

General Equilibrium Asset Price

Example from Section III and Figure 1 of Brunnermeier Parker , AER September 2005

This program solves a two –

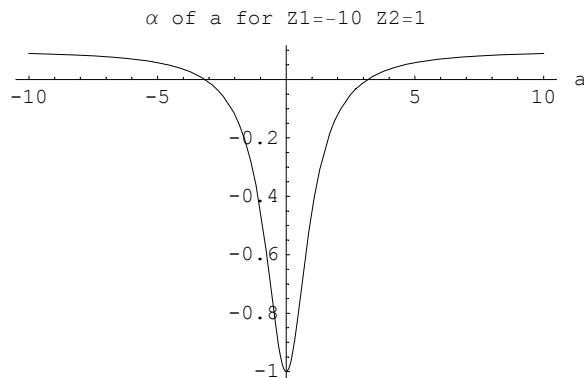
state skewed GE example. We specify the true probability of state 1, risk aversion, and the two epsilons for the risky asset (assuming $R = 1$). We choose these so that price of one implies that the asset has zero rationally expected excess return and so no one holds the asset in a RE equilibrium. Then we guess and verify the optimal expectations price, which is the one that has a bimodal lifetime wellbing function with the two local optima of equal height. In equilibrium, two populations of agents hold beliefs at each local optimal in shares such that markets clear.

(* 1. Define utility,
marginal utility and transformation for aplha and pi and pihat*)

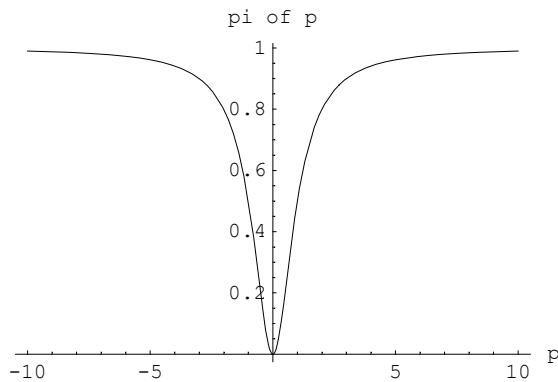
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(* Utility function*)
uof[ $\alpha z$ _,  $\gamma$ _]:= ((1 +  $\alpha z$ )^(1 -  $\gamma$ )) / (1 -  $\gamma$ )

(* Marginal utility function*)
duof[ $\alpha z$ _,  $\gamma$ _]:= (1 +  $\alpha z$ )^(- $\gamma$ )

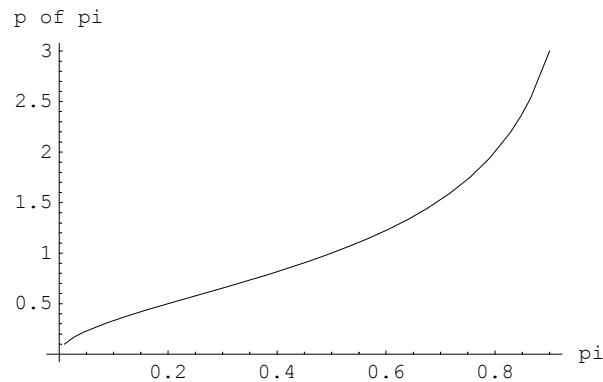
(* Transformations to keep probabilities between 0 and 1 and
portfolio choice so that consumption is always greater than zero*)
aof[a_, z1_, z2_]:= (1/z2 - 1/z1) (aa/(1+aa)) - 1/z2
Plot[aof[a, -10, 1], {a, -10, 10}, AxesLabel -> {"a", "a of a for z1=-10 z2=1"}]
piof[p]:= pp/(1+pp)
Plot[piof[p], {p, -10, 10}, AxesLabel -> {"p", "pi of p"}]
pof[pi]:=  $\sqrt{pi/(1-pi)}$ 
Plot[pof[pi], {pi, 0.01, 0.9}, AxesLabel -> {"pi", "p of pi"}]
```



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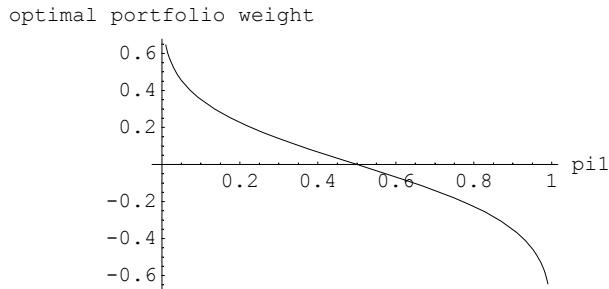
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(* 2. First–Order Ccondition for optimal portfolio choice *)

FOC[α _, $\hat{\pi}$ _, γ _, z_1 _, z_2 _] := ($\hat{\pi} z_1$ duof[α z_1 , γ]) + ((1 - $\hat{\pi}$) z_2 duof[α z_2 , γ])

(* 3. Implicitly define optimal portfolio choice as a function of the true probability, risk aversion and the payoffs in each state*)

```
αstar[pihat_, γ_, z1_, z2_] :=
  αof[a /. FindRoot[FOC[αof[a, z1, z2], pihat, γ, z1, z2] == 0, {a, √[-z2/z1]}], z1, z2]
(*Initial guess is alpha=0*)
Plot[αstar[pil, 3, -1, 1], {pil, 0.01, 0.99},
  AxesLabel → {"pil", "optimal portfolio weight"}]
```



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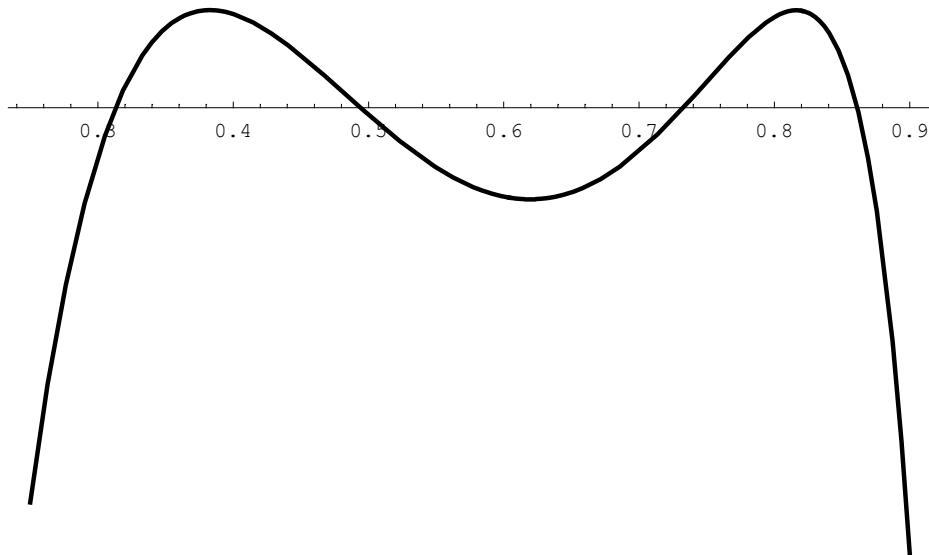
(* 4. Define anticipatory utility and expected actual utility, and Wellbeing *)

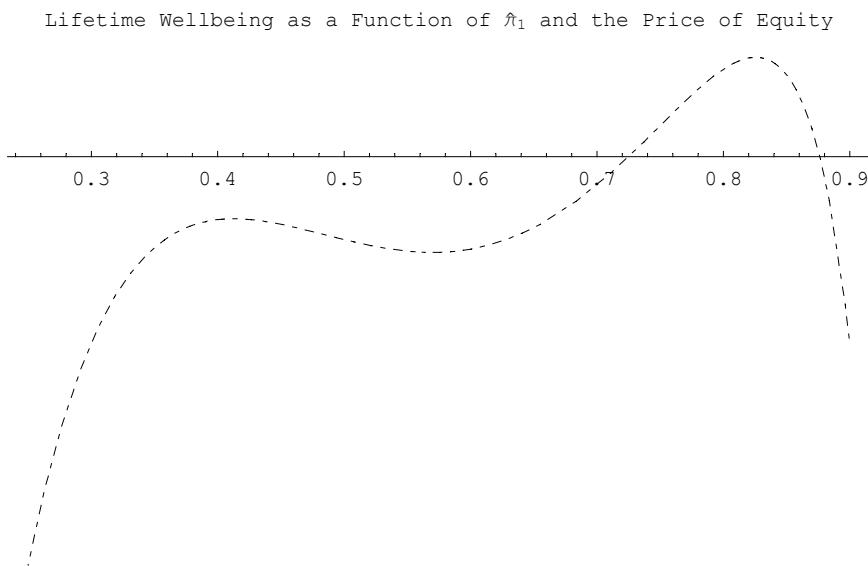
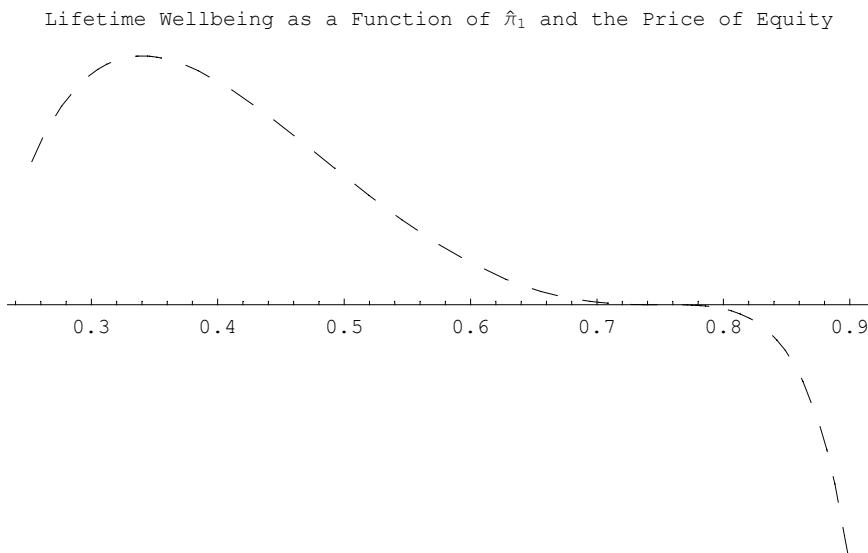
```
EhatU[pihat_, γ_, z1_, z2_] := (pihat uof[z1 αstar[pihat, γ, z1, z2], γ]) +
  ((1 - pihat) uof[z2 αstar[pihat, γ, z1, z2], γ]);
EU[pi_, pihat_, γ_, z1_, z2_] := (pi uof[z1 αstar[pihat, γ, z1, z2], γ]) +
  ((1 - pi) uof[z2 αstar[pihat, γ, z1, z2], γ]);
W[pi_, pihat_, γ_, z1_, z2_] := EhatU[pihat, γ, z1, z2] + EU[pi, pihat, γ, z1, z2]
(*Fig1=Plot[W[0.500000000001,pihat,3,-1,1], {pihat,.4997,.5003},
  AxesLabel->{"pilhat","W"}, PlotLabel->"Fair Symmetric Truth"]
Fig2=Plot[W[0.500000000001,pihat,3,-1,1], {pihat,.4997,.5003},
  AxesLabel->{"pilhat","W"}, PlotLabel->"p1hat TINY bit larger than truth"]
Fig3=Plot[W[0.5,pihat,3,-1,1.001], {pihat,.4,.6},
  AxesLabel->{"pilhat","W"}, PlotLabel->"Z2 larger than fair"]*)
(*Fig3=Plot[W[0.4999,pihat,3,-1,1], {pihat,.4,.6},
  AxesLabel->{"pilhat","W"}, PlotLabel->"p1 smaller than fair"]*)
```

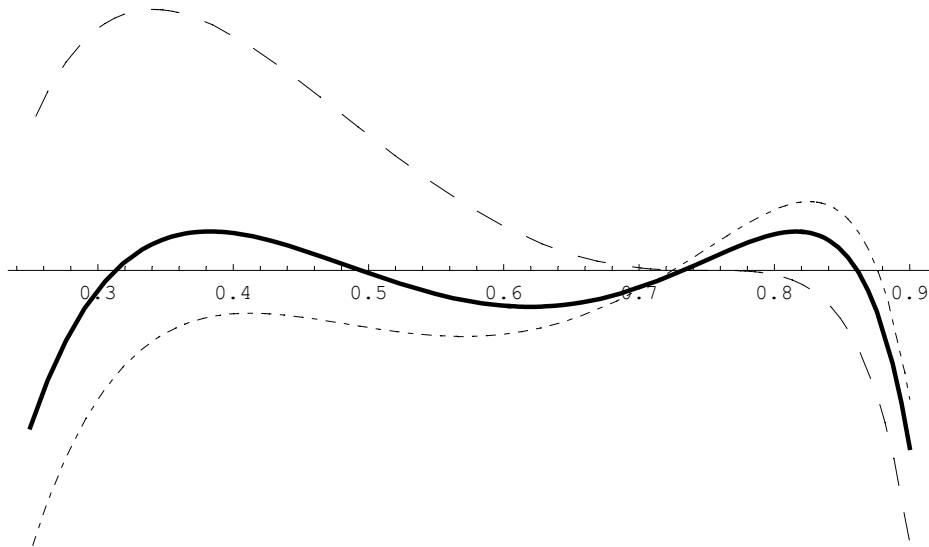
(* 5. Functions to find OPTIMAL pi-hat and alpha, but only find only one local optimum*)

```
pihatoe[pi_, γ_, z1_, z2_, guess_] :=
  piof[phat] /. Part[FindMaximum[W[pi, piof[phat], γ, z1, z2], {phat, guess}], 2]
αstroe[pi_, γ_, z1_, z2_, guess_] := αof[a /. FindRoot[FOC[αof[a, z1, z2],
  piof[phat] /. Part[FindMaximum[W[pi, piof[phat], γ, z1, z2], {phat, guess}], 2],
  γ, z1, z2] == 0, {a, 1}], z1, z2]
```

```
(* 6. Construct the example *)
exgamma = 3
expi1 = 0.25
eps1 = -0.6
eps2 = 0.2
price = 1 - (1 - .993088) * 2
(*This price is found by guess and verify, inelegant, I know*)
FigW = Plot[W[expi1, 1 - pihat2, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1],
{pihat2, .25, .9}, Axes → {True, False},
AxesLabel → {"", ""}, PlotStyle → {Thickness[0.005]}]
FigWhigh = Plot[W[expi1, 1 - pihat2, exgamma, (1 + eps1) / 1 - 1, (1 + eps2) / 1 - 1],
{pihat2, .25, .9}, Axes → {True, False},
AxesLabel → {"", ""}, PlotStyle → {Dashing[{0.03, 0.03}]},
PlotLabel → "Lifetime Wellbeing as a Function of  $\hat{\pi}_1$  and the Price of Equity"]
FigWlow = Plot[W[expi1, 1 - pihat2, exgamma, (1 + eps1) / 0.98 - 1, (1 + eps2) / 0.98 - 1],
{pihat2, .25, .9}, Axes → {True, False}, AxesLabel → {"", ""},
PlotStyle → {Dashing[{0.005, 0.01, 0.01, 0.01}]},
PlotLabel → "Lifetime Wellbeing as a Function of  $\hat{\pi}_1$  and the Price of Equity"]
Show[FigW, FigWhigh, FigWlow]
(*, Axes→{True, False} PlotRange→{-200.025,-199.990}
PlotRange→{-199.9968,-199.9966} GridLines→Automatic PlotLabel→
"Lifetime Wellbeing as a Function of  $\hat{\pi}_1$  and the Price of Equity"*)
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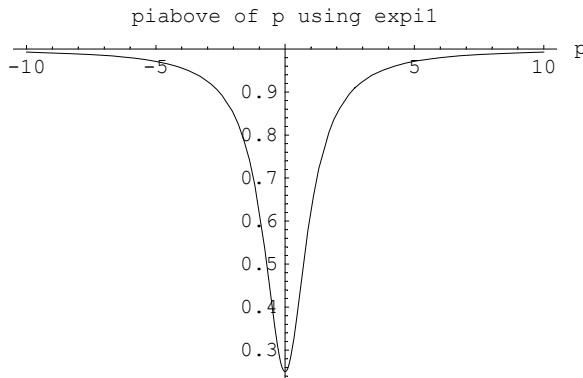




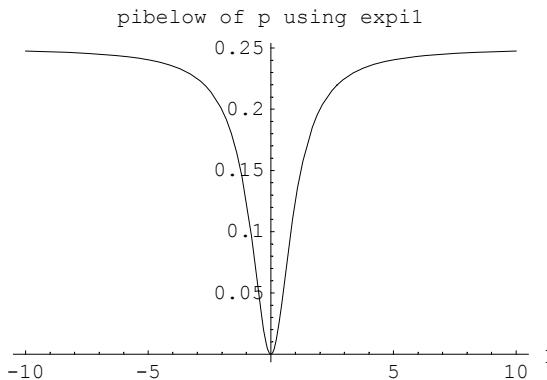


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(*Functions that look for probabilities only above and below expi1 *)
piaboveof[p_, pi_] := pi + (pp / (1 + pp)) (1 - pi)
pibelowof[p_, pi_] := (pp / (1 + pp)) pi
Plot[piaboveof[p, expi1], {p, -10, 10}, AxesLabel -> {"p", "piabove of p using expi1"}]
Plot[pibelowof[p, expi1], {p, -10, 10}, AxesLabel -> {"p", "pibelow of p using expi1"}]
```



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```
pihataboveoe[pi_, γ_, z1_, z2_] := piaboveof[
  phat /. Part[FindMaximum[W[pi, piaboveof[phat, pi], γ, z1, z2], {phat, 3}], 2], pi]
pihatbelowoe[pi_, γ_, z1_, z2_] := pibelowof[
  phat /. Part[FindMaximum[W[pi, pibelowof[phat, pi], γ, z1, z2], {phat, 3}], 2], pi]

αstaraboveoe[pi_, γ_, z1_, z2_] :=
  αof[a /. FindRoot[FOC[αof[a, z1, z2], piaboveof[phat /. Part[
    FindMaximum[W[pi, piaboveof[phat, pi], γ, z1, z2], {phat, 3}], 2], pi]
    , γ, z1, z2] == 0, {a, 1}], z1, z2]
αstarbelowoe[pi_, γ_, z1_, z2_] :=
  αof[a /. FindRoot[FOC[αof[a, z1, z2], pibelowof[phat /. Part[FindMaximum[W[pi, pibelowof[
    phat, pi], γ, z1, z2], {phat, 3}], 2], pi], γ, z1, z2] == 0, {a, 1}], z1, z2]
```

```
(*Solve for the key variables for the example *)
Part[FindMaximum[W[expil, piaboveof[phat, expil],
    exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1], {phat, 1}], 1]
Part[FindMaximum[W[expil, pibelowof[phat, expil], exgamma,
    (1 + eps1) / price - 1, (1 + eps2) / price - 1], {phat, 1}], 1]
pihataboveoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1]
pihatbelowoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1]
αstaraboveoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1]
αstarbelowoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1]
(*The share of agents who short*)
αstarbelowoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1] /
(αstarbelowoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1] -
 αstaraboveoe[expil, exgamma, (1 + eps1) / price - 1, (1 + eps2) / price - 1])
```

-0.998044

-0.998046

0.617276

0.183816

-0.674867

0.190613

0.22024