Appendix of "Evolving Monetary/Fiscal Policy Mix in the United States"

Francesco Bianchi

Duke University

Abstract

This appendix contains a description of the model, the priors, and estimates of all the parameters for the paper *Evolving Monetary/Fiscal Policy Mix in the United States* published in the *American Economic Review, Papers and Proceedings, May 2012.*

1 The Model

The model used in this paper is a small size DSGE similar to the one used by Lubik and Schorfheide (2004), augmented with a fiscal block and a series of additional features as habit persistence and inflation indexation.

The representative household maximizes the following utility function:

$$E_0 \left[\sum_{s=0}^{\infty} \beta^s e^{\widetilde{d}_s} \left[\log \left(C_s - \Phi C_{s-1}^A \right) - h_s \right] \right] \tag{1}$$

subject to the budget constraint:

$$C_t + \frac{B_t}{P_t} + \frac{T_t}{P_t} = W_t h_t + R_{t-1} \frac{B_{t-1}}{P_t} + D_t + \frac{TR_t}{P_t}$$

where D_t stands for dividends paid by the firms, C_t is consumption, h_t is hours, W_t is the real wage, T_t stands for taxes, TR_t stands for transfers, and C_s^A represents the average level of consumption in the economy. The parameter Φ captures the degree of external habit. The preference shock $\widetilde{d}_s$ evolves according to an AR(1) process:

$$\widetilde{d}_t = \rho_d \widetilde{d}_{t-1} + \epsilon_{d,t}$$

Each of the monopolistically competitive firms face a downward-sloping demand curve:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-1/\upsilon} Y_t \tag{2}$$

The parameter $1/\upsilon$ is the elasticity of substitution between two differentiated goods. The firms take as given the general price level, P_t, and level of real activity, Y_t. Whenever a firm wants to change its price, it faces quadratic adjustment costs represented by an output loss:

$$AC_t(j) = \frac{\varphi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi_{t-1}\right)^2 Y_t(j)\frac{P_t(j)}{P_t} \tag{3}$$

where Π_{t-1} is the gross inflation rate that prevailed in the previous period.

Labor is the only input in a linear production function:

$$Y_t(j) = A_t h_t(j) \tag{4}$$

where total factor productivity A_t evolves according to an exogenous process:

$$\ln A_t = \gamma + \ln A_{t-1} + \widetilde{a}_t \tag{5}$$

$$\widetilde{a}_t = \rho_a \widetilde{a}_{t-1} + \epsilon_{a,t} \tag{6}$$

The Central Bank moves the FFR according to the rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_{R,\xi_t^{sp}}} \left[\left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_{\pi,\xi_t^{sp}}} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_{y,\xi_t^{sp}}}\right]^{\left(1-\rho_{R,\xi_t^{sp}}\right)} e^{\epsilon_{R,t}}$$

where R^* is the steady-state (gross) nominal rate, Y_t^* is the target for output, Π^* is the target level for gross inflation, and ξ_t^{sp} is a hidden variable that determines the regime in place at time t.

Total government purchases represent a fraction ζ_t of total output and it is equally divided among the J different goods. We define $g_t = 1/(1-\zeta_t)$ and we assume that $\widetilde{g}_t = \ln(g_t/g^*)$ follows an AR(1) process (this assumption is relaxed in Bianchi and Ilut (2011)):

$$\widetilde{g}_t = \rho_g \widetilde{g}_{t-1} + \epsilon_{g,t} \tag{7}$$

Federal expenditure, i.e. the sum of federal government expenditure and transfers, (as a fraction of GDP and in deviations from the steady state) follows an exogenous process:

$$e_t - e^* = \rho_e\left(e_{t-1} - e^*\right) + \epsilon_{e,t}$$

Finally, the government collects a lump-sum tax according to the rule:

$$\tau_t - \tau^* = \rho_{\tau,\xi_t^{sp}} (\tau_{t-1} - \tau^*)$$
$$+ \left(1 - \rho_{\tau,\xi_t^{sp}}\right) \left[\delta_{b,\xi_t^{sp}} (b_{t-1} - b^*) + \delta_{e,\xi_t^{sp}} (e_t - e^*) + \delta_y \widetilde{y}_t\right] + \epsilon_{\tau,t}$$

The Government budget constraint is given by:

$$e_t + \frac{R_{t-1,t}^b}{\Pi_t G_t} b_{t-1} = b_t + \tau_t$$

where b_t and G_t are the debt to GDP ratio and the gross GDP per capita growth rate, respectively. $R_{t-1,t}^b$ is the return on government bonds between period $t-1$ and period t:

$$\widetilde{R}_{t-1,t}^b = \widetilde{R}_{t-1} + \sum_{i=2}^T x^i \left(\widetilde{R}_{t-1,t}^i - \widetilde{R}_{t-1}\right)$$

where x^i represents the fraction of government bonds with maturity i. The maturity structure is assumed to be constant over time.

The unobserved state variable ξ_t^{sp} captures the monetary/fiscal policy combination that is in place at time t. The unobserved state takes on a finite number of values $j = 1, ..., m^{sp}$ and follows a Markov chain that evolves according to the transition matrix H^{sp}. The target for inflation and debt are assumed to be constant over time. What changes is the strength with which the Government tries to pursue its goals, not the goals themselves. This is in line with the idea that policy makers might find high inflation or high debt acceptable under some circumstances, perhaps in order to preserve output stability, but not desirable in itself. Agents in the model know the probability of moving across regimes and they use this information when forming expectations. The probability of moving across regimes depends only on the regime that is in place at time t.

Heteroskedasticity is modelled as an independent Markov-switching process:

$$\epsilon_t \sim N\left(0, Q\left(\xi_t^{vo}\right)\right), Q\left(\xi_t^{vo}\right) = diag\left(\theta^{vo}\left(\xi_t^{vo}\right)\right)$$

where ξ_t^{vo} is a hidden variable that evolves according to the transition matrix H^{vo}. The benchmark model considers three regimes for the policy rules and two regimes for the stochastic volatilities.

3

2 Solving the MS-DSGE model

First order conditions are derived and all the variables are then rescaled in order to obtain stationarity. The model is then linearized with respect to taxes, government expenditure (e_t), and debt, whereas it is loglinearized with respect to all the other variables. If we define the vector θ containing the parameters of the model and the DSGE state vector S_t, then we can rewrite the system of equations characterizing the linearized model as:

$$\Gamma_0 \left(\xi_t^{sp}, \theta\right) S_t = \Gamma_1 \left(\xi_t^{sp}, \theta\right) S_{t-1} + \Psi \left(\xi_t^{sp}, \theta\right) \epsilon_t + \Pi \eta_t \tag{8}$$

with η_t a vector containing the expectations errors.

A model in this form in which there are no regime changes could be easily solved using the solution method for linear rational expectations models described in Sims (2002). In the current context computations become more complicated because the model is quasi-linear, i.e. it is linear when conditioning on ξ_t^{sp}.

The solution method used in this paper is based on the work of Farmer et al. (2009). The authors show that when a MSV solution exists, it can be characterized as a regime switching vector-autoregression of the kind studied by Hamilton (1989) and Sims and Zha (2006):

$$S_t = T \left(\xi_t^{sp}, \theta^{sp}, H^{sp}\right) S_{t-1} + R \left(\xi_t^{sp}, \theta^{sp}, H^{sp}\right) \epsilon_t \tag{9}$$

It is worth emphasizing that the law of motion of the DSGE states depends on the structural parameters (θ^{sp}), the regime in place (ξ_t^{sp}), and the probability of moving across regimes (H^{sp}). This means that what happens under regime i does not only depend on the structural parameters describing that particular regime, but also on what agents expect is going to happen under alternative regimes and on how likely it is that a regime change will occur in the future. In other words, agents' beliefs matter for the law of motion governing the economy. From now on, a more compact notation will be used: $T \left(\xi_t^{sp}\right) = T \left(\xi_t^{sp}, \theta^{sp}, H^{sp}\right)$ and $R \left(\xi_t^{sp}\right) = R \left(\xi_t^{sp}, \theta^{sp}, H^{sp}\right)$.

3   Estimation strategy and data

The law of motion (9) can be combined with a system of observation equations. The result is a model cast in state space form:

$$X_t = D(\theta^{ss}) + ZS_t + v_t \tag{10}$$

$$S_t = T\left(\xi_t^{sp}\right)S_{t-1} + R\left(\xi_t^{sp}\right)\epsilon_t \tag{11}$$

$$\epsilon_t \sim N\left(0, Q\left(\xi_t^{vo}\right)\right),\ Q\left(\xi_t^{vo}\right) = diag\left(\theta^{vo}\left(\xi_t^{vo}\right)\right)^2 \tag{12}$$

$$v_t \sim N\left(0, U\right),\ U = diag\left(0, \sigma_\pi^2, 0, \sigma_b^2, 0\right) \tag{13}$$

$$p\left(\xi_t^{sp} = i | \xi_{t-1}^{sp} = j\right) = H^{sp}(i,j),\ p\left(\xi_t^{vo} = i | \xi_{t-1}^{vo} = j\right) = H^{vo}(i,j) \tag{14}$$

where:

$$X_t = \begin{bmatrix} \Delta\log\left(GDP_t\right) \\ INFL_t \\ FFR_t^A \\ B_t/GDP_t \\ T_t/GDP_t \end{bmatrix},\ D(\theta^{ss}) = \begin{bmatrix} 0 \\ 100\pi^* \\ 400(\pi^* + r^*) \\ 100b^* \\ 100\tau^* \end{bmatrix}$$

and v_t is a vector containing observation errors. The vector X_t contains five observables: real GDP growth rate, GDP deflator quarterly inflation, annualized quarterly FFR, debt to GDP ratio on a quarterly basis, tax revenues to GDP ratio. Real GDP, the GDP deflator, and the series for tax revenues are obtained from the Bureau of Economic Analysis (NIPA tables: BEA T1.1.6 L1, BEA T1.1.9 L1, total receipt T3.2, L37). The series for the FFR is obtained averaging monthly figures downloaded from the St. Louis Fed website, whereas the series for debt is downloaded from the Dallas Fed website. The sample spans from 1954:III up to 2009:IV.

For a DSGE model with fixed parameters the likelihood can be easily evaluated using the Kalman filter and then combined with a prior distribution for the parameters. When dealing with a MS-DSGE model the Kalman filter cannot be applied in its standard form because, given an observation for X_t, the estimate of the underlying DSGE state vector distribution is not unique. Furthermore, the standard Bayesian updating procedure that is generally used to evaluate the likelihood of Markov-switching models, cannot be applied because it relies on the assumption that Markov states are history independent. This does not occur here: Given that we do not observe S_t, the probability assigned to a particular Markov state depends on the value of S_{t-1}, whose distribution depends on the realization of $\xi^{sp,t-1}$.[1] These issues are handled using the methods described in Bianchi (2011).

---

[1] Here and later on $\xi^{sp,t-1}$ stands for $\{\xi_s^{sp}\}_{s=1}^{t-1}$.

Para	Mean	std.	type	Para	Mean	std.	type	Para	Mean	std.	type
$\psi_{\pi,i}$	1.30	0.50	G	κ	0.50	0.2	G	$\sigma_{R,i}$	0.31	0.40	IG
$\psi_{y,i}$	0.25	0.15	G	ρ_g	0.50	0.2	B	$\sigma_{g,i}$	0.38	0.40	IG
$\rho_{R,i}$	0.50	0.20	B	ρ_a	0.50	0.2	B	$\sigma_{a,i}$	1.00	0.80	IG
$\delta_{b,1}$	0.01	0.005	N	ρ_d	0.50	0.2	B	$\sigma_{\tau,i}$	2.00	2.00	IG
$\delta_{b,2}, \delta_{b,3}$	0.00	0.005	N	ρ_e	0.50	0.2	B	$\sigma_{d,i}$	1.00	0.80	IG
$\delta_{e,i}$	0.25	0.35	N	δ_y	0.20	0.2	G	$\sigma_{e,i}$	2.00	2.00	IG
$\rho_{\tau,i}$	0.50	0.20	B	Φ	0.50	0.2	G	σ_π	0.05	0.15	IG
p_{ii}	$Dir(19,2)$			γ	0.52	0.3	N	σ_b	1.00	2.00	IG

Table 1: Prior distributions for DSGE model parameters. All the volatilities and the growth rate of technology have been rescaled by 100.

4 Estimates

The benchmark model allows for a total of six regimes, *three* for the structural parameters and *two* for the stochastic volatilities. The transition matrix that enters the model and is used by agents to form expectations, H^m, is assumed to coincide with the one observed by the econometrician, H^{sp}. In order to facilitate the search for the posterior mode and reduce the number of parameters to be estimated, I fix some parameters and I assume a circular structure for the transition matrix H^{sp}:

$$
H^{sp} = \begin{bmatrix} H^{sp}_{11} & & 1 - H^{sp}_{33} \\ 1 - H^{sp}_{11} & H^{sp}_{22} & \\ & 1 - H^{sp}_{22} & H^{sp}_{33} \end{bmatrix}
$$

The parameters that are fixed are the steady state for government debt ($b^* = 1$), the inflation target ($\pi^* = .5\%$), and the discount rate ($\beta = .9985$). Tight priors on these parameters would deliver similar results, but they would not help in reducing the computational burden and in facilitating the search for the posterior mode.

The priors reported in Table 1 and are symmetric across regimes, except for the response to debt under Regime 1 ($\delta_{b,1}$), that is a Normal centered on .01, instead of 0. Results obtained under perfectly symmetric priors are virtually identical. However, the asymmetric prior on $\delta_{b,1}$ makes the implementation of the normalization necessary to identify the regimes easier. The results shown below are based on 600,000 Gibbs sampling replications. The first 50,000 draws are disregarded as burn-in and of the remaining 450,000 one every twenty draws is retained.

Table 2 reports the posterior mode estimates and 90% error bands for the DSGE parameters and the transition matrices diagonal elements. Following the terminology introduced

Para	Mode	5%	95%	Para	Mode	5%	95%
$\psi_{\pi,1}$	1.6000	1.5574	2.3422	κ	0.0147	0.0124	0.0326
$\psi_{y,1}$	0.5825	0.4602	0.9408	ρ_g	0.9074	0.8896	0.9597
$\rho_{R,1}$	0.8612	0.8111	0.9107	ρ_a	0.4299	0.3796	0.5678
$\delta_{b,1}$	0.0062	0.0036	0.0176	ρ_d	0.8407	0.8167	0.9023
$\delta_{e,1}$	-0.0491	-0.3644	0.3819	ρ_e	0.8278	0.7469	0.8619
$\rho_{\tau,1}$	0.9180	0.8991	0.9738	δ_y	0.1165	0.0597	0.1519
$\psi_{\pi,2}$	0.9451	0.8250	1.0806	Φ	0.4795	0.2780	0.5006
$\psi_{y,2}$	0.2378	0.1824	0.3654	γ	0.4666	0.4265	0.5485
$\rho_{R,2}$	0.6604	0.4916	0.7187	$\sigma_{R,1}$	0.0636	0.0561	0.0753
$\delta_{b,2}$	-0.0007	-0.0062	0.0066	$\sigma_{g,1}$	0.2626	0.1470	0.3446
$\delta_{e,2}$	0.3355	0.2341	0.4406	$\sigma_{a,1}$	0.4910	0.3849	0.5394
$\rho_{\tau,2}$	0.6392	0.5959	0.7728	$\sigma_{\tau,1}$	0.3248	0.3090	0.3562
$\psi_{\pi,3}$	1.2503	1.1978	2.1995	$\sigma_{d,1}$	2.0364	1.6963	2.3063
$\psi_{y,3}$	0.5849	0.1742	0.6178	$\sigma_{e,1}$	1.2902	1.2334	1.4301
$\rho_{R,3}$	0.2517	0.1039	0.3508	$\sigma_{R,2}$	0.2713	0.2311	0.3543
$\delta_{b,3}$	-0.0036	-0.0078	0.0016	$\sigma_{g,2}$	1.0615	0.7592	1.2473
$\delta_{e,3}$	-0.1766	-0.2674	-0.0600	$\sigma_{a,2}$	0.8236	0.6453	1.0854
$\rho_{\tau,3}$	0.5100	0.3605	0.7324	$\sigma_{\tau,2}$	0.7635	0.6740	0.8863
H_{11}^{sp}	0.9865	0.9734	0.9956	$\sigma_{d,2}$	3.8684	3.1260	4.5286
H_{22}^{sp}	0.9801	0.9029	0.9864	$\sigma_{e,2}$	2.5068	2.0058	3.7142
H_{33}^{sp}	0.9761	0.9330	0.9901	σ_π	0.2221	0.2037	0.2439
H_{11}^{vo}	0.9565	0.9166	0.9705	σ_b	1.9940	1.7747	2.2262
H_{22}^{vo}	0.8608	0.8032	0.8843				

Table 2: Posterior mode estimates and 90% error bands of the DSGE parameters.

by Leeper (1991), Regime 1 corresponds to Active Monetary Policy/Passive Fiscal policy (AM/PF), the regime in which the central bank controls inflation and the fiscal authority is committed to stabilize debt, Regime 2 is characterized by Passive Monetary Policy and Active Fiscal policy (PM/AF), whereas the last regime implies that both policies are active (AM/AF). If taken in isolation, the first two regimes would imply determinacy, whereas the third one would imply non-existence.

Concerning the parameters of the Taylor rule, under Regime 1 ($\xi_t^{sp} = 1$) the Federal Funds rate reacts strongly to deviations of inflation from its target. The opposite occurs under Regime 2. The degree of interest rate smoothing turns out to be quite different across the three regimes and it is remarkably low under Regime 3. This might be explained by accounting for the change in the conduct of monetary policy that occurred during the early years of the Volcker chairmanship.

As for the stochastic volatilities, Regime 2 is the High Volatility regime. Please refer to the paper for impulse responses and regime smoothed probabilities.

References

Bianchi, F.: 2011, Regime switches, agents' beliefs, and post-world War II U.S. Macroeconomic Dynamics, Duke University, working paper.

Bianchi, F. and Ilut, C.: 2011, Monetary/Fiscal Policy Mix and Agents' Beliefs, Duke University, working paper.

Farmer, R. E. A., Waggoner, D. and Zha, T.: 2009, Understanding Markov-Switching Rational Expectations Models, *Journal of Economic Theory* **144**, 1849–1867.

Hamilton, J. D.: 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica* **57**, 357–384.

Leeper, E. M.: 1991, Equilibria Under Active and Passive Monetary And Fiscal Policies, *Journal of Monetary Economics* **27**, 129–147.

Lubik, T. and Schorfheide, F.: 2004, Testing for Indeterminacy: An Application to U.S. Monetary Policy, *American Economic Review* **94**(1), 190–217.

Sims, C. A.: 2002, Solving linear rational expectations models, *Computational Economics* **20**(1), 1–20.

Sims, C. A. and Zha, T.: 2006, Were there Regime Switches in US Monetary Policy?, *American Economic Review* **91**(1), 54–81.