Web Appendices of
Selling to Overconfident Consumers

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A   Option Pricing Intuition

This appendix provides additional intuition based on option pricing for the result in Proposition 2.

Consider the case of monopoly. At time one, the monopolist is selling a series of call options, or equivalently units bundled with put options, rather than units themselves. The marginal price charged for a unit q at time two is simply the strike price of the option sold on unit q at time one. The series of call options being sold are interrelated; a call option for unit q can't be exercised unless the call option for unit $q-1$ has already been exercised. However, it is useful to consider the market for each option independently. According to Proposition 2 (equation 8), when there is no pooling and the satiation constraint is not binding, the optimal marginal price for a unit q is:

$$\hat{P}_q(q) = C_q(q) + V_{q\theta}(q, \hat{\theta}(q)) \frac{F^*(\hat{\theta}(q)) - F(\hat{\theta}(q))}{f(\hat{\theta}(q))} \tag{14}$$

It turns out that this is exactly the strike price that maximizes the net value of a call or put option on unit q given the difference in priors between the two parties.[4]

To show this explicitly, write the net value NV of a call option on minute q as the difference between the consumers' value of the option CV and the firm's cost of providing that option, FV. The option will be exercised whenever the consumer values unit q more than the strike price p, that is whenever $V_q(q, \theta) \geq p$. Let $\theta(p)$ denote the minimum type who exercises the call option, characterized by the equality $V_q(q, \theta(p)) = p$.

The consumers' value for the option is their expected value received upon exercise, less the expected strike price paid, where expectations are based on the consumers' prior $F^*(\theta)$:

$$CV(p) = \int_{\theta(p)}^{\bar{\theta}} V_q(q, \theta) f^*(\theta) d\theta - [1 - F^*(\theta(p))] p$$

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[4]This parallels Mussa and Rosen's (1978) finding in their static screening model, that the optimal marginal price for unit q is identical to the optimal monopoly price for unit q if the market for unit q were treated independently of all other units.

The firm's cost of providing the option is the probability of exercise based on the firm's prior $F(\theta)$ times the difference between the cost of unit q and the strike price received:

$$FV(p) = [1 - F(\theta(p))](c - p)$$

Putting these two pieces together, the net value of the call option is equal to the consumers' expected value of consumption less the firms expected cost of production plus an additional term due to the gap in perceptions:

$$NV(p) = \int_{\theta(p)}^{\bar{\theta}} V_q(q, \theta) f^*(\theta) d\theta - [1 - F(\theta(p))] c + [F^*(\theta(p)) - F(\theta(p))] p$$

The additional term $[F^*(\theta(p)) - F(\theta(p))] p$ represents the difference between the exercise payment the firm expects to receive and the consumer expects to pay. The term $[F^*(\theta(p)) - F(\theta(p))]$ represents the disagreement between the parties about the probability of exercise.

As a monopolist selling call options on unit q earns the net value $NV(p)$ of the call option by charging the consumer $CV(p)$ upfront, a monopolist should set the strike price p to maximize $NV(p)$. By the implicit function theorem, $\frac{d}{dp}\theta(p) = \frac{1}{V_{q\theta}(q,\theta(p))}$, so the first order condition which characterizes the optimal strike price is:

$$\frac{f(\theta(p))}{V_{q\theta}(q, \theta(p))} [p - C_q(q)] = [F^*(\theta(p)) - F(\theta(p))] \tag{15}$$

As claimed earlier, this is identical to the characterization of the optimal marginal price $\hat{P}_q(q)$ for the complete nonlinear pricing problem when monotonicity and satiation constraints are not binding (equation 14).

Showing that the optimal marginal price for unit q is given by the optimal strike price for a call option on unit q is useful, because the first order condition $\Psi_q(q, \theta) = 0$ can be interpreted in the option pricing framework. Consider the choice of exercise price p for an option on unit q. A small change in the exercise price has two effects. First, if a consumer is on the margin, it will change the consumers' exercise decision. Second, it changes the payment made upon exercise by all infra-marginal consumers. In a common-prior model, the infra-marginal effect would net to zero as the payment is a transfer between the two parties. This is not the case here, however, as the two parties disagree on the likelihood of exercise by $[F^*(\theta(p)) - F(\theta(p))]$.

Consider the first order condition as given above in equation (15). On the left hand side, the term $\frac{f(\theta(p))}{V_{q\theta}(q,\theta(p))}$ represents the probability that the consumer is on the margin and that a marginal increase in the strike price p would stop the consumer exercising. The term $[p - C_q(q)]$ is the cost

to the firm if the consumer is on the margin and no longer exercises. There is no change in the consumer's value of the option by a change in exercise behavior at the margin, as the margin is precisely where the consumer is indifferent to exercise ($V_q (q, \theta (p)) = p$).

On the right hand side, the term $[F^* (\theta (p)) - F (\theta (p))]$ is the firm's gain on infra-marginal consumers from charging a slightly higher exercise price. This is because consumers believe they will pay $[1 - F^* (\theta (p))]$ more in exercise fees, and therefore are willing to pay $[1 - F^* (\theta (p))]$ less upfront for the option. However the firm believes they will actually pay $[1 - F (\theta (p))]$ more in exercise fees, and the difference $[F^* (\theta (p)) - F (\theta (p))]$ is the firm's perceived gain.

The first order condition requires that at the optimal strike price p, the cost of losing marginal consumers $\frac{f(\theta(p))}{V_{q\theta}(q, \theta(p))} [p - C_q (q)]$ is exactly offset by the "perception arbitrage" gain on infra-marginal consumers $[F^* (\theta (p)) - F (\theta (p))]$.

Setting the strike price above or below marginal cost is always costly because it reduces efficiency. In the discussion above, referring to $[F^* (\theta (p)) - F (\theta (p))]$ as a "gain" to the firm for a marginal increase in strike price implies that the term $[F^* (\theta (p)) - F (\theta (p))]$ is positive. This is the case for $\theta (p) > \theta^*$, when consumers underestimate their probability of exercise. In this case, from the firm's perspective, raising the strike price above marginal cost increases profits on infra-marginal consumers, thereby effectively exploiting the perception gap. On the other hand, for $\theta (p) < \theta^*$, the term $[F^* (\theta (p)) - F (\theta (p))]$ is negative and consumers overestimate their probability of exercise. In this case reducing the strike price below marginal cost exploits the perception gap between consumers and the firm.

Fixing θ and the firm's prior $F(\theta)$, the absolute value of the perception gap is largest when the consumer's prior is at either of two extremes, $F^* (\theta) = 1$ or $F^* (\theta) = 0$. When $F^* (\theta) = 1$, the optimal marginal price reduces to the monopoly price for unit q where the market for minute q is independent of all other units. This is because consumers believe there is zero probability that they will want to exercise a call option for unit q. The firm cannot charge anything for an option at time one; essentially the firm must wait to charge the monopoly price until time two when consumers realize their true value.

Similarly, when $F^* (\theta) = 0$, the optimal marginal price reduces to the monopsony price for unit q. Now rather than thinking of a call option, think of the monopolist as selling a bundled unit and put option at time one. In this case consumers believe they will consume the unit for sure and exercise the put option with zero probability. This means that the firm cannot charge anything for the put option upfront, and must wait until time two when consumers learn their true values and buy units back from them at the monopsony price. The firm's ability to do so is of course limited by free disposal which means the firm could not buy back units for a negative price.

Marginal price can therefore be compared to three benchmarks. For all quantities q, the marginal price will lie somewhere between the monopoly price $p^{ml}(q)$ and the maximum of the monopsony price $p^{ms}(q)$ and zero, hitting either extreme when $F^*(\theta) = 1$ or $F^*(\theta) = 0$, respectively. When $F^*(\theta) = F(\theta)$, marginal price is equal to marginal cost. To illustrate this point, the equilibrium marginal price for the running example with positive marginal cost $c = \$0.035$ and low overconfidence $\Delta = 0.25$ previously shown in Figure 4, panel C is replotted with the monopoly and monopsony prices for comparison in Figure 10.
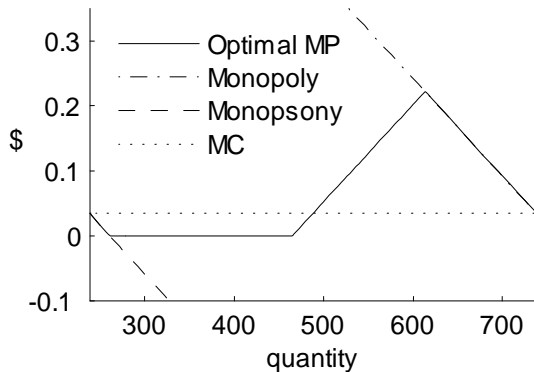

Figure 10: Equilibrium pricing for Example 1 given $c = \$0.035$ and $\Delta = 0.25$: Marginal price is plotted along with benchmarks: (1) marginal cost, (2) ex post monopoly price - the upper bound, and (3) ex post monopsony price - the lower bound.

B Pooling

As it was omitted from the main text, a characterization of pooling quantities when the monotonicity constraint binds is provided below in Lemma 4. This is useful because it facilitates the calculation of pooling quantities in numerical examples.

Lemma 4 *On any interval $[\theta_1, \theta_2]$ such that monotonicity (but not non-negativity) is binding inside, but not just outside the interval, the equilibrium allocation is constant at some level $\hat{q} > 0$ for all $\theta \in [\theta_1, \theta_2]$. Further, the pooling quantity $\hat{q}$ and bounds of the pooling interval $[\theta_1, \theta_2]$ must maintain continuity of $\hat{q}(\theta)$ and satisfy the first order condition on average: $\int_{\theta_1}^{\theta_2} \Psi_q(\hat{q}, \theta) f(\theta) d\theta = 0$. Non-negativity binds inside, but not just above, the interval $[\underline{\theta}, \theta_2]$ only if $\int_{\underline{\theta}}^{\theta_2} \Psi_q(0, \theta) f(\theta) d\theta \leq 0$ and $q^R(\theta_2) = 0$.*

Proof. Given the result in Lemma 2, the proof is omitted as it closely follows ironing results for the standard screening model. The result follows from the application of standard results in

optimal control theory (Seierstad and Sydsæter 1987). See for example the analogous proof given in Fudenberg and Tirole (1991), appendix to chapter 7. ∎

Note that because the virtual surplus function is strictly quasi-concave, but not necessarily strictly concave, optimal control results yield necessary rather than sufficient conditions for the optimal allocation. For the special case $V_{qq\theta} = 0$, virtual surplus is strictly-concave and applying the ironing algorithm suggested by Fudenberg and Tirole (1991) using the conditions in Lemmas 2 and 4 is sufficient to identify the uniquely optimal allocation.

Proposition 2 characterizes marginal pricing at quantities for which there is no pooling, and states that marginal price will jump discretely upwards at quantities where there is pooling. It is therefore interesting to know when $q^R(\theta)$ will be locally decreasing, so that the equilibrium allocation $\hat{q}(\theta)$ involves pooling. First, a preliminary result is helpful. Proposition 5 compares the relaxed allocation to the first best allocation, showing that the relaxed allocation is above first best whenever $F(\theta) > F^*(\theta)$ and is below first best whenever $F(\theta) < F^*(\theta)$:

Proposition 5 *Given maintained assumptions:*

$$
q^R(\theta) \begin{cases} \geq^\blacklozenge q^{FB}(\theta) & F(\theta) > F^*(\theta) \\ = q^{FB}(\theta) & F(\theta) = F^*(\theta) \\ < q^{FB}(\theta) & F(\theta) < F^*(\theta) \end{cases}
$$

(♦) strict iff $C_q\left(q^{FB}(\theta)\right) > 0$

Proof. See Section B.1 at the end of this appendix. ∎

Given that $F^*(\theta)$ crosses $F(\theta)$ once from below, the relationship between the relaxed allocation and the (strictly increasing) first best allocation given in Proposition 5 leads to the conclusion that $q^R(\theta)$ is strictly increasing near the bottom $\underline{\theta}$ and near the top $\bar{\theta}$. More can be said about pooling when consumers either have nearly correct beliefs, or are extremely overconfident. When consumers' prior is close to that of the firm, the relaxed solution is close to first best, and like first best is strictly increasing. In this case the equilibrium and relaxed allocations are identical.

When consumers are extremely overconfident such that their prior is close to the belief that $\theta = \theta^*$ with probability one, the relaxed solution is strictly decreasing at or just above θ^*. In this case ironing will be required and an interval of types around θ^* pool at the same quantity $\hat{q}(\theta^*)$. The intuition is that when the consumers' prior is exactly the belief that $\theta = \theta^*$ with probability one, $[F(\theta) - F^*(\theta)]$ falls discontinuously below zero at θ^*. Thus, by Proposition 5, the relaxed solution must drop discontinuously from weakly above first best just below θ^* to strictly below first best at θ^*. These results and the notion of "closeness" are made precise in Proposition 6.

Proposition 6 *(1) There exists $\varepsilon > 0$ such that if $|f^*(\theta) - f(\theta)| < \varepsilon$ for all θ then $q^R(\theta)$ is strictly increasing for all θ. (2) There exists a finite constant γ, such that if $f^*(\theta^*) > \gamma$ and $f^*(\theta)$ is continuous at θ^* then $q^R(\theta)$ is strictly decreasing just above θ^*.*

Proof. If neither non-negativity nor satiation constraints bind at θ, Lemma 2 and the implicit function theorem imply that $\frac{d}{d\theta}q^R(\theta) = -\frac{\Psi_{q\theta}(q^R(\theta),\theta)}{\Psi_{qq}(q^R(\theta),\theta)}$. In this case the sign of the cross partial derivative $\Psi_{q\theta}$ determines whether $q^R(\theta)$ is increasing or decreasing. For any θ at which F^* is sufficiently close to F in both level and slope, $\Psi_{q\theta} > 0$, but when $f^*(\theta)$ is sufficiently large $\Psi_{q\theta} < 0$. See the proof at the end of this appendix for details. ■

B.1 Pooling Appendix Proofs

B.1.1 Small Lemma 5

Lemma 5 *Satiation $q^S(\theta)$ and first best $q^{FB}(\theta)$ quantities are continuously differentiable, strictly positive, and strictly increasing. Satiation quantity is higher than first best quantity, and strictly so when marginal costs are strictly positive at q^{FB}.*

Proof. Satiation and first best quantities are strictly positive because by assumption: $V_q(0,\theta) > C_q(0) \geq 0$. Therefore given maintained assumptions, $q^S(\theta)$ and $q^{FB}(\theta)$ exist, and are continuous functions characterized by the first order conditions $V_q(q^S(\theta),\theta) = 0$ and $V_q(q^{FB}(\theta),\theta) = C_q(q^{FB}(\theta))$ respectively. Zero marginal cost at $q^{FB}(\theta)$ implies $q^S(\theta) = q^{FB}(\theta)$. When $C_q(q^{FB}(\theta)) > 0$, $V_{qq}(q,\theta) < 0$ implies that $q^S(\theta) > q^{FB}(\theta)$. The implicit function theorem implies $\frac{d}{d\theta}q^S = -\frac{V_{q\theta}}{V_{qq}} > 0$ and $\frac{d}{d\theta}q^{FB} = -\frac{V_{q\theta}}{V_{qq}-C_{qq}} > 0$. ■

B.1.2 Proof of Proposition 5

Proof. The relaxed allocation maximizes virtual surplus $\Psi(q,\theta)$ within the constraint set $[0, q^S(\theta)]$ and $\Psi(q,\theta)$ is strictly quasi-concave in q (See proof of Lemma 2). Moreover, $q^{FB} \in (0, q^S(\theta)]$ and $\Psi_q(q^{FB}(\theta),\theta) = V_{q\theta}(q^{FB}(\theta),\theta)\frac{F(\theta)-F^*(\theta)}{f(\theta)}$ since the first best allocation satisfies $V_q(q^{FB}(\theta),\theta) = C_q(q^{FB}(\theta))$ (Lemma 5). Therefore there are three cases to consider:

1. $F(\theta) = F^*(\theta)$: In this case virtual surplus and true surplus are equal so $q^R(\theta) = q^{FB}(\theta)$.

2. $F(\theta) > F^*(\theta)$: In this case $\Psi_q(q^{FB}(\theta),\theta) > 0$ and therefore $q^R(\theta) \geq q^{FB}(\theta)$. When $C_q(q^{FB}(\theta)) = 0$, Lemma 5 shows that $q^S(\theta) = q^{FB}(\theta)$ and therefore the satiation constraint binds: $q^R(\theta) = q^{FB}(\theta) = q^S(\theta)$. When $C_q(q^{FB}(\theta)) > 0$, Lemma 5 shows that satiation is not binding at first best, and therefore the comparison is strict: $q^R(\theta) > q^{FB}(\theta)$.

3. $F(\theta) < F^*(\theta)$: In this case $\Psi_q\left(q^{FB}(\theta),\theta\right) < 0$ and therefore $q^R(\theta) < q^{FB}(\theta)$ since, by Lemma 5, non-negativity is not binding at first best $(q^{FB}(\theta) > 0)$.

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B.1.3 Proof of Proposition 6

Proof. If, over the interval (θ_1,θ_2), neither non-negativity nor satiation constraints bind and $\Psi_q(q,\theta)$ is continuously differentiable, then in the same interval $\Psi_q\left(q^R,\theta\right) = 0$ (Lemma 2) and the implicit function theorem implies $\frac{d}{d\theta}q^R(\theta) = -\frac{\Psi_{q\theta}\left(q^R(\theta),\theta\right)}{\Psi_{qq}\left(q^R(\theta),\theta\right)}$. (Given maintained assumptions, $\Psi_q(q,\theta)$ is continuously differentiable at θ where $f^*(\theta)$ is continuous.) Therefore, in the same interval $q^R(\theta)$ will be strictly increasing if $\Psi_{q\theta}\left(q^R(\theta),\theta\right) > 0$ and strictly decreasing if $\Psi_{q\theta}\left(q^R(\theta),\theta\right) < 0$.

Part (1): As the non-negativity constraint is not binding at $\underline{\theta}$ $(q^R(\underline{\theta}) = q^{FB}(\underline{\theta}) > 0)$, the upper bound $q^S(\theta)$ is strictly increasing, and $q^R(\theta)$ is continuous, $q^R(\theta)$ will be strictly increasing for all θ if the cross partial derivative $\Psi_{q\theta}(q,\theta)$ is strictly positive for all $(q,\theta) \in \left[0,q^S(\bar{\theta})\right] \times [\underline{\theta},\bar{\theta}]$.

Define $\varphi(q,\theta)$ and ε: (Note that ε is well defined since $\left[0,q^S(\bar{\theta})\right] \times [\underline{\theta},\bar{\theta}]$ is compact and $F \in C^2$ and $V(q,\theta) \in C^3$ imply $\varphi(q,\theta)$ is continuous.)

$$\varphi(q,\theta) \equiv \frac{1}{f(\theta)} + \frac{(\bar{\theta}-\underline{\theta})}{f(\theta)}\left(\frac{|V_{q\theta\theta}(q,\theta)|}{V_{q\theta}(q,\theta)} + \frac{\left|\frac{d}{d\theta}f(\theta)\right|}{f(\theta)}\right) > 0$$

$$\varepsilon \equiv \min_{(q,\theta)\in\left[0,q^S(\bar{\theta})\right]\times[\underline{\theta},\bar{\theta}]} \frac{1}{\varphi(q,\theta)}$$

By differentiation:

$$\Psi_{q\theta}(q,\theta) = V_{q\theta\theta}(q,\theta)\frac{F(\theta)-F^*(\theta)}{f(\theta)} + V_{q\theta}(q,\theta)\left(1 + \frac{f(\theta)-f^*(\theta)}{f(\theta)} - \frac{\frac{d}{d\theta}f(\theta)}{f^2(\theta)}[F(\theta)-F^*(\theta)]\right)$$

As $f(\theta) > 0$ and $V_{q\theta}(q,\theta) > 0$:

$$\Psi_{q\theta}(q,\theta) \geq -|V_{q\theta\theta}(q,\theta)|\frac{|F(\theta)-F^*(\theta)|}{f(\theta)} + V_{q\theta}(q,\theta)\left(1 - \frac{|f(\theta)-f^*(\theta)|}{f(\theta)} - \frac{\left|\frac{d}{d\theta}f(\theta)\right|}{f^2(\theta)}|F(\theta)-F^*(\theta)|\right)$$

The assumption $|f(\theta)-f^*(\theta)| < \varepsilon$ implies that $|F(\theta)-F^*(\theta)| < \varepsilon(\bar{\theta}-\underline{\theta})$ and therefore that:

$$\Psi_{q\theta}(q,\theta) > V_{q\theta}(q,\theta)(1 - \varepsilon\varphi(q,\theta))$$

By the definition of ε, this implies $\Psi_{q\theta}(q,\theta) > 0$ for all $(q,\theta) \in \left[0,q^S(\bar{\theta})\right] \times [\underline{\theta},\bar{\theta}]$.

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Part (2): The first step is to show that $\Psi_{q\theta}(q, \theta^*) < 0$. Define γ:

$$\gamma \equiv \max_{q \in [0, q^S(\bar{\theta})]} \left(\frac{|V_{q\theta\theta}(q, \theta^*)|}{V_{q\theta}(q, \theta^*)} + 2f(\theta^*) + \frac{\left|\frac{d}{d\theta}f(\theta^*)\right|}{f(\theta^*)} \right)$$

As $f(\theta) > 0$ and $V_{q\theta}(q, \theta) > 0$:

$$\Psi_{q\theta}(q, \theta) \leq |V_{q\theta\theta}(q, \theta)| \frac{|F(\theta) - F^*(\theta)|}{f(\theta)} + V_{q\theta}(q, \theta) \left(2 - \frac{f^*(\theta)}{f(\theta)} + \frac{\left|\frac{d}{d\theta}f(\theta)\right|}{f^2(\theta)} |F(\theta) - F^*(\theta)| \right)$$

since $|F(\theta) - F^*(\theta)| \leq 1$:

$$\Psi_{q\theta}(q, \theta) \leq \frac{V_{q\theta}(q, \theta)}{f(\theta)} \left(\frac{|V_{q\theta\theta}(q, \theta)|}{V_{q\theta}(q, \theta)} + 2f(\theta) + \frac{\left|\frac{d}{d\theta}f(\theta)\right|}{f(\theta)} - f^*(\theta) \right)$$

By definition of γ and $f^*(\theta^*) > \gamma$, it follows that for all $q \in [0, q^S(\bar{\theta})]$: $\Psi_{q\theta}(q, \theta^*) < 0$. Given continuity of $f^*(\theta)$ at θ^*, $\Psi_{q\theta}(q, \theta)$ is also continuous at θ^*. Therefore for some $\delta_1 > 0$, $\Psi_{q\theta}(q, \theta) < 0$ just above θ^* in the interval $\theta \in [\theta^*, \theta^* + \delta_1)$.

The second step is to show that for some $\delta_2 > 0$ neither satiation nor non-negativity constraints are binding in the interval $(\theta^*, \theta^* + \delta_2)$. First, satiation is not binding just above θ^* as q^R is below first best here (Proposition 5), which is always below the satiation bound (Lemma 5). Second, non-negativity is not binding just to the right of θ^* because $q^R(\theta^*) = q^{FB}(\theta^*) > 0$ (Proposition 5 and Lemma 5) and $q^R(\theta)$ is continuous (Lemma 2).

Steps one and two imply that $q^R(\theta)$ is strictly decreasing in the interval $(\theta^*, \theta^* + \min\{\delta_1, \delta_2\})$ just above θ^*. Therefore, $q^R(\theta)$ is either strictly decreasing at θ^* or has a kink at θ^* and is strictly decreasing just above θ^*. In either case, monotonicity is violated at θ^* and the equilibrium allocation will involve pooling at θ^*. ∎

C   Monopoly Multi-Tariff Menu Extension

In this appendix, I extend the single tariff model explored in detail in the main paper to a multi-tariff monopoly model. The model is described in Section 5 of the paper. (Note that I replace equation (1) by the stricter assumption $V_{qq\theta} = 0$.) The model and solution methods are closely related to Courty and Li (2000). To work with the new problem, first define:

Definition 2 *(1) Let $q^c(s, \theta, \theta') \equiv \min\{q(s, \theta'), q^S(\theta)\}$ be the consumption quantity of a consumer who chooses tariff s, is of type θ, and reports type θ'. Consumption quantity of a consumer who honestly reports true type θ is $q^c(s, \theta) \equiv q^c(s, \theta, \theta)$. (2) $u(s, \theta, \theta') \equiv V(q^c(s, \theta, \theta'), \theta) - P(s, \theta')$*

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is the utility of a consumer who chooses tariff s, is of type θ, and reports type θ'. (3) $u(s, \theta)$
$\equiv u(s, \theta, \theta)$ is the utility of a consumer who chooses a tariff s, and honestly reports true type θ.
(4) $U(s, s') \equiv \int_{\underline{\theta}}^{\bar{\theta}} u(s', \theta) f(\theta|s) d\theta$ is the true expected utility of a consumer who receives signal s,
chooses tariff s', and later reports θ honestly. $U^*(s, s') \equiv \int_{\underline{\theta}}^{\bar{\theta}} u(s', \theta) f^*(\theta|s) d\theta$ is the analogous
perceived expected utility. (5) $U(s) \equiv U(s, s)$ is the expected utility of a consumer who honestly
chooses the intended tariff s given signal s, and later reports θ honestly. $U^*(s) \equiv U^*(s, s)$ is the
analogous perceived expected utility.

Invoking the revelation principle, the monopolist's problem may then be written as:

$$\max_{\substack{q(s,\theta) \geq 0 \\ P(s,\theta)}} E\left[\int_{\underline{\theta}}^{\bar{\theta}} (P(s, \theta) - C(q(s, \theta))) f(\theta|s) d\theta \right]$$

such that

1. Global IC-2 $u(s, \theta, \theta) \geq u(s, \theta, \theta') \quad \forall s \in S, \forall \theta, \theta' \in \Theta$
2. Global IC-1 $U^*(s, s) \geq U^*(s, s') \quad \forall s, s' \in S$
3. Participation $U^*(s) \geq 0 \quad \forall s \in S$

As the signal s does not enter the consumers value function directly, the second period incentive compatibility constraints may be handled just as they are in the single tariff model (See part 1 of the proof of Proposition 1). In particular, second period local incentive compatibility ($\frac{\partial}{\partial \theta} u(s, \theta) = V_\theta(q^c(s, \theta), \theta)$) and monotonicity ($\frac{\partial}{\partial \theta} q^c(s, \theta) \geq 0$) are necessary and sufficient for second period global incentive compatibility. Moreover, Lemma 1 naturally extends to the multi-tariff setting. Applying the same satiation refinement $q(s, \theta) \leq q^S(\theta)$, the distinction between $q^c(s, \theta)$ and $q(s, \theta)$ may be dropped.

The next step is to express payments in terms of consumer utility, and substitute in the second period local incentive constraint, just as was done in the single-tariff model (See parts 2 and 3 of the proof of Proposition 1). This yields analogs of equations (3) and (5):

$$U^*(s, s') = u(s', \underline{\theta}) + E\left[V_\theta(q(s', \theta), \theta) \left. \frac{1 - F^*(\theta|s)}{f(\theta|s)} \right| s \right] \tag{16}$$

$$U^*(s) - U(s) = E\left[V_\theta(q(s, \theta), \theta) \left. \frac{F(\theta|s) - F^*(\theta|s)}{f(\theta|s)} \right| s \right] \tag{17}$$

$$P(s, \theta) = V(q(s, \theta), \theta) - \int_{\underline{\theta}}^{\theta} V_\theta(q(s, z), z) dz - u(s, \underline{\theta}) \tag{18}$$

In the single tariff model, the utility of the lowest type $\underline{\theta}$ was determined by the binding participation constraint. To pin down the utilities $u(s, \underline{\theta})$ in the multi-tariff model requires consideration

of both participation and first-period incentive constraints given an assumed ordering of the signal space S. I will consider signal spaces S which are ordered either by FOSD, or a more general reverse second order stochastic dominance (RSOSD).[5] Note that I assume the ordering, either FOSD or RSOSD, applies to consumer beliefs $F^*(\theta|s)$.

In the standard single period screening model the participation constraint will bind for the lowest type, and this guarantees that it is satisfied for all higher types. The same is true here given the assumed ordering of S, as stated in Lemma 6.

Lemma 6 *If S is ordered by FOSD then the participation constraint binds at the bottom ($U^*(\underline{s}) = 0$). This coupled with first period incentive constraints are sufficient for participation to hold for all higher types $s > \underline{s}$. The same is true if S is ordered by RSOSD and $V_{\theta\theta} \geq 0$.*

Proof. It is sufficient to show that $U^*(s, s')$ is non-decreasing in s. If this is true, then IC-1 and $U^*(\underline{s}) \geq 0$ imply participation is satisfied: $U^*(s, \underline{s}) \geq U^*(\underline{s}, \underline{s}) \geq 0$. Hence if $U^*(\underline{s}) \geq 0$ were not binding, profits could be increased by raising fixed fees of all tariffs. Now by definition, $U^*(s, s') \equiv E[u(s', \theta)|s]$. By local IC-2 and $V_\theta \geq 0$ (which follows from $V(0, \theta) = 0$ and $V_{q\theta} > 0$), it is clear that conditional on tariff choice s', consumers' utility is non-decreasing in realized θ:

$$u_\theta\left(s', \theta\right) = V_\theta\left(q\left(s', \theta\right), \theta\right) \geq 0$$

Given a FOSD ordering, this is sufficient for $U^*(s, s')$ to be non-decreasing in s (Hadar and Russell 1969, Hanoch and Levy 1969).

Taking a second derivative of consumer second period utility shows that conditional on tariff choice s', consumers' utility is convex in θ if $V_{\theta\theta} \geq 0$. This follows from increasing differences $V_{q\theta} > 0$, monotonicity $q_\theta(s', \theta) \geq 0$, and local IC-2:

$$u_{\theta\theta}\left(s', \theta\right) = \underbrace{V_{q\theta}\left(q\left(s', \theta\right), \theta\right)}_{(+)} \underbrace{q_\theta\left(s', \theta\right)}_{(+) \text{ by IC-2}} + \underbrace{V_{\theta\theta}\left(q\left(s', \theta\right), \theta\right)}_{\geq 0 \text{ (assumption)}} \geq 0$$

Under RSOSD ordering, this implies that $U^*(s)$ is increasing in s. (This result is analogous to the standard result that if X second order stochastically dominates Y then $E[h(X)] \geq E[h(Y)]$ for any concave utility h (Rothschild and Stiglitz 1970, Hadar and Russell 1969, Hanoch and Levy 1969). The proof is similar and hence omitted.) ∎

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[5] Courty and Li (2000) restrict attention primarily to orderings by first order stochastic dominance or mean preserving spread. However, in their two type model, they do mention that orderings can be constructed from the combination of the two which essentially allows for the more general reverse second order stochastic dominance orderings I consider here.

Given Lemma 6 and the preceding discussion, the monopolist's problem can be simplified as described in Lemma 7.

Lemma 7 *Given a FOSD ordering of S, or a RSOSD ordering of S and $V_{\theta\theta} \geq 0$, the monopolist's problem reduces to the following constrained maximization over allocations $q(s, \theta)$ and utilities $u(s, \underline{\theta})$ for $U^*(s) = U^*(s, s)$ and $U^*(s, s')$ given by equation (16):*

$$\max_{\substack{q(s,\theta)\in[0,q^S(\theta)] \\ u(s,\underline{\theta})}} E\left[\Psi\left(s, q\left(s, \theta\right), \theta\right)\right] - E\left[U^*(s)\right]$$

such that

1. Monotonicity $q(s, \theta)$ non-decreasing in θ

2. Global IC-1 $U^*(s, s) \geq U^*(s, s')$ $\forall s, s' \in S$

3. Participation $U^*(\underline{s}) = 0$

$$\Psi(s, q, \theta) \equiv V(q, \theta) - C(q) + V_\theta(q, \theta) \frac{F(\theta|s) - F^*(\theta|s)}{f(\theta|s)}$$

Payments $P(s, \theta)$ are given as a function of the allocation $q(s, \theta)$ and utilities $u(s, \underline{\theta})$ by equation (18). Second period local incentive compatibility $(\frac{d}{d\theta}u(q, \theta) = V_\theta(q(s, \theta), \theta))$ always holds for the described payments.

Proof. Firm profits can be re-expressed as shown in (equation 19).

$$E\left[\Pi(s, \theta)\right] = E\left[S(s, \theta)\right] + E\left[U^*(s) - U(s)\right] - E\left[U^*(s)\right] \tag{19}$$

Substituting equation (17) for fictional surplus gives the objective function. The participation constraint follows from Lemma 6. For handling of the satiation constraint and second period incentive compatibility, refer to discussion in the text and proofs of Lemma 1 and Proposition 1. ∎

Now, assume that there are two possible first-period signals $s \in \{L, H\}$, and that the probability of signal H is α. There are two first-period incentive constraints. Given Lemma 6, the downward incentive constraint $U^*(H, H) \geq U^*(H, L)$ (IC-H) must be binding. Otherwise the monopolist could raise the fixed fee $P(H, \underline{\theta})$, and increase profits. Together with equation (16) and the binding participation constraint $U^*(L) = 0$ (IR-L), this pins down $U^*(H)$ as a function of the allocation $q(L, \theta)$. Substituting the binding IR-L and IC-H constraints for $u(L, \underline{\theta})$ and $u(H, \underline{\theta})$ into both the monopolist's objective function described in Lemma 7 and the remaining upward incentive constraint $U^*(L, L) \geq U^*(L, H)$ (IC-L) delivers a final simplification of the monopolist's problem in Proposition 7.

Proposition 7 *Define a new virtual surplus* $\Phi\left(q_L, q_H, \theta\right)$ *and function* $\zeta\left(q_L, q_H, \theta\right)$ *by equations (20) and (21) respectively.*

$$\Phi\left(q_L, q_H, \theta\right) \equiv \begin{array}{c} \alpha\Psi\left(H, q_H, \theta\right) f_H\left(\theta\right) + \left(1 - \alpha\right)\Psi\left(L, q_L, \theta\right) f_L\left(\theta\right) \\ -\alpha V_\theta\left(q_L, \theta\right)\left(F_L^*\left(\theta\right) - F_H^*\left(\theta\right)\right) \end{array} \tag{20}$$

$$\zeta\left(q_L, q_H, \theta\right) \equiv \left[F_L^*\left(\theta\right) - F_H^*\left(\theta\right)\right]\left[V_\theta\left(q_L, \theta\right) - V_\theta\left(q_H, \theta\right)\right] \tag{21}$$

Given either a FOSD ordering of S, or a RSOSD ordering and $V_{\theta\theta} \geq 0$:

1. *A monopolist's optimal two-tariff menu solves the reduced problem:*

$$\max_{q_L\left(\theta\right), q_H\left(\theta\right) \in \left[0, q^S\left(\theta\right)\right]} \int_{\underline{\theta}}^{\bar{\theta}} \Phi\left(q_L\left(\theta\right), q_H\left(\theta\right), \theta\right) d\theta$$

such that

IC-2 $q_L\left(\theta\right)$ *and* $q_H\left(\theta\right)$ *non-decreasing in* θ

IC-L $-\int_{\underline{\theta}}^{\bar{\theta}} \zeta\left(q_L\left(\theta\right), q_H\left(\theta\right), \theta\right) d\theta \geq 0$

2. *Payments are given by equations (18), and (22-23):*

$$u\left(L, \underline{\theta}\right) = -E\left[V_\theta\left(q_L\left(\theta\right), \theta\right)\frac{1 - F_L^*\left(\theta\right)}{f_L\left(\theta\right)}\middle| L\right] \tag{22}$$

$$u\left(H, \underline{\theta}\right) = E\left[\left[V_\theta\left(q_L\left(\theta\right), \theta\right) - V_\theta\left(q_H\left(\theta\right), \theta\right)\right]\frac{1 - F_H^*\left(\theta\right)}{f_H\left(\theta\right)}\middle| H\right] + u\left(L, \underline{\theta}\right) \tag{23}$$

Proof. Beginning with the result in Lemma 7, the simplification is as follows: Equation (16) and $U^*\left(L\right) = 0$ imply equation (22). Equation (16) and $U^*\left(H, H\right) = U^*\left(H, L\right)$ imply equation (23). By equation (16) and equations (22-23) $U^*\left(H\right)$ is $\int_{\underline{\theta}}^{\bar{\theta}} V_\theta\left(q_L\left(\theta\right), \theta\right)\left[F_L^*\left(\theta\right) - F_H^*\left(\theta\right)\right] d\theta$. As $U^*\left(L\right) = 0$, this means $E\left[U^*\left(s\right)\right] = \alpha\int_{\underline{\theta}}^{\bar{\theta}} V_\theta\left(q_L\left(\theta\right), \theta\right)\left[F_L^*\left(\theta\right) - F_H^*\left(\theta\right)\right] d\theta$, which leads to the new term in the revised virtual surplus function Φ. Further, as $U^*\left(L\right) = 0$, the upward incentive constraint (IC-L) is $-U^*\left(L, H\right) \geq 0$. By equation (16) and equations (22-23) $U^*\left(L, H\right)$ is $\int_{\underline{\theta}}^{\bar{\theta}} \zeta\left(q_L\left(\theta\right), q_H\left(\theta\right), \theta\right) d\theta$ which gives the new expression for the upward incentive constraint. ∎

Given the problem described by Proposition 7, a standard approach would be to solve a relaxed problem which ignores the upward incentive constraint, and then check that the constraint is satisfied. Given a FOSD ordering, a sufficient condition for the resulting relaxed allocation to solve the full problem is for $q\left(s, \theta\right)$ to be non-decreasing in s. (Sufficiency follows from IC-H binding and $V_{q\theta} > 0$.) This is not the most productive approach in this case, because the sufficient condition is likely to fail with high levels of overconfidence. An alternative approach is to directly incorporate the upward incentive constraint into the maximization problem using optimal control techniques.

Proposition 8 *Given either a FOSD ordering or a RSOSD ordering and $V_{\theta\theta} \geq 0$: The equilibrium allocations $\hat{q}_L(\theta)$ and $\hat{q}_H(\theta)$ are continuous and piecewise smooth. For fixed $\gamma \geq 0$, define relaxed allocations (which ignore monotonicity constraints):*

$$q_L^R(\theta) = \arg\max_{q_L \in [0, q^S(\theta)]} \left\{ \Psi(L, q_L, \theta) - \frac{\gamma + \alpha}{1 - \alpha} V_\theta(q_L, \theta) \frac{F_L^*(\theta) - F_H^*(\theta)}{f_L(\theta)} \right\} \tag{24}$$

$$q_H^R(\theta) = \arg\max_{q_H \in [0, q^S(\theta)]} \left\{ \Psi(H, q_H, \theta) + \frac{\gamma}{\alpha} V_\theta(q_H, \theta) \frac{F_L^*(\theta) - F_H^*(\theta)}{f_H(\theta)} \right\} \tag{25}$$

The relaxed allocations $q_L^R(\theta)$ and $q_H^R(\theta)$ are continuous and piecewise smooth functions characterized by their respective first order conditions except where satiation or non-negativity constraints bind. Moreover, there exists a non-negative constant $\gamma \geq 0$ such that:

1. *On any interval over which a monotonicity constraint is not binding, the corresponding equilibrium allocation is equal to the relaxed allocation: $\hat{q}_s(\theta) = q_s^R(\theta)$.*

2. *On any interval $[\theta_1, \theta_2]$ such that the monotonicity constraint is binding inside, but not just outside the interval for tariff s, the equilibrium allocation is constant at some level $\hat{q}_s(\theta) = q'$ for all $\theta \in [\theta_1, \theta_2]$. Further, the pooling quantity q' and bounds of the pooling interval $[\theta_1, \theta_2]$ must satisfy $q_s^R(\theta_1) = q_s^R(\theta_2) = q'$ and meet the first order condition from the relaxed problem in expectation over the interval. For instance, for $q_L(\theta)$ this second condition is:*

$$\int_{\theta_1}^{\theta_2} \left\{ \Psi_q(L, \hat{q}, \theta) - \frac{\gamma + \alpha}{1 - \alpha} V_{q\theta}(\hat{q}, \theta) \frac{F_L^*(\theta) - F_H^*(\theta)}{f_L(\theta)} \right\} f_L(\theta) \, d\theta = 0$$

3. *Complementary slackness: $\gamma = 0$ or IC-L binds with equality ($\int_{\underline{\theta}}^{\bar{\theta}} \zeta(\hat{q}_L(\theta), \hat{q}_H(\theta), \theta) \, d\theta = 0$).*

Proof. I apply Seierstad and Sydsæter (1987) Chapter 6 Theorem 13, which gives sufficient conditions for a solution. To apply the theorem, I first restate the problem described by Proposition 7 in the optimal control framework. This includes translating the upward incentive constraint into a state variable $k(\theta) = \int_{\underline{\theta}}^{\theta} -\zeta(q_L(z), q_H(z), z) \, dz$ with endpoint constraints $k(\underline{\theta}) = 0$ (automatically satisfied) and $k(\bar{\theta}) \geq 0$. Remaining state variables are $q_L(\theta)$ and $q_H(\theta)$, which have free endpoints. Control variables are $c_L(\theta)$, $c_H(\theta)$, and $c_k(\theta)$, and the control set is R^3. Costate variables are $\lambda_L(\theta)$, $\lambda_H(\theta)$, and $\lambda_k(\theta)$.

$$\max_{q_L, q_H} \int_{\underline{\theta}}^{\bar{\theta}} \Phi(q_L(\theta), q_H(\theta), \theta) \, d\theta$$

State	Control	Costate	Control - State Relation
$q_L(\theta)$	$c_L(\theta)$	$\lambda_L(\theta)$	$\dot{q}_L(\theta) = c_L(\theta)$
$q_H(\theta)$	$c_H(\theta)$	$\lambda_H(\theta)$	$\dot{q}_H(\theta) = c_H(\theta)$
$k(\theta)$	$c_k(\theta)$	$\lambda_k(\theta)$	$\dot{k}(\theta) = -\zeta(q_L, q_H, \theta)$

	End Point Constraints	Lagrangian Multipliers
(by definition)	$k(\underline{\theta}) = 0$	$\underline{\gamma}$
IC-L	$k(\bar{\theta}) \geq 0$	$\bar{\gamma}$

	Control Constraints	Lagrangian Multipliers
Monotonicity	$c_L(\theta) \geq 0$	$\mu_L(\theta)$
	$c_H(\theta) \geq 0$	$\mu_H(\theta)$

	State Constraints	Lagrangian Multipliers
Non-negativity	$q_L(\theta) \geq 0$	$\eta_L(\theta), \underline{\eta}_L, \bar{\eta}_L$
	$q_H(\theta) \geq 0$	$\eta_H(\theta), \underline{\eta}_H, \bar{\eta}_H$
Satiation	$\left[q^S(\theta) - q_L(\theta)\right] \geq 0$	$\sigma_L(\theta), \underline{\sigma}_L, \bar{\sigma}_L$
	$\left[q^S(\theta) - q_H(\theta)\right] \geq 0$	$\sigma_H(\theta), \underline{\sigma}_H, \bar{\sigma}_H$

The Hamiltonian and Lagrangian for this problem are:

$$H = \Phi(q_L, q_H, \theta) + \lambda_L(\theta) c_L(\theta) + \lambda_H(\theta) c_H(\theta) - \lambda_k(\theta) \zeta(q_L, q_H, \theta)$$

$$
\begin{aligned}
L = \ & H + \mu_L(\theta) c_L(\theta) + \mu_H(\theta) c_H(\theta) + \eta_L(\theta) q_L(\theta) + \eta_H(\theta) q_H(\theta) \\
& + \sigma_L(\theta) \left[q^S(\theta) - q_L(\theta)\right] + \sigma_H(\theta) \left[q^S(\theta) - q_H(\theta)\right]
\end{aligned}
$$

The following 7 conditions are sufficient conditions for a solution:

1. Control and state constraints are quasi-concave in states q_L, q_H, and k, as well as controls c_L, c_H, and c_k for each θ. Endpoint constraints are concave in state variables. This is satisfied because all constraints are linear.

2. The Hamiltonian is concave in states q_L, q_H, and k, as well as controls c_L, c_H, and c_k for each θ. This is satisfied because H is constant in k and c_k, linear in c_L and c_H, strictly concave in q_L and q_H (equations 26-27), and all other cross partials are zero. This relies on $V_{qq\theta} = 0$

and $S_{qq}(q, \theta) < 0$.

$$\frac{\partial^2 H}{\partial q_L^2} = (1 - \alpha) S_{qq}(q_L, \theta) < 0 \tag{26}$$

$$\frac{\partial^2 H}{\partial q_H^2} = \alpha S_{qq}(q_H, \theta) < 0 \tag{27}$$

3. State and costates are continuous and piecewise differentiable in θ. Lagrangian multiplier functions as well as controls are piecewise continuous in θ. All constraints are satisfied.

4.
$$\frac{\partial \hat{L}}{\partial c_L} = \frac{\partial \hat{L}}{\partial c_H} = \frac{\partial \hat{L}}{\partial c_k} = 0$$

5.
$$\frac{d}{d\theta}\lambda_L = -\frac{\partial \hat{L}}{\partial q_L}; \quad \frac{d}{d\theta}\lambda_H = -\frac{\partial \hat{L}}{\partial q_H}; \quad \frac{d}{d\theta}\lambda_k = -\frac{\partial \hat{L}}{\partial k}$$

6. Complementary Slackness conditions

   (a) For all θ:
   $$\mu_L, \mu_H, \eta_L, \eta_H, \sigma_L, \sigma_H \geq 0$$
   $$\mu_L c_L = \mu_H c_H = \eta_L q_L = \eta_H q_H = \sigma_L \left[q^S - q_L\right] = \sigma_H \left[q^S - q_H\right] = 0$$

   (b) For $\bar{\theta}$
   $$\bar{\eta}_L, \bar{\eta}_H, \bar{\sigma}_L, \bar{\sigma}_H \geq 0$$
   $$\bar{\eta}_L q_L(\bar{\theta}) = \bar{\eta}_H q_H(\bar{\theta}) = \bar{\sigma}_L \left[q^S(\bar{\theta}) - q_L(\bar{\theta})\right] = \bar{\sigma}_H \left[q^S(\bar{\theta}) - q_H(\bar{\theta})\right] = 0$$

   (c) For $\underline{\theta}$
   $$\underline{\eta}_L, \underline{\eta}_H, \underline{\sigma}_L, \underline{\sigma}_H \geq 0$$
   $$\underline{\eta}_L q_L(\underline{\theta}) = \underline{\eta}_H q_H(\underline{\theta}) = \underline{\sigma}_L \left[q^S(\underline{\theta}) - q_L(\underline{\theta})\right] = \underline{\sigma}_H \left[q^S(\underline{\theta}) - q_H(\underline{\theta})\right] = 0$$

   (d) For IC-L
   $$\bar{\gamma} \geq 0$$
   $$\underline{\gamma}k(\underline{\theta}) = \bar{\gamma}k(\bar{\theta}) = 0$$

7. Transversality Conditions

$$\lambda_L(\bar{\theta}) = \bar{\eta}_L - \bar{\sigma}_L; \quad \lambda_H(\bar{\theta}) = \bar{\eta}_H - \bar{\sigma}_H; \quad \lambda_k(\bar{\theta}) = \bar{\gamma}$$
$$\lambda_L(\underline{\theta}) = -\underline{\eta}_L + \underline{\sigma}_L; \quad \lambda_H(\underline{\theta}) = -\underline{\eta}_H + \underline{\sigma}_H \quad \lambda_k(\underline{\theta}) = -\underline{\gamma}$$

Fix $\gamma \geq 0$. Let $\lambda_k(\theta) = \bar{\gamma} = -\underline{\gamma} = \gamma$. As $\lambda_k(\theta)$ is constant, it is continuous and differentiable, and $\frac{d}{d\theta}\lambda_k = 0$. Neither the state $k(\theta)$ nor the control $c_k(\theta)$ enter the Lagrangian, so $\frac{\partial \hat{L}}{\partial c_k} = -\frac{\partial \hat{L}}{\partial k} = 0$. For any allocation, $k(\underline{\theta}) = 0$ by definition, so $\underline{\gamma}k(\underline{\theta}) = 0$. Putting aside for now the constraint $k(\bar{\theta}) \geq 0$ and the complementary slackness condition $\bar{\gamma}k(\bar{\theta}) = 0$, all other conditions concerning state $k(\theta)$ are satisfied. Moreover, the Hamiltonian and Lagrangian are both additively separable in q_L and q_H. Hence the remaining conditions are identical to those for two independent maximization problems. Namely:

$$\max_{\substack{q_L \in [0, q^S(\theta)] \\ \frac{d}{d\theta}q_L \geq 0}} \left\{ \Psi(L, q_L, \theta) - \frac{\gamma + \alpha}{1 - \alpha} V_\theta(q_L, \theta) \frac{F_L^*(\theta) - F_H^*(\theta)}{f_L(\theta)} \right\}$$

$$\max_{\substack{q_H \in [0, q^S(\theta)] \\ \frac{d}{d\theta}q_H \geq 0}} \left\{ \Psi(H, q_H, \theta) + \frac{\gamma}{\alpha} V_\theta(q_H, \theta) \frac{F_L^*(\theta) - F_H^*(\theta)}{f_H(\theta)} \right\}$$

For fixed $\gamma \geq 0$, the solution to these problems exists, satisfies Proposition 8 parts 1-2, and meets all the relevant conditions in 3-7 above. The proof for this statement is omitted, because the solution for each subproblem given any fixed $\gamma \geq 0$, is similar to the single-tariff case, and closely parallels standard screening results. The idea is that for regions (θ_1, θ_2) where monotonicity is not binding for allocation $\hat{q}_s(\theta)$, the $\hat{q}_s(\theta)$ subproblem conditions are the Kuhn-Tucker conditions for the relaxed solution $q_s^R(\theta)$. For regions in which monotonicity is binding, the characterization closely parallels Fudenberg and Tirole's (1991) treatment of ironing in the standard screening model. For the nice properties of the relaxed solution stated in the proposition, refer to the analogous proof of Proposition 2 part 1.

All that remains to show is that there exists a $\gamma \geq 0$, such that $k(\bar{\theta}) \geq 0$ and $\gamma k(\bar{\theta}) = 0$ given $q_L(\theta)$ and $q_H(\theta)$ that solve the respective subproblems for that γ. If $k(\bar{\theta}) \geq 0$ for $\gamma = 0$ there is no problem. If $k(\bar{\theta}) < 0$ for $\gamma = 0$, the result follows from the intermediate value theorem and two observations: (1) $k(\bar{\theta})$ varies continuously with γ, and (2) for γ sufficiently large $k(\bar{\theta}) > 0$. The latter point follows because each subproblem is a maximization of $E[\Phi(q_L, q_H, \theta)] + \gamma k(\bar{\theta})$. As γ increases, the weight placed on $k(\bar{\theta})$ in the objective increases, and $k(\bar{\theta})$ moves towards its maximum. Given FOSD, $k(\bar{\theta})$ is maximized (given non-negativity, satiation, and monotonicity) at $\{q_L(\theta), q_H(\theta)\} = \{0, q^S(\theta)\}$, for which $q_H > q_L$ and therefore, by binding IC-H and $V_{q\theta} > 0$, IC-L must be strictly satisfied. The argument is more involved, but the constrained maximum of $k(\bar{\theta})$ is also strictly positive given RSOSD. ∎

D Additional Tables

Tables 6-7 replicate Tables 3-4 based on customer-weighted rather than bill-weighted average mistake sizes.

Table 6: Frequency and size of ex post "mistakes" (fall 2002 menu).

	Plan 0 Customers	Plan 1 Customers	Plan 2 Customers
Customers	393 (62%)	92 (15%)	124 (20%)
Bills	5,495	893	1,185
Alternative Considered	Plan 1, 2, or 3	Plan 0	Plan 0
Alternative Lower Cost Ex Post	5%	65%	49%
Conditional Avg. Saving†	25%**	68%**	54%**
Unconditional Avg. Saving†	NA	39%**	5%

†Customer-weighted average per month, as a percentage of Plan 1 monthly fixed-fee.
** 99% confidence.

Table 7: Underusage versus overusage for customers who could have saved on plan 0.

	Plan 1 Customers (60)		Plan 2 Customers (61)	
	Bills (523)	Potential Saving†	Bills (578)	Potential Saving†
Underusage	56%	15%	59%	31%
Intermediate	28%	(7%)	33%	(13%)
Overusage	16%	60%	9%	35%
Total	100%	68%	100%	53%

†Customer-weighted average per month as a percentage of Plan 1 monthly fixed-fee.