Estimating Time-Varying Hedge Ratios with A Range-Based Multivariate Volatility Model

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Abstract

This paper proposes a new range-based multivariate volatility model to estimate the time-varying optimal hedge ratios. We compare the out-of-sample performance of this model with those of other alternatives using data of ten futures contracts. The alternative methods include traditional static hedge method and other dynamic hedge methods using the return-based multivariate GARCH model. The result suggests that our model outperforms all of these methods in most cases. The average efficiency gain over the traditional OLS method measured by the percentage variance reduction is about 27 percent.

Keyword: Hedge ratio, Minimum variance hedge, Range, CARR model, DCC model

Ⅰ. Introduction

Recent research has made significant contributions to the knowledge of futures hedging, both in terms of the academic support and the implementation of the strategies. In early history, the prices are assumed to follow a random walk with price changes being identically and independently distributed. However, many commodity price changes appeared not to be independent but rather to be characterized by quiet and volatile periods as variances change over time, following Mandelbrot (1963) and Fama (1965). The unconditional distributions of commodity price changes were also found to be fat-tailed, or leptokurtic.

For a given level of expected return, the purpose of hedging is to minimize the risk of the portfolio. Causes that affect the hedge construction and its effectiveness include basis risk, hedging horizon, and correlation between changes in the futures price and the spot price. Traditionally, the classical regression method was proposed, which assumed that a time-invariant hedge ratio, to compute the hedge ratio. Recently, the surge of the conditional volatility literature has provided many models that capture the time-varying variance and covariance of the cash and the futures. Several investigators adopted the framework of the GARCH model (Engle, 1982; Bollerslev, 1986). Particularly, the bivariate GARCH models were widely used to explain the behavior of the spot and futures prices which produced the dynamic hedge strategy

The results from the performance of the GARCH hedge ratios in comparing with the traditional methods are mixed. Some studies have found that the dynamic hedge strategies constructed by the GARCH methods outperform those of the static methods, e.g., Baillie and Myers (1991) and Kroner and Sultan (1993). On the other hand, Lien and Tse (2002) and Lien, Tse and Tsui (2002) report that while the dynamic hedging generated better performance for in-sample comparisons, but out-of-sample comparisons are mostly in favor of the conventional hedge strategy. Our paper intends to provide further evidence in this debate by introducing ranges in the bi-variate GARCH models.

In estimating volatility, the range data of asset prices perform better than the return data with close-to-close price (Parkinson, 1980; Wiggins, 1991; Alizadeh, Brandt and Diebold, 2002; Chou, 2005, and Brandt and Jones, 2006). Chou (2005) proposed the Conditional Autoregreesive Range (CARR) model to estimate the volatility process. Compared with GARCH model, CARR model obtained superior volatility forecast. Moreover, Chou, Wu and Liu (2006) extended it to a multivariate context using the Dynamic Conditional Correlation (DCC) model proposed by Engle (2002b). The DCC model is a kind of two steps forecasting model which estimates univariate GARCH models for each asset and then calculates its time-varying correlation by using the transformed standardized residuals from the first step. Chou, Wu and Liu (2006) found that the range-based DCC model performs better than the return-based model in forecasting covariances and correlations. In this paper, we test the range-based volatility model on hedging performance.

This paper applies new volatility models to exercise the optimal futures hedge. In addition to rollover OLS model, these methods were all based on the frameworks of CCC model and DCC model. The remainder of this paper is organized as follow. In section 2, we describe the methodologies. Section 3 presents our data analysis and out-of sample results of the optimal hedging ratios constructed by the above models. We conclude in the final section.

Ⅱ. Hedging Methodology

The application of portfolio theory to hedges has attracted a great deal of attention from academics and market participants. Johnson (1960) and Stein (1961) introduced the concept of portfolio theory through hedging the spot position with futures. Edrington (1979) applied this concept in determining a minimum-variance hedge ratio and proposed a measure of hedging effectiveness. Furthermore, the hedging portfolio has usually been adopted as the returns of holding the spot asset on the returns together with the hedging instruments like futures contracts. Lence (1995) argued that the benefit of sophisticated estimation techniques of the hedge ratio is small. He also promotes that hedgers may feel better by using the simpler and more intuitive hedge models. This view is supported by Lien, Tse and Tsui (2002). In this case, the optimal hedge ratio can be defined as the amount of futures position for bearing one unit of spot position such that we have minimum variance hedge portfolio. The Optimal hedge ratio is given by

$$h = \mathrm{cov}(s_t, f_t) / \mathrm{var}(f_t), \tag{1}$$

where s_t and f_t are the returns of the spot and futures between time $t-1$ and t.

The simplest way to hedge the spot price risk is through the naïve hedge strategy. This strategy suggests that an investor who has a long position in the spot market should sell a unit of futures today and buy it back when he sells the spot. If the spot and futures prices both change by the same amount at all times, this will be a perfect hedge. Except for the naïve hedge, a conventional method estimates the following linear regression model:

$$s_t = \alpha + \beta f_t + \varepsilon_t . \tag{2}$$

The coefficient β is an OLS estimator which provides an estimate for the minimum-variance hedge ratio. This method has been broadly applied in the literature. A major shortcoming of the OLS hedge ratio is its dependence on the unconditional second moments. In the presence of an environment with changing conditional second moments, this method may not provide an effective hedge using the futures instruments.

In equation (2), the joint distribution of spot and futures price is constant over time.

We could expand this model to the solution for the sequence of hedge ratios calculated as the least squares estimator from a time-series regression of s_t on f_t. In fact, the distribution of spot and futures prices may be time-varying. Recent studies suggest that the time-varying volatility prevails in many time series. The risk of assets changes because new information is continuously received by the markets (Bollerslev, 1990; Kroner and Sultan, 1993). Therefore, the hedge ratio should be time-varying because it depends on the conditional moments of the spot and futures returns. On account of it, we want to find the optimal hedge ratio estimated from different models.

The CARR model

Parkinson (1980) presented that the extreme value theories imply that the range is an effective estimator of the local volatility. Chou (2005) proposed the CARR model to capture the dynamic evolution of volatilities by using the range data as a measure of volatility. The CARR model and the Autoregressive Conditional Duration (ACD) model of Engle and Russell (1998) are both special cases of the multiplicative error models of Engle (2002a).

Let P_t be the logarithm of the price of some speculative asset at time t and let $G_t = \ln P_t^{High} - \ln P_t^{Low}$ be the high-low range in the price path during the interval of [t-1, t]. The CARR (p, q) model can be expressed by

$$G_t = \lambda_t v_t$$

$$\lambda_t = \omega + \sum_{i=1}^{q} \alpha_i G_{t-i} + \sum_{j=1}^{p} \beta_j \lambda_{t-j} \qquad (3)$$

$$v_t | \Omega_{t-1} \overset{iid}{\sim} f(1, \cdot) .$$

In (3), λ_t is the conditional mean of the range based on all information up to time t, or $\lambda_t \equiv E(G_t | \Omega_{t-1})$, $\lambda_t \geq 0$. The innovation v_t is assumed to be distributed with a density function $f(.)$ with a unit mean. The coefficients α_i and β_j are indicators measuring the responses of short-run effect and long-run effect respectively. Chou (2005) also indicated that the α estimated by the CARR(1,1) is bigger than GARCH(1,1) model. Hence, the CARR model is better than GARCH model to capture the short term dynamic movements.

With some particular specifications for the distribution, f_t, (e.g., exponential or Weibull distribution) the CARR model can be estimated using the Quasi-Maximum-Likelihood Estimation method (QMLE). For inferences, the covariance matrix is computed by the robust method of Bollerslev and Wooldridge (1992).

The DCC model

The DCC model proposed by Engle (2002b) is a new model for measuring and forecasting correlation as well as volatilities. The DCC estimators have the adaptability of the univariate GARCH model but not the complexity of the conventional multivariate GARCH model. Let r_t be a vector of returns, the DCC model is formulated as the following specification:

$$
\begin{aligned}
& r_t|\Omega_{t-1} \sim N(0,H_t)\,,\quad H_t = D_t R_t D_t \\
& R_t = diag\{Q_t\}^{-1/2} Q_t\, diag\{Q_t\}^{-1/2} \\
& Q_t \quad = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1} Z_{t-1} + B \circ Q_{t-1}\,,\quad Z_t = D_t^{-1} \times r_t\,.
\end{aligned} \tag{4}
$$

In this model, the covariance matrix H_t is characterized as a product of the conditional correlation matrix R_t, pre and post multiplied by the matrix D_t - a diagonal matrix with the volatility of return $r_{i,t}$, or $\sqrt{h_{i,t}}$ on the i$^{\text{th}}$ diagonal. In the third equation in (4), ι is a vector of ones and $\circ$ is the Hadamard product of two identically sized matrices for an element-by-element multiplication. Q_t is the conditional standardized residual covariance matrix. The CCC model of Bollerslev (1990) is a special case of this model that the conditional correlation is specified to be a constant.

For its log-likelihood function, we can express it as

$$
L(\psi,\phi) \quad = -\frac{1}{2}\sum_t (n\log(2\pi) + 2\log|D_t| + r_t' D_t^2 r_t) - \frac{1}{2}\sum_t (-r_t' D_t^2 r_t + \log|R_t| + \varepsilon_t' R_t^{-1} \varepsilon_t)
$$

$$= L_v(\psi) + L_c(\psi, \phi) \,. \tag{5}$$

Let the parameters in D_t be denoted by ψ and the other parameters in R_t to be denoted as ϕ. The log-likelihood can be rewritten as the sum of a volatility part ($L_v(\psi)$) and a correlation part ($L_c(\psi, \phi)$).

When the specific GARCH model is fitted, the term of volatility in the likelihood function can demonstrated as below:

$$L_V(\theta) = -\frac{1}{2}\sum_t \sum_{i=1}^{k}\left(\log(2\pi) + \log(h_{i,t}) + \frac{r_{i,t}^2}{h_{i,t}} \right). \tag{6}$$

By the same token, if D_t is determined by a CARR specification, then the likelihood function of the volatility term will be

$$L_V(\theta) = -\frac{1}{2}\sum_t \sum_{i=1}^{k}\left(\log(2\pi) + 2\log(\lambda_{i,t}^*) + \frac{r_{i,t}^2}{\lambda_{i,t}^{*2}} \right), \tag{7}$$

Where $\lambda_{i,t}^*$ denotes the conditional standard deviation as computed from a scaled expected range, using the CARR model of Chou (2005).

Out of sample forecasts of the DCC models for correlation can be obtained using the standard backward iterative approach; given T as the sample size, T+1st observation is obtained. At time T for the bivariate case, the out of sample forecast for conditional correlation in the period (T+1) is presented by

$$\begin{bmatrix} q_{1,T+1} & q_{12,T+1} \\ q_{12,T+1} & q_{2,T+1} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,T}^2 & z_{1,T}z_{2,T} \\ z_{1,T}z_{2,T} & z_{2,T}^2 \end{bmatrix} + b\begin{bmatrix} q_{1,T} & q_{12,T} \\ q_{12,T} & q_{2,T} \end{bmatrix} \tag{8}$$

where $\rho_{T+1} = q_{12,T+1} / \sqrt{q_{1,T+1} q_{2,T+1}}$.

Ⅲ. Data Analysis and Comparisons of Various Hedging Models

For empirical study, we use the prices of the spot and futures contracts for ten different assets or commodities: S&P500 (SP), Swiss Franc (SF), Gold, Coffee, Corn, Wheat, Cotton, Sugar, Soybean oil and Soybean. The first two contracts are of financial futures while the remaining eight are of metal or agricultural commodities. There are all very actively traded. The data are obtained from Datastream. The sample period is from January, 4, 1988 to December, 30, 2005 with 939 weekly observations. Weekly frequency is adopted since a higher frequency such as daily will incur much higher transaction costs in constructing rebalancing portfolios. We select the nearest contract to deliver but rolled it over to the next nearest contract on the first day of the delivery month in order to avoid thin trading and expiration effects.

In Figure 1, we plot the closing prices of these ten contracts in spot and futures. In general, the futures prices tract the spot market fairly closely. However, noticeable deviations can be clearly seen in some markets such as corn, wheat, cotton and sugar. Given the clearly nonstationary pattern in the price series, we focus our analysis in the returns. Table 1 gives summary statistics for returns of each spot and futures commodity prices. The mean of the returns are almost identical for all of the series, and very close to zero. As is noted by Fama (1965), this martingale behavior is often interpreted as being consistent with a weak form efficient market. In most of cases, the volatility of futures returns is somewhat higher than the volatility of spot returns, which is a common empirical finding (see, for example, Kroner and Sultan, 1993). It appears that all of the spot and futures returns are highly leptokurtic and negatively or positively skewed.

Table 1: The Statistics for Return of Cash and Futures Prices, 1988-2005

Spot	SP	SF	Gold	Coffee	Corn	Wheat	Cotton	Sugar	Soyoil	Soybean
Mean	0.174	0.002	0.007	-0.016	0.012	0.000	-0.026	0.057	-0.004	-0.003
Median	0.309	-0.067	0.026	0.086	0.185	0.000	0.000	0.231	-0.252	0.115
Maximum	7.492	6.075	13.005	40.143	16.594	18.207	13.794	21.268	14.266	13.337
Minimum	-12.330	-6.832	-6.564	-33.173	-21.052	-23.278	-30.538	-20.641	-15.415	-24.506
Std. Dev.	2.095	1.598	1.708	5.198	3.369	3.796	3.244	4.304	3.233	3.318
Skewness	-0.470	0.105	0.486	0.314	-0.227	0.033	-1.213	-0.222	0.109	-0.691
Kurtosis	5.905	3.677	7.504	10.398	6.283	5.279	15.476	5.181	4.504	8.449
Futures	**SP**	**SF**	**Gold**	**Coffee**	**Corn**	**Wheat**	**Cotton**	**Sugar**	**Soyoil**	**Soybean**
Mean	0.176	0.002	0.007	-0.019	0.012	0.003	-0.020	0.045	-0.006	-0.005
Median	0.294	-0.071	0.000	-0.064	0.000	-0.125	0.000	0.291	-0.099	0.099
Maximum	8.124	6.440	12.806	43.788	18.297	16.420	17.638	30.111	12.855	13.337
Minimum	-12.395	-6.457	-8.376	-23.732	-24.890	-24.288	-34.617	-22.445	-16.145	-17.721
Std. Dev.	2.165	1.626	1.855	5.518	3.390	3.521	3.671	4.837	3.195	3.234
Skewness	-0.479	0.112	0.272	0.717	-0.079	0.045	-0.809	-0.226	0.071	-0.301
Kurtosis	5.928	3.671	7.180	9.052	8.098	6.876	12.371	6.455	4.484	6.040

Notes: The returns for commodity prices are computed by $100 \times \log(p_t^{close} / p_{t-1}^{close})$. There are 939 weekly sample observations. All data are extracted from Datastream.

Table 2 gives summary statistics for ranges of each spot and futures commodity prices. The order of the magnitudes for the means of the range is roughly the same as that for the standard deviations of the returns in Table 1. This reflects the fact that both range and standard deviations are measures of volatilities. Given that the range data are non-negative, positive skewness is present for all commodities.

Table 2: The Statistics for Range of Cash and Futures Prices, 1988-2005

Spot	SP	SF	Gold	Coffee	Corn	Wheat	Cotton	Sugar	Soyoil	Soybean
Mean	2.269	1.765	1.757	4.727	3.475	4.202	2.575	4.650	3.541	3.294
Median	1.954	1.577	1.486	3.939	2.892	3.647	1.951	4.011	3.047	2.734
Maximum	12.330	6.832	13.005	40.143	21.052	25.766	30.538	24.737	17.007	24.506
Minimum	0.207	0.181	0.096	0.000	0.437	0.394	0.000	0.000	0.389	0.261
Std. Dev.	1.410	0.920	1.218	3.925	2.406	2.550	2.646	2.890	2.016	2.337
Skewness	1.901	1.352	2.376	2.901	2.383	2.394	3.193	1.848	1.768	2.719
Kurtosis	9.098	6.051	15.057	19.352	12.209	14.918	24.034	8.997	8.196	16.395
Futures	**SP**	**SF**	**Gold**	**Coffee**	**Corn**	**Wheat**	**Cotton**	**Sugar**	**Soyoil**	**Soybean**
Mean	2.352	1.792	1.875	5.636	3.328	3.738	3.733	5.004	3.476	3.246
Median	2.022	1.586	1.592	4.701	2.715	3.229	3.205	4.073	3.050	2.732
Maximum	12.395	6.765	13.981	43.788	24.890	24.288	34.617	43.178	16.252	17.721
Minimum	0.195	0.266	0.129	0.616	0.099	0.305	0.000	0.000	0.456	0.393
Std. Dev.	1.500	0.939	1.273	3.945	2.378	2.235	2.469	3.625	1.973	2.211
Skewness	1.865	1.410	2.388	2.877	2.436	2.430	3.184	2.993	1.736	2.195
Kurtosis	8.597	6.314	15.253	19.018	14.245	14.886	30.688	21.717	7.853	10.125

Notes: The ranges for commodity prices are computed by $100\times\log(p_t^{high}/p_t^{low})$. P_t^{high} and P_t^{low} are the maximum and minimum price respectively among the daily close prices in the t^{th} week and the last trading day close price in the $t\text{-}1^{th}$ week. There are 939 weekly sample observations. All data are extracted from Datastream.

Out-of-sample hedge

In this section, we compare the performances of several different methods. The variance of portfolio returns are computed under the following six alternative models: naïve hedging; hedging with optimal hedge ratios (OHRs) using rollover OLS methods of returns; hedging with time-varying OHRs using the return-based CCC model; hedging with time-varying OHRs using the return-based DCC model; hedging with time-varying OHRs using the range-based DCC model; hedging with time-varying OHRs using the range-based DCC model.

In order to formally compare the performances of each kind of hedge method, we construct the portfolios implied by the computed hedge ratios each week and calculate the variance of the returns to these portfolios, i.e., $Var\ (s_t - b_{t-1}^* f_t)$, where b_t^* are estimated OHRs from different hedge methods. Because most people are more interested in knowing how well they can do in the future with a different hedging strategy, we highlight the out-of-sample performances. In-sample forecasting results are similar and hence are not reported here.

The one period ahead out-of-sample forecasting results are reported in Table 3. In the upper panel, we calculate the variances of the hedge portfolios and the in the lower panel. To further gauge the hedging efficiency among various methods, we report in the lower panel the percentage variance improvement of each of the alternative methods compared with the OLS method. A positive (negative) number indicates an efficiency gain (loss) of the method relative to that of the OLS method.

<u>Table 3: Comparisons of Out-of-Sample Hedging Effectiveness for Different Methods, 1988-2005</u>

Variances	SP	SF	Gold	Coffee	Corn	Wheat	Cotton	Sugar	Soyoil	Soybean
Naïve	0.154	0.129	0.988	12.790	2.551	6.331	17.560	9.340	1.124	2.594
Roll-over OLS	0.149	0.126	0.837	10.497	2.631	6.304	10.806	7.799	1.152	2.551
Return CCC	0.138	0.121	0.670	8.214	2.110	5.831	10.456	7.103	0.930	1.870
Return DCC	0.140	0.120	<u>0.630</u>	7.932	1.700	<u>5.549</u>	8.872	5.213	0.846	<u>1.534</u>
Range CCC	<u>0.127</u>	0.103	0.719	7.155	<u>1.571</u>	6.263	9.187	6.048	0.857	2.534
Range DCC	0.130	<u>0.102</u>	0.635	<u>6.862</u>	1.795	5.564	<u>7.632</u>	<u>4.191</u>	<u>0.732</u>	1.978

Percentage Variance Improvement Compared with Roll-over OLS Hedge

$(Var_{OLS} - Var_{Model})/Var_{OLS}$

	SP	SF	Gold	Coffee	Corn	Wheat	Cotton	Sugar	Soyoil	Soybean	Average
Naïve	-0.028	-0.028	-0.180	-0.218	0.031	-0.004	-0.625	-0.198	0.024	-0.017	-0.124
Return CCC	0.073	0.041	0.199	0.217	0.198	0.075	0.033	0.089	0.193	0.267	0.139
Return DCC	0.062	0.033	<u>0.247</u>	0.244	0.354	<u>0.120</u>	0.179	0.332	0.265	<u>0.399</u>	0.224
Range CCC	<u>0.151</u>	0.183	0.141	0.318	<u>0.403</u>	0.006	0.150	0.224	0.256	0.007	0.184
Range DCC	0.129	<u>0.190</u>	0.242	<u>0.346</u>	0.318	0.117	<u>0.294</u>	<u>0.463</u>	<u>0.365</u>	0.225	<u>0.269</u>

Note:

1. There are 500 observations, about 10 years, in each of the estimated models. Additionally, the rolling sample method provides 439 one period ahead out-of-sample forecasting values for comparison. The first forecasted value occurs on the week of August 1, 1997.
2. The number with an underline stands for the smallest hedging portfolio variance or the largest percentage variance improvement in each commodity column.
3. The Optimal Hedge Ratios (OHRs) of the Naïve model are always one; The OHRs of the rollover OLS model are the coefficient of the regression of spot price on futures by rolling sample. The OHRs estimated by the return-based CCC model and the return-based CCC model are based on the constant conditional correlations. The OHRs estimated by the return-based DCC model and range-based DCC model are based on the dynamic conditional correlations.

Several observations can be made from the reading of Table 3. First, the portfolio variances are much smaller in the two financial futures (S&P500 and Swiss Franc) than in other eight futures contracts in metal or agricultural products. This result may be reflecting the fact that financial futures markets are more liquid and information flows are efficient than in the commodity markets. Hence, risks in the spot prices can be more effectively hedged.

We next turn to the comparison of the six hedging methods. The Naïve method is the worst of all. This is not surprising as the assumption of perfect correlation between the cash and the futures returns underlying this method is clearly not supported empirically. Next, all the four methods with time-varying hedge ratios outperform the static OLS method indicating that the traditional method assuming constant hedge ratio has a lot of room for improvements.

Among the dynamic hedge methods, how does the range-based method compare with the return-based method? The results seem to suggest that the range-based methods are better than their corresponding opponents with return-based methods. Specifically, the variances of the hedging portfolio derived from range-based volatility models are smaller than return-based volatility models in seven out of ten commodities. In the two cases, Gold and Wheat, where return-based models perform better, the differences are trivial. Only in the soybean case, the return-based DCC model has more notable dominance over the range-based DCC model.

The over all performance is summarized by the average percentage variance improvement reported in the last column in the lower panel. The range-based DCC model is the clear winner of all methods, with an efficiency gain of about 27% over the OLS method. The next best model is the return-based DCC model with a 22.4% efficiency gain. This result confirms the finding of Chou (2005) that range is indeed a better measure of volatility than return. The CCC methods are also better than the OLS method, although their efficiency gains are lower than the DCC specifications. This finding suggests that the additional effort in modeling the time-varying pattern of the conditional correlation is not without rewards.

For illustration, we also plot the estimated hedge-ratios using different methods for two cases, the S&P500 and the soybean oil respectively in figures 2 and 3. The OLS has the smoothest pattern in both cases. It is a static hedge method but still varies over time because a rolling-sample of ten years is used in these out-of-sample comparisons. In both cases, the dynamic methods provide wide variations of the hedge ratios around the OLS estimates. A more flexible range of hedge ratios seems to be necessary in order to obtain a more effective hedge strategy.

Ⅳ. Conclusion

In this paper, we adopt a range-based multivariate volatility model in computing the time-varying hedge ratios for ten futures contracts. We compare the hedging performance of this method with other alternatives with a static hedge or with dynamic hedges based on other methods. For the one-period, out-of--sample forecast, we find that the hedging result of our model is significantly better than the OLS model. The efficiency gain over the OLS method is 46% for sugar, and the average gain is about 27%. The alternative dynamic hedge method using a return-based volatility model also outperforms the OLS method but is dominated by our range-based model in most cases. Modeling the dynamics of the conditional correlations also helps improving the hedging performances.

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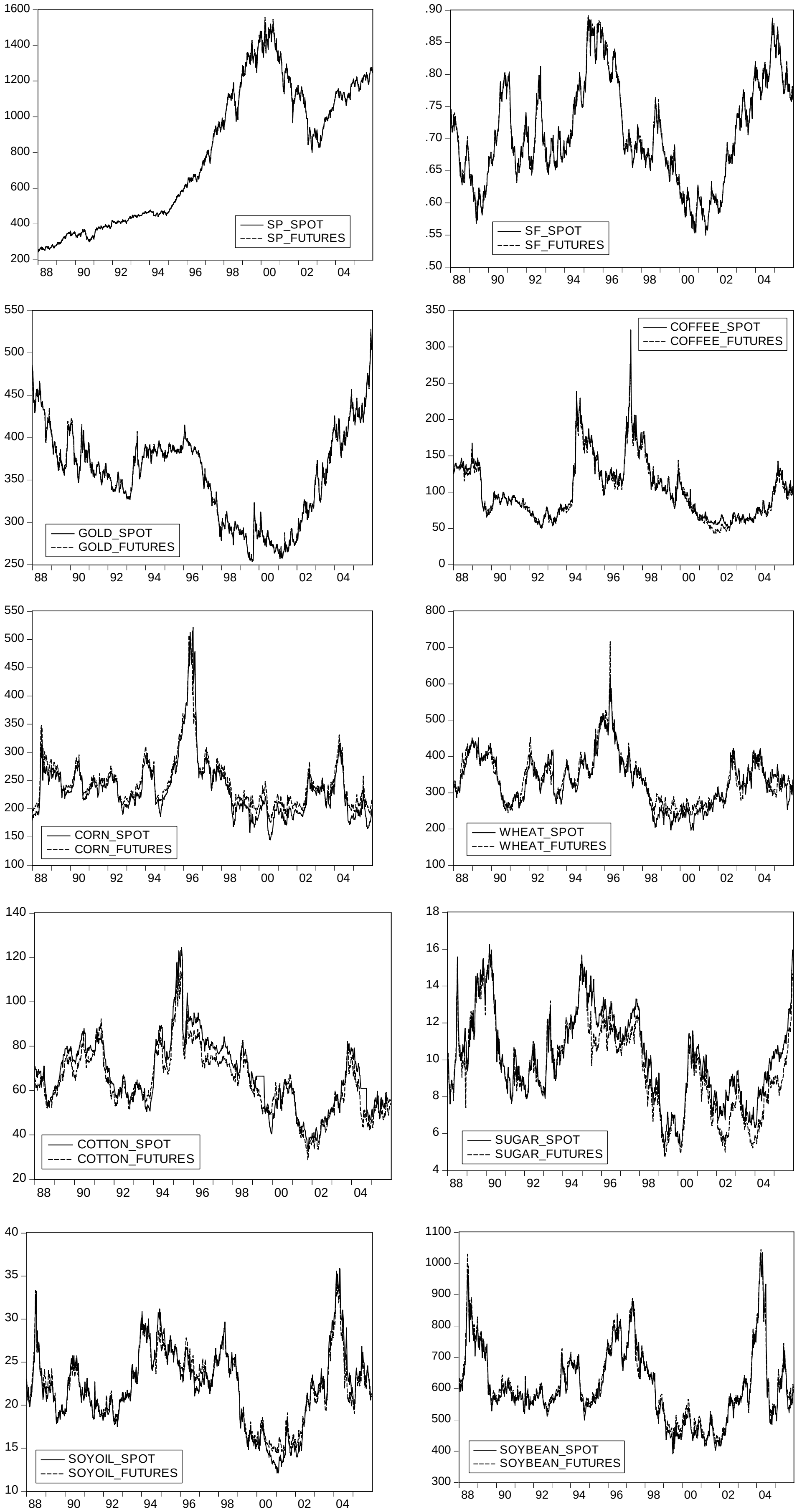


Figure1: Close prices of spot and futures for all commodities

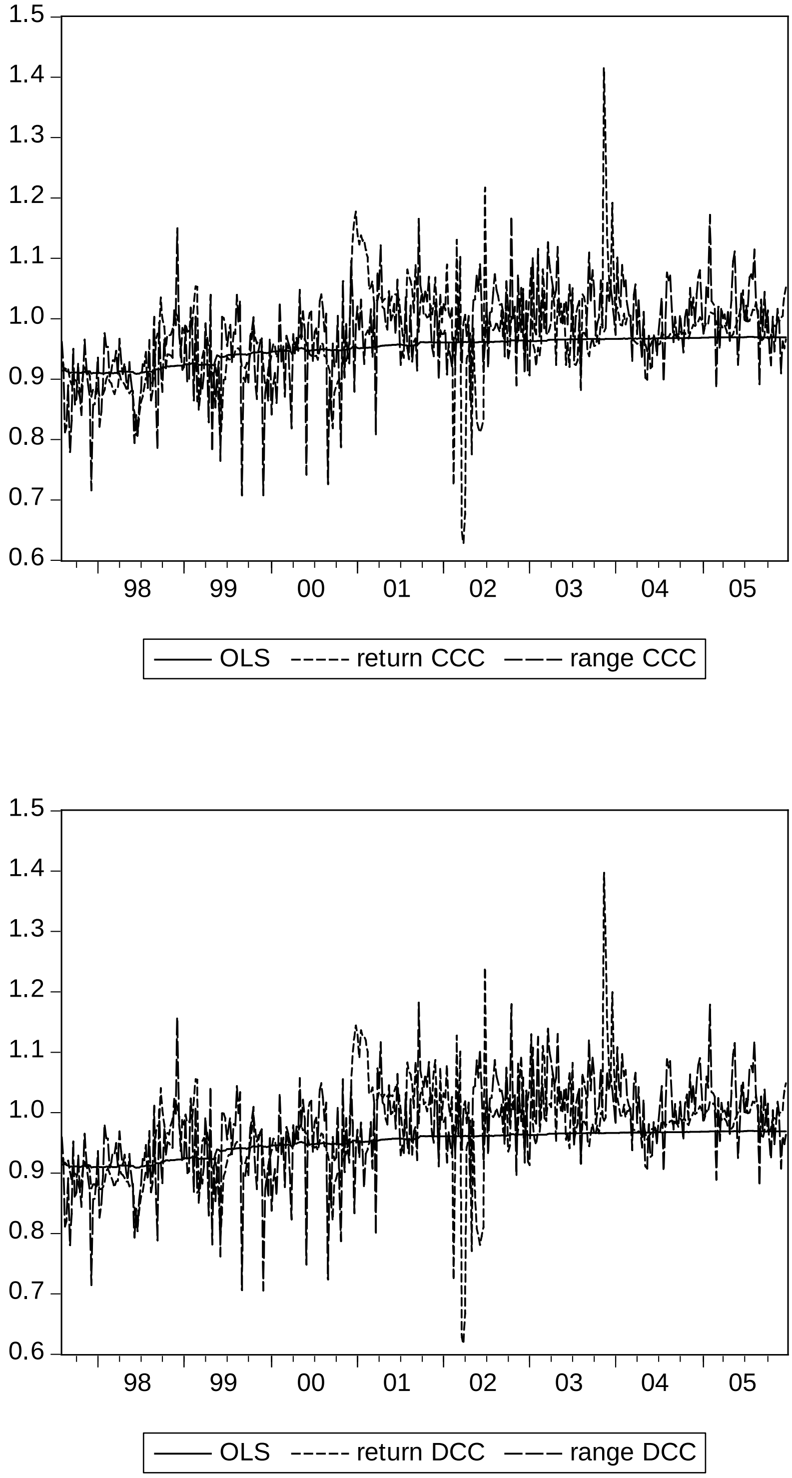
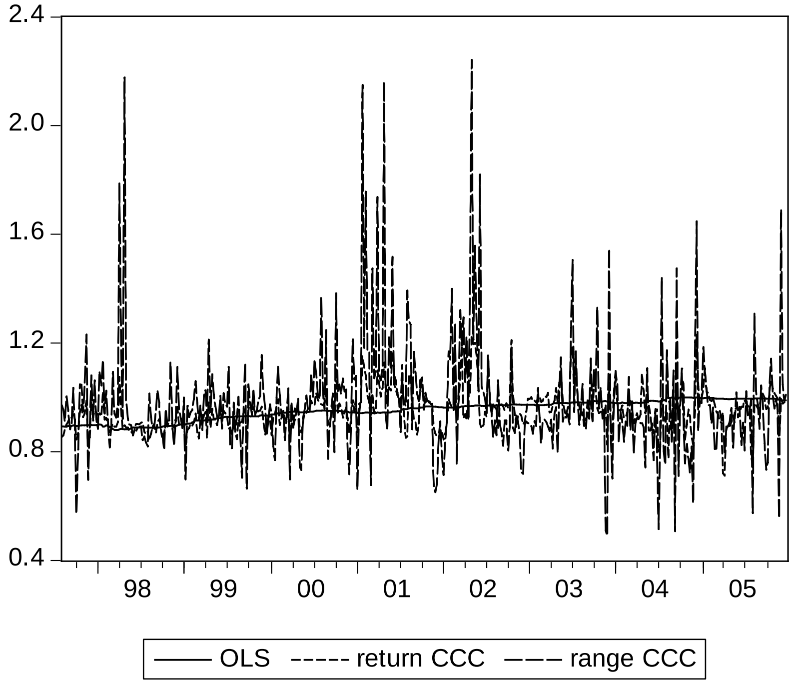


Figure 2: Comparison of optimal hedge ratios for S&P 500



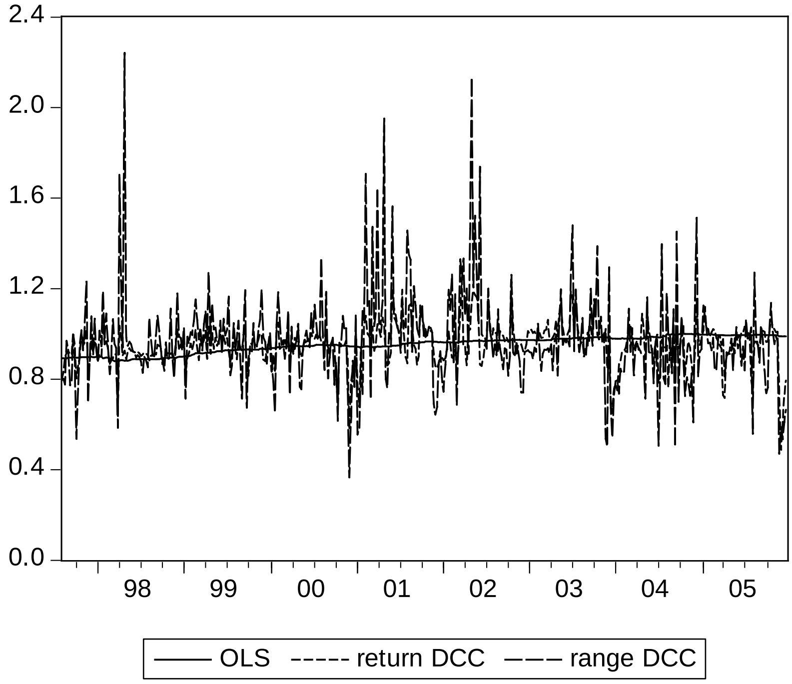


Figure 3: Comparison of optimal hedge ratios for Soybean Oil