The Marginal Worker and the Aggregate Elasticity of Labor Supply*

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Abstract

This paper attempts to reconcile the large volatility of aggregate employment with small micro estimates of the elasticity of labor supply. Our model of indivisible labor with complete markets shows that the aggregate intertemporal elasticity of labor supply depends on the homogeneity of the workforce at the margin. Hence, the shape of the distribution of reservation wages is the key determinant of the elasticity of aggregate labor supply. Even if most workers are wage-inelastic, the aggregate elasticity can be large if sufficiently many agents are close to their reservation wage. These agents are indeed likely to enter or exit the labor market upon a small change in their wage. To evaluate this hypothesis, we estimate the model using monthly panel data drawn from the NLSY. This allows us to measure the aggregate elasticity implied by realistic heterogeneity. We estimate a Frisch elasticity around 1.5. Our model also has a natural cross-sectional implication: workers who are nearly indifferent between working or not are more sensitive to aggregate fluctuations. We find empirical support for this prediction: on average, the group of marginal workers, which makes up 22% of the population, accounts for 49% of aggregate fluctuations in employment.

Keywords: indivisible labor, reservation wage distribution, labor supply, business cycles.

JEL Codes: E24, E32, J21, J22.

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1 Introduction

This paper attempts to reconcile the large volatility of aggregate hours with small estimates of the Frisch (or intertemporal) elasticity of labor supply. There is a wide consensus among labor economists that the intertemporal elasticity of hours worked per worker is low. MaCurdy (1981) found an elasticity between 0.1 and 0.4 for men who are continuously working. Further research has confirmed this finding, while suggesting that the elasticity may be higher for women. By contrast, macroeconomic models typically use much higher estimates for the Frisch elasticity of labor supply. For instance, King and Rebelo (1999) in their survey of RBC models use an elasticity of 4 in their basic model and an infinite elasticity in an extension of their model. New Keynesian models also require a high elasticity: Rotemberg and Woodford (1997; see also Woodford 2003 p.341) use an elasticity close to 9. Any macroeconomic model which is to generate variation in aggregate labor with a wage-taking household needs to have an elastic labor supply.[1]

This discrepancy between microeconomic estimates and the macroeconomic parameters can probably be explained by the importance of the extensive margin. As noted by Coleman (1984), short-term fluctuations in aggregate hours worked are mostly accounted for by changes in employment, rather than by changes in hours per worker. This explains why some microeconomic estimates, such as MaCurdy's, are not directly relevant for macroeconomics. When labor supply is indivisible, most agents either work or not, irrespective of the level of the aggregate wage, and only a few agents actually shift in and out of the workforce in response to a change in the aggregate wage. One can imagine that a small group of agents ("marginal workers") who adjust their labor discretely does all the cyclical adjustment in the workforce, generating what looks like a highly elastic *aggregate* labor supply, while the majority of the workforce is irresponsive to macroeconomic conditions.

In this paper, we study the implications of indivisible labor when agents are heterogeneous in tastes and abilities. We show that the aggregate Frisch elasticity of labor supply depends on the homogeneity of the workforce at the margin: the aggregate elasticity is the hazard rate

[1] Search and matching models of the labor market similarly fail to generate enough employment volatility (Shimer 2005), except when the implicit elasticity of labor supply is high (Hagedorn and Manovskii 2005 and the discussion in Hall 2006).

of the distribution of reservation wages, evaluated for the worker who is indifferent between working and not working this period. This formula implies that the *shape* of the distribution of reservation wages around the marginal worker is the crucial determinant of the elasticity. In Rogerson's (1988) seminal paper, all agents are indifferent to working, so that the hazard rate is very high (actually infinite), and a small change in the wage is enough to induce these agents to enter the workforce, making the aggregate labor supply very (infinitely) elastic.[2] Hence when workers are very homogenous at the margin, the elasticity is very high. The key question is: what is the elasticity implied by realistic heterogeneity, i.e. by a realistic distribution of reservation wages?

To answer this question, we estimate our model on panel data from the NLSY. We allow for non-parametric heterogeneity by using fixed effects that control for wealth and permanent differences in tastes or abilities. On the other hand, we need to assume independent normal shocks to tastes and abilities to estimate the model. Importantly, we use monthly data for which the indivisibility is a good approximation. We find a Frisch elasticity of labor supply around 1.5. This is an average across populations with different elasticities. These populations can first be defined in terms of observable characteristics: for instance, young people, women, and less-educated workers are more elastic.[3] But there are also individual unobservable characteristics which make some agents more elastic. We consider an interesting cross-sectional implication of our model: agents which are nearly indifferent between working and not - the "marginal workers" - are more sensitive to aggregate fluctuations. These agents are indeed likely to enter or exit the labor market upon a small change in their wage. We find empirical support for this prediction in the NLSY: on average, the group of marginal workers, which makes up 22% of the population, accounts for 49% of aggregate fluctuations in employment. (In a previous draft, we obtained similar results using the PSID.)

We assume complete markets. This simplifies our derivations and allows us to study more clearly the effect of the shape of the distribution, which we demonstrate in some simple examples. Further work will examine the quantitative importance of market incompleteness.

[2] We show, as Mulligan (2001) does, that Rogerson's result does not rely on the availability of lotteries. This result does not seem to be widely known.

[3] As noted by many authors including Clark and Summers (1980), Kydland (1984), Heckman (1993), and Gomme, Rogerson, Rupert and Wright (2005).

Our paper is related to the large macroeconomic literature on the extensive margin of labor supply (Rogerson 1988, Cho and Rogerson 1988, Cho 1995, Mulligan 2001, and Chang and Kim 2006a and 2006b). Some of our theoretical results are similar to Mulligan (2001), but we draw different empirical implications from these results. On the substance, our work is probably most closely related to Chang and Kim (2006a). Our work complements theirs by emphasizing the importance of distributions, while they emphasize the role of incomplete markets. This leads us to a different empirical strategy. Finally, to implement our estimation procedure, we borrow from the vast "micro-labor" literature (e.g. Heckman and MaCurdy 1980, 1982, Kimmel and Kniesner 1998). Our paper is also related to the recent discussion of labor supply' response to taxes (Prescott 2002 and Ljungqvist and Sargent 2005). Ljungqvist and Sargent consider the steady-state compensated elasticity of labor supply, which is not the intertemporal elasticity. They emphasize the role of human capital accumulation in increasing the cost of interrupting employment temporarily.[4]

A point of semantics: in this paper, when we refer to an elasticity, it is always the Frisch (or intertemporal) elasticity, i.e. the response of aggregate hours to an increase in the wage, holding marginal utility of wealth constant. This elasticity is the relevant one for business cycle analysis.

The rest of the paper is organized as follows. Section 2 develops our model of indivisible labor supply with heterogeneity and complete markets. Section 3 discusses the estimation strategy. Section 4 presents our empirical results, both for the aggregate labor supply and for the cross-sectional implications. Section 5 concludes.

2 A Model of Indivisible Labor Supply with Complete Markets

In this Section we analyze a model of indivisible labor supply with complete markets and heterogeneity. There are three main results. First we derive an equation for labor force participation, which we estimate in Section 4. Second, we show that the aggregate labor supply is related to the distribution of agents' abilities and tastes, i.e. to the distribution of reservation

[4] See also Imai and Keane (2004) for an analysis of the intensive margin of labor supply which incorporates human capital accumulation.

wages. As a result, the aggregate Frisch elasticity of labor supply is the hazard rate of this distribution, evaluated at the marginal worker. Finally, we derive a cross-sectional prediction: some workers are more sensitive to aggregate shocks.[5]

A. Economic Environment

Time is discrete indexed by $t = \{0, 1, 2, ...\}$. There is a unit measure of agents, indexed by $i \in [0, 1]$. Labor is indivisible: each agent works either $\overline{n}$ hours or not at all. We abstract from the intensive margin because it is relatively unimportant for business cycles and because microeconomic estimates of the elasticity of hours are low.

We assume separable preferences: working does not affect the utility $u(c_{it})$ derived from consumption. Individual preferences are characterized by the function $u(\cdot)$ and the disutility of work $v(\overline{n})$. We normalize $v(0) = 0$.

At each date t, agents are subject to four types of shocks: aggregate and idiosyncratic productivity and taste shocks. Specifically, there is an aggregate stochastic productivity $z_t \in \mathbb{R}^{++}$; there is also an aggregate stochastic disutility of work, $d_t \in \mathbb{R}^{++}$. "Taste shocks" d_t can also be interpreted as shocks to the productivity of the household production sector, or as changes in tax rates.

We denote agent i productivity (or earning capability) by $\pi_{it} \in \mathbb{R}^{++}$; π_{it} is a stochastic endowment of efficiency units of labor. Finally, agent's i has an idiosyncratic disutility for labor $\theta_{it} \in \mathbb{R}^{++}$. To ease notations, the state of an agent i at date t is summarized by the vector $s_{it} = (\pi_{it}, \theta_{it}, z_t, d_t) \in S = (\mathbb{R}^{++})^4$. In this state of the world, the cost of working in terms of utility is $\theta_{it} d_t v(\overline{n})$, while the benefit of working in terms of goods is $\overline{n}\pi_{it} w_t(z^t, d^t)$, where $w_t(z^t, d^t)$ stands for the marginal product of an efficiency unit of labor in aggregate state (z^t, d^t).

To sum up, preferences are:

$$E \sum_{t \geq 0} \beta^t \left(u(c_{it}) - \theta_{it} d_t v(n_{it}) \right).$$

Idiosyncratic and aggregate shocks $\pi_{it}, \theta_{it}, z_t, d_t$ are all stationary Markov processes, and are independent. They are described by the functions $p_\pi^t, p_\theta^t, p_z^t, p_d^t$: these are densities over shock

[5] Some of our results are already in Mulligan (2001).

histories. For instance, $p^t_\pi(\pi^t)$ is the probability of history π^t.

Finally, we assume a neoclassical production function: $Y_t\left(z^t,d^t\right)=F\left[K_t\left(z^{t-1},d^{t-1}\right),z_tN_t\left(z^t,d^t\right)\right],$ where aggregate labor – in efficiency units – is:

$$N_t\left(z^t,d^t\right)=\int_i\int_{\mathbb{R}^{++}}\int_{\mathbb{R}^{++}}\left[\pi_{it}n_{it}\left(\pi^t,\theta^t,z^t,d^t\right)\right]p^t_\pi\left(\pi^t\right)p^t_\theta\left(\theta^t\right)d\theta^t d\pi^t di,$$

and we also consider a standard capital accumulation equation:

$$K_{t+1}(z^t,d^t)=(1-\delta)K_t(z^{t-1},d^{t-1})+I_t(z^{t-1},d^{t-1}).$$

Note that the assumptions about the production function and capital accumulation are not necessary for our results: we make them for concreteness only.

In order to consider a convex problem, we will allow randomization of an agent's labor supply: $\alpha_i\left(s^t\right)$ denotes the probability to work for agent i in state s^t. However, labor supply will turn out to be almost always deterministic. Hence lotteries are only an analytical device and their presence does not affect the equilibrium.

B. Pareto Optimum: Participation Equation

Since we allow the randomization of labor supply, our economy is convex. We thus know that there exists a competitive equilibrium and that any competitive equilibrium is Pareto Optimal. This allows us to use a social planner problem to characterize competitive equilibria; this approach is simpler. (In the appendix we describe the corresponding competitive equilibrium.) It is sometimes convenient to rename and index the agents using their Pareto weights μ (instead of i). We assume these Pareto weights are distributed according to H, a strictly increasing and differentiable cumulative distribution function over $\mathbb{R}^{++}$.

The social planner chooses a sequence of functions $\left\{c_t\left(s^t,\mu\right),\alpha_t\left(s^t,\mu\right)\right\}$ to maximize

$$\sum_{t\geq 0}\beta^t\int_{\mathbb{R}^{++}}\int_{S^t}\mu\left[u\left(c_t\left(s^t,\mu\right)\right)-\theta_t d_t v\left(\overline{n}\right)\alpha_t\left(s^t,\mu\right)\right]p^t_s\left(s^t\right)ds^t dH\left(\mu\right),$$

such that the following constraints are satisfied: (1) an initial capital stock K_0 is given, (2) lotteries satisfy $\alpha_t\left(s^t,\mu\right)\in[0,1]$ for all $\left(s^t,\mu\right)$, and (3) the following resource constraint holds: for all t, and all $\left(z^t,d^t\right)$,

$$\begin{aligned}&\int_\mu\int_{\pi^t}\int_{\theta^t}c_t\left(s^t,\mu\right)p^t_\pi\left(\pi^t\right)d\pi^t p^t_\theta\left(\theta^t\right)d\theta^t dH\left(\mu\right)+K_{t+1}\left(z^t,d^t\right)\\ \leq\;& F\left[K_t\left(z^{t-1},d^{t-1}\right),z_tN_t\left(z^t,d^t\right)\right]+(1-\delta)K_t\left(z^{t-1},d^{t-1}\right),\end{aligned}\tag{2.1}$$

where aggregate labor N_t is measured in efficiency units:

$$N_t\left(z^t,d^t\right)\equiv\int_\mu\int_{\theta^t}\int_{\pi^t}\left[\pi_t\alpha_t\left(s^t,\mu\right)\overline{n}\right]p_\pi^t\left(\pi^t\right)d\pi^t p_\theta^t\left(\theta^t\right)d\theta^t dH\left(\mu\right). \tag{2.2}$$

Denote by w_t the marginal product of labor:

$$w_t\left(z^t,d^t\right)\equiv z_tF_2\left[K_t\left(z^{t-1},d^{t-1}\right),z_tN_t\left(z^t,d^t\right)\right]. \tag{2.3}$$

The planner problem is convex, and hence characterized by first-order conditions. Let $\lambda_t(z^t,d^t)$ be the multiplier on the resource constraint (2.1). The first-order condition with respect to consumption is, for all $t,s^t,\mu:$

$$\mu\cdot u'\left(c_t\left(s^t,\mu\right)\right)=\lambda_t\left(z^t,d^t\right), \tag{2.4}$$

and the first-order condition with respect to $\alpha_t(s^t,\mu)$ is, for all $t,s^t,\mu:$

$$\lambda_t\left(z^t,d^t\right)w_t\left(z^t,d^t\right)\pi_t\overline{n}-d_t\theta_tv\left(\overline{n}\right)\mu\begin{cases}>0\text{ if }\alpha_t\left(s^t,\mu\right)=1,\text{ i.e. }n\left(s^t,\mu\right)=\overline{n},\\=0\text{ if }\alpha_t\left(s^t,\mu\right)\in(0,1),\\<0\text{ if }\alpha_t\left(s^t,\mu\right)=0,\text{i.e. }n\left(s^t,\mu\right)=0,\end{cases} \tag{2.5}$$

The first-order condition for lottery choice implies that actual randomization, i.e. $\alpha\left(s^t,\mu\right)\in(0,1)$ is a zero-probability event, if the distributions for π_t,θ_t,μ are atomless. Indeed,

$$\alpha_t\left(s^t,\mu\right)\in(0,1)\Longleftrightarrow\frac{\lambda_t\left(z^t,d^t\right)w_t\left(z^t,d^t\right)\overline{n}}{d_tv\left(\overline{n}\right)}=\frac{\theta_t\mu}{\pi_t},$$

so that if H,p_θ^t,p_π^t are atomless, this event has measure zero because $\frac{\lambda_t\left(z^t,d^t\right)w_t\left(z^t,d^t\right)\overline{n}}{d_tv\left(\overline{n}\right)}$ is equal across all agents. The intuition for this result is that any lottery – any extraneous randomization device – can be replaced by actual exogenous randomness: the purification argument relies on the economic environment containing "as much risk" as any artificial lottery.[6]

Result 1: Labor Force Participation

[6] "Purification" refers to the fact that mixed strategies are actually degenerate, and hence equivalent to pure strategies with probability one in this environment. But even if the economy has no uncertainty, lotteries are not needed: workers can average over time, switching in and out of the labor force to choose the average time that they work. This is however different than purification because the *dates* where an agent works may be indeterminate, even though the total time the agent works is not.

From (2.5), we obtain the participation equation: agent μ works in state s^t, i.e. $n_t(s^t,\mu) = \overline{n}$, if and only if the benefit is greater than the cost:

$$\lambda_t\left(z^t,d^t\right) w_t\left(z^t,d^t\right) \pi_{it}\overline{n} - d_t\theta_{it} v\left(\overline{n}\right)\mu \geq 0. \tag{2.6}$$

This is the equation that we will estimate in Section 4.

A more intuitive way to state this result is the following. Define the individual wage w_{it} and the reservation wage w_{it}^R for agent i at date t, from the equations:

$$\begin{aligned} w_{it} &= w_t\left(z^t,d^t\right)\pi_{it}\overline{n}, \\ w_{it}^R &= \frac{d_t}{\lambda_t\left(z^t,d^t\right)}\theta_{it}\mu_i v\left(\overline{n}\right), \end{aligned}$$

The participation equation (2.6) then amounts to $w_{it} \geq w_{it}^R$. The reservation wage w_{it}^R takes into account the aggregate and idiosyncratic shocks to the disutility of labor, but also the marginal utility of consumption, as measured by $\lambda_t(z^t,d^t)$, and the wealth of an agent, as measured by his Pareto-weight μ_i.

C. Aggregate Implications

Define aggregate employment $\widetilde{N}_t(z^t,d^t)$ as the physical quantity of aggregate hours worked, or equivalently as the number of agents agents employed times the workweek $\overline{n}$ (since there is no intensive margin):

$$\widetilde{N}_t\left(z^t,d^t\right) = \int_\mu\int_{\theta^t}\int_{\pi^t} \mathbf{1}_{[n_t(s^t,\mu)=\overline{n}]} p_\pi^t\left(\pi^t\right) p_\theta^t\left(\theta^t\right) d\pi^t d\theta^t dH\left(\mu\right),$$

where $\mathbf{1}_A$ is a characteristic function. Note that this is different from aggregate labor $N_t(z^t,d^t)$, which is defined in efficiency units. Now rewrite the participation equation (2.6) as[7]

$$n_{it}\left(s^t\right) = \overline{n} \text{ if only if } x_{it} \overset{def}{=} \frac{\theta_{it} v\left(\overline{n}\right)\mu_i}{\pi_{it}\overline{n}} \leq \frac{\lambda_t\left(z^t,d^t\right) w_t\left(z^t,d^t\right)}{d_t} \overset{def}{=} x_t^*\left(z^t,d^t\right),$$

where we introduce the variable x_{it} to summarize individual heterogeneity: we collapse various dimensions of heterogeneity – tastes, abilities, wealth – into a single variable x. Then the participation decision rule is summarized by a simple cutoff x_t^*, such that agent i works in

[7] It is a slight abuse to write "if and only if" since in the case of an equality, randomization occurs, and $n_t(s^t,\mu) = 0$ could happen.

period t if and only if $x_{it} \leq x_t^*$. To compute aggregate hours, define $G(\cdot)$ as the cumulative distribution function of $\log x_{it} = \log \frac{\theta_{it} v(\overline{n})\mu_i}{\pi_{it}\overline{n}}$. $G(\cdot)$ is constructed from the distribution H of Pareto-weights and from the invariant distributions of the Markov processes for π and θ, which we denote λ_π and λ_θ :

$$\begin{aligned} G(x) &= \Pr\left[\log \frac{\theta v(\overline{n})\mu}{\pi \overline{n}} \leq x\right] \\ &= \Pr\left[\frac{\theta v(\overline{n})\mu}{\pi \overline{n}} \leq \exp x\right] \\ &= \Pr\left[\mu \leq \frac{\pi \overline{n}}{\theta v(\overline{n})} \exp x\right] \\ &= \int_\Theta \int_\Pi H\left(\frac{\pi \overline{n}}{\theta v(\overline{n})} \exp x\right) \lambda_\pi(\pi) \lambda_\theta(\theta)\, d\pi d\theta. \end{aligned}$$

Then aggregate employment is:

$$\widetilde{N}_t\left(z^t, d^t\right) = G\left(\log\left(\frac{\lambda_t\left(z^t, d^t\right)}{d_t}\right) + \log w_t\left(z^t, d^t\right)\right) = G\left(x_t^*\left(z^t, d^t\right)\right). \tag{2.7}$$

Equivalence with a representative agent economy

We now follow Mulligan (2001) and show that this economy has the same aggregate implications as a representative agent economy with divisible labor as in Lucas and Rapping (1969), with aggregate utility function $u(C_t) - d_t V\left(\widetilde{N}_t\right)$ where $u(.)$ is the same utility function for the representative agent than for the underlying individuals, and $V(.)$ is defined as a function of the underlying distributions:

$$V\left(\widetilde{N}_t\right) = \int_0^{\widetilde{N}_t} \exp\left(G^{-1}(n)\right) dn.$$

The proof consists simply in checking that with these definitions, the first order conditions of the representative agent $u'(c_t) = \lambda_t$ and $\frac{d_t V'\left(\widetilde{N}_t\right)}{u'(c_t)} = w_t$ coincide with the formula (2.7) derived from the model with heterogeneity: since $V'\left(\widetilde{N}_t\right) = \exp\left(G^{-1}\left(\widetilde{N}_t\right)\right)$,

$$\frac{d_t V'\left(\widetilde{N}_t\right)}{u'(c_t)} = w_t = \frac{d_t \exp\left(G^{-1}\left(\widetilde{N}_t\right)\right)}{u'(c_t)} \Leftrightarrow \widetilde{N}_t = G\left(x_t^*\right) = G\left(\log \frac{\lambda_t w_t}{d_t}\right).$$

Result 2: Aggregate Frisch Elasticity of Labor Supply

Equation (2.7) characterizes the Frisch labor supply schedule in this economy, which depends on the marginal utility of aggregate consumption λ_t, the marginal product of labor w_t, and the stochastic disutility of labor d_t.[8] The Frisch elasticity of aggregate labor is:

$$\frac{\partial \log \widetilde{N}_t}{\partial \log w_t} = \frac{g\left(\log\left(\frac{\lambda_t\left(z^t, d^t\right)}{d_t}\right) + \log w_t\left(z^t, d^t\right)\right)}{G\left(\log\left(\frac{\lambda_t\left(z^t, d^t\right)}{d_t}\right) + \log w_t\left(z^t, d^t\right)\right)} = \frac{g\left(\log x_t^*\left(z^t, d^t\right)\right)}{G(\log x_t^*\left(z^t, d^t\right))}. \tag{2.8}$$

At the aggregate level, the Frisch elasticity of labor supply is a measure of homogeneity of the workforce at the margin (akin to a hazard rate). In this model, therefore, the elasticity can be anything, depending on the shape of the distribution of wages and reservation wages around the marginal worker. If the density g is high, then there are many workers that are indifferent between working and not, and small fluctuations in wages generate large aggregate fluctuations in labor.

Relation to Rogerson (1988)

To see that Rogerson's (1988) infinite Frisch elasticity is a special case, consider this simple example. Assume that G is the uniform distribution over $[a-\varepsilon, a+\varepsilon]$ with density $\frac{1}{2\varepsilon}$. Any allocation where an interior fraction of the population works has:

$$\left\{\log\left(\frac{\lambda_t\left(z^t, d^t\right)}{d_t}\right) + \log w_t\left(z^t, d^t\right)\right\} \in \left(a-\varepsilon, a+\varepsilon\right),$$

so that the Frisch Elasticity is:

$$\frac{1}{2\varepsilon}\left(\frac{\log\left(\frac{\lambda_t\left(z^t, d^t\right)}{d_t}\right) + \log w_t\left(z^t, d^t\right) - (a-\varepsilon)}{2\varepsilon}\right)^{-1} > \frac{1}{2\varepsilon},$$

which obviously becomes infinite as ε tends to zero, that is as the economy gets homogeneous. In this case, all workers are marginal, hence aggregate labor supply is infinitely elastic.

The importance of distributional assumptions

In this section, we show the importance of the distributions used to calibrate tastes, abilities and wealth. To make this comparison as transparent as possible, we assume tastes are homogeneous, wealth is equal across households, and we choose a stationary distribution F for

[8] Frisch demands (for leisure in this case) are defined as functions of the price system and the marginal utility of consumption. In this case however, the proper measure of a compensated labor supply function also controls for aggregate "taste shocks" d_t, since they shift the supply of labor, given a wage w_t and a marginal utility of consumption λ_t.

the logarithm of individual productivity $\log \pi_i$. We consider several distributions, each of which is characterized by two parameters. We choose these two parameters to obtain $E(\log \pi) = 0$ and $Var(\log \pi) = 0.6964$. These two moments are chosen to replicate the observed properties of the residuals in the wage regression, i.e. the idiosyncratic wage shocks. (The precise number for the variance is taken from Chang and Kim 2006b.) We then choose the cutoff π^* to match an employment rate of 60% on average. Table 2 reports the implied elasticities for various distributions.

Distribution of $\log \pi$	Implied Aggregate Frisch Elasticity
Normal	1.16
Logistic	1.31
Uniform	0.60
Pareto (distribution of π)	1.79
Mixture of two normals ($r = .1$)	1.04
Mixture of two normals ($r = 1$)	1.45
Mixture of two normals ($r = 10$)	10.81

Table 2: Model-implied Frisch Elasticity in Complete Markets, for various distributions. All the distributions have the same variance and have mean 0, with an employment rate of 60%.

The last three rows of the table refer to the case where there is a mixture of two normals, i.e.:

$$F(x) = \alpha \Phi\left(\frac{x - \mu_1}{\sigma_1}\right) + (1 - \alpha)\Phi\left(\frac{x - \mu_2}{\sigma_2}\right),$$

where Φ is the standard normal cumulative distribution function. The three examples share the same parameters, except for the ratio of standard deviations $r = \sigma_2/\sigma_1$.[9]

We find it interesting that reasonable variations in the precise shape of individual heterogeneity, as represented by these distribution functions, can change the implied elasticity by a large amount. Even without considering mixtures, the elasticity ranges from 0.60 for the uniform distribution to 1.79 for the Pareto distribution. With mixtures, it is easy to see that

[9] Clearly, there are more parameters than moments. We set arbitrarily $\alpha = .50$ and $\mu_1 = -0.8$, and choose μ_2 to satisfy $E(\log \pi) = 0$. Next, for a given r, we choose σ_1 (and thus $\sigma_2 = r\sigma_1$) to match $Var(\log \pi)$.

any elasticity can be generated. From a mathematical point of view, this should not be surprising: the elasticity is determined by a specific value of the hazard rate, which has no direct relation to the three conditions used for the calibration. These computations suggest that the shape of heterogeneity is quantitatively important. Moreover, what matters for the elasticity, namely marginal homogeneity, cannot be summarized easily using two moments of the whole distribution. In what follows, we exploit the tractability of complete markets to develop an empirical model that allows for richer heterogeneity.

The importance of microdata

Following Mulligan (2001), we note that in this model, a representative agent exists and the usual first-order condition which equates the marginal utility of leisure over the marginal utility of consumption to the wage holds, for some utility function which reflects the underlying heterogeneity. Mulligan (2001, 2002) concludes that there is little point in estimating the aggregate labor supply elasticity with microeconomic data, and he uses the aggregate marginal condition to estimate the aggregate labor supply elasticity.

While the equivalence makes this approach appealing, we think it has also some disadvantages. First, this aggregate condition relies on the absence of aggregate labor supply shocks (d_t in our notation). But a wide empirical literature in macroeconomics argues that these shocks are important,[10] and it is not easy to find valid instruments (Hall 1980). In contrast, by using microeconomic data we are able to take into account these aggregate shocks in our estimation. Second, using aggregate data requires to confront the compositional effect in average wage or productivity series (Solon, Barsky and Parker 1994). Third, we are able to evaluate the elasticity at any point in time. This allows us to check if the elasticity varies over time. The model suggests this may be true: after a long expansion, many potential workers enter the workforce, and the reservation wage of the remaining potential workers may be high. This could make the elasticity of labor supply become lower after a long boom such as the late 1990s, and conversely we might expect the elasticity of labor supply to become high in recessions.[11] (Of course, since

[10] See e.g. Benhabib, Rogerson and Wright (1991), Christiano and Eichenbaum (1992), Hall (1997).

[11] For instance, the New York Time reported on December 20, 1999, that "As labor pools shrinks, a new supply is tapped". The article discusses how some individuals who were out of the labor force (students and retired people) were agressively recruited by expanding businesses. On the other hand, then-Chairman of the Federal Reserve Alan Greenspan worried at the same time about the size of the "shrinking pool of available

this depends on the shape of the distribution, the effect could in principle go either way.)

D. Cross-Sectional Implications and Marginal Workers

Since the participation decision (2.6) is almost never interior, only the marginal worker (who is just indifferent to working or not) would react to a marginal change in z_t or d_t. Hence ex post – after all idiosyncratic shocks are realized – the marginal effect of w_t is nil except for this marginal worker. Hence some workers are more sensitive to aggregate shocks. However, this ex-post prediction has no content unless we "identify" the marginal workers.

A natural solution is to consider the marginal effect of w_t, *before x_{it} is known.* That is, consider our one-dimensional measure of idiosyncratic risk $x_{it} \equiv \theta_{it} v\left(\overline{n}\right) \mu_i / \left(\pi_{it} \overline{n}\right)$, and suppose we observe X_{it}, a variable which is correlated with x_{it}. In practice, we will use estimated fixed effects or covariates (i.e. observable characteristics) as X_{it}. However, X_{it} could also include lagged employment n_{it}, or lagged values of x_{it}, as in the example below. Our model immediately yields the following formula for the marginal effect of a change in aggregate wage (i.e. the elasticity of the probability of working to the wage rate), as a function of X_{it} :

Result 3: Heterogeneous Elasticities

$$\frac{\partial \log \Pr\left[n_{it} = 1 \mid X_{it}\right]}{\partial \log w_t} = \frac{g\left(\log \frac{\lambda_t\left(z^t, d^t\right)}{d_t} + \log w_t\left(z^t, d^t\right) \mid X_{it}\right)}{G\left(\log \frac{\lambda_t\left(z^t, d^t\right)}{d_t} + \log w_t\left(z^t, d^t\right) \mid X_{it}\right)} = \frac{g\left(\log x_t^* | X_{it}\right)}{G\left(\log x_t^* | X_{it}\right)},$$

where $g(x|y)$ (resp. $G(x|y)$) is the conditional probability density function (resp. cumulative distribution function) of x_{it} given $X_{it} = y$, evaluated at x.

This equation implies that the *level* change in the probability is

$$\frac{\partial \Pr\left[n_{it} = 1 \mid X_{it}\right]}{\partial \log w_t} = g\left(\log x_t^* | X_{it}\right).$$

Assuming that $G(x|y)$ is increasing in y, (i.e. that "x_{it} is monotone in X_{it}"), and assuming that the density g, which is defined on all $\mathbb{R}$, is single-peaked, we can see that this change in probability $\frac{\partial \Pr[n_{it}=1|X_{it}]}{\partial \log w_t}$ will be low for very high or very low X_{it}, for which agents are almost surely working or not working, and high for intermediate values of X_{it}. The marginal

workers" (Remarks on April 5, 2000).

effect $\frac{\partial \log \Pr[n_{it}=1|X_{it}]}{\partial \log w_t}$ will also be low for high X_{it}, but it may be high for low X_{it} because the denominator $G(\log x_t^*|X_{it})$ is small. This is illustrated in Figure 1.

Importantly, the aggregate elasticity, given in (2.8), is a weighted average of these marginal effects. Indeed, since $\widetilde{N}_t = \int \Pr(n_{it} = 1|X_{it})dF(X_{it})$,

$$\begin{aligned}\frac{\partial \log \widetilde{N}_t}{\partial \log w_t} &= \int \frac{\partial \log \Pr[n_{it} = 1 \mid X_{it}]}{\partial \log w_t} \frac{\Pr[n_{it} = 1 \mid X_{it}]}{\widetilde{N}_t} dF(X_{it}), \\ &= \frac{1}{\widetilde{N}_t} \int \frac{\partial \Pr[n_{it} = 1 \mid X_{it}]}{\partial \log w_t} dF(X_{it}).\end{aligned}$$

The first line states that the aggregate elasticity is a weighted sum (by the probability to working) of individual marginal effects. The second line states that the elasticity is also the sum of the level effect on probabilities. Hence if there are enough "marginal workers", for whom the elasticity is high, the aggregate elasticity will be high. Clearly, the cross-sectional prediction is closely related to our aggregate implications, as the similarity of the formulas suggests.

Are the marginal workers constant?

An extreme version of our theory is that marginal workers are constant - they are a fixed set of people. This is the case when π and θ are fixed over time. The workers whose π and θ make them close to the reservation wage are marginal. In this case, the people with high π or low θ always work, the people with low π or high θ never work, and intermediate π or θ people are the only ones whose employment changes over time. This version of the theory makes sharp predictions on who adjusts over the business cycle.

The other extreme version is that π and θ are i.i.d. In this case,all workers are equally likely to enter or exit the workforce, and we have no prediction on who adjusts over the business cycle.

Importantly, whether marginal workers are the same people or not over time is irrelevant for our aggregate estimate. To put it another way, what matters to our "marginal homogeneity" argument is the *total* (unconditional **XXXX**) amount of heterogeneity at the margin, which can come from either fixed effects, covariates, or idiosyncratic shocks. (In particular, while it is reasonable to argue that idiosyncratic shocks are approximately log-normal, there is no reason to expect this for fixed effects or covariates.) The downside is that unless we are specific about the stochastic structure of idiosyncratic shocks, it is hard to get predictions regarding the pattern of marginal effects and individual sensitivities to aggregate shocks.

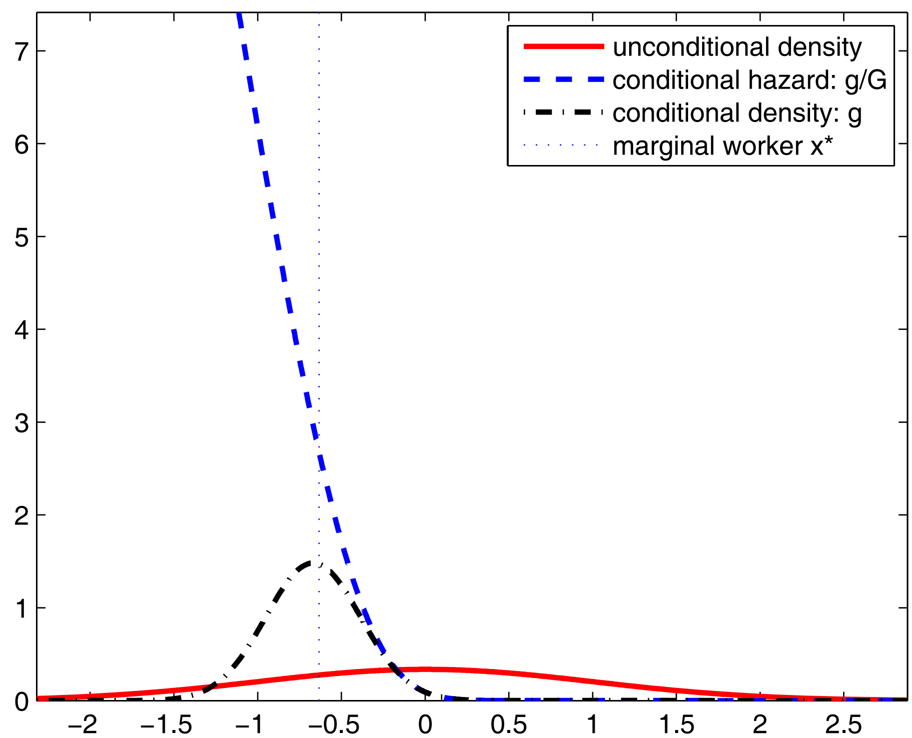

Figure 1: The dashed line represents the marginal effects $\partial \log \Pr\left[n_{it} = 1 \mid \pi_{it-1}\right] / \partial \log w_t$. The dashed-dotted line represents $\partial \Pr\left[n_{it} = 1 \mid \pi_{it-1}\right] / \partial \log w_t$. The solid line represents the unconditional density of $\log \pi_{it-1}$.

As a more realistic example, consider the case where the only heterogeneity is in productivity π_{it}. Productivity follows an AR(1) process: $\log \pi_{it} = \rho \log \pi_{it-1} + \sigma \varepsilon_t$. We use $X_{it} = \pi_{it-1}$ as a predictor of π_{it}. We compute the marginal effect of a change in w_t on the probability of working at t given π_{it-1}. Figure 1 shows the heterogeneity in marginal effects $\frac{\partial \log \Pr[n_{it}=1|X_{it}]}{\partial \log w_t}$: the dotted line shows that most agents have zero marginal effects, while a few agents with the lowest wages have high marginal effects. However, the level changes in probabilities $\frac{\partial \Pr[n_{it}=1|X_{it}]}{\partial \log w_t}$ (i.e. the marginal effects weighted by their actual probability of employment) are hump-shaped, so that the marginal worker is actually in the middle of the distribution, where both the marginal effect and the employment probability are substantial. The aggregate elasticity is the integral of the dashed-dotted line.

In a previous draft (Gourio and Noual 2006), we used the PSID to quantify if agents in the middle of the distribution were actually more responsive to aggregate fluctuations. We used observable characteristics such as hours worked last year, education, age, etc., to sort people and we found that intermediate agents were usually more sensitive to aggregate fluctuations.

3 Estimation Strategy

We first discuss the equation that we estimate, then we discuss the estimation method, the data and exact specification, and finally the elasticities we measure. Section 4 presents the results.

A. The participation and the wage equations

We use a balanced panel of I individual agents over T periods. We observe employment $n_{it} \in \{0,1\}$, and if $n_{it} = 1$ we also observe wages w_{it}. Our model asserts that $n_{it} = 1$ if and only if $w_{it} \geq w_{it}^R$: the reservation wage w_{it}^R is never observed, while the wage rate w_{it} is observed only for workers.

In our model, the reservation wage is $w_{it}^R = \frac{d_t}{\lambda_t} \theta_{it} v(\overline{n})$ and the wage rate is $w_{it} = w_t \pi_{it} \overline{n}$. We generalize this model slightly in two respects. First, we take into account observable factors which affect wages on one side, and reservation wages on the other side. Second, we add fixed effects to taste or productivity. While neither of these features is present in our theoretical model, it is easy to incorporate them, at a great expense of notation. Formally, we assume that productivity and taste satisfy:

$$\begin{aligned} \log \pi_{it} &= a_i + \mathbf{x}_{it}\gamma + u_{it}, \\ \log \theta_{it} &= \widetilde{a}_i^R + \mathbf{y}_{it}\eta + e_{it}, \end{aligned}$$

where $\mathbf{x}$ and $\mathbf{y}$ are vectors of observable characteristics. We assume that the vector (u_{it}, e_{it}) is independent across i and t and jointly normal. Our participation equation reads:

$$\begin{aligned} \log w_{it} &\geq \log w_{it}^R, \\ \text{i.e. } \log w_t + \log \pi_{it} + \log \overline{n} &\geq \log \frac{d_t}{\lambda_t} + \log \theta_{it} v\left(\overline{n}\right) \mu_i, \\ b_t + a_i + \mathbf{x}_{it}\gamma + u_{it} &\geq b_t^R + \widetilde{a}_i^R + \mathbf{y}_{it}\eta + e_{it} + \log \mu_i, \end{aligned} \tag{3.1}$$

where we replace the aggregate wage and the disutility of labor over marginal utility of wealth by time effects, which also absorb the constant,[12] with $b_t^R = \log \frac{d_t}{\lambda_t}$. In the end, the wage equation reads:

$$\begin{aligned} \log w_{it} &= \log w_t + \log \pi_{it} + \log \overline{n}, \\ &= \log w_t + a_i + \mathbf{x}_{it}\gamma + u_{it} + \log \overline{n}, \\ &= b_t + a_i + \mathbf{x}_{it}\gamma + u_{it}. \end{aligned} \tag{3.2}$$

These specifications allow for unobservable nonparametric heterogeneity with fixed effects, but on the other hand the fact that (u_{it}, e_{it}) is i.i.d. and normal is restrictive. However, some persistence can be contained in the vectors of observable covariates $\mathbf{x}_{it}, \mathbf{y}_{it}$. We plan to work on relaxing this assumption using simulation methods (for instance as in Hyslop 1999). Another restrictive assumption is that there are no costs of a job transition (e.g. job search costs).

B. Estimation method

We assume a joint normal distribution for transitory shocks on wages and reservation wages (u_{it}, e_{it}):

$$\begin{pmatrix} u_{it} \\ e_{it} \end{pmatrix} \overset{i.i.d.}{\sim} N\left(0, \Sigma\right) \text{ with } \Sigma = \begin{pmatrix} \sigma_u^2 & \sigma_{ue} \\ \sigma_{ue} & \sigma_e^2 \end{pmatrix}.$$

[12] The variable d_t/λ_t is not observed. The aggregate wage w_t may be observed, but there is a composition bias, so we also prefer to use time effects.

The likelihood of our observations on $\{n_{it}, w_{it}\}$ is:

$$L = \prod_{i=1}^{N} \prod_{t=1}^{T} \{\Pr[n_{it} = 0 \mid \mathbf{x}_{it}, \mathbf{y}_{it}]\}^{1-n_{it}} \{\phi(w_{it}, n_{it} = 1 \mid \mathbf{x}_{it}, \mathbf{y}_{it})\}^{n_{it}}, \tag{3.3}$$

where ϕ stands for the joint density of wages w_{it} and participation n_{it}, which can be recovered from the joint density of shocks u_{it} and e_{it}, as detailed in the Appendix. Identification of parameters $\left(\sigma_u^2, \sigma_{ue}, \sigma_e^2, \gamma, \eta, \{a_i^R\}_{i=1}^N, \{b_t\}_{t=1}^T, \{b_t^R\}_{t=1}^T\right)$ follows from the maximization of the likelihood. We experimented with this likelihood maximization, using the tools developed by Greene (2001), who shows how to exploit the sparsity of Hessians matrices to estimate a large number of fixed effects. However, the likelihood is not concave, which makes it hard to attain the global maximum. For this reason, we use instead the standard Heckman (1979) two-step estimator.[13] We still use the tools of Greene (2001) to perform the first step (the Probit model).

Rephrasing the first-order condition (2.6) in terms of our statistical model,

$$\begin{aligned} \Pr[n_{it} = 1] &= \Pr\left[w_{it}^R \leq w_{it}\right] = \Pr\left[b_t^R + a_i^R + \mathbf{y}_{it}\eta + e_{it} \leq b_t + a_i + \mathbf{x}_{it}\gamma + u_{it}\right], \\ &= \Pr\left[e_{it} - u_{it} \leq b_t + a_i + \mathbf{x}_{it}\gamma - \left(b_t^R + a_i^R + \mathbf{y}_{it}\eta\right)\right], \\ &= \Phi\left(\frac{b_t + a_i + \mathbf{x}_{it}\gamma - \left(b_t^R + a_i^R + \mathbf{y}_{it}\eta\right)}{\sigma}\right), \end{aligned} \tag{3.4}$$

where $\sigma^2 = V(e_{it} - u_{it}) = \sigma_u^2 + \sigma_e^2 - 2\sigma_{ue}$, and Φ is the cumulative function of the standard normal distribution. This participation equation (3.4) is a Probit model which can be estimated by MLE. The Probit allows to recover estimates of $\left\{\left\{\frac{b_t - b_t^R}{\sigma}\right\}_t, \left\{\frac{a_i - a_i^R}{\sigma}\right\}_i, \frac{\gamma}{\sigma}, \frac{\eta}{\sigma}\right\}$.

However, to identify σ separately from the other parameters $\left\{\gamma, \eta, \{a_i\}_{i=1}^N, \{a_i^R\}_{i=1}^N, \{b_t\}_{t=1}^T, \{b_t^R\}_{t=1}^T\right\}$, the wage equation (3.2) needs to be estimated as well. Therefore, after the probit, we estimate the wage regression for workers, controlling for selection. As a result, our second step consists of the following OLS regression:

$$\log w_{it} = b_t + a_i + x_{it}\gamma + \frac{\sigma_u^2 - \sigma_{ue}}{\sigma} \frac{\phi(c_{it})}{\Phi(c_{it})} + u_{it}, \tag{3.5}$$

where the last term is the inverse Mills ratio, evaluated at the index c_{it} which determines participation:

$$c_{it} = \frac{b_t + a_i + \mathbf{x}_{it}\gamma - \left(b_t^R + a_i^R + \mathbf{y}_{it}\eta\right)}{\sigma}. \tag{3.6}$$

[13]Matlab programs which perform our estimation are available upon request.

In this second step, all parameters are identified from an exclusion restriction: some variables in $\mathbf{y}_{it}$ are not in $\mathbf{x}_{it}$, that is there are determinants of the reservation wages (such as the age of children) that have no effect on market wages: as a result, the Mills ratio is not colinear to the other determinants of wages.

From our estimates of $\widehat{\frac{\gamma}{\sigma}}$ in the first step (probit), and our estimates of $\widehat{\gamma}$ in the second step (wage regression), we recover σ. Because $\mathbf{x}_{it}$ is not a scalar, we cannot recover σ simply as $\widehat{\gamma}/\widehat{\frac{\gamma}{\sigma}}$: we construct a minimum distance estimator which minimizes a weighted average of $\left(\widehat{\gamma}_k - \widehat{\sigma}\widehat{\frac{\gamma_k}{\sigma}}\right)^2$ for all variables k that we take as regressors for wages. From our estimate of σ, we then recover all other parameters (see the Appendix for details).

C. Data and Specification

We use the National Longitudinal Survey of Youth 1979 (NLSY 79). (See the appendix for a more detailed description of this well-known data set.) This gives us a panel with $N = 5571$ agents and $T = 168$ months. Our sample starts in January 1979 and ends in December 1992.[14] To estimate the probit with fixed effects, we need to exclude agents who are either always working or never working. However, we take these agents into account in our computations of aggregate elasticities.

We use the following variables in our specification. The vector $\mathbf{x}_{it}$ includes determinants of wages: experience, experience squared, and schooling. The vector $\mathbf{y}_{it}$ includes determinants of reservation wages: a dummy if the youngest child is less than 2 years old, a dummy if the yougest child is between 3 and 6 years old, and a dummy if the youngest child is between 7 and 14 years old; a dummy for the marital status (interacted with gender), and a dummy for the answer to a question: "do health problems limit the amount or type of work [you] can perform?". In unreported results, we tried to add more covariates, including interactions, but the aggregate results did not change markedly; however, this might require further investigation.

[14] Our data cover 1979-1998 in fact, yet we are much less confident about the wage data after 1993, due to a change in methodology in the NLSY survey.

D. Measurement of Elasticities

The following subsection explains in detail how we use our estimates to compute macroeconomic or group-level elasticities. Consider the latent index c_{it} that determines participation:

$$c_{it} \equiv \frac{b_t + a_i + \mathbf{x}_{it}\gamma - \left(b_t^R + a_i^R + \mathbf{y}_{it}\eta\right)}{\sigma}.$$

The index c_{it} is the difference between the predicted wage rate w_{it} and reservation wage rate w_{it}^R, scaled by the variance of i.i.d. shocks. It is estimated from the first step of our estimation, i.e. the probit model for participation. Predicted aggregate employment is: $\widehat{N}_t = \sum_{i=1}^{I} \Pr\left[n_{it} = 1\right] = \sum_{i=1}^{I} \Phi\left(c_{it}\right).$

To compute the aggregate elasticity of labor supply, we evaluate the derivative with respect to b_t, since b_t captures the aggregate wage in our equation:

$$\frac{\partial \log \widehat{N}_t}{\partial \log b_t} = \frac{1}{\sigma} \frac{\sum_{i=1}^{I} \phi\left(c_{it}\right)}{\sum_{i=1}^{I} \Phi\left(c_{it}\right)}. \tag{3.7}$$

Importantly, this definition of the aggregate elasticity is not always the measure reported by researchers, who sometimes report the response of the average or median individual, i.e. $\frac{1}{\sigma}\phi(\overline{c}_i)/\Phi\left(\overline{c}_i\right)$. This difference may be important, as we demonstrate in Section 4. Note how

$$\frac{\sum_{i=1}^{I} \phi\left(c_{it}\right)}{\sum_{i=1}^{I} \Phi\left(c_{it}\right)} = \frac{\sum_{i=1}^{I} \frac{\phi(c_{it})}{\Phi(c_{it})} \Phi(c_{it})}{\sum_{i=1}^{I} \Phi\left(c_{it}\right)}$$

is simply a weighted sum of hazard rates (or marginal effects, once scaled by σ^{-1}). One can interpret this formula as an average of marginal workers: for each value of the index, there is a continuum of workers and $\sigma^{-1}\phi\left(c_{it}\right)$ of them are "marginal". The elasticity is just a weighted sum of these numbers of marginal workers. When i.i.d. shocks are larger, as measured by σ, there is more heterogeneity and the aggregate elasticity falls.

We also present results by groups: in this case, the index i in each of the two sums in (3.7) ranges over all the i in the group rather than $i = 1$ to I. For instance, the elasticity of men is:

$$\frac{\partial \log \widehat{N}_t^{men}}{\partial \log b_t} = \frac{1}{\sigma} \frac{\sum_{i \in men} \phi\left(c_{it}\right)}{\sum_{i \in men} \Phi\left(c_{it}\right)}.$$

Obviously, we can then break down the aggregate elasticity into the weighted sum of each group's elasticities, where the weight is the share of employment of each group:

$$\frac{\partial \log \widehat{N}_t}{\partial \log b_t} = \frac{\partial \log \widehat{N}_t^{men}}{\partial \log b_t} \frac{\widehat{N}_t^{men}}{\widehat{N}_t} + \frac{\partial \log \widehat{N}_t^{women}}{\partial \log b_t} \frac{\widehat{N}_t^{women}}{\widehat{N}_t}.$$

4 Empirical Results

We now present the results obtained from the estimation procedure on our NLSY sample. We first present our estimated wage and participation equations. We next compute the Frisch elasticity implied by the model and we illustrate how the elasticity differs by groups formed on observable characteristics. Finally, we measure the contribution of "marginal workers" to aggregate fluctuations.

A. Estimation results

Table 3 reports the coefficients of the wage regression (3.5). (All coefficients and their standard errors are expressed in percentages to facilitate the interpretation.) The return to one year of schooling is around 10%, consistent with the usual results of Mincer regressions. Similarly, the coefficients on experience and experience squared are similar to known results about the determinants of earnings (e.g. Heckman, Lochner and Todd 2003).

	point estimate ($\times$100)	standard error ($\times$100)
experience	12.16	0.18
experience squared	-0.26	0.01
schooling	9.88	0.09

Table 3: Estimates of coefficients γ
on observables $\mathbf{x}_{it}$ in the wage regression for w_{it}

Table 4 reports our estimates of the determinants of reservation wages. The results are sensible: being married raises the reservation wage for a woman, yet it decreases the reservation wage for men. Having a medical condition that limits the type or amount of work one can perform raises the reservation wage. Finally, the reservation wage is much higher for women with young children, and the younger the children the higher the reservation wage.

	point estimate (×100)	standard error (×100)
married (men)	-4.63	0.13
married (women)	3.31	0.08
health limit	6.33	0.17
youngest kid 0-2 years	17.61	0.46
youngest kid 3-6 years	8.39	0.24
youngest kid 7-14 years	3.86	0.16

Table 4: Estimates of coefficients η

on observables $\mathbf{y}_{it}$ in the reservation wage w_{it}^R

An important determinant of wages and reservation wages is unobserved permanent heterogeneity: this is captured by the fixed effects a_i, a_i^R (for wages and reservation wages respectively). Since labor force participation depends on the net effect $a_i - a_i^R$, we present its histogram in figure 2. There is a substantial amount of permanent heterogeneity that cannot be attributed to observables: the standard deviation of $\left(a_i - a_i^R\right)$ is 12.65%. Figure 2 also shows that this distribution is more concentrated than a normal distribution of same mean and variance (represented as 'fitted normal'). Finally, a crucial parameter is σ, the standard deviation of transitory shocks to wages and reservation wages w_{it}^R and w_{it}. (Recall $\sigma = Std\left(e_{it} - u_{it}\right)$.) We estimate $\sigma = 17.63\%$, with a standard error of 0.26%. Hence, i.i.d. shocks are large: their standard deviation is 50% larger than the standard deviation of fixed effects. We also report in Table 5 the structure of shocks to wages and reservations wages.

	point estimate (×100)
σ_u^2	12.40
σ_e^2	15.31
σ_{ue}	12.30

Table 5: Estimates of the variance of shocks

$\sigma^2 = Var\left(e_{it} - u_{it}\right) = \sigma_u^2 + \sigma_e^2 - 2\sigma_{ue}$

In terms of goodness of fit, the average probability of correct prediction of employment status n_{it} is 77%.[15] Unsurprisingly given that we include time effects, the model's prediction

[15] This measure has been suggested by Ben-Akiva and Lerman (1985). Another measure of goodness

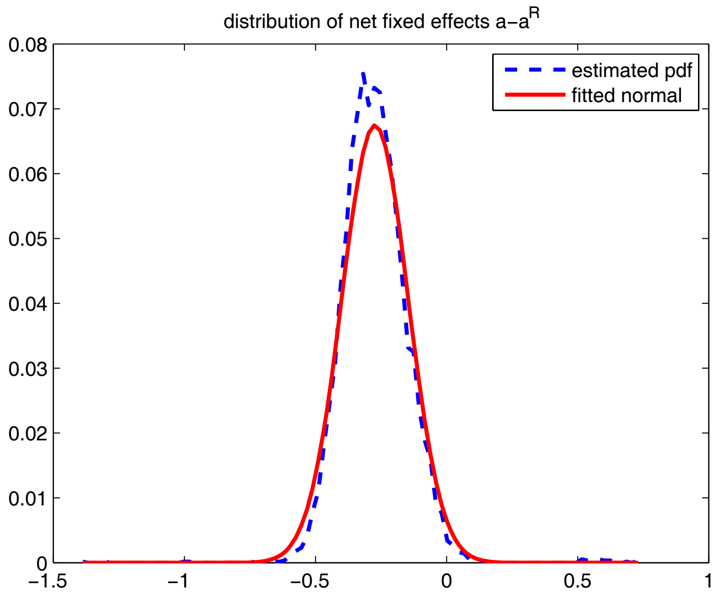

Figure 2: The distribution of net fixed effects $a_i - a_i^R$ in our estimates: a_i is a fixed effect in individual wage rate w_{it}, while a_i^R is a fixed effect in reservation wage rate w_{it}^R : participation depends, in part, on the net fixed effect represented in this histogram.

for aggregate employment fits the data very well. Overall, our results in these dimensions are consistent with the existing literature. We now turn to the aggregate implications of these estimates.

B. The estimated elasticity of labor supply

Figure 3 depicts the Frisch aggregate elasticity at each date, together with the plus and minus two-standard errors bands. Figure 4 plots the aggregate Frisch elasticity together with aggregate employment. The employment series has two noticeable features. First, it has an important seasonal component, especially in the ear early part of the sample. Second, it trends up. These features are due to the changing age composition of our data. Because the NLSY follows some initial cohorts over time (people aged between 14 and 22 in 1979), their labor supply grows along the life-cycle.

The Frisch elasticity is defined using our formula above:

$$\frac{\partial \log \widehat{N}_t}{\partial \log b_t} = \frac{1}{\sigma} \frac{\sum_{i=1}^{I} \phi\left(c_{it}\right)}{\sum_{i=1}^{I} \Phi\left(c_{it}\right)},$$

where c_{it} is our estimate for the latent index that determines participation (i.e. $n_{it} = \overline{n}$ if $c_{it} \geq \varepsilon_{it}$ where $\varepsilon_{it} \sim N\left(0,1\right)$, see Section 3D for details on measurement given our estimates). This elasticity is rather precisely estimated. The elasticity at the median date is 1.27, and the average elasticity over the whole sample is 1.50, which is higher than estimates based on the intensive margin, but still lower than the number of 3 or 4 required for macroeconomics (King and Rebelo 1999, Prescott 2006). As we noted already, our measure of the elasticity differs from the marginal effects for the median (or average) agent which is often reported in the discrete choice literature. For instance, had we computed the marginal effect of w_t at the median date in our sample, evaluated at the mean c_{it}, we would report an elasticity of 1.83, while the true Frisch elasticity is 1.27 at this date.

Our estimate for the elasticity exhibits large fluctuations over time. These fluctuations are highly negatively correlated with aggregate employment. This is not very surprising: if the density does not decrease too fast, we would expect fluctuations in employment (the denom-

of fit, Efron's (1978), is 38% in our case. Efron's measure is close to an R^2, since it is computed as $1 - \sum_{i,t}\left(n_{it} - p_{it}\right)^2 / \sum_{i,t}\left(n_{it} - \overline{n}\right)^2$, where p_{it} is our estimate for $\Pr\left[n_{it} = 1\right]$.

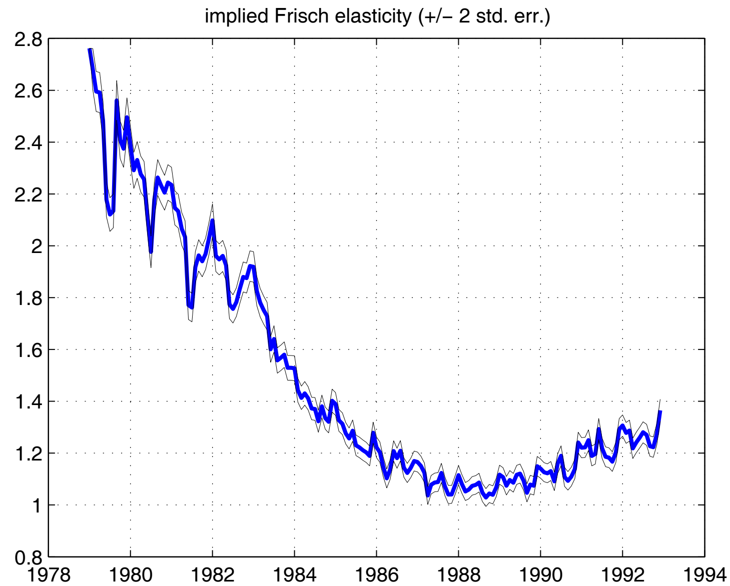

Figure 3: The Aggregate Elasticity of Labor Supply, measured at each date. The figure represents our point estimate, and a 95% confidence interval around it.

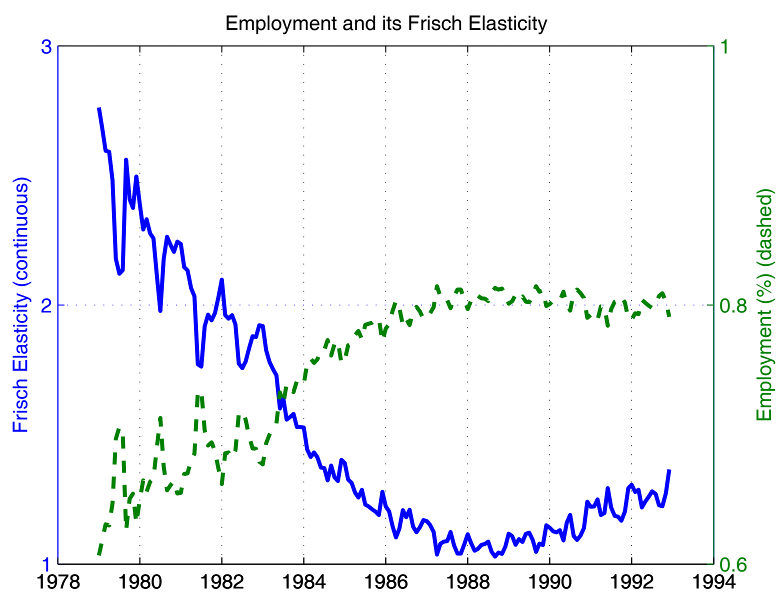

Figure 4: **Left scale**: the Frisch elasticity over time (thick plain line). **Right scale**: employment over time for our NLSY sample (dashed).

inator in our measure of the elasticity) to make the elasticity countercyclical. Indeed, most of the hazard rates of the usual distributions are decreasing, as long as the distributions are log-concave.[16]

This negative correlation between the Frisch elasticity and employment has two facets. The first one is the life-cycle

component. It is well known that young workers are more cyclical. For instance, Gomme, Rogerson, Rupert and Wright (2004), note that teenagers and young adults have more volatile employment, and are more elastic, than prime age workers.

The second facet of this negative correlation is the business cycle. To separate the effect of age and macroeconomic conditions, we compute aggregate employment and the Frisch elasticity for the population of workers aged between 28 and 30, over the period 1987-1992: for each date t between 1987 and 1992, we include an individual i in our computation of the Frisch elasticity and employment only if his or her current age is 28, 29 or 30. Figure **??** 5 presents the results. This figure demonstrates that the Frisch elasticity displays fluctuations even when age is held constant. The Frisch elasticity is higher during the NBER recession of 1990-1991 and the ensuing 'jobless recovery' than in earlier years.

Of course, the changing composition of the NLSY is a problem for us, since we would like to have a representative sample of the entire US population. In earlier work, we used the PSID which is better in this respect. However, the NLSY is unique in providing monthly information on employment.

In this respect, it is interesting to see that the NLSY results match standard national statistics. Figure 6 compares the ratio of employment to population in our sample for respondents aged 28 to30 and the total number of employees according to the CES survey of the BLS. (The two series are normalized to have the same average.) The high correlation, in particular during the recession of 1990-1991 suggests that the life-cycle dimension of the NLSY is not necessarily an important problem: the two series decline by a similar amount.

Constructing labor supply schedules and hazard rates

Our estimation method allows us to construct the entire labor supply schedule. Indeed, we can compute a counterfactual latent index c_{it} for different hypothetical realizations of the

[16] If F is log-concave, then the hazard rate $f/F = d(\log F)$ is decreasing since $d^2 \log F < 0$.

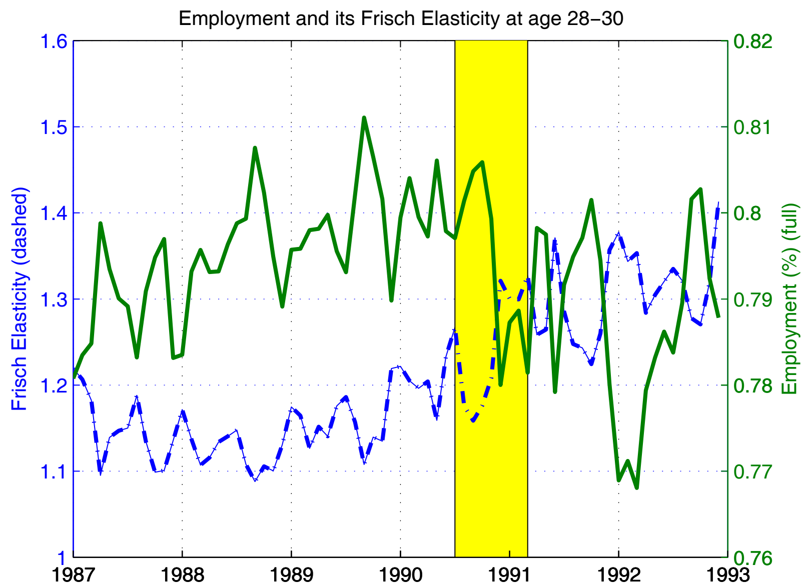

Figure 5: **Employment and the estimated Frisch elasticity for respondents aged 28 to 30.** This figure displays a measure of the elasticity at a constant age (Thus the respondents included in these aggregate measures changes accordingly.) The dashed zone represents the 1991 recession, according the NBER. As employment (dashed and green) is low during the recession and jobless recovery, the elasticity (plain and blue) is higher.

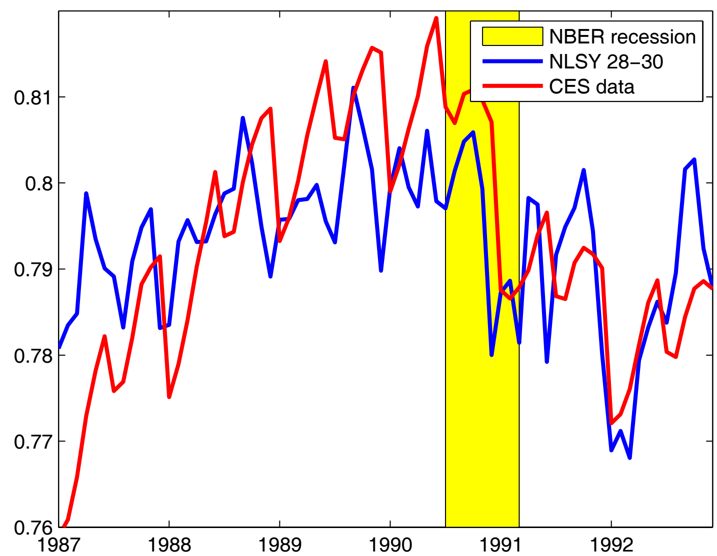

Figure 6: **Employment for respondents aged 28 to 30 and the CES number of employees.** The CES series is normalized to have the same mean as the NLSY series.

aggregate wage rate w_t (the marginal product of an efficiency unit of labor). This amounts to varying b_t, the time effect that captures these fluctuations in w_t. Remember that

$$c_{it} \equiv \frac{b_t + a_i + \mathbf{x}_{it}\gamma - \left(b_t^R + a_i^R + \mathbf{y}_{it}\eta\right)}{\sigma},$$

and predicted employment for a given wage is just the sum of employment probabilities $\Phi\left(c_{it}\right)$ evaluated at a given b_t.

Figure 7 depicts the aggregate labor supply schedule, i.e. aggregate employment at different wage rates, at a given date t (we choose the median date of our sample). The right panel represents the hazard rate of the distribution, evaluated at each wage rate. The vertical line represents the realized wage rate w_t at this date. The predicted employment at this date (respectively Frisch elasticity) can be read off the graph, on the left panel (resp. right panel), at the intersection of the cumulative distribution (resp. hazard rate) and the vertical line. As mentioned earlier, our formula for the Frisch elasticity is naturally decomposed across subpopulations. That is, we can compute an elasticity for each subsample of our population, and – weighting these elasticities by average employment probabilities – recover the aggregate elasticity. Figures 8 and 9 perform this exercise by separating our sample by gender, and by

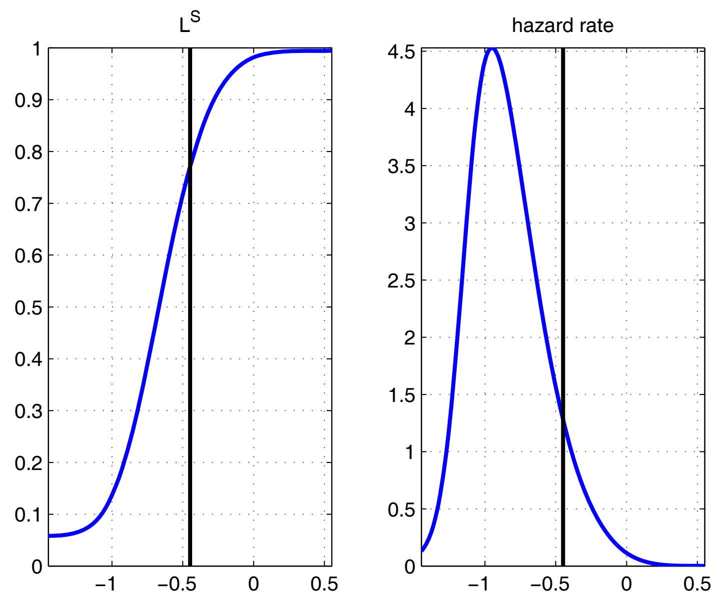

Figure 7: **Aggregate Labor Supply Curve and Hazard Rate.** *Left panel*: aggregate employment for various macro wage rates w_t. *Right panel*: the corresponding hazard rate, i.e. the Frisch elasticity. The vertical line represents the realized wage rate at this date. Computation performed at the median date of our sample (July 1985).

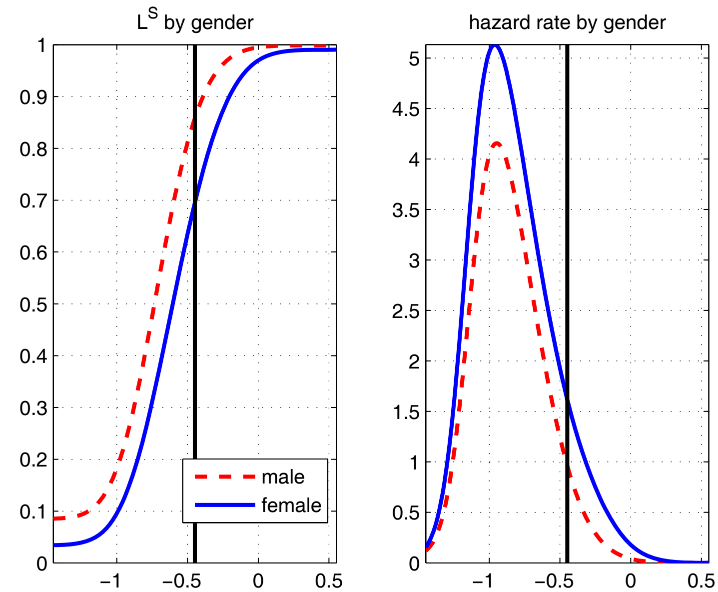

Figure 8: **Comparison of employment and elasticities by gender**: for both men and women, employment (resp. the Frisch elasticity) can be read on the y-axis, at the intersection of the CDF (resp. the hazard rate curve) and the vertical line which represents the realized wage rate. Computation performed at the median date of our sample (July 1985).

education. These figures are exact analogs to figure 7, except that we sum employment shares (left panel) and marginal effects (right panel) over different populations, as explained in Section 3D. These graphs show that women and the less-educated work less and are more elastic respectively than men and the more educated. This is a standard result in the labor literature (e.g. Heckman 1993, and Gomme, Rogerson, Rupert and Wright 2005). The Frisch elasticity can be read off on the hazard rate at the threshold value. For the median date (July 1985), the Frisch elasticity for female is 1.62 and the Frisch elasticity for male is 0.96. The averages over the whole sample are respectively 1.82 and 1.19. The Frisch elasticities for college graduates, high-school graduates, and high-school dropouts at the median date are 1.13, 1.22 and 2.05. In each of these decompositions of population across observable characteristics, we can check that the aggregate elasticity at this date is a weighted average of the different groups' elasticities, where the weights are the relative employment shares.

Our results are consistent with an estimate of the aggregate Frisch elasticity around 1 for

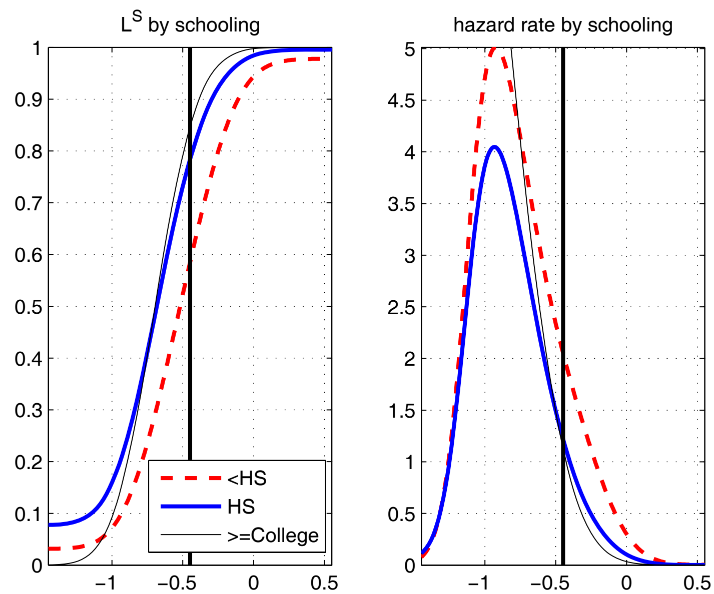

Figure 9: **Comparison of employment and elasticities by schooling level**. <HS means no high-school degree, HS means high school degree and possibly some college, and College means a BA or higher. For each education level, employment (resp. the Frisch elasticity can be read at the intersection of the CDF (resp. the hazard rate) and the vertical line which represents the realized wage rate. Computation performed at the median date of our sample (July 1985).

prime-age workers (i.e. at the end of our sample). This estimate takes only into account the extensive margin. To give a definite answer regarding the Frisch elasticity of the whole population, we would need a sample representative of the US population: the main drawback of our dataset is that the average age of respondents rise continuously. Furthermore, we have no workers aged more than 35. Workers older than 65 are known to be volatile. In further work, we plan to extend the analysis to a longer sample (until 1998).

C. The Marginal Worker

A large part of heterogeneity is not attributable to observable characteristics. This motivates us to use fixed effects and to go beyond simple decompositions by demographic group. In this subsection, we use the latent index c_{it}, which determines participation, to identify "marginal workers". More precisely the combination of fixed effects and observable covariates $\mathbf{x}_{it}, \mathbf{y}_{it}$ allows to identify the agent who is closest to being indifferent between working or not: the latent index c_{it} is nearly equal to zero for this agent. We define marginal workers as workers whose c_{it} is small enough, and we choose a threshold $|c_{it}| \leq 0.5$. This is equivalent to defining marginal workers at time t as the individuals for which the model predicts a probability of working higher than $0.305\% = \Phi(-0.5)$ and less than $0.695\% = \Phi(0.5)$.

Figure 10 plots the share of the population which is 'marginal' by this definition. This share falls steeply at the beginning of our sample before stabilizing around 12%. This is clearly in large part driven by age and experience effects. Figure 11 shows the evolving distribution of the index c_{it} over time. The vertical lines at 0 represent the marginal worker. In 1979, when agents are all young, they are very homogeneous, and most agents have c_{it} close to zero. As time goes by, c_{it} drifts the right, most agents are permanently employed, and the marginal workers are not any more the mode of the distribution. The figure shows that the distribution gets skewed, and that the variance first increases, probably as people enter the workforce at different ages.

We can then test the hypothesis that these marginal workers account for a large share of aggregate fluctuations. We decompose changes in aggregate employment into the sum of the changes of the marginal worker population and the non-marginal worker population:

$$\Delta \widetilde{N}_t = \Delta \widetilde{N}_t^{\text{marginal workers}} + \Delta \widetilde{N}_t^{\text{non marginal workers}},$$

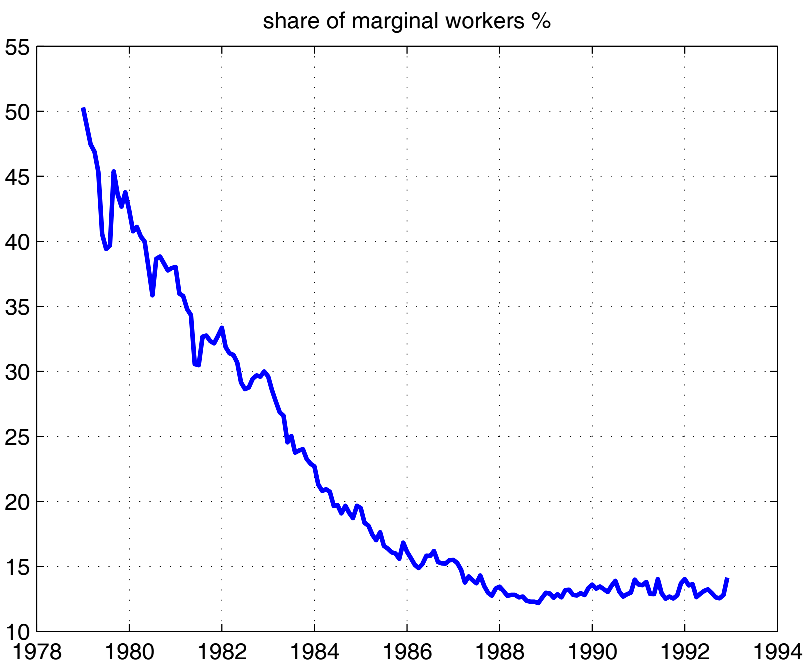

Figure 10: **Share of population who are "marginal workers"**. Marginal workers are defined by $|c_{it}| \leq 0.5$.

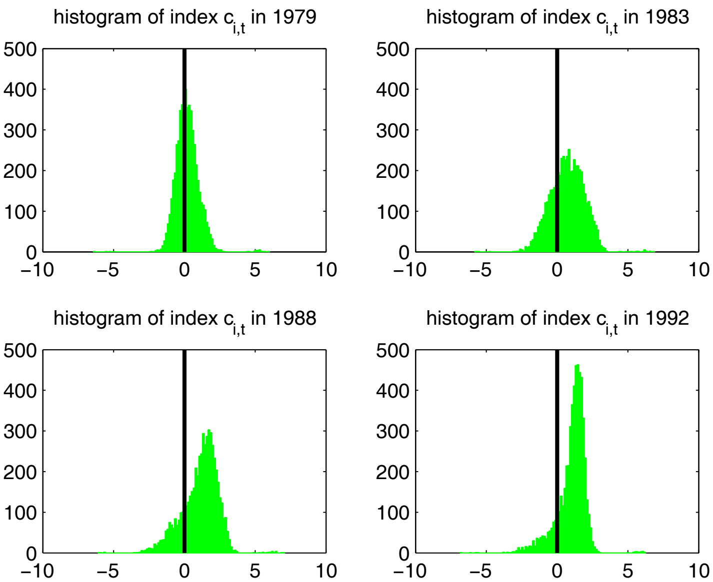

Figure 11: **The distribution of the index c_{it} at various dates**: 1979, 1983, 1988, 1992. The figure illustrates how the distribution fans out and becomes skewed over time.

and we measure the contribution of marginal workers to aggregate fluctuations as their share in the variance, i.e.:

$$\frac{Cov\left(\Delta\widetilde{N}_t, \Delta\widetilde{N}_t^{\text{marginal workers}}\right)}{V\left(\Delta\widetilde{N}_t\right)}.$$

Over 1979-92, the group of marginal workers (who represents 21.8% of the population on average) accounts for 48.6% of aggregate fluctuations in employment. Marginal workers also account for 32.6% of transitions between employment and non-employment. The c_{it} we measure is only an estimate of the latent index, before current shocks are realized. Therefore the variance of transitory shocks – which is approximately as big as the variance of c_{it} – introduces some 'noise' and implies that our marginal workers do not account for 100% of the variance of aggregate employment.

As explained above, the degree of persistence of being marginal has no direct effect on the aggregate elasticity. Hence, these calculations about marginal workers are really a joint test of the indivisible labor model (with complete markets) and a particular process for tastes and abilities: fixed effect plus i.i.d. To test our joint hypothesis more precisely, we simulate data from the estimated model and apply the same procedure to define marginal workers. The model predicts a share of variance of aggregate employment accounted for by marginal workers of 45.8%. This is close to that obtained from the data (48.6%). The model predicts however that marginals do more transitions (41.5%) than they do in the data (32.6%).

Table 6 presents some summary statistics on the population of marginal workers. Marginal workers are less experienced, slightly less schooled, and are often women with young children. Marginal workers are more likely to feel constrained by health in their choice of work.

Means	whole population	marginal workers
experience (yrs)	10.15	7.58
schooling (yrs)	13.29	12.74
gender (male=1)	0.46	0.20
marital Male	0.26	0.05
marital Female	0.34	0.61
health limit	0.07	0.15
kid 0-2 (% of total pop that are women with kid-0-2)	0.10	0.32
kid 3-6	0.14	0.24
kid 7-14	0.13	0.17

Table 6: Summary statistics for the population of marginal workers and for the whole population in December 1991

5 Conclusion

This paper makes two points. First, agents differ in their individual elasticity of labor supply: only a limited fringe of agents react to aggregate shocks. Second, the aggregate elasticity of labor supply is related to the homogeneity of the workforce at the margin, i.e. to the number of "marginal workers": these marginal workers are nearly indifferent between work and leisure at a given point in time, hence fluctuations in the aggregate wage drive them in and out of the workforce.

We develop an empirical framework to measure the elasticity implied by this heterogeneity. Our estimate for the Frisch elasticity of aggregate labor is about 1.5 over our whole sample.

This elasticity varies over the life cycle and over the business cycle: it is countercyclical.

One test of the model is to identify some "marginal workers" and compute their contribution to aggregate fluctuations. Over 1979-1992, 22% of agents, the closest to the marginal worker, account for about 49% of aggregate fluctuations in employment. However, this number gives little information by itself on the importance of the mechanism or the aggregate labor supply. Further work will investigate in more detail the cross-sectional implications.

There are two natural directions in which to extend this work. First, it may seem important to improve the fit of the econometric model by allowing for persistent shocks or costs of job search (Altug and Miller 1998, Hyslop 1999). Second, a limitation of our approach is that maximum likelihood estimation requires to make distributional assumptions on the unobserved stochastic heterogeneity, even if we identify permanent heterogeneity nonparametrically with fixed effects. Because we estimate idiosyncratic shocks to be large, the hazard rates that give the Frisch elasticity inherit the gaussian shape of the i.i.d. shock. The assumptions we make are thus too strong. It would be interesting to measure the distribution of agents' wages and reservation wages around the marginal worker in more generality.

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6 Appendix

The first subsection of this Appendix describes how to derive a competitive equilibrium for the economy analyzed in section 2.

The second subsection discusses our dataset, and describes our treatment of the data, in greater detail than in sections 3 and 4.

A. Competitive Equilibrium Allocation

We consider an environment with complete markets, where idiosyncratic shocks are perfectly insured. Agents are born with wealth k_0 distributed according to H.

We introduce lotteries to handle the indivisibility in labor supply and obtain a convex problem, just as Rogerson (1988). Because preferences are separable in leisure and consumption, and because $u(c)$ is concave, we can restrict to allocations where consumption does not depend on the lottery outcome. Following Mulligan (2001), we can show that labor supply is deterministic with probability one.

Let $q_t(s^t) = q_t(\pi^t, \theta^t, z^t, d^t)$ denote the price of a contingent claim to one unit of consumption at date t after history s^t. (For simplicity, we restrict from the start to allocations where others' histories do not matter for the price of this contingent claim.) Let $w_t(z^t, d^t)$ denote the aggregate wage rate, and let $r_t(z^t, d^t)$ denote the aggregate rental rate of capital.

We first describe the agent's time zero problem, given his initial wealth k_0, and given sequences of prices $r_t(z^t, d^t)$, $w_t(z^t, d^t)$, $q_t(s^t)$:

$$V(k_0) = \max_{\{c_t(\cdot), \alpha_t(\cdot), k_{t+1}(\cdot)\}} \sum_{t \geq 0} \beta^t \int_{S^t} \left[u\left(c_t\left(s^t, k_0\right)\right) - \theta_t d_t v(\overline{n}) \alpha_t\left(s^t, k_0\right) \right] dP_s^t\left(s^t\right), \qquad (P(k_0))$$

such that the following intertemporal budget constraint holds:

$$0 \geq \sum_{t \geq 0} \int_{S^t} q_t\left(s^t\right) \left[\begin{array}{c} c_t\left(s^t, k_0\right) - \overline{n}\pi_t w_t\left(z^t, d^t\right) \alpha_t\left(s^t, k_0\right) + k_{t+1}\left(s^t, k_0\right) \\ -\left(1 - \delta + r_t\left(z^t, d^t\right)\right) k_t\left(s^{t-1}, k_0\right) \end{array} \right] ds^t \qquad (\eta(k_0))$$

This is a convex problem, hence the agents' optimal decision rules can be characterized from the first-order conditions. For individual capital holdings, we find

$$-q_t\left(s^t\right) + \int_{s_{t+1}} \left(1 - \delta + r_{t+1}\left(z^{t+1}, d^{t+1}\right)\right) q_{t+1}\left(s^{t+1}\right) ds_{t+1} = 0$$

And for individual consumption, we obtain

$$p_s^t\left(s^t\right) u'\left(c_t\left(s^t, k_0\right)\right) = \eta\left(k_0\right) q_t\left(s^t\right),$$

and we use the aggregation result that $u'\left(c_t\left(s^t, k_0\right)\right) = \eta\left(k_0\right) \lambda_t\left(z^t, d^t\right)$, where λ depends only on aggregate shocks. Equivalently, $q_t\left(s^t\right) = p_s^t\left(s^t\right) \lambda_t\left(z^t, d^t\right)$: the price of a contingent claims equals the probability of the corresponding state s^t times a price that depends only on aggregate shocks, $\lambda_t\left(z^t, d^t\right)$.

Finally, for individual labor supply, i.e. for the choice of the lottery $\alpha_t\left(s^t, k_0\right)$,

$$\eta\left(k_0\right) q_t\left(s^t\right)\left[\overline{n}\pi_t w_t\left(z^t, d^t\right)\right] - p_s^t\left(s^t\right)\theta_t d_t v\left(\overline{n}\right) \begin{cases} > 0 \text{ if } \alpha_t\left(s^t, k_0\right) = 1, \text{ that is, } n\left(s^t, k_0\right) = \overline{n}, \\ = 0 \text{ if } \alpha_t\left(s^t, k_0\right) \in (0,1), \\ < 0 \text{ if } \alpha_t\left(s^t, k_0\right) = 0, \text{ that is, } n\left(s^t, k_0\right) = 0. \end{cases}$$

At this point, it is easy to see that there is only a measure zero of agents whose labor supply is actually randomized (just as we do in Section 2). So finally, the relevant decision rule is to work, i.e. set $n\left(s^t, k_0\right) = \overline{n}$, if and only if

$$\begin{aligned} \frac{q_t\left(s^t\right)}{p_s^t\left(s^t\right)} \frac{w_t\left(z^t, d^t\right)}{d_t} &= \frac{\theta_t}{\pi_t \eta\left(k_0\right)} \frac{v\left(\overline{n}\right)}{\overline{n}} \\ \text{that is, } \frac{\lambda_t\left(z^t, d^t\right) w_t\left(z^t, d^t\right)}{d_t} &= \frac{\theta_t}{\pi_t \eta\left(k_0\right)} \frac{v\left(\overline{n}\right)}{\overline{n}} \end{aligned}$$

which amounts to the decision rule for employment in Section 2, with $\mu = \eta\left(k_0\right)^{-1}$: as usual in Negishi aggregation, Pareto weights are the inverse of agents' Lagrange multipliers on their intertemporal budget constraint.

We now describe the production side: the model is closed with a neoclassical production function $F(K, zN)$, where zN is aggregate labor in efficiency units. This technology is operated by a single firm, without loss of generality since F has constant returns to scale. That is, in each aggregate state $\left(z^t, d^t\right)$,

$$z_t N_t\left(z^t, d^t\right) = \int_{\mathbb{R}} \int_{\mathbb{R}^{++}} \int_{\mathbb{R}^{++}} \pi_t n\left(\pi^t, \theta^t, z^t, d^t, k_0\right) dp_\pi\left(\pi_t\right) dp_\theta\left(\theta_t\right) dH\left(k_0\right),$$

hence the following resource constraint in each aggregate state of the world $\left(z^t, d^t\right)$:

$$\begin{aligned} & \int_{\mathbb{R}} \int_{\mathbb{R}^{++}} \int_{\mathbb{R}^{++}} c\left(\pi^t, \theta^t, z^t, d^t, k_0\right) dp_\pi\left(\pi_t\right) dp_\theta\left(\theta_t\right) dH\left(k_0\right) + K_{t+1}\left(z^t, d^t\right) \\ \leq \; & F\left(K_t\left(z^{t-1}, d^{t-1}\right), z_t N_t\left(z^t, d^t\right)\right) + (1-\delta) K_t\left(z^{t-1}, d^{t-1}\right) \end{aligned}$$

Definition 1 *A competitive equilibrium is a set of decision rules* $\{c_t(s^t, k_0), \alpha_t(s^t, k_0)\}$ *and a set of prices* $\{w_t(z^t, d^t), q_t(s^t)\}$ *such that:*

1. *the decision rules solve each agent's problem* $P(k_0)$*,*

2. *the labor market, the rental market for capital, and the market for consumption claims clear in each aggregate state of the world* (z^t, d^t)*.*

In particular, market clearing for labor implies

$$w_t\left(z^t, d^t\right) = \frac{\partial}{\partial N_t} F\left(K_t\left(z^{t-1}, d^{t-1}\right), z_t N_t\left(z^t, d^t\right)\right).$$

Existence of a competitive equilibrium is guaranteed in this convex economy. Moreover, we just showed that the marginal product of labor and employment are the same as in the Pareto Optimal allocation described in Section 2.

B. Our dataset

Our empirical exercise uses panel data from the National Longitudinal Survey of Youth 1979 (NLSY 79). The sample consists of a cohort of men and women, born between 1957 and 1964, surveyed annually from 1979 until 1994, and then every other year until now.

These data provide a detailed account of each individual's work history, including precise dates for each employment spell, with the associated wage rate and hours worked. In addition, detailed demographic information is presented. This dataset is extremely valuable for our purpose, especially for the availability of high frequency data: we take the unit of time to be one month, where it seems reasonable to focus on the extensive margin. In related work, we studied the PSID, where annual hours of work are reported, which makes it less appropriate in our framework (In earlier work, we developped a calibration exercise, matching moments of annual data on hours worked, compargin them to time-aggregated predictions of our model).

Our sample only excludes the military supplements and the supplements for poor whites.[17] We analyze the period between January 1979 and December 1992: hence $T = 168$, for $N = 5571$ agents. We use sampling weights as provided by the NLSY.

[17] That is, we include the following subsamples: cross male white, cross male white poor, cross male black, cross male hispanic, cross female white, cross female white poor, cross female black, cross female Hispanic, supplement male black, supplement male Hispanic, supplement female black, supplement female hispanic.

Our employment variable is constructed from weekly labor force status (employed, unemployed, or out of the labor force). Monthly labor force status is set as "employed" if the respondent was employed in any week of this month. Labor force status is interpreted as $n_{it} = 1$ if employed, and $n_{it} = 0$ if unemployed or out of the labor force. We balance our sample according to this employment measure.

Data on wages w_{it} come from information on hourly wage rates for up to five jobs for each survey year. Matching this information with labor force status is sometimes difficult and results in missing wage data (w_{it} is missing for approximately 15% of observations with $n_{it} = 1$). Moreover, the quality of wage data seems to deteriorate after 1993 (when the survey methodology changed for these questions), which is why we restricted to 1979-1992. In the case of simultaneous jobs, we weight wage rates by average hours worked at each job. Information on hours worked is not used otherwise.

The construction of most observable covariates $\mathbf{x}_{it}, \mathbf{y}_{it}$ is straightforward from the data that are available online. We constructed experience by summing cumulated months worked at each point in time. The highest grade achieved at school is given in the dataset. The age of a respondent's youngest kid can be constructed very accurately from the data.

Our empirical specification of wages and participation thus conforms to the general practice in labor economics. In their study on married women. Heckman and MaCurdy (1980, 1982) have a participation equation with the number of children and the number of children less than six years old, family income excluding the wife's earnings, the wife's age, her husband's hours unemployed, and whether he is retired or disabled. Hyslop (1999) includes race, age of youngest child, nonlabor income, marital status as well.