

CAPITAL THEORY AND INVESTMENT BEHAVIOR*

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Introduction

There is no greater gap between economic theory and econometric practice than that which characterizes the literature on business investment in fixed capital. According to the neoclassical theory of capital, as expounded for example by Irving Fisher, a production plan for the firm is chosen so as to maximize utility over time. Under certain well-known conditions this leads to maximization of the net worth of the enterprise as the criterion for optimal capital accumulation. Capital is accumulated to provide capital services, which are inputs to the productive process. For convenience the relationship between inputs, including the input of capital services, and output is summarized in a production function. Although this theory has been known for at least fifty years, it is currently undergoing a great revival in interest. The theory appears to be gaining increasing currency and more widespread understanding.

By contrast, the econometric literature on business investment consists of *ad hoc* descriptive generalizations such as the "capacity principle," the "profit principle," and the like. Given sufficient imprecision, one can rationalize any generalization of this type by an appeal to "theory." However, even with the aid of much ambiguity, it is impossible to reconcile the theory of the econometric literature on investment with the neoclassical theory of optimal capital accumulation. The central feature of the neoclassical theory is the response of the demand for capital to changes in relative factor prices or the ratio of factor prices to the price of output. This feature is entirely absent from the econometric literature on investment.

It is difficult to reconcile the steady advance in the acceptance of the neoclassical theory of capital with the steady march of the econometric literature in a direction which appears to be diametrically opposite. It is true that there have been attempts to validate the theory. Both profits and capacity theorists have tried a rate of interest here or a price of investment goods there. By and large these efforts have been unsuccessful; the naïve positivist can only conclude, so much the worse for the theory. I believe that a case can be made that previous attempts to "test" the neoclassical theory of capital have fallen so far

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short of a correct formulation of this theory that the issue of the validity of the neoclassical theory remains undecided. There is not sufficient space to document this point in detail here; but I will try to illustrate what I would regard as a correct formulation of the theory in what follows.

Stated baldly, the purpose of this paper is to present a theory of investment behavior based on the neoclassical theory of optimal accumulation of capital. Of course, demand for capital is not demand for investment. The short-run determination of investment behavior depends on the time form of lagged response to changes in the demand for capital. For simplicity, the time form of lagged response will be assumed to be fixed. At the same time a more general hypothesis about the form of the lag is admitted than that customary in the literature. Finally, it will be assumed that replacement investment is proportional to capital stock. This assumption, while customary, has a deep justification which will be presented below. A number of empirical tests of the theory is presented, along with an analysis of new evidence on the time form of lagged response and changes in the long-run demand for capital resulting from changes in underlying market conditions and in the tax structure.

Summary of the Theory

Demand for capital stock is determined to maximize net worth. Net worth is defined as the integral of discounted net revenues; all prices, including the interest rate, are taken as fixed. Net revenue is defined as current revenue less expenditure on both current and capital account, including taxes. Let revenue before taxes at time t be $R(t)$, direct taxes, $D(t)$, and r the rate of interest. Net worth, say W , is

$$W = \int_0^{\infty} e^{-rt} [R(t) - D(t)] dt.$$

We will deduce necessary conditions for maximization of net worth for two inputs—one current and one capital—and one output. The approach is easily generalized to any number of inputs and outputs.

Let p be the price of output, s the wage rate, q the price of capital goods, Q the quantity of output, L the quantity of variable input, say labor, and I the rate of investment; net revenue is

$$R = pQ - sL - qI.$$

Let u be the rate of direct taxation, v the proportion of replacement chargeable against income for tax purposes, w the proportion of interest,

and x the proportion of capital losses chargeable against income; where K is capital stock and δ the rate of replacement, direct taxes are

$$D = u[pQ - sL - (v\delta q + wrq - xq)K]$$

Maximizing net worth subject to a standard neoclassical production function and the constraint that the rate of growth of capital stock is investment less replacement, we obtain the marginal productivity conditions

$$\frac{\partial Q}{\partial L} = \frac{s}{p},$$

$$\frac{\partial Q}{\partial K} = \frac{q \left[\frac{1-uv}{1-u} \delta + \frac{1-uw}{1-u} r - \frac{1-ux}{1-u} \frac{\dot{q}}{q} \right]}{p}.$$

The numerator of the second fraction is the "shadow" price or implicit rental of one unit of capital service per period of time. We will call this price the user cost of capital. We assume that all capital gains are regarded as "transitory," so that the formula for user cost, say c , reduces to

$$c = q \left[\frac{1-uv}{1-u} \delta + \frac{1-uw}{1-u} r \right].$$

Second, we assume that output and employment on the one hand and capital stock on the other are determined by a kind of iterative process. In each period, production and employment are set at the levels given by the first marginal productivity condition and the production function with capital stock fixed at its current level; demand for capital is set at the level given by the second marginal productivity condition, given output and employment. With stationary market conditions, such a process is easily seen to converge to the desired maximum of net worth. Let K^* represent the desired amount of capital stock, if the production function is Cobb-Douglas with elasticity of output with respect to capital, γ ,

$$K^* = \gamma \frac{pQ}{c}.$$

We suppose that the distribution of times to completion of new investment projects is fixed. Let the proportion of projects completed in time τ be w_τ . If investment in new projects is I_t^E and the level of starts of new projects is I_t^N , investment is a weighted average of past starts:

$$I_t^E = \sum_{\tau=0}^{\infty} w_{\tau} I_{t-\tau}^N = w(L) I_t^N,$$

where $w(L)$ is a power series in the lag operator, L . We assume that in each period new projects are initiated until the backlog of uncompleted projects is equal to the difference between desired capital stock, K_t^* , and actual capital stock, K_t :

$$I_t^N = K_t^* - [K_t + (1 - w_0) I_{t-1}^N + \dots],$$

which implies that:

$$I_t^E = w(L) [K_t^* - K_{t-1}^*].$$

It is easy to incorporate intermediate stages of the investment process into the theory. For concreteness, we consider the case of two intermediate stages, which will turn out to be anticipated investment, two quarters hence, and anticipated investment, one quarter hence. A similar approach can be applied to additional intermediate stages such as appropriations or commitments. The distribution of completions of the first stage, given new project starts, may be described by a sequence, say $\{v_{0\tau}\}$; similarly, the distribution of completions of a second stage, given completion of the first stage, may be described by a sequence $\{v_{1\tau}\}$. Finally, the distribution of investment expenditures, given completion of a second intermediate stage is described by a sequence $\{v_{2\tau}\}$. Where $I_t^{S_1E}$ represents completions of the first stage, $I_t^{S_2E}$ completions of the second stage, and I_t^E actual investment, as before, we have:

$$\begin{aligned} I_t^{S_1E} &= \sum_{\tau=0}^{\infty} v_{0\tau} I_{t-\tau}^N = v_0(L) I_t^N, \\ I_t^{S_2E} &= \sum_{\tau=0}^{\infty} v_{1\tau} I_{t-\tau}^{S_1E} = v_1(L) I_t^{S_1E}, \\ I_t^E &= \sum_{\tau=0}^{\infty} v_{2\tau} I_{t-\tau}^{S_2E} = v_2(L) I_t^{S_2E}. \end{aligned}$$

where $v_0(L)$, $v_1(L)$, and $v_2(L)$ are power series in the lag operator.

Up to this point we have discussed investment generated by an increase in desired capital stock. Total investment, say I_t , is the sum of investment for expansion and investment for replacement, say I_t^R :

$$I_t = I_t^E + I_t^R.$$

We assume that replacement investment is proportional to capital

stock. The justification for this assumption is that the appropriate model for replacement is not the distribution of replacements for a single investment over time but rather the infinite stream of replacements generated by a single investment; in the language of probability theory, replacement is a recurrent event. It is a fundamental result of renewal theory that replacements for such an infinite stream approach a constant proportion of capital stock for (almost) any distribution of replacements for a single investment and for any initial age distribution of capital stock. This is true for both constant and growing capital stocks. Representing the replacement proportion by δ , as before,

$$I_t^R = \delta K_t;$$

combining this relationship with the corresponding relationship for investment in new projects, we have:

$$I_t = w(L)[K_t^* - K_{t-1}^*] + \delta K_t.$$

Using the assumption that capital stock is continued in use up to the point at which it is replaced, we obtain the corresponding relationships for gross investment at each of the intermediate stages, say $I_t^{S_1}$ and $I_t^{S_2}$:

$$\begin{aligned} I_t^{S_1} &= v_0(L)[K_t^* - K_{t-1}^*] + \delta K_t, \\ I_t^{S_2} &= v_1(L)v_0(L)[K_t^* - K_{t-1}^*] + \delta K_t; \end{aligned}$$

we can also derive the following;

$$\begin{aligned} I_t^{S_2} &= v_1(L)[I_t^{S_1} - \delta K_t] + \delta K_t, \\ I_t &= v_2(L)[I_t^{S_2} - \delta K_t] + \delta K_t, \\ I_t &= v_2(L)v_1(L)[I_t^{S_1} - \delta K_t] + \delta K_t. \end{aligned}$$

For empirical implementation of the theory of investment behavior, it is essential that each of the power series— $v_0(L)$, $v_1(L)$, $v_2(L)$ —have coefficients generated by a rational function; for example,

$$w(L) = v_2(L)v_1(L)v_0(L) = \frac{s(L)}{t(L)},$$

where $s(L)$ and $t(L)$ are polynomials. We will call the distribution corresponding to the coefficients of such a power series a rational power series distribution. The geometric and Pascal distributions are among the many special instances of the rational power series distribution.

Empirical Results

To test the theory of investment behavior summarized in the preceding section, the corresponding stochastic equations have been fitted to quarterly data for U. S. manufacturing for the period 1948-60. The data on investment are taken from the OBE-SEC Survey; first and second anticipations of investment expenditure as reported in that Survey are taken as intermediate stages.¹ With two intermediate stages, six possible relationships may be fitted. First, for actual investment and both intermediate stages, the level of investment is determined by past changes in desired capital stock. Second, investment is determined by past values at each intermediate stage and the second anticipation is determined by past values of the first anticipation. The first test of the theory is the internal consistency of direct and derived estimates of the coefficients of each of the underlying power series in the lag operator.

The results of the fitting are given in Table 1. For each of the fitted relationships coefficients of the polynomials $s(L)$ and $t(L)$ in the ex-

¹ Data on capital stock were obtained by interpolating the capital stock series for total manufacturing given in the U.S. national accounts between 1949 and 1959, using the formula

$$K_{t+1} = I_t + (1 - \delta)K_t.$$

Given an investment series, a unique value of δ may be determined from the initial and terminal values of capital stock. Investment data from the OBE-SEC Survey were used for the interpolation. For desired capital stock, the quantity pQ was taken to be sales plus changes in inventories, both from the *Survey of Current Business*. User cost depends on a number of separate pieces of data. The quantity q is an investment deflator, δ is, of course, a fixed parameter (taken to be equal to .025), r is the U.S. government long-term bond rate. The tax functions vary with time; as an example, the tax rate u is the ratio between corporate income tax payments and corporate profits before taxes as reported in the U.S. national accounts. A detailed description of the data underlying this study will be reported elsewhere.

² To derive the form of the functions used in the actual fitting, we take $v_2(L)v_1(L)V_0(L)$ as an example. First:

$$I_t = \frac{s(L)}{t(L)} [K_t^* - K_{t-1}^*] + \delta K_t.$$

Secondly,

$$I_t = s(L)[K_t^* - K_{t-1}^*] + [1 - t(L)][I_t - \delta K_t] + \delta K_t.$$

The coefficient t_0 may be normalized at unity so that:

$$1 - t(L) = -t_1L - t_2L^2 - \dots$$

The a priori value $\delta = .025$ was used to compute $I_{t-\tau} - \delta K_{t-\tau}$. An estimate of δ is given by the coefficient of K_t . If δ is different from its a priori value, the process of estimation can be reiterated, using a second approximation to the value of δ .

The parameter γ is estimated using the constraint:

$$\sum_{\tau=0}^{\infty} w_{\tau} = 1.$$

TABLE 1

REGRESSION COEFFICIENTS AND GOODNESS OF FIT STATISTICS, UNRESTRICTED ESTIMATES

Regression	γ_{s_0}	γ_{s_1}	γ_{s_2}	h_1	h_2	δ	R^2	s	Δ^2/s^2
$v_2 v_1 v_0$.00102 (.00049)	-1.51911 (.09945)	.63560 (.10098)	.02556 (.00163)	.94265	.10841	2.14039
$v_1 v_0$.00132 (.00073)		-1.25242 (.12667)	.36656 (.12977)	.02618 (.00240)	.89024	.16229	2.00431
v_0	.00109 (.00085)			-1.26004 (.13044)	.37281 (.13138)	.02549 (.00278)	.87227	.18974	2.37298
$v_2 v_1$.81357 (.03492)			.01962 (.00175)	.92729	.11955	1.16294
v_1		.90024 (.02722)				.02295 (.00127)	.96234	.08604	1.47693
v_0		.89462 (.03145)				.02337 (.00135)	.95121	.10597	1.70179

REGRESSION COEFFICIENTS AND GOODNESS OF FIT STATISTICS, RESTRICTED ESTIMATES

$v_2 v_1 v_0$.00106 (.00049)	-1.52387 (.09925)	.63100 (.10074)		.94156	.10830	2.10778
$v_1 v_0$.00133 (.00073)		-1.25704 (.12509)	.36769 (.12862)		.88986	.16087	2.00549
v_0	.00109 (.00084)			-1.26395 (.12942)	.37300 (.13051)		.87128	.18848	2.53442
$v_2 v_1$.82764 (.04037)				.89995	.13883	.87127
v_1		.91545 (.02933)					.95406	.09409	1.23560
v_0		.90271 (.03276)					.94538	.11100	1.53759

pression for each power series as a rational function are given.² For example, the power series $v_2(L)v_1(L)v_0(L)$ is expressed as:

$$v_2(L)v_1(L)v_0(L) = \frac{.00106L^2}{1 - 1.52387L + .63100L^2}$$

The value of the replacement proportion δ estimated from data on capital stock is .025. Two sets of regressions were run, one with δ fitted from the data (unrestricted), the other with $\delta = .025$ (restricted). Throughout, the coefficient of multiple determination R^2 , the standard error of estimate for the regression s , and the VonNeumann ratio Δ^2/s^2 are presented as measures of goodness of fit.

The first set of tests of the theory is the comparison of alternative estimates of each of the fundamental power series. As an example, one may take the hypothesis that the direct estimates of the power series $v_2(L)$ and $v_1(L)$, when combined, give an estimate of $v_2(L)v_1(L)$ which is close to that obtained by direct estimation. Using the unrestricted estimates, the result of this comparison is:

$$.91545L \odot .90271L = .82639L^2;$$

the derived estimate, which may be compared with the direct estimate, $.82764L^2$. The difference between the two estimates is slightly over .03 standard errors. A similar test of the hypothesis that the direct estimates of the power series $v_1(L)$ and $v_0(L)$, when combined, yield an estimate of $v_1(L)v_0(L)$ which is close to that obtained by direct estimation results in

$$.00109 \cdot \frac{.90271L}{1 - 1.26395L + .37300L^2} = \frac{.00098L}{1 - 1.26395L + .37300L^2},$$

which may be compared with the direct estimate,

$$\frac{.00133L}{1 - 1.25704L + .36769L^2}.$$

The coefficient of the numerator is within half a standard error of the derived estimate. The coefficients of the denominator are within .06 and .04 standard errors of the derived estimates. The similarity of derived and direct estimates for the power series $v_2(L)v_1(L)v_0(L)$ is less striking. The three possible derived estimates are extremely similar to each other, but they differ considerably from the direct estimate. Nevertheless, using any of the derived estimates as the null hypothesis for a test of the direct estimates would probably lead to acceptance of the null hypothesis. In general, the theory of investment behavior is strongly confirmed by the set of tests of internal consistency. Of course, given the internal consistency of the alternative estimates, it is possible to improve efficiency of estimation for the model as a whole by combining information from the various sources.

The tests of internal consistency just described are tests of the theory of investment in new projects. A test of the theory of replacement investment is a test of the consistency of the empirical results with the hypothesis $\delta = .025$. This hypothesis is borne out in two ways. First, for all but one of the regressions, the usual null hypothesis is accepted; a much stronger result is that for the first three regressions, estimates of the relationships under the restriction that $\delta = .025$ results in a reduction in the standard error of estimate for the regression. Finally, each of the standard errors of the estimates of δ is less than one-tenth the size of the corresponding regression coefficient. We conclude that the hypothesis that replacement is a constant fraction of capital stock, specifically, that $\delta = .025$, is strongly validated by the empirical results.

We turn now to comparisons of the fitted regressions with some simple alternatives. First, as alternatives for the first three regressions, we take the naïve models:

$$I_t = I_{t-1},$$

$$I_t^{S_2} = I_{t-1}^{S_2},$$

$$I_t^{S_1} = I_{t-1}^{S_1}.$$

Simple as these models may be, they are quite stringent standards for comparison for seasonally adjusted quarterly data, much more stringent, for example, than the corresponding models for annual data. The appropriate statistics for comparison are the standard errors of estimate and the VonNeumann ratios. Results of this comparison are given separately for the periods 1948–60 and second quarter 1955 to 1960 in Table 2.³ For the period as a whole, each of the regression models has a standard error well below that for the corresponding naïve model. For the later subperiod the advantage of the regression models is even greater. Turning to the VonNeumann ratios, there is practically no evidence of autocorrelated errors for the fitted models and very clear evidence of autocorrelation for the naïve models. Of course, this test is biased in favor of the fitted regressions. Even with this qualification, the fitted regressions are clearly superior in every respect to the corresponding naïve models.

As a standard of comparison for the second three regressions, we take the forecasts actually used by the Department of Commerce in presenting the results of the OBE-SEC Survey. These alternative models take the form:

$$I_t = I_{t-2}^{S_1},$$

$$I_t = I_{t-1}^{S_2},$$

$$I_t^{S_2} = I_{t-1}^{S_1}.$$

Despite the high level of performance of the OBE-SEC anticipations data, the fitted regressions constitute a substantial improvement in both goodness of fit as measured by standard error of estimate and absence of autocorrelation of residuals. The test for autocorrelation is not biased in favor of the fitted regressions, so that the evidence is unequivocal; the fitted relationships are clearly superior to the corresponding forecasting models for the period as a whole and for the subperiod since second quarter 1955.

A further comparison of the fitted regressions with the corresponding naïve and forecasting models is given in the second half of Table 2, where an analysis of the conformity of turning points of each of the

³ Data for both anticipations and actual expenditures on a revised basis are available from the Department of Commerce only since the second quarter of 1955. Anticipations data for the earlier period were revised by multiplying each observation by the ratio of revised to unrevised actual investment for the period in which the observation was made.

TABLE 2
GOODNESS OF FIT STATISTICS: FITTED, NAIVE, AND FORECASTING MODELS

Model	1948I-1960IV			1955II-1960IV			1948I-1960IV			1955II-1960IV		
	R ²	s	Δ ² /s ²	R ²	s	Δ ² /s ²	TP Error	Over-estimate	Under-estimate	TP Error	Over-estimate	Under-estimate
$I_t = f(\Delta K_t^*)$.94156	.10830	2.10778	.94298	.11378	1.95800	29%	47%	24%	23%	41%	36%
$I_t^* = f(\Delta K_t^*)$.88986	.16087	2.00549	.94757	.09368	1.84699	29	43	27	41	32	27
$I_t^* = f(\Delta K_t^*)$.87128	.18848	2.53442	.91921	.14372	2.25394	39	33	27	41	27	32
$I_t = I_{t-1}$.86193	.15950	.66900	.81929	.18410	.52078	22	39	39	23	32	45
$I_t^* = I_{t-1}^*$.84966	.18058	1.04366	.81297	.19435	.94465	24	37	39	32	27	41
$I_t^* = I_{t-1}^*$.83854	.20282	1.31901	.81161	.19947	.83580	35	25	39	32	27	41
$I_t = f(I_t^*)$.92729	.11955	1.16294	.92169	.12996	1.09855	16	43	41	23	41	36
$I_t = f(I_t^*)$.96234	.08604	1.47693	.95931	.09368	1.65030	14	45	41	18	45	36
$I_t^* = f(I_t^*)$.95121	.10597	1.70179	.96477	.09045	2.26525	16	45	39	23	41	36
$I_t = I_t^*$.83673	.17391	.69933	.77138	.20707	.46505	25	47	27	27	50	23
$I_t = I_t^*$.93504	.10969	.99342	.91854	.12359	.91146	14	51	35	18	50	32
$I_t^* = I_{t-1}^*$.93380	.11983	1.45737	.92833	.12031	1.25062	20	49	31	18	45	36

NOTE: Total percentages may not add to 100% because of rounding error.

“forecasts” to the turning points of the actual data is presented. In general, the first set of fitted regressions is slightly inferior to the naïve models and the second set slightly superior to the forecasting models on the basis of this criterion. A final comparison is between the fitted regression of investment on changes in desired capital stock and the forecast of investment from its second anticipation. The comparison favors the fitted regression; however, the anticipations data used in a fitted relationship between investment and second anticipation provide a model which is superior to the simple forecasting model and to the fitted regression of investment on changes in desired capital stock.

Structure of the Investment Process

In the preceding sections, only those aspects of the theory of investment behavior relevant to testing the theory were presented. In this section certain further implications of the theory are developed. Specifically, we will characterize the long-term response of investment to changes in the underlying market conditions and the tax structure and the time pattern of response of investment to changes in demand for capital.

First, using the facts that gross investment is determined by the relationship:

$$I_t = w(L)[K_t^* - K_{t-1}^*] + \delta K_t$$

and that capital stock is determined by past investments, we obtain:

$$\begin{aligned} I_t &= [1 - (1 - \delta)L]w(L)K_t^* \\ &= y(L)K_t^* \end{aligned}$$

where $y(L)$ is a power series in the lag operator. We define the τ -period response of investment to a change in market conditions or tax structure as the change in gross investment resulting from a change in the underlying conditions which persists for τ periods. More precisely, suppose that desired capital remains at a fixed level for τ periods to the present; then,

$$K_t^* = K_{t-\nu}^* \quad (\nu = 1, 2 \dots \tau),$$

and

$$\begin{aligned} I_t &= \sum_{\nu=0}^{\infty} y_{\nu} K_{t-\nu}^* \\ &= z_{\tau} K_t^* + \sum_{\nu=\tau+1}^{\infty} y_{\nu} K_{t-\nu}^* \end{aligned}$$

where $\{z_\tau\}$ is the sequence of cumulative sums of the coefficients of $y(L)$. As an example, the response of gross investment to a change in the rate of interest is:

$$\frac{\partial I}{\partial r} = z_\tau \frac{\partial K^*}{\partial r}.$$

The coefficients $\{z_\tau\}$ characterize the time pattern of response. Obviously,

$$\lim_{\tau \rightarrow \infty} z_\tau = \lim_{\tau \rightarrow \infty} \sum_{\nu=0}^{\tau} y_\nu = \delta,$$

so that the long-term response of gross investment to changes in, say, the rate of interest, is

$$\frac{\partial I}{\partial r} = \delta \frac{\partial K^*}{\partial r}.$$

Clearly, the short-term responses approach the long-term response as a limit; the approach is not necessarily monotone, since the coefficients of the power series $y(L)$ are not necessarily non-negative.

Long-term response and elasticities of gross investment with respect to the price of output, price of capital goods, and the rate of interest are given in the top half of Table 3. The corresponding responses and elasticities for the income tax rate, the proportion of replacement and the proportion of interest chargeable against income for tax purposes are given in the bottom half of Table 3. It should be noted that the rate of interest and the tax rate are measured as proportions, not percentages. For example, a decrease in the rate of interest by 1 per cent increases manufacturing gross investment by \$.15178 billions per quarter in the long run, at least to a first approximation.

The time pattern of response is presented in Table 4, where the functions $w(L)$, $y(L)$, and $z(L)$ are derived from the fitted regressions. The

TABLE 3
RESPONSES AND ELASTICITIES OF INVESTMENT WITH RESPECT TO CHANGES IN MARKET
CONDITIONS AND TAX STRUCTURE

	RESPONSE		ELASTICITY	
	Average	End of Period	Average	End of Period
Market Conditions				
Price of output.....	.35830	.35299	1.00000	1.00000
Price of capital goods....	-.35273	-.32106	-1.00000	-1.00000
Rate of interest.....	-14.23653	-15.17789	-.29143	-.37866
Tax Structure				
Income tax rate.....	-.37487	-.33016	-.50959	-.42064
Proportion of replacement	.18729	.20502	.39181	.48565
Proportion of interest....	.55656	.79840	.19428	.32659

TABLE 4
TIME FORM OF LAGGED RESPONSE

Lag	$w(L)$	$y(L)$	$z(L)$
0	0	0	0
1	0	0	0
2	.11277	.11277	.11277
3	.14209	.03214	.14491
4	.13700	-.00154	.14337
5	.11965	-.01393	.12944
6	.09969	-.01697	.11247
7	.08101	-.01619	.09628
8	.06491	-.01407	.08221
9	.05159	-.01170	.07051
10	.04081	-.00949	.06102
11	.03219	-.00760	.05342
12	.02535	-.00604	.04738
13	.01994	-.00478	.04260
14	.01567	-.00377	.03883
15	.01231	-.00297	.03586
16	.00967	-.00233	.03353
17	.00760	-.00183	.03170
18	.00597	-.00144	.03026
19	.00469	-.00113	.02913
20	.00368	-.00089	.02824
Remaining.....	.01346	-.00325	
Rate of decline.....	.78531	.78531	

average lag between change in demand for capital stock and the corresponding net investment is, roughly, 6.5 quarters or about a year and a half. Of course, this estimate is affected by the essentially arbitrary decision to set the proportion of the change invested in the same period and period immediately following the change at zero. The coefficients of the power series $z(L)$ are of interest for computation of short-period responses of investment to changes in the demand for capital stock. For example, the 2-period response of manufacturing gross investment to a change in the rate of interest of 1 per cent is:

$$z_r \frac{\partial K^*}{\partial r} = \frac{.11277}{.02500} \cdot .15178 = .68465 \text{ billions/quarter.}$$

By comparison, the corresponding 10-period response is .37046 billions per quarter. The response dies out, almost to its long-term level of .15178 billions/quarter, by twenty periods from the initial change in demand for capital stock. Similar calculations of the response of gross investment to changes in market conditions or the tax structure may be made for any of the six determinants of demand for capital by combining the responses given in Table 3 with the time pattern presented in Table 4.