Web Appendices

A. Proofs of Lemmas

Lemma 1 *For all $p > p'$, $G(\cdot\,; p)$ first-order stochastically dominates $G(\cdot\,; p')$.*

Proof. Recall that

$$G(q) = (1 - m_B)\,G_A(q) + m_B G_B(q)$$

where

$$G_\gamma(q) = p\Phi\left(\frac{s_\gamma(q) - 1}{\sigma_\gamma}\right) + (1-p)\,\Phi\left(\frac{s_\gamma(q)}{\sigma_\gamma}\right)$$

$\gamma = A, B$.

Now, $\frac{d}{dp}G_\gamma(q) =$

$$= \Phi\left(\frac{s_\gamma(q) - 1}{\sigma_\gamma}\right) - \Phi\left(\frac{s_\gamma(q)}{\sigma_\gamma}\right) + \left(p\phi\left(\frac{s_\gamma(q) - 1}{\sigma_\gamma}\right) + (1-p)\,\phi\left(\frac{s_\gamma(q)}{\sigma_\gamma}\right)\right)\frac{\partial s_\gamma(q)}{\partial p} < 0$$

because $\frac{\partial s_\gamma(q_\gamma)}{\partial p} = -\frac{\sigma_\gamma^2}{p(1-p)} < 0$ and $\Phi\left(\frac{s_\gamma(q)-1}{\sigma_\gamma}\right) < \Phi\left(\frac{s_\gamma(q)}{\sigma_\gamma}\right)$.

Since $G(q)$ is a convex combination of $G_A(q)$ and $G_B(q)$, it follows that $\frac{d}{dp}G(q) < 0$ for all $q \in (0,1)$. This proves the lemma. ■

Lemma 2 *$G_A(\cdot)$ is a mean preserving spread of $G_B(\cdot)$. And, for all $m_B < m'_B$, $G(\cdot\,; m_B)$ is a mean preserving spread of $G(\cdot\,; m'_B)$.*

Proof. First, we verify that $E_{G_A}[Q_A] = E_{G_B}[Q_B] = p$.

By definition,

$$E_{G_\gamma}[Q_\gamma] = \int_0^1 q g_\gamma(q)\,dq$$

where $\gamma \in \{A, B\}$. Changing the integration variable from probability q to signal s, we get

$$E_{G_\gamma}[Q_\gamma] = \int_{-\infty}^{\infty} q_\gamma(s)\,g_\gamma(s)\,\frac{dq_\gamma(s)}{ds}ds$$

where $q_\gamma(s) = \frac{p\phi\left(\frac{s-1}{\sigma_\gamma}\right)}{p\phi\left(\frac{s-1}{\sigma_\gamma}\right)+(1-p)\phi\left(\frac{s}{\sigma_\gamma}\right)}$, $\frac{\partial q_\gamma(s)}{\partial s} = \frac{p(1-p)}{\sigma_\gamma}\frac{\phi\left(\frac{s}{\sigma_\gamma}\right)\phi\left(\frac{s-1}{\sigma_\gamma}\right)}{\left(p\phi\left(\frac{s-1}{\sigma_\gamma}\right)+(1-p)\phi\left(\frac{s}{\sigma_\gamma}\right)\right)^2}$

and $g_\gamma(s) = \left(p\phi\left(\frac{s-1}{\sigma_\gamma}\right) + (1-p)\,\phi\left(\frac{s}{\sigma_\gamma}\right)\right)\frac{\sigma_\gamma}{q_\gamma(s)(1-q_\gamma(s))}$. Hence,

$$\begin{aligned} E_{G_\gamma}[Q_\gamma] &= \int_{-\infty}^{\infty} q_\gamma(s)\, g_\gamma(s)\, \frac{dq_\gamma(s)}{ds} ds \\ &= p\int_{-\infty}^{\infty} \phi\left(\frac{s-1}{\sigma_\gamma}\right) ds \\ &= p \end{aligned}$$

This proves that $E_{G_A}[Q_A] = E_{G_B}[Q_B] = p$. For later use, note that $E_{G(\cdot;m_B)}[Q] = E_{G\left(\cdot;m'_B\right)}[Q] = p$.

To prove that $G_A(\cdot)$ is a mean preserving spread of $G_B(\cdot)$ it now suffices to show that, on the interval $(0,1)$, $G_B(\cdot)$ crosses $G_A(\cdot)$ only once and from below. We do this by establishing that the difference $D(q) \equiv G_A(q) - G_B(q)$ has two extrema: starting from zero at $q = 0$, $D(q)$ first reaching a maximum—at which $D(q)$ is strictly positive—and then a minimum—at which $D(q)$ is strictly negative.

Let

$$\zeta = \ln\left(\frac{1-q}{q}\frac{p}{1-p}\right)$$

such that

$$\begin{aligned} D &= G_A(q) - G_B(q) \\ &= p\Phi\left(\frac{-\frac{1}{2} - \sigma_A^2\zeta}{\sigma_A}\right) + (1-p)\,\Phi\left(\frac{\frac{1}{2} - \sigma_A^2\zeta}{\sigma_A}\right) \\ &\quad -p\Phi\left(\frac{-\frac{1}{2} - \sigma_B^2\zeta}{\sigma_B}\right) - (1-p)\,\Phi\left(\frac{\frac{1}{2} - \sigma_B^2\zeta}{\sigma_B}\right) \end{aligned}$$

Relying on the fact that ζ is a monotone function of q, we now ask when $\frac{dD}{d\zeta} = 0$:

$$\begin{aligned} \frac{dD}{d\zeta} &= -\sigma_A p\phi\left(\frac{-\frac{1}{2} - \sigma_A^2\zeta}{\sigma_A}\right) - \sigma_A(1-p)\,\phi\left(\frac{\frac{1}{2} - \sigma_A^2\zeta}{\sigma_A}\right) \\ &\quad +\sigma_B p\phi\left(\frac{-\frac{1}{2} - \sigma_B^2\zeta}{\sigma_B}\right) + \sigma_B(1-p)\,\phi\left(\frac{\frac{1}{2} - \sigma_B^2\zeta}{\sigma_B}\right) \\ &= 0 \end{aligned}$$

$\iff$

$$
\begin{aligned}
\frac{\sigma_A}{\sigma_B} &= \frac{\phi\left(\frac{-\frac{1}{2}-\sigma_B^2\zeta}{\sigma_B}\right)+\frac{1-p}{p}\phi\left(\frac{\frac{1}{2}-\sigma_B^2\zeta}{\sigma_B}\right)}{\phi\left(\frac{-\frac{1}{2}-\sigma_A^2\zeta}{\sigma_A}\right)+\frac{1-p}{p}\phi\left(\frac{\frac{1}{2}-\sigma_A^2\zeta}{\sigma_A}\right)} \\
&= \frac{e^{-\frac{1}{2}\left(\frac{-\frac{1}{2}-\sigma_B^2\zeta}{\sigma_B}\right)^2}+\frac{1-p}{p}e^{-\frac{1}{2}\left(\frac{\frac{1}{2}-\sigma_B^2\zeta}{\sigma_B}\right)^2}}{e^{-\frac{1}{2}\left(\frac{-\frac{1}{2}-\sigma_A^2\zeta}{\sigma_A}\right)^2}+\frac{1-p}{p}e^{-\frac{1}{2}\left(\frac{\frac{1}{2}-\sigma_A^2\zeta}{\sigma_A}\right)^2}}
\end{aligned}
$$

Now consider the right-hand side, which we denote by Ψ, as a function of ζ.

$$
\Psi \equiv \frac{e^{-\frac{1}{2}\left(\frac{-\frac{1}{2}-\sigma_B^2\zeta}{\sigma_B}\right)^2}+\frac{1-p}{p}e^{-\frac{1}{2}\left(\frac{\frac{1}{2}-\sigma_B^2\zeta}{\sigma_B}\right)^2}}{e^{-\frac{1}{2}\left(\frac{-\frac{1}{2}-\sigma_A^2\zeta}{\sigma_A}\right)^2}+\frac{1-p}{p}e^{-\frac{1}{2}\left(\frac{\frac{1}{2}-\sigma_A^2\zeta}{\sigma_A}\right)^2}}
$$

$$
\begin{aligned}
&= e^{-\frac{1}{2}\left(\frac{1}{4\sigma_B^2}+\sigma_B^2\zeta^2\right)+\frac{1}{2}\left(\frac{1}{4\sigma_A^2}+\sigma_A^2\zeta^2\right)} \\
&= e^{\left(\frac{1}{8}\left(\frac{1}{\sigma_A^2}-\frac{1}{\sigma_B^2}\right)+\frac{1}{2}\left(\sigma_A^2-\sigma_B^2\right)\zeta^2\right)}
\end{aligned}
$$

Thus, D takes on extrema at values of ζ that solve

$$
\frac{\sigma_A}{\sigma_B} = e^{\left(\frac{1}{8}\left(\frac{1}{\sigma_A^2}-\frac{1}{\sigma_B^2}\right)+\frac{1}{2}\left(\sigma_A^2-\sigma_B^2\right)\zeta^2\right)}
$$

Taking logs

$$
\ln\frac{\sigma_A}{\sigma_B} = \frac{1}{8}\left(\frac{1}{\sigma_A^2}-\frac{1}{\sigma_B^2}\right)+\frac{1}{2}\left(\sigma_A^2-\sigma_B^2\right)\zeta^2
$$

Therefore, the solutions to ζ are roots of the function

$$
\frac{1}{8}\left(\frac{1}{\sigma_A^2}-\frac{1}{\sigma_B^2}\right)+\frac{1}{2}\left(\sigma_A^2-\sigma_B^2\right)\zeta^2-\ln\frac{\sigma_A}{\sigma_B}
$$

These roots are

$$
\zeta = \frac{-1}{2\sigma_A\sigma_B\ln\frac{\sigma_B}{\sigma_A}},\ \zeta = \frac{1}{2\sigma_A\sigma_B\ln\frac{\sigma_B}{\sigma_A}}
$$

The existence of two distinct roots for ζ (and hence for q) implies that G_A and G_B cross each other exactly once. It remains to verify that G_B crosses G_A from below and not from above. Now,

$$
\begin{aligned}
D &= G_A(q) - G_B(q) \\
&= p\left(G_{A1}-G_{B1}\right)+(1-p)\left(G_{A0}-G_{B0}\right)
\end{aligned}
$$

At $q = \underline{q}^I = \frac{1}{1+\frac{1-p}{p}e^{\frac{1}{2\sigma_A \sigma_B}}}$, $G_{A1} - G_{B1} = 0$ while $G_{A0} - G_{B0} > 0$. Hence, $D\left(\underline{q}^I\right) > 0$.

At $q = \underline{q}^{II} = \frac{1}{1+\frac{1-p}{p}e^{-\frac{1}{2\sigma_A \sigma_B}}}$, $G_{A0} - G_{B0} = 0$ while $G_{A1} - G_{B1} < 0$. Hence, $D\left(\underline{q}^{II}\right) < 0$.

Now, because $\underline{q}^I < \underline{q}^{II}$, this implies that G_B crosses G_A from below.

This completes the proof that $G_A(\cdot)$ is a mean-preserving spread of $G_B(\cdot)$.

Finally, to prove that $G(\cdot; m_B)$ is a mean preserving spread of $G(\cdot; m'_B)$ for all $m_B < m'_B$, it remains to show that $G(\cdot; m_B)$ second-order stochastically dominates $G(\cdot; m'_B)$. Or,

$$\int_0^{\hat{q}} G(q, m_B)\, dq - \int_0^{\hat{q}} G(q, m'_B)\, dq \leq 0$$

for all $\hat{q} \in (0,1)$, with strict inequality for some $\hat{q}$. Now,

$$\begin{aligned} & \int_0^{\hat{q}} G(q, m_B)\, dq - \int_0^{\hat{q}} G(q, m'_B)\, dq \\ = \;& (m'_B - m_B) \int_0^{\hat{q}} \left(G_A(q) - G_B(q)\right) dq \leq 0 \end{aligned}$$

where the weak inequality for all $\hat{q}$, and the strict inequality for some $\hat{q}$, follow from the fact that $G_B(\cdot)$ second-order stochastically dominates $G_A(\cdot)$.

This completes the proof. ■

Lemma 3 *There exists a unique threshold,* $\underline{q}^I \equiv \frac{1}{1+\frac{1-p}{p}e^{\frac{1}{2\sigma_A \sigma_B}}} < p$, *where the probability of type I error is the same for both kinds of candidates.*

Proof.

$$G_{A1}\left(\underline{q}\right) = G_{B1}\left(\underline{q}\right)$$

$\Leftrightarrow$

$$\Phi\left(\frac{s_A\left(\underline{q}\right) - 1}{\sigma_A}\right) = \Phi\left(\frac{s_B\left(\underline{q}\right) - 1}{\sigma_B}\right)$$

$\Leftrightarrow$

$$\frac{s_A\left(\underline{q}\right) - 1}{\sigma_A} = \frac{s_B\left(\underline{q}\right) - 1}{\sigma_B}$$

where

$$s_\gamma(q) = \frac{1}{2} - \sigma_\gamma^2 \ln\left(\frac{1-q}{q}\frac{p}{1-p}\right)$$

Hence,

$$\frac{\frac{1}{2} - \sigma_A^2 \ln\left(\left(\frac{1}{\underline{q}} - 1\right)\frac{p}{1-p}\right) - 1}{\sigma_A} = \frac{\frac{1}{2} - \sigma_B^2 \ln\left(\left(\frac{1}{\underline{q}} - 1\right)\frac{p}{1-p}\right) - 1}{\sigma_B}$$

$$\sigma_A Z + \frac{1}{2\sigma_A} = \sigma_B Z + \frac{1}{2\sigma_B}$$

$$\begin{aligned}
\sigma_B \sigma_A^2 \ln\left(\left(\frac{1}{\underline{q}} - 1\right)\frac{p}{1-p}\right) + \frac{1}{2}\sigma_B &= \sigma_A \sigma_B^2 \ln\left(\left(\frac{1}{\underline{q}} - 1\right)\frac{p}{1-p}\right) + \frac{1}{2}\sigma_A \\
Z &= \frac{1}{2\sigma_B \sigma_A}
\end{aligned}$$

$\Leftrightarrow$

$$\underline{q} = \frac{1}{1 + \frac{1-p}{p} e^{\frac{1}{2\sigma_A \sigma_B}}}$$

■

Lemma 4 *Suppose* $\underline{q} > p$. *Then:*

1. *The distribution* G_{A1} *dominates* G_{B1} *in terms of the likelihood ratio.*
2. *The distribution* G_{A0} *dominates* G_{B0} *in terms of the likelihood ratio.*

Proof. To establish this, it is sufficient to show that $\frac{\partial^2 \ln g_{A1}}{\partial\sigma\partial q} > 0$.

$$\begin{aligned}
\frac{\partial^2 \ln g_{A1}}{\partial\sigma\partial q} &= \frac{\partial^2 \ln \phi\left(\frac{s(q)-1}{\sigma}\right)\frac{\sigma}{q(1-q)}}{\partial\sigma\partial q} \\
&= \frac{\partial^2 \ln\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s(q)-1}{\sigma}\right)^2} \frac{\sigma}{q(1-q)}\right)}{\partial\sigma\partial q} \\
&= \frac{2}{q}\sigma \frac{\ln\left(\frac{1-q}{q}\frac{p}{1-p}\right)}{1-q} > 0
\end{aligned}$$

where the inequality holds since $q > p$. The proof of part 2 of the Lemma is virtually identical. ■

Lemma 5 *Suppose* $\underline{q} > p$. *Then:*

1. *The distribution* G_{A1} *dominates* G_{B1} *in terms of the hazard rate.*
2. *The distribution* G_{A0} *dominates* G_{B0} *in terms of the hazard rate.*

Proof. Lemma 4 implies that

$$\frac{g_{B1}(q')}{g_{B1}(q)} < \frac{g_{A1}(q')}{g_{A1}(q)}$$

for all $p < q < q'$.

Hence,

$$\int_q^1 \frac{g_{A1}(t)}{g_{A1}(q)}dt > \int_q^1 \frac{g_{B1}(t)}{g_{B1}(q)}$$
$$\frac{1-G_{A1}(q)}{g_{A1}(q)} > \frac{1-G_{B1}(q)}{g_{B1}(q)}$$

or, equivalently,

$$\frac{g_{A1}(q)}{1-G_{A1}(q)} < \frac{g_{B1}(q)}{1-G_{B1}(q)}$$

The proof of part 2 of the lemma is virtually identical. ■

B. Proofs of Propositions

Proposition 1 *The optimal threshold, $\underline{q}^*$, is the unique interior solution to*

$$\underline{q}^* = \frac{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 qdG(q)\right)\right)c}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k}$$

Proof. Recall that

$$\begin{aligned} V\left(\underline{q}\right) &= \frac{\delta\int_{\underline{q}}^1 (qv+(1-q)(-c))\,dG(q)-k}{1-\delta\left(1-\int_{\underline{q}}^1 qdG(q)\right)} \\ &= \frac{\delta v\int_{\underline{q}}^1 qdG(q)-\delta c\left(1-G\left(\underline{q}\right)\right)+\delta c\int_{\underline{q}}^1 qdG(q)-k}{1-\delta\left(1-\int_{\underline{q}}^1 qdG(q)\right)} \end{aligned}$$

It is useful to represent this as numerator and denominator components for purposes of differentiation. Hence, define

$$N \equiv \delta\int_{\underline{q}}^1 (qv+(1-q)(-c))\,dG(q)-k$$

and

$$D \equiv 1-\delta\left(1-\int_{\underline{q}}^1 qdG(q)\right)$$

Thus, the first-order necessary condition for optimality, $\frac{\partial V(\underline{q})}{\partial \underline{q}}=0$, may be expressed as

$$\frac{DN'-ND'}{D^2}=0$$

Therefore,

$$\begin{aligned}\frac{\partial V\left(\underline{q}\right)}{\partial \underline{q}} &= \frac{D\left(-\delta g\left(\underline{q}\right)\left(\left(v+c\right)\underline{q}-c\right)\right)-N\left(-\delta \underline{q} g\left(\underline{q}\right)\right)}{D^2} \\ &= \delta g\left(\underline{q}\right)\frac{-D\left(v+c\right)\underline{q}+Dc+N\underline{q}}{D^2}\end{aligned}$$

Hence,

$$-D\left(v+c\right)\underline{q}+Dc+N\underline{q}=0$$

and this implies that

$$\underline{q}^* = \frac{Dc}{D\left(v+c\right)-N}$$

Substituting for D and N, and simplifying, we get the following implicit characterization of $\underline{q}^*$:

$$\begin{aligned}\underline{q}^* &= \frac{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 qdG\left(q\right)\right)\right)c}{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 qdG\left(q\right)\right)\right)\left(v+c\right)-\delta\int_{\underline{q}^*}^1\left(qv+\left(1-q\right)\left(-c\right)\right)dG\left(q\right)+k} \\ &= \frac{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 qdG\left(q\right)\right)\right)c}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+\left(1-\delta\right)v+k}\end{aligned}$$

and this yields the expression in Lemma 1.

Having derived the necessary first-order condition for an interior solution $\underline{q}^* \in (0,1)$, we now prove its actual existence.

At $\underline{q}^*=0$, LHS $<$ RHS. At $\underline{q}^*=1$, LHS $>$ RHS. Hence, by continuity and the intermediate value theorem, there must be a $\underline{q}^* \in (0,1)$ such that LHS $=$ RHS.

Next, we prove uniqueness by showing that there is at most one $\underline{q}^* \in (0,1)$ that satisfies the necessary first-order condition.

To see this, first notice that we may rewrite the first-order condition as follows:

$$\underline{q}^*\left(c+\left(1-\delta\right)v+k\right)=c-c\delta\left(1-\int_{\underline{q}^*}^1 qdG\left(q\right)\right)+\delta G\left(\underline{q}^*\right)c\underline{q}^*$$

Integrating by parts, we obtain

$$\underline{q}^*\left(c+\left(1-\delta\right)v+k\right)=c-c\delta\int_{\underline{q}^*}^1 G\left(q\right)dq$$

Adding and subtracting $c\delta\int_0^{\underline{q}^*} G(q)\,dq$ to the right-hand side yields

$$\underline{q}^*(c+(1-\delta)v+k) = c - c\delta\int_0^1 G(q)\,dq + c\delta\int_0^{\underline{q}^*} G(q)\,dq$$

Finally, noting that $\int_0^1 G(q)\,dq = 1-p$ and substituting, we obtain

$$\underline{q}^*(c+(1-\delta)v+k) = c(1-\delta) + c\delta\left(p + \int_0^{\underline{q}^*} G(q)\,dq\right)$$

Hence,

$$\underline{q}^* = \frac{(1-\delta)c + c\delta p}{(c+(1-\delta)v+k)} + \frac{c\delta}{(c+(1-\delta)v+k)}\int_0^{\underline{q}^*} G(q)\,dq$$

Note that the right-hand side is monotonically increasing in $\underline{q}^*$ at a speed < 1, for all $\underline{q}^* \in (0,1)$. This implies, however, that the right-hand side can cross the 45-degree line, which corresponds to the left-hand side, at most once. Hence, there is at most one $\underline{q}^* \in (0,1)$ that satisfies the necessary first-order condition.

Finally, we show that at the unique interior $\underline{q}^*$, the value function reaches a global maximum. This follows from the observation that $\lim_{\underline{q}\to 1} V(\underline{q}) \to -\infty$, and that there exists an $\varepsilon > 0$ such that for all $0 < \underline{q} < \varepsilon$, $\frac{\partial V(\underline{q})}{\partial \underline{q}} > 0$. To see that the latter assertion is indeed true, recall that

$$V(\underline{q}) = \frac{\delta\int_{\underline{q}}^1 (qv + (1-q)(-c))\,dG(q) - k}{1-\delta\left(1-\int_{\underline{q}}^1 q\,dG(q)\right)}$$

and that

$$\frac{\partial V(\underline{q})}{\partial \underline{q}} = \delta g(\underline{q})\frac{-D(v+c)\underline{q} + Dc + N\underline{q}}{D^2}$$

where N and D denote the numerator and the denominator of $V(\underline{q})$, respectively.

Now we rewrite $\frac{\partial V(\underline{q})}{\partial \underline{q}}$ to get

$$\frac{\partial V(\underline{q})}{\partial \underline{q}} = \delta g(\underline{q})\left(\frac{c}{D} + \frac{V(\underline{q}) - (v+c)}{D}\underline{q}\right)$$

Written in this form, it is obvious that, for sufficiently small $\underline{q} > 0$, both factors in the last expression are strictly positive. This proves the proposition. ■

Proposition 2 *For all $\underline{q} \in (0,1)$, there exist parameter values such that $\underline{q}^* = \underline{q}$.*

Proof. Fix $k = 0$. In that case, the employer will always wish to participate by interviewing candidates rather than eschewing the employment market. When $c \downarrow 0$, the right-hand side of equation (2) goes to zero; hence, $\underline{q}^* \downarrow 0$. When $c \to \infty$, the right-hand side of equation (2) goes to 1 as the following argument shows:

$$\begin{aligned} & \lim_{c\to\infty} \frac{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 q dG(q)\right)\right)c}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v} \\ \geq & \lim_{c\to\infty} \frac{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 dG(q)\right)\right)c}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v} \\ = & \lim_{c\to\infty} \frac{\left(1-\delta G\left(\underline{q}^*\right)\right)c}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v} = 1 \end{aligned}$$

Hence, $\lim_{c\to\infty} \underline{q}^* = 1$. Finally, since the right-hand side of equation (2) is continuous in c, it follows that there exist parameter values such that $\underline{q}^* = \underline{q}$ for all $\underline{q} \in (0,1)$. ■

Proposition 3

1. Minorities are overrepresented in the workplace iff the employer's optimal search strategy leads to lower Type I error for minorities than for majorities, i.e., $0 < \underline{q}^* < \underline{q}^I$.

2. Minorities are underrepresented iff the employer's optimal search strategy leads to higher Type I error for minorities than for majorities, i.e., $\underline{q}^I \leq \underline{q}^* < 1$.

Proof. Under a uniform threshold success probability $\underline{q}$, $\frac{r_B}{m_B} = 1$ iff $G_{A1}\left(\underline{q}\right) = G_{B1}\left(\underline{q}\right)$. As we saw in Lemma 3, this corresponds to $\underline{q} = \underline{q}^I = \frac{1}{1+\frac{1-p}{p}e^{\frac{1}{2\sigma_A\sigma_B}}}$. To prove the proposition, we show that at the critical point $\underline{q}^I$, raising $\underline{q}$ leads to strict underrepresentation of minorities. That is, we calculate the derivative of

$$G_{A1}(q) - G_{B1}(q) = \Phi\left(\frac{s_A(q)-1}{\sigma_A}\right) - \Phi\left(\frac{s_B(q)-1}{\sigma_B}\right)$$

with respect to q, evaluate it at $\underline{q}^I = \frac{1}{1+\frac{1-p}{p}e^{\frac{1}{2\sigma_A\sigma_B}}}$ and show that it is strictly negative.

The derivative is equal to

$$g_{A1}(q) - g_{B1}(q) = \phi\left(\frac{s_A(q)-1}{\sigma_A}\right)\frac{\sigma_A}{q(1-q)} - \phi\left(\frac{s_B(q)-1}{\sigma_B}\right)\frac{\sigma_B}{q(1-q)}$$

Multiplying by $q(1-q)$ and evaluating at $\underline{q}^I$, we get

$$\begin{aligned}
&= \phi\left(\frac{\frac{1}{2}\frac{\sigma_B-\sigma_A}{\sigma_B}-1}{\sigma_A}\right)\sigma_A - \phi\left(\frac{-\frac{1}{2}\frac{\sigma_B-\sigma_A}{\sigma_A}-1}{\sigma_B}\right)\sigma_B \\
&= \phi\left(-\frac{1}{2}\frac{\sigma_B+\sigma_A}{\sigma_A\sigma_B}\right)\sigma_A - \phi\left(-\frac{1}{2}\frac{\sigma_B+\sigma_A}{\sigma_A\sigma_B}\right)\sigma_B \\
&= (\sigma_A-\sigma_B)\,\phi\left(\frac{1}{2}\frac{\sigma_B+\sigma_A}{\sigma_A\sigma_B}\right) < 0
\end{aligned}$$

This proves the proposition. ■

Proposition 4 *Suppose that the employer is "selective," i.e., $\underline{q}^* > p$, then:*

1. As the employer becomes more selective, minority representation in the workplace decreases. Formally, r_B is decreasing in $\underline{q}^*$.

2. As the employer becomes arbitrarily selective, minorities vanish from the workplace. Formally, $\lim_{\underline{q}^*\to 1} r_B = 0$.

Proof. To prove part 1, differentiate r_B with respect to $\underline{q}$:

$$\begin{aligned}
\frac{\partial r_B}{\partial \underline{q}} &= \frac{-m_B g_{B1}(1-m_B G_{B1}-m_A G_{A1}) - (-m_B g_{B1} - m_A g_{A1})\, m_B (1-G_{B1})}{(1-m_B G_{B1}-m_A G_{A1})^2} \\
&= \frac{m_B m_A (g_{A1}(1-G_{B1}) - g_{B1}(1-G_{A1}))}{(1-m_B G_{B1}-m_A G_{A1})^2}
\end{aligned}$$

Notice that the sign of $\frac{\partial r_B}{\partial \underline{q}}$ depends only on the hazard rates of G_{A1} and G_{B1}. And by Lemma 5 it then follows that $\frac{\partial r_B}{\partial \underline{q}} < 0$.

To prove part 2 of the proposition, notice that (via L'Hôpital's rule)

$$\lim_{\underline{q}\to 1} r_B = \lim_{\underline{q}\to 1} \frac{m_B}{m_B + m_A \frac{g_{A1}}{g_{B1}}}$$

and this limit depends solely on the limit of the likelihood ratio, $\frac{g_{A1}}{g_{B1}}$. Finally, it may be readily shown that:

$$\begin{aligned}
\lim_{\underline{q}\to 1} \frac{g_{A1}}{g_{B1}} &= \lim_{\underline{q}\to 1} \frac{\phi\left(\frac{s_A(q)-1}{\sigma_A}\right)\sigma_A}{\phi\left(\frac{s_B(q)-1}{\sigma_B}\right)\sigma_B} \\
&= \lim_{\underline{q}\to 1} e^{\frac{1}{8\sigma_A^2\sigma_B^2}\left(4\sigma_A^2\sigma_B^2\ln^2\left(\frac{q}{1-q}\right)-1\right)\left(\sigma_B^2-\sigma_A^2\right)}\frac{\sigma_A}{\sigma_B} \to \infty
\end{aligned}$$

Hence,

$$\lim_{\underline{q}\to 1} r_B = 0$$

■

Proposition 5 *Minorities are fired at higher rates than majorities.*

Proof. Because hires are fired if and only if they turn out to be incompetent, we have to prove that

$$\Pr\left(\theta_A = 0 \mid q_A \geq \underline{q}\right) = \frac{(1 - G_{A0})(1-p)}{1 - G_A} < \frac{(1 - G_{B0})(1-p)}{1 - G_B} = \Pr\left(\theta_B = 0 \mid q_B \geq \underline{q}\right)$$

for all $\underline{q} \in (0,1)$.

This is equivalent to showing that

$$\frac{1 - G_{A0}}{1 - G_A} < \frac{1 - G_{B0}}{1 - G_B}$$

or

$$\frac{1 - G_B}{1 - G_{B0}} < \frac{1 - G_A}{1 - G_{A0}}$$

Now,

$$\begin{aligned}
\frac{1 - G_B}{1 - G_{B0}} &< \frac{1 - G_A}{1 - G_{A0}} \iff \\
\frac{1 - pG_{B1} - (1-p)\,G_{B0}}{1 - G_{B0}} &< \frac{(1 - pG_{A1} - (1-p)\,G_{A0})}{1 - G_{A0}} \iff \\
\frac{1 - G_{B1}}{1 - G_{B0}} &< \frac{1 - G_{A1}}{1 - G_{A0}}
\end{aligned}$$

Hence, showing that $\Pr\left(\theta_A = 0 \mid q_A \geq \underline{q}\right) < \Pr\left(\theta_B = 0 \mid q_B \geq \underline{q}\right)$ is equivalent to showing that the ratio of good hiring decisions over bad hiring decisions, $\frac{1-G_{\gamma 1}}{1-G_{\gamma 0}}$, is greater for kind A hires than for kind B hires. To prove the latter, we show that

$$\frac{d}{d\sigma_\gamma}\left[\frac{1 - G_{\gamma 1}\left(\underline{q}\right)}{1 - G_{\gamma 0}\left(\underline{q}\right)}\right] < 0$$

Now, $\frac{d}{d\sigma_\gamma}\left[\frac{1-G_{\gamma 1}\left(\underline{q}\right)}{1-G_{\gamma 0}\left(\underline{q}\right)}\right]$

$$\begin{aligned}
&= \frac{d}{d\sigma_\gamma}\left[\frac{\int_{\underline{q}}^{1} g_{\gamma 1}(q)\,dq}{\int_{\underline{q}}^{1} g_{\gamma 0}(q)\,dq}\right] \\
&= \frac{d}{d\sigma_\gamma}\left[\frac{\int_{\underline{q}}^{1} \phi\left(\frac{s_\gamma(q)-1}{\sigma_\gamma}\right)\frac{\sigma_\gamma}{q(1-q)}dq}{\int_{\underline{q}}^{1} \phi\left(\frac{s_\gamma(q)}{\sigma_\gamma}\right)\frac{\sigma_\gamma}{q(1-q)}dq}\right]
\end{aligned}$$

Using that $\frac{ds_\gamma(q)}{d\sigma_\gamma} = \frac{2\left(s_\gamma(q)-\frac{1}{2}\right)}{\sigma_\gamma}$, straightforward algebra leads to the conclusion that the sign of $\frac{d}{d\sigma_\gamma}\left[\frac{1-G_{\gamma 1}\left(\underline{q}\right)}{1-G_{\gamma 0}\left(\underline{q}\right)}\right]$ is equal to the sign of

$$\int_{\underline{q}}^{1} g_{\gamma 1}(q)\, dq \int_{\underline{q}}^{1} s_\gamma(q)\left(s_\gamma(q)-1\right) g_{\gamma 0}(q)\, dq - \int_{\underline{q}}^{1} g_{\gamma 0}(q)\, dq \int_{\underline{q}}^{1} s_\gamma(q)\left(s_\gamma(q)-1\right) g_{\gamma 1}(q)\, dq$$

Changing variables of integration from q to s, we get

$$\begin{aligned}&\int_{s_\gamma\left(\underline{q}\right)}^{1} g_{\gamma 1}(s)\frac{\partial q_\gamma(s)}{\partial s} ds \int_{s_\gamma\left(\underline{q}\right)}^{1} s(s-1)\, g_{\gamma 0}(s)\frac{\partial q_\gamma(s)}{\partial s} ds \\ &- \int_{s_\gamma\left(\underline{q}\right)}^{1} g_{\gamma 0}(s)\frac{\partial q_\gamma(s)}{\partial s} ds \int_{s_\gamma\left(\underline{q}\right)}^{1} s(s-1)\, g_{\gamma 1}(s)\frac{\partial q_\gamma(s)}{\partial s} ds\end{aligned}$$

Substituting for $g_{\gamma 0}$, $g_{\gamma 1}$, and $\frac{\partial q_\gamma(s)}{\partial s}$,

$$\int_{s_\gamma\left(\underline{q}\right)}^{1} \phi\left(\frac{s-1}{\sigma_\gamma}\right) ds \int_{s_\gamma\left(\underline{q}\right)}^{1} s(s-1)\,\phi\left(\frac{s}{\sigma_\gamma}\right) ds - \int_{s_\gamma\left(\underline{q}\right)}^{1} \phi\left(\frac{s}{\sigma_\gamma}\right) ds \int_{s_\gamma\left(\underline{q}\right)}^{1} s(s-1)\,\phi\left(\frac{s-1}{\sigma_\gamma}\right) ds$$

Expanding $s(s-1)$,

$$\begin{aligned}&\int_{s_\gamma\left(\underline{q}\right)}^{1} \phi\left(\frac{s-1}{\sigma_\gamma}\right) ds \left(\int_{s_\gamma\left(\underline{q}\right)}^{1} s^2\phi\left(\frac{s}{\sigma_\gamma}\right) ds - \int_{s_\gamma\left(\underline{q}\right)}^{1} s\phi\left(\frac{s}{\sigma_\gamma}\right) ds\right) \\ &- \int_{s_\gamma\left(\underline{q}\right)}^{1} \phi\left(\frac{s}{\sigma_\gamma}\right) ds \left(\int_{s_\gamma\left(\underline{q}\right)}^{1} s^2\phi\left(\frac{s-1}{\sigma_\gamma}\right) ds - \int_{s_\gamma\left(\underline{q}\right)}^{1} s\phi\left(\frac{s-1}{\sigma_\gamma}\right) ds\right)\end{aligned}$$

Writing in terms of conditional expectations,

$$\begin{aligned}&\left(1-\Phi\left(\frac{s_\gamma\left(\underline{q}\right)-1}{\sigma_\gamma}\right)\right)\left(1-\Phi\left(\frac{s_\gamma\left(\underline{q}\right)}{\sigma_\gamma}\right)\right)\left(E\left[S_{\gamma 0}^2 \mid S_{\gamma 0} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 0} \mid S_{\gamma 0} \geq s_\gamma\left(\underline{q}\right)\right]\right) \\ &- \left(1-\Phi\left(\frac{s_\gamma\left(\underline{q}\right)}{\sigma_\gamma}\right)\right)\left(1-\Phi\left(\frac{s_\gamma\left(\underline{q}\right)-1}{\sigma_\gamma}\right)\right)\left(E\left[S_{\gamma 1}^2 \mid S_{\gamma 1} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 1} \mid S_{\gamma 1} \geq s_\gamma\left(\underline{q}\right)\right]\right)\end{aligned}$$

Dividing by the common positive factor $\left(1-\Phi\left(\frac{s_\gamma\left(\underline{q}\right)-1}{\sigma_\gamma}\right)\right)\left(1-\Phi\left(\frac{s_\gamma\left(\underline{q}\right)}{\sigma_\gamma}\right)\right)$:

$$E\left[S_{\gamma 0}^2 \mid S_{\gamma 0} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 0} \mid S_{\gamma 0} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 1}^2 \mid S_{\gamma 1} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 1} \mid S_{\gamma 1} \geq s_\gamma\left(\underline{q}\right)\right]$$

Now, the moment generating function, mgf, of a left-truncated standard normal random variable U with truncation point d is (see, for example, Heckman and Honoré, 1990):

$$mgf(\beta) = e^{\frac{1}{2}\beta^2}\frac{\int_{d-\beta}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}u^2\right) du}{\int_{d}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}u^2\right) du}$$

Hence,

$$\begin{aligned} E[U \mid U \geq d] &= \frac{\partial mgf}{\partial \beta}|_{\beta=0} \\ &= \frac{\phi(d)}{1-\Phi(d)} \end{aligned}$$

while

$$\begin{aligned} E\left[U^2 \mid U \geq d\right] &= \frac{\partial^2 mgf}{\partial \beta^2}|_{\beta=0} \\ &= 1 + d\frac{\partial mgf}{\partial \beta}|_{\beta=0} \\ &= 1 + \frac{d\phi(d)}{1-\Phi(d)} \end{aligned}$$

For $X \sim N(\mu, \sigma^2)$, this implies

$$\begin{aligned} E[X \mid X \geq d'] &= \mu + \frac{\sigma\phi\left(\frac{d'-\mu}{\sigma}\right)}{1-\Phi\left(\frac{d'-\mu}{\sigma}\right)} \\ E\left[X^2 \mid X \geq d'\right] &= \sigma^2 + (\mu + d')\frac{\sigma\phi\left(\frac{d'-\mu}{\sigma}\right)}{1-\Phi\left(\frac{d'-\mu}{\sigma}\right)} + \mu^2 \end{aligned}$$

Now, recall that $S_{\gamma 0} \sim N(0, \sigma_\gamma)$ and $S_{\gamma 1} \sim N(1, \sigma_\gamma)$. Hence,

$$E\left[S_{\gamma 0}^2 \mid S_{\gamma 0} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 0} \mid S_{\gamma 0} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 1}^2 \mid S_{\gamma 1} \geq s_\gamma\left(\underline{q}\right)\right] - E\left[S_{\gamma 1} \mid S_{\gamma 1} \geq s_\gamma\left(\underline{q}\right)\right]$$

$$\begin{aligned} &= \sigma_\gamma^2 + s_\gamma\left(\underline{q}\right)\frac{\sigma_\gamma\phi\left(\frac{s_\gamma(\underline{q})}{\sigma_\gamma}\right)}{1-\Phi\left(\frac{s_\gamma(\underline{q})}{\sigma_\gamma}\right)} - \frac{\sigma_\gamma\phi\left(\frac{s_\gamma(\underline{q})}{\sigma_\gamma}\right)}{1-\Phi\left(\frac{s_\gamma(\underline{q})}{\sigma_\gamma}\right)} \\ &\quad - \left(\sigma_\gamma^2 + \left(1 + s_\gamma\left(\underline{q}\right)\right)\frac{\sigma_\gamma\phi\left(\frac{s_\gamma(\underline{q})-1}{\sigma_\gamma}\right)}{1-\Phi\left(\frac{s_\gamma(\underline{q})-1}{\sigma_\gamma}\right)} + 1 - 1 - \frac{\sigma_\gamma\phi\left(\frac{s_\gamma(\underline{q})-1}{\sigma_\gamma}\right)}{1-\Phi\left(\frac{s_\gamma(\underline{q})-1}{\sigma_\gamma}\right)}\right) \end{aligned}$$

Dividing by σ_γ and collecting terms, we get

$$\left(s_\gamma\left(\underline{q}\right) - 1\right)\frac{\phi\left(\frac{s_\gamma(\underline{q})}{\sigma_\gamma}\right)}{1-\Phi\left(\frac{s_\gamma(\underline{q})}{\sigma_\gamma}\right)} - s_\gamma\left(\underline{q}\right)\frac{\phi\left(\frac{s_\gamma(\underline{q})-1}{\sigma_\gamma}\right)}{1-\Phi\left(\frac{s_\gamma(\underline{q})-1}{\sigma_\gamma}\right)}$$

Hence, the question is whether

$$(s-1)\frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)} - s\frac{\phi\left(\frac{s-1}{\sigma}\right)}{1-\Phi\left(\frac{s-1}{\sigma}\right)} < 0$$

$$\frac{s-1}{\sigma}\frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)} - \frac{s}{\sigma}\frac{\phi\left(\frac{s-1}{\sigma}\right)}{1-\Phi\left(\frac{s-1}{\sigma}\right)} < 0$$

for all $s \in \mathbb{R}$ and $\sigma > 0$.

Denote hazard rate $\frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)}$ by $\lambda\left(\frac{s}{\sigma}\right)$. The expression then becomes

$$(s-1)\lambda\left(\frac{s}{\sigma}\right) - s\lambda\left(\frac{s-1}{\sigma}\right)$$

Graphically, when $s-1<0$, [**Figure 3 Here**]

Hence, for all $s-1<0$, it is obvious that

$$(s-1)\lambda\left(\frac{s}{\sigma}\right) - s\lambda\left(\frac{s-1}{\sigma}\right) < 0$$

When $s-1>0$, graphically, [**Figure 4 Here**].

Here, in principle, it could go either way.

Now, for $s-1>0$,

$$\begin{aligned} & (s-1)\lambda\left(\frac{s}{\sigma}\right) - s\lambda\left(\frac{s-1}{\sigma}\right) \\ = \; & (s-1)\left(\lambda\left(\frac{s}{\sigma}\right) - \lambda\left(\frac{s-1}{\sigma}\right)\right) - (s-(s-1))\lambda\left(\frac{s-1}{\sigma}\right) \\ \leq \; & \int_{\lambda\left(\frac{s-1}{\sigma}\right)}^{\lambda\left(\frac{s}{\sigma}\right)} \lambda^{-1}(l)\,dl - \int_{s-1}^{s} \lambda\left(\frac{x}{\sigma}\right)dx \end{aligned}$$

where the inequality follows from the convexity of $\lambda\left(\frac{s}{\sigma}\right)$.

Changing the variable of integration in the first term from hazard rate l to signal x, the last expression becomes

$$\begin{aligned} &= \int_{s-1}^{s} x\frac{\partial l}{\partial x}dx - \int_{s-1}^{s}\lambda\left(\frac{x}{\sigma}\right)dx \\ &= \int_{s-1}^{s} \frac{x}{\sigma}\lambda'\left(\frac{x}{\sigma}\right)dx - \int_{s-1}^{s}\lambda\left(\frac{x}{\sigma}\right)dx \\ &= \int_{s-1}^{s}\left(\frac{x}{\sigma}\lambda'\left(\frac{x}{\sigma}\right) - \lambda\left(\frac{x}{\sigma}\right)\right)dx \end{aligned}$$

Finally, we show that the integrand, which we write as

$$z\lambda'(z) - \lambda(z)$$

is negative for all $z \geq 0$.

First, note that

$$\begin{aligned}
\lambda'\left(\frac{s}{\sigma}\right) &= \frac{d}{d\frac{s}{\sigma}}\lambda\left(\frac{s}{\sigma}\right) = \frac{d}{d\frac{s}{\sigma}}\left[\frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)}\right] \\
&= \frac{-\frac{s}{\sigma}\phi\left(\frac{s}{\sigma}\right)\left(1-\Phi\left(\frac{s}{\sigma}\right)\right)+\phi^2\left(\frac{s}{\sigma}\right)}{\left(1-\Phi\left(\frac{s}{\sigma}\right)\right)^2} \\
&= \frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)}\left(\frac{\phi\left(\frac{s}{\sigma}\right)-\frac{s}{\sigma}\left(1-\Phi\left(\frac{s}{\sigma}\right)\right)}{\left(1-\Phi\left(\frac{s}{\sigma}\right)\right)}\right) \\
&= \frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)}\left(\frac{\phi\left(\frac{s}{\sigma}\right)}{1-\Phi\left(\frac{s}{\sigma}\right)}-\frac{s}{\sigma}\right) \\
&= \lambda\left(\frac{s}{\sigma}\right)\left(\lambda\left(\frac{s}{\sigma}\right)-\frac{s}{\sigma}\right)
\end{aligned}$$

Hence, the integrand can be written as

$$\begin{aligned}
& z\lambda'(z) - \lambda(z) \\
&= z\lambda(z)(\lambda(z)-z)-\lambda(z) \\
&= \lambda(z)(z(\lambda(z)-z)-1)
\end{aligned}$$

Dividing by $\lambda(z)$, The question becomes whether

$$z(\lambda(z)-z) < 1$$

for $z \geq 0$.

Now, note that $\lambda'(z) < 1$ for all z, as the derivative of the hazard rate of the standard Normal distribution converges to 1 from below when $z \to \infty$. Hence, it suffices to show that

$$z(\lambda(z)-z) \leq \lambda(z)(\lambda(z)-z) = \lambda'(z)$$

Now,

$$z(\lambda(z)-z) \leq \lambda(z)(\lambda(z)-z)$$

is equivalent to

$$0 \leq (\lambda(x)-x)^2$$

where the last inequality is obviously true. ■

C. Proofs of Implications

Implication 1 *Reducing the cost of firing increases workplace diversity.*

Proof. We prove the implication by showing that $\underline{q}^*$ is increasing in c. Recall that optimality of the threshold strategy implies that

$$(V(\underline{q}^*) - v)\underline{q}^* + (1 - \underline{q}^*)c = 0 \tag{3}$$

Implicitly differentiating with respect to c while noting that $\frac{\partial V(\underline{q}^*)}{\partial \underline{q}^*} = 0$ gives

$$(V(\underline{q}^*) - v)\frac{d\underline{q}^*}{dc} + \frac{\partial V(\underline{q}^*)}{\partial c}\underline{q}^* + (1 - \underline{q}^*) - c\frac{d\underline{q}^*}{dc} = 0$$

Solving for $\frac{d\underline{q}^*}{dc}$:

$$\frac{d\underline{q}^*}{dc} = \frac{\left(\frac{\partial V(\underline{q}^*)}{\partial c} - 1\right)\underline{q}^* + 1}{v + c - V(\underline{q}^*)}$$

It is easily checked that

$$\frac{\partial V(\underline{q}^*)}{\partial c} = \frac{-\delta \int_{\underline{q}^*}^1 (1-q)\,dG(q)}{1 - \delta\left(1 - \int_{\underline{q}^*}^1 q\,dG(q)\right)}$$

Substituting into the expression for $\frac{d\underline{q}^*}{dc}$ and simplifying, one obtains

$$\frac{d\underline{q}^*}{dc} = \left(\frac{\delta G(\underline{q}^*) - 1}{1 - \delta\left(1 - \int_{\underline{q}^*}^1 q\,dG(q)\right)}\right)\underline{q}^* + 1$$

To establish that the right-hand side of this expression is positive requires that we show

$$(1 - \delta G(\underline{q}^*))\underline{q}^* - \left(1 - \delta\left(1 - \int_{\underline{q}^*}^1 q\,dG(q)\right)\right) < 0$$

To see this, notice that

$$\begin{aligned} & (1 - \delta G(\underline{q}^*))\underline{q}^* - \left(1 - \delta\left(1 - \int_{\underline{q}^*}^1 q\,dG(q)\right)\right) \\ < \;& (1 - \delta G(\underline{q}^*))\underline{q}^* - (1 - \delta(1 - \underline{q}^*(1 - G(\underline{q}^*)))) \\ = \;& -(1-\delta)(1 - \underline{q}^*) \\ < \;& 0 \end{aligned}$$

■

Implication 2 *Reducing the cost of interviewing decreases workplace diversity.*

Proof. To establish the implication, we show that $\underline{q}^*$ is decreasing in k. Implicitly differentiating equation (3) with respect to k while noting that $\frac{\partial V(\underline{q}^*)}{\partial \underline{q}^*} = 0$, we obtain

$$\left(V\left(\underline{q}^*\right) - v\right)\frac{d\underline{q}^*}{dk} + \frac{\partial V\left(\underline{q}^*\right)}{\partial k}\underline{q}^* - c\frac{d\underline{q}^*}{dk} = 0$$

Solving for $\frac{d\underline{q}^*}{dk}$,

$$\frac{d\underline{q}^*}{dk} = \frac{\frac{\partial V\left(\underline{q}^*\right)}{\partial k}\underline{q}^*}{v + c - V\left(\underline{q}^*\right)}$$

Hence, $\frac{d\underline{q}^*}{dk}$ and $\frac{\partial V(\underline{q}^*)}{\partial k}$ have the same sign, while it is easily checked that $\frac{\partial V(\underline{q}^*)}{\partial k} < 0$. ■

Implication 3 *Diversity is procyclical.*

Proof. From Implication 1, we already know that q^* is increasing in k. It remains to prove that q^* is decreasing in v.

Implicitly differentiating equation (3) with respect to v while noting that $\frac{\partial V(\underline{q}^*)}{\partial \underline{q}^*} = 0$, we obtain

$$\left(V\left(\underline{q}^*\right) - v\right)\frac{d\underline{q}^*}{dv} + \left(\frac{\partial V\left(\underline{q}^*\right)}{\partial v} - 1\right)\underline{q}^* - c\frac{d\underline{q}^*}{dv} = 0$$

Solving for $\frac{d\underline{q}^*}{dv}$:

$$\frac{d\underline{q}^*}{dv} = \frac{\left(\frac{\partial V\left(\underline{q}^*\right)}{\partial v} - 1\right)\underline{q}^*}{v + c - V\left(\underline{q}^*\right)}$$

It is easily checked that

$$\frac{dV\left(\underline{q}^*\right)}{dv} = \frac{\delta \int_{\underline{q}^*}^1 (q)\, dG(q)}{1 - \delta\left(1 - \int_{\underline{q}^*}^1 q dG(q)\right)}$$

Substituting this back into $\frac{d\underline{q}^*}{dv}$ and simplifying, one obtains

$$\begin{aligned}\frac{d\underline{q}^*}{dv} &= -\frac{\frac{1-\delta}{1-\delta\left(1-\int_{\underline{q}^*}^1 qdG(q)\right)}\underline{q}^*}{v + c - V\left(\underline{q}^*\right)} \\ &< 0\end{aligned}$$

■

Implication 4 *Riskier firms are more diverse.*

Proof. Implicitly differentiating equation (3) with respect to δ while noting that $\frac{\partial V\left(\underline{q}^*\right)}{\partial \underline{q}^*} = 0$, we obtain

$$\left(V\left(\underline{q}^*\right) - v\right)\frac{d\underline{q}^*}{d\delta} + \left(\frac{dV\left(\underline{q}^*\right)}{d\delta} - 1\right)\underline{q}^* - c\frac{d\underline{q}^*}{d\delta} = 0$$

Solving for $\frac{d\underline{q}^*}{d\delta}$:

$$\frac{d\underline{q}^*}{d\delta} = \frac{\left(\frac{dV\left(\underline{q}^*\right)}{d\delta} - 1\right)\underline{q}^*}{v + c - V\left(\underline{q}^*\right)}$$

It is easily checked that:

$$\frac{dV\left(\underline{q}^*\right)}{d\delta} = \frac{Z\left(1-\delta X\right) + X\left(\delta Z - k\right)}{\left(1-\delta X\right)^2}$$

where

$$\begin{aligned} Z &\equiv \int_{\underline{q}}^{1}\left(qv + (1-q)(-c)\right)dG(q) \\ X &\equiv \left(1 - \int_{\underline{q}}^{1} q\,dG(q)\right) \end{aligned}$$

To show that $\frac{d\underline{q}^*}{d\delta} > 0$, it is sufficient to show that $\frac{dV\left(\underline{q}^*\right)}{d\delta} - 1 > 0$, or equivalently

$$Z\left(1-\delta X\right) + X\left(\delta Z - k\right) - \left(1-\delta X\right)^2 > 0$$

To see this, simplify the left-hand side of the above expression and recall that, since the employer finds it optimal to search in the first place, $\delta Z - k \geq 0$. This yields

$$\begin{aligned} & Z - Xk + \left(1 - X\delta\right)^2 \\ \geq\ & Z - X\delta Z + \left(1 - X\delta\right)^2 \\ =\ & \left(1 - X\delta\right)\left(Z + 1 - X\delta\right) \\ >\ & 0 \end{aligned}$$

where the last inequality follows from the fact that $Z > 0$ and X, $\delta \in (0,1)$. ■

Implication 5 *The larger the minority, the smaller its degree of underrepresentation.*

Proof. Recall that $\underline{q}^*$ satisfies

$$\begin{aligned}\underline{q}^* &= \frac{\left(1-\delta\left(1-\int_{\underline{q}^*}^1 qdG(q)\right)\right)c}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k} \\ &= \frac{(1-\delta)c+c\delta\int_{\underline{q}^*}^1 qdG(q)}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k}\end{aligned}$$

$$\begin{aligned}&= \frac{(1-\delta)c+c\delta\left(\int_{\underline{q}^*}^1 qdG(q)+\int_0^{\underline{q}^*} qdG(q)-\int_0^{\underline{q}^*} qdG(q)\right)}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k} \\ &= \frac{(1-\delta)c+c\delta\left(E_G[Q]-\int_0^{\underline{q}^*} qdG(q)\right)}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k}\end{aligned}$$

$$\begin{aligned}&= \frac{(1-\delta)c+c\delta\left(E_G[Q]-\left(\underline{q}^*G\left(\underline{q}^*\right)-\int_0^{\underline{q}^*} G(q)\,dq\right)\right)}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k} \\ &= \frac{(1-\delta)c+c\delta\left(E_G[Q]-\underline{q}^*G\left(\underline{q}^*\right)+\int_0^{\underline{q}^*} G(q)\,dq\right)}{\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k}\end{aligned}$$

Therefore,

$$\begin{aligned}\underline{q}^*\left(\left(1-\delta G\left(\underline{q}^*\right)\right)c+(1-\delta)v+k\right) &= (1-\delta)c+c\delta\left(E_G[Q]-\underline{q}^*G\left(\underline{q}^*\right)+\int_0^{\underline{q}^*} G(q)\,dq\right) \\ (c+(1-\delta)v+k)\,\underline{q}^*-c\delta\underline{q}^*G\left(\underline{q}^*\right) &= (1-\delta)c+c\delta E_G[Q]-c\delta\underline{q}^*G\left(\underline{q}^*\right)+c\delta\int_0^{\underline{q}^*} G(q)\,dq \\ (c+(1-\delta)v+k)\,\underline{q}^* &= (1-\delta)c+c\delta\left(E_G[Q]+\int_0^{\underline{q}^*} G(q)\,dq\right)\end{aligned}$$

Now, from Lemma 2, we know that if $m_B < m'_B$, then $G(q, m_B)$ is a mean preserving spread of $G(q, m'_B)$. Hence, if we go from m_B to m'_B, $E_G[Q]$ remains unchanged in the RHS of the last equation but, by definition of second-order stochastic dominance, $\int_0^{\underline{q}^*} G(q; m_B) > \int_0^{\underline{q}^*} G(q; m'_B)$. Hence, the LHS also increases. Therefore, it must be that $\underline{q}^*(m_B) > \underline{q}^*(m'_B)$, because c, δ, v, and k are all constants. We conclude that $\frac{\partial \underline{q}^*}{\partial m_B} < 0$. ■

Implication 6 *In jobs that require rare skills, minorities will be underrepresented. In jobs that require common skills, minorities will be overrepresented. Formally, there exist* $0 < p_0 < p_1 < 1$ *such that for all* $p \in (0, p_0)$, $\frac{r_B}{m_B} < 1$, *while for all* $p \in (p_1, 1)$, $\frac{r_B}{m_B} > 1$.

Proof. First, we establish that $\lim_{p\uparrow 1} \underline{q}^* < 1$ and $\lim_{p\downarrow 0} \underline{q}^* > 0$. To see this, note that $\underline{q}^*$ is monotone in p since, by implicitly differentiating equation (3),

$$\frac{d\underline{q}^*}{dp} = \frac{\frac{\partial V(\underline{q}^*)}{\partial p}\underline{q}^*}{v - V\left(\underline{q}^*\right) + c} > 0$$

where the inequality follows from the fact that $v > V(\underline{q}^*)$ and, by Lemma 1, $\frac{\partial V(\underline{q}^*)}{\partial p} > 0$.

Since $\underline{q}^*$ is bounded and monotone function of p we know that both limits must exist.

To establish that $\lim_{p\uparrow 1} \underline{q}^* < 1$, suppose, to the contrary, that $\lim_{p\uparrow 1} \underline{q}^* = 1$. Then the right-hand side of equation (2) becomes:

$$\begin{aligned} & \lim_{p\uparrow 1} \frac{\left(1 - \delta\left(1 - \int_1^1 q dG(q)\right)\right) c}{(1 - \delta G(1)) c + (1-\delta) v + k} \\ = & \frac{(1-\delta) c}{(1-\delta) c + (1-\delta) v + k} \neq 1 \end{aligned}$$

which is a contradiction.

To establish that $\lim_{p\downarrow 0} \underline{q}^* > 0$, recall that $\underline{q}^*$ is implicitly defined by equation (2). Taking limits:

$$\begin{aligned} \lim_{p\downarrow 0} \underline{q}^* &= \lim_{p\downarrow 0} \frac{\left(1 - \delta\left(1 - \int_{\underline{q}^*}^1 q dG(q)\right)\right) c}{\left(1 - \delta G\left(\underline{q}^*\right)\right) c + (1-\delta) v + k} \\ &> \lim_{p\downarrow 0} \frac{(1-\delta) c}{c + (1-\delta) v + k} > 0 \end{aligned}$$

To complete the proof, it remains to show that $\underline{q}^{II}$ and $\underline{q}^{I}$ are monotone in p with limits $\lim_{p\downarrow 0} \underline{q}^{II} = 0$ and $\lim_{p\downarrow 1} \underline{q}^{I} = 1$. Monotonicity may be readily verified by differentiating the expressions for $\underline{q}^{II}$ and $\underline{q}^{I}$. Likewise, the limit results are trivial to obtain. ■