Some Resource Economics of Invasive Species

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Abstract: An emerging problem for environmental policy is how to design efficient strategies for the prevention and control of invasive species. However, the literature has mostly focused either on pre-introduction prevention or post-introduction control of an invasive. The benefits of prevention cannot be understood or estimated without knowing the costs of post-introduction control. This paper provides an integrated framework where optimal prevention is combined with optimal pest removal.

Keywords: Invasive species, optimal prevention, integrated biological management, optimal stochastic control

1. Introduction:

Forest resources, especially tropical forests, are at risk of invasion by exotic species, often of an irreversible nature. An emerging problem for environmental policy is how to design efficient strategies for the prevention and control of invasive species. However, most of the literature has focused either on pre-introduction prevention or post-introduction control (see e.g., Carter et al., 2004; Eiswerth and Johnson, 2002; Horan et al., 2002; Kaiser and Roumasset, 2004; Olson 2004; Olson and Roy, 2002; Settle and

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Shogren, 2002; Perrings and Dalmazzone, 2000). However, the benefits of prevention cannot be understood or estimated without knowing the costs of post-introduction control. In order to provide an integrated framework, we solve for optimal prevention in a model wherein the probability of invasion can be reduced by prevention expenditures and with optimal pest removal determining the damage costs of invasion. We first consider a general model and then add some restrictions in order to facilitate the analysis.

2. The Model

Let N_0 be the initial invasive population (ranging from 0 to $N_{\max}$), N_t be the stock of the invasive at time t, $g(N_t)$ be the growth rate of the stock, $D(N_t)$ be the resulting damage at time t, $C(N_t, x_t)$ be the cost[1] of harvesting x_t from the stock, y_t be the prevention expenditures at time t, and r be the discount rate. The stock changes according to the stochastic equation $dN_t = [g(N_t) - x_t + \rho(y_t)]dt + \sigma(y_t)dz$, where $\rho(y_t)$ is the mean entry of the invasive at time t, $\sigma(y_t)$ represents the variability of entry, and dz is an increment of a Brownian motion stochastic process (z). We can assume

$\rho(y_t) > 0, \rho'(y_t) < 0,$ for $y_t \in [0, y_{\max}),$ and $\rho(y_{\max}) = 0$ so that at the maximum prevention level, mean entry is zero. Similarly, we can assume

$\sigma(y_t) \geq 0, \sigma'(y_t) < 0,$ for $y_t \in [0, y_{\max}]$ so that the uncertainty about the entry is reduced

[1] When an invasive population is introduced, its stock causes damage, and its removal incurs costs. Optimal removal minimizes the present value of damages and control costs. One important feature of invasive control is the search needed to find the invasive before it can be removed. Cost of the search usually depends on the invasive stock. The greater the size of the stock in a particular area, the easier it is to find it and hence lower the search costs. Therefore, total control costs including search cost depend on stock level at any time. This feature has been ignored in some studies of optimal invasive control, which assume that control costs depend only on the amount of removal (see, e.g., Eiswerth and Johnson, 2002; Olson and Roy, 2002).

with greater prevention. Our objective is to minimize the present value of the costs of control and damage as follows:

$$\text{V}(N_0) = \underset{x_t, y_t}{\text{Min}} \int_0^\infty e^{-rt} \left[D(N_t) + C(N_t, x_t) + y_t \right] dt$$

subject to:

$$dN_t = \left[g(N_t) - x_t + \rho(y_t) \right] dt + \sigma(y_t) dz, \quad 0 \le N_0 \le N_{\max} \qquad \ldots(1)$$

The optimality condition is (following Stengel, 1994; Kamien and Schwartz, 1991):

$$rV(N_t)dt = \underset{x_t, y_t}{Min} \left\{ \left[D(N_t) + C(N_t, x_t) + y_t \right] dt + EdV \right\} \qquad \ldots (2)$$

This yields the first order conditions:

$$C_x(N_t, x_t)dt = -\frac{\partial EdV}{\partial x_t} \qquad \ldots (3)$$

$$1 \cdot dt = -\frac{\partial EdV}{\partial y_t} \qquad \ldots (4)$$

The first equation (3) indicates that at the optimum, the cost of removing a unit of the invasive, incurred for an instant *dt*, must equal the resulting reduction in the expected instantaneous change in the value of the objective function. Further insight may be obtained by using Itô's lemma to re-write (2) as:

$$rV(N_t) = \underset{x_t, y_t}{Min} \left\{ D(N_t) + C(N_t, x_t) + y_t + V'(N_t) \left[g(N_t) - x_t + \rho(y_t) \right] + 1/2 \sigma^2(y_t) \cdot V''(N_t) \right\} \ldots(5)$$

The first order condition (3) is now:

$$C_x(N_t, x_t) = V'(N_t) \qquad \ldots (6)$$

The R.H.S. of (6) is the reduction in the present value of future damage and control costs due to a smaller stock (N_t). Equation (4) has a similar interpretation. The L.H.S. in (4) is the instantaneous cost of increasing prevention expenditures by one unit, and the R.H.S. is the reduction in the expected instantaneous change in the value of the objective function.

A policy maker, using specific functional forms and calibrations for $D(.), C(.), g(.), \rho(.), \sigma(.)$, can solve (3) or (6), and (4). Using the solutions, one can solve the differential equation (5) for $V(N_t)$. Once the function $V(N_t)$ has been obtained, optimizing control laws ϕ_1, ϕ_2 such that $x_t = \phi_1(N_t), y_t = \phi_2(N_t)$ can be obtained from (3) or (6), and (4). The solution is usually complicated and its limited tractability affords few insights into the inherent tradeoffs involved. Next, we simplify the framework in order to gain an understanding of the optimal decision problem without eliminating the crucial interdependence of prevention and control.

2.1. Simplified Model

The problem of an invasive can be thought of as the problem of a natural resource stock that is introduced in an area, grows, causes damage, and can be partially or wholly removed. It can be imperfectly prevented from entry by appropriate measures. If prevention fails and introduction occurs, the stock can be harvested according to an optimal control program that leads to a steady-state stock level.

Optimal management of such a stock would require one to choose an optimal path of control (harvest) to minimize the control costs and the damage from the invasive. Depending on the initial stock level, the cost of control, and the damages from the stock, the optimal path may entail doing nothing (zero control level) and letting the stock grow to its carrying capacity (or natural steady state), eradicating the stock completely (zero stock level), or achieving a steady state with a positive control and stock level. Minimized control and damage costs are obtained from this solution.

Once the control problem has been solved, it needs to be embedded in the optimal prevention problem. Optimal prevention minimizes the expected present value of prevention costs and the minimized control and damage costs determined in the control problem.

We first examine the control of an invasive that has already arrived. It can be wholly or partially removed or left alone. The objective of management is to minimize the costs of control and damage from the invasive. To this end, a social planner chooses the optimal harvest path leading to a steady-state population level, which may be zero or greater. We then examine prevention before the introduction of the invasive. Prevention efforts are meant to reduce the probability of an introduction. The costs associated with the optimal control path are the costs resulting from prevention failure. The social planner chooses a prevention level to minimize the expected costs of prevention and prevention failure.

2.2. Optimal "Harvest"

As before, suppose a certain population of an invasive (N_0) is introduced (ranging from 0 to $N_{\max}$). Let N_t be the stock of the invasive at time *t,* g(N_t) be the growth rate of the stock, $D(N_t)$ be the resulting damage at time *t*, $C(N_t, x_t)$ be the cost of harvesting x_t from the stock, *r* be the discount rate. However, uncertainty will be in the effects of prevention; there is no uncertainty in control now. Then we minimize the present value of the costs of control and damage as follows:

$$\underset{x_t}{\text{Min}} \quad \text{V}, \quad \text{where V} = \int_0^{\infty} e^{-rt}\left[D(N_t) + C(N_t, x_t)\right]dt \qquad \text{.....(7)}$$

subject to:

$$\dot{N}_t = g(N_t) - x_t, \quad 0 \le N_0 \le N_{\max}$$

The Maximum principle (Pontryagin et al., 1962) provides the following Hamiltonian and first-order necessary conditions:

$$H = \left[-D(N_t) - C(N_t, x_t)\right] + \lambda_t\left[g(N_t) - x_t\right] \qquad \text{.....(8)}$$

$$\frac{\partial H}{\partial x_t} = -C_{x_t}(N_t, x_t) - \lambda_t \le 0, \qquad \frac{\partial H}{\partial x_t} x_t = 0 \qquad \text{.....(9)}$$

$$\frac{\partial H}{\partial N_t} = -D_{N_t}(N_t) - C_{N_t}(N_t, x_t) + \lambda_t g_{N_t}(N_t) = r\lambda_t - \dot{\lambda}_t \qquad \text{.....(10)}$$

$$\frac{\partial H}{\partial \lambda_t} = g(N_t) - x_t = \dot{N}_t \qquad \text{.....(11)}$$

Manipulation of the above conditions yields the following equation of motion for control:

$$\dot{x}_t = -\frac{1}{(g_N - r)C_{xx}}\left[D_N + C_N + (g_N - r)C_x + (g - x)C_{xN}\right] \qquad \text{.....(12)}$$

Equations (11) and (12) specify the necessary conditions for optimal state and control paths over time. Steady state population, N^* and harvest rate, x^*, can be obtained by setting $\dot{x}_t = \dot{N}_t = 0$. The resulting condition is:

$$r \cdot C_x(N^*, x^*) - C_N(N^*, x^*) - g_N(N^*) \cdot C_x(N^*, x^*) = D_N(N^*) \qquad .....(13)$$

This equates the one-period opportunity cost of harvesting a unit of stock ($r\ C_x$>0), the cost increase ($-C_N$>0) due to stock reduction by one unit, and the increase (decrease) in cost ($-g_N C_x$) due to the resulting increased (decreased) growth, on the L.H.S. with the resulting benefit of reduced damage (D_N >0) on the R.H.S. Depending on the costs and damages, the value of N^* may be positive or zero (implying that eradication of the invasive is optimal). The above conditions provide the optimal time paths of N_t and x_t that minimize V. We denote the minimized value of V for a given introduced population (N_0) by $V^*(N_0)$. Next, we imbed this optimal control solution in the optimal prevention problem to determine the efficient level of prevention expenditures.

2.3. Optimal Prevention

Let the prevention expenditure in period t be y, and the resulting probability of an introduction of size N_0 in period $t+1$ be $p(N_0, y)$. If an introduction of size N_0 occurs, the optimal control program derived in the previous section is followed, i.e., a total cost of

$V^*(N_0)$ is incurred. Thus, the expected value of the costs and damages is

$$EV(y) = \sum_{N_0=0}^{N_{max}} \left[p(N_0, y) \cdot V^*(N_0) \right] \qquad .....(14)$$

and $$EV'(y) = \sum_{N_0=0}^{N_{max}} \left[p_y(N_0, y) \cdot V^*(N_0) \right] \qquad .....(15)$$

We assume that $p(N_0, y)$ is such that $EV'(y) < 0, EV''(y) > 0$, and that the control problem for each introduction is independent of every other introduction.[2] The effect of prevention (*y*) at time *t=0*, is that, at time *t=1,* we incur the expected present value of *EV(y).* Similarly, the effect of prevention (*y*) at time *t=1*, is that, at time *t=2,* we incur the expected present value of *EV(y),* and so on. The problem of prevention in each period and at each branch is identical as shown in Figure 1.

[2] This would be the case if each introduction takes place at a spatially distinct location (e.g., a pair of brown tree snakes entering Oahu in Hawaii might reside and start a population in Schofield while another might do the same in Dillingham area). Alternatively, damages, and growth might be approximately linear in stock. This ensures that a new introduction does not modify a control regime already underway to mitigate a previous introduction.

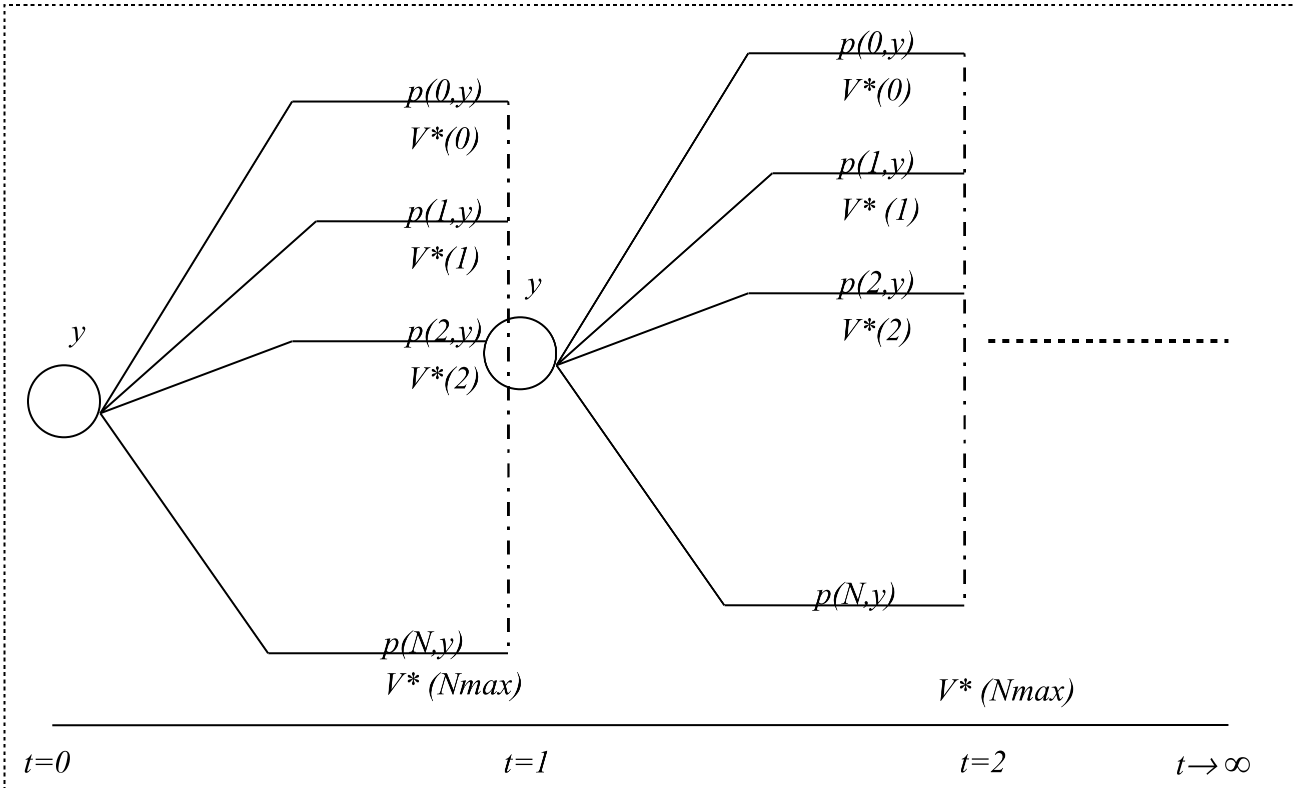

Fig.1: Prevention (y) with given control costs (V^*)
(To avoid clutter, the figure shows only one branch expanded from *t=1* to *t=2*. All the other branches would expand identically.)

The expected present value (W) of prevention, control, and damage costs is given by:

$$W = \left(EV + [1+r]\, y \right) / r \qquad .....(16)$$

Choosing y to minimize W yields the first-order condition:

$$\frac{\partial W}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}\left[\left(EV + [1+r]\, y \right) / r \right] = 0 \qquad .....(17)$$

This gives us the following condition[3] for optimal y (denoted y^*):

[3] Since $EV'(y) < 0, EV''(y) > 0$, the second order condition ($\frac{\partial^2 W}{\partial y^2} > 0$) is also met.

$$- \frac{EV'(y^*)}{(1+r)} = 1 \qquad .....(18)$$

The L.H.S. is the reduction in the expected present value of control and damage costs due to a unit increment in prevention expenditures and the R.H.S. is the unit increment in prevention expenditures. Differentiating (18) w.r.t *r* yields:

$$- EV''(y^*)\frac{\partial y^*}{\partial r} = 1 \qquad .....(19)$$

Thus, the optimal prevention expenditures (y^*) increase as the interest rate (r) falls. Also, using (14), (18) can be re-written as:

$$\sum_{N_0=0}^{N_{max}} \left[p_y(N_0, y^*) \cdot V^*(N_0) \right] = -(1+r) \qquad .....(20)$$

That is, y^* must increase if $-p_y(N_0, y)$ increases for all y (probability becomes more sensitive to prevention) or if $V^*(N_0)$ increases for all N_0. Thus, the optimal prevention expenditures increase with increasing effectiveness prevention and with increasing control and damage costs.

2.4. When first introduction is all that matters

So far, we have discussed the case where each introduction causes damages and costs depending on the size of the entering population. In some cases, once an introduction has occurred, further entry does not matter. If the first-time introduction and its control path involve large enough population levels, then further entry may have negligible effects on concave damages. This case requires modifying the above analysis as follows.

If there is non-zero entry in a period, we stop prevention and simply incur the minimized present value of damage and control costs derived previously (V^*). In reality, entering population of different sizes is possible and we would incur an expected value similar to (14). However, since in the present case we are concerned only with large invasions, we do not distinguish entering population sizes and assume all the costs arising from entry are built into V^*. Let the prevention expenditure in each period be y, and the resulting probability of introduction be $p(y)$. If there is no introduction, we continue to spend on prevention. The resulting infinite probability tree is given in Figure 2.

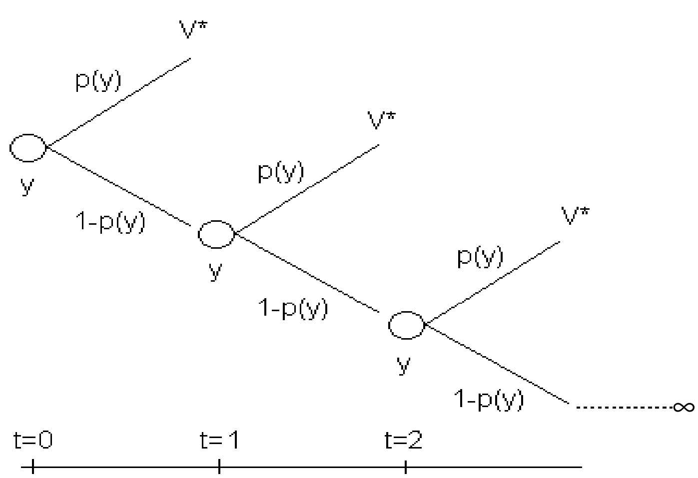

Fig.2: Prevention (y) with given minimized damage and control costs (V^*)

The expected present value of prevention and control costs (including damage) is:

$$W = \left(\frac{p(y)\, V^* + [1+ r]\, y}{r + p(y)} \right) \qquad .....(21)$$

We choose *y* to minimize *W*. The first-order condition is:

$$\frac{\partial W}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}\left[\left(\frac{p(y)\, V^* + [1+ r]\, y}{r + p(y)} \right)\right] = 0 \qquad .....(22)$$

This gives us the following condition[4] for optimal *y*:

$$(1+ r)\left[r + p(y)\right] + \left[r\, V^* - (1+ r)y\right]\, p'(y) = 0 \qquad .....(23)$$

Using the definition of *W* from (9) and re-arranging, we get:

$$\underbrace{-\frac{p'(y)\, V^*}{(1+ r)}}_{\text{MB of prevention}} = 1 + \underbrace{\frac{1}{(1+ r)}\frac{\partial}{\partial y}\{[1- p(y)]\, W\}}_{\text{MC due to increased probability of further prevention and control}} \qquad .....(24)$$

This implies that if the control and damage costs in the case of introduction were large (large V^*), optimal prevention expenditures (*y*) would also be large (ceteris paribus). Similarly, the more sensitive the probability of introduction ($p'(y)$) is to prevention expenditures, the bigger the expenditures. The prevention expenditures would also be bigger, the smaller the interest rate (*r*) is.

3. Conclusion

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[4] For a minimum, we also require $\frac{\partial^2 W}{\partial y^2} > 0$. Some manipulation of this condition in combination with (22) shows that it will be met if $p'' > 0$.

We provide a framework to combine optimal pre-introduction prevention and post-introduction control of invasive species. Optimal prevention depends on the minimized costs that would result when prevention fails to stop an invasion, including damage costs. Higher control and/or damage costs required after the species is introduced result in higher optimal prevention expenditures. Similarly lower interest rates and greater prevention effectiveness also increase optimal prevention expenditures. The stochastic optimal control model discussed at the beginning of section 2 can be used as the basis of numerical studies.

References

Carter, C., J. Chalfant, R. Goodhue, and G. Mckee "A Chemical Resistant Invader: Greenhouse Whitefly in Strawberries." University of California-Davis, 2004. http://www.farmfoundation.org/projects/documents/carterppt.ppt. Date accessed: 10/22/04.

Eiswerth, Mark E. and Wayne S. Johnson, "Managing Nonindigenous Invasive Species: Insights from Dynamic Analysis," *Environmental and Resource Economics* 23: 319–342, 2002.

Hanley, N., J. F. Shogren, and B. White. *Environmental economics in theory and practice.* New York, NY: Oxford University Press. 1997.

Horan, Richard D; Charles Perrings, Frank Lupi, and Erwin H. Bulte "Biological Pollution Prevention Strategies under Ignorance: The Case of Invasive Species." *American Journal of Agricultural Economics*, vol. 84, no. 5, 2002, pp. 1303-10.

Kaiser, Brooks, and James Roumasset "Integrating Prevention and Control of Invasive Species: Lessons from Hawaii." USDA workshop. August 2004. http://www.ers.usda.gov/briefing/invasivespecies/wkshopagenda804.htm, Date accessed: 10/22/04.

Kamien, M. I. and N. L. Schwartz. *Dynamic Optimization*. New York, NY: Elsevier Science. 1991. Sec. 22.

Olson, Lars J; Roy, Santanu "The Economics of Controlling a Stochastic Biological Invasion." *American Journal of Agricultural Economics*, vol. 84, no. 5, 2002, pp. 1311-16.

Olson, L. "Randomly Introduced Biological Invasions: The Economics of Prevention and Control." University of Maryland Working Paper. 2004.

Pontryagin, L. S., V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko. *The Mathematical Theory of Optimal Processes*, (Translated by K. N. Trirogoff, L. W. Neustadt, ed.), Hoboken, NJ: John Wiley. 1962.

Settle, Chad; Shogren, Jason F. "Modeling Native-Exotic Species within Yellowstone Lake" *American Journal of Agricultural Economics*, vol. 84, no. 5, 2002, pp. 1323-28.

Stengel, R. Optimal Control and Estimation (Dover Books on Advanced Mathematics). Courier Dover Publications. 1994. Sec. 5.

Perrings C., Williamson M., and Dalmazzone S. (eds). *The Economics of Biological Invasions.* Cheltenham: Edward Elgar. 2000.