

# Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts

Emmanuel Farhi and Jean Tirole

## Web Appendix

### Proof of Proposition 5

Ex-post (date-1) welfare  $W^{\text{ex post}}(R; R^*)$  is given by

$$W^{\text{ex post}}(R; R^*) = -L(R) + \int \frac{[\beta - (1-R)\frac{\rho_0}{R}] \frac{1-\frac{\rho_0}{R^*}}{1-\frac{\rho_0}{R}} A (1-n(\beta, A))}{1-\pi-\alpha\rho_0+(1-\alpha)(1-\frac{\rho_0}{R^*})} dF(\beta, A)$$

and the equilibrium correspondance  $\mathcal{R}$  is defined accordingly. The corresponding planning problem is

$$(1) \quad \underline{K} \equiv \min_{\{n(\beta, A)\}} K$$

s.t.

$$0 \leq n(\beta, A) \leq 1$$

and

$$R^* \notin \mathcal{R}(R^*) \text{ for all } R^* \in [\rho_0, 1).$$

The condition for  $R^* \in [\rho_0, 1)$  not to be an equilibrium is that there exists  $R \in (R^*, 1]$  such that

$$W^{\text{ex post}}(R; R^*) - W^{\text{ex post}}(R^*; R^*) > 0$$

or equivalently

$$\int_{R^*}^R \frac{\partial W^{\text{ex post}}(\hat{R}; R^*)}{\partial \hat{R}} d\hat{R} > 0.$$

For any given set of values  $\Delta(R, R^*)$  and consider the following subproblem

$$(2) \quad \underline{K}(\{\Delta(R, R^*)\}) \equiv \min_{\{n(\beta, A)\}, \{\Delta(R, R^*)\}} K$$

s.t.

$$0 \leq n(\beta, A) \leq 1$$

and

$$\int_{R^*}^R \frac{\partial W^{\text{ex post}}(\hat{R}; R^*)}{\partial \hat{R}} d\hat{R} \geq \Delta(R, R^*) \text{ for all } R^* \in [\rho_0, 1) \text{ and } R \in [R^*, 1].$$

Then the original planning problem (1) and the subproblem (2) are related in the following way:

$$(3) \quad \underline{K} = \min_{\{\Delta(R, R^*)\}} \underline{K}(\{\Delta(R, R^*)\})$$

s.t. the constraint that for all  $R^* \in [\rho_0, 1)$ , there exists  $R \in (R^*, 1]$  such that  $\Delta(R, R^*) > 0$ . Moreover the solution  $\{n(\beta, A)\}$  of (1) coincides with the solution of (2) when  $\{\Delta(R, R^*)\}$  is set as the solution of (3).

Turning back to (2) and take  $\{\Delta(R, R^*)\}$  to be the solution of (3), the constraint set and the objective function are linear in  $\{n(\beta, A)\}$  so the first order conditions are necessary and sufficient for optimality. Let  $\mu_{R^*, R} \geq 0$  be the multiplier on the constraint

$$\int_{R^*}^R \frac{\partial W^{\text{ex post}}(\hat{R}; R^*)}{\partial \hat{R}} d\hat{R} \geq \Delta(R, R^*).$$

Let  $\bar{\nu}_{\beta, A} dF(\beta, A)$  be the multiplier on the constraint  $n(\beta, A) \leq 1$  and  $\underline{\nu}_{\beta, A} dF(\beta, A)$  be the multiplier on the constraint  $n(\beta, A) \geq 0$ . Finally, let

$$\mu = \sum \mu_{R^*, R} \int_{R^*}^R \frac{1 - \frac{\rho_0}{R^*}}{1 - \pi - \alpha \rho_0 + (1 - \alpha) \left(1 - \frac{\rho_0}{R^*}\right)} \frac{\frac{\rho_0}{R^2}}{\left(1 - \frac{\rho_0}{R}\right)^2} d\hat{R}.$$

The first-order condition for  $n(\beta, A)$  is

$$c(m(1))^\lambda A^\lambda = \underline{\nu}_{\beta, A} - \bar{\nu}_{\beta, A} + A(\beta + \rho_0 - 1)\mu.$$

The result follows directly from this first-order condition and the complementary slackness conditions  $\underline{\nu}_{\beta, A}n(\beta, A) = \bar{\nu}_{\beta, A}[1 - n(\beta, A)] = 0$ .

## Proof of Proposition 6

Let us first focus on values of  $(R_0, R)$  such that banking entrepreneur choose to hoard enough liquidity to continue at full scale in case of a crisis, i.e. values that satisfy equation (??). For such values, we have

$$j(R_0, R) = i(R_0, R) = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha\rho_0}{R_0} + \frac{(1-\alpha)}{R_0} \left(1 - \frac{\rho_0}{R}\right)}.$$

Plugging these expressions into equation (??), we verify that that  $W^{\text{ex ante}}(R, R_0)$  increases in  $R$  if and only if Assumption ?? holds. For any couple  $(R_0, R)$  satisfying equation (??), so does  $(R_0, 1)$ . We therefore have that

$$W^{\text{ex ante}}(R_0, R) \leq W^{\text{ex ante}}(R_0, 1).$$

It is then easy to verify that  $W^{\text{ex ante}}(R_0, 1)$  is increasing in  $R_0$  as long as Assumption ?? holds.

Let us now turn to values of  $(R_0, R)$  such that banking entrepreneurs choose to hoard no liquidity and instead load up on short-term debt, i.e. values such that equation (??) is violated. We only have to consider two values for  $R$ :  $\rho_0$  and 1. We have

$$j(R_0, \rho_0) = i(R_0, 1) = i(R_0, \rho_0) = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha\rho_0}{R_0}}$$

and

$$j(R_0, 1) = 0.$$

Let us first consider the case where  $R = 1$ . Plugging these expressions in equation (??), it can be verified that  $W^{\text{ex ante}}(R_0, 1)$  is increasing in  $R_0$  if and only if  $\alpha\beta \leq 1 - \pi - \alpha\rho_0$ . This condition is implied by Assumptions ?? and ??. Turning now to the case where  $R_0 = \rho_0$ , we

find that  $W^{\text{ex ante}}(R_0, \rho_0)$  is increasing in  $\rho_0$  if and only if Assumption ?? holds.

## Proof of Proposition 9

The proposition follows easily from the following two lemmas. The first one characterizes the form of the optimal bailout given an interest rate  $R$ . The second one derives the optimal ex-ante liquidity choices of banks. The first lemma follows easily from the linearity of the ex-post bailout program in  $j(i, x)$ . The second lemma follows from a simple calculation of the banking entrepreneur's welfare given his liquidity choice  $x$ .

**Lemma 1** (*optimal bailout*). *Under the optimal bailout:*

- (i) if  $R \geq \underline{R}(\gamma)$ , then  $t(i, x) = 0$  and  $j(i, x) = x/(1 - \rho_0/R)$ ;
- (ii) if  $R < \underline{R}(\gamma)$  then  $t(i, x) = (1 - x - \rho_0/R)i$  and  $j(i, x) = i$ .

Lemma 1 shows that given  $R$ , whether direct transfers are used depends on whether or not  $R < \underline{R}(\gamma)$ . When  $R < \underline{R}(\gamma)$ , it is optimal to transfer funds to banks that claim to be distressed, even though a fraction will end up in banks that are truly intact. Enough funds are then transferred so that distressed banks can continue at full scale. When  $R \geq \underline{R}(\gamma)$ , it is preferable not to engage in direct transfers because too high a fraction would end up in banks that are truly intact. This is intuitive: when the interest rate  $R$  is low, distressed banks can lever up the direct transfers more (and intact banks perceived as distressed cannot), which makes direct transfers more attractive. That the threshold interest rate  $\underline{R}(\gamma)$  for direct transfers increases with  $\gamma$  makes sense: The asymmetric information problem is worse when  $\gamma$  is low so that the proportion of false positives  $(1 - \gamma)\nu$  is high.

**Lemma 2** (*liquidity choice*). *The optimal scale and liquidity choice of banks when they expect the interest rate to be  $R$  in the event of a crisis is:*

- (i) if  $R \geq \underline{R}(\gamma)$ , then  $i/A = 1/[1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \rho_0/R)]$ ,  $x = 1 - \rho_0/R$  and  $d = \pi - (1 - \rho_0/R)$ ;
- (ii) if  $R < \underline{R}(\gamma)$ , then  $i/A = 1/[1 - \pi - \alpha\rho_0]$ ,  $x = 0$  and  $d = \pi$ .

Lemma 2 shows that the ex-ante liquidity choices also depends on whether or not  $R < \underline{R}(\gamma)$ . When  $R < \underline{R}(\gamma)$ , the government provides a big enough direct transfer to banks that claim to be distressed so that they can continue at full scale if they are truly

distressed. Therefore, hoarding liquidity is useless. It only reduces the investment scale and total leverage  $i/A$ . As a result, banks opt to be completely illiquid ( $x = 0$ ) and choose a maximal level of short-term debt ( $d = \pi$ ). When  $R \geq \underline{R}(\gamma)$ , the government does not provide any direct transfer. Continuation scale therefore increases with liquidity and decreases with the amount of short-term debt. Banks choose to hoard enough liquidity ( $x = 1 - \rho_0/R$ ) and take on only as much short-term debt ( $d = \pi - (1 - \rho_0/R)$ ) so as to be able to continue at full scale in case of a crisis.