

Optimal Inattention to the Stock Market

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Inattentive agents update their information sporadically, rather than continuously, and thus respond belatedly to news. We generate optimally inattentive behavior by assuming that to observe the value of his investment portfolio the consumer must pay a cost that is proportional to the portfolio's contemporaneous value. It is optimal for the consumer to check his investment portfolio at equally spaced points in time, consuming from a riskless transactions account in the interim. The riskless transactions account that finances consumption guarantees that funds are never unwittingly exhausted.¹ We show that the optimal interval of time between consecutive observations of the value of the portfolio is the unique positive solution to a nonlinear equation. Quantitatively, even a small observation cost (one basis point of wealth) implies a substantial (8 month) decision interval under conventional parameter values.

Darrell Duffie and Tong-sheng Sun (1990) analyze a consumption and portfolio problem with transactions costs that nest our formulation of transactions costs. However, we assume that the investment portfolio of riskless bonds and risky stocks is managed by a portfolio manager who continuously rebalances the portfolio, whereas Duffie and Sun assume that interest payments are reinvested in bonds and dividends are reinvested in equity during periods of inattention. The assumption of continuous rebalancing simplifies the solution considerably and enables us to characterize the optimal inattention span as the unique positive root of a nonlinear equation. Xavier Gabaix and David I. Laibson (2002) analyze a model with inattention and continuous rebalancing of the investment portfolio, but they specify the observation cost to be constant in terms of utility,² which prevents them from being able to solve the consumer's optimization exactly. They approximate the consumer's objective function and derive the (approximately) optimal interval of time between observations.

I The Consumer's Optimization Problem

The consumer maximizes

$$E_t \left\{ \int_0^\infty \frac{1}{1-\alpha} c_{t+s}^{1-\alpha} e^{-\rho s} ds \right\}, \quad (1)$$

where $0 < \alpha \neq 1$ and $\rho > 0$.

The consumer's wealth is held in an investment portfolio and in a riskless liquid asset used for transactions. The investment portfolio holds a riskless bond with rate of return $r \geq 0$ and a non-dividend-paying stock with price P_t that follows a geometric Brownian motion

$$\frac{dP_t}{P_t} = \mu dt + \sigma dz, \quad (2)$$

where $\mu > r$ and $\sigma > 0$. The consumer can observe the value of the investment portfolio only by paying a fraction θ , $0 \leq \theta < 1$, of the contemporaneous value of the investment portfolio. The consumer can withdraw funds from the investment portfolio only at times when the value of this portfolio is observed.

In addition to the investment portfolio, the consumer holds a riskless liquid asset, which pays a rate of return $0 \leq r^L < r$, to finance consumption. We assume that r^L is lower than r to reflect the return associated with the liquidity of this asset.

Let t_j , $j = 1, 2, 3, \dots$ be the discrete times at which the consumer observes the value of the investment portfolio. At time t_j , the consumer chooses (1) the next date, $t_{j+1} = t_j + \tau$, at which to observe the value of the investment portfolio; (2) the amount of riskless liquid asset $X_{t_j}(\tau)$ to finance consumption from time t_j to time $t_{j+1} = t_j + \tau$; and (3) the fraction, ϕ , of the investment portfolio to hold in stocks.

Recall that $X_{t_j}(\tau)$ is the amount of the riskless liquid asset used to finance consumption from time t_j to time $t_{j+1} = t_j + \tau$, so

$$X_{t_j}(\tau) = \int_0^\tau c_{t_j+s} e^{-r^L s} ds. \quad (3)$$

For ease of readability, we will write $X_{t_j}(\tau)$ as simply X_{t_j} . When time $t_j + \tau$ arrives, the amount held in the riskless liquid asset will just have reached zero since $r^L < r$, and the value of wealth, after paying the cost of observing the value of the investment portfolio is

$$W_{t_j+\tau} = (1 - \theta) (W_{t_j} - X_{t_j}) R(t_j, t_j + \tau), \quad (4)$$

where $R(t_j, t_j + s)$ is the gross rate of return on the investment portfolio from time t_j to time $t_j + s$, and $R(t_j, t_j) = 1$.

The investment portfolio is managed by a portfolio manager who continuously rebalances the portfolio to maintain a constant fraction ϕ of the investment portfolio in stock, so

$$\frac{dR(t_j, t_j + s)}{R(t_j, t_j + s)} = [r + \phi(\mu - r)] ds + \phi \sigma dz. \quad (5)$$

We will solve the consumer's problem in four steps: (1) Given τ and X_{t_j} , the consumer chooses consumption from time t_j to time $t_j + \tau$ to maximize utility over this interval of time; (2) Given τ , the consumer chooses the optimal values of X_{t_j} and ϕ ; (3) Given the optimal values of X_{t_j} and ϕ conditional on τ , the consumer computes the value function as a function of τ ; and (4) The consumer chooses τ to maximize the value function.

II Step 1: Given τ and X_t , choose c_{t+s} , $0 \leq s \leq \tau$

Given X_{t_j} and τ , define

$$U_{t_j}(\tau) \equiv \max_{\{c_{t_j+s}\}_{s=0}^\tau} \int_0^\tau \frac{1}{1 - \alpha} c_{t_j+s}^{1-\alpha} e^{-\rho s} ds, \quad (6)$$

subject to equation (3). Optimality requires that the product of the intertemporal marginal rate of substitution between times t_j and $t_j + s$, $\left(\frac{c_{t_j+s}}{c_{t_j}}\right)^{-\alpha} e^{-\rho s}$, and the gross rate of return between these times, $e^{r^L s}$, equals one, which implies that

$$c_{t_j+s} = c_{t_j} e^{-\frac{\rho-r^L}{\alpha}s}, \text{ for } 0 \leq s \leq \tau. \quad (7)$$

Substituting c_{t_j+s} from equation (7) into the expression for $X_{t_j}(\tau)$ in equation (3) yields

$$X_{t_j} = c_{t_j} h(\tau). \quad (8)$$

where

$$h(\tau) = \int_0^\tau e^{-\omega s} ds = \frac{1 - e^{-\omega\tau}}{\omega}, \quad (9)$$

and we assume that

$$\omega \equiv \frac{\rho - (1 - \alpha)r^L}{\alpha} > 0. \quad (10)$$

Use equation (7) to substitute for c_{t_j+s} in equation (6) and use equation (8) to rewrite the resulting equation as

$$U_{t_j}(\tau) = \frac{1}{1 - \alpha} X_{t_j}^{1-\alpha} [h(\tau)]^\alpha. \quad (11)$$

III Step 2: Given τ , choose X_{t_j} and ϕ

Given τ , the consumer's problem becomes Paul A. Samuelson's (1969) classic discrete-time lifetime portfolio selection problem, with the period rate of return $R(t_j, t_j + \tau)$ multiplied by a constant $1 - \theta$. At times t_j at which the consumer observes the portfolio value, the value function $V(W_{t_j})$ satisfies

$$V(W_{t_j}) = \max_{X_{t_j}, \phi} U_{t_j}(\tau) + e^{-\rho\tau} E_{t_j} \left\{ V((1 - \theta)(W_{t_j} - X_{t_j})R(t_j, t_j + \tau)) \right\}, \quad (12)$$

where ϕ is the share of equity in the investment portfolio. Hypothesize that

$$V(W_{t_j}) = \frac{1}{1-\alpha} \gamma W_{t_j}^{1-\alpha}, \quad (13)$$

where γ is a positive constant to be determined. Substituting equations (11) and (13) into equation (12) yields

$$\begin{aligned} \frac{1}{1-\alpha} \gamma W_{t_j}^{1-\alpha} &= \max_{X_{t_j, \phi}} \frac{1}{1-\alpha} X_{t_j}^{1-\alpha} [h(\tau)]^\alpha \\ &+ e^{-\rho\tau} \frac{1}{1-\alpha} \gamma (W_{t_j} - X_{t_j})^{1-\alpha} (1-\theta)^{1-\alpha} E_{t_j} \{ [R(t_j, t_j + \tau)]^{1-\alpha} \}. \end{aligned} \quad (14)$$

The optimal allocation of the investment portfolio maximizes

$$\frac{1}{1-\alpha} E_{t_j} \{ [R(t_j, t_j + \tau)]^{1-\alpha} \} = \frac{1}{1-\alpha} \exp \left[(1-\alpha) \left(r + \phi(\mu - r) - \frac{1}{2} \alpha \phi^2 \sigma^2 \right) \tau \right]. \quad (15)$$

Differentiating equation (15) with respect to ϕ and setting the derivative equal to zero yields the optimal share of equity in the investment portfolio, denoted ϕ^* ,

$$\phi^* = \frac{\mu - r}{\alpha \sigma^2}. \quad (16)$$

Substituting equation (16) into equation (15) implies that

$$\max_{\phi} \frac{1}{1-\alpha} \exp(-\rho\tau) E_{t_j} \{ [R(t_j, t_j + \tau)]^{1-\alpha} \} = \frac{1}{1-\alpha} \exp[-\alpha\lambda\tau], \quad (17)$$

where

$$\Omega(\alpha) \equiv r + \frac{1}{2} \frac{1}{\alpha} \left(\frac{\mu - r}{\sigma} \right)^2 > r^L \geq 0 \quad (18)$$

and

$$\lambda \equiv \frac{\rho - (1 - \alpha) \Omega(\alpha)}{\alpha} > 0. \quad (19)$$

The restriction that $\lambda > 0$ is an additional assumption that keeps the present value in equation (17) finite as τ approaches infinity.

Substitute equation (17) into equation (14) to obtain

$$\frac{1}{1 - \alpha} \gamma W_{t_j}^{1 - \alpha} = \max_{X_{t_j}} \frac{1}{1 - \alpha} X_{t_j}^{1 - \alpha} [h(\tau)]^\alpha + \frac{1}{1 - \alpha} \gamma (W_{t_j} - X_{t_j})^{1 - \alpha} \chi^\alpha e^{-\alpha \lambda \tau}, \quad (20)$$

where

$$\chi \equiv (1 - \theta)^{\frac{1 - \alpha}{\alpha}}. \quad (21)$$

Differentiate the right hand side of equation (20) with respect to X_t and set the derivative equal to zero to obtain

$$X_{t_j} = \gamma^{-\frac{1}{\alpha}} h(\tau) \chi^{-1} e^{\lambda \tau} (W_{t_j} - X_{t_j}). \quad (22)$$

Now define

$$A \equiv \gamma^{-\frac{1}{\alpha}} h(\tau) \chi^{-1} e^{\lambda \tau}. \quad (23)$$

Use the definition of A in equation (23) to rewrite equation (22) as

$$X_{t_j} = \frac{A}{1 + A} W_{t_j}. \quad (24)$$

IV Step 3: Given τ , compute the value function

Substitute X_{t_j} from equation (24) into the value function in equation (20) and simplify to obtain

$$\gamma(\tau) = \left(\frac{A}{1 + A} \right)^{1 - \alpha} [h(\tau)]^\alpha + \gamma \left(\frac{1}{1 + A} \right)^{1 - \alpha} \chi^\alpha e^{-\alpha \lambda \tau}. \quad (25)$$

Equations (23) and (25) are two equations in γ and A . Solving these equations simultaneously, and using the definition of $h(\tau)$ in equation (9), yields³

$$A = \chi^{-1}e^{\lambda\tau} - 1 \quad (26)$$

and

$$\gamma(\tau) = \left[\frac{1 - e^{-\omega\tau}}{1 - \chi e^{-\lambda\tau}} \right]^\alpha \omega^{-\alpha}. \quad (27)$$

Note that if the consumer decides never to use a portfolio manager and simply holds all wealth in the liquid asset, the value of $\gamma(\tau)$ would be $\hat{\gamma} = \omega^{-\alpha}$.⁴

V Step 4: Choose τ to maximize the value function

The next step is to choose τ to maximize the value function in equation (13), which is equivalent to choosing τ to maximize

$$F(\tau) \equiv \frac{\gamma(\tau)}{1 - \alpha} \quad (28)$$

subject to $\chi e^{-\lambda\tau} < 1$, so that equation (26) implies that $A > 0$, and hence equation (24) implies that $X_{t_j} > 0$. Observe from the definitions of ω and λ in equations (10) and (19), respectively, that

$$\omega - \lambda = \frac{1 - \alpha}{\alpha} [\Omega(\alpha) - r^L]. \quad (29)$$

Now define

$$M(\tau) \equiv [\omega - \lambda + \lambda e^{\omega\tau}] \frac{1}{\omega} e^{-\lambda\tau} > 0, \quad (30)$$

and observe that

$$M'(\tau) = (\omega - \lambda) (e^{\omega\tau} - 1) \frac{1}{\omega} \lambda e^{-\lambda\tau} \quad (31)$$

and

$$M''(\tau) = \left[e^{\omega\tau} - (e^{\omega\tau} - 1) \frac{\lambda}{\omega} \right] (\omega - \lambda) \lambda e^{-\lambda\tau}. \quad (32)$$

Lemma 1 Define τ^* as the unique positive value of τ that satisfies $M(\tau^*) = \chi^{-1}$. Then τ^* maximizes $F(\tau)$ over positive τ , subject to $\gamma(\tau) > 0$.

Proof. Use equations (27) and (29) to rewrite equation (28) as $F(\tau) \equiv [(\omega - \lambda) \alpha \omega^\alpha]^{-1} \times [\Omega(\alpha) - r^L] (1 - e^{-\omega\tau})^\alpha (1 - \chi e^{-\lambda\tau})^{-\alpha}$. Since $F(\tau)$ is twice differentiable for τ satisfying $\chi e^{-\lambda\tau} < 1$, the maximum value of $F(\tau)$ is characterized by $F'(\tau) = 0$. Differentiate $F(\tau)$ to obtain $F'(\tau) = (\omega - \lambda)^{-1} v(\tau) [\chi^{-1} - M(\tau)]$, where $v(\tau) \equiv \chi \omega^{1-\alpha} e^{-\omega\tau} [\Omega(\alpha) - r^L] \times (1 - e^{-\omega\tau})^{\alpha-1} (1 - \chi e^{-\lambda\tau})^{-(1+\alpha)} > 0$. Since $\chi^{-1} - M(\tau^*) = 0$, $F'(\tau^*) = 0$ and $F''(\tau^*) = -(\omega - \lambda)^{-1} v(\tau) M'(\tau^*)$. Use equation (31) to obtain $F''(\tau^*) = -v(\tau) (e^{\omega\tau} - 1) (\lambda/\omega) e^{-\lambda\tau} < 0$ ■

Lemma 1 implies that the optimal value of τ , denoted τ^* , satisfies

$$M(\tau^*) \chi = 1. \quad (33)$$

Corollary 1 $\chi e^{-\lambda\tau^*} < 1$ and $\gamma(\tau^*) / (1 - \alpha) > \hat{\gamma} / (1 - \alpha)$.

Proof. Equation (33) implies that $\chi e^{-\lambda\tau^*} = e^{-\lambda\tau^*} [M(\tau^*)]^{-1}$, which, along with equation (30), implies $\chi e^{-\lambda\tau^*} = \omega / [\omega + (e^{\omega\tau} - 1) \lambda] < 1$, which implies that $(1 - e^{-\omega\tau^*}) (1 - \chi e^{-\lambda\tau^*})^{-1} = [1 + e^{-\omega\tau^*} (\omega - \lambda) / \lambda]$. Therefore, since equation (27) and the fact that $\hat{\gamma} = \omega^{-\alpha}$ imply that $\gamma(\tau^*) = (1 - e^{-\omega\tau^*})^\alpha (1 - \chi e^{-\lambda\tau^*})^{-\alpha} \hat{\gamma}$, we have $\gamma(\tau^*) = [1 + e^{-\omega\tau^*} (\omega - \lambda) / \lambda]^\alpha \hat{\gamma}$. This equation, along with equation (29), implies $[\gamma(\tau^*) - \hat{\gamma}] / (1 - \alpha) = ([1 + (1 - \alpha) \delta]^\alpha - 1) \hat{\gamma} / (1 - \alpha)$, where $\delta \equiv [\Omega(\alpha) - r^L] e^{-\omega\tau^*} / (\alpha \lambda) > 0$. ■

Corollary 1 implies that the value function is higher when the consumer holds an investment portfolio of stocks and bonds than if the consumer simply held the liquid asset. The following propositions demonstrate properties of the optimal value of τ .

Proposition 1 $d\tau^*/d\theta > 0$.

Proof. Totally differentiate equation (33) with respect to τ and χ to obtain $d\tau^*/d\chi = -M(\tau^*) / [\chi M'(\tau^*)]$. Differentiate χ with respect to θ to obtain $d\chi/d\theta = -(1 - \alpha) \chi [\alpha (1 - \theta)]^{-1}$. Then use equation (31), along with equations (29) and (33) to obtain $d\tau^*/d\theta = (d\tau^*/d\chi) (d\chi/d\theta) = \omega e^{\lambda\tau^*} / [\lambda\chi(1 - \theta) [\Omega(\alpha) - r^L] (e^{\omega\tau^*} - 1)] > 0$. ■

Proposition 2 $d\tau^*/dr^L > 0$.

Proof. Applying the implicit function theorem to $M(\tau^*)\chi = 1$ implies that $d\tau^*/dr^L = -(d\omega/dr^L) (M_\omega/M'(\tau^*))$ where $M_\omega \equiv \partial M(\tau^*; \lambda, \omega) / \partial \omega$. Differentiating equation (30) with respect to ω yields $M_\omega = [1 - (1 - \tau\omega) e^{\omega\tau}] \lambda e^{-\lambda\tau} / \omega^2$, which, along with equations (10), (29), and (31), implies $d\tau^*/dr^L = [1 - (1 - \tau\omega) e^{\omega\tau}] [\Omega(\alpha) - r^L]^{-1} (e^{\omega\tau} - 1)^{-1} \omega^{-1}$, which is positive for $\omega\tau > 0$. ■

Lemma 2 *If $\alpha > 1$, then $d\tau^*/d\Omega(\alpha) < 0$.*

Proof. Suppose that $\alpha > 1$. Define $M_\lambda \equiv \partial M(\tau^*; \lambda, \omega) / \partial \lambda$ and differentiate equation (30) with respect to λ to obtain $M_\lambda = (1/\lambda) M(\tau^*) e^{-\lambda\tau^*} [-[M(\tau^*)]^{-1} + (1 - \lambda\tau^*) e^{\lambda\tau^*}]$. Since $M(\tau^*)\chi = 1$, $(1 - \lambda\tau^*) e^{\lambda\tau^*} < 1$ for $\lambda\tau > 0$, and $M(\tau^*) > 0$ we have $M_\lambda < (1/\lambda) M(\tau^*) e^{-\lambda\tau^*} (-\chi + 1)$. The definition of χ in equation (21) implies that if $\alpha > 1$, then $\chi > 1$. Hence, $M_\lambda < 0$.⁵ Since $\alpha > 1$, equation (29) implies that $\omega - \lambda < 0$, so equation (31) implies that $M'(\tau^*) < 0$. Applying the implicit function theorem to $M(\tau^*)\chi = 1$ implies that $d\tau^*/d\lambda = -M_\lambda/M'(\tau^*) < 0$. The definition of λ implies that $d\lambda/d\Omega(\alpha) = (\alpha - 1)/\alpha > 0$ for $\alpha > 1$. Therefore, $d\tau^*/d\Omega(\alpha) = (d\tau^*/d\lambda) (d\lambda/d\Omega(\alpha)) < 0$. ■

Proposition 3 *If $\alpha > 1$, then $d\tau^*/d\mu < 0$ and $d\tau^*/d\sigma^2 > 0$.*

Proof. Use Lemma 2 and the definition of $\Omega(\alpha)$ in equation (18), which implies that $\Omega(\alpha)$ is increasing in μ (recall that $\mu > r$) and decreasing in σ . ■

Proposition 4 *If $\alpha > 1$, then $d\tau^*/dr \leq 0$ as $\phi^* \leq 1$.*

Proof. Differentiate $\Omega(\alpha)$ with respect to r , and use the expression for ϕ^* to obtain $d\Omega(\alpha)/dr = 1 - \phi^*$. Then apply Lemma 2. ■

If $\phi^* > 1$ the investment portfolio has negative holding of bonds, and an increase in r decreases $\Omega(\alpha)$.

VI Quadratic Approximation

Observe from equation (31) that $M(0) = 1$, $M'(0) = 0$ and from equation (32) that $M''(0) = (\omega - \lambda)\lambda \neq 0$. Therefore, the function $M(\tau)$ is locally quadratic at $\tau = 0$. The second-order Taylor expansion of $M(\tau)$ at $\tau = 0$, denoted $\widehat{M}(\tau)$, is

$$\widehat{M}(\tau) \equiv 1 + \frac{1}{2}(\omega - \lambda)\lambda\tau^2. \quad (34)$$

Let $\widehat{\tau}$ be the (approximately) optimal value of τ that satisfies $\widehat{M}(\widehat{\tau}) = \chi^{-1}$. Substituting equation (34) into this expression and rearranging yields

$$\widehat{\tau} = \sqrt{\frac{2(\chi^{-1} - 1)}{(\omega - \lambda)\lambda}}. \quad (35)$$

VII Illustrative Calculations

Consider the baseline case with $\theta = 0.0001$, $\alpha = 4$, $\rho = 0.01$, $r^L = 0.01$, $r = 0.02$, $\mu = 0.06$ and $\sigma^2 = (0.16)^2$, where ρ , r^L , r , μ , and σ are rates per year. As shown in Table 1, even when θ is only one basis point, the optimal value of τ is 0.696 years. The rows following the baseline row vary the parameters one at a time from their baseline values.

	τ^*	$\widehat{\tau}$
Baseline	0.696	0.693
$\theta = 0.001$	2.223	2.193
$\rho = 0.02$	0.662	0.659
$\alpha = 2$	0.587	0.585
$r^L = 0$	0.557	0.555
$r = 0.03$	0.541	0.538
$\mu = 0.07$	0.584	0.581
$\sigma = 0.20$	0.796	0.792

Table 1: Optimal and approximate decision intervals, in years

VIII Conclusion

We have solved the consumption/portfolio problem of an inattentive consumer who faces proportional transaction costs. We plan to extend this model to allow occasional large shocks that capture consumers' attention, so that the time between adjustments will be a state-dependent random interval, rather than a constant. This will allow a study of adjustment to aggregate shocks by consumers who are at different points in their adjustment cycles.

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Notes

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We thank Xavier Gabaix and David Laibson for helpful detailed correspondence, and Adriano Rampini and the Penn Macro Lunch Group for helpful comments.

¹In a related framework, Ricardo A.M.R. Reis (2006) deals with this potential problem by using CARA utility and allowing negative consumption.

²In private correspondence, Gabaix and Laibson (November 25, 2006) clarify their observation cost by stating “that the utility cost is always $qw_0e^{-\rho t}$, not $qw_t e^{-\rho t}$,” so that the cost is not proportional to the contemporaneous value of wealth.

³In this step, we are choosing the optimal X_{t_j} given τ , and we restrict attention to values of τ for which optimal X_{t_j} in equation (24) is strictly positive. This restriction, along with $1 + A > 0$, which follows from equation (26), implies that $A > 0$. Therefore, equation (26) implies that $\chi e^{-\lambda\tau} < 1$. Corollary 1 verifies that this condition holds at the optimal value of τ .

⁴Formally, set $\theta = 0$ (which implies $\chi = 1$) and set $\Omega(\alpha) = r^L$ (which implies $\omega = \lambda$) in equation (27). Alternatively, but equivalently, set $W_{t_j} = X_{t_j}$ and $\tau = \infty$ in equation (11) to obtain $U_{t_j}(\infty) = \frac{1}{1-\alpha} W_{t_j}^{1-\alpha} [h(\infty)]^\alpha = \frac{1}{1-\alpha} W_{t_j}^{1-\alpha} \omega^{-\alpha}$.

⁵For the case with $\alpha < 1$, M_λ could be either positive or negative. For instance, using the baseline parameters from Table 1 ($\theta = 0.0001$, $\rho = 0.01$, $r^L = 0.01$, $r = 0.02$, $\mu = 0.06$, and $\sigma = 0.16$), $M_\lambda < 0$ for $\alpha = 0.9$, but $M_\lambda > 0$ for $\alpha = 0.85$.