Identification and Estimation of Bidders' Risk Aversion in First-Price Auctions

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Session Title: Empirical Industrial Organization

Session Chair: Patrick Bajari

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Risk aversion is a fundamental concept in economics used to explain agents' behavior under uncertainty. Risk aversion in auctions has been justified through the many uncertainties faced by bidders in auctions and through the large value of bids relative to bidders' assets. In first-price auctions, risk aversion renders bidding more aggresssive, while bidding in ascending auctions is not affected leading to the dominance of the sealed-bid mechanism over the ascending one. Risk aversion has been extensively tested on experimental data as overbidding relative to the Nash Equilibrium is frequently observed. In view of recent developments in the structural estimation of auction models, Patrick Bajari and Ali Hortacsu (2005) show that the risk aversion model provides the best fit to experimental data over several competing models.

Given larger financial stakes, it is likely that bidders' risk aversion is present in field auction data as well. The theoretical auction literature does not provide simple implications that can be tested on bidding data. Consequently, detecting risk aversion in auctions is difficult if not impossible. More generally with microeconomic data, risk aversion can be detected only when diversification occurs such as in portfolio management and in auctions with diversification across species as in Susan Athey and Jonathan Levin (2001). This calls for the necessity of a structural modeling to evaluate bidders' risk aversion. In the structural approach, observed bids are assumed to be the outcome of the Bayesian Nash equilibrium of a particular model. Though a tight structure is imposed to explain observed bids, assessing risk aversion is a challenging problem. As a matter of fact, the model with risk averse bidders is not identified in general from observed bids. Moreover, in view of bids only we cannot distinguish several well known specifications of risk aversion such relative risk aversion (RRA) or absolute risk aversion (ARA) though they lead to different economic implications.

Thus, recovering risk aversion requires to impose some restrictions. We discuss three possible restrictions: (i) partial parameterization of the auction model, (ii) exogeneity of the number of bidders, and (iii) additional bidding data. In the first case, parameterizing the utility function is not sufficient as an increase in the risk aversion parameter such as constant RRA (CRRA) can be compensated by a shrinkage of the quantiles of the valuation distribution leading to the same bid observations. Similarly, parameterizing the underlying valuation distribution does not allow to recover the utility function. A combination of both is necessary to identify the model, i.e. parameterization of the utility function and parameterization of a conditional quantile of the underlying distribution. In the second case, assuming exogeneity of the number of bidders allows to recover the bidders' utility function without any further assumption. This exclusion restriction is in the spirit of instrumental variables. In the third case, the availability of ascending auction data allows to recover the underlying distribution, which can be used in first-price auction data to recover the bidders' utility function. This paper reviews these results while relying on three articles, namely Sandra Campo, Emmanuel Guerre, Isabelle Perrigne and Quang Vuong (2006), Guerre, Perrigne and Vuong (2006) and Jingfeng Lu and Perrigne (2006).

A first section introduces the auction model and discusses the nonindentification of risk aversion. A second section presents the identification of risk aversion under parametric restrictions and briefly discusses the estimation of risk aversion. A third section is devoted to the identification of risk aversion under exclusion restrictions, while a fourth section considers the identification of risk aversion using additional data.

I. Nonidentification of Risk Aversion

As a benchmark, we consider the first-price sealed-bid auction with independent private values distributed as $F(\cdot)$ on $[\underline{v}, \overline{v}]$ and a $I \geq 2$ symmetric bidders. Bidders are potentially risk averse with a utility function $U(\cdot)$ satisfying $U(0) = 0$, $U'(\cdot) > 0$ and $U''(\cdot) \leq 0$. Bidder i maximizes his expected utility $\mathrm{E}\Pi_i = U(v_i - b_i)\Pr(b_i \geq b_j, j \neq i)$ with respect to his bid b_i given his private value v_i. Denoting by $s(\cdot)$ the Bayesian Nash equilibrium strategy, the solution of the game is given by the differential equation $s'(v_i) = (I-1)[f(v_i)/F(v_i)]\lambda(v_i - b_i)$, where $\lambda(\cdot) = U(\cdot)/U'(\cdot)$ with the boundary condition $s(\underline{v}) = \underline{v}$.

The observations consist of the pair (I, G), where $G(\cdot)$ is the distribution of equilibrium bids. Following Guerre, Perrigne and Vuong (2000), the previous differential equation can then be rewritten as

$$v_i = b_i + \lambda^{-1}\left(\frac{1}{I-1}\frac{G(b_i)}{g(b_i)}\right), \tag{1}$$

since $\lambda(\cdot)$ is strictly increasing. This equation expresses each bidder's private value as a function of his corresponding bid, the bid distribution and density, the number of bidders and the utility function. Note that (1) provides an expression of the inverse of the bidding strategy $s(\cdot)$. We now address the problem of identification of the model structure (U, F) from observables (I, G), namely whether the structure can be recovered uniquely from the observations. Guerre et al. (2006) provide some smoothness conditions on $U(\cdot)$ and $F(\cdot)$, which in turn imply some smoothness conditions on $s(\cdot)$ and $G(\cdot)$. See Definitions 1–3 and Theorem 1 in that paper.

Proposition 1: *The model with risk aversion is not identified.*

The proof of such a result is obtained by constructing a structure $[\tilde{U}, \tilde{F}]$ such that $\tilde{U}(\cdot) = [U(\cdot/\delta)/U(1/\delta)]^\delta$, with $\delta \in (0,1)$ leading to the same bid distribution $G(\cdot)$ as $[U, F]$. It then remains to check that $[\tilde{U}, \tilde{F}]$ satisfies the smoothness conditions and leads to an increasing inverse equilibrium strategy. As such, $[\tilde{U}, \tilde{F}]$ and $[U, F]$ are observationally equivalent. Since the risk neutral model with $U(\cdot)$ being the identity function is identified, this suggests that nonidentification arises from the unknown $U(\cdot)$. As a matter of fact, any bid distribution $G(\cdot)$ satisfying some smoothness conditions can be rationalized by a CRRA or a constant ARA (CARA) structure. By allowing risk aversion, one can explain almost any bid distribution. This problem arises because of the weakness of the restrictions imposed by the model on observables. In particular, in view of observations we cannot discriminate a model with arbitrary risk aversion from a CRRA or a CARA model. Since a risk neutral model is a special risk averse model, any risk neutral model may lead a priori to the same bid observations than some CRRA or CARA model. It is worthnoting that these results still hold when considering a binding reserve price, a random reserve price, affiliated private values and asymmetric bidders.

II. Identification of Risk Aversion Under Parametric Restrictions

A first identifying strategy is to introduce some partial parametric restrictions as a fully parametric model is almost always identified. Though the source of the problem comes from the bidders' utility function, a first natural choice would be to parameterize the private value distribution $F(\cdot)$ as $F(\cdot;\gamma)$. A second natural choice would be to parameterize the utility function $U(\cdot)$ as $U(\cdot;\theta)$ though little is known on the nature of bidders' risk aversion.

Proposition 2: *The semiparametric model composed of structures* $[U(\cdot), F(\cdot;\gamma)]$ *is not identified. The semiparametric model composed of structures* $[U(\cdot,\theta), F(\cdot)]$ *is not identified.*

Hence, parameterizing either $U(\cdot)$ or $F(\cdot)$ is not sufficient to identify the model. Regarding the first part of Proposition 2, it is straightforward to construct a counterexample with a uniform distribution. On the other hand, the proof of the second part of Proposition 2 provides an interesting fact. By considering the CRRA model, one can construct an observationally equivalent structure $[\tilde{U}, \tilde{F}]$ with a CRRA parameter $\tilde{c}$ larger than c in the original structure $[U, F]$. The increase in risk aversion is compensated by a shrinkage of all the quantiles of the private value distribution. Thus, the support of $\tilde{F}(\cdot)$ is $[\underline{v}, \tilde{\overline{v}}]$ with $\tilde{\overline{v}} < \overline{v}$. It is worhtnoting that this result extends to a binding reserve price, a random reserve price, affiliated private values and asymmetry in private values in which bidders draw their private values from different distributions $F_1, \ldots, F_I$. On the other hand, asymmetry in preferences in which bidders have different utility functions $U_1, \ldots, U_I$ provides an interesting identification result when $U_1, \ldots, U_I$ are parameterized. Let $b_{i\alpha}$ be the α-quantile of the bid distribution $G_i(\cdot)$ for bidder i and v_α the α-quantile of $F(\cdot)$. Because all the quantiles $(b_{1\alpha}, \ldots, b_{I\alpha})$ correspond to the same quantile v_α, (1) evaluated at a quantile for an arbitrary pair (i,j) gives $b_{i\alpha} + \lambda_i^{-1}(1/H_i(b_{i\alpha})) = b_{j\alpha} + \lambda_j^{-1}(1/H_j(b_{i\alpha}))$, where $\lambda_i(\cdot) = U_i(\cdot)/U_i'(\cdot)$ and $H_i(\cdot) = \sum_{j\neq i} g_j(\cdot)/G_j(\cdot)$. This leads to some compatibility conditions that can be used to achieve identification of the asymmetric auction model.

Given these nonidentification results, more parametric restrictions need to be made to achieve identification. On one hand, parameterizing $F(\cdot)$ leaves little choice as additional parameterization of $U(\cdot)$ would lead to a fully parametric model. On the other hand, parameterizing $U(\cdot)$ leaves some flexibility as $F(\cdot)$ can be partially parameterized through one quantile. The preceding phenomenon of compensation between risk aversion and the quantiles of $F(\cdot)$ suggests that this parameterization will pin down that quantile thereby identifying the model. This idea is exploited further by considering heterogeneity across auctions embodied in some observed characteristics $\mathbf{Z}$ or in the number of bidders I. This leads to the structure $[U(\cdot;\theta), F(\cdot|\mathbf{Z}, I)]$ as $U(\cdot)$ is parameterized but independent of $(\mathbf{Z}, I)$. By construction, the bid distribution becomes $G(\cdot|\mathbf{Z}, I)$.

Let $b_\alpha(\mathbf{Z}, I)$ and $v_\alpha(\mathbf{Z}, I)$ be the α-quantiles of the distributions $G(\cdot|\mathbf{Z}, I)$ and $F(\cdot|\mathbf{Z}, I)$, respectively with $\alpha \in [0, 1]$. The idea is to impose a parametric conditional quantile restriction, i.e. for some $\alpha \in (0, 1]$, $v_\alpha(\mathbf{z}, I) = v_\alpha(\mathbf{z}, I; \gamma)$. For instance, $v_\alpha(\mathbf{z}, I; \gamma)$ can be chosen as a constant or a polynomial to allow for more flexibility, where γ is the vector of coefficients and α can be chosen as 0.5 giving the median. This condition is crucial in identifying the model. Equation (1) becomes

$$g(b_\alpha(\mathbf{z}, I)|\mathbf{z}, I) = \frac{1}{I-1} \frac{\alpha}{\lambda(v_\alpha(\mathbf{z}, I; \gamma) - b_\alpha(\mathbf{z}, I); \theta)}. \tag{2}$$

Identification requires to have enough variation in $\mathbf{Z}$ or I to identify the parameters θ and γ. For instance, if θ and γ are both one-dimensional, two different values of $\mathbf{Z}$ or I are sufficient to achieve identification.

Proposition 3: *The semiparametric model composed of structures* $[U(\cdot, \theta), F(\cdot)]$ *is identified, when some quantile of* $F(\cdot)$ *satisfies the condition* $v_\alpha(\mathbf{z}, I) = v_\alpha(\mathbf{z}, I; \gamma)$.

This result extends to a binding reserve price, a random reserve price, affiliated private values and asymmetry in private values. It remains to discuss the semiparametric estimation of such a model. For simplicity, Guerre et al. (2006) consider a constant restriction at the upper quantile. It is easy to rewrite (2) for $\alpha = 1$ and $v_1(\mathbf{z}, I) = \gamma$. A preliminary step consists in estimating nonparametrically the upper boundary of the bid distribution and the bid density at this upper boundary. With the latter replacing $g(b_1(\mathbf{z}, I)|\mathbf{z}, I)$ in (2), nonlinear least squares are applied to estimate (θ, γ). From these estimates, (1) can be used to recover the private values and estimate nonparametrically $f(\cdot|\cdot,\cdot)$. Guerre et al. (2006) show that the risk aversion parameter is estimated at the optimal rate, which is slower than $\sqrt{N}$ and independent of the dimension of $\mathbf{Z}$ thereby avoiding the curse of dimensionality.

III. Identification of Risk Aversion Under Exclusion Restrictions

There is no general agreement among economists on which concept of risk aversion is the most appropriate to explain behavior under uncertainty. Moreover, little is known on the shape of agents' utility function. In view of these, it is interesting to exploit other restrictions beyond parameterization to identify bidders' utility function. Exclusion restrictions are common in econometrics such as in the wage equation with instrumental variables to solve for endogeneity of education. More recently, exclusion restrictions such as exogenous bidders' participation in auctions have been used to test for common value. A similar idea can be exploited to identify bidders' untility function. Specifically, exogenous bidders' participation, which leads to a latent private value distribution independent of the number of bidders, is considered. Variations of the bid distribution in the number of bidders while the private value distribution remains the same identify the bidders' utility function since $F(\cdot|I) = F(\cdot)$. For simplicity, consider two levels of competition with $I_2 > I_1$ leading to the bid distributions $G_2(\cdot)$ and $G_1(\cdot)$.

Proposition 4: *Under exogeneity of the number of bidders, $\lambda^{-1}(\cdot)$ is identified on the range of the function $[1/(I_1-1)][G_1(b)/g_1(b)]$.*

Because v_α does not vary with I, (2) leads to the compatibility condition $\lambda^{-1}[[1/(I_1 - 1)][\alpha/g_1(b_{1\alpha})]] = \lambda^{-1}[[1/(I_2-1)][\alpha/g_2(b_{2\alpha})]] + \Delta b_\alpha$, where $\Delta b_\alpha = b_{2\alpha} - b_{1\alpha}$. By continuity of the function $[1/(I_1-1)][\cdot/g_1(b_{1\cdot})]$, there exists a value $\tilde{\alpha}$ such that $[1/(I_1-1)][\tilde{\alpha}/g_1(b_{1\tilde{\alpha}})] = [1/(I_2-1)][\alpha/g_2(b_{2\alpha})]$, which can be used to rewrite the previous compatibility condition as $\lambda^{-1}[[1/(I_1-1)][\alpha/g_1(b_{1\alpha})]] = \lambda^{-1}[[1/(I_2-1)][\tilde{\alpha}/g_2(b_{2\tilde{\alpha}})]] + \Delta b_{\tilde{\alpha}} + \Delta b_\alpha$. Continuing the same exercise gives a sequence of values for α, namely $\{\alpha_t\}$ such that

$$\lambda^{-1}(u) = \sum_{t=0}^{+\infty} \Delta b_{\alpha_t}, \tag{3}$$

with $u = [1/(I_1 - 1)][\alpha_0/g_1(b_{1\alpha_0})]$ and α_0 arbitraty in $[0, 1]$. This corresponds to a monetary gain of $v_{\alpha_0} - b1\alpha_0$. As α_0 is arbitrary, $\lambda^{-1}(\cdot)$ is identified on the range of the function $[1/(I_1 - 1)][G_1(b)/g_1(b)]$. It is easy to show that once $\lambda^{-1}(\cdot)$ is identified, $F(\cdot)$ is identified. Proposition 4 extends to a binding reserve price, affiliated private values and asymmetric bidders. It is worthnoting that asymmetry in both private values and preferences can be entertained for bidders participating to both auctions. This result can be also extended to endogenous bidders' participation provided instruments are available. In particular, one can model bidders' participation as $I = I(\mathbf{Z}, \mathbf{W}) + \epsilon$, where ϵ is an additive error term. The variable $\mathbf{W}$ plays the role of an instrument as $F(v|\mathbf{Z}, \mathbf{W}, \epsilon) = F(v|\mathbf{Z}, \epsilon)$, which corresponds to an exclusion restriction. Under such a restriction, the bidders' utility function is identified nonparametrically. Estimating $\lambda^{-1}(\cdot)$, however, is difficult because of the infinite series of differences (3) in quantiles as the estimated b_{α_t} are serially correlated.

IV. Identification of Risk Aversion Using Additional Data

Another identification strategy exploits additional bidding data. Specifically, data from two auction designs can be used to identify the bidders' utility function. A striking feature of risk aversion is that it does not affect bidding in an ascending auction as bidding his private value remains a dominant strategy. Thus, ascending auction data can be exploited to identify the underlying distribution of private values. In particular, the observed winning bids in ascending auctions can be interpreted as the second-highest private values. Using distribution of order statistics, it becomes straightforward to recover the distribution $F(\cdot)$ as shown by Athey and Philip Haile (2002). Once the latter is identified, (2) can be used to recover the bidders' utility function on $[0, \max_\alpha v_\alpha - b_\alpha]$ using the quantiles of $F(\cdot)$ and $G(\cdot)$ from first-price sealed-bid auction data.

Proposition 5: *Any structure $[U, F]$ is identified when bidding data from ascending and first-price sealed-bid auctions are combined.*

The result extends to a binding reserve price and asymmetric bidders. On the other hand, when private values are affiliated, the observation of the winning bid is not sufficient to recover the distribution of private values as the affiliation among private values cannot be captured by a single bid. Using two auction designs, however, is valid if the same set of bidders participate to both auctions. Otherwise, we may face two different underlying private value distributions. This crucial assumption needs to be checked in the data. The resulting estimation procedure combines order statistics, nonparametric estimators for distribution and density as well as quantiles.

This paper offers a review of the identification problem of risk aversion in auction models. Various restrictions such as partial parameterization of the structure, exclusion restrictions and the use of additional data are used to achieve identification of the bidders' utility function. The results extend to more general models including a binding reserve price, affiliated private values and asymmetric bidders. The above methods have been applied to timber auction data, where significant risk aversion has been found. Uncertainty may affect bidders' private values such as in a model with stochastic private values, in which a random shock realized ex post changes the winner's private value. Such an uncertainty affects bidding behavior under risk aversion. Given the expost uncertainties faced by bidders in construction procurements, extension of the current results to stochastic private values may have interesting economic applications.

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