APPENDIX: A MODEL WITH PRODUCTIVE CAPITAL

In this appendix, we present results from a two-sector sticky-price model in which the durable is productive capital. Households get utility from the nondurable and get disutility from working. Using our earlier notation, the households maximize

$$E_t\left[\sum_{i=0}^{\infty}\beta^i\left(u\left(C_{t+i}\right)-v\left(N_{t+i}\right)\right)\right]$$

subject to the constraints

$$P_{c,t}C_t+P_{x,t}X_t+M_t\le W_tN_t+\Pi_t+T_t+\left(1+i_{t-1}\right)S_{t-1}-S_t+M_{t-1}+R_tK_t,$$

$$K_{t+1}=X_t+K_t\left(1-\delta\right).$$

The first order conditions for *C*, *N* and *X* are the same as before with the one exception that now γ_t is the shadow value of an additional unit of capital. If MP_t^K is the marginal product of capital at time *t*, then γ_t is

$$\gamma_t=\beta MP_{t+1}^K+\beta^2\left(1-\delta\right)MP_{t+2}^K+\beta^3\left(1-\delta\right)^2MP_{t+3}^K+...=\beta\sum_{i=0}^{\infty}\left[\beta\left(1-\delta\right)\right]^iMP_{t+i+1}^K$$

The remainder of the model is the same with the exception that the capital market clearing condition is now $K_t=K_{x,t}+K_{c,t}$, which reflects the fact that the capital stock is no longer fixed. For the simulations, we use the same production functions and parameter values as before ($u(C_t)=\ln(C_t)$).

Figure A.1 shows the impulse responses to a permanent 1.00% increase in the money supply for three cases: symmetric price rigidity, sticky nondurable goods prices, and sticky investment goods prices. Overall, the impulse responses are very similar to those reported in Figure 1 in the text. There are small differences however. The

differences are due to the fact that capital is complementary with labor in the model with capital while it was assumed to be separable in the model with the consumer durable.

In the model with productive capital for instance, with equally sticky prices, the real rate of return for the durable rises by roughly 16 basis points in the quarter after the shock. With flexible investment goods prices, output remains below trend by roughly 4 basis points three years after the shock. Neither of these effects was observed in the model with consumer durables. The reason for both of these differences is the complementarity between capital and labor. The real rate of return is tied to the marginal product of capital. Thus, when production increases in the short-run (when prices are equally sticky) so does the real interest rate. (Note that a 1 percent change in MP_t^K implies an $(r+\delta)$ percentage point change in the real interest rate.) With flexible investment prices, capital remains slightly below trend after the shock because of the huge collapse in investment in the short-run. This depresses labor supply and output.

That there are differences is not surprising. The limiting result (for a purely transitory shock and for infinitely lived durables) is not an exact result but is rather an extremely useful approximation. For instance, in the limit, the nominal interest rate should exactly reflect inflation in the investment goods price. When prices are equally sticky in the capital model, the increase in the average nominal interest rate in the first quarter is 1.67%; the average rate of inflation in the durables price over this period was 1.52%. The difference is the 15 basis point change in the real rate which is due to the fact that we are not in the limiting case. This discrepancy is bigger in the capital model because of the complementarity but will vanish in *any model* for a sufficiently temporary shock or for a sufficiently long-lived durable and sufficiently patient households.

FIGURE A.1: A PERMANENT ONE PERCENT INCREASE IN THE MONEY SUPPLY: PRODUCTIVE CAPITAL.

Note: Each panel reports a different variable. The heavy grey line is the model with both investment goods and consumption goods and equally sticky prices in each sector. The dark dashed line is the model with sticky consumption goods prices but flexible investment goods prices. The dark solid line is the model with flexible consumption goods prices but sticky investment goods prices. Time (in quarters) is on the horizontal axis.