Labor Supply, Self-insurance and Knightian Uncertainty

Hening Liu[†]

Northern Illinois University

Abstract

The focus of this paper is on the implications of income uncertainty for the optimal labor supply. The meaningful distinction between Knightian uncertainty, which is often attributed to Frank Knight (1921), and risk is allowed. Agents are both uncertainty averse and risk averse. The labor-leisure and the labor-leisure-saving choices are studied under Knightian uncertainty in one-period and two-period setting, respectively. The multiple-priors utility model is adopted. The effects of income uncertainty on optimal labor supply are analyzed by deriving closed form solutions. Using data from Panel Survey of Income Dynamics (PSID) on self-employed American males, we find that, income uncertainty, proxied by the wage dispersion in different industries, plays an important role in determining male self-employed labor supply.

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A probability measure cannot adequately represent both the relative likelihoods of events and the amount, type, and reliability of the information underlying those likelihoods. —Epstein and Wang (1994)

1 Introduction

Recent empirical and theoretical studies have attempted to investigate the effect on the optimal labor supply of wage[1] uncertainty. For example, Parker et al. (2005), by estimating a labor supply specification, studies how past variations in wages can affect self-employed American male workers' labor supply. Hartwick (2000) considers a one period model with a constant elasticity of substitution utility function. Through numerical simulation, he finds that, under some restrictive conditions, a mean preserving spread of anticipated stochastic wage induces an agent to supply more work hours. In a two-period model, Floden (2005) also finds that the uncertainty about future random wages raises current labor supply when preferences are consistent with balanced growth. In general, the existing literature conforms to the expected utility model in that they assume uncertainty can be reduced to risk. That is, there exists a single objective probability measure governing the states of the world, and agents are able to assign specific probabilities to the states precisely. Alternatively, in the framework of Bayesian approach, an agent's beliefs about the likelihoods of future states of the world are summarized by a single subjective probability measure (or Bayesian prior). As a result, no meaningful distinction is made between risk, where precise probabilities are available to guide choice, and uncertainty, where information is too imprecise to be summarized adequately by specific probabilities. In contrast, Knight (1921) emphasized the distinction between risk and uncertainty and argued that uncertainty is more important in economic decision-making. A notable experimental evidence is the Ellsberg Paradox, which suggests that people prefer to act on known probabilities rather than unknown or vague probabilities. The story is as follows: one urn contains 30 red balls and 60 other balls that are either black or yellow. Moreover, the exact number of black or yellow balls is unknown. When people are given a choice between the following two gambles, Gamble A: you receive

[1] The 'wage' is usually computed by dividing a measure of total income by work hours (Hamilton 2000).

100 dollars if you draw a red ball; and Gamble B: you receive 100 dollars if you draw a black ball, most people strongly prefer to bet on Gamble A rather than Gamble B. However, the standard expected utility model and/or any Bayesian prior's prediction of an agent's behavior will be contradicted by the Ellsberg-type behavior in the real world. Thus any single prior underlying choices cannot be supported, and people dislike uncertainty, where we cannot express randomness in terms of precise probabilities.

In this paper, we distinguish uncertainty[2] from risk and focus on the implication of income uncertainty for the optimal labor supply. To accomodate distinction between risk and Knightian uncertainty, we adopt the multiple-priors utility model axiomatized by Gilboa and Schmeidler (1989). In the model, the agent's beliefs about future states are represented by a set of priors. The set of priors captures both the degree of Knightian uncertainty and uncertainty aversion. The set of priors is constructed by statistically perturbing a single reference probability measure[3] through a Radon-Nikodym derivative, where there are a set of probability measures and the agent is uncertain which probability measure is the true one. The perturbation reflects the amount of the agent's uncertainty about the surrounding environment. The size of the set of priors is determined through the relative entropy criterion. The broader the set of priors, the more uncertainty averse the agent is. In making decisions, the agent evaluates his/her action under the probability measure in the set of priors which delivers the worst-case utility. In general, the outcome of the decision can be formalized as a two-player game: for every action chosen by an agent, a second fictitious player selects the worst-case belief from the family of potential beliefs.

This paper analyzes the optimal labor supply choice in one-period and two-period setting before income risk and uncertainty are resolved. The agent only values two goods: consumption and leisure. To obtain transparent results, solvable utility specifications, including CRRA (constant relative risk aversion), CARA (constant absolute risk aversion) and quadratic utility functions, are considered. In all examples studies below, we conduct comparative static analysis for the family of normal distributions with identical variances. The uncertainty is reflected in terms of the distortion to the mean. All the comparative

[2] 'Knightian uncertainty', 'uncertainty' and 'ambiguity' are often interchangably used in the existing literature.

[3] In the language of robust control theory, the reference probability measure is also called 'approximating model'. See Anderson, Hansen and Sargent (2003).

static results focus on the effect of the degree of Knightian uncertainty on the optimal labor supply.

In one-period model, the solved examples show that the optimal labor supply depends crucially on the way in which the wage uncertainty enters the model. Wage uncertainty can enter various specifications either in an additive or a multiplicative way[4]. With multiplicative wage uncertainty, we obtain the clear-cut result that higher wage risk, which is defined as a mean preserving spread in the wage distribution, induces the agent to work more hours. Uncertainty, however, has different implications for the optimal labor supply. For CRRA utility function, if relative risk aversion parameter is less than one, then the agent with multiple priors utility works less than the agent with standard expected utility. Further, the agent with higher degree of uncertainty aversion supplies less work hours. Conversely, if relative risk aversion parameter is greater than one, the opposite conclusion is obtained.

When wage uncertainty appears in an additive way, greater risk and/or higher degree of uncertainty will unambiguously increase the optimal labor supply. Moreover, there is a separate self-insuring component in the optimal labor supply that cannot be attributed to risk. This component is a first order function of the standard deviation of the stochastic wage, and increases with respect to the agent's uncertainty aversion. This is in contrast to the standard model where self-insuring is of second order concern. In the standard expected utility model, for quadratic utility, the optimal labor supply is invariant to the level of income risk and thus no self-insuring motive due to risk exists[5]. However, it is worth noting that the self-insuring component due to Knightian uncertainty can arise even for quadratic utility.

In the two-period model, the agent chooses the optimal labor supply in anticipation of future exogenous income uncertainty. In a similar fashion, Miao (2004) analyzes the problem of consumption-saving choice under Knightian uncertainty in a two-periods model. He finds that under income uncertainty, there is an extra component of precautionary savings due to uncertainty. By incorporating labor supply choice, we show that this extra component of precautionary savings leads to a corresponding self-insuring component in the optimal labor supply. Still, in contrast to the standard expected utility model (Kimball (1990)) where

[4] The formal definitions of the multiplicative uncertainty and the additive uncertainty is given in the next section.

[5] see, for example, Parker et al.(2005) for details.

precautionary savings and self-insuring components are of second order and cannot arise for quadratic utility, in the multiple priors utility model these two components become first order concern and can exist for quadratic utility.

A testable implication from the theoretical discussion could be that the variation of labor supply observed in data can be partially explained by income uncertainty. To test the model's implication, we choose the PSID data from 1968-93 on self-employed American males. The ideal data should include individuals who can flexibly adjust hours of labor supply and are simultaneously exposed to comparable amount of income uncertainty and/or risk. There is now clear evidence that self-employed workers have more variable and unequal incomes than regular employees do (Carrington et al., (1996); Parker, (1997)). Parker et al. (2005) studies how income risk, which is measured by the past variations of individual wages, influences self-employed workers' labor supply choice. In this paper, however, we introduce another factor: income uncertainty. We exploit the panel nature of the PSID data to construct the measure of income uncertainty. We use the wage dispersion in different industries as the proxy of income uncertainty. It is found that when risk and uncertainty are treated differently, uncertainty is the key determinant of male self-employed labor supply, with a significant positive effect. This result potentially provides an explanation of the long-standing self-employed labor supply puzzle, that is, why self-employed individuals work longer average hours than their employee counterparts do[6] (Carrington et al. (1996), Hamilton (2000)).

Recently, Knightian uncertainty (or model uncertainty) has become an important topic in finance and macroeconomics. A number of papers discuss the role of model uncertainty in asset market and macroeconomic policy. Two different approaches, multiple priors utility model[7] and robust control approach, are usually formulated. The latter approach is developed by Hansen and Sargent and their coauthors (e.g., Anderson, Hansen and Sargent (2003) and Hansen and Sargent (2000). In this paper, we apply the multiple priors utility model to

[6] The interest of the empirical analysis is not to consider the entry/exit decision of self-employed workers; nor shall we explain why individuals participate self-employment in face of lower average wages. For the discussion of the decision to participate in self-employment instead of paid employment, see Miao and Wang (2005).

[7] See Epstein and Wang (1994, 1995), Chen and Epstein (2002), Epstein and Miao (2003), Miao and Wang (2003), Epstein and Schneider (2002).

analyze the role of wage uncertainty in determining labor supply choice, an issue that none of previous studies has considered.

The remainder of paper is organized as follows. Section 2 provides a theoretical model of the relationship between wage uncertainty and labor supply, and derives the closed form solutions. Section 3 discusses the econometrical issues in estimating the empirical specification, describes the features of data sample and presents the empirical results. Section 4 concludes. Section 5(Appendix) includes proofs of several propositions in the paper.

2 Theoretical Model

In this section, we analyze the following problems: (1) In the one-period model, the agent needs to determine, in advance, a specific amount of labor supply without knowing the future uncertain wage. Because the agent's information about the wage rate is too imprecise to be summarized by a single probability distribution, the beliefs can be represented by a set of priors. (2) in the two-period setting, it is assumed that the agent knows the deterministic wage in period 0 while has a set of priors of the uncertain exogenous income in period 1[8]; the agent needs to determine the period 0's optimal labor supply, consumption and savings, and period 1's consumption. Suppose the measurable state space is $(\Omega, \mathcal{F})$, with a reference probability measure P on this space. In the standard expected utility model, the random wage is defined on the probability space $(\Omega, \mathcal{F}, P)$.

2.1 Multiple priors utility and information structure

The utility function is given by the multiple-priors utility model axiomatized by Gilboa and Schmeidler (1989). In the atemporal model, the utility function is given by

$$U(c,l) = \min_{Q \in \mathcal{P}} E_Q[u(c,l)] \tag{2.1}$$

where $u(c,l)$ is a vNM utility function where c is consumption and l is leisure. In the two-period model (period 0 and 1), the utility function is

$$U(c_0, l_0; c_1) = u_0(c_0, l_0) + \beta \min_{Q \in \mathcal{P}} E_Q[u_1(c_1)] \tag{2.2}$$

[8] There is no other source of income in period 1.

where $u_0(\cdot)$ and $u_1(\cdot)$ are vNM utility functions in two periods, respectively; $\beta \in (0,1)$ is a discount factor, c_0 is period 0's consumption, l_0 is period 0's labor supply and c_1 is period 1's consumption. The special feature of the multiple priors utility function is that the agent has a set of priors $\mathcal{P}$ over $(\Omega, \mathcal{F})$, instead of a single prior in the standard expected utility model. Intuitively, the size of the set of priors $\mathcal{P}$ includes the neighborhoods of the reference probability measure P, and reflects the agent's uncertainty about the environment. The agent evaluates the action under the worst-case probability measure. As a result, in the multiple-priors utility, the minimum delivers uncertainty aversion. The larger size of $\mathcal{P}$ implies higher degree of uncertainty aversion. Suppose that $\mathcal{P}$ contains P, and is compact in the weak convergence topology. This assumption ensures that an agent's view of the world is not contradicted by the data. When the set $\mathcal{P}$ collapes to a singleton P, $\mathcal{P} = \{P\}$, the model is reduced to the standard expected utility model.

In order to obtain more transparent results, we need to impose some specific structure on $\mathcal{P}$. One tractable specification is based on the entropy criterion. Formally, the set of priors is defined as

$$\mathcal{P}(P,\phi) = \{Q \in \mathcal{M}(\Omega) : E_Q[\log(\frac{dQ}{dP})] \leq \phi^2\},\ \phi > 0, \tag{2.3}$$

where $\mathcal{M}(\Omega)$ is the set of probability measures on Ω and $\frac{dQ}{dP}$ denotes Radon-Nikodym derivative. This specification is consistent with the concept of relative entropy, which Anderson, Hansen and Sargent(2003) use in their paper to develop robust control theory in continuous-time. In the standard rational expectation theory, agents' subjective probability distributions equate to the objective one because it is assumed that agents have access to an infinite history of observations. However, in reality, agents can only have finite observations. Thus they have concerns about model's specification and fear their 'approximating model' P being misspecified. The misspecification is parameterized by the relative entropy $E_Q[\log(\frac{dQ}{dP})]$, and the set of misspecified models can be characterized by the set of priors $\mathcal{P}$. By evaluating actions under the worst-case probability, agents seek decision rules which are robust to the misspecification of the approximating model.

In this paper, let us consider a formulation studied by Kogan and Wang (2002). Let P be the probability measure corresponding to a normal distribution with mean μ and variance σ^2. Assume all probability measures in $\mathcal{P}(P,\phi)$ have normal distributions. Moreover, each measure Q in $\mathcal{P}(P, \phi)$ has a fixed variance and a mean $\mu - v$ for some $v \in \mathcal{R}$. As shown in Kogan and Wang (2002), $\mathcal{P}(P, \phi)$ is isomorphic to the set

$$\mathcal{V}(\phi) = \{v \in \mathcal{R} : \frac{1}{2}\frac{v^2}{\sigma^2} \leq \phi^2\} \tag{2.4}$$

The parameter $\phi > 0$ models the degree of Knightian uncertainty. It can also be interpreted as an uncertainty aversion parameter.

2.2 One-period Model

In this section, the agent's labor income is given by

$$I = E + \alpha L + B; \tag{2.5}$$

where E is the autonomous component of the earnings from labor supply, $\alpha > 0$ is the marginal product of labor, L is the chosen number of hours worked, B is the unearned income. For simplicity, the unearned income B is assumed to be zero throughout the theoretical discussion, though it is included in the empirical analysis. For employees, the autonomous component E may include the corporate bonus in the profit-sharing plans (on the positive side) and social security payment (on the negative side). For self-employed, E may include high inventory sales in times of high demand (on the positive side) and lump-sum business operating costs (on the negative side). Within this specification, when E is deterministic and α is uncertain and defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, it is called the *multiplicative uncertainty* and the computed wage is $w = \tilde{\alpha} + E/L$. When α is deterministic and E_0 is uncertain and defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, it is called the *additive uncertainty* and the computed wage is $w = \alpha + \tilde{E}/L$. (The notation $\tilde{z}$ means z is uncertain). The set of priors specified in (4) will guide the agent's optimal labor supply decision. This set reflects the potential distortions to the agent's belief of probability distribution of uncertain income.

In order to obtain closed form solutions, we consider three different separable utility specifications. The agent must commit, in advance, to a deterministic labor supply when the information set contains a set of priors of the uncertain wage. Wage uncertainty enters either in a multiplicative or an additive way. The agent's preference is represented by the multiple priors utility. We first present a benchmark model without Knightian uncertainty where the agent has CRRA (power) utility and wage uncertainty is multiplicative. We show that a mean preserving spread in the stochastic wage unambiguously increases the optimal labor supply. Then Knightian uncertainty is introduced into the model, and it is shown that uncertainty has different effects on the optimal labor supply depending on the relative risk aversion parameter. Then we present an example with quadratic utility and additive uncertainty. Unlike standard results, the optimal labor supply responds to Knightian uncertainty, though risk has no effect. For CARA (exponential) utility and additive uncertainty, both risk and uncertainty have determinate effects on the optimal labor supply. Interior solutions are assumed throughout the analysis[9].

CRRA utility, multiplicative uncertainty

Assume the utility function is:

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{l^{1-\gamma}}{1-\gamma} \tag{2.6}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. For simplicity, we assume no autonomous component in the labor income specification. That is, $E = 0$ and $w = \tilde{\alpha}$. The computed wage enters the model in the multiplicative way. Then the agent's budget constraint is

$$c = (T - l)w \tag{2.7}$$

where T is the total time endowment, w is the uncertain wage. We further assume w follows a lognormal distribution, that is, $\ln(w) \sim N(\mu_w, \sigma_w^2)$.

Standard expected utility case

In the absence of uncertainty, the agent's problem is

$$\max_{c,l} E[u(c,l)] \tag{2.8}$$

subject to the budget constraint (6), where $u(c,l)$ is given by (5).

Proposition 1: Suppose $\gamma \neq 1$, a mean preserving spread (an increase in σ_w^2) in $\ln(w)$ increases the optimal labor supply. When $\gamma = 1$, a mean preserving spread has no effect on the optimal labor supply.

[9] With corner solutions, issue becomes exogenous labor supply.

Proof: see Appendix.

According to Rothchild and Stiglitz (1971), for an increase in wage risk (a mean preserving spread) to result in more labor supply, it is sufficient that the agent's first-order condition is convex in the random wage and that the slope of the first order condition, with respect to the wage, has a single sign. Although neither of these two conditions is satisfied here, we obtain the clear-cut result.

The example sheds light on the inconclusive nature of previous studies such as Block and Heineke (1973), Hartwick (2000) and Parker et al. (2005). The first two studies use non-separable utility functions. In neither case were unambiguous results discovered, except for a particular special case considered by Hartwick (2000), where stochastic wage(wage enters in a multiplicative way) outcomes are clustered closely together and individuals' elasticities of substitution are greater than unity. In such a case, a mean preserving spread in the stochastic wage results in greater labor supply. Parker et al. (2005) consider the separable utility, but still cannot obtain any clear prediction about the effect of greater wage risk on the optimal labor supply.

Multiple priors utility case

Now let us introduce Knightian uncertainty into the model, and consider multiple priors utility formulation. The agent's problem is:

$$\max_{c,l} U(c,l) \tag{2.9}$$

subject to the budget contraint (6), where $U(c,l)$ is given in (1) and $u(c,l)$ in (5). The set of priors is specified by (4). The next proposition shows that an increase in the degree of uncertainty has different effects (depending on whether $\gamma > 1$ or $\gamma < 1$) on the optimal labor supply.

Proposition 2: When $0 < \gamma < 1$*, higher degree of uncertainty decreases the optimal labor supply; when* $\gamma > 1$*, higher degree of uncertainty increases the optimal labor supply; when* $\gamma = 1$*, uncertainty has no effect on the optimal labor supply. Specifically, for* $0 < \gamma < 1$*, the optimal labor supply is given by*

$$L^{**} = \frac{1}{1 + e^{\left(\frac{1-\gamma}{\gamma}\sqrt{2}\sigma_w \phi\right)} e^{-\frac{1-\gamma}{\gamma}\mu_w - \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2}} T \tag{2.10}$$

and for $\gamma > 1$, the optimal labor supply is given by

$$L^{**} = \frac{1}{1 + e^{(-\frac{\gamma-1}{\gamma}\sqrt{2}\sigma_w\phi)} e^{-\frac{1-\gamma}{\gamma}\mu_w - \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2}} T \tag{2.11}$$

Proof: see Appendix

In the presence of wage uncertainty, giving up a unit of time of leisure does not necessarily lead to a certain increase in income. Thus, the impact of wage uncertainty on the optimal labor supply depends on two opposite income and substitution effects. Knightian uncertainty, as distinct from risk, has different implications in this example. An increase in the degree of Knightian uncertainty makes the agent less inclined to expose his labor income to the possibility of loss, since the agent also values the leisure. As a result, the substitution effect tends to reduce the labor supply. On the other hand, higher degree of uncertainty induces the agent to work more and earn more labor income in order to protect oneself against possible low levels of future wealth. This result in the positive income effect on labor supply. The overall effect depends on the relative risk aversion γ. When the agent is slight risk averse ($\gamma < 1$), substitution effect outweights income effect and labor supply decreases in response to higher degree of uncertainty. When the agent is strongly risk averse ($\gamma > 1$), income effect dominates substitution effect and labor supply increases as a result of higher degree of uncertainty.

Quadratic utility, additive uncertainty

Assume the utility function is

$$u(c, l) = -(b-c)^2 - (T-l)^2 \quad b > c \tag{2.12}$$

and the constraint is

$$c = \tilde{E}_0 + \alpha(T-l) \tag{2.13}$$

where α is deterministic, $\tilde{E}_0$ is stochastic and follows a normal distribution with mean μ_E and variance σ_E^2.

In the standard expected utility model where the set of priors collapes to a singleton P, it is well-known that the optimal labor supply is invariant to the level of risk if utility is separable and quadratic. For the impact of risk on the optimal labor supply to exist in the expected utility model, the third derivative $u'''(c)$ cannot be equal to 0. Nevertheless, quadratic utility does not satisfy this condition. Here we present an example with quadratic utility, in which optimal labor supply does responds to Knightian uncertainty. Thus, we obtain the result that the third derivative is irrelevant for self-insuring component due to uncertainty to exist in the optimal labor supply. The agent's problem is given by

$$\max_{c,l} U(c,l) \tag{2.14}$$

subject to the constraint (9), where $U(c,l)$ is given in (1) and $u(c,l)$ is in (8).

Proposition 3: Assume the set of priors is described by (4) and b is large enough ($b > \mu_E + \alpha L^{**}$). *Then the optimal labor supply is increasing in the degree of uncertainty:* $\dfrac{\partial L^{**}}{\partial \phi} > 0$. *The optimal labor supply is given by*

$$L^{**} = \frac{\alpha(b - \mu_E + \sqrt{2}\sigma_E \phi)}{\alpha^2 + 1} \tag{2.15}$$

Proof: see Appendix.

It is obvious to see that variance σ_E^2 plays no role in determining the optimal labor supply. The presence of the term $\sqrt{2}\sigma_E\phi$ is a special feature of the model and arises because of Knightian uncertainty. This term disappears in the standard expected utility model. This example indicates that the self-insuring motive is more likely to arise because of the uncertainty rather than the risk.

Exponential utility, additive uncertainty

Proposition 4: Suppose the utility function is given by

$$u(c,l) = -\frac{1}{\theta}e^{-\theta c} - \frac{1}{\theta}e^{-\theta l} \quad \theta > 0, \tag{2.16}$$

and budget constraint is given by (9). If the set of priors is described by (4), the optimal labor supply increases in the presence of greater risk and/or the higher degree of uncertainty. The optimal labor supply is given by

$$L^{**} = \frac{\ln\alpha + \theta(T - \mu_E) + \frac{1}{2}\theta^2\sigma_E^2 + \sqrt{2}\sigma_E\phi}{\theta(1+\alpha)} \tag{2.17}$$

Proof: see Appendix

Again, the last term in the nominator captures the effect of the uncertainty. The optimal labor supply under the uncertainty exceeds that in the standard expected utility model. Moreover, the component of labor supply due to uncertainty is first-order in the sense that it is proportional to the standard deviation σ_E, rather than the variance σ_E^2 as in the standard expected utility model. With the low variation of stochastic wage income, the effect of uncertainty is of higher magnitude than that of risk.

2.3 Two-period Model

In this section, we consider a two-period model where the agent makes consumption, leisure and savings choices in the first period, and chooses consumption in the second period. In the second period the agent does not work but receives uncertain exogenous income[10](such as social security benefits). Two solvable utility specifications, quadratic and CARA utility, are adopted. It is shown that under future income uncertainty, there is an extra precautionary savings component and a corresponding self-insuring component in the optimal labor supply which cannot be attributed to the risk.

Suppose the period 1's exogenous income y is normal with mean μ_y and variance σ_y^2. The family of the agent's beliefs is again described in (4). The agent's budget constraints are

$$c_0 = w_0(T - l_0) - s \tag{2.18}$$

$$c_1 = y + s \tag{2.19}$$

where w_0 is period 0's wage rate and assumed to be deterministic, and s is period 0's savings. For simplicity, the discount factor β is assumed to be 1.

In the standard expected utility model, the positivity of the third derivative of the vNM utility function ($u''' > 0$) is necessary for the precautionary saving motive to exist. However, when the agent is both risk averse and uncertainty averse, the precautionary savings component is still present even for quadratic utility. Moreover, this component results in a self-insuring component in the optimal labor supply. The agent's decision problem is given

[10] The assumption is for analytical simplicity. More complicated cases may require numerical solutions and are left as future research topics.

by

$$\max_{c_0,l_0,s} U(c_0,l_0;c_1) \tag{2.20}$$

subject to (10) and (11), where multiple priors utility $U(c_0,l_0;c_1)$ is given in (2), and

$$u_0(c_0,l_0) = -(b-c_0)^2 - (T-l_0)^2 \tag{2.21}$$

$$u_1(c_1) = -(b-c_1)^2 \tag{2.22}$$

Proposition 5: Assume the set of priors is described by (4) and b is large enough ($b+c_0-\mu_y-w_0(T-l_0)>0$). Then the optimal labor supply in period 0 is increasing in the uncertainty aversion parameter ϕ, $\frac{\partial L_0^*}{\partial \phi}>0$.
Proof: see Appendix

Now, let us consider CARA utility

$$u_0(c_0,l_0) = -\frac{1}{\theta}e^{-\theta c_0} - \frac{1}{\theta}e^{-\theta l_0} \tag{2.23}$$

and

$$u_1(c_1) = -\frac{1}{\theta}e^{-\theta c_1},\ \theta>0. \tag{2.24}$$

Proposition 6: Assume the set of priors is described by (4), and the agent's multiple priors utility is again given in (2). The optimal labor supply in period 0 is increasing in both risk and uncertainty. That is, $\frac{\partial L_0^*}{\partial \sigma_y}>0$ and $\frac{\partial L_0^*}{\partial \phi}>0$.
Proof: see Appendix

In the two-period setting, the third derivative of the separable utility function is again irrelevant for us to obtain both the precautionary savings component and the self-insuring component. Knightian uncertainty is sufficient to generate these two components given that agents are uncertainty averse. Thus, both the one-period and two-period model motivate a testable implication: wage uncertainty may exert a significant impact on work hours, which is discussed in the next section.

3 Empirical Evidence

The focus of the empirical analysis is on the question whether we can find a positive and statistically significant effect of wage uncertainty on individuals' work hours when we regress work hours on wage uncertainty together with other related variables. That is, how much variation in hours of labor supply can be explained by the variations of wage uncertainty. In the regression model, we should include two variables: wage risk which is measured by the variance of stochastic wage income, as well as wage uncertainty, whose measure will be discussed below. The wage uncertainty is captured by the distortion term v, which is computed as the product of the standard deviation of stochastic wage income and the uncertainty aversion parameter ϕ. The possible explanation is that in the presence of high standard deviation of wage, it is more difficult for the agent to learn about the mean[11]. The dependence of the uncertainty on the risk arises from the relative entropy criterion (3). However, in reality it is possible that the agent has very volatile past wages but little doubt (uncertainty) about the expected wage. As a result, we allow for the possibility that the uncertainty depends on the state more flexibly. Thus for the empirical estimation purpose, we break up the relation when we study the effect of wage uncertainty on labor supply[12]. Let us consider an informal transformation of the relative entropy criterion (4),

$$\mathcal{V}(\phi) = \{v \in \mathcal{R} : v^2 \leq \phi^2\}. \tag{3.1}$$

Under this specification, one should obtain the result that the optimal labor supply relies on the variance of stochastic wage income and the uncertainty parameter ϕ alone, where ϕ is determined by some state variables.

[11] For example, see Anderson, Hansen and Sargent (2003), Hansen, Sargent, and Tallarini (1999) and Hansen, Sargent, Turmuhambetova, and Williams (2004).

[12] For example, Anderson, Ghysels and Juergens (2006) analyze the impact of risk and uncertainty on expected returns. They do not restrict the uncertainty to necessarily be tied to volatility of returns.

3.1 Estimating specification

We can derive an estimating specification from the previous theoretical discussions. In Proposition 2, assume $\gamma > 1$, the optimal labor supply under condition (12) is given by

$$L^{**} = \frac{1}{1 + e^{(-\frac{\gamma-1}{\gamma}\phi)} e^{-\frac{1-\gamma}{\gamma}\mu_w - \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2}} T \tag{3.2}$$

Taking logs for both sides and then the first-order Taylor expansion gives us

$$\ln L^{**} = \ln T + \frac{1}{2}(\frac{1-\gamma}{\gamma}\mu_w + \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2 + \frac{\gamma-1}{\gamma}\phi) \tag{3.3}$$

where μ_w is the mean of log wage, σ_w^2 is the variance and ϕ is the uncertainty. Moreover, the above specification implies certain parameter restrictions. Let us first estimate the model without restrictions and then test the validity of the restrictions.

Let L_{it} be the observed number of hours supplied by individual i at time t; let w_{it} be their computed hourly wage[13]; let var_{it} be a measure of wage risk; and let unc_{it} be a measure of wage uncertainty. Moreover, let X_{it} denote a vector of personal and job-specific characteristics including the unearned income, which is denoted as B_{it} for individual i at time t. Consider the following labor supply specification[14]:

$$\ln L_{it} = \beta_1 \times \ln w_{it} + \beta_2 \times var_{it} + \beta_3 \times unc_{it} + \mathbf{X}_{it}' \times \boldsymbol{\psi} + \alpha_i + \varepsilon_{it}, \tag{3.4}$$

where β_1, β_2 and β_3 are scalars, $\boldsymbol{\psi}$ is a $k-$dimensional vector of coefficients, α_i are individual fixed effects, and ε_{it} is a disturbance assumed to be normally and independently distributed, with mean zero and variance σ_ε^2.

In the empirical specification, the measurements of the uncertainty and the risk are two important issues. The risk, var_{it}, can be measured by the variance of individual i's log wages in contiguous time period prior to t[15]. This measure captures the historical income variation pertaining to the individual i. The uncertainty, unc_{it}, can be measured by the wage dispersion (ϕ_{kt}) in different industries. That is,

$$unc_{it} = \phi_{kt}. \tag{3.5}$$

[13] w_{it} is computed as the total income from work divided by hours of work. Further, negative bias may exist if labor supply L_{it} is estimated as a function of w_{it} by least squares, because $cov(w_{it}, L_{it}) < 0$.Thus, w_{it} needs to be instrumented. See Parker et. al. (2005).

[14] Parker, et al.(2005) uses a similar specification without wage uncertainty term.

[15] This measure is also adopted by Parker, et al. (2005).

For example, if individual i at time t is in industry k, ϕ_{kt} is measured by the standard deviation of all realized wages in industry k at time t. The economic rationale is that self-employed workers in the same industry are exposed to the similar uncertainty of economic conditions. It is reasonable to believe that the degree of dispersion of realized wages in a particular industry can characterize the uncertainty pertaining to that industry during certain period. Moreover, because self-employed workers' productivity is mainly influenced by the time-varying economic conditions, the wage distribution within an industry is also time varying. Thus ϕ_{kt} adequately captures self-employed workers' uncertainty about the surrounding economic environment, which can subsequently affect their wage income.

There are two estimation problems in the specification (14). First, $\ln w_{it}$ is likely to be endogenous, which may cause inconsistent estimators. we use $t-1$ and $t-2$ lagged wages as instruments in the empirical analysis. Lagged wages cannot depend on current work hours and hence are uncorrelated with the current period disturbance term. Second, the problem of heteroscedasticity may exist, which results in inefficient estimates. To improve the efficiency, a two-stage feasible GLS estimator can be used to estimate parameters of interest. Specifically, according to Greene (2003), we consider two forms of heteroscedasicity.

1. known form of heteroscedasicity: the problem of heteroscedasticity may be caused by the deviations of sample values of wage risk ($\widetilde{var}_{it}$) and uncertainty ($\widetilde{unc}_{it}$) from their population values, var_{it} and unc_{it}. To specify the structure of the variance of the disturbance, assume $\widetilde{var}_{it} = var_{it} + \nu_{it}$ and $\widetilde{unc}_{it} = unc_{it} + \vartheta_{it}$, where ν_{it} and ϑ_{it} are normally distributed with zero-mean, and $cov(\nu_{it}, \vartheta_{it}) = 0$. Then the true disturbance term is $\Delta_{it} = \varepsilon_{it} + \beta_2\nu_{it} + \beta_3\vartheta_{it}$, and its variance can be consistently estimated as[16]

$$\hat{\xi}_{it}^2 := V(\hat{\Delta}_{it}) = \hat{\sigma}_{\varepsilon}^2 + \frac{2\widehat{\beta}_2^{\,2}\widehat{\widetilde{var}_{it}}^{\,4}}{n_{it}-1} + \frac{2\widehat{\beta}_3^{\,2}\widehat{\widetilde{unc}_{it}}_{it}^{\,4}}{m_{it}-1} \tag{3.6}$$

   where individual i's wages have been observed over n_{it} periods, and there are m_{it} realized wages in individual i's industry at time t.Thus greater n_{it} and m_{it} implies more precise estimates of $\widetilde{var}_{it}$ and $\widetilde{unc}_{it}$, and namely, less deviation of their samples values from population values.

[16] See Wetherill (1981).

2. unknow form of heteroscedasticity: Suppose the functional form of heteroscedasticity, after controlling for unobserved effects, is

$$\sigma_{it}^2 = \exp(\boldsymbol{H}_{it} \otimes \varphi) \tag{3.7}$$

where $\boldsymbol{H}_{it}$ is the set of regressors which covaries with the heteroscedasticity, and we assume $\boldsymbol{H}_{it}\colon = [var_{it}, unc_{it}, \ln w_{it}]$ and φ is the corresponding coefficient vector. Then the predictions from the above estimation give the weights for performing the weighted least square estimation in the second stage. The weights are given by $\hat{\xi}_{it}^2 = \hat{\sigma}_{it}^2$.

After instrumenting the log wage to get $\ln w_{it}^I$, the two-stage GLS estimator, namely $\hat{\Theta}\colon = (\beta_1, \beta_2, \beta_3, \boldsymbol{\psi}')'$, is

$$\hat{\Theta} = [\sum_{i,t} (\frac{1}{\hat{\xi}_{it}^2}) \boldsymbol{z}_{it} \boldsymbol{z}_{it}']^{-1} [\sum_{i,t} (\frac{1}{\hat{\xi}_{it}^2}) \boldsymbol{z}_{it} \ln L_{it}], \tag{3.8}$$

where $\boldsymbol{z}_{it}$ are the columns of independent variables $\boldsymbol{Z}_{it}\colon = [\ln w_{it}^I, var_{it}, unc_{it}, \mathbf{X}_{it}]$. In summary, the first step is to estimate individual disturbance and the second step is to estimate $\hat{\Theta}$ from (18).

3.2 Description of data

The data sample are collected from the Panel Study of Income Dynamics (PSID), including a panel of 26 waves of cleaned observations, from 1968 to 1993. The sample comprises self-employed male working-age household heads. Data on self-employed workers is desirable for performing the empirical analysis for the following two reasons. First, self-employed workers have more flexibility in choosing work hours than employees do. Accordingly, their hours of labor supply are more likely to be endogenous choices. Second, the labor income of self-employed workers is more variable and unequal, as found in Carrington et. al.(1996) and Parker (1997). As a result, distinguishing wage uncertainty from risk is meaningful for self-employed workers.

The self-employed are individuals who were working at the time of the interview; who were self-employed in their main jobs; and who were not also working for other employers contemporaneously. The sample only includes self-employed individuals who earned positive incomes and worked positive numbers of hours, who had at least three consecutive years of income data, and single continuous spells in self-employment. Wages are computed by dividing annual earned income by annual hours worked. Both earned and unearned incomes are deflated to 1975 dollars using the CPI. The variable $\ln w_{it}$ was defined as the natural logarithm of the hourly wage; $\ln B_{it}$ was defined as the natural logarithm of one plus the respondent's investment income plus wife's income, if any; the squared unearned income equals the square of $\ln B_{it}$; var_{it} is calculated as the sample variance of the natural logarithm of the wages in the preceding contiguous years; and unc_{it} is calculated according to (15). The sample correlation between var_{it} and unc_{it} is 0.08.

Several other explanatory variables, **X**, include the number of years in the present job (tenure), square tenure, a dummy variable for disability status, age and its square, and the number of children in the household. The summary statistics are reported in Table 1.

Table 1: Summary statistics

Variable names	Description	Mean	Std. Dev.	Minimum	Maximum
LHOURS	Log of annual work hours	7.77	0.43	2.48	9.19
WAGE	Log of real wage	1.61	0.74	0.00	7.56
RISK	Variance of log past wages	0.21	0.43	0.00	6.25
UNC	Wage dispersion in industries	0.66	0.21	0.06	1.72
UINCOME	Log of real unearned income	6.66	3.79	0.00	11.52
UINCOME^2	UINCOME squared	58.77	37.90	0.00	132.64
TENURE	No. years in the current job	13.94	9.90	0.08	50.00
TENURE^2	TENURE squared/100	2.92	3.88	0.00	25.00
DISAB	Dummy variable =1 if disabled	0.09	0.29	0.00	1.00
AGE	Age	46.46	10.85	21.00	65.00
AGE^2	Age squared/100	22.76	9.96	4.41	42.25
NCHILD	No. of children in the household	1.15	1.26	0.00	7.00

3.3 Results

Because the selection bias may exist, the computed wage is likely to be endogenous. Following Parker et al. (2005), we use wages at $t-1$ and $t-2$ as instrumental variables for wage at t. obtain statistically significant coefficient of 0.149 on wage at $t-1$, which suggests positive wage persistence. The R^2 and F statistics for the instrumented wage equation were 0.733 and 10.45, showing these instruments are sufficient.

Table 2 presents the main results. The Hausman test statistics are 20.61 and 46.27 for the estimation with and without year dummies respectively, and is 48.10 for the model with an alternative measure of wage uncertainty (explained below). This indicates that the fixed effect model, rather than random effect model, be fitted. Estimated results with known form of heteroscedasticity (16) are included in Model 1, Model 2 and Model 5, and results with unknown form of heteroscedasticity (17) are included in Model 3, Model 4 and Model 6. The estimator is the two-stage feasible GLS. It is worth noting that among all the coefficient estimates for different models, wage uncertainty exerts a positive and significant effect on self-employed work hours. Apart from the instrumented log wages, age and squared age, the positive significant impact of wage uncertainty is robust The significance is pronounced with p-value less than 0.01 for all models considered. Model 1 is estimated without year dummies, in which the effect of wage uncertainty is 0.282 and the standard error 0.077. The effect increases to 0.642 (standard error 0.083) in Model 2, in which year dummies are included. Model 5 is another robustness check, where we replace the measure of uncertainty in Model 2 with the variance, instead of standard deviation, of realized wages in a particular industry in certain year. The estimate of effect of wage uncertainty is 0.322 with standard error 0.064. The results are virtually unchanged from those of Model 1 and 2. Under the unknown form of heteroscedasticity, the impact of wage uncertainty is still positive significant, as in Model 3 and Model 4 (including year dummies), with the effects 0.330 (standard error 0.075) and 0.988 (standard error 0.080) respectively. Model 6 uses the alternative measure of wage uncertainty discussed above, and results are essentially the same, with coefficient 0.594 (standard error 0.076). Our results show that the positive significant effect on self-employed work hours of wage uncertainty is robust to different heteroscedastic assumptions, and alternative estimating specification and measure of wage uncertainty. Thus far the empirical estimation provides sound evidences that self-employed individuals self-insure themselves by working more hours to compensate for greater income uncertainty, which is consistent with the theoretical prediction about the effect of income uncertainty. This positive effect on work hours can be the key to explain the self-employed labor supply puzzle.

In all models considered, the instrumented wage rate has a significant negative effect, which implies a backward bending labor supply schedule for self-employed individuals. This result is also robust to different heteroscedastic assumptions and estimating specifications. The risk variable is negative and insignificant except in Model 6, suggesting that wage risk, measured by the past variations of individual wages, has little power in explaining the self-employed labor supply behavior. The effect of age is significantly positive, which is opposite to that of squared age. This suggests that on the one hand, self-employed individuals adjust their work hours upwards as they age; on the other hand, they tend to lower work hours when aging. The net effect on work hours is ambiguous and depends on specific individuals' age. Moreover, the negative effect of squared age implies that the downward effect is more pronounced for older workers than younger ones. Similarly, the effects on work hours of job experiences and unearned income rely on the values of these two variables for each individual.

Test for parameter restrictions

The specification (13) implies the following parameter restrictions for the estimating model

1. $\beta_1 = \beta_3$

2. $\frac{1}{2}\beta_1^2 = \beta_1\beta_2 + \beta_2$

We perform the Wald test to test the following two null hypotheses: 1. restrictions 1 and 2 hold jointly; 2. only restriction 2 holds. The results are presented in Table 3. The implication is mixed in that whether to reject the null hypothesis largely depends on the estimating specifications. For Model 1, where no year dummies are included, we cannot reject the null that both restrictions are satisfied, while that only restriction 2 holds can be rejected at 5% significance level. When we add year dummies and/or use the alternative measure of the uncertainty, we can reject both null hypotheses. Under unknown heteroscedasticity assumption, we cannot reject both two hypotheses except in Model 5, where year dummies are added to Model 4.

4 Conclusion

In this paper, we distinguish income uncertainty from risk and analyze the effect of income uncertainty on the optimal labor supply in one-period and two-periods settings, respectively. In the theoretical discussion, agents are assumed to be uncertainty averse and risk averse. The uncertainty aversion is modeled through the adoption of multiple priors utility. For several types of utility specifications, we show that the uncertainty of stochastic wage income has impacts on the optimal labor supply. An increase in the degree of uncertainty can induce agents to supply more labor hours to protect themselves against future uncertainty. The model's implication is tested using PSID data on American male self-employed workers. The empirical analysis indicates that the wage uncertainty, which is proxied by the wage dispersion in different industries, appears to be more important in determining self-employed labor supply than risk, which is measured by the variance of individuals' past wages. Thus these results can potentially provide explanation of the long-standing puzzle of why the self-employed work longer hours than their employee counterpart.

5 Appendix

Proof of Proposition 1:

$$\begin{aligned}\max_{c,l} E[u(c,l)] &= \max_{l} E[\frac{(T-l)^{1-\gamma}}{1-\gamma} w^{1-\gamma} + \frac{l^{1-\gamma}}{1-\gamma}] \\ &= \max_{l} [\frac{(T-l)^{1-\gamma}}{1-\gamma} E(w^{1-\gamma}) + \frac{l^{1-\gamma}}{1-\gamma}]\end{aligned}$$

$$E(w^{1-\gamma}) = E[e^{\ln(w)\times(1-\gamma)}] = e^{(1-\gamma)\mu_w + \frac{1}{2}(1-\gamma)^2\sigma_w^2}$$

The first order condition is

$$(T-l)^{-\gamma} e^{(1-\gamma)\mu_w + \frac{1}{2}(1-\gamma)^2\sigma_w^2} = l^{-\gamma}$$

Simplifying yields optimal labor supply, L^*,

$$L^* = T - l^* = \frac{1}{1 + e^{-\frac{1-\gamma}{\gamma}\mu_w - \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2}} T$$

It is obvious that an increase in σ_w^2, which represents a mean preserving spread, unambiguously increases the optimal labor supply. That is $\frac{\partial L^*}{\partial \sigma_w} > 0$.
Moreover, for a log utility agent, labor supply is independent of wage risk. This is due to the additivity of a log function of products:

$$\ln[(T-l)w] = \ln w + \ln(T-l)$$

where the first term $\ln w$ does not appear in the first order condition. Q.E.D
Proof of Proposition 2:
For $\gamma \in (0,1)$

$$U(c,l) = \min_{Q\in\mathcal{P}} E_Q[\frac{(T-l)^{1-\gamma}}{1-\gamma} w^{1-\gamma}] = \frac{(T-l)^{1-\gamma}}{1-\gamma} \min_{Q\in\mathcal{P}} E_Q[w^{1-\gamma}].$$

The first order condition is

$$(T-l)^{-\gamma} \min_{Q\in\mathcal{P}} E_Q[w^{1-\gamma}] = l^{-\gamma}$$

Under wage uncertainty, the optimal labor supply L^{**} is given by

$$L^{**} = T - l^{**} = \frac{1}{1 + [\min_{Q\in\mathcal{P}} E_Q(w^{1-\gamma})]^{-1/\gamma}} T$$

Thus, $L^{**} < L^{*}$[17], which implies higher degree of Knightian uncertainty decreases the optimal labor supply. Applying the distributional assumption of w to the above expression of L^{**} gives us the closed form of solution

$$L^{**} = \frac{1}{1 + e^{(\frac{1-\gamma}{\gamma}\sqrt{2}\sigma_w \phi)} e^{-\frac{1-\gamma}{\gamma}\mu_w - \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2}} T$$

For $\gamma > 1$,

$$\begin{aligned} U(c,l) &= \min_{Q\in\mathcal{P}} E_Q[\frac{(T-l)^{1-\gamma}}{1-\gamma} w^{1-\gamma}] \\ &= \frac{(T-l)^{1-\gamma}}{1-\gamma} \max_{Q\in\mathcal{P}} E_Q[w^{1-\gamma}]. \end{aligned}$$

The first order condition is

$$(T-l)^{-\gamma} \max_{Q\in\mathcal{P}} E_Q[w^{1-\gamma}] = l^{-\gamma}.$$

The optimal labor supply is

$$L^{**} = \frac{1}{1 + [\max_{Q\in\mathcal{P}} E_Q(w^{1-\gamma})]^{-1/\gamma}} T$$

Thus, $L^{**} > L^{*}$, which implies higher degree of Knightian uncertainty increases the optimal labor supply. Applying the distributional assumption of w to the above expression of L^{**} gives us the closed form solution

$$L^{**} = \frac{1}{1 + e^{(-\frac{\gamma-1}{\gamma}\sqrt{2}\sigma_w \phi)} e^{-\frac{1-\gamma}{\gamma}\mu_w - \frac{1}{2}\frac{(1-\gamma)^2}{\gamma}\sigma_w^2}} T$$

Again, for log utility, Knightian uncertainty has no effect on labor supply because labor supply choice is independent of wage. Q.E.D

Proof of Proposition 3. By substituting the constraint into agent's problem, it can be rewritten as

$$\max_{l} \min_{Q\in\mathcal{P}} E_Q[-(b - \tilde{E}_0 - \alpha L)^2 - L^2]$$

Let us solve the inner minimization problem first:

$$\begin{aligned} \min_{Q\in\mathcal{P}} E_Q[-(b - \tilde{E}_0 - \alpha L)^2 - L^2] &= \min_{Q\in\mathcal{P}} E_Q[-(b - \tilde{E}_0 + \mu_E - v_E - \mu_E + v_E - \alpha L)^2] - L^2 \\ &= -\sigma_E^2 - \max_{v_E \in \mathcal{V}(\phi)} [(b - \mu_E + v_E - \alpha L)^2] - L^2 \\ &= -\sigma_E^2 - [(b - \mu_E + \sqrt{2}\sigma_E \phi - \alpha L)^2] - L^2 \end{aligned}$$

[17] L^{*} is the derived optimal labor supply in the standard expected utility model.

The second equality uses the facts

$$E_Q[(\tilde{E}_0 - (\mu_E - v_E))^2] = \sigma_E^2$$

and

$$E_Q[\tilde{E}_0 - (\mu_E - v_E)] = 0$$

where $Q \in \mathcal{P}(P, \phi)$. The third equality uses the fact that b is large enough, $b-(\mu_E+\alpha L) > 0$. For the maximization problem, the first order condition is that

$$\alpha(b - \mu_E + \sqrt{2}\sigma_E\phi - \alpha L) = L.$$

The optimal labor supply is then given by

$$L^{**} = \frac{\alpha(b - \mu_E + \sqrt{2}\sigma_E\phi)}{\alpha^2 + 1}$$

and $\dfrac{\partial L^{**}}{\partial \phi} > 0$ follows immediately. Q.E.D

Proof of Proposition 4: Substituting the budget constraint into the multiple priors utility function yields

$$\begin{aligned} U(c,l) &= \min_{Q\in\mathcal{P}} E_Q[-\frac{1}{\theta}e^{-\theta(\tilde{E}_0+\alpha L)} - \frac{1}{\theta}e^{-\theta(T-L)}] \\ &= -\frac{1}{\theta}e^{-\theta(\alpha L)} \max_{Q\in\mathcal{P}} E_Q[e^{-\theta\tilde{E}_0}] - \frac{1}{\theta}e^{-\theta(T-L)} \\ &= -\frac{1}{\theta}e^{-\theta(\alpha L)} \max_{v_E\in\mathcal{V}(\phi)} (e^{-\theta(\mu_E-v_E)+\frac{1}{2}\theta^2\sigma_E^2}) - \frac{1}{\theta}e^{-\theta(T-L)} \\ &= -\frac{1}{\theta}e^{-\theta(\alpha L)}(e^{-\theta(\mu_E-\sqrt{2}\sigma_E\phi)+\frac{1}{2}\theta^2\sigma_E^2}) - \frac{1}{\theta}e^{-\theta(T-L)} \end{aligned}$$

The first order condition is given by

$$\alpha e^{(-\theta(\mu_E-\sqrt{2}\sigma_E\phi)+\frac{1}{2}\theta^2\sigma_E^2-\theta(\alpha L))} = e^{(-\theta(T-L))}.$$

Simplifying it gives us the optimal labor supply

$$L^{**} = \frac{\ln\alpha + \theta(T - \mu_E) + \frac{1}{2}\theta^2\sigma_E^2 + \sqrt{2}\sigma_E\phi}{\theta(1+\alpha)}.$$

Thus $\dfrac{\partial L^{**}}{\partial \sigma_E^2} > 0$ and $\dfrac{\partial L^{**}}{\partial \phi} > 0$. Q.E.D

Proof of Proposition 5.

$$\begin{aligned}\min_{Q\in\mathcal{P}} E_Q[-(b-c_1)^2] &= \min_{Q\in\mathcal{P}} E_Q\{-[b-y-w_0(T-l_0)+c_0]^2\} \\ &= \min_{Q\in\mathcal{P}} E_Q\{-[b-y+\mu_y-v_y-\mu_y+v_y-w_0(T-l_0)+c_0]^2\} \\ &= -\sigma_y^2 - \max_{v_y\in\mathcal{V}(\phi)}[b-\mu_y+v_y-w_0(T-l_0)+c_0]^2 \\ &= -\sigma_{E_1}^2 - [b-\mu_y+\sqrt{2}\sigma_y\phi-w_0(T-l_0)+c_0]^2\end{aligned}$$

The fact $b+c_0-\mu_y-w_0(T-l_0)>0$ is used in the last equality.

The first order conditions are given by

$$b-c_0 = b-\mu_y+\sqrt{2}\sigma_y\phi-w_0L_0+c_0$$

$$L_0 = w_0(b-\mu_y+\sqrt{2}\sigma_y\phi-w_0L_0+c_0)$$

The optimal consumption in period 0 is

$$c_0^* = \frac{w_0^2b+\mu_y-\sqrt{2}\sigma_y\phi}{2+w_0^2}$$

The optimal labor supply in period 0 is:

$$L_0^* = \frac{w_0(2b-\mu_y+\sqrt{2}\sigma_y\phi)}{2+w_0^2}$$

The optimal saving in period 0 is:

$$s^* = \frac{w_0^2b-(w_0^2+1)(\mu_y-\sqrt{2}\sigma_y\phi)}{2+w_0^2}$$

As a result, $\dfrac{\partial s^*}{\partial\phi}>0$ and $\dfrac{\partial L_0^*}{\partial\phi}>0$. Q.E.D

Proof of Proposition 6.

$$\begin{aligned}\min_{Q\in\mathcal{P}} E_Q[-\frac{1}{\theta}e^{-\theta c_1}] &= -\frac{1}{\theta}\max_{Q\in\mathcal{P}} E_Q[e^{-\theta(y+w_0L_0-c_0)}] \\ &= -\frac{1}{\theta}e^{-\theta(w_0L_0)}e^{\theta c_0}\max_{Q\in\mathcal{P}} E_Q[e^{-\theta y}] \\ &= -\frac{1}{\theta}e^{-\theta(w_0L_0)}e^{\theta c_0}\max_{v_y\in\mathcal{V}(\phi)}[e^{-\theta(\mu_y-v_y)+\frac{1}{2}\theta^2\sigma_y^2}] \\ &= -\frac{1}{\theta}e^{-\theta(w_0L_0)}e^{\theta c_0}e^{-\theta(\mu_y-\sqrt{2}\sigma_y\phi)+\frac{1}{2}\theta^2\sigma_y^2}]\end{aligned}$$

The first order conditions are

$$e^{-\theta c_0} = e^{-\theta(w_0 L_0) - \theta(\mu_y - \sqrt{2}\sigma_y \phi) + \frac{1}{2}\theta^2\sigma_y^2 + \theta c_0}$$

$$e^{-\theta(T-L_0)} = w_0 e^{-\theta(w_0 L_0) - \theta(\mu_y - \sqrt{2}\sigma_y \phi) + \frac{1}{2}\theta^2\sigma_y^2 + \theta c_0}$$

The optimal labor supply is

$$L_0^* = \frac{\ln w_0^2 + 2\theta T - \theta(\mu_y - \sqrt{2}\sigma_y \phi) + \frac{1}{2}\theta^2\sigma_y^2}{\theta(w_0 + 2)}$$

The optimal saving in period 0 is given by

$$s^* = (w_0 + 1)L_0^* - (T + \frac{1}{\theta}\ln w_0)$$

It is obvious that $\frac{\partial L_0^*}{\partial \phi} > 0$ and $\frac{\partial s^*}{\partial \phi} > 0$. Q.E.D

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Table 2: Estimates of the labor supply model(Dependent Variable: lnL); standard errors are in parentheses; $*$: p-value less than 0.05; $**$: p-value less than 0.01

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
WAGE (lnw^I)	-0.171^{**}	-0.173^{**}	-0.166^{**}	-0.148^{**}	-0.145^{**}	-0.344^{**}
	(0.025)	(0.024)	(0.024)	(0.026)	(0.023)	(0.027)
RISK	-0.078	-0.085	-0.077	0.025	-0.085^{*}	-0.268^{**}
	(0.047)	(0.047)	(0.047)	(0.034)	(0.040)	(0.049)
UNC	0.282^{**}	0.642^{**}	0.322^{**}	0.330^{**}	0.988^{**}	0.594^{**}
	(0.077)	(0.083)	(0.064)	(0.075)	(0.080)	(0.076)
UINCOME	0.071^{**}	0.050^{**}	0.051^{**}	0.067^{**}	0.019	0.039^{*}
	(0.016)	(0.016)	(0.016)	(0.016)	(0.013)	(0.017)
UINCOME^2	-0.007^{**}	-0.004^{*}	-0.004^{**}	-0.006^{*}	-0.001	-0.001
	(0.002)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
TENURE	-0.015^{**}	-0.017^{**}	-0.018^{**}	-0.021^{**}	-0.005	0.005
	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.005)
TENURE^2	0.045^{**}	0.043^{**}	0.040^{**}	0.050^{**}	0.040^{**}	0.012
	(0.012)	(0.011)	(0.010)	(0.010)	(0.010)	(0.014)
DISAB	-0.054	-0.029	-0.023	-0.016	0.284^{**}	-0.087
	(0.051)	(0.049)	(0.050)	(0.052)	(0.034)	(0.051)
AGE	0.367^{**}	0.329^{**}	0.344^{**}	0.363^{**}	0.347^{**}	0.381^{**}
	(0.004)	(0.005)	(0.005)	(0.041)	(0.004)	(0.005)
AGE^2	-0.402^{**}	-0.361^{**}	-0.376^{**}	-0.396^{**}	-0.399^{**}	-0.440^{**}
	(0.006)	(0.007)	(0.006)	(0.006)	(0.005)	(0.006)
NCHILD	-0.027^{*}	-0.003	-0.007	0.026	-0.137^{**}	-0.125
	(0.013)	(0.013)	(0.013)	(0.014)	(0.011)	(0.015)
Hausman test	20.81	46.27	48.10	20.81	46.27	48.10
R^2	0.728	0.737	0.739	0.728	0.737	0.739
$F(k, n-k)$	9.54	9.06	9.17	9.48	9.06	9.17

Table 3: Wald test of parameter restrictions: the table contains the Wald test statistics and the corresponding p-values (in parentheses) for Chi-square distribution.

Model	Restriction 1 and 2	Restriction 2
Model 1	5.693	4.117
	(0.058)	(0.042)
Model 2	33.791	4.926
	(0.000)	(0.026)
Model 3	8.733	3.971
	(0.013)	(0.046)
Model 4	5.748	0.122
	(0.056)	(0.727)
Model 5	115.73	5.852
	(0.002)	(0.016)
Model 6	3.934	0.039
	(0.140)	(0.843)