Web Appendix to "Do Vertical Mergers Facilitate Upstream Collusion?"

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1. Secret Offers

While it is very natural to assume that downstream firms' *retail* prices are publicly observed (indeed, in most retail markets, prices are publicly posted), the details of a *wholesale* contract between an upstream and a downstream firm may not be easily verified by other firms. In this section, we will assume that only the downstream prices are publicly observed at the pricing stage, while upstream firm Ui's contract offer (w_{ij}, F_{ij}) to downstream firm Dj is private information to Ui and Dj. As we will show, the main conclusion of our paper is robust: even when contracts are secret, a (single) vertical merger facilitates upstream collusion.

To simplify the analysis, we will continue to assume that there is a publicly observed randomization device (so as to allow firms to share the market in an arbitrary way).[1] We will also assume that all contract offers (and acceptance decisions) are publicly revealed at the end of each period (after profits are realized).

At the acceptance stage, each downstream firm Dj must form beliefs about the secret contract offers to its downstream rivals. In perfect Bayesian equilibrium, these beliefs are pinned down by equilibrium play. However, if an integrated upstream-downstream pair, say $U1$-$D1$, deviates from equilibrium by charging a different retail price (or by setting a different quantity) – which is publicly observed – then, at the acceptance stage, each downstream firm Dj, $j > 1$, may hold arbitrary beliefs about the contracts that the deviant integrated firm has offered to any rival downstream firm Di, $i > 1$, $i \neq j$.[2] Similarly, if upstream firm Ui deviates by offering a different contract to downstream firm Dj, then perfect Bayesian equilibrium does not pin down Dj's beliefs about Ui's offer to Dk, $k \neq j$.

In the literature on foreclosure and vertical restraints with secret contracts, where this problem arises, it is customary to impose restrictions on the set of out-of-equilibrium beliefs; see Oliver Hart and Jean Tirole (1990), R. Preston McAfee and Marius Schwartz (1994), Ilya Segal (1999), and Patrick Rey and Tirole (2003). Our assumption that upstream and downstream firms set offers and prices (quantities) simultaneously implies that, unlike most of the literature, we do *not* need to restrict downstream firms' beliefs in the event that they receive an out-of-equilibrium contract offer from an unintegrated upstream firm, or from the integrated $U1$-$D1$ (unless $U1$-$D1$ deviates by also changing p_1 or q_1). Our model requires us to restrict downstream firms' beliefs only for the subgame in which the integrated firm, $U1$-$D1$, has deviated by changing the price (or quantity) of its downstream affiliate, $D1$. Having observed such a deviation, we assume that an (unintegrated) downstream firm Dj forms *wary beliefs* (see McAfee and Schwartz (1994)) at the acceptance stage: Dj believes that the cheating $U1$-$D1$'s secret contract offers to the other downstream firms maximize $U1$-$D1$'s profit, given $U1$-$D1$'s offer to Dj. An appealing implication of this assumption on beliefs is that no downstream firm will ex post regret accepting an upstream firm's optimal deviation

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[1] Instead of assuming the existence of a public correlating device, we could alternatively allow firms to share the collusive profits by making secret side payments at the end of the period, after profits are realized.

[2] If an unintegrated downstream firm Dj were to deviate at the pricing stage, then perfect Bayesian equilibrium implies that, at the acceptance stage, all downstream firms would continue to believe that their downstream rivals were offered their equilibrium contracts.

contract, and so an upstream firm's deviation profit is bounded from above by the monopoly profit.[3]

It is straightforward to show that, for *any* set of out-of-equilibrium beliefs about a deviant upstream firm's contract offers, it is possible to sustain the symmetric noncollusive outcome, where all upstream firms offer $(0,0)$ to all (unintegrated) downstream firms, and each downstream firm makes profit $\pi^{NC} \geq 0$, as a perfect Bayesian equilibrium. This claim is based on two observations. First, at the acceptance stage, a downstream firm observes its downstream rivals' prices (or quantities), and given these prices, its profit does not directly depend on its rivals' contracts. Second, each downstream firm can always make a non-negative profit by accepting the contract $(0,0)$ offered by one of the non-deviant upstream firms, no matter whether or not an unintegrated upstream or the integrated $U1$-$D1$ has deviated.

Consider now the collusive strategy profile where upstream firms extract all of the monopoly rents, and any deviation by an upstream firm or by an integrated upstream-downstream pair is followed, in all future periods, by the infinite play of the (symmetric) noncollusive equilibrium. Along the collusive equilibrium path, contract offers and downstream prices (or quantities) are as in the case of public offers.

We claim that upstream firms' deviation profits, and hence incentive constraints, are as in the case of public offers. The intuition is as follows. Recall that at the acceptance stage each downstream firm Dj observes its downstream rivals' prices/quantities. It then follows that downstream firm Dj's out-of-equilibrium beliefs about the secret offers made by the deviant Ui to its rivals affect Dj's profit only insofar as these secret offers may or may not lead to Dj's rivals rejecting all of their offers – since in this case, Dj would face additional demand from the rationed consumers. Further, note that if Ui deviates but all retail prices/quantities remain unchanged (which is necessarily the case if Ui is an unintegrated upstream firm), then each downstream firm would make zero profit by accepting the contract (w^M, F^M) offered by Ui's nondeviant rivals. Hence, as long as downstream prices are unchanged, each Dj will correctly believe that its downstream rivals' will not reject all of their offers, and so Dj will not face additional demand from rationed consumers. When downstream prices remain unchanged, each Dj is thus willing to accept the same set of deviant contracts as in the case of public offers. Finally, out-of-equilibrium beliefs matter when the integrated $U1$-$D1$ deviates by changing its downstream price p_1 (or quantity q_1). In particular, if $p_1 < p^M$ (or $q_1 > q^M$), downstream firms would make a loss if all of them were to accept the nondeviant contract (w^M, F^M). What our assumption of wary beliefs ensures is that Dj cannot be exploited by falsely believing that its downstream rivals are rejecting all of their contracts (in which case Dj would face additional demand from rationed consumers).

PROPOSITION 1. *Under secret offers, a vertical merger between an upstream and a downstream firm facilitates upstream collusion.*

Proof: The assertion obviously holds if we can show that upstream firms' incentive constraints are as under public offers. The argument proceeds in two steps.

First, we claim that each upstream firm can obtain at least the same deviation profit under secret offers as under public offers. To see this, observe that by making the same deviant contract offers as under public offers, a deviant upstream firm can make the same deviation profit as under public offers. Indeed, suppose an upstream firm offers the deviant contract $(w^M - \varepsilon, F^M - \varepsilon)$ to all unintegrated downstream firms, where (w^M, F^M) is the collusive equilibrium contract. Since each downstream firm Dj observes each rival's price (p^M) or quantity (Q^M/N), it is optimal for Dj to accept this deviant offer, independently of its downstream rivals' acceptance decisions.

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[3]In the above-cited literature, which is concerned only with static models, the restriction of *passive beliefs* is often imposed: receiving an out-of-equilibrium offer, a downstream firm continues to believe that its rivals received their equilibrium offers. This restriction is typically justified by the argument that, from the upstream firm's point of view, downstream firms operate in separate markets. However, in our context of collusion in a repeated game, this argument no longer holds: conditional on deviating (i.e., conditional on triggering the punishment phase), it is optimal for the deviant upstream firm to deviate through *each* downstream firm. Under passive beliefs, an upstream firm's maximum deviation profit would be larger than the monopoly profit, and downstream firms would ex post regret accepting deviant offers. See also Rey and Tirole (2004) and Rey and Thibaud Vergé (2004) for an alternative argument as to why wary beliefs may sometimes be preferred to passive or symmetric beliefs.

Second, we claim that an upstream firm cannot obtain a larger deviation profit than under public offers. To see this, note that each firm has the option of rejecting all contracts, and making a profit of zero. Hence, a deviant upstream firm can extract a higher deviation profit only if at least one downstream firm, say Dj, makes a loss ex post. Since downstream prices are publicly observed, this can occur only if Dj incorrectly believes that one or more of its downstream rivals reject all of their contracts – in which case Dj would face a larger demand (or, under quantity competition, would be able to fetch a higher price) than otherwise.

Suppose first that the deviant firm is an unintegrated upstream firm. In this case, all downstream prices are unchanged, and so each downstream firm can make a profit of zero by accepting the equilibrium offer of a nondeviant upstream firm (which is what equilibrium prescribes in this case). Hence, when a downstream firm receives a deviant offer from an unintegrated upstream firm, it will correctly believe that its downstream rivals will continue to be active in this period. The same is true in the event where the integrated $U1$-$D1$ deviates and $U1$-$D1$ charges retail price $p_1 \geq p^M$ (or sets a quantity $q_1 \leq q^M$).

Suppose now that the integrated $U1$-$D1$ deviates and sets $p_1 < p^M$ (or $q_1 > q^M$). Clearly, conditional on setting p_1 (or q_1) and offering the deviant contract (w'_{1j}, F'_{1j}) to the unintegrated downstream firm Dj, $U1$-$D1$ will optimally offer contracts to the other unintegrated downstream firms so as to maximize its deviation profit. But having wary beliefs, this is exactly what Dj believes about $U1$-$D1$'s other contract offers. Hence, Dj will accept only those contracts that ensure that it makes a nonnegative profit if $U1$-$D1$ behaves optimally vis-à-vis the other downstream firms. Since this applies to each unintegrated Dj, $U1$-$D1$'s deviation profit is bounded from above by Π^M. □

As should be clear from our discussion above, we could dispense with *any* restriction on out-of-equilibrium beliefs – even in the event of a deviation involving the integrated firm's downstream affiliate – and still obtain the same result if we were to assume that downstream firm Dj's demand would not increase if its downstream rival Dk rejects all of its offers and is thus unable to serve its demand.

In our analysis, we have assumed that contract offers become common knowledge at the end of each period. Would our results still hold if contract offers were never publicly revealed? For the case of homogeneous final goods and price competition downstream, it is possible to construct a collusive equilibrium in which firms' incentive constraints (and the critical discount factor) are as under public offers, as we briefly describe below. If final goods are differentiated (or downstream competition is in quantities), however, then it will in general not be possible for upstream firms to extract all of the monopoly rents along the collusive equilibrium path.

Assume that final goods are homogeneous and retail competition is in prices, and that downstream firms have wary beliefs off the equilibrium path. To see that it is possible to construct a collusive equilibrium where the incentive constraints are as under public offers, suppose upstream firm Ui has deviated by undercutting Uj's offer to downstream firm Dk, and that Dk has accepted this deviant offer in preference to the equilibrium offer of Uj. The collusive strategy profile then prescribes that: (i) in all future periods, both Ui and Uj offer $(0,0)$ to all (unintegrated) downstream firms forever; (ii) in all future periods, downstream firm Dk charges a retail price of zero; (iii) whenever an off-equilibrium retail price is observed, all firms switch to the symmetric noncollusive equilibrium (involving contract offers of $(0,0)$ and retail prices of zero) in all future periods; here, this means that two periods after Ui has deviated, the whole industry has reverted to the symmetric noncollusive equilibrium.

Clearly, neither Ui nor Uj have an incentive to deviate from this collusive strategy profile: given that one upstream firm offers $(0,0)$ to all downstream firms, it is optimal for the other to do so as well. (For this, it is irrelevant that Uj does not observe the identity of the upstream firm that has undercut its offer to Dk.) Further, Dk has no incentive to deviate: having wary beliefs, Dk believes that Ui has also deviated through all other downstream firms (since this is in Ui's best interest) and will therefore expect all other downstream firms to charge a price of zero in the next period (and all periods thereafter). Therefore, it is optimal for Dk to charge a price of zero in the next period (and thereafter). Finally, when observing an off-equilibrium retail price, it is optimal for each upstream firm to offer the contract $(0,0)$ to all downstream firms forever since it (correctly) expects its upstream rivals to do the same.

However, if final goods are differentiated (or downstream competition is in quantities), then it will in general not be possible for upstream firms to extract all of the monopoly rents along the collusive equilibrium path if offers are never publicly revealed. This follows from two observations. First, in this case, downstream firms are better off in the punishment phase than along the collusive equilibrium path since $\pi^{NC} > 0$ in the symmetric noncollusive equilibrium. Second, since contract offers are never publicly revealed, firms face an "inference problem" when Dj rejects Ui's offer: Did Dj reject the offer because some other upstream firm deviated by undercutting Ui's contract (which should trigger a punishment phase) or because Dj deviated on its own account (which should not trigger punishment) in the hope of triggering the punishment phase?

2. Optimal Punishment

In our analysis in the main text, we have assumed that, in the collusive equilibrium, a deviation by an upstream firm triggers an infinite reversion to a noncollusive equilibrium in all subsequent periods. This may, however, not be the worst possible punishment that can be inflicted on the deviator. In the existing literature on collusion, which has focused on repeated *normal*-form games, inflicting the worst possible punishment on a deviator consists in playing, from the following period onward, the subgame-perfect equilibrium that yields the lowest payoff to the deviator. In the context of our model, a deviating unintegrated upstream firm cannot be punished any more harshly than by reversion to the noncollusive equilibrium where it receives zero profits. But it may be feasible to sustain a per-period payoff for an integrated upstream-downstream pair that is less than the noncollusive profit π^{NC}. If so, this would further reduce the critical discount factor under vertical integration, and only strengthen our main result that (single) vertical integration facilitates upstream collusion by reducing the size of the punishment effect.

Our model is a repeated *extensive*-form game, in which it may be possible to use *within-period* punishment. In repeated extensive-form games, inflicting the lowest feasible payoff in all *future* periods does not generally constitute the worst possible punishment for a deviator. For instance, in our model, an upstream firm can profitably deviate only if at least one downstream firm accepts the deviant offer. An optimal punishment scheme may therefore provide incentives for downstream firms to reject deviant offers. Studying optimal punishment schemes in general is beyond the scope of this paper, but we are able to do so for the case where final goods are homogeneous.

Here, we consider the optimal punishment scheme for the case when downstream firms produce a homogeneous final good and compete in either prices or quantities. We have two aims. First, we show that our conclusion – that a vertical merger between an upstream and a downstream firm facilitates upstream collusion – continues to hold under the optimal punishment scheme. Second, not much is known about optimal punishment in repeated *extensive*-form (rather than *normal*-form) games, and so the punishment scheme we derive is of independent interest. Indeed, we show that the logic of *simple penal codes* (Dilip Abreu, 1988) breaks down in our repeated-extensive form game: upstream collusion may be sustainable only under a strategy profile with the property that the continuation play after an upstream firm's deviation depends not only on the identity of the deviator, but also on the details of the deviation.

We assume that there is a public randomization device not only between the pricing and acceptance stages (so as to allow an optimal sharing of collusive profits between upstream firms) as before, but also at the beginning of each period (so as to allow coordination on a particular equilibrium of the stage game). For simplicity, we will confine attention to the case of two downstream firms, $N = 2$. But it should be clear from our discussion below that the qualitative features of our results do not depend on this restriction.

Asymmetric Noncollusive Equilibria. In addition to the symmetric noncollusive equilibrium, there exist asymmetric subgame-perfect equilibria of the stage game. Suppose no firm is vertically integrated. In this case, for any downstream firm Dj, there exists a subgame-perfect equilibrium of the stage game, denoted $\widehat{\sigma}_j$, where Dj receives the monopoly profit Π^M, while all other firms make zero profits. Furthermore, when no firm is vertically integrated, there exists an equilibrium of the stage game, denoted $\widehat{\sigma}_0$, in which all upstream and downstream firms make zero profits: under price competition this is the symmetric noncollusive equilibrium; under quantity competition, this equilibrium is generated by a "coordination failure" between

the quantity choices of and offers to the downstream firms. If $U1$-$D1$ is vertically integrated (and no other firm is vertically integrated), there exists an equilibrium where the integrated firm obtains the monopoly rent.

LEMMA 1. *Suppose final goods are homogeneous. If no firm is vertically integrated, there exists a subgame-perfect equilibrium of the stage game, denoted $\widehat{\sigma}_j$, in which an (arbitrary) downstream firm Dj captures all of the monopoly rents Π^M. Moreover, in this case, there exists a subgame-perfect equilibrium of the stage game, denoted $\widehat{\sigma}_0$, in which industry profits are zero. If only one usptream-downstream pair, say $U1$-$D1$, is vertically integrated, $\widehat{\sigma}_1$ forms a subgame-perfect equilibrium in which $U1$-$D1$ obtains Π^M.*

Proof: Suppose first that no firm is vertically integrated. To see that there exists an equilibrium of the stage game where Dj obtains all of the monopoly profits, consider the following strategy profile, denoted $\widehat{\sigma}_j$. At the pricing stage, each upstream firm offers the contract $(0,0)$ to Dj, and a contract with a sufficiently large wholesale price, say $(p^M + \varepsilon, F)$ with $\varepsilon > 0, F \geq 0$, to all other downstream firms Di, $i \neq j$. At the same time, downstream firm Dj sets price $p_j = p^M$ (or quantity $q_j = Q^M$), while Di, $i \neq j$, sets price $p_i > p^M$ (or quantity $q_i = 0$). At the acceptance stage, each downstream firm accepts any contract that leaves it with nonnegative rents, given the acceptance decisions of the other firms. Along the equilibrium path, Dj accepts at least one of the contracts offered to it, and makes the monopoly profit Π^M. It is straightforward to verify that no firm has an incentive to deviate unilaterally. To see that there exists an equilibrium of the stage game where all firms make zero profits, suppose all upstream firms make contract offers of the form (∞, ∞), and each downstream firm Dj sets a quantity $q_j = 0$ or a price $p_j = \infty$. Clearly, no firm has an incentive to deviate. It is easy to check that $\widehat{\sigma}_1$ still forms a subgame-perfect equilibrium if $U1$-$D1$ is vertically integrated. □

As we will now show, the asymmetric noncollusive equilibria are used in the optimal punishment scheme for rewarding those downstream firms that have rejected deviant offers, and the zero-profit equilibrium $\widehat{\sigma}_0$ for punishing downstream firms in the event where all of them have accepted deviant offers.

Collusive Equilibrium: Non-Integration. We now derive the optimal collusive scheme under non-integration, which differs from the collusive strategy profile described before only in the subgames following a deviation by an upstream firm. For simplicity, we restrict attention to the case where each of the two unintegrated downstream firms sells half of the monopoly output along the collusive equilibrium path (i.e., each downstream firm sets a quantity of $Q^M/2$, or charges a price of p^M and consumer demand is divided equally between the two retailers). Of course, the strategy profile will prescribe asymmetric downstream behavior in certain subgames off the equilibrium path.

The worst possible punishment that might be inflicted upon a deviant upstream firm is that – in addition to the play of one of the noncollusive equilibria in all future periods – all of its deviant contracts are rejected by the downstream firms in the period of deviation, leaving the unintegrated upstream firm with a deviation profit of zero. By playing one of the asymmetric noncollusive equilibria, $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, in all future periods, the maximum joint "reward" R that can feasibly be shared between downstream firms if they do reject deviant offers is:

$$R \equiv \frac{\delta}{1-\delta}\Pi^M. \tag{1}$$

For downstream firm Dj to accept upstream firm Ui's deviant offer (w'_{ij}, F'_{ij}), the deviator needs to leave rents to Dj, in the form of a reduced wholesale price and/or a reduced fixed fee. Let

$$b_{ij} \equiv [p^M - w'_{ij}]\frac{Q^M}{2} - F'_{ij}$$

denote Ui's "bribe" to downstream firm Dj.

An important feature of the optimal punishment scheme (discussed further below) is that downstream firm Dj's reward for rejecting upstream firm Ui's deviant offer (and accepting the equilibrium contract

offered by a nondeviant upstream firm) depends on the bribes b_{i1} and b_{i2} offered by Ui, and on both downstream firms' acceptance decisions.[4] Specifically, the optimal punishment scheme is of the following form. *Case (i):* Suppose the bribes satisfy $\max\{b_{i1}b_{i2}\} \leq \Pi^M/2$. (a) If only one downstream firm, say Dj, rejects Ui's offer, then Dj receives the maximum reward R. That is, the asymmetric equilibrium $\widehat{\sigma}_j$ will be played in all future periods. (b) If both downstream firms reject the deviant offers, then $D1$ receives a fraction $\gamma_{i1} \in [0,1]$ of the maximum reward R, and $D2$ receives the remaining fraction $\gamma_{i2} = 1 - \gamma_{i1}$, where γ_{ij} will be chosen optimally as a function of b_{i1} and b_{i2}. That is, in any future period, the asymmetric equilibrium $\widehat{\sigma}_j$, $j = 1,2$, will be played with probability γ_{ij} (using the public randomization device at the beginning of each period). (c) If both downstream firms accept the deviant offers, then the zero-profit equilibrium $\widehat{\sigma}_0$ will be played in all future periods. *Case (ii):* Suppose the bribes satisfy $b_{ij} < \Pi^M/2 < b_{ik}$. In this case, the deviant Ui would make a loss on its offer to Dk, and so Dk should not be rewarded for rejecting Ui's offer. Instead, downstream firm Dj will receive the maximum reward R for rejecting Ui's offer, independently of Dk's acceptance decision. *Case (iii):* Suppose the bribes satisfy $\min\{b_{i1}, b_{i2}\} > R$. In this case, the deviant Ui will make a loss on each contract if accepted, and so no downstream firm will be offered a reward for rejecting the offer. (The strategy profile may prescribe, for example, the play of the zero-profit equilibrium $\widehat{\sigma}_0$ in all future periods.)

Clearly, it will never be optimal for Ui to offer a bribe $b_{ij} > \Pi^M/2$, and so we will from now on confine attention to case (i) where $b_{ij} \leq \Pi^M/2$ for each $j = 1,2$. Observe that the optimal punishment scheme is such that whenever a downstream firm accepts a deviant offer, it will receive zero profits in all future periods. Consequently, Dj will accept Ui's deviant offer if and only if the offered bribe b_{ij} exceeds Dj's expected reward for rejecting Ui's offer. We now derive the optimal "choice" of γ_{ij} as a function of the offered bribes b_{i1} and b_{i2}.

First, suppose that Ui deviates by offering bribes $b_{i1} > R$ and $b_{i2} > R$. In this case, even the maximum feasible reward R is insufficient to prevent any one downstream firm from accepting the deviant offer. Equilibrium thus prescribes that both downstream firms accept Ui's offers, independently of the value of γ_{ij}, and Ui's deviation profit is $\Pi^M - (b_{i1} + b_{i2})$. Hence, conditional on offering such bribes, Ui can obtain a maximum deviation profit of (approximately) $\Pi^M - 2R$ by setting $b_{i1} = b_{i2} = R + \varepsilon$ with ε being arbitrarily small.

Second, suppose that Ui deviates by offering bribes $0 < b_{ij} \leq R$ and $b_{ik} \geq R$. In this case, the available rewards are insufficient to induce both downstream firms to reject the Ui's offer; in fact, if $b_{ik} > R$, downstream firm Dk would accept Ui's offer even if offered the maximum reward R. The optimal punishment therefore involves setting $\gamma_{ij} = 1$, which induces downstream firm Dj to reject Ui's offer (and instead to accept the equilibrium contract (w^M, F^M) offered by a nondeviant upstream firm). The strategy profile then prescribes that Dj accepts and Dk rejects the deviant offer, and Ui's deviation profit is $\Pi^M/2 - b_{ik}$. Hence, conditional on offering such bribes, Ui can obtain a maximum deviation profit of $\Pi^M/2 - R$ by setting $b_{ij} \in (0,R]$ and $b_{ik} = R$.

Third, suppose that Ui deviates by offering bribes $b_{i1} \leq R$ and $b_{i2} \leq R$ with $b_{i1} + b_{i2} \leq R$. In this case, both downstream firms can be induced to reject Ui's deviant offer (and accept the equilibrium contract offered by a nondeviant upstream firm) by setting $\gamma_{ij} = b_{ij}/(b_{i1} + b_{i2})$. Hence, by offering such bribes, Ui's deviation profit is zero.

Fourth, suppose that Ui deviates by offering bribes $b_{i1} < R$ and $b_{i2} < R$ with $b_{i1} + b_{i2} > R$. (Given that $b_{i1}, b_{i2} \leq \Pi^M/2$, this case can arise only if $R < \Pi^M$, or $\delta < 1/2$.) In this case, the feasible rewards are insufficient to induce both downstream firms to reject Ui's offers for sure. But it is possible to offer rewards such that one downstream firm will reject for sure, or else that each downstream firm will reject with positive probability. By setting $\gamma_{ij} = 1$, downstream firm Dj can be induced to reject (and its rival Dk

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[4]Formally, the optimal reward depends on the deviant contract offers (w'_{i1}, F'_{i1}) and (w'_{i2}, F'_{i2}) (in addition to downstream firms' acceptance decisions). But since downstream firms' prices/quantities are set at the same time as upstream firms' offers are made, what matters for downstream firms' incentives are not the offered wholesale prices and fixed fees *per se* but the induced bribes b_{i1} and b_{i2}.

to accept) Ui's offer for sure. Alternatively, if $0 \le \gamma_{i1} < b_{i1}$ and $0 \le \gamma_{i2} < b_{i2}$ (with $\gamma_{i1} + \gamma_{i1} = 1$), there is a multiplicity of equilibria at the acceptance stage: (i) two pure-strategy equilibria where Dj accepts Ui's contract, while Dk rejects it, and Ui's deviation profit is $\Pi^M/2 - b_{ij}$, and (ii) a mixed-strategy equilibrium where each downstream firm accepts with positive probability. The collusive strategy profile prescribes that the worst equilibrium from the deviant Ui's point of view will be played: as we will now show, this is the mixed strategy equilibrium.

LEMMA 2. *Suppose upstream firm Ui deviates by offering a weakly larger bribe to Dj than to Dk, $b_{ik} \le b_{ij} < \min\{R, \Pi^M/2\}$, and $b_{i1} + b_{i2} > R$. (The restrictions imply $R < \Pi^M$, i.e., $\delta < 1/2$.) Then, the optimal punishment scheme is such that $\gamma_{ij} = b_{ij}/R$ and $\gamma_{ik} = 1 - b_{ij}/R$. The strategy profile prescribes that Dk rejects Ui's offer with probability one and that Dj rejects the deviant offer with probability $(R - b_{ik})/b_{ij}$. Conditional on offering such bribes, Ui's optimal deviation consists in setting*

$$b_{i1} = b_{i2} = \frac{\sqrt{R\Pi^M}}{2} = \frac{\Pi^M}{2}\sqrt{\frac{\delta}{1-\delta}},$$

which results in a deviation profit of

$$\left[1 - \sqrt{\frac{R}{\Pi^M}}\right]^2 \Pi^M = \left[1 - \sqrt{\frac{\delta}{1-\delta}}\right]^2 \Pi^M.$$

  Proof: The proof proceeds in steps. (1) Assuming that Ui has deviated by offering bribes satisfying $b_{ik} \le b_{ij} < \min\{R, \Pi^M/2\}$, and $b_{i1} + b_{i2} > R$, we derive the optimal choice of γ_{ij} such that the equilibrium at the acceptance is in *mixed* strategies. (2) We show that *any* choice of γ_{ij} (not necessarily the one derived in step (1)) that is followed by the play of a *pure*-strategy equilibrium at the acceptance stage, will lead to a higher deviation profit, and is hence not an optimal punishment. (3) We derive Ui's optimal deviation under the stated restrictions on the bribes.

  (1) For at least one downstream firm to use a mixed strategy at the acceptance stage, the distribution of rewards must satisfy $1 - b_{ik}/R \le 1 - \gamma_{ik} \equiv \gamma_{ij} \le b_{ij}/R$ (where one inequality must be strict since $b_{ij} + b_{ik} > R$ by assumption). Suppose first that both inequalities are strict so that there exists an equilibrium in which both Dj and Dk use a mixed strategy. Let θ_k denote the probability that Dk accepts Ui's deviant offer. For Dj to use a mixed strategy, it has to be indifferent between accepting and rejecting, and so

$$b_{ij} = \theta_k R + (1 - \theta_k)\gamma_{ij}R,$$

where the l.h.s. gives Dj's payoff upon accepting Ui's offer, and the r.h.s. the expected payoff from rejecting. Solving for θ_k, we obtain

$$\theta_k = \frac{b_{ij} - \gamma_{ij}R}{(1 - \gamma_{ij})R}.$$

Deriving θ_j in the same fashion, we can write Ui's expected deviation profit as

$$\pi_i^{dev}(b_{ij}, b_{ik}; \gamma_{ij}) = \left(\frac{b_{ij} - \gamma_{ij}R}{(1 - \gamma_{ij})R}\right)\left[\frac{\Pi^M}{2} - b_{ik}\right] + \left(\frac{b_{ik} - (1 - \gamma_{ij})R}{\gamma_{ij}R}\right)\left[\frac{\Pi^M}{2} - b_{ij}\right] \qquad (2)$$

The optimal division of rewards, γ_{ij}, minimizes this expression, subject to the constraint $1 - b_{ik}/R \le \gamma_{ij} \le b_{ij}/R$. The deviation profit is strictly concave in γ_{ij} over the relevant range, and so we obtain a corner solution $\gamma_{ij} \in \{b_{ij}/R, 1 - b_{ik}/R\}$. It can easily be verified that $\pi_i^{dev}(b_{ij}, b_{ik}; b_{ij}/R) \le \pi_i^{dev}(b_{ij}, b_{ik}; 1 - b_{ik}/R)$ if and only if $b_{ij} \ge b_{ik}$, which holds by assumption. Hence, the deviation profit in (2) is minimized if $\gamma_{ij} = b_{ij}/R$. In the limit as $\gamma_{ij} \to b_{ij}/R$, downstream firm Dk rejects Ui's offer with probability one, while Dj accepts Ui's offer with probability $(b_{ij} + b_{ik} - R)/b_{ij} \in (0,1)$. (In fact, if $\gamma_{ij} = b_{ij}/R$, there exists a continuum of mixed-strategy equilibria. In all of these, Dk rejects Ui's offer with probability one, and Dj

accepts the offer with probability larger than or equal to $(b_{ij} + b_{ik} - R)/b_{ij}$. Hence, the afore-mentioned equilibrium is the worst mixed-strategy equilibrium from the deviant Ui's point of view.) The minimized deviation profit is given by

$$\pi_i^{dev}(b_{ij}, b_{ik}) = \left(1 - \frac{R - b_{ik}}{b_{ij}}\right)\left[\frac{\Pi^M}{2} - b_{ij}\right]. \tag{3}$$

(2) In step (1), we have assumed that γ_{ij} is chosen such that a mixed-strategy equilibrium at the acceptance stage exists, and that downstream firms do indeed play this equilibrium. May the same or a different choice of γ_{ij}, followed by a pure-strategy equilibrium at the acceptance stage, lead to a lower deviation profit? The answer is, no. Since Ui offers a weakly larger bribe to Dj than to Dk (and, so if accepted, Ui's makes a weakly lower profit on its contract with Dj than with Dk), the worst possible (from Ui's point of view) pure-strategy equilibrium at the acceptance stage entails Dj accepting Ui's offer for sure, and Dk rejecting for sure. But in the mixed-strategy equilibrium derived in step (1), Dk also rejects Ui's offer for sure, but in addition Dj rejects the deviant offer with positive probability as well.

(3) In the previous steps, we have shown that $\gamma_{ij} = b_{ij}/R$, followed by the play of a mixed-strategy equilibrium (where Dk rejects for sure and Dj rejects with probability $(R - b_{ik})/b_{ij}$) is the most severe punishment. Given this punishment, we now derive the deviant Ui's optimal choice of bribes, under the conditions of the lemma, i.e., $b_{ik} \le b_{ij} < \min\{R, \Pi^M/2\}$, and $b_{i1} + b_{i2} > R$. From equation (3), Ui's deviation profit $\pi_i^{dev}(b_{ij}, b_{ik})$ under the optimal punishment scheme is strictly increasing in b_{ik}, given that $b_{ik} \le b_{jk}$. Hence, Ui will optimally set $b_{ik} = b_{jk} = b$. Maximizing $\pi_i^{dev}(b, b)$ with respect to b, yields Ui's profit-maximizing bribe, $b = (1/2)\sqrt{R\Pi^M}$. The resulting deviation profit is $\left[1 - \sqrt{R/\Pi^M}\right]^2 \Pi^M$. $\square$

When it is infeasible to induce both downstream firms to reject the deviant offer with probability one, the optimal punishment scheme provides rewards in such a way to permit "miscoordination" between downstream firms at the acceptance stage. To see why such miscoordination is detrimental for the profits of the deviant upstream firm, suppose Ui offers bribes $b_{i1} = b_{i2} = (1 + \varepsilon)R/2 < \Pi^M/2$, where ε is small. The worst possible punishment involving a pure-strategy equilibrium at the acceptance stage is such that one downstream firm, say Dk, rejects the offer, while its downstream rival Dj accepts it. (This holds for any choice of $0 \le \gamma_{i1} = 1 - \gamma_{i2} \le 1$.) The resulting deviation profit is $\left[\Pi^M - (1 + \varepsilon)R\right]/2$. However, if $\gamma_{ij} = (1 + \varepsilon)/2$ and $\gamma_{ik} = (1 - \varepsilon)/2$, there exists a mixed-strategy equilibrium at the acceptance stage where downstream firm Dk rejects the deviant offer for sure, while Dj randomizes and accepts the deviant offer with probability $2\varepsilon/(1 + \varepsilon)$. Ui's deviation profit is only $\left[\Pi^M - (1 + \varepsilon)R\right]\varepsilon/(1 + \varepsilon)$, which is close to zero for ε small.

To summarize, by optimally choosing the bribes b_{i1} and b_{i2}, upstream firm Ui can get a maximum deviation profit of

$$\begin{aligned}
\pi_{optimal}^{dev} &= \max\left\{\Pi^M - 2R, \Pi^M/2 - R, 0, \Pi^M\left[\max\left\{1 - \sqrt{\frac{R}{\Pi^M}}, 0\right\}\right]^2\right\} \\
&= \max\left\{\left(\frac{1 - 3\delta}{1 - \delta}\right)\Pi^M, \left(\frac{1 - 3\delta}{1 - \delta}\right)\frac{\Pi^M}{2}, 0, \Pi^M\left[\max\left\{1 - \sqrt{\frac{\delta}{1 - \delta}}, 0\right\}\right]^2\right\}
\end{aligned} \tag{4}$$

where the arguments are listed in the order of our discussion above. (As regards the final argument, the maximum-term reflects that this fourth case can arise only if $R < \Pi^M$, or $\delta < 1/2$.) The incentive constraint under non-integration can then be written as

$$\frac{\Pi^M}{M(1 - \delta)} \ge \pi_{optimal}^{dev}. \tag{5}$$

We obtain the following result.

LEMMA 3. *Suppose no firm is vertically integrated. Then, under the optimal punishment scheme, the critical discount factor above which monopoly profits upstream can be sustained is given by*

$$\widehat{\delta}^{NI}_{optimal} = \begin{cases} \frac{M-1}{3M} & \text{if } 2 \leq M \leq 13, \\ \frac{M-\sqrt{2M-1}}{2M} & \text{if } M \geq 13. \end{cases}$$

Proof: The assertion follows from equations (4) and (5). Observe that the second argument in (4) is one half times the first argument, and so this argument can be dropped. The first argument is positive only if $\delta < 1/3$, and the final argument is positive only if $\delta < 1/2$. That is, $\pi^{dev} = 0$ if $\delta \geq 1/2$. Inserting the first argument into the incentive constraint (5), we obtain

$$\frac{\Pi^M}{M(1-\delta)} \geq \left(\frac{1-3\delta}{1-\delta}\right)\Pi^M,$$

or

$$\delta \geq \frac{M-1}{3M}.$$

Inserting the final argument into the incentive constraint, yields

$$\frac{\Pi^M}{M(1-\delta)} \geq \left[\max\left\{1-\sqrt{\frac{\delta}{1-\delta}},0\right\}\right]^2 \Pi^M,$$

or

$$\delta \geq \frac{M-\sqrt{2M-1}}{2M}.$$

Hence,

$$\widehat{\delta}^{NI}_{optimal} = \max\left\{\frac{M-1}{3M}, \frac{M-\sqrt{2M-1}}{2M}\right\}.$$

$\square$

Comparing the critical discount factor under "Nash reversion", $\widehat{\delta}^{NI} = (M-1)/M$, with that under the optimal punishment scheme, $\widehat{\delta}^{NI}_{optimal}$, we observe that the former goes to one as the number of upstream firms becomes large, $\widehat{\delta}^{NI} \to 1$ as $M \to \infty$, while the latter is bounded from above by $1/2$, $\widehat{\delta}^{NI}_{optimal} \to 1/2$ as $M \to \infty$. Under the optimal punishment scheme, the more patient are players (or the less frequent prices can be adjusted), the greater is the present value of future rewards. If $\delta \geq 1/2$, there exists a division of these future rewards and an equilibrium at the acceptance stage such that an upstream firm's maximum deviation profit is actually zero.

In the foregoing analysis, we have derived the critical discount factor assuming that each downstream firm produces one half of the monopoly output along the collusive equilibrium path. Intuitively, this arrangement facilitates upstream collusion: downstream asymmetries would make upstream collusion more difficult. To see this, suppose one downstream firm, say Dj, were to produce the whole monopoly output in the collusive equilibrium. Then, a deviant Ui would need to offer a bribe of only $b_{ij} = R + \varepsilon$ (with ε being arbitrarily small) to that downstream firm Dj, and obtain a deviation profit of (almost) $\Pi^M - R$. The resulting critical discount factor would be $(M-1)/(2M)$, which is clearly larger than the one when downstream firms share the market symmetrically.

The existing literature on collusion has focused almost exclusively on repeated *normal*-form games. As Abreu (1988) has shown, when deriving the optimal punishment scheme in such games, one can confine attention to *simple penal codes*. Any subgame-perfect outcome can be supported by a profile with the property that any deviation by a player from the current prescribed path is punished by the same punishment path (penal code). That is, the continuation play after a deviation by a player is independent of the details of the deviation, depending only on the identity of the deviator.

The optimal punishment scheme derived above is *not* a simple penal code. First, depending on the deviant upstream firm's contract offers, the strategy profile prescribes that different downstream firms reject the offers, and the associated "rewards" mean that different outcome paths are played in future periods. For example, suppose that the deviant Ui offers bribes $b_{i1} = b_{i2} = R/2 < \Pi^M/2$. Then, the strategy profile prescribes that both downstream firms reject the offer, and that in all future periods each one of the two asymmetric equilibria, $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, is played with probability $1/2$. In contrast, suppose that Ui offers bribes $b_{i1} = R < b_{i2}$. In this case, the strategy profile prescribes that $D1$ rejects Ui's offer (and that $D2$ accepts the offer), and that in all future periods, the asymmetric equilibrium $\widehat{\sigma}_1$ is played with probability one. Hence, the prescribed outcome path in all future periods depends on the details of Ui's deviant contract offers. Second, even in the event when both downstream firms reject Ui's deviant offers, the continuation play optimally depends on the details of these offers. Indeed, as lemma 2 shows, when the offered bribes satisfy $b_{i1} \le b_{i2} < R$ and $b_{i1} + b_{i2} > R$, downstream firm $D2$ should obtain a fraction b_{i2}/R of the total reward R (and $D1$ the remaining fraction) when both $D1$ and $D2$ reject Ui's offer. Hence, the probability distribution over the two asymmetric equilibria, $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, in all future periods depends on the details of Ui's contract offers, even after both downstream firms have taken the same action at the acceptance stage.

Our results show that the logic of simple penal codes breaks down in repeated *extensive*-form games. In our model, upstream collusion may be sustainable only under a strategy profile with the property that the prescribed continuation play following a deviation not only depends on the identity of the deviant upstream firm, but also on the details of its contract offers.[5]

PROPOSITION 2. *Under non-integration, the optimal punishment scheme is not a simple penal code.*

Collusive Equilibrium: Single Integration. Suppose now that one upstream-downstream pair, say $U1$-$D1$, is vertically integrated. As shown in lemma 1, monopoly profits upstream are then sustainable for any discount factor $\delta \ge 0$. Hence, the main result of our paper continues to hold under the optimal punishment scheme.

PROPOSITION 3. *Suppose final goods are homogeneous. Under the optimal punishment scheme, (single) vertical integration facilitates upstream collusion.*

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[5]We provide some further discussion of the failure of simple penal codes in a separate note, George J. Mailath, Volker Nocke, and Lucy White (2004).