Web Appendix – Proof of Theorem 1

Search and Satisficing
Andrew Caplin, Mark Dean, and Daniel Martin

PROOF:

We establish the result first for the final period, and then induct backward. Suppose that search continues until period T and that there is at least one unsearched item left. Let $\bar{u}^T = (\bar{u}_0, ..., \bar{u}_{T-1}) \in \mathbb{R}^T$ denote the vector of highest utility objects encountered in prior periods, and let $H(T, \bar{u}^T)$ capture the expected utility in hand at time T based on the possibility that the search clock had stopped strictly prior to period T,

$$H(T, \bar{u}^T) = \sum_{s=0}^{T-1} [J(s) - J(s+1)]\bar{u}_s.$$

If no search is conducted in period T, the payoff from stopping is $\pi^S(T, \bar{u}^T)$,

$$\pi^S(T, \bar{u}^T) = H(T, \bar{u}^T) + J(T)\bar{u}_{T-1}.$$

If search continues for one last period, then the payoff for the final period is still $\bar{u}_{T-1}$ unless a new object is identified (probability q), that object has utility higher than $\bar{u}_{T-1}$, and the random choice time continues to period T,

$$\pi^C(T, \bar{u}^0) = H(T, \bar{u}^T) - \kappa + J(T) \left[\bar{u}_{T-1} + q \int_{\bar{u}_{T-1}}^{\infty} [z - x] \, dF(z) \right].$$

Hence continued search is an optimal strategy if and only if,

$$\int_{\bar{u}_{T-1}}^{\infty} [z - x] \, dF(z) \geq \frac{K}{qJ(T)} \equiv \int_{u^R(T)}^{\infty} [z - x] \, dF(z).$$

Thus, continued search is optimal if and only if $\bar{u}_{T-1} \leq u^R(T)$, stopping search is optimal if and only if $\bar{u}_{T-1} > u^R(T)$, establishing the result for period T.

Now assume that the identified strategy is optimal if search continues in period $t+1 \geq 2$, and consider the optimal strategy in period t with prior maxima $\bar{u}^t = (\bar{u}_0, ..., \bar{u}_{t-1})$ and with $H(t, \bar{u}^t)$ the fixed expected utility should the search clock have stopped prior to period t. Continued search for one and only one period costs κ, yielding the expected surplus above $\bar{u}_{\tau-1}$ if the new search is effective. Hence it is worthwhile if and only if,

$$\int_{\bar{u}_{t-1}}^{\infty} [z - x] \, dF(z) \geq \frac{K}{qJ(t)} \equiv \int_{u^R(t)}^{\infty} [z - x] \, dF(z).$$

Given that ρ is strictly increasing, one and only one additional period of search dominates stopping if $\bar{u}_{t-1} < u^R(t)$, stopping immediately is strictly superior if $\bar{u}_{t-1} > u^R(t)$, while they are indifferent if $\bar{u}_{t-1} = u^R(t)$. To establish the induction hypothesis requires only that an individual for whom it is optimal to stop when considering one period continuation will not continue on the basis of expected gains in later periods. This can be ruled out, since if $\bar{u}_{t-1} \geq u^R(t)$, then since $u^R(t) > u^R(t+1)$ the induction hypothesis implies that search will certainly not continue beyond period $t+1$, making the single period argument in favor of stopping definitive.