

The Political Economy of Housing Supply: Homeowners, Workers, and Voters*

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Abstract

Equilibrium on the housing market depends on a complex set of interactions between: (1) individual location decisions; (2) individual housing investment; (3) collective decisions on urban growth. We embed these elements in a model of a dynamic economy with two sources of friction: ill-defined property rights on future land development and uninsurable shocks affecting labor productivity. We characterize the feedback between the households' desire to invest in housing as a hedge against the risk of rent fluctuations and their support for supply restrictions once they own housing. The model generates inefficiently low supply of housing in equilibrium. The model also rationalizes persistence in housing undersupply: the more restricted the initial housing supply, the smaller the city size selected by the voting process. We use the model study the effects of a number of policies and institutional changes.

1 Introduction

An increasing body of evidence points to the importance of supply restrictions in understanding housing price dynamics. Glaeser, Gyourko and Saks (2005a) report that changes in regulatory regimes explain the scarcity of land for housing development in what are today the most expensive US housing markets. They point to differences in man-made scarcity as the key driver for the explosion of the dispersion in housing prices across US housing markets since the mid-Seventies. Quigley and Raphael (2005) also blame housing regulation for the recent housing price boom in California.¹ Green, Malpezzi and Mayo (2005) find that

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¹For further evidence on the critical impact of housing regulations on housing supply, see Ozanne and Thibodeau (1983), Rose (1989), Malpezzi, Chun and Green (1998), Mayer and Somerville (2000), Glaeser and Gyourko (2003), Glaeser and Ward (2006), Glaeser, Gyourko and Saks (forthcoming).

housing supply regulations are the key driver of differences in housing supply elasticities across US metropolitan areas. For the UK, Barker (2003, 2005) identifies the regulatory constraints on the release of land for housing development as the primary reason behind the unresponsiveness of housing supply to price increases.

What are the determinants of housing supply regulations? Restrictive supply regulations cannot survive without political support. To understand the political economy of housing supply, we need to understand who participates in the decision process, what are the stakes of the participants, and the mechanism whereby participants' preferences translate into policies. We then need, at a minimum, a *location choice model* to determine who lives in a particular area and a *housing investment model* to predict what real estate assets the residents own. We also need a *collective choice model* to map the identities and preferences of local residents into political decisions over urban growth.

Our goal in this paper is to provide a first step towards a theory that encompasses these three elements: housing consumption, housing investment and collective choice over housing supply regulation. Each of these three elements is quite complex and is linked to the others through multiple channels. We do not try to capture in one model the richness of the institutions that regulate these three phenomena. Instead, we offer a parsimonious and tractable framework to gain some insight into the basic issues, and to link areas of research that have traditionally been separate.

In this spirit, we assume only two deviations from complete markets. The first is a key feature of housing markets: building permits are needed for new construction and they are issued by the local government. Hence, while property rights on existing buildings are relatively well-defined, property rights on future construction are blurred. The second deviation is a staple assumption in macroeconomics: households cannot insure against future labor income shocks. As we shall see, the combination of these two imperfections is sufficient to generate undersupply of housing in equilibrium and persistence in undersupply.

In the baseline model, we consider a country with one city and a vast countryside.² A continuum of agents live for two periods. In the first period, every agent is assigned a productivity level. Productivity and location are complementary: the more productive agents are even more productive if they live in the city rather than in the countryside.³

²In Section 6 we will allow for multiple cities.

³Although we interpret our model in terms of labor productivity, there is a mathematically equivalent interpretation in terms of quality of life (discussed in detail on page 8). In that interpretation, the benefit of living in the city (perhaps a beach resort) is due to local amenities, which are more valuable to certain people. Within both interpretations, housing is a hedge against rent risk.

Agents' productivity may change from the first to the second period. In particular, there may be a technological innovation with two effects: an increase both in average productivity and in the turbulence in the productivity levels of individual agents. Turbulence involves a reordering of individual productivity levels. For instance, the IT revolution that occurred in the San Francisco Bay Area in the Nineties boosted overall productivity but had a more positive effect for certain workers (e.g. software engineers) than for others (e.g. nurses and teachers).

Agents who move to the city in the first period have the option to buy or to rent their home. All houses are identical and can only accommodate one agent. We also assume that homeownership is a continuous choice variable, going from unboundedly negative (short-selling city real estate) to unboundedly positive (owning multiple homes in the city, or derivatives on the city housing price index).⁴

At the end of the first period, city residents vote over housing supply in the second period: they select the number of construction licenses to be issued. A key element of the model is the institutional mechanism that regulates the distribution of new licenses. The windfall gain deriving from new construction can accrue to homeowners (e.g. licenses are sold to developers and the revenues distributed to local homeowners), residents (licenses are sold to developers and the revenues are used for local services), or to a set of measure zero of the population (lucky or clever developers in case of an arbitrary mechanism, or well-connected developers and perhaps corrupt public officials in case of favoritism). We are particularly interested in situations where only a small portion of the gain accrues to residents or homeowners.⁵

Equilibrium is determined by two orders of interactions. On the housing market, agents choose how much to invest in real estate. A city resident who does not own housing faces a "rent risk". A positive productivity shock increases the average wage in the city, and hence rent. The resident can insure against this risk by purchasing housing. As productivity shocks are associated with income turbulence leading to mean reversion, the rent hedging

⁴In Section 6, we study the effect of legal restrictions which make home ownership a binary choice between renting one home or buying one home.

⁵To our knowledge, there is little systematic evidence on the distribution of windfall gains. In her comprehensive review of housing supply in Britain, Barker (2003, Chapter 5) argues that: (1) Developers hold option agreements on large tracts of land currently without building permission; (2) Developers have significant local market power; (3) While local authorities have a legal avenue to demand monetary transfers in exchange for issuing building permission, the payments obtained in this way are quite low (of the order of £2000/8000 per unit built – See Table 8.2 in Barker). These three facts taken together seem to indicate that most of the windfall gains accrue to developers. Things may be different in Hong Kong where the government uses an auction mechanism to sell land for development (Wall Street Journal, 10/24/2006) .

motive is stronger for agents who currently enjoy high labor productivity. In equilibrium, investment in housing is an increasing function of the agent's current productivity level.

On the political side, agents who are more invested in housing are more likely to favor a restrictive licensing regime. This leads to a first, natural result: in equilibrium, housing is under-supplied. Voters support artificial supply restrictions in order to protect their investment. In turn, they invest because they expect the value of their housing investment to be protected by urban growth restrictions. This generates an unambiguous welfare loss because the city remains too small. All citizens, before investing in housing (but after learning their individual productivity level), would support a commitment to increasing housing supply to the maximum possible level.

More importantly, the model displays persistence in housing undersupply. The degree of undersupply in the second period is an increasing function of undersupply in the first period. This is not due to construction costs (there are none) but to the interaction between hedging demand and politics. A city with a low initial housing supply is a city with an initial population of high-productivity agents who pay a high housing price or rent relative to their income. The median voter is then highly invested in housing and he is keen to keep the city small and housing expensive. Under plausible assumptions, there is no new construction, which leads to the most extreme form of size persistence. The opposite occurs in a city that starts from a relatively high housing supply and thus low housing costs relative to income.

Once we identify the potentially vicious circle between homeownership and housing supply, we can begin to discuss the effects of a number of institutional reforms that have been suggested. First and foremost, one needs to question the current mechanism for allocating housing permits. While the allocation system varies widely across countries (and even within countries), it typically does not take the form of an auction (except for the Hong Kong example mentioned in footnote 5). Our model formally identifies a strong link between the housing undersupply and the share of windfall gains that accrues to the median voter. The most natural way to break the vicious circle of housing undersupply is to create simple legal instruments through which local communities can appropriate windfall gains.⁶

⁶This point seems to have escaped governments concerned with housing affordability. For instance, the recent comprehensive report sponsored by the UK Treasury (Barker 2005) uses a wealth of information to show that housing inflation in the UK is due to undersupply of land, which in turn is due to the unwillingness of local authorities to make more land available. However, the policy recommendation is to tax windfall gains and transfer the proceeds to central government, and to deprive local government of the only existing channel to appropriate some of the developers' rent (the so-called "section 106" – see Barker (2005, p.7, recommendation 29)).

Second, we study the effect of making city planning decisions at a more or less centralized level. We have assumed that housing supply decisions are taken at a level that corresponds to the local labor market (i.e. a metropolitan area). In practice, city planning may occur at a different level. At one extreme, the UK Town and Country Act of 1948 and subsequent laws give the national government an enormous power on planning decisions. The government can in practice force local communities to accept large-scale land development. On the other extreme, a number of metropolitan areas around the world (this is true for most large US cities) are not under a unified jurisdiction: planning decisions are made by a number of autonomous local governments. Our paper shows that there exists a U-shaped relation between the degree of centralization and equilibrium housing supply. A very centralized system and a very decentralized one result in more construction than a situation where local government coincides with local labor markets. A centralized government wants more housing supply because it takes into account the welfare of countryside residents (who may move to the city if more houses are built). A very decentralized system falls into a beggar-thy-neighbor equilibrium, whereby local residents do not internalize the negative price effect that construction in their community imposes on the rest of the metropolitan area.

Third, we examine the effects of subsidizing homeownership. Encouraging households to own more housing gives them an obvious incentive to restrict urban growth. This is what happens in equilibrium: when homeownership is subsidized, households vote for a more restricted housing supply in the second period and housing is more expensive.

Fourth, we study the effect of imposing restrictions on fractional ownership. Caplin et al. (1997) argue that current rules make it difficult for people to share ownership of their home with other agents. We compare the baseline case with a set-up where people can only own zero or one home. Besides the direct portfolio effect discussed by Caplin et al., we identify an indirect supply effect of restrictions to fractional ownership. The elimination of such restrictions is likely to make housing more affordable through increased political support for urban growth.

Our paper brings together two influential strands of literature on housing, which have hitherto remained mostly separate.

The first strand moves from the premise that a meaningful discussion of the housing market must include an endogenous housing supply function. Glaeser, Gyourko and Saks (2005b) provide a detailed model of the decision process involved in authorizing housing development. They study the effects of changing judicial tastes, decreasing ability to bribe regulators, rising incomes and demand for public amenities, and improvements in the ability

of homeowners to organize and influence local decisions. They find the main driver for the rise in urban growth restrictions to be a significant increase in the ability of local residents to block new projects. They conclude that cities have changed from urban growth machines to homeowners' cooperatives.

Our paper foregoes most of the political process complexity of Glaeser et al. in order to endogenize the composition of the local population, their tenure decision, and hence their preferences for urban growth. Our approach to the voting decision builds on the work of Fischel (2001), who provides detailed arguments and empirical evidence that downside risk motivates homeowners to participate in the planning process. He argues that “homevoters” are politically motivated by the risk of loss on their home because of the difficulty to diversify this risk away. Households in our model are motivated not only by the potential losses due to new constructions but also by the prospects of enjoying capital gains when aggregate demand increases.

The second strand of literature endogenizes housing investment by modeling the tenure choice of risk averse households in a stochastic environment (e.g., Ortalo-Magné and Rady, 2002, Sinai and Souleles, 2005, Hilber, 2005, Davidoff, 2006, Shore and Sinai, 2006). This strand of literature links housing prices with expected future rents and shows that homeownership provides a hedge against future housing expenditures. Sinai and Souleles (2005) develop a stylized model of dynamic investment decisions by households facing stochastic rent fluctuations and endogenous house prices, and they show that it can account for a number of observed patterns in tenure decisions.

Our model incorporates this key insight about housing demand in a market equilibrium model. The rent risk is now endogenous, and it is the combined effect of labor productivity shocks, households' location decisions, and collective supply decisions. As in the contributions cited above, the possibility of rent fluctuations gives housing a hedging value, which pins down homeownership patterns and, in turn, determines political support for urban growth.⁷

While our model brings together these two frameworks, it neglects – by necessity – a number of important issues related to housing supply. In particular, we abstract from the

⁷One may wonder whether the link between preferences over urban growth and individual labor productivity levels could be obtained, in a deterministic model, through a simple wealth effect. However, the fact that rich people consume more housing than poor people is not enough. One must also argue that rich people have a reason to spend proportionally more on purchases rather than rentals, which – in a world without uncertainty – requires ad-hoc assumptions (e.g. rental-related transaction costs are relatively higher than purchase-related transaction costs for expensive properties or the tax advantage to owning is increasing in income).

issue of local taxation for the provision of local public goods, peer effects and agglomeration economies.⁸ As several papers have shown, externalities are essential to understanding the political economy of housing supply. In this paper, we abstract from them in order to clarify the dynamic connection between homeownership and housing supply. We will mention where appropriate how our model could incorporate local externalities. Also, our model considers only one – stylized – form of growth restriction. We do not consider regulation pertaining to height, density, use, etc.⁹ This choice, again, is in the interest of parsimony. Our methodology can easily be extended to other forms of regulation.

The plan of the paper is as follows. Section 2 lays out the model. For expositional purposes, we first analyze the model holding housing supply fixed (Section 3) and we then endogenize supply and study the political equilibrium (Section 4). We discuss the persistence result in Section 5. Section 6 studies the effect of institutional reforms. Section 7 concludes. All proofs are in the Appendix.

2 Model

Consider a two-period model of an open economy with two locations, the City and the Countryside. There are two commodities, housing and a numeraire consumption good. For simplicity, we abstract from housing construction costs. Housing in both communities consists in homogeneous plots of land. The economy is populated by a mass 1 of agents with identical CARA utility defined over consumption of second period numeraire only, $u(c) = -e^{-ac}$.

The endowment of numeraire good each agent receives every period depends on his location and his productivity index $i \in [0, 1]$. We normalized the productivity in the Countryside to zero.¹⁰ Let y^i be a random variable uniformly distributed on $[0, 1]$. In the first

⁸See Fernandez and Rogerson (1997) and Calabrese, Epple and Romano (2006) for studies of the political economy of zoning regulation and its interaction with the provision of local public goods.

A number of papers analyze growth controls in a static setting. They focus on issues related to the creation of amenities and the distribution of the value generated by the location of economic activity within cities; e.g., Brueckner (1995), and Helsley and Strange (1995), Brueckner and Lai (1996). As mentioned in Brueckner and Lai, a dynamic model is necessary to capture the motivation for growth controls that comes from the prospects of capital gains on one's home.

⁹For references, see for example Wheaton (1998), and Bertaud and Brueckner (2004). Note that Glaeser and Ward (2006) find that a variety of regulations, which act as very effective barriers against new construction, generate little price effects beyond their effect on housing density. For example, if high minimum lot sizes were used to “improve” the mix of a community and such “improvements” had a significant economic impact, their housing price effect would be greater than that justified by their restrictive effect on the number of homes that can be built

¹⁰Our results could easily be extended to a more general setting in which the productivity of agent i at time t is y_t^i in the City and ay_t^i in the countryside, where $a \in (0, 1)$.

period, if agent i works in the City, his productivity is $y_1^i = y^i$. In period 2, the distribution of earnings evolves as follows. With probability $1 - \pi$, no shock occurs: if agent i works in the City, his productivity is $y_2^i = y_1^i = y^i$. With probability π , there is a positive shock to aggregate productivity: average productivity increases by $g > 0$. Conditional on this shock occurring, with probability $1 - \gamma$, the productivity of agent i in the City is $y_2^i = y^i + g$. With probability γ , all agents draw a new productivity parameters \tilde{y}^i from the initial productivity distribution. The productivity of agent i in the City is then $y_2^i = \tilde{y}^i + g$. The expected aggregate growth rate of the economy between periods 1 and 2 is therefore πg . The greater the probability γ , the more insecure are the agents about their future productivity.

Our key assumption about the labor market is that there is positive correlation between growth and turbulence. If the economy grows faster there is a higher probability that the income ranking in period 1 is changed in period 2. Such a positive correlation could be due to technological change. Every time an important innovation is introduced, most agents reach higher productivity levels but certain agents, whose skills become obsolete, lose out, at least in relative terms.¹¹

This same mathematical formulation also has a different interpretation in terms of leisure rather than productivity. The variable y^i represents the utility that agent i gets if he lives in the ‘City’, which can now be seen as an area with certain amenities. Such amenities may be natural (a coastal region, a ski resort) or man-made (a historical town, a vibrant metropolis). Shocks in y^i derive from social phenomena that determine shifts in preferences for amenities (the desire to retire to sunnier climates, the feasibility of telecommuting, reduced crime rates in large cities). Such shifts create both a higher average utility of living in the ‘City’ and turbulence in the utility rankings of agents. For concreteness, the rest of the paper will refer to the productivity interpretation rather than the leisure interpretation.

Working in the City requires consuming one unit of City housing. We denote l_t^i the housing consumption by agent i in period t where $l_t^i = 1$ if the agent locates in the City, $l_t^i = 0$ otherwise.

Independently of their housing consumption choice, agents may also invest in City housing. Let $h_t^i \in (-\infty, \infty)$ be the measure of City housing that agent i owns in period t . We do not restrict this measure to be a positive integer. A non-integer h_t^i indicates fractional

¹¹An alternative model, which would yield qualitatively similar results, is one where a positive shock corresponds to random waves of new groups of skilled professionals. Such arrivals produce a twofold effect: the City reaches a higher average income but the previous City residents are now relatively poorer than the newcomers.

property. A negative h_t^i means that the agent has sold City housing short for period t . In practice, there are serious obstacles to fractional property and to shortselling properties.¹²

In the Countryside, the supply of housing is perfectly elastic at a cost normalized to zero. There are no moving costs between City and Countryside.

There is a measure N_1 of housing in the City at the start of period 1 owned initially by a large number of international real estate investment trusts (REITs), which maximize the expected value of their real estate investment. At the end of period 1, City residents choose the measure N_2 of houses available in the City in period 2, . We assume that existing houses cannot be destroyed and no depreciation of the housing stock, $N_2 \geq N_1$. City residents therefore vote on the number of building permits that will be issued, $N_2 - N_1 \geq 0$.

Permits are assumed to be identical, divisible, and immediately tradeable. Given the parameters $\phi \in [0, 1]$ and $\tau \in [0, 1]$, the permits are allocated as follows:

- A proportion $1 - \phi$ of the permits goes to a set of measure zero of the population. We think of this as the classical system of allocating permits to certain developers without asking them for a payment corresponding to the capital gain they will experience. These developers may in turn return some of the windfall gain to city officials in the form of bribes or campaign contributions. We assume that developers and officials represent a negligible proportion of the city population (and hence the median voter never benefits from this share of permits).¹³
- A proportion $\phi\tau$ is allocated to current residents in equal parts. This is the case where permits are allocated to developers, but in exchange for a payment or for the provision of certain goods or services (building a park, paying for infrastructure, etc).
- A proportion $\phi(1 - \tau)$ goes to the owners of existing properties in the City in proportion to the number of properties they own in the City. This case may arise when the city pursues a policy of densification.

As soon as they are issued, building permits can be freely traded. Let b represent the total market value of permit issued by the City.

¹²We thus abstract for the time being from a number of potential imperfections of the housing market (later, we will discuss the effect of restrictions to fractional properties and the introduction of fiscal distortions).

¹³The writer Tom Wolfe (New York Times, 2006) offers a vivid account of the permit allocation process in New York City. A key role is played by the Landmarks Preservation Commission, a body that decides whether a designated landmark building can be altered or replaced. As landmarks tend to be located in highly desirable areas, the commission's decisions can generate large windfall gains for the owners of the buildings. We would argue that Tom Wolfe's description can be represented by an allocation system with $\phi = 1$.

The property markets open at the start of each period. Let r_t denotes the rent and p_t denote the unit price of housing in the City at period t . Competition among REITs ensures that the price of housing is equal to its expected rent return plus any benefit that accrues from the sale of building permits. We assume an exogenous interest rate of zero between periods 1 and 2 for ease of exposition.

To sum up, the timing of the model is as follows:

1. Period $t = 1$ begins, agents learn y^i .
2. The property market opens, agents choose h^i and l^i .
3. City residents vote on the measure of City houses N_2 to be made available in period 2. A measure $N_2 - N_1$ of building permit is issued in proportion $\phi\tau$ to City residents (in equal parts) and in proportion $\phi(1 - \tau)$ to owners. These permits can be traded immediately.
4. Each City resident receives y_1^i and pays the rent r_1 . Permit holders can build new houses in the City at zero cost.
5. Period $t = 2$ begins, both aggregate and idiosyncratic shocks are realized, agents learn y_2^i
6. The property market opens, agents choose l_2^i .
7. Each City residents receives y_2^i and pays the rent r_2 .
8. Agents consume their accumulated wealth.

To summarize, the accumulated wealth of agent i at the end of the game is

$$w^i = l_1^i \left(y_1^i + \phi\tau \frac{N_2 - N_1}{N_1} b - r_1 \right) + l_2^i (y_2^i - r_2) + h^i \left(r_2 + r_1 + \phi(1 - \tau) \frac{N_2 - N_1}{N_1} b - p_1 \right). \quad (1)$$

We need parameter assumptions $g < \frac{1}{a}$ and

$$\phi(1 - \tau) \frac{1}{N_1} (1 - N_1 + \pi g) < 1. \quad (2)$$

The rationale for these assumptions will become clearer in the next sections.

3 Housing Market Equilibrium – Exogenous Housing Supply

To make the analysis more transparent, we proceed in two steps. In this section, we assume that the city size is exogenously given in both periods and we focus on location decisions and tenure decisions. We provide a full characterization of the individual housing investment and location decision and we derive the unique market equilibrium. In the next section, we will endogenize the size of the city in period 2.

Suppose that N_1 and N_2 are exogenously given and known to the agents with $N_2 \geq N_1$. Under this condition, we characterize the set of City housing prices and rents such that the REITs are indifferent between renting and selling properties and the housing markets clear in each period when the agents' housing consumption and investments policies maximize their utility taking the prices and rents as given. We proceed by backward induction.

Once the period-two shock is realized, agent i faces a simple one period deterministic location choice problem. He lives in the City if and only if City earnings more than compensate the rent: $y_2^i \geq r_2$. In equilibrium the N_2 most productive agent live in the City. The second-period market rent is then given by the earnings of the N_2 th most productive agent:

$$r_2 = \begin{cases} 1 - N_2 & \text{if there is no aggregate shock} \\ 1 - N_2 + g & \text{with an aggregate shock} \end{cases}$$

The price of a building permit issued at time 1 is the expected value of a unit of housing in period 2 evaluated prior to the realization of the shock

$$b = E[r_2] = 1 - N_2 + \pi g.$$

The location choice in period 1 is simple as well. The N_1 most productive agents live in the City. The market rent is determined by the willingness to pay of the N_1 th most productive agent:

$$r_1 = 1 - N_1 + \phi\tau \frac{N_2 - N_1}{N_1} b,$$

where the third term account for the fact that city residents receive a proportion $\phi\tau$ of the building permits.

Given the presence of REITs, which are risk neutral and deep pockets, the price of a house in period 1 is the expected present values of period 1 and period 2 rents plus the value of the building permits that ownership of a house gives rights to:

$$p_1 = E \left[r_2 + r_1 + \phi(1 - \tau) \frac{N_2 - N_1}{N_1} b \right] = 1 - N_1 + \left(1 + \phi \frac{N_2 - N_1}{N_1} \right) b + E[r_2].$$

The period 1 price of a house captures the benefits of the building permits allocated to both renters and owners. The payoff of buying a unit of real estate in period 1 depends on the realization of the shock in period 2: investors lose πg if there is no aggregate shock and gain $(1 - \pi)g$ if there is a shock. Given that period 2 is the last period of the model, $p_2 = r_2$.

Substituting equilibrium rents and prices into equation (1) yields the following expression for final wealth:

$$w^i = l_1^i (y_1^i - 1 + N_1) + l_2^i (y_2^i - 1 + N_2) + h^i D,$$

where $D = -\pi g$ if there is no aggregate productivity shock and $D = (1 - \pi)g$ otherwise.

Now that we know how agents choose where to locate and the benefits of ownership, we can write the final utility of agent i conditional on his choice of h^i as follows:

- If there is no shock:

$$U_{NN} = u(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2) - h^i \pi g)$$

- If only the aggregate shock occurs:

$$U_{SN} = u(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2) + h^i (1 - \pi)g)$$

- If both the aggregate and the idiosyncratic shock occur:

$$U_{SS} = E_{\tilde{y}} [u(\max(0, y^i - 1 + N_1) + \max(0, \tilde{y}^i - 1 + N_2) + h^i (1 - \pi)g)]$$

When no idiosyncratic shock occurs, the income of city residents increases by the same amount as the city rent. This explains why city earnings in period 2 are identical in the first two expressions above. When an idiosyncratic shock occurs, agents face the possibility of reduced earnings in a state when second period rents are high because of the aggregate shock. Such a realization makes housing a useful asset to own because it delivers gains $(1 - \pi)g$ at the time when the agents face the risk of a decrease in earnings, at the cost of losses in the state when the agents face constant earnings.

To decide how much housing to buy, agent i solves

$$\max_{h^i} (1 - \pi) U_{NN} + \pi (1 - \gamma) U_{SN} + \pi \gamma U_{SS}$$

Proposition 1 *Given $0 < N_1 \leq N_2 < 1$, there is a unique market equilibrium with the following properties:*

(i) In period t , agent i lives in the City if and only if $y_t^i \geq 1 - N_t$;

(ii) An agent with first-period income y^i buys \hat{h}^i units of housing, where \hat{h}^i is the unique solution of

$$-U'_{NN}(\hat{h}^i, y^i) + (1 - \gamma)U'_{SN}(\hat{h}^i, y^i) + \gamma U'_{SS}(\hat{h}^i, y^i) = 0,$$

with

$$\begin{aligned} U'_{NN}(h^i, y^i) &= u'(\max(y^i - 1 + N_1, 0) + \max(y^i - 1 + N_2, 0) - h^i \pi g) \\ U'_{SN}(h^i, y^i) &= u'(\max(y^i - 1 + N_1, 0) + \max(y^i - 1 + N_2, 0) + h^i(1 - \pi)g) \\ U'_{SS}(h^i, y^i) &= E_{\tilde{y}^i} [u'(\max(y^i - 1 + N_1, 0) + \max(\tilde{y}^i - 1 + N_2, 0) + h^i(1 - \pi)g)] \end{aligned}$$

Besides the location part, this is a classical insurance result. Agents use housing investment to transfer utility across states of the world. Given the standard properties of the utility function, this equilibrium is unique. Solving the equilibrium basically amounts to solving a first order condition for every agent. We now use these conditions to characterize the comparative statics of the market equilibrium:

Proposition 2 *In equilibrium:*

- (1) *Housing investment is nondecreasing in y^i (strictly increasing for $y^i > 1 - N_2$);*
- (2) *There exists $y^* > 1 - N_2$ such that agents with $y^i < y^*$ choose $h^i < 0$ and agents with $y^i > y^*$ choose $h^i > 0$;*
- (3) *If γ is sufficiently high with respect to g , $h^i > 1$ for all agents with $y^i > y^* + \varepsilon$, $\varepsilon > 0$;*
- (4) *A marginal increase in N_2 induces agents with $y^i < 1 - N_2$ to buy less housing and agents with $y^i > 1 - N_2$ to buy more housing.*
- (5) *A marginal increase in N_1 keeping $N_2 > N_1$ has no effect on housing investment decisions.*

To understand property (1), note first that if an agent chooses to live in the city in period 1, he remains in the city in period 2 unless he suffers an idiosyncratic income shock that decreases his period 2 productivity below the new rent. We saw above that investing in city housing generates a loss if no aggregate productivity shock occurs, and a gain otherwise. Investing in city housing therefore allows residents to transfer wealth from the state when no aggregate shock occurs to the states when an aggregate shock occurs possibly concurrently with an idiosyncratic shock. The higher an agent's productivity in the first period, the greater the probability that an idiosyncratic shock will result in a loss of earnings, therefore

the greater his demand for insurance against idiosyncratic shocks and the greater his housing investment.¹⁴

Property (2) builds from the fact that city residents with the lowest productivity pay a rent equal to their earnings. They get no benefit from living and working in the city in period 1. They also get not benefit from the city in period 2 if no idiosyncratic shock occurs. If the idiosyncratic shock occurs, any marginal agent who draws a lower productivity moves to the countryside and gets a surplus of zero. Any marginal agent who draws a higher productivity stays in the city and enjoys a positive surplus. Due to risk aversion, marginal city residents wants to shift resources away from the state in which an idiosyncratic shock occurs. He therefore sells city housing short in the first period. At the other extreme, the agents who start with the highest productivity can only lose from an idiosyncratic shock. To insure against this loss, they take a long position in city housing in period 1. By monotonicity of the optimal housing investment policy, there must be city residents who do not own any city housing. Any resident with lower productivity goes short on housing, and vice versa.¹⁵

Property (3) exploits the fact that housing investment is increasing in the agent's productivity and in the probability they suffer an idiosyncratic shock. The greater an agent's productivity and the greater the probability of an idiosyncratic shock, the more the agent stands to loose if the idiosyncratic shock occurs. Therefore, the greater the hedging demand of this agent for housing, hence the greater his purchase of housing in period 1.

A higher N_2 yields a second period rent lower by the same amount in all states and a lower city housing price in the first period. A same quantity of housing investment in the first period therefore yields less resource transfers in the second period across states. Since a change in N_2 does not bring about a change in the income risk faced by agents, they need to take larger positions in the housing market. Agents who short the housing market take a bigger negative position. Agents who take a long position in the market buy more housing. This explains property (4).

Finally, property (5) follows from the fact that the housing supply in period 1, N_1 affects the rent in period 1 and the price in period 1. It does not change the extent to which housing

¹⁴The fact that agents with higher endowment hold more housing is critical for our results. Here we obtain this prediction from the specification of the income process. Note that a number of alternative – and complementary – assumption could generate a similar housing investment in equilibrium. For example, we would expect housing investment to increase with income in a world with heterogenous property size and moral hazard on the rental market that makes it optimal to own rather than rent large homes.

¹⁵Imposing a no-short-sale constraint on housing investment would not take anything away from our results but would complicate the analysis.

investment allows agents to shift resources across states in period 2. This explains why per se a change in N_1 does not affect housing investment.

Example

We will use $N_1 = N_2 = \frac{1}{2}$, $\pi = \frac{1}{2}$, $g = \frac{1}{12}$, $\gamma = \frac{1}{2}$. The CARA coefficient is 2.

If $y^i \geq \frac{1}{2}$, the agent lives in the City and his marginal utilities are:

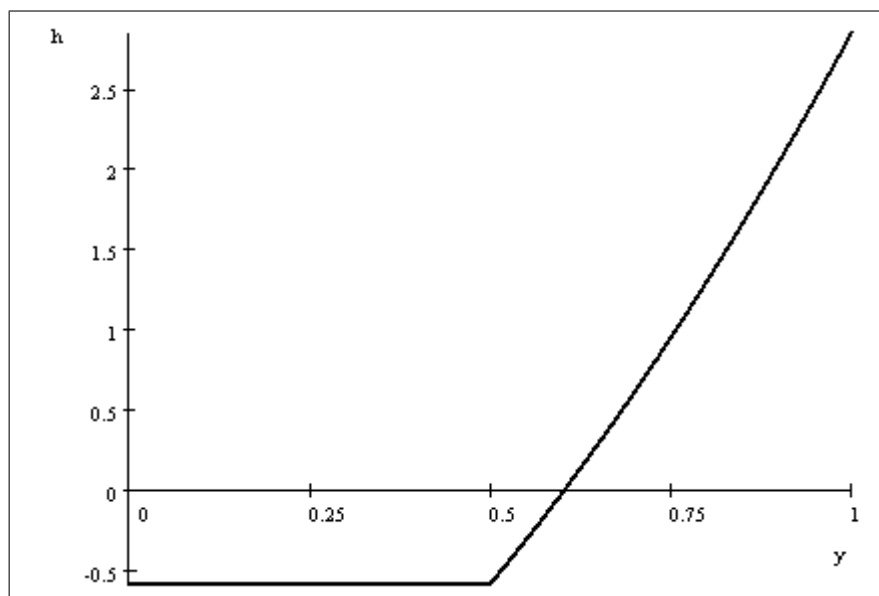
$$\begin{aligned} U'_{NN}(h^i, y^i) &= \exp \left[-2 \left(2 \left(y^i - \frac{1}{2} \right) - \frac{1}{24} h^i \right) \right] \\ U'_{SN}(h^i, y^i) &= \exp \left[-2 \left(2 \left(y^i - \frac{1}{2} \right) + \frac{1}{24} h^i \right) \right] \\ U'_{SS}(h^i, y^i) &= \int_{1-N_2}^1 \exp \left[-2 \left(y^i - \frac{1}{2} + \max \left(0, \tilde{y}^i - \frac{1}{2} \right) + \frac{1}{24} h^i \right) \right] d\tilde{y}^i \end{aligned}$$

The first-order condition for i is

$$\begin{aligned} 0 &= -\exp \left[-2 \left(2 \left(y - \frac{1}{2} \right) - \frac{1}{24} h^i \right) \right] + \frac{1}{2} \exp \left[-2 \left(2 \left(y - \frac{1}{2} \right) + \frac{1}{24} h^i \right) \right] \\ &\quad + \frac{1}{2} \exp \left[-2 \left(y - \frac{1}{2} + \frac{1}{24} h^i \right) \right] \left(1 - \frac{1}{2} \exp[-1] \right) \end{aligned}$$

If $y^i < \frac{1}{2}$, the agent lives in the Countryside and buys an amount of housing that is independent of y^i and equal to the amount of housing bought by the marginal city resident (If $y^i = \frac{1}{2}$, the optimal amount of housing is $h^i = -0.578$).

The optimal amount of housing is plotted below:



4 Endogenous Housing Supply

We now revert to the full-fledge game introduced in Section 2. The size of the city in period 2 is decided by residents through a vote. A certain size is a political equilibrium if there is no majority of voters who want to deviate to a smaller or to a bigger city. Together with the political equilibrium condition, we will also have the conditions on location choice and housing investment that we derived in the previous section.

We focus attention on equilibria in undominated pure strategies.¹⁶ We shall see that such an equilibrium always exists.

Suppose the game has an equilibrium where voters select a certain city size N_2 . We let \hat{N}_2 denote a possible deviation from N_2 . The agents take as given the equilibrium p_1 and r_1 that correspond to the supply N_2 . They internalize the fact that changing housing supply affects the rent in period 2 and the value of the housing permits.

The final wealth of agent i is

$$w^i = l_1^i \left(y_1^i + \phi\tau \frac{\hat{N}_2 - N_1}{N_1} \hat{b} - r_1 \right) + l_2^i (y_2^i - \hat{r}_2) + h^i \left(\hat{r}_2 + r_1 + \phi(1 - \tau) \frac{\hat{N}_2 - N_1}{N_1} \hat{b} - p_1 \right)$$

where $\hat{b} = 1 - \hat{N}_2 + \pi g$ and $\hat{r}_2 = 1 - \hat{N}_2$ if there is no aggregate shock, and $\hat{r}_2 = 1 - \hat{N}_2 + g$ otherwise.

Proposition 3 *Given deviation \hat{N}_2 , agent i 's final wealth is*

$$w^i = l_1^i (y_1^i - 1 + N_1 + \phi\tau\Omega) + l_2^i (y_2^i - 1 + \hat{N}_2) + h^i (D + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega),$$

where $D = -\pi g$ if there is no shock and $D = (1 - \pi)g$ if there is a shock, and Ω represents the effect of deviation to \hat{N}_2 on the total market value of new housing permits per unit of existing housing:

$$\Omega = \frac{\hat{N}_2 - N_1}{N_1} (1 - \hat{N}_2 + \pi g) - \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g).$$

To understand Proposition 3, note that on the equilibrium path $\hat{N}_2 = N_2$, $\Omega = 0$, and the expression for i 's wealth boils down to:

$$w^i = l_1^i (y_1^i - 1 + N_1) + l_2^i (y_2^i - 1 + N_2) + h^i D.$$

¹⁶As usual in majority voting, there can be equilibria in dominated strategies involving a coordination failure, which do not select the median voter's preferred policy. The restriction to pure strategies is made to keep notation light.

In equilibrium, the number of new permits $N_2 - N_1$ is fully internalized in house prices and rents. The number of houses in the second period only affects agent i through its real effect: more people will be able to move to the city if N_2 goes up.

Off the equilibrium path, a deviation from the conjectured number of houses creates a potential windfall gain for city residents (in proportion $\phi\tau$) and home owners (in proportion $\phi(1 - \tau)$), but it also generates a price drop for home owners, which is captured by the term $N_2 - \hat{N}_2$.

We now need to resort to the technical assumption (2). This restriction is satisfied if N_1 or τ are not too small or ϕ is not too large. For example, this condition is satisfied when none of the benefits from the building permits accrue to the residents or homeowners. The assumption guarantees that the building permit revenues that accrue to homeowners when supply increases from N_1 to $N_1 + \epsilon$ do not more than compensate the capital losses they incur on the homes they own due to the same rise in supply.

With this assumption, the preferences of voters over \hat{N}_2 are single-peaked and we can apply Downs Theorem. The equilibrium amount of housing supply in period 2 corresponds to the \hat{N}_2 preferred by the median city resident, which we call m . The median city resident has the median income among city residents: i.e. $y^m = 1 - \frac{N_1}{2}$.

The final wealth of the median voter is then given by

$$w^m = \frac{N_1}{2} + \phi\tau\Omega + l_2^m \left(y_2^m - 1 + \hat{N}_2 \right) + h^m \left(D + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega \right).$$

A certain city size is an equilibrium if and only if it maximizes the median voter's welfare. We can now state:

Proposition 4 *The necessary and sufficient conditions for the existence of an equilibrium in which $N_1 < N_2 < 1$ are:*

- (i) *The conditions for a market equilibrium*
- (ii) *the no-political-deviation condition:*

$$\left(h^m - \phi(\tau + h^m(1 - \tau)) \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2 = N_2} \right) U' = U'_{city} \quad (3)$$

where

$$U' = (1 - \pi) U'_{NN} + \pi(1 - \gamma) U'_{SN} + \pi\gamma U'_{SS}$$

and

$$U'_{city} = (1 - \pi) U'_{NN} + \pi(1 - \gamma) U'_{SN} + \pi\gamma \int_{1-N_2}^1 u' \left(\frac{N_1}{2} + \phi\tau\Omega + \max(0, \tilde{y}^m - 1 + N_2) + h^m((1 - \pi)g + \phi(1 - \tau)\Omega) \right) d\tilde{y}^m$$

The necessary and sufficient conditions for an equilibrium with $N_1 = N_2$ are as above but (3) is replaced with

$$\left(h^m - \phi(\tau + h^m(1 - \tau)) \frac{\partial \Omega}{\partial \tilde{N}_2} \Big|_{\tilde{N}_2 = N_2} \right) U' \geq U'_{city}. \quad (4)$$

The equilibrium with endogenous housing supply must satisfy the conditions set out in Proposition 1. Every agent i must choose the optimal housing consumption and investment given the number of houses in the two periods, N_1 and N_2 . However, there is now an additional condition. The number of houses in the second period is endogenously determined by the preference of the median city resident. In turn, the median voter's preferences depend on the amount of housing h^m he owns. Thus, the conditions in (i) and (ii) constitute a system of equations which are necessary and sufficient.

The left-hand side of equation (3) captures the marginal cost in terms of utility from the capital gain/loss for the median voter as a result of an increase in the housing stock. There are two components: the term h^m is due to the linear and negative effect of an increase in N_2 on $r_2 = 1 - N_2$; the other, more complex term consists of the revenue that accrues to the median voter as a homeowner and as a resident from the sale of housing permits. Part of the benefits accrue to the agents because they are city residents, $\phi\tau$. The remainder accrue to the agents as a proportion of the properties they own, $\phi h^m(1 - \tau)$. The right-hand side of the equation represents the marginal benefit of a change in city size computed at the equilibrium size. This benefit comes from two sources. First, a bigger city means a smaller second period rent. Second, a bigger city means a greater probability of earning a surplus from living and working in the city if the idiosyncratic shock occurs.¹⁷

In an equilibrium where city size does not expand to its maximum value, the marginal benefit of increasing the size of the city cannot be greater than the marginal cost of increasing the size of the city. However, because citizens are not allowed to reduce the size of their city, it can happen that the marginal cost of city expansion exceeds the marginal benefit. This explains the inequality in 4.

Finally, one can draw clear welfare implications. Compare the equilibrium housing supply (in the second period), N_2 , with a very high housing supply $N_2 = 1 - \epsilon$, where ϵ is infinitesimal.¹⁸ The hedging properties of housing are the same, but in the latter case

¹⁷At this point in the discussion, it should be clear that allowing for congestion effects in the city, or agglomeration economies, or peer effects would only change the right-hand side of equation (3). This would not affect any of the insights we derive from the model beyond the obvious effects such as, for example, the stronger the negative congestion externality, the smaller the city size.

¹⁸There is a discontinuity between $N_2 = 1 - \epsilon$ and $N_2 = 1$. In the former case, the rent in the second period is $r_2 = \epsilon$ if there is no shock and $r_2 = g + \epsilon$ if there is a shock: thus, the house is still a valuable

the second-period rent is lower and the disposable income in the second period is higher for every level of productivity y^i . Hence, the choice set at time 1 improves for every agent.

In other words, suppose that, after y^i is revealed but before agents choose h^i , agent i is asked what $N_2 \in [N_1, 1)$ he prefers. Every agent would select $N_2 \rightarrow 1$. This result holds a fortiori before y^i is revealed. Hence, the supply level determined in Proposition 4 is inefficiently low, in a very strong sense.

For expositional purposes, our findings so far have been expressed in terms of a generic utility function $U(\cdot)$. However, as our agents have CARA utility, the same results can be re-written in a pure parametric form:

Proposition 5 *Let $a > 1$ be the risk-aversion coefficient. The first-order condition for a housing market equilibrium is:*

$$(1 - \gamma) \exp[-ah^m g] + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) = 1,$$

and the first-order condition for a political equilibrium is

$$\begin{aligned} & 1 - h^m + \phi(\tau + h^m(1 - \tau)) \left(\frac{1}{N_1}(1 + N_1 - 2N_2 + \pi g)\right) \\ = & \pi \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2). \end{aligned}$$

All the variables in this proposition can in principle be replaced by real-world values. Our two-period model is too simple to be used for empirical work. But this result suggests that a suitable infinite-horizon version could provide a useful platform to perform empirical analysis in this area.

5 Persistence in Housing Supply

Now that we have a full characterization of the equilibrium, we can use it to answer a number of natural questions. In this section, we study the dynamic properties of our equilibrium. Namely, how does this period's city size affect next period's size?.

Proposition 6 *The equilibrium number of houses in the second period N_2 is a strictly increasing function of the number of houses in the first period N_1 .*

hedge. In the latter case, the rent in the second period is always zero and the house cannot be used as a hedge. The welfare comparison between $N_2 = 1 - \epsilon$ and $N_2 = 1$ is ambiguous.

There are two cases. If $N_2 = N_1$ in equilibrium, then it is immediate that a higher increase in N_1 causes a higher. If instead the initial N_1 yielded an interior solution, we can use the first-order conditions to determine the effect of an increase in N_1 . We know from point (5) of Proposition 2 that an increase in N_1 does not make agent i change his housing investment. However, an increase in N_1 changes the identity of the median city resident, who is now poorer and buys less housing. Hence, the voting outcome changes and allows for a strictly higher N_2 .

This proposition captures a phenomenon of great practical importance. Consider a community that, for exogenous reasons (natural barriers) or past events (a period of very fast local growth), is today characterized by a low housing supply (recall that low housing supply in the model is equivalent to high housing price to income ratio). Our analysis predict that this unbalance will persist over time. Its residents choose to be highly invested in local housing and will vote against growth.

This observation could guide empirical work: if we observe a strong correlation between the existence of natural barriers and house prices, we should not infer that high prices are due exclusively to the barriers. The vicious cycle between ownership and supply plays a role. This is important for policy purposes: as the next section argues, we can find a number of instruments to mitigate this effect.

Example of Persistence

Before moving to the analysis of institutional reform, we briefly discuss a numerical example of persistence in housing supply. Assume that

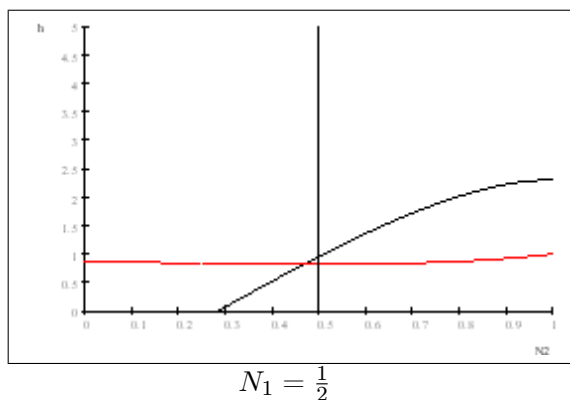
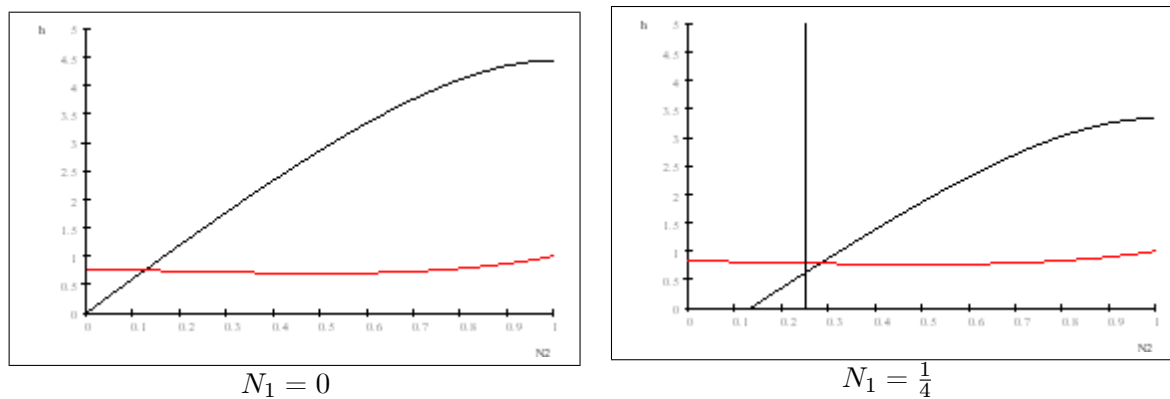
$$a = 2, \gamma = \frac{1}{2}, g = \frac{1}{12}, \phi = 0, \pi = \frac{1}{2}.$$

Then, the two first-order conditions become

$$\begin{aligned} -1 + \frac{1}{2} \exp \left[-2 \left(\frac{1}{12} h \right) \right] + \frac{1}{2} \exp \left[-2 \left(\frac{n}{2} - N + \frac{1}{12} h \right) \right] \left(\frac{3}{2} - N - \frac{1}{2} \exp [-2N] \right) &= 0, \\ 1 - h &= \frac{1}{4} \exp \left[-2 \left(\frac{n}{2} - N + \frac{1}{12} h \right) \right] (1 - N). \end{aligned}$$

We now plot the locus of the market equilibrium (black line) and the locus of the political equilibrium (red line) for various values of N_1 . The intersection of the two loci is the solution to the unconstrained problem (disregarding the requirement that $N_2 \geq N_1$). If the intersection is to the right of N_1 , then it is the solution of the problem. If it is to the left,

then the solution is given by the intersection of N_1 and the market equilibrium locus.



In the first plot ($N_1 \rightarrow 0$), the market equilibrium locus intersects the political equilibrium locus at approximately $N_2 = 0.13$; in the second plot ($N_1 = 1/4$) at 0.27; and in the third plot ($N_1 = 1/2$) at 0.48. In the first two cases, the $N_2 \geq N_1$ constraint is not binding. In the third case it is, and the equilibrium value of N_2 is then 0.50. As the proposition predicts, these three values are strictly increasing.

6 Institutional Reform

This section uses the model to study the effects of a number of policies that have been proposed.

6.1 Housing Permit Allocation

It is clear that the distribution of building permits plays an important part in determining equilibrium supply of housing. The next proposition characterizes the effect of allocating less permits to cronies and more permits to citizens (i.e. an increase in ϕ). Policy proposals in this sense have been made by Glaeser, Schuetz, and Ward (2006) in the context of Greater Boston.

Proposition 7 *Start from a situation where the sale of housing permit does not benefit citizens directly ($\phi = 0$) and the initial city size is sufficiently large ($N_1 > \frac{1}{2}$). Then, a marginal increase in ϕ causes an increase in housing supply.*

Now, compare the effect of an increase in ϕ when owners get all the benefit ($\tau = 0$) or residents get all the benefit ($\tau = 1$). Start from an equilibrium where $\phi = 0$, $N_2 < \frac{1+N_1}{2}$ and the median voter owns \hat{h}^m units of housing. A marginal increase in ϕ causes an increase in housing supply that is greater when $\tau = 0$ rather than $\tau = 1$ if and only if $\hat{h}^m > 1$.

The first part of this proposition says that giving the median voter a larger share of housing windfalls (either as a resident or as a homeowner) will make him want to vote for more urban growth.

The second part states that assigning permits to homeowners rather than to city residents is better for growth if and only if the median resident owns more than a unit of housing. If the median voter is highly invested in housing, handing out permits on the basis of ownership will make him want to issue a larger number of permits. In practice, we do not expect the choice between residents and owners to be crucial because in most communities the median voter owns exactly one home (probably because of indivisibilities – see discussion below).

6.2 Incentives for homeownership

Suppose that the tax system creates distortions between renting and buying. What happens in the second period is irrelevant because it gets discounted. Suppose that there is a tax or a subsidy on house purchases on the part of citizens (but not REITs). For every dollar of housing purchased the state offers a subsidy of s cents (or a tax if s is negative).

Assume that $\phi = 0$ (for notational simplicity only: the proof would go through for any ϕ). The final wealth is now

$$w^i = l_1^i (y_1^i - r_1) + l_2^i (y_2^i - r_2) + h^i (r_2 + r_1 - (1 - s)p_1).$$

That is,

$$w^i = l_1^i (y_1^i - 1 + N_1) + l_2^i (y_2^i - 1 + N_2) + h^i (D + s(2 - N_1 - N_2)).$$

It is easy to see that, ceteris paribus, a higher s increases h^i . Hence, in equilibrium the median voters buys more housing and votes for less housing expansion.

Proposition 8 *A subsidy (tax) on house ownership reduces (boosts) housing supply.*

Most Western countries have tax regimes that favor owning over renting. Apart from portfolio allocation distortions, we find that these regimes generate less housing supply. This highlights a tension between the goal of making homeownership more affordable in the short term (by encouraging purchase by families rather than by institutional investors) and the goal of making it more affordable in the long term (by encouraging voters to allow for new construction).

6.3 Multiple Cities and Decentralized Decision-Making

Our core model was build on a two-fold assumption: there is only one city and only city residents participate in the political process that determines housing supply. Both parts of the assumptions can be questioned in practice. On the one hand, metropolitan areas in the US and in Europe tend to be divided in a number of independent jurisdictions. On the other hand, state governments in the US and national governments in Europe often try to affect local housing supply. It is then natural to ask what are the effect of centralizing or decentralizing housing policy decisions.

We begin with decentralization. Suppose that there are m independent cities. The cities are initially of identical size: each with a stock of housing $\frac{N_1}{m}$. The model is as before (in particular, the productivity shock is the same across cities), but now at the end of period 1 each of the m cities hold a separate election to decide the local housing supply.¹⁹

If $\phi = 0$, there is no difference with the core model: the benefit from a deviation is unchanged and the equilibrium is exactly as in Proposition 4. If however $\phi > 0$, things change. If the city is smaller, the median voter in m gets a larger share the revenues accruing from selling construction licenses in his city. Formally, the marginal benefit of an

¹⁹We assume that the m independent cities are still part of the same labor market. It can be argued that this is indeed the case for different municipalities in a metropolitan area. It would be extremely interesting to look at multiple labor markets, with shocks that are nor perfectly correlated, but this is left to future research.

extra license becomes

$$\frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2=N_2} = \frac{m}{N_1} \left(1 + \frac{N_1}{m} - 2N_2 + \pi g \right),$$

which is increasing in m (and tends to 1 when the number of cities tends to infinity). It is then immediate from the first-order political economy condition of Proposition 4 that the second period supply must increase (in an interior equilibrium):

Proposition 9 *If city planning decisions are made at a more decentralized level, housing supply in the city increases.*

Let us now turn to a more centralized planning system. Instead of assuming that housing supply is decided by city residents, suppose that all citizens vote on housing supply; i.e., the political process takes into consideration not only the interest of city residents but also those of potential future residents of the city. By proposition 2, the amount of housing is an increasing function of productivity. The median citizen owns less housing than the median city residents. Hence

Proposition 10 *If city planning decisions are made at a national level, housing supply in the city is higher.*

In sum, there appears to be an interesting U-shaped relation between housing supply and planning centralization. The lowest housing supply is achieved when planning decisions are made by a polity that corresponds to the relevant labor market. This is due to the fact that housing price externalities operate at the labor market level. A discrepancy between the scope of externalities and the scope of policy is likely to make policy less responsive to the desire of maintaining high housing prices.

6.4 Barriers to Fractional Ownership

In practice, there are serious barriers to fractional ownership. Caplin et al. (1997) have argued that this imposes a sizeable cost on households. In our baseline case, fractional ownership is allowed. In this section, we shall assume that fractional ownership is impossible.

Suppose $h^i \in \{0, 1\}$. Our first-order condition on the amount of housing owned by the median voter is replaced by a comparison between the level of wealth that the median voter gets if he buys a house rather than no house.²⁰

²⁰It is trivial to extend this discussion to the case where $h^i \in \{0, 1, 2, \dots\}$.

Proposition 11 *If allowing for fractional ownership reduces (increases) the amount of housing that the median voter owns, housing supply increases (decreases).*

Consider a community where fractional ownership is impossible and, in the current equilibrium, the median voter owns a house. Suppose that, if fractional ownership is introduced, the median voter would indeed take advantage of it by releasing some of the home equity to third parties. In this plausible case, the support for urban growth will increase. Citizens will hence receive two benefits from the introduction of fractional ownership, a direct one coming from the ability to fine-tune their portfolio allocation and an indirect one from the additional housing supply.

7 Conclusions

Our goal in this paper was to build a simple housing model with endogenous housing consumption, housing investment, and supply regulation. We have shown that this model generates undersupply of housing, and that this undersupply is persistent over time. The model also allows us to analyze a number of policy changes.

Ours is an extremely stylized model. There are only two departures from complete markets: future land development is non-contractible and future income shocks are non-insurable. While it is important to know that these two frictions can by themselves generate a number of interesting phenomena, we also acknowledge the need for future work to enrich this basic model with more features of actual housing markets.

An obvious extension concerns local externalities; e.g., congestion costs and agglomeration economies. Our framework can easily be amended to include an interaction term in people's utility functions, which depends on who lives in the city. This term will be reflected in the first-order condition which determines housing supply (Equation 3 on page 17) with obvious effects on the equilibrium outcome.

Issues related to the provision of local public goods financed by local taxes are more complex. They introduce a trade-off of a different nature than the ones already incorporated in our model: limiting growth reduces the tax base.

One can also introduce frictions in the housing market. For example, suppose that people must pay a moving cost if they move between period 1 and period 2. The effect of this form of friction on the rent risk is ambiguous. On one hand, city residents are somewhat protected against the influx of outsiders. On the other, if rents do go up current residents do not have a free option of moving to the countryside. It is unclear what the overall effect

would be on the hedging motive for homeownership – and hence on the political support for urban growth.

Another relevant form of housing market frictions are credit constraints. Such constraints are likely to reinforce the positive relation between income and housing investment already in our model, especially if we allow for heterogeneous homes within locations.

Our theoretical results lead to a number of testable implications. A key insight we draw from the model concerns the persistence of housing undersupply. This theoretical prediction sheds new light on the empirical finding that historical housing density predicts current tightness in housing supply (e.g., Evenson and Wheaton, 2003, Glaeser and Ward, 2006). We also note that the U.S. metropolitan areas that have experienced the greatest housing price growth over the past three decades are areas that were already expensive in the Seventies. Furthermore, the households who moved into these areas recently have tended to be richer than the ones who were already there, pushing early owners to a lower rank within the local income distribution (Gyourko, Mayer and Sinai, 2005, Ortalo-Magné and Rady, 2005) – the type of labor income risk we focus on in our model.

Our model yields testable implications on voter behavior. The preferences of voters for supply restrictions depend on voters' investment in housing. Dubin, Kiewiet and Noussair (1985) analyze voting data on urban growth control measures on the ballot in the city and county of San Diego in 1988. They take advantage of cross-sectional differences in the socio-economic make-up of precincts to tease out the factors correlated with support for growth controls. They find strong support for the hypothesis that homeowners are more likely to favor growth controls.²¹

Finally, the model highlights a potentially important link between housing supply, the welfare system and labor market regulations. We assumed that labor income risk is uninsurable. However, partial insurance may exist if the state provides a safety net through welfare payments or job protection. In our framework, a reduction in idiosyncratic labor income risk would lead to a lower desire for homeownership and higher housing supply. It is interesting to note that Australia, the UK and the US have much higher ownership rates than Germany and Switzerland and have experienced much stronger housing price growth since the Seventies. According to Evans and Hartwich (2005) the differences in housing

²¹Using the US National Election Study, Scheve and Slaughter (2001) show that, controlling for other individual characteristics, people who live in areas with a concentration of firms in industries vulnerable to international competition are more likely to oppose free trade agreements if they own their home rather than if they rent it.

prices growth rates across these countries are due primarily to differences in supply regulations. These facts appear to be consistent with a situation where German and Swiss workers face a lower uninsurable labor risk than their Anglo-Saxon counterparts. It is an intriguing possibility that the same combination of labor market turbulence and unemployment policies at the heart of the work of Ljungqvist and Sargent (1998, 2005) on difference in unemployment trends across Western countries, could also explain the differences in the trends of their housing prices.

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Appendix

Proof of Proposition 1

The first-order condition on h^i is

$$-(1-\pi)\pi gU'_{NN} + \pi(1-\gamma)(1-\pi)gU'_{SN} + \pi\gamma(1-\pi)gU'_{SS} = 0.$$

We write

$$\Psi(h^i, y^i) \equiv -U'_{NN} + (1-\gamma)U'_{SN} + \gamma U'_{SS}.$$

The first-order condition is satisfied if and only if $\Psi(h^i, y^i) = 0$. Note also that

$$\Psi_h(h^i, y^i) = \pi gU''_{NN} + (1-\pi)g(1-\gamma)U''_{SN} + (1-\pi)g\gamma U''_{SS} < 0.$$

Hence, for every y^i there exists a unique h_1 such that $\Psi(h^i, y^i) = 0$. ■

Proof of Proposition 2

To prove (1), note that

$$\Psi_y(h^i, y^i) = \begin{cases} 0 & \text{if } y^i < 1 - N_2 \\ -U''_{NN} + (1-\gamma)U''_{SN} & \text{if } 1 - N_2 < y^i < 1 - N_1 \\ -2U''_{NN} + 2(1-\gamma)U''_{SN} + \gamma U''_{SS} & \text{if } y^i > 1 - N_1 \end{cases}$$

Re-write the last expression as

$$\Psi_y(h^i, y^i) = 2\left(-\frac{U''_{NN}}{U'_{NN}}\right)U'_{NN} - 2(1-\gamma)\left(-\frac{U''_{SN}}{U'_{SN}}\right)U'_{SN} - \gamma\left(-\frac{U''_{SS}}{U'_{SS}}\right)U'_{SS}.$$

As the utility is CARA, we can write

$$\begin{aligned} \Psi_y(h^i, y^i) &\propto 2U'_{NN} - 2(1-\gamma)U'_{SN} - \gamma U'_{SS} \\ &= U'_{NN} - (1-\gamma)U'_{SN} \\ &= \gamma U'_{SS} > 0 \end{aligned}$$

where the two equalities are due to the foc. Similarly, the second expression can be rewritten as

$$\Psi_y(h^i, y^i) = U'_{NN} - (1-\gamma)U'_{SN} = \gamma U'_{SS} > 0.$$

Recall that Ψ_h was derived above and was found to be always negative. By the implicit function theorem,

$$\frac{\partial h^i}{\partial y^i} = -\frac{\Psi_y(h^i, y^i)}{\Psi_h(h^i, y^i)} \begin{cases} = 0 & \text{if } y^i < 1 - N_2 \\ > 0 & \text{if } y^i > 1 - N_2 \end{cases}$$

To prove (2), examine

$$\Psi(0, y^i) \equiv -U'_{NN}(0, y^i) + (1-\gamma)U'_{SN}(0, y^i) + \gamma U'_{SS}(0, y^i).$$

Note that

$$\begin{aligned} U'_{NN}(0, y^i) &= u'(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2)) \\ U'_{SN}(0, y^i) &= u'(\max(0, y^i - 1 + N_1) + \max(0, y_1^i - 1 + N_2)) \\ U'_{SS}(0, y^i) &= E_{\tilde{y}}[u'(\max(0, y^i - 1 + N_1) + \max(0, \tilde{y}^i - 1 + N_2))] \end{aligned}$$

Recall that $\Psi_y (h^i, y^i)$ is non-decreasing in y^i . First, if $y^i < 1 - N_2$,

$$\begin{aligned} U'_{NN} (0, y^i) &= u' (0) \\ U'_{SN} (0, y^i) &= u' (0) \\ U'_{SS} (0, y^i) &= E_{\tilde{y}^i} [u' (\max (0, \tilde{y}^i - \bar{y}_{N_2}))] < u' (0) \end{aligned}$$

Hence, if $y^i < 1 - N_2$,

$$\Psi (0, y^i) < -u' (0) + (1 - \gamma) u' (0) + \gamma u' (0) = 0.$$

Second, it is easy to see that there exists a threshold such that, for a higher y^i ,

$$\begin{aligned} &u' (\max (0, y^i - 1 + N_1) + \max (0, y_1^i - 1 + N_2)) \\ &< E_{\tilde{y}} [u' (\max (0, y^i - 1 + N_1) + \max (0, \tilde{y}^i - 1 + N_2))] . \end{aligned}$$

In that case, $\Psi (0, y^i) > 0$. As $\Psi (0, y^i)$ is non-decreasing in y^i , there exists a threshold $y^* \geq 1 - N_2$ such that $\Psi (0, y^i) \geq 0$ if and only if $y^i \geq y^*$. This means that agents below (above) y^* choose a negative (positive) amount of housing.

Now, turn to (3). For any positive ε , consider an agent with $y^i = y^* + \varepsilon$. Consider $h^i = 1$ and note that

$$\begin{aligned} U_{NN} (1, y^i) &= u (\max (0, y^i - 1 + N_1) + \max (0, y^i - 1 + N_2) - \pi g) \\ U_{SN} (1, y^i) &= u (\max (0, y^i - 1 + N_1) + \max (0, y_1^i - 1 + N_2) + (1 + \pi) g) \\ U_{SS} (1, y^i) &= E_{\tilde{y}^i} [u (\max (0, y^i - 1 + N_1) + \max (0, \tilde{y}^i - 1 + N_2) + (1 + \pi) g)] \end{aligned}$$

Note that $U_{NN} (1, y^i)$, $U_{SN} (1, y^i)$, and $U_{SS} (1, y^i)$ do not depend on γ , and that

$$U'_{NN} (1, y^* + \varepsilon) < U'_{SN} (1, y^* + \varepsilon).$$

Hence, for γ sufficiently high,

$$\Psi (1, y^* + \varepsilon) > 0,$$

and the optimal h^i must be greater than 1.

To prove (4), consider

$$\begin{aligned} \Psi_{N_2} (h^i, y^i) &= -U''_{NN} + (1 - \gamma) U''_{SN} \\ &\quad + \gamma \int_{1-N_2}^1 u'' (\max (0, y^i - 1 + N_1) + \tilde{y}^i - 1 + N_2 + (1 + \pi) g) d\tilde{y}^i \quad \text{if } y_1^i > 1 - N_2 \end{aligned}$$

and

$$\Psi_{N_2} (h^i, y^i) = \gamma \int_{1-N_2}^1 u'' (\max (0, y^i - 1 + N_1) + \tilde{y}^i - 1 + N_2 + (1 + \pi) g) d\tilde{y}^i \quad \text{if } y_1^i < 1 - N_2.$$

If $y_1^i > 1 - N_2$, CARA implies

$$\begin{aligned} \Psi_{N_2} (h^i, y^i) &\propto U'_{NN} - (1 - \gamma) U'_{SN} - \gamma \int_{1-N_2}^1 u' (\max (0, y^i - 1 + N_1) + \tilde{y}^i - 1 + N_2 + (1 + \pi) g) d\tilde{y}^i \\ &> U'_{NN} - (1 - \gamma) U'_{SN} - \gamma U'_{SS} = 0. \end{aligned}$$

Instead, if $y_1^i < 1 - N_2$, $\Psi_{N_2} (h^i, y^i) < 0$. We then have

$$\frac{\partial h^i}{\partial N_2} \begin{cases} < 0 & \text{if } y^i > 1 - N_2 \\ > 0 & \text{if } y^i < 1 - N_2 \end{cases}$$

To prove (5), note that, if $y^i < 1 - N_1$, $\Psi_{N_1}(h^i, y^i) = 0$. If $y^i > 1 - N_1$,

$$\Psi_{N_1}(\hat{h}^i, y^i) = -U''_{NN}(\hat{h}^i, y^i) + (1 - \gamma)U''_{SN}(\hat{h}^i, y^i) + \gamma U''_{SS}(\hat{h}^i, y^i),$$

which, by CARA, can be re-written as $\Psi(\hat{h}^i, y^i)$ and, by the first-order condition, is equal to zero. ■

Proof of Proposition 3

Given the conjectured level of N_2 , the agents face first period rent and price as follows:

$$r_1 = 1 - N_1 + \phi\tau \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g)$$

$$p_1 = 1 - N_1 + \left(1 + \phi \frac{N_2 - N_1}{N_1}\right) (1 - N_2 + \pi g)$$

The payoff of living and working in the city for agents y^i assuming a deviation to \hat{N}_2 is

$$\begin{aligned} & y_1^i + \phi\tau \frac{\hat{N}_2 - N_1}{N_1} \tilde{b} - r_1 \\ = & y_1^i - 1 + N_1 + \phi\tau \left(\frac{\hat{N}_2 - N_1}{N_1} (1 - \hat{N}_2 + \pi g) - \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g) \right) \end{aligned}$$

We denote Ω the change in the total value of the housing permits per unit of existing housing due to a political deviation from N_2 to \hat{N}_2 :

$$\Omega \equiv \frac{\hat{N}_2 - N_1}{N_1} (1 - \hat{N}_2 + \pi g) - \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g).$$

The expected gain from investing in a unit of city housing is affected by a deviation from N_2 to \hat{N}_2 as follows

$$\tilde{r}_2 + r_1 + \phi(1 - \tau) \frac{\hat{N}_2 - N_1}{N_1} \tilde{b} - p_1 = D + N_2 - \hat{N}_2 + \phi(1 - \tau) \Omega$$
■

Proof of Proposition 4

The median city resident decides the election. His wealth in the three possible outcomes is

- If there is no shock:

$$\begin{aligned} w_{NN} &= y^m - 1 + N_1 + \phi\tau\Omega + y^m - 1 + \hat{N}_2 + h^m \left(-\pi g + N_2 - \hat{N}_2 + \phi(1 - \tau) \Omega \right) \\ &= \hat{N}_2 + \phi\tau\Omega + h^m \left(-\pi g + N_2 - \hat{N}_2 + \phi(1 - \tau) \Omega \right) \end{aligned}$$

- If the aggregate shock alone occurs

$$w_{SN} = \hat{N}_2 + \phi\tau\Omega + h^m \left((1 - \pi)g + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega \right)$$

- If both the aggregate and the idiosyncratic shock occur

$$w_{SS} = \frac{N_1}{2} + \phi\tau\Omega + \max\left(0, \tilde{y}^m - 1 + \hat{N}_2\right) + h^m \left((1 - \pi)g + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega \right)$$

Let

$$U^m = (1 - \pi)u(w_{NN}) + \pi(1 - \gamma)u(w_{SN}) + \pi\gamma u(w_{SS})$$

The condition for N_2 to be the equilibrium choice is that

$$\left. \frac{dU^m}{d\hat{N}_2} \right|_{N_2=\hat{N}_2} = 0$$

That is

$$\frac{dU^m}{d\hat{N}_2} = (1 - \pi)u'(w_{NN})\frac{dw_{NN}}{d\hat{N}_2} + \pi(1 - \gamma)u'(w_{SN})\frac{dw_{SN}}{d\hat{N}_2} + \pi\gamma E \left[u'(w_{SS})\frac{dw_{SS}}{d\hat{N}_2} \right].$$

Note that

$$\begin{aligned} \frac{dw_{NN}}{d\hat{N}_2} &= \frac{dw_{SN}}{d\hat{N}_2} = 1 + \phi\tau \frac{d\Omega}{d\hat{N}_2} + h^m \left(-1 + \phi(1 - \tau) \frac{d\Omega}{d\hat{N}_2} \right) \\ &= 1 - K \end{aligned}$$

where

$$K = h^m - \phi(\tau + h^m(1 - \tau)) \frac{d\Omega}{d\hat{N}_2}$$

and

$$\begin{aligned} \frac{dw_{SS}}{d\hat{N}_2} &= \phi\tau \frac{d\Omega}{d\hat{N}_2} - \frac{d}{d\hat{N}_2} \max(0, \tilde{y}^m - 1 - N_2) + h^m \left(-1 + \phi(1 - \tau) \frac{d\Omega}{d\hat{N}_2} \right) \\ &= \phi\tau \frac{d\Omega}{d\hat{N}_2} + I_{\tilde{y}^m > 1 - \hat{N}_2} + h^m \left(-1 + \phi(1 - \tau) \frac{d\Omega}{d\hat{N}_2} \right) \\ &= \begin{cases} 1 - K & \text{if } \tilde{y}^m > 1 - \hat{N}_2 \\ -K & \text{if } \tilde{y}^m < 1 - \hat{N}_2 \end{cases} \end{aligned}$$

Note also that

$$\left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} = \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g).$$

Hence

$$\begin{aligned}
& \left. \frac{dU^m}{d\hat{N}_2} \right|_{\hat{N}_2=N_2} \\
&= (1-\pi) u'(w_{NN})(1-K) + \pi(1-\gamma) u'(w_{SN})(1-K) + \pi\gamma \int_{1-N_2}^1 u'(w_{SS}(h_m, \tilde{y}^m))(1-K) d\tilde{y}^m \\
&\quad + \pi\gamma \int_0^{1-N_2} u'(w_{SS}(h_m, 0))(-K) d\tilde{y}^m \\
&= -((1-\pi) u'(w_{NN}) + \pi(1-\gamma) u'(w_{SN}) + \pi\gamma u'(w_{SS})) K \\
&\quad + (1-\pi) u'(w_{NN}) + \pi(1-\gamma) u'(w_{SN}) + \pi\gamma \int_{1-N_2}^1 u'(w_{SS}(h_m, \tilde{y}^m)) d\tilde{y}^m
\end{aligned}$$

■

Proof of Proposition 5

Start with the condition for a political equilibrium:

$$\left(h^m - \phi(\tau + h^m(1-\tau)) \left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} \right) U' = U'_{\text{city}} \quad (5)$$

First, note that the first-order condition on the housing market implies

$$\begin{aligned}
U' &= (1-\pi) U'_{NN} + \pi(1-\gamma) U'_{SN} + \pi\gamma U'_{SS} \\
&= U'_{NN} \\
&= \exp[-a(N_2 - h^m \pi g)].
\end{aligned}$$

Next,

$$\begin{aligned}
U'_{\text{city}} &= (1-\pi) U'_{NN} + \pi(1-\gamma) U'_{SN} + \pi\gamma \int_{1-N_2}^1 u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m \\
&= U'_{NN} - \pi\gamma \int_0^{1-N_2} u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m.
\end{aligned}$$

Note that

$$\begin{aligned}
& \int_0^{1-N_2} u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m \\
&= \int_0^{1-N_2} \exp[-a(y^m - 1 + N_1 + h^m(1-\pi)g)] d\tilde{y}^m \\
&= \int_0^{1-N_2} \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] d\tilde{y}^m \\
&= \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] (1-N_2)
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{U'_{\text{city}}}{U'} &= \frac{U'_{NN} - \pi\gamma \int_0^{1-N_2} u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m}{U'_{NN}} \\
&= \frac{\exp[-a(N_2 - h^m\pi g)] - \pi\gamma \exp[-a(\frac{N_1}{2} + h^m(1-\pi)g)](1-N_2)}{\exp[-a(N_2 - h^m\pi g)]} \\
&= 1 - \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right](1-N_2)
\end{aligned}$$

Thus, the political equilibrium condition is:

$$h^m - \phi(\tau + h^m(1-\tau)) \left. \frac{\partial \Omega}{\partial \tilde{N}_2} \right|_{\tilde{N}_2=N_2} = 1 - \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right](1-N_2),$$

which can immediately be re-written as in the statement of the proposition.

The other first-order condition is:

$$-U'_{NN} + (1-\gamma)U'_{SN} + \gamma U'_{SS} = 0$$

where

$$\begin{aligned}
U'_{NN} &= \exp[-a(N_2 - h^m\pi g)] \\
U'_{SN} &= \exp[-a(N_2 + h^m(1-\pi)g)] \\
U'_{SS} &= \int_0^1 \exp[-a((y^m - 1 + N_1) + \max(\tilde{y}^m - 1 + N_2, 0) + h^m(1-\pi)g)] d\tilde{y}^m \\
&= \exp[-a((y^m - 1 + N_1) + h^m(1-\pi)g)] \int_0^1 \exp[-a \max(\tilde{y}^m - 1 + N_2, 0)] d\tilde{y}^m \\
&= \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] \int_0^1 \exp[-a \max(\tilde{y}^m - 1 + N_2, 0)] d\tilde{y}^m \\
&= \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-2N_2]\right)
\end{aligned}$$

because

$$\begin{aligned}
&\int_0^1 \exp[-a \max(\tilde{y}^m - 1 + N_2, 0)] d\tilde{y}^m \\
&= \int_0^{1-N_2} \exp[0] d\tilde{y}^m + \int_{1-N_2}^1 \exp[-a(\tilde{y}^m - 1 + N_2)] d\tilde{y}^m \\
&= 1 - N_2 - \frac{1}{2} \exp[-a(1-1+N_2)] + \frac{1}{2} \exp[-a(1-N_2-1+N_2)] \\
&= 1 - N_2 - \frac{1}{2} \exp[-aN_2] + \frac{1}{2} \\
&= \frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]
\end{aligned}$$

which re-writes as

$$\begin{aligned}
&-\exp[-a(N_2 - h^m\pi g)] + (1-\gamma) \exp[-a(N_2 + h^m(1-\pi)g)] \\
&+ \gamma \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\
&= 0
\end{aligned}$$

and can be simplified to

$$-1 + (1 - \gamma) \exp[-ah^m g] + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) = 0.$$

■

Proof of Proposition 6

Starting from the first-order conditions in Proposition 5, let

$$\begin{aligned} f &= -1 + (1 - \gamma) \exp[-ah^m g] + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\ g &= (1 - h^m) - \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) \end{aligned}$$

Compute partial derivatives and simplify using the first-order conditions:

$$\begin{aligned} f_h &= -ag(1 - \gamma) \exp[-ah^m g] - ag\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\ &= -ag < 0 \end{aligned}$$

$$\begin{aligned} g_h &= -1 + ag\pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) \\ &= -1 + ag(1 - h^m) = -1 + ag - agh^m < 0 \text{ since } g < 1/a \end{aligned}$$

$$\begin{aligned} f_{N_2} &= a\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\ &\quad + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (-1 + \exp[-aN_2]) \\ &= \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(a\left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) + (-1 + \exp[-aN_2])\right) \\ &= a\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) > 0 \end{aligned}$$

$$\begin{aligned} g_{N_2} &= -a\pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) + \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \\ &= \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (-a(1 - N_2) + 1) \\ &= \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (aN_2 - a + 1) \leq 0 \\ &\quad \exp\left[-2\left(\frac{n}{2} - N + \frac{1}{12}h\right)\right] (2N - 1) \end{aligned}$$

$$f_{N_1} = -\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) < 0$$

$$g_{N_1} = \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) > 0$$

The implicit function theorem says that

$$\begin{bmatrix} \frac{\partial h^m}{\partial N_1} \\ \frac{\partial N_2}{\partial N_1} \end{bmatrix} = \begin{bmatrix} f_h & f_{N_2} \\ g_h & g_{N_2} \end{bmatrix}^{-1} \begin{bmatrix} -f_{N_1} \\ -g_{N_1} \end{bmatrix}$$

That is

$$\begin{bmatrix} \frac{\partial h^m}{\partial N_1} \\ \frac{\partial N_2}{\partial N_1} \end{bmatrix} = \begin{bmatrix} -\frac{g_{N_2}}{\Delta} & \frac{f_{N_2}}{\Delta} \\ \frac{g_h}{\Delta} & -\frac{f_h}{\Delta} \end{bmatrix} \begin{bmatrix} -f_{N_1} \\ -g_{N_1} \end{bmatrix}$$

where $\Delta = -f_h g_{N_2} + g_h f_{N_2}$ is the Jacobian, which must be negative if the second-order condition is satisfied. Then

$$\begin{aligned} \frac{\partial N_2}{\partial N_1} &= -\frac{g_h}{\Delta} f_{N_1} + \frac{f_h}{\Delta} g_{N_1} \\ &= \frac{1}{(-)} (-(-)(+) + (-)(-)) > 0 \end{aligned}$$

This shows that the unconstrained N_2 is increasing in N_1 . If we denote the unconstrained value with \bar{N}_2 , it is immediate to see that the constrained value $\max(\bar{N}_2, N_1)$ is increasing in N_1 a fortiori. ■

Proof of Proposition 7

Start with

$$\left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} = \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g)$$

$$\begin{aligned} f &= -1 + (1 - \gamma) \exp[-2h^m g] + \gamma \exp \left[-2 \left(\frac{N_1}{2} - N_2 + h^m g \right) \right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-2N_2] \right) \\ g &= \left(1 - h^m + \phi \left(\tau + \hat{h}^m (1 - \tau) \right) \left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} \right) - \pi \gamma \exp \left[-2 \left(\frac{N_1}{2} - N_2 + h^m g \right) \right] (1 - N_2) \end{aligned}$$

Assume that $\phi = 0$ and look at the effect of an increase in ϕ .

By the Implicit Function Theorem

$$\begin{bmatrix} \frac{\partial h^m}{\partial \phi} \\ \frac{\partial N_2}{\partial \phi} \end{bmatrix} = \begin{bmatrix} -\frac{g_{N_2}}{\Delta} & \frac{f_{N_2}}{\Delta} \\ \frac{g_h}{\Delta} & -\frac{f_h}{\Delta} \end{bmatrix} \begin{bmatrix} -f_\phi \\ -g_\phi \end{bmatrix}$$

Note that

$$g_\phi = \left(\tau + \hat{h}^m (1 - \tau) \right) \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g) > 0$$

if $N_2 < \frac{1+N_1}{2}$. In turn, $N_2 < \frac{1+N_1}{2}$ holds if $N_1 > \frac{1}{2}$. As $f_\phi = 0$, we have

$$\begin{aligned} \frac{\partial h^m}{\partial \phi} &= -\frac{f_{N_2}}{\Delta} g_\phi = -\frac{(+)}{(-)} (+) > 0 \\ \frac{\partial N_2}{\partial \phi} &= \frac{f_h}{\Delta} g_\phi = \frac{(-)}{(-)} (+) > 0. \end{aligned}$$

The proofs of Propositions 8–11 follow directly from the arguments in the main text. ■