \beta_2) - \alpha_g(\beta_2 - \beta_4) + (\beta_2 - \beta_4) \\
&= \alpha_g(\beta_1 - \beta_2) + (1-\alpha_g)(\beta_2 - \beta_4) \\
&> 0.
\end{aligned} \tag{28}$$

And by Assumption 1 and 2,

$$\begin{aligned}
&(1-\alpha_g)\beta_2 - (1-\alpha_b)\beta_4 \\
&= p(C_1^L \mid g)\, p(C_2^H \mid C_1^L \text{and } g) - p(C_1^L \mid b)\, p(C_2^H \mid C_1^L \text{and } b) \\
&= p(C_2^H \text{ and } C_1^L \mid g) - p(C_2^H \text{ and } C_1^L \mid b) \\
&> 0.
\end{aligned} \tag{29}$$

So W > 0. To (27), because $E_b < \dfrac{C_1^L + X_b}{n}$,

$$\begin{aligned}
&\beta_4\{\alpha_b(\beta_3 C_2^H + (1-\beta_3)C_2^L) + (1-\alpha_b)(\beta_4 C_2^H + (1-\beta_4)C_2^L) - \varepsilon\} + (1-\beta_4)C_2^L - \lambda(C_1^L + X_b) \\
&< \beta_4(P_2^* - \varepsilon) + (1-\beta_4)C_2^L - \lambda(C_1^L + X_b) \\
&< 0.
\end{aligned} \tag{30}$$

With W > 0 and Z < 0, the solution of m in Inequality (24) is

$$\frac{-Y-\sqrt{Y^2-4WZ}}{2W} < m < \frac{-Y+\sqrt{Y^2-4WZ}}{2W} \quad (31)$$

$\frac{-Y-\sqrt{Y^2-4WZ}}{2W}$ is a negative solution and $\frac{-Y+\sqrt{Y^2-4WZ}}{2W}$ is positive solution. In the model, m is between 0 and 1. To prove $\frac{-Y+\sqrt{Y^2-4WZ}}{2W} < 1$, I need the condition, $-Z-W < Y$.

If $-Z-W < Y$,

$$4W(W+Y) + Y^2 > Y^2 - 4WZ$$

$$\Rightarrow \quad (2W+Y)^2 > Y^2 - 4WZ$$

$$\because \quad 2W + Y > -Z > 0$$

$$\therefore \quad 2W + Y > \sqrt{Y^2-4WZ}$$

$$\Rightarrow \quad \frac{-Y+\sqrt{Y^2-4WZ}}{2W} < 1.$$

$-Z-W < Y$ implies $\lambda < \omega$,

where $\omega =$

$$\frac{\{\alpha_b[\beta_3 C_2^H + (1-\beta_3)C_2^L] + (1-\alpha_b)[\beta_4 C_2^H + (1-\beta_4)C_2^L] - \varepsilon\}[\alpha_b\beta_4 + (1-\alpha_g)\beta_2] + C_2^L[\alpha_b(1-\beta_4) + (1-\alpha_g)(1-\beta_2)] + (1-\alpha_g)\beta_2(C_2^H - C_2^L)[\alpha_g\beta_1 + (1-\alpha_g)\beta_2 - \alpha_b\beta_3 - (1-\alpha_b)\beta_4]}{(1-\alpha_g)(C_1^L + X_g) + \alpha_b(C_1^L + X_b)} \quad (32)$$

Therefore, if $\lambda < \omega$, $\frac{-Y+\sqrt{Y^2-4WZ}}{2W} < 1$ and the solution of m is between 0 and $\frac{-Y+\sqrt{Y^2-4WZ}}{2W}$. ■

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