Uncertainty and the Dynamics of R&D*

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Uncertainty about future productivity and demand conditions appears to vary strongly over time, rising by 50% to 100% during recessions, and by 100% to 200% after major political and economic shocks.[1] These fluctuations in uncertainty appear to generate fluctuations in investment, hiring and productivity as higher uncertainty generates a temporary slowdown and bounceback as firms postpone activity and wait for uncertainty to resolve (Bloom, 2006).

An omitted factor from this analysis, however, is R&D which may respond differently from investment to uncertainty because of different adjustment costs. Investment in the capital stock typically incurs *stock* adjustment costs from changing the capital stock, while R&D investment in the knowledge stock typically incur *flow* adjustment costs from changing the flow rate of the knowledge stock (see section 1). I show that these different adjustment costs lead to different predicted dynamics under uncertainty. R&D rates should have a much greater persistence over time, especially at higher uncertainty. When R&D levels are costly to change and uncertainty rises firms will become more cautious in responding to changing business conditions. The response of R&D to uncertainty will also differ from investment, being dependent on both lagged R&D levels and as well as current business conditions.

These uncertainty effects can help to explain the high persistence of R&D across time, which at the firm-level is about three times more autocorrelated than investment. They may

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[1]Schwert (1989) shows that measures of uncertainty over future industrial production, stock prices and bond prices fluctuate over the business cycle, increasing by 50% to 100% in recessions. Bloom (2006) shows stock-market volatility jumps 100% to 200% after economic and political major shocks like the Cuban Missile crisis, the assassination of JFK and the 9/11 attack.

also help to explain why across-business cycles R&D is highly persistent and responds to recessions with a lag (Barlevy, 2007). The higher uncertainty in downturns will reduce the responsiveness of R&D, delaying its response to worsening business conditions. Finally the results imply firms will be much less responsive to technology policies during periods of high uncertainty, for example if the policy change itself increases uncertainty.

1. Time varying uncertainty with stock and flow adjustment costs

The traditional real options models assumed time constant uncertainty in order to derive analytical solutions.[2] They assumed some driving process, for example price (P), evolved as a Geometric Brownian motion with a constant drift μ and constant volatility σ

$$dP_t = P_t(\mu + \sigma dW_t) \qquad \text{where } dW_t \sim N(0,1) \qquad (1.1)$$

Since volatility (σ) is fixed investigating the impact of time-varying fluctuations in uncertainty is not possible in these models.

A small literature has tried to extend these models to incorporate time varying uncertainty, finding temporary increases in uncertainty cause a drop and rebound in investment, employment and productivity growth due to a "delay-effect". At high levels of uncertainty firms postpone making decisions so aggregate investment and employment activity slows down. Productivity growth also slows down as reallocation of factors of production from low to high productivity firms slows.[3] Higher uncertainty also induces a "caution effect" whereby

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[2]See, for example, MacDonald and Siegel (1986), Dixit and Pindyck (1994), Bertola and Caballero (1994) or Abel and Eberly (1996).

[3]See Bernanke 1983 and Hassler 1996 for a single agent single factor model, and Bloom 2006 for a micro to macro model with capital, labor and productivity and empirical evidence.

firms are less responsive to any given shock because higher uncertainty increases the chances of making a costly mistake, so responsiveness is lower (Bloom, Bond and Van Reenen, 2007).

These extensions, however, have yet to examine the impact of time varying uncertainty on R&D and the knowledge stock. In the productivity and innovation literature the knowledge stock (G_t) is usually modelled as the accumulation of R&D expenditures (R_t) over time, in a similar way that capital stocks (K_t) are modelled as the accumulation of investment expenditures (I_t) over time (Grilliches, 1996):

$$K_{t+1} = (1 - \delta_K)K_t + I_t \tag{1.2}$$

$$G_{t+1} = (1 - \delta_G)G_t + R_t \tag{1.3}$$

Although uncertainty and real-options are not modelled, one could speculate that R&D will be affected in the same way by uncertainty as investment.[4] But, this turns out not to be true due to the different adjustment costs for capital and knowledge stocks.

Capital stock adjustment costs are typically assumed to arise from direct changes to their stocks, for example from resale losses for capital goods. This can be written as

$$C_K(I_t) \approx C_K(\Delta K_t) \tag{1.4}$$

Knowledge stocks, however, are intangible and can not typically be bought or sold.[5] Instead, knowledge stocks are adjusted (more slowly) by changing the level of R&D, which changes the growth rate of the knowledge stock. The adjustment costs for R&D - for example resale losses on R&D equipment or scientists hiring/firing costs - are similar, however, to the adjustment

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[4]There is a literature looking at "R&D real-options", which uses stochastic calculus to value complex multi-stage R&D projects, but these assume no R&D adjustment costs and no change in uncertainty over time, so are focused on R&D valuations rather than dynamics (see for example, Schwartz, 2003).

[5]Patents are one exception to this, although these cover a small fraction of R&D as they are only available on innovative codified knowledge, typically with a few years lag due to delays in the patenting process.

costs for capital in that they depend on the change in R&D levels. Given the law of motion

for the knowledge stock (1.3) this implies

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\begin{aligned}
C_G(\Delta R_t) &= C_G(\Delta\Delta G_t - \delta_G \Delta G_t) \quad (1.5)\\
&\approx C_G(\Delta\Delta G_t)
\end{aligned}
$$

Comparing (1.4) to (1.5) the adjustment costs for the knowledge stock are one order of

difference apart from the adjustment cost for the capital stock. This distinction arises because

the costs of adjustment for capital arises directly from changing its stock. The costs of

changing knowledge stocks arise not from changing its stock, but from changing the rate of

change of its stock (R&D). Thus, adjustment costs arise in changing the *level* of the capital

stock and changing the *flow* rate of the knowledge stock.[6] This distinction plays a critical

role in shaping the response of investment and R&D to uncertainty.

2. Simulation results for R&D and uncertainty

I first present a set of simulation results to provide some initial intuition, and then present

the full model of R&D used to generate the simulations in section (4).

Figure 1 plots the optimal rates of R&D as a function of current business conditions for

low uncertainty ($\sigma_t = 5\%$), medium uncertainty ($\sigma_t = 20\%$) and high uncertainty ($\sigma_t = 50\%$).

There are two key results from the simulation.

First, the adjustment costs for changing R&D generates a zone of inaction in the response

of R&D to changes in business (demand and productivity) conditions. Given the costs of

changing R&D rates firms only incur this when the gap between the actual and desired R&D

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[6]Interestingly, Christiano, Eichenbaum and Evans (2005) assume a *flow* cost for changing investment rates
between periods, $C_K(\Delta I_t)$, so that my results for R&D would extend to capital under their assumptions.

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rate is above a certain threshold, generating a central region of inaction. This creates a dynamic link between current and past R&D rates, consistent with the empirical evidence that R&D rates change only slowly over time, and are more persistent then sales growth, employment growth or investment rates.[7]

Second, the zone of inaction is larger for higher values of uncertainty, and the response is more muted when it does occur. This is the "caution effect" of uncertainty on R&D behavior. When uncertainty is high the probability of business conditions changing are greater, and since it is costly to change R&D rate the option value of waiting is greater.

Figures 2a and 2b plot the optimal rates of R&D expenditures at low, medium and high uncertainty for low prior values of R&D and high prior values of R&D.[8] The key result is that the direct impact of uncertainty depends on the difference between optimal R&D and lagged R&D. If optimal R&D is higher than lagged R&D (the right side of both figures) - so that firms want to raise R&D - then higher uncertainty reduces R&D, a negative "delay-effect". If optimal R&D is below lagged R&D (the left side of both figures) - so the firms want to cut R&D - then higher uncertainty increases R&D, a positive "delay-effect". Thus, the impact of the "delay effect" depends on the relationship between desired R&D and lagged R&D.[9]

Figure 3 plots the optimal rates of R&D expenditure for low, medium and high uncertainty assuming only quadratic adjustment costs for R&D. The effects of uncertainty almost

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[7]For example, in Compustat data (1990 to 1999, all manufacturing) the correlation between current and two-year lagged sales growth rates are 0.082, labor growth rates are 0.095, investment rates are 0.274, and R&D rates are 0.690. The aggregate figures show a similar patter, with Comin and Gertler (2006) reporting annual autocorrelations of 0.65 for output, 0.65 for hours, 0.76 for investment and 0.83 for R&D.

[8]These low and high values are 1.875% and 7.5%, chosen as half and twice the steady-state rate of R&D expenditure, $r_t = \frac{R_t}{G_t}$, noting that with 15% depreciation in G_t steady state implies quarterly $r_t = 3.75\%$.

[9]Note that if R&D labor and equipment depreciate over time (due to scientists quitting and equipment wearing out for example), and the adjustment costs are in gross changes in R&D then the effects of temporary increases in uncertainty on R&D can be signed as negative at the steady state since the inherited level of R&D will always have depreciated below the optimal level of R&D.

completely disappear. With quadratic adjustment costs no real options effects arise, and the assumed homogeneity on revenue function in demand conditions and knowledge stocks minimizes any Jensen's effects from a concave/convex marginal revenue product of R&D in demand conditions. Hence, the impact of uncertainty on R&D also depends on the adjustment costs for R&D.

3. Implications of uncertainty for micro and macro R&D

At the micro level the "caution-effect" of uncertainty on R&D implies much lower responsiveness of firms in periods of high uncertainty. This could be investigated by estimating (with appropriate instrumentation) the following type of regression[10]

$$r_{i,t} = \alpha_0 + \beta_1 r_{i,t-1} + \beta_2 \Delta y_{i,t} + \beta_3 \sigma_{i,t} + \beta_4 r_{i,t-1}\sigma_{i,t} + \beta_5 y_{i,t}\sigma_{i,t} + X_{i,t} + \varepsilon_{i,t}$$

where $r_{i,t}$ is firm i year t (R&D/sales), $y_{i,t}$ is firm i year t log(sales) and $X_{i,t}$ are a full set of controls including fixed effects and year dummies. The empirical implication from section (2) for R&D is that higher uncertainty should reduce the responsiveness of firms to sales growth ($\beta_5 < 0$) and increase the responsiveness to lagged R&D expenditure ($\beta_4 < 0$).

At the macro level the "delay-effect" of uncertainty on R&D is highlighted in table 1 below, with uncertainty effects on investment in table 2 for comparison. The two columns in Table 1 reflect the result that higher uncertainty increases R&D if the current period R&D is a downward adjustment ($R_t < R_{t-1}$), and reduces R&D if the current period R&D is an upward adjustment ($R_t > R_{t-1}$). Looking across the two rows shows that after controlling for

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[10]Micro data is particularly suitable for testing the "caution effect" of uncertainty because of the large samples of impulses and responses in firm panel data. Macro data is particularly suitable for testing the "delay-effect" of uncertainty because of the role of re-allocation across firms in driving the productivity component of the "delay-effect" which only arises under aggregation.

the change in R&D the current knowledge stock has no effect on the response to uncertainty

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Table 1: The marginal impact of an increase in uncertainty on R&D

	R&D decreasing*	R&D increasing
Knowledge stock decreasing	+	−
Knowledge stock increasing	+	−

* If R&D rates depreciate at rate δ_R, then the condition is $R_t < (1 - \delta_R)R_{t-1}$

In contrast Table 2 shows that lagged investment plays no role in determining current investment. Instead comparing across the two rows shows that uncertainty increases current investment if the capital stock is decreasing after controlling for depreciation ($K_t < (1 - \delta_K)K_{t-1}$), and reduces it if capital stock is increasing ($K_t > K_{t-1}$).

Table 2: The marginal impact of an increase in uncertainty on investment

	Investment decreasing	Investment increasing
Capital stock decreasing*	+	+
Capital stock increasing	−	−

* After controlling for depreciation, $K_t < (1 - \delta_K)K_{t-1}$

Thus, while higher uncertainty generates a lower responsiveness in the *rate of change* of the knowledge stock to demand conditions, it generates a lower responsiveness in the *stock* of capital to demand conditions. Hence, at a macro-level this implies uncertainty would reduce [increase] investment and hiring when capital and labor are below [above] their equilibrium levels, and reduce [increase] knowledge capital growth when R&D is below [above] its equilibrium rate.

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4. A brief model of R&D and uncertainty

The model underlying the simulations assumes firms have a revenue function $F(X, K, L, G) = X^\phi K^{\alpha(1-\epsilon)} L^{\beta(1-\epsilon)} G^{(1-\alpha-\beta)(1-\epsilon)}$ which nests a Cobb-Douglas production function in capital (K), labor (L) and the knowledge stock (G), and an iso-elastic demand curve with elasticity (ϵ). Demand and productivity conditions are combined into an index (X), henceforth called "business conditions".

Business conditions evolve as an augmented geometric random walk with mean μ and variance σ_t

$$dX_t = X_t(\mu + \sigma_t dZ_t) \qquad \text{where } dZ_t \sim N(0,1) \tag{4.1}$$

The uncertainty process (σ_t) is modelled for simplicity as an AR(1) process, noting this could easily be generalized

$$\sigma_t = \sigma_{t-1} + \rho_\sigma(\sigma^* - \sigma_{t-1}) + \sigma_S S_t \qquad \text{where } dS_t \sim N(0,1) \tag{4.2}$$

In the model for analytical tractability I assume the adjustment costs for capital and labor are zero and focus on the implications of R&D adjustment costs. This should not change the stylized results for R&D[11], but facilitates a numerical solution to the model since the state and control spaces are both reduced by two dimensions.

There are no structural estimates of R&D adjustment costs in the literature. But there is a long literature on capital and labor adjustment costs which I use as a starting point for modelling R&D adjustment costs[12]. This literature focuses on three cost terms - linear costs reflecting per unit adjustment costs: $C(\Delta R_t) = \Delta R_t * (\kappa_+[\Delta R_t > 0] - \kappa_-[\Delta R_t < 0])$,

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[11] As noted in Bloom (2006) in a Cobb-Douglas production function with iso-elastic demand each factor responds most to its own adjustment costs, with limited cross-factor response.

[12] See Cooper and Haltiwanger (2007) or Bloom (2006).

quadratic adjustment costs reflecting higher costs of rapid changes: $C(\Delta R_t) = \kappa_Q G_t (\Delta \frac{R_t}{G_t})^2$, and fixed costs reflecting the revenue loss from disruption involved in changing factors of production: $C(\Delta R_t) = \kappa_F * F(X, K, L, G)_t (\Delta R_t \neq 0)$. I report results for linear and quadratic R&D adjustment costs, but also investigated a range of other adjustment costs and found real options effects whenever linear or fixed costs were present.

With fully flexible capital and labor I can optimize these out, and then normalize the business conditions process, to derive the concentrated revenue function $\widetilde{F}(Y, G) = AY^{1-\theta}G^{\theta}$.[13] The Bellman equation can then be stated as follows

$$V(Y_t, G_t, R_{t-1}, \sigma_t) = \max_{R_t} \widetilde{F}(Y_t, G_t) - C(R_t, R_{t-1}) - wR_t + \frac{1}{1+r} E[V(Y_{t+1}, G_{t+1}, R_t, \sigma_{t+1})]$$

where w is the cost of R&D and r is the discount rate. The problem is jointly homogeneous of degree one in (Y_t, G_t, R_t, R_{t-1}) so can be normalized by G_t

$$V(y_t, 1, r_{t-1}, \sigma_t) = \max_{r_t} \widetilde{F}(y_t, 1) - C(r_t, r_{t-1}) - wr_t + \frac{(1-\delta_K)(1-r_t)}{1+r} E[V(y_{t+1}, 1, r_t, \sigma_{t+1})]$$

where $y_t = (Y_t/G_t)$ and $r_t = (R_t/G_t)$. Analytical results can show a unique solution to the firm's optimization problem exists, with numerical methods used to solve for exact values.[14]

5. Conclusions

Uncertainty varies strongly over time, persistently rising by 50% to 100% during recessions, and temporarily rising by 100% to 200% after major political and economic shocks. The

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[13] Where $\varphi = \frac{\phi}{1-\alpha(1-\epsilon)-\beta(1-\epsilon)}$, $\theta = \frac{(1-\alpha-\beta)(1-\epsilon)}{1-\alpha(1-\epsilon)-\beta(1-\epsilon)}$, A is a constant and $Y^{1-\theta} = X^{\varphi}$.

[14] The simulations are run quarterly with the following set of parameter values (at annualized rates): $\theta = 0.31$, $\mu = 0.02$, $\rho_\sigma = 0.9$, $\sigma^* = 20$, $w = 25$, and $\delta_G = 0.15$, for the linear adjustment cost specifications $\kappa_+ = 25$ and $\kappa_- = 25$, and for the quadratic adjustment cost specifications $\kappa_2 = 1000$. In experiments I ran the results appear broadly robust to modifications to the assumped parameters. The full code is available on http://www.stanford.edu/~nbloom.

impact of these changes in uncertainty on investment and labor-demand appears to be two-fold: first, higher uncertainty makes firms less responsive to any changes in their environment, a "caution effect"; and second, higher uncertainty typically reduces aggregate investment, hiring and productivity growth due to a "delay effect". These effects have been shown to be both analytically and empirically important in micro and macro data.

This paper extends these results on time varying uncertainty to R&D by modelling the *flow* adjustment costs of knowledge stocks and contrasting this to the *stock* adjustment costs of capital and labor. I show that higher uncertainty reduces the responsiveness of R&D to changes in demand conditions and increases the persistence of R&D over-time, the R&D equivalent to the "caution-effect". I also show that if firms are increasing R&D then the marginal effect of uncertainty on R&D will be negative, while if firms are reducing R&D then the marginal effect of uncertainty on R&D will be positive. Thus, the R&D equivalent to the "delay-effect" depends on the desired change in R&D. I then present micro and macro predictions, with the hope that future empirical research will make progress in testing these.

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Figure 1: Higher uncertainty makes R&D less responsive to current business conditions and more persistent over time
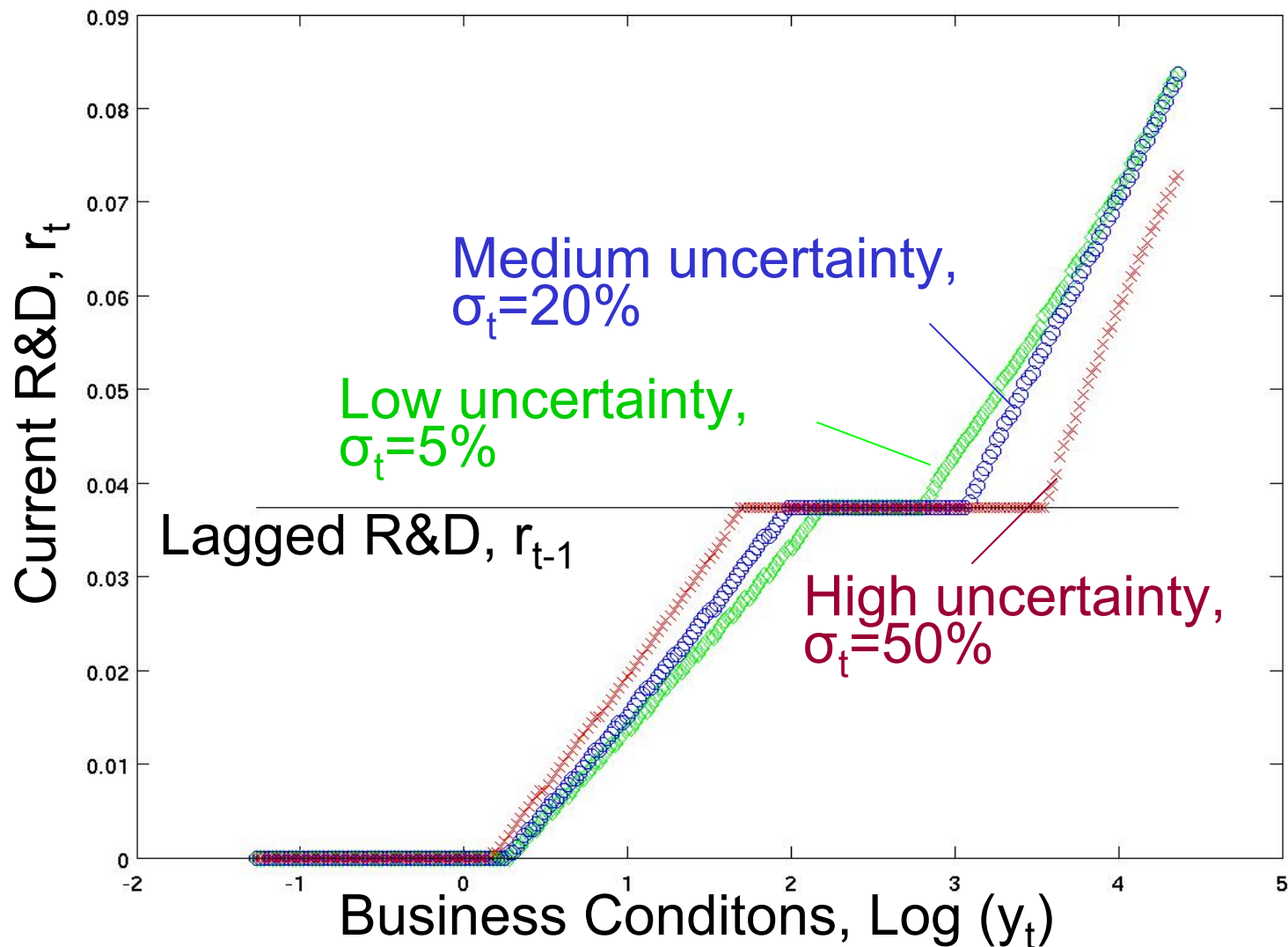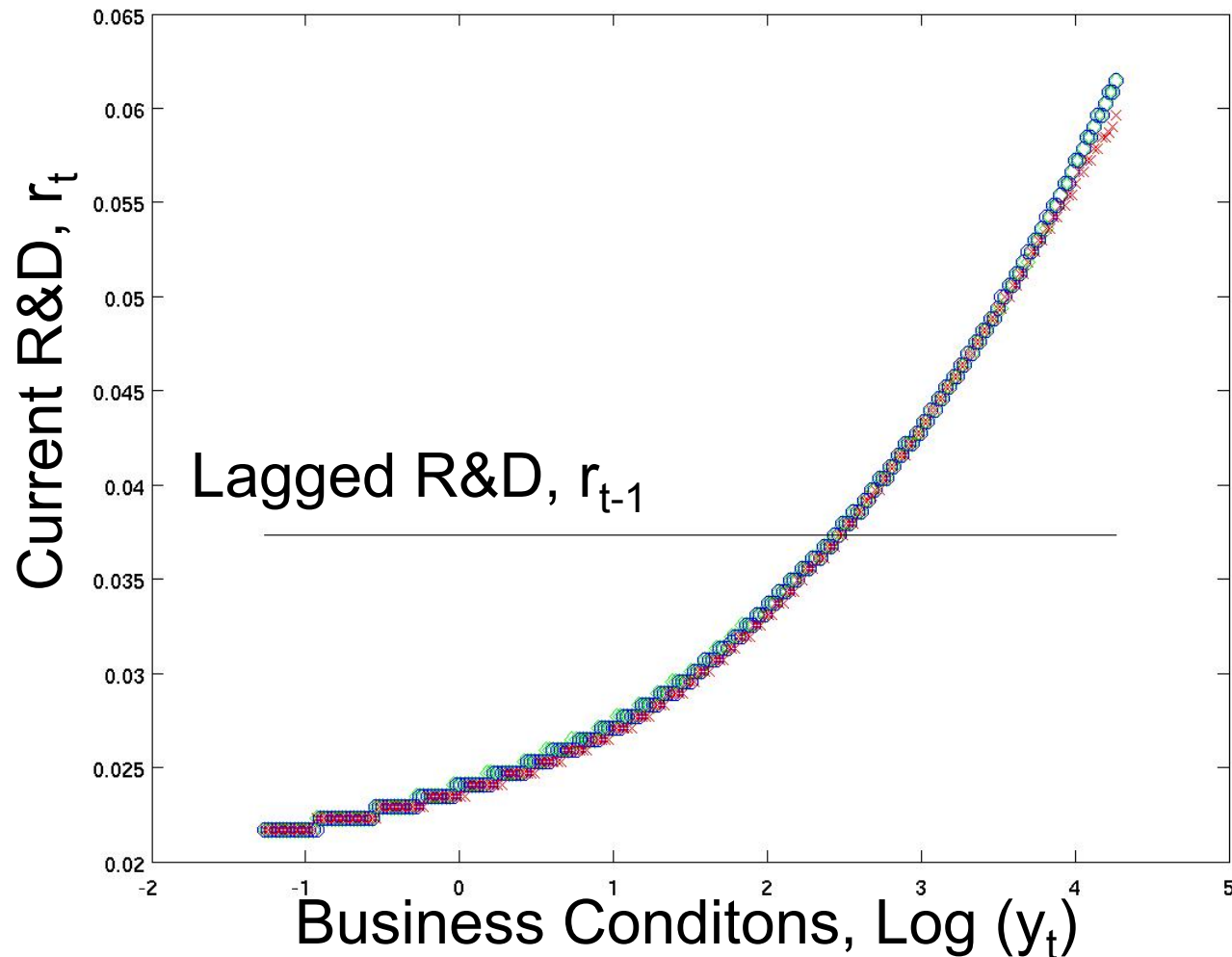

Plots the numerical solution to the firm's optimisation problem. The control variable, current R&D (r_t), is a function of the three state variables: lagged R&D (r_t), current business conditions (y_t), and the uncertainty over future business conditions (σ_t).

Figures 2a and 2b: The effect of uncertainty on R&D is negative if R&D is increasing, and positive if R&D is falling
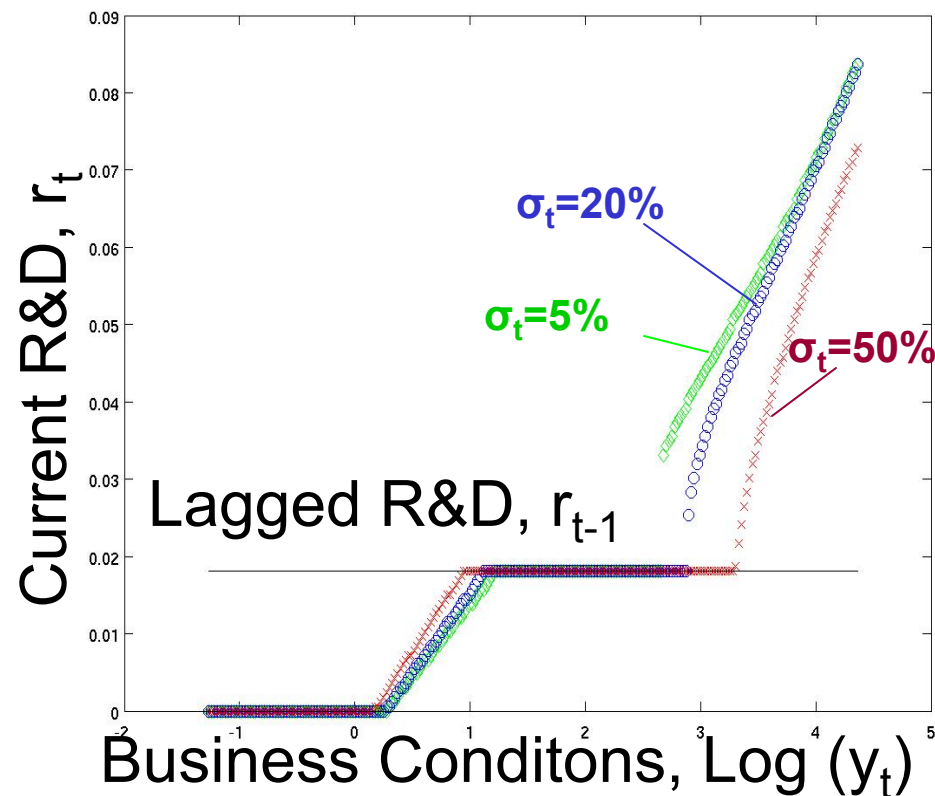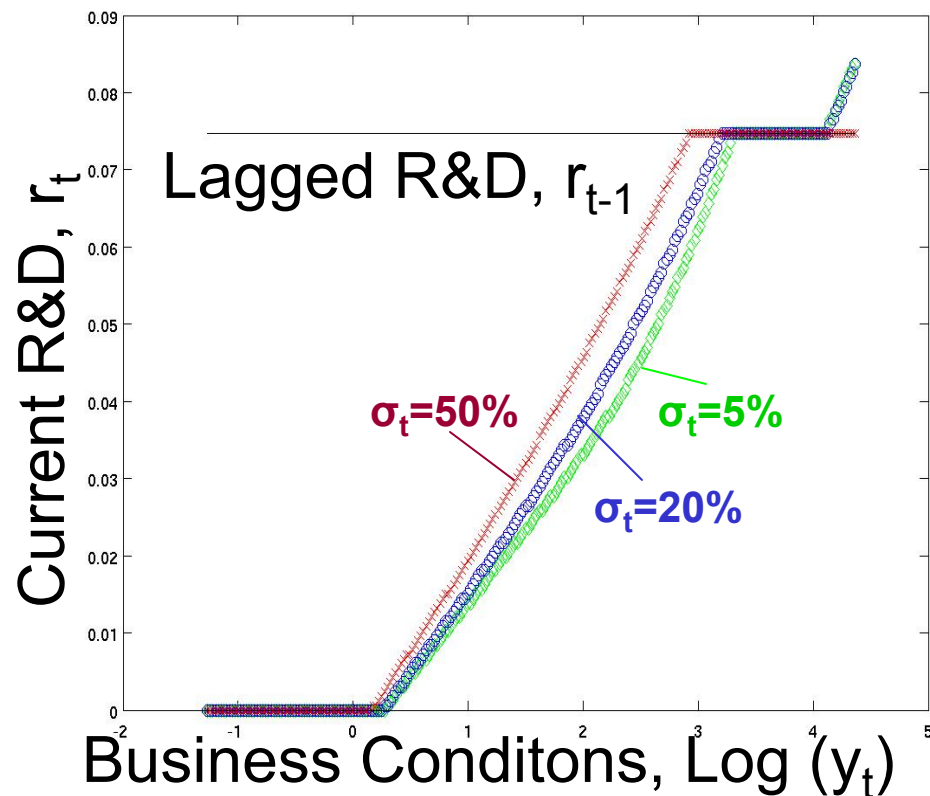
Plots the numerical solution to the firm's optimisation problem. The control variable, current R&D (r_t), is a function of the three state variables: lagged R&D (r_t), current business conditions (y_t), and the uncertainty over future business conditions (σ_t).

Figure 3: With only quadratic adjustment costs there are no real options effects of uncertainty on R&D

Plots the numerical solution to the firm's optimisation problem. The control variable, current R&D (r_t), is a function of the three state variables: lagged R&D (r_t), current business conditions (y_t), and the uncertainty over future business conditions (σ_t).