Spatial Modeling in Transportation II: Railroad Pricing, Alternative Markets, and Capacity Constraints[*]

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Abstract

We describe competition over space between a competitive shipping industry (truck-barge) and one with market power (the railroad). The latter prices so as to "beat the competition" in equilibrium, or else at the monopoly price, if that is lower. The monopoly price rises more slowly than do the costs of transportation (freight absorption) if the spatial demand at each point is log-concave. With log-convex demand, price rises quicker than the costs of transportation. The model readily extends to allow for multiple destination points. Introducing capacity constraints for the railroad indicates that only the furthest locations will be served by the railroad since they yield the highest mark-ups per mile.

KEYWORDS: Spatial equilibrium, market power, transportation networks, mode/destination choice, equilibrium mode price, capacity constraints.

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1 Introduction

Much has been written in the Economics literature about spatial competition between firms. The actual shipping of commodities is usually assumed competitive, however, and is typically not further discussed. In this paper, we describe the shipping companies (such as railroads) as wielding market power. In practice, the shipping industries are responsible for a considerable fraction of economic activity, and some have considerable market clout. While some, such as trucking or river transportation by barge, may reasonably be modeled as approximately competitive, others, like rail, have power by dint of their ownership of the relevant infrastructure (the railway tracks) and an absolute cost advantage over some traffic routes.

This paper reflects this (rather crude) depiction of market structure by providing an explicit description of shipping companies. We determine the equilibrium for a market in which a monopolized sector (rail) faces competition from a competitive sector (truck-barge). This involves finding the equilibrium in three related markets. The truck and barge markets are complementary, in that trucks need to be used to get shipments to the river system to be loaded onto barges. The railroad is a substitute for the joint truck-barge mode and provides an alternative way of shipping to the final destination (or terminal) market. A further contribution of the paper from our previous work is to introduce alternative terminal markets. This extension allows shippers to choose the terminal market as well as the mode. This may be particularly important in the case of railroads because the railroad tracks may be linked to a network with multiple terminal markets. By contrast, the geography of the river system may limit the terminal market shipping options.

The simplest version of the model is derived when shipper demand is totally inelastic regarding the decision of whether to ship or not (the shipper chooses the cheaper mode if shipping). This forms a useful benchmark case by showing clearly that the railroad's pricing policy takes the form of just beating the competition from truck barge (or else choosing the shipper's reservation price, if that is below the truck-barge cost). This set-up implies that the consequences of an improvement in the transport technology in the truck-barge system (reducing the cost of barge traffic, say) will be purely distributional in those areas that remain served by the railroad. Namely, railroad prices will have to go down (and shippers will benefit) even if they still do not ship by truck-barge. This is because the railroad now has to beat a tougher competitor. Shippers gain, and railroads lose out.

There are additional welfare gains once we allow for a downward sloping shipper demand function. We show how such a demand can be derived from a simple cost function for production (of grain, for example). The properties of the demand function determine the properties of the railroad's equilibrium pricing policy. Once again, its actions are constrained by the existence of the competing shipping option (truck-barge). However, when that constraint is not binding, the railroad's pricing behavior can exhibit several interesting patterns, depending on the shape of the demand curve. First, when unconstrained, the price rises with distance.[1] The price, though, may rise faster or slower than the rate of increase in the costs of shipping. Equivalently, the markup may go up faster or slower than the cost. The latter case is called freight absorption, and comes about when demand is not "too" convex. Indeed, most of the demand curves usually considered in economics fall into this class. For linear demand, for example, the railroad's price rises at half the rate that costs rise.[2] For

[1] When constrained, the price may fall even if the distance shipped by the railroad rises. This case arises if the competing sector that the railroad has to beat out, the competitive truck-barge sector, is closer to the terminal market even as the railroad distance to its terminal market is increasing. Think, for example, of a case when truck-barge ships one direction, and the railroad ships in the opposite direction.

[2] Put another way, the incidence on the producer is the same as on the consumer for the linear case. The cost increase due to higher distance is shared equally between them.

sufficiently convex demand curves, the pass-on rate of costs exceeds 100%. In such cases, the welfare cost of market power can be quite large (see Anderson and Renault, 2002).

When the railroad's pricing is locally constrained by the competing truck-barge option, there may be a hidden benefit to an reduction in costs from using the truck-barge system, a benefit that is not identified by just looking at the shippers who actually use trucks and barges for shipping. This benefit accrues because the constraint on the railroad's pricing tightens, and the railroad reduces its price in response. With a downward sloping shipper demand, this causes more than a simple redistribution from railroads to shippers; it also yields a greater volume of production and an increased social surplus. That is, there is an efficiency gain to shippers who do not even use the truck-barge system, and it is larger than the reduction in profits to the railroad. Indeed, this efficiency gain could potentially be quite large.

The stock of railroad trucks and trains is not unlimited. Especially at peak periods, railroads have to choose prices to ration off the number of shippers wishing to use their services. A further contribution of the paper is to introduce capacity constraints in the rail sector. This is done by assuming that there is a fixed number of train-hours available in total (as opposed to assuming that a fixed number of trips can be made). Entering the constraint in this manner allows us to take account of the fact that some trips (longer ones) make more intensive use of the limited rail capacity. Railroad pricing, and the railroad's corresponding allocation of railcars to trips, must be made accordingly. We show that the railroad will choose to serve locations associated with the highest mark-ups per mile, where the operative concept for mark-up per mile is calculated as the total trip mark-up over the distance traversed. The locations that generate the highest mark-up per mile tend to be furthest from the restraining force of the competing rival shipping industry.

The next section sets out the basic model and determines the equilibrium pricing behavior in the three sectors. Section 3 allows for different possible terminal markets for the railroad industry's shipping destination. Section 4 goes through the analysis for downward-sloping demand, and indicates sources of deadweight loss. It also indicates where welfare gains are possible due to improvements in barge efficiency, even on account of shippers who do not use truck-barge to ship. Section 5 then introduces the possibility of capacity constraints in the railroad industry and finds the locations served by rail and the corresponding prices. Section 6 concludes and suggests some directions for future research.

2 The Model

The objective of the research is to model interaction between a competitive transport sector (truck-barge) and one with market power (rail). In keeping with our previous work in the river-canal context (Anderson and Wilson, 2004), we suppose that shipments by river-canal must first be transported by truck to a river terminal, and then loaded onto barges. The river-canal system runs from North to South, and terminates at the final transshipment port. Let the EW distance to the river-canal be in the x direction, and let the NS direction up and down the river-canal be denoted $y \geq 0$. Shipping by truck and by barge is perfectly competitive, and constant per unit per unit distance shipped, at rates t and b respectively. Hence, the cost of shipping from coordinate (y, x) to the terminal market location at $(0, 0)$ is $tx + by$.

The alternative shipping method is by rail. By rail, the shipper incurs costs of r per unit per mile shipped. However, the railroad controls the rails – it has market power over its transport mode, though it is constrained in its exercise of its market power by the existence of the truck-barge option. Rail transport is assumed to follow the "Manhattan metric," meaning that distances must be traversed EW and NS only. We assume that $t > r > b$, so that, if transport modes are priced at cost, the combination of truck and barge is the cheaper option for locations close to the river since the high per mile cost of trucking is offset by the low per unit cost of barge (see Anderson and Wilson, 2004).

However, since the railroad has market power, it will typically exercise that power by pricing above cost. Denote the railroad's price per unit (since it is a price setter) by $p(y, x)$: this is the price for shipping the commodity to the terminal port $(0, 0)$ from coordinate (y, x). Our next objective is to characterize this shipping price.

First assume that each shipment point (i.e., coordinate (y, x)) is associated to a shipment of unit size up to a reservation value that is "high enough" that it plays no role in what immediately follows.[3] In the sequel, we address the case of a binding cap and introduce a downward sloping demand at each point in space. Let the space of shipment points be denoted Ω. This space may be simply thought of as the land cultivated for corn in the mid-West, for example.

We define the "natural market area" for barge shipping as that area for which barge shipping is cheaper than rail shipping. This means that the natural market area is the set

[3] We are thinking here of farmers (say) who directly ship their produce themselves down to the terminal market. Equivalently, they could sell to an intermediary that would then ship the product down to the final market. If the intermediary makes no profit (for example, if there were competing intermediaries offering their services to farmers) then farmers would make exactly the same revenues whether or not they shipped the product themselves or via intermediaries. In that case, none of the results of the model would be altered.

In practice however, there are intermediaries in the agricultural market, and they may have some degree of market power. They are the grain elevators, and they are left out of the model. Grain elevators buy up produce locally from farmers and then take care of shipping. They have some local monopsony power due to spatial proximity insofar as rival elevators are further from nearby farmers. Since farmers have the option of taking care of shipping themselves, the grain elevators might be seen as having countervailing market power with railroads. They also might act to smooth shipping patterns across the season. A fuller description of their behavior remains an objective for further research.

$$\mathcal{B}=\{(y,x)\in\Omega : t\,|x| + by \leq r\,|x| + ry\}$$

The set $\mathcal{B}$ is a cone with vertex at $(0,0)$, and is illustrated in Figure 1. This shape reflects the increasing advantage for points further North of the cheaper river mode in overcoming the higher truck rate with which it is bundled.

INSERT FIGURE 1

Clearly, the railroad will not entertain the prospect of pricing below marginal cost, so that the full area of $\mathcal{B}$ is served by river-canal (i.e., truck-barge). Note that it is optimal to serve all of $\mathcal{B}$ by truck-barge from a cost-minimizing social optimum perspective: the cheaper shipment mode ought to be used. What is perhaps more surprising is that the railroad also serves everything outside $\mathcal{B}$, i.e., it serves its own natural market, $\mathcal{R}$, defined by

$$\mathcal{R}=\{(y,x)\in\Omega : t\,|x| + by > r\,|x| + ry\}$$

The objective of the railroad is to maximize profits over all its natural market, $\mathcal{R}$ – it clearly does not need to serve any point in $\mathcal{B}$ and does not want to either. Since we have assumed that shipping costs are constant per unit shipped and that $t > r$,[4] the railroad can simply choose its price at each point without regard to its price elsewhere. This independence property greatly simplifies the railroad's problem.[5] Given the pricing independence property, the railroad's problem is simply to "beat the competition." For technical reasons (having to do with maximization over an open set), we assume that the railroad can price

[4] The latter condition ensures the railroad has an advantage for locations far enough from the river. Arbitrage (by shipping by truck to iron out anomalies in spatial pricing by the railroad) will not be a concern for the railroad since shipping costs are constant per unit distance shipped.

[5] When we analyze capacity constraints, decisions at one point do affect profits elsewhere. Understanding the current case renders the later one manageable.

to meet the competition and still retain the shipping contract.[6] Formally, we invoke the efficient serving rule introduced by Lederer and Hurter (1987). This means that the commodity is shipped by the more efficient mode (in terms of actual transport cost) in the case of a quoted price tie. The rationale is that the more efficient mode could always profitably undercut the less efficient one.[7] The price charged by the railroad is, therefore, equal to the shipping cost of the rival mode, so that

$$p(y,x) = t\,|x| + by \text{ for } (y,x) \in \mathcal{R}. \tag{1}$$

This is understood as meeting the competition and still making the sale. This price schedule has the property that the railroad's profit per unit shipped is greatest the further the location is distant from the river-canal, *ceteris paribus*, because there it is least constrained. To see this property formally, note that the railroad's profit per unit shipped is given by $p(y,x) - r\,|x| - ry$ for $(y,x) \in \mathcal{R}$, which is simply its price minus its shipping cost. Rewriting and using (1), this means that its profit per unit is $t\,|x| + by - r\,|x| - ry$ for $(y,x) \in \mathcal{R}$. For any given y, each extra mile further from the river (i.e., a rise in $|x|$) raises profit by $t - r > 0$.

The discussion above is summarized in the following Proposition, and the equilibrium prices and costs are illustrated in Figure 2 for a given ordinate, $\bar{y}$.

Proposition 1 *The equilibrium shipping mode is truck-barge for all $(y,x) \in \mathcal{B}$, which is the natural market for truck-barge, with corresponding price $t\,|x| + by$.*

[6] Wilson (1996) presents a model of disaggregate railroad pricing based on such a condition in his model of market dominance.

[7] There is a further technical issue in the case of rival oligopolistic railroads. We then also need that equilibrium play is in strictly non-dominated strategies – otherwise there may be multiple equilibria. In sum, there are some technical difficulties associated to the standard Bertrand homogenous goods pricing model with asymmetric goods that arise here. Dealing with these carefully allows us to pick the "natural" equilibrium: the market at any point is served by the lowest cost shipper, at a price equal to the serving cost of the second lowest cost shipper. This is "meeting the competition."

The equilibrium shipping mode is by rail for all $(y, x) \in \mathcal{R}$, which is the natural market for rail, with price $p(y, x) = t\,|x| + by$ per unit shipped. The railroad's profit per unit rises linearly with distance from the river.

INSERT FIGURE 2

Corollary 2 *The equilibrium allocation is efficient.*

This property follows because each mode serves it natural market and demand is totally inelastic at each point. In other words, there is no deadweight loss because all shipping is done by the cheapest mode. The price level *per se* is not a source of deadweight loss but rather has distributional consequences only. This strong welfare property is relaxed in some of the following extensions. In this model, a reduction in the barge rate causes a reduction in the equilibrium rail price too because the competition is stronger for the alternative option to rail.[8] Hence, a reduction yields a larger natural market for truck-barge, $\mathcal{B}$. The lower barge rate induces a transfer from the railroad to farmers.

We can now see what are the effects of a reservation price "cap" that limits the price that can be charged.[9] Such a price will simply act as a cap on the price charged in the market. Referring to Figure 2, think of a horizontal line at reservation price $\bar{p}$, across the top of the Figure. This represents the price cap, and is drawn in Figure 3.

INSERT FIGURE 3

[8] In a recent survey by one of the authors, *rail* shippers routinely commented on the need for an efficient waterway to temper and restrain railroad pricing. The survey, and related work, is reported in Trrain and Wilson (2004).

[9] One (rather loose) interpretation is that this reservation price reflects the possibility that shippers may store the commodity if the price is currently too low.

Introducing such a cap then means that the mark-up on rail starts diminishing as soon as the truck-barge rate hits the cap. Thereafter, the binding constraint is the reservation price rather than the competing truck-barge industry rate. The mark-up for the rail shipper then decreases with distance away from the river[10] all the way to the point where it hits zero. At this point, the costs of railroad shipping have reached the reservation price for using that mode. Thereafter, there are no shipments that are profitable. If the reservation price represents a long-run value (as opposed to a transitory one, say waiting for an alternative shipping date with higher expected prices at the terminal market), then the land will not be farmed beyond this point.

Proposition 3 *Let the reservation price be $\bar{p}$. The equilibrium shipping mode is truck-barge for all $(y, x) \in \mathcal{B}$, with corresponding price $t\,|x| + by$. The equilibrium shipping mode is by rail for all $(y, x) \in \mathcal{R}$ such that $r\,|x| + ry \leq \bar{p}$ with price $p(y, x) = \min\{t\,|x| + by, \bar{p}\}$ per unit shipped. The railroad's profit per unit rises linearly with distance from the river up to the point where $t\,|x| + by = \bar{p}$ and thereafter diminishes linearly.*

The strong efficiency property of the preceding set-up is retained here.

Corollary 4 *The equilibrium allocation is efficient.*

This property follows since demand still has no elasticity and it is furthermore efficient to serve up to the point where serving cost reaches the reservation price. The rail shipper does this because such shipments still generate profits.

3 Alternative markets

In the previous analysis, we assumed that rail shipments have the same final destination as truck-barge shipments, namely the terminal market at the South

[10] Indeed, if the reservation price is $\bar{p}$, the profit per unit is $\bar{p} - t\,|x| - by$, which decreases with $|x|$.

end of the river. In practice though, railroads are not as constrained by geography as is river transport. The river can only deliver to locations along its banks and at its mouth, if we except costly trucking and in the absence of a canal system. The railroad tracks are typically part of a large network and so railroads can deliver output to any node on a large and expansive grid. This gives the railroad another strategic advantage not enjoyed by the river users. Namely, the railroad has the choice of different possible terminal markets.

If, indeed an alternative terminal market is available for the railroad (for example, a different port, such as the Pacific Northwest, or Baltimore), then the analysis is modified in the following manner. Assume for simplicity that the price received by the farmer is the same at either final market, so that the objective of the farmer at any (y, x) coordinate is still to choose the least cost transport mode.[11]

The same principles apply as above, namely that the railroad will meet the competition in its pricing. However, its natural market will be extended (and the other will contract) with the availability of the second option. Consider the situation illustrated in Figure 4, with a second terminal market, M, at $(\tilde{y}, \tilde{x})$ in the North-East quadrant.

INSERT FIGURE 4

The railroad will find it less costly to ship to M from points (y, x) such that[12]

$$rx + ry > r\,|x - \tilde{x}| + r\,|y - \tilde{y}|\,.$$

Assume, as in the Figure, that $x < \tilde{x}$ for all $(y, x) \in \Omega$. Then, for $y \geq \tilde{y}$, the port M is preferred for $x > [\tilde{x} - \tilde{y}]\,/2$. For $y < \tilde{y}$, the port M is preferred for $y > \frac{[\tilde{x}+\tilde{y}]}{2} - x$.

[11] Different prices at different terminal modes are readily added.

[12] With different prices at different terminal modes, the price advantage of M may be simply added to the RHS.

Suppose that the boundary between $\mathcal{B}$ and $\mathcal{R}$ (using the original definitions, i.e., before we allow for the existence of M) intersects the locus of indifference between M and O for rail shipping at a level below $\tilde{y}$, as illustrated in the Figure. The intersection is the intersection of the two lines $r(x+y) = tx + by$ with $y = \frac{[\tilde{x}+\tilde{y}]}{2} - x$, and so the relevant y value is given by $y = \frac{[\tilde{x}+\tilde{y}]}{2} - \frac{(r-b)}{(t-r)}y$, which simplifies to $y = \frac{(t-r)}{(t-b)}\frac{[\tilde{x}+\tilde{y}]}{2}$. The condition that this is below $\tilde{y}$ is then $[\tilde{x}+\tilde{y}] < \tilde{y}\frac{2(t-b)}{(t-r)}$ or $\tilde{x} < \tilde{y}\frac{t-2b+r}{(t-r)}$, so we assume this parameter condition holds.

Hence for $y \leq \frac{(t-r)}{(t-b)}\frac{[\tilde{x}+\tilde{y}]}{2}$, the equilibrium entails truck-barge shipping for $x \leq \frac{(r-b)}{(t-r)}y$, rail shipping to O for $x > \frac{(r-b)}{(t-r)}y$ and $x < \frac{[\tilde{x}+\tilde{y}]}{2} - y$, and rail shipping to M for $x > \frac{[\tilde{x}+\tilde{y}]}{2} - y$.

For $y \in \left(\frac{(t-r)}{(t-b)}\frac{[\tilde{x}+\tilde{y}]}{2}, \tilde{y}\right)$, the boundary between truck-barge and rail shipping to M is determined from the condition $tx+by = r[\tilde{x} - x + \tilde{y} - y]$, or $y = \frac{r[\tilde{x}+\tilde{y}]}{(b+r)} - x\frac{(t+r)}{(b+r)}$, so that truck-barge is preferred for lesser y values in this range. Note this critical locus is downward-sloping because shipping by rail to M becomes relatively more attractive for higher $y < \tilde{y}$.

Finally, for $y \geq \tilde{y}$, the boundary between truck-barge and rail shipping to M is determined from the condition $tx + by = r[\tilde{x} - x + y - \tilde{y}]$, or $y = \frac{r[\tilde{x}-\tilde{y}]}{(b+r)} + x\frac{(t+r)}{(r-b)}$, so that truck-barge is preferred for lesser y values in this range. This critical locus is upward sloping because the relative benefit of barge rises further North than $\tilde{y}$.

Proposition 5 *The equilibrium shipping choice is the one with least social cost even when there are multiple terminal markets.*

In the model so far, there are no total social surplus consequences to there being market power in the railroad sector. The effects are instead purely distributional. Comparing a monopoly railroad to a benchmark case of a railroad that prices at cost, market power shifts farmer surplus to railroad profit. Another facet of this property is illustrated by considering the effects of an improvement in the barge rate (a reduction in b). Such an improvement will broaden the area served by truck-barge (the truck-barge catchment area, $\mathcal{B}$), and welfare improvements may be measured by the reduced costs of shippers in $\mathcal{B}$. However, shippers outside also benefit from the improved barge rate, because it reduces the railroad price. Since this is purely a transfer from railroads to shippers, there are no implications for social surplus, other than the redistribution.

These results stem from the assumption that demand at each point in space is perfectly inelastic. Introducing demand elasticity in the amount shipped as a function of the price paid for shipping will introduce direct welfare losses from supra-competitive pricing. We now turn to this complication.

4 Downward-sloping demand

We return now to the case of a single terminal market, so the railroad also delivers to O. Instead of a rectangular demand, as above, suppose now that each point generates a downward-sloping demand for shipping services.[13] It is worth spending a little time on the provenance of this demand. In particular, the shippers themselves (those who produce or farm the commodity that is being shipped) are assumed price takers in the shipping market and in the final downstream market. They do, though, decide how much to produce and ship. Accordingly, suppose that production of Q units of output is associated with a production cost of $C(Q)$ (assumed twice continuously differentiable, increasing, and convex) and, for simplicity, assume that this cost is independent of location. If the price paid per unit for the commodity at the terminal market is p_T and

[13] These points could form a continuum (for example, if there are many farms over the landscape), or they could be isolated spots (like coal mines). Under some caveats, the isolated points could be grain elevators. Ideally though, one would like to model more finely the behavior of grain elevators, and to derive their behavior in shipping from a full-fledged model of their competition for farmers' produce in local markets.

the price paid to ship that unit there is p, then the shipper's profit is

$$\pi = [p_T - p]\, Q - C(Q).$$

The shipper will then optimally choose Q to maximize this profit. The shipper's problem is concave. The solution is to product nothing if $C'(0) \geq [p_T - p]$. If this does not hold, the shipper produces at the unique solution, Q^*, to $C'(Q^*) = [p_T - p]$. Equivalently,

$$Q^* = C'^{-1}(p_T - p).$$

This is then the demand for shipping services. Note that it is declining in p because a higher shipping price reduces revenue per unit and so elicits a lower supply response. The marginal cost curve, $C'(.)$, and the derivation of the demand curve from it, are illustrated in Figure 5. Note by construction that the demand curve mirrors the properties of the marginal cost curve. As p changes, then the supply of the produced good changes according to the marginal cost schedule, and this supply is simultaneously the demand for shipping services. In the sequel, we refer to the demand for shipping services as $D(p)$ and we suppress the terminal price p_T. It should be clear that (for constant p_T) we can write $D(p) = C'^{-1}(p_T - p)$.

INSERT FIGURE 5

We impose some regularity properties on the demand curve, $D(p)$, where p is the price paid per unit for delivering the commodity to O (e.g., $p = tx + by$).[14] In particular, let $D(.)$ be strictly (-1)-concave, meaning that $1/D(p)$ is a strictly convex function, and let $D(.)$ be twice continuously differentiable. The differentiability property implies that the strict convexity of $1/D(p)$ is equivalent to $D"D - 2(D')^2 < 0$ almost everywhere. Recalling that $D'(.) < 0$

[14] In the case of different prices at different terminal markets, we might subtract the terminal price (received by the shipper) from these costs, in order to get a net cost.

(demand slopes down), then this property is clearly satisfied by any concave demand. It also allows for demand to be convex, as long as it is not "too" convex. The class of log-concave functions is necessarily (-1)-concave: the property means that $\ln D(.)$ is concave, so that D'/D is decreasing.[15] This in turn implies that $D"D - (D')^2 \leq 0$, which implies that $D"D - 2(D')^2 < 0$. The concept of log-convexity is also employed below, and is defined analogously. That is, $\ln D(.)$ is convex, so that D'/D is increasing and so $D"D - (D')^2 \geq 0$.

Further insight into these functions is given by noting that the function $D(p) = \exp(a - bp)$, with $b > 0$ is log-linear, and so forms a boundary case. Anything "more concave" than a negative exponential function is therefore log-concave. Any function "more convex" is log-convex. Likewise, the function $D(p) = b/p$, with $b > 0$ is (-1)-linear, so that any function "more concave" than this is (-1)-concave.

For what follows, we need some properties of the railroad's (unconstrained) profit function at each point in space, as well as its monopoly price. These properties are determined by considering the behavior of the railroad's price derivative. The profit earned from any point is $\pi = (p - c) D(p)$, where we let c denote the railroad's marginal cost from that point (and we temporarily drop the dependence of this marginal cost on the location under consideration). The desired profit derivative is then

$$\frac{d\pi}{dp} = (p - c) D'(p) + D(p),$$

which we can rewrite as

$$\frac{d\pi}{dp} = -D'(p)\left[-(p - c) - \frac{D(p)}{D'(p)}\right].$$

The profit derivative, therefore, has the sign of the term in square brackets.

[15] A log-concave function is concave, but the converse is not necessarily true. Intuitively, since ln is an increasing and concave operator, then applying it to a concave function also yields a concave function. As long as it is applied to a function no more convex than an exponential function, taking the logarithm also gives a concave function.

This may be graphed as a function of p as the intersection of the two positive functions $(p-c)$ and $-\frac{D(p)}{D'(p)}$ as in Figure 6. The former has slope 1, so as long as the latter has slope less than 1, then there is a single intersection, and profit is rising left of the intersection (because the profit derivative is positive) and falling right of the intersection (because the profit derivative is negative). That is, the profit function is quasi-concave. The desired condition is therefore that the derivative of $\frac{D(p)}{D'(p)}$ be greater than -1: writing this out shows it to be satisfied if $D(p)$ is strictly (-1)-concave, which we have assumed.

INSERT FIGURE 6

We are also interested in how the monopoly price changes as a function of the serving cost, c. In the present spatial context, following Phlips (1983), we shall say there is *freight absorption* if the price rises by less than does the serving cost. Conversely, if the price rises faster, there is *freight over-pass-on*.[16]

The monopoly price is given implicitly by the solution, p^m, to the first-order condition,

$$(p-c)\,D'(p)+D(p)=0,$$

as described above. Applying the implicit function theorem gives the rate at which cost increases are passed on as

$$\frac{dp^m}{dc}=\frac{D'(p)}{(p-c)\,D''(p)+2D'(p)}.$$

[16] These properties apply too to transportation improvements in the railroad sector. That is, in the case of freight absorption, a change that reduces the cost by a dollar (to the railroad) of serving a location is passed on by less than 100% to the shipper. The benefits are shared in that sense. In Public Finance, these are essentially incidence questions, and the analysis is applied to who "bears" a tax (see Anderson, de Palma, and Kreider, 2001a and 2001b). A tax is analogous to a cost increase, and a subsidy to a cost decrease. Notice that in the case of cost over pass-on, a reduction in the cost to the railroad of $1 in serving a given location will lead to the price charged to shippers going down by more than $1. In that sense, the shippers could be said to benefit directly by even more than the magnitude of any transport reduction. Even stronger, insofar as shippers are induced to ship more when prices go down, their benefits would be even greater still due to additional shipper surplus.

The denominator is negative by the second order condition for a maximum, and the numerator is negative, so that (clearly) the monopoly price rises with cost. Further insight is provided by substituting the first-order condition into this expression, to give

$$\frac{dp^m}{dc} = \frac{\left[D'(p)\right]^2}{-D(p)\,D\text{''}(p) + 2\left[D'(p)\right]^2}. \tag{2}$$

The denominator is positive by the condition that $D(.)$ be strictly (-1)-concave.

We are interested in whether the monopoly price rises more or less quickly than costs. That is, whether $\frac{dp^m}{dc}$ is greater or less than one. Since the denominator of (2) is positive, it is a simple matter to cross-multiply and check this condition: i.e., $\frac{dp^m}{dc} \lesseqgtr 1$ as $\left[D'(p)\right]^2 \lesseqgtr -D(p)\,D\text{''}(p) + 2\left[D'(p)\right]^2$. This readily simplifies to $0 \lesseqgtr -D(p)\,D\text{''}(p) + \left[D'(p)\right]^2$. It is worth breaking out the possibilities one at a time.

Proposition 6 *If $D(.)$ is strictly log-concave, then $\frac{dp^m}{dc} < 1$ and there is freight absorption.*

This means that the monopoly price rises more slowly than the cost rises. All concave demands thus exhibit this property. In the special case of linear demand, it is readily verified that costs are passed on 50 cents on the dollar. The freight absorption property holds for even some convex demands (but not "too convex").

Proposition 7 *The monopoly price satisfies $\frac{dp^m}{dc} = 1$ if and only if demand, $D(.)$, is log-linear.*

A log-linear demand is, therefore, the demand function for which costs are passed on dollar for dollar.

Proposition 8 *If $D(.)$ is strictly log-convex, then $\frac{dp^m}{dc} > 1$, and there is more than unitary cost pass-on.*

The three Propositions above underscore the key properties in describing the behavior of the railroad's price when unconstrained by the truck-barge rate. Otherwise, the same principles apply as above, but with now the monopoly rail price forming a cap. Therefore, consider the behavior of the equilibrium shipping price at a given y, as a function of x. The truck-barge cost curve at any given y rises with x at rate t from a (low) base level of by at $x = 0$. The rail rate starts out higher but rises more slowly. The intersection is the boundary between the natural areas, as in Figure 2 above. Furthermore, the "conditional monopoly price" (i.e. if the railroad was an unconstrained monopolist) is $p^m(y,x) = \arg\max[p - (rx + ry)]D(p)$, where the term $(rx + ry)$ is the marginal cost, denoted c previously.

Proposition 9 *For a downward sloping point demand function, $D(p)$, the equilibrium rail price per unit is given by $p(y,x) = \min\{(tx + by), p^m(y,x)\}$ for $(y,x) \in \mathcal{R}$.*

The situation is illustrated in Figure 7.

INSERT FIGURE 7

For linear demand, as illustrated, the monopoly price increases at half the rate that costs increase. Close by the river, truck-barge is the equilibrium modal choice. Further out, the choice is rail, but the rail price is constrained by the truck-barge cost. Thereafter, the unconstrained monopoly price is charged, given that this is below the truck-barge rate (that is, for locations such that $p^m(y,x) \leq (tx + by)$). The market is no longer served beyond the point at which the rail cost exceeds the demand curve intercept, that is, where $D(rx + ry) = 0$.[17]

[17] This treatment abstracts from non-price features that may affect choices, such as time of

The case illustrated in Figure 7 is important for hidden welfare gains from barge cost reductions. In particular, there is the direct benefit from the cost reduction in the natural truck-barge market. Then, however, there is the induced reduction in the rail rate over the next interval. Part of the latter benefit accrues as a transfer from railroads, but with downward sloping demand, there is also a reduction in deadweight loss. The same observation applies to the other cases considered in this section.

Proposition 10 *For a downward sloping point demand function, $D(p)$, the equilibrium rail price per unit generates deadweight loss for $(y, x) \in \mathcal{R}$. A reduction in the barge cost generates additional surplus in the natural market for truck-barge exceeding the cost reduction applied to the existing volume of shipping, it generates additional surplus in expanding the natural market, and it generates additional surplus in reducing the rail price in the natural rail market.*

Figure 8 illustrates the analogous case for costs and prices when truck-barge and rail shipping go to different terminal transshipment points. In this case, the railroad's price actually rises further away from the river even though its cost of serving the market are falling. This is the case of the cost of meeting the competition and the cost of serving the market going in different directions.

INSERT FIGURE 8

The case when the railroad can choose whether to ship to O or to M is illustrated in Figure 9. In this case, the railroad's cost first rises with x while shipping to O is cheaper, and then falls with x.

INSERT FIGURE 9

transit and reliability. In accord with a long history in transportation research, and as noted by Train and Wilson (2004), these factors may be extremely important in demand decisions. In the present case, such complications may be allowed for by redefining rates to include non-price factors along with the monetary price.

The next section addresses the possibility that there is a limited capacity in rail shipping.

5 Capacity constraints

The analysis above has supposed that the capacity of the railroad is unlimited. However, the rolling stock is limited, and it takes time to make deliveries. Not all profitable contracts may be taken in view of these limitations. We now address this issue, and with it some more general pointers for a full (long-run) analysis including the costs of increasing railroad capacity. We suppose throughout that there is no capacity constraint for barges or barge pricing.[18]

Consider the original case where points in space generate unit demand. For simplicity, and to fix ideas, we start by considering a given y ordinate. Suppose too that rail shipping at that y level is constrained, and less than the natural market for rail would prescribe. The railroad would then choose to serve all the locations farthest away, up to capacity, since these give the highest profit. Then a reduction in the truck-barge cost would give the same type of transfers that we saw above, as well as directly reducing the barge shipping costs.

The sketch in the preceding paragraph takes the number of trips *per se* as a constraint. The more relevant concept would be to take the number of miles that can be traveled as representing the capacity constraint. This constraint makes sense when the railroad capacity is construed as a fixed total amount of time available (for example, train-hours per month). We now analyze such a constraint, namely that the number train hours is fixed exogenously by the capacity of the rail sector.

The economic principle involves considering the opportunity cost of a trip at various distances from the river. Clearly, with a tight capacity constraint

[18] Capacity constraints in the barge market are analyzed in Anderson and Wilson (2004). Future work will integrate the possibility of limited barge capacity with the current analysis.

the railroad does not want the trips right near the "competitive boundary" but those are also the ones that can be "turned around" quicker (i.e., the rolling stock can get back faster to pick up more cargo). A trip taking half the time as another needs to take in less than half the markup in order to not supplant the other one. The proper criterion for the current context is to choose the trips with the highest mark-up per mile. The main question then boils down to whether those further out deliver better mark-ups per mile. The mark-up earned at any point is $(tx+by)-(rx+ry)$, and hence, dividing by the distance traversed, $(x+y)$, gives the mark-up per mile as

$$m(y,x)=\frac{(tx+by)-(rx+ry)}{(x+y)}.$$

Differentiating shows how this mark-up changes with distance from the river, x:

$$\begin{aligned} \frac{dm}{dx} &\overset{s}{=} (t-r)(x+y)-[(tx+by)-(rx+ry)] \\ &= (t-b)y>0, \end{aligned}$$

where the notation $\overset{s}{=}$ denotes the sign of the expression. Thus, the mark-up per mile is higher further out, so that these are the locations that are served by the railroad, up to its capacity. This allocation also forms the optimal pattern from a social perspective. The intuition is that the hinterlands are further from the river and so less constrained by potential competition. They then generate higher mark-ups per ton-mile shipped.

We now consider the full two-dimensional picture. We have first to specify the nature of the capacity constraints more carefully. In particular, suppose that there is a fixed number of ton-miles that can be shipped in a given period, and that the demand from each location is still fixed. The criterion for a location to be served is that it yield higher mark-up per mile than any location left unserved by the railroad, given that the railroad must meet the going truck-barge rate at any location. As above, the mark-up per mile is $[(tx+by)-(rx+ry)]/(x+y)$.

We have shown that this rises with x. It varies with y according to:

$$\frac{dm}{dy} \overset{s}{=} (b-r)(x+y) - [(tx+by)-(rx+ry)]$$
$$= \quad (b-t)x < 0.$$

This indicates that the mark-up per mile is higher lower down. Pulling this together, there is a big "fixed" cost disadvantage to rail up-river; and more advantage further out (the "fixed" cost disadvantage from is then offset by the advantage over trucking). This means that the set of locations served will look like the natural market space for rail illustrated in Figure 1, except further out.

Another way to see this is to determine a locus of constant mark-up per unit. For a level of mark-up, k, this means that

$$\frac{(tx+by)-(rx+ry)}{(x+y)} = k$$

or $[(tx+by)-(rx+ry)] = k(x+y)$. Rearranging, $x(t-r-k) = y(-b+r+k)$, or

$$y = \frac{t-r-k}{-b+r+k}x$$

These are rays emanating from the origin. The higher is k, the lower the y value for given x. This observation suggests an algorithm that may be useful in the more complex cases. Since the relation is monotonic, take a value for k and find the ray of constant mark-up per unit. Then see if capacity is met. If not, reduce k, so that the y value rises at each x. This implies that the catchment area rises and capacity is closer to being met. Continue until capacity is met: clearly the process is stable and unique.[19]

[19] In the current case, the problem can be solved directly algebraically given any specification for Ω.

Proposition 11 *Let demand at each point in space be totally inelastic, and suppose that the railroad is capacity constrained. Then, for each location* (y, x)*, the equilibrium shipping mode is rail if* $y \geq \frac{t-r-k}{-b+r+k}x$*, where* k *is determined from the capacity constraint that all points served by rail completely exhaust the railroad's capacity. The equilibrium rail price at such points is* $p(y, x) = tx + by$*, and the allocation of the given capacity is socially efficient.*

The principles above are readily extended to other variants of the problem, involving relaxing some of the assumptions made. First of all, consider endogenous capacity choice by the railroad. What this will in effect do is to determine the appropriate level of k in the above analysis. Indeed, suppose that a unit of extra capacity has an amortized purchase cost of \$$Z$ per year and enables W unit-miles to be shipped in the year. This translates into a cost of \$$Z/W$ per unit per mile: when capacity is optimally chosen, this is therefore the equilibrium value of k. It is interesting that it is also the optimal value. It is also noteworthy that equating Z/W to the marginal benefit is the marginal condition for a maximum in profits and that infra-marginal locations generate positive profits. That is, the railroad earns positive profits.

If demand is no longer fixed, and is downward-sloping, as in Section 4, the appropriate principle that determines the allocation of a fixed capacity is that marginal revenue be equalized at all locations served. Due deference must be taken to locations where the railroad is constrained by the truck-barge rate (thus implying a fixed marginal revenue), and those that are unconstrained. When capacity is endogenous, as above, the price of capacity determines the relevant level of marginal revenue.

6 Further directions

This paper is the second of a sequence. In the previous work, we developed an equilibrium model of the barge market. The model has shippers located over geographic space and deciding how to ship their product to market. This model explicitly allows for transportation infrastructure. Specifically, we introduce flow constraints on the waterway in the form of locks. The cost of using the waterway increases with the level of traffic. The model results in a unique equilibrium wherein barge rates, quantities, and congestion are determined endogenously for *given* rail and truck rates. The model allows for shippers to by-pass locks and points to a stacking property of pool level demands that requires evaluation of lock improvements to be made at a system level.

In this second work, we extend the basic framework to allow for the determination of railroad prices over geographic space. In the benchmark model, the railroad sets prices so as to beat the competition. In this model, the effects of waterway improvements are purely distributional. The railroads must lower price to meet tougher competition, and shippers will gain the rent. In this benchmark model, shipper demands at each point in space are assumed completely inelastic (as in our earlier paper). However, we show that if there are downward sloping demands at each shipping point, then waterway improvements may generate additional welfare gains as shippers experience lower prices and expand output. These basic insights also apply when shippers may choose the final shipping point from a menu of possible options. In both of these cases, rail capacity is unlimited. In the final section, capacity constraints are introduced with the result that the railroad prefers to serve the shippers from which it can receive the highest markups. Such locations are the captive shippers to railroads i.e., the shippers located furthest from the waterway.

There are a number of potential extensions of our work. Army Corps of

Engineers planning models, generally, assume that the pool level demand is perfectly inelastic up to a threshold point (where the alternative mode dominates). A somewhat similar assumption is used in our basic framework, but at a much more disaggregate level (so that the pool level demand is not inelastic). In particular, we started with the assumption that demand at each point in space is totally inelastic, but nevertheless is served by the mode offering the cheaper rate. Even though the point demand is inelastic (up to a reservation price), and switches from truck-barge to rail at a critical indifference price, the aggregate demand (i.e., once we account for the total demand as the sum over space of the point demands) is not inelastic. Indeed, the demand for the river will only be inelastic in this framework if the extensive margin is locally empty. By this we mean that small changes in the margin (as would be induced by cost changes of using the river) would bring in no new economic activity to using the river. Second, even if this condition were met, if there were some local elasticity in the demand for river transport at points in space, the aggregate would exhibit some price sensitivity. Only if each point demand for river transport were totally inelastic and if the extensive margin were empty would aggregate demand be totally inelastic.

It remains to be seen what would be reasonable estimates of the aggregate elasticity taking into account both effects. The model presented in this paper could be usefully calibrated in this regard. It would also be important to integrate the model with the demand system for the goods being shipped. Specifically, our model provides modal demands, but it also determines the supply of goods by mode and terminal market. This supply of goods to the terminal market (e.g., New Orleans) can then be integrated with the demand for goods (import demand at New Orleans) to determine the port price of the commodity. By so doing, there would be a direct link between the inland transportation markets (by mode and terminal), defined over space and with endogenous congestion emanating from capacity constraints in the rail market, the barge market (locks), and the highway system. Indeed, in such an extended model, the terminal markets would no longer be the final market, but rather the transshipment point (or points, in the case of multiple ports) to markets further afield . Doing this would allow a full determination of the beneficiaries of a transport improvement. For example, such a study would enable us to determine how much of any transportation improvement is captured by foreign consumers in terms of lower prices for the goods they receive, as well as how much is gained by domestic shippers, and how much is lost by domestic railroads.

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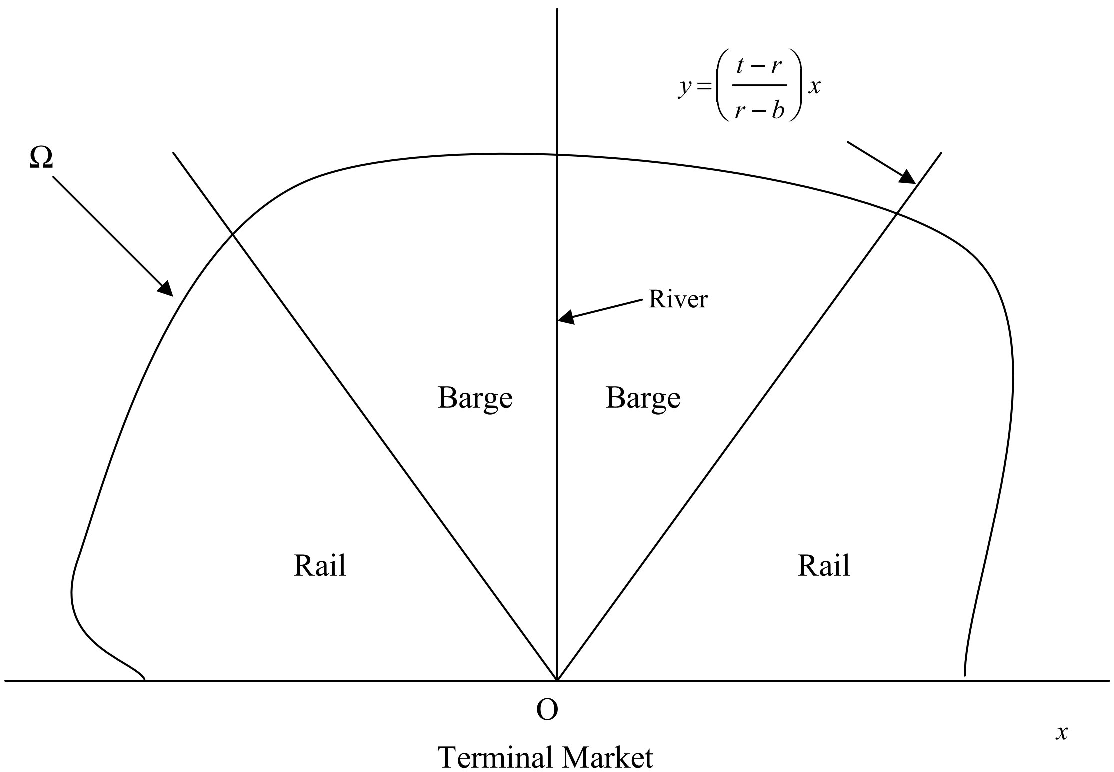
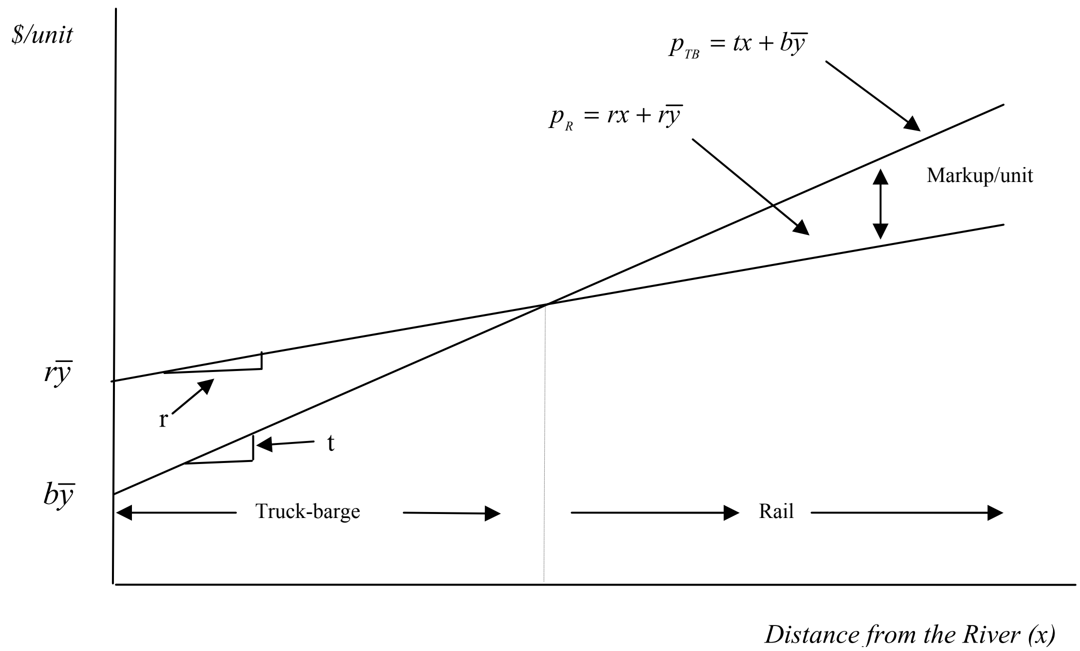
Figure 1



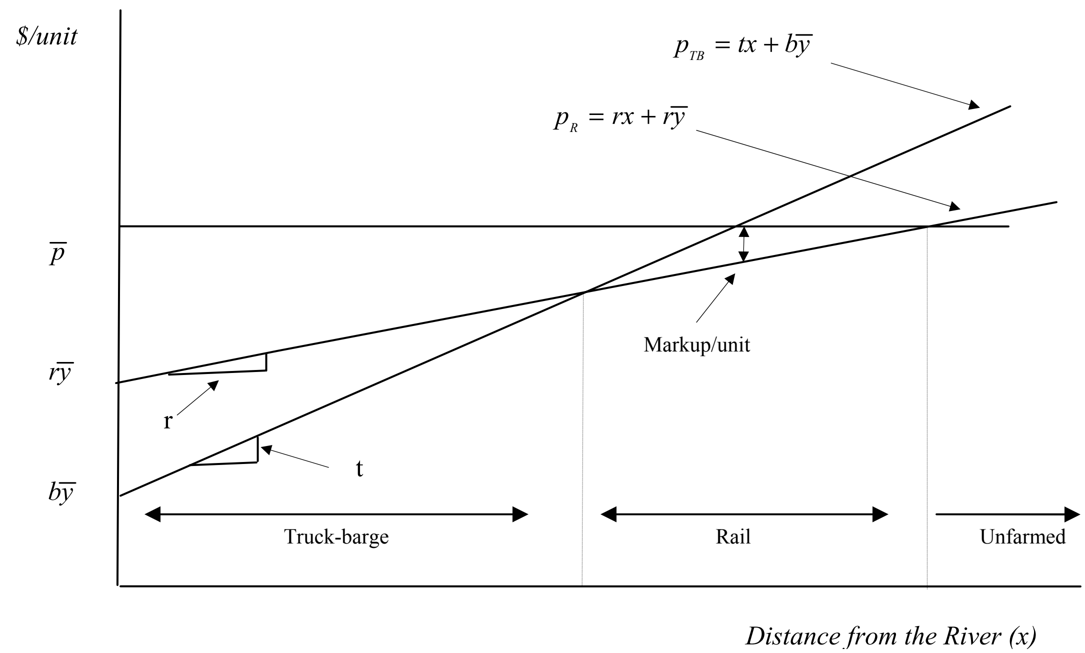
$y = \left(\frac{t-r}{r-b}\right)x$
Ω
River
Barge
Barge
Rail
Rail
O
Terminal Market
x

Figure 2.



$/unit
$p_{TB} = tx + b\overline{y}$
$p_R = rx + r\overline{y}$
Markup/unit
$r\overline{y}$
r
t
$b\overline{y}$
Truck-barge
Rail
Distance from the River (x)

Figure 3.



$/unit
$p_{TB} = tx + b\overline{y}$
$p_R = rx + r\overline{y}$
$\overline{p}$
$r\overline{y}$
$b\overline{y}$
Markup/unit
r
t
Truck-barge
Rail
Unfarmed
Distance from the River (x)

Figure 4. Alternative Port

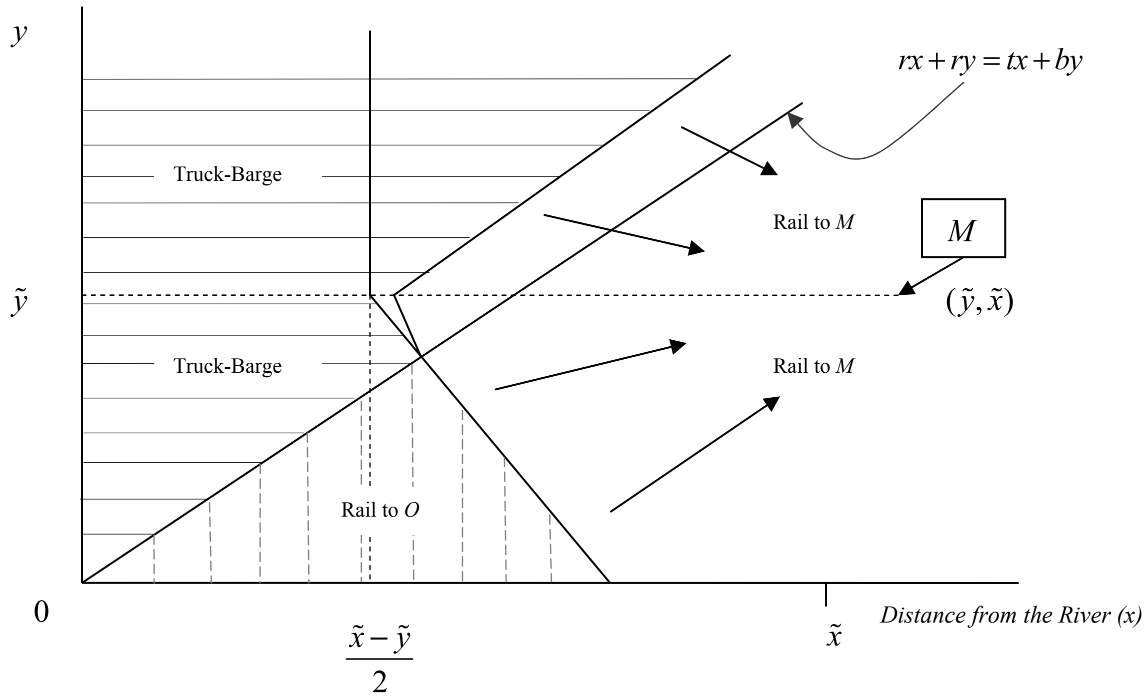


Figure 5.

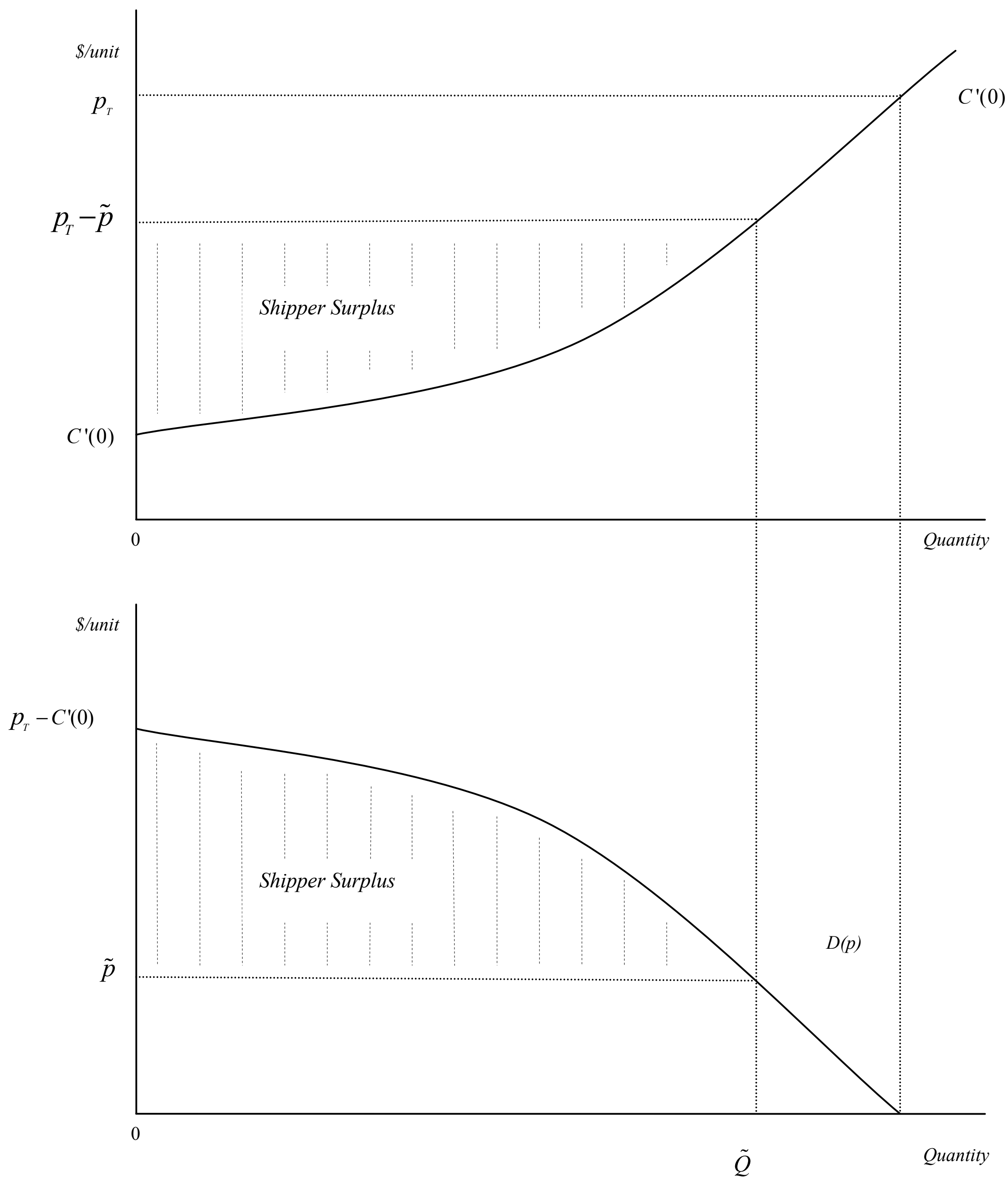

$/unit
p_T
$C'(0)$
$p_T - \tilde{p}$
Shipper Surplus
$C'(0)$
0
Quantity
$/unit
$p_T - C'(0)$
Shipper Surplus
D(p)
$\tilde{p}$
0
$\tilde{Q}$
Quantity

Figure 6.

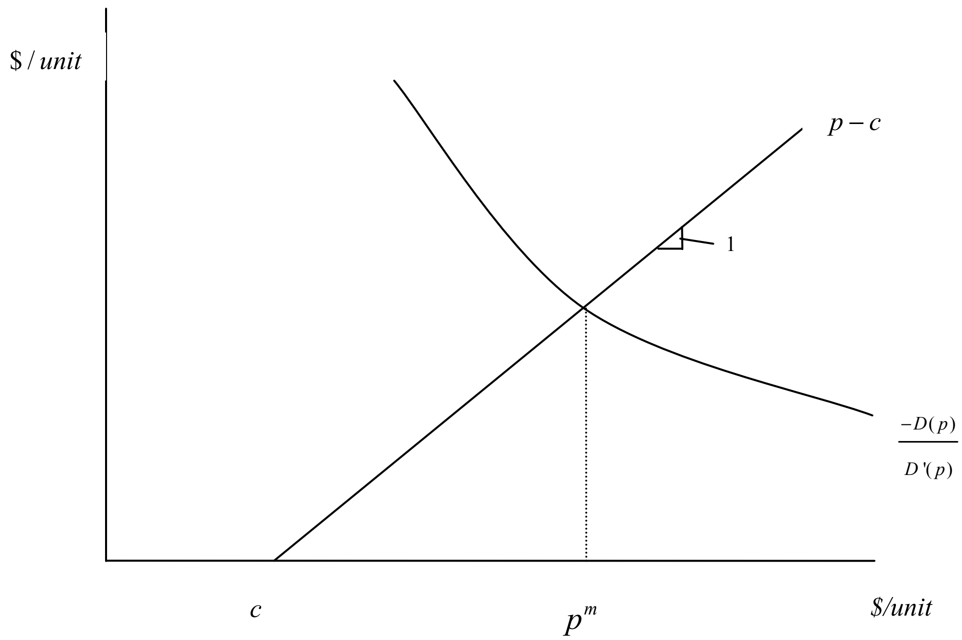
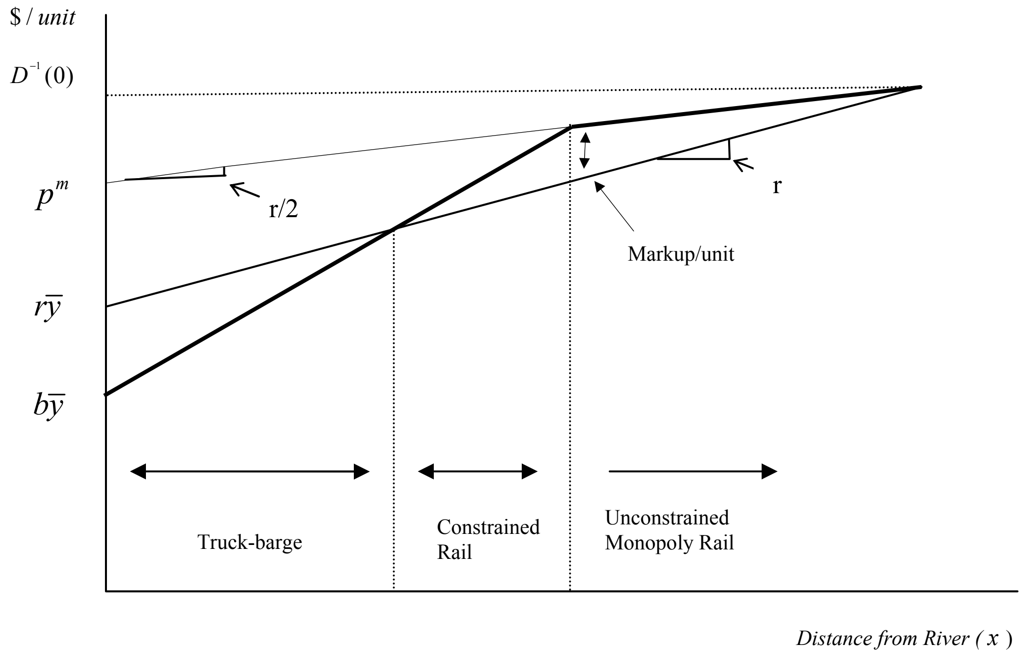
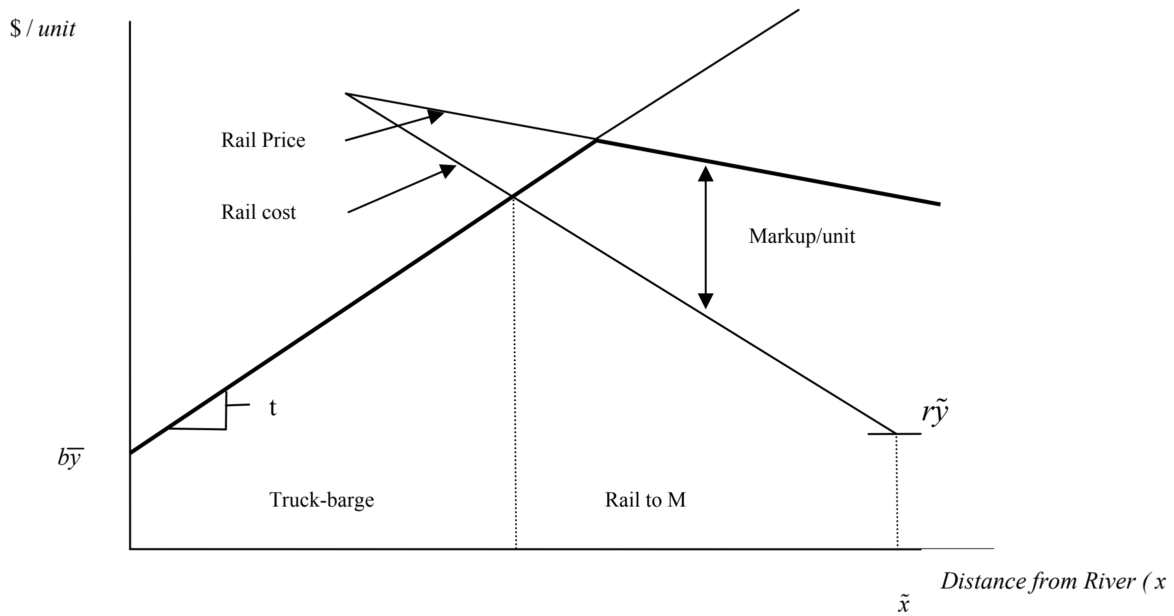


Figure 7.



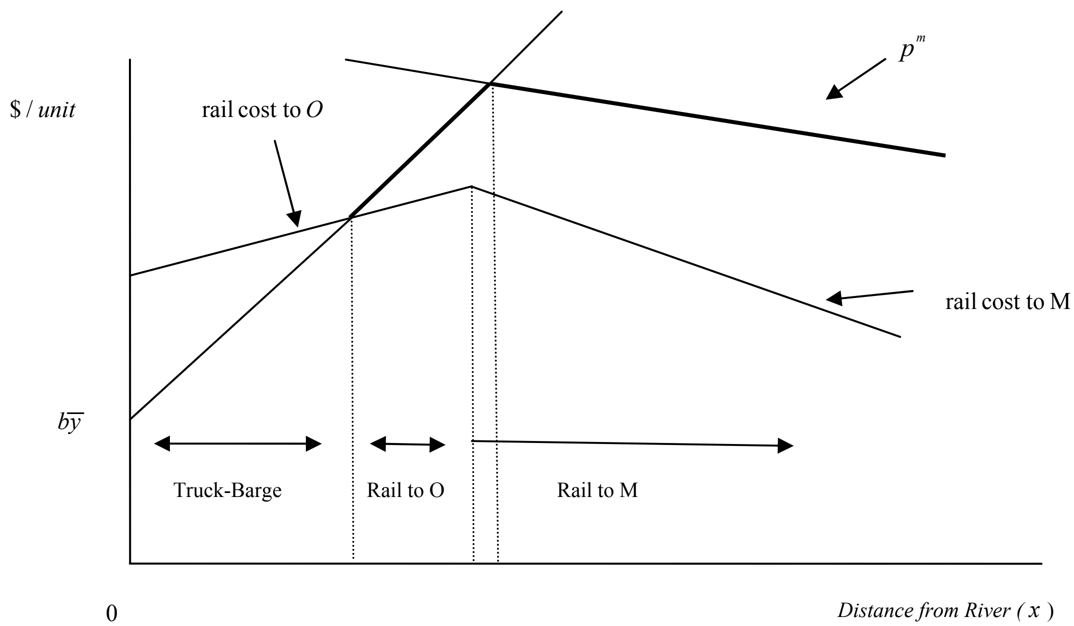
$ / unit
$D^{-1}(0)$
p^m
r/2
r
Markup/unit
$r\overline{y}$
$b\overline{y}$
Truck-barge
Constrained
Rail
Unconstrained
Monopoly Rail
Distance from River (x)

Figure 8



$ / unit
Rail Price
Rail cost
Markup/unit
t
$r\tilde{y}$
$b\overline{y}$
Truck-barge
Rail to M
Distance from River (x)
$\tilde{x}$

Figure 9



p^m
\$ / *unit*
rail cost to *O*
rail cost to M
$b\overline{y}$
Truck-Barge
Rail to O
Rail to M
0
Distance from River (x)