

Online Appendices

Durable consumption and asset management with transactions
and observation costs

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APPENDIX A: DATA SOURCES AND VARIABLES' DEFINITIONS

A1. The Survey of Household Income and Wealth (SHIW)

The Bank of Italy Survey of Household Income and Wealth (SHIW) collects detailed data on demographics, households' consumption, income, transaction habits and money holdings and household financial and real assets. It started to be run in the mid 1960s but is available on tape only since 1984. Since 1989 it has been conducted biannually and sampling methodology, sample size and broad contents of the information collected is unchanged. It is one of the few household surveys that collects consumption information, separately for non durable expenditures and purchases of durable categories, together with wealth and income data. In this study we rely on the 2004 wave which matches closely the 2003 UCS. Each survey covers about 8,000 households, constituting a representative sample of the Italian resident population. Sampling is in two stages, first municipalities and then households. Households are randomly selected from registry office records. Households are defined as groups of individuals related by blood, marriage or adoption and sharing the same dwelling. The head of the household is conventionally identified with the husband, if present. Brandolini and Cannari (1994) present a detailed discussion of sample design, attrition, response rates and other measurement issues, and comparisons of the SHIW variables with the corresponding aggregates. The 2004 wave interviewed 8,012 households with a response rate of 36%. The SHIW is publicly accessible; an English version of the questionnaire is available and data can be downloaded at www.bancaditalia.it/statistiche/indcamp/bilfait.

A2. The Unicredit Survey (UCS)

The Unicredit Investors Survey (UCS) draws on the population of clients of one of the three largest European banking groups, with over 4 million accounts in Italy. Two waves of the survey, which is proprietary, are currently available, the first was run in 2003, the second in 2007. The first wave interviewed 1,834 individuals (1,716 the second) with a checking account in one of the banks that are part of the Unicredit Group based in Italy. The sample is representative of the eligible population of customers, excluding customers less than 20 years old or older than 80, and those who hold accounts of less than 1,000 euro (less than 10,000 euros in the 2007 wave) or more than 2.5 million euro.

UCS goal is to study retail customers' financial behavior and expectations. The survey has detailed information on households' demographic structure, individuals financial assets holding (both within and outside the bank), real wealth components and income. It has data on attitudes towards saving and financial investment, propensity to take financial risk, retirement saving and life insurance as well as data relevant for financial decision taking such as financial information activity, financial literacy, trading experience and practice, assets knowledge

and confidence in markets. Interviews for the first wave have been administered between September 2003 and January 2004 (in the first half of 2007 for the second wave) by an Italian leading poll agency, which also serves the Bank of Italy for the Survey on Household Income and Wealth (SHIW). Most interviewers had substantial experience in administering the Bank of Italy SHIW (see below). The Computer Assisted Personal Interview (CAPI) methodology was employed for all interviews. Before the interview, each customer was contacted by phone.

The sampling design is similar to that of the Bank of Italy SHIW. As in SHIW, the population of account holders is stratified along geographical area of residence (North-East, North-West, Central and Southern Italy), city size (less than 30,000 inhabitants and more), and wealth held with Unicredit (as of December 31, 2003). The use of the same company to run the field and the similar sample design facilitates the comparison between UCS and SHIW. The questionnaire was designed with the help of field experts and academic researchers. It has several sections, dealing with household demographic structure, occupation, propensity to save, to invest and to risk, financial information and literacy, individual and household financial portfolio and investment strategies, real estate, entrepreneurial activities, income and expectations, life insurance and retirement income. The wealth questions match those in the Bank of Italy SHIW, which allows interesting comparison between the wealth distributions in the two surveys.

An important feature of the UCS is that sample selection is based on individual clients of Unicredit. The survey, however, contains detailed information also on the household head – defined as the person responsible for the financial matters of the family – and spouse, if present. Financial variables are elicited for both respondents and household.

A3. UCS assets data and wealth definition

UCS contains detailed information on ownership of real and financial assets, and amounts invested. Real assets refer to the household. Financial assets refer to both the account holder and the household. For real assets, UCS reports separate data on primary residence, investment real estate, land, business wealth, and debt (mortgage and other debt). Real asset amounts are elicited without use of bracketing.

Two definitions of financial wealth are available. One refers to the individual account holder, and the other to the entire household. The two can differ because some customers keep financial wealth also in different banks or financial institutions (multi-banking) and/or because different household members have different accounts. In this study we only rely on household level variables.

Calculation of financial assets amounts requires some imputation. First of all, respondents report ownership of financial assets grouped in 10 categories. Respondents are then asked to report financial assets amounts; otherwise, they are asked to report amounts in 16 predetermined brackets and if the stated amount is closer to the upper or lower interval within each bracket. In the 2003 wave the

questions are the same used in the Bank of Italy SHIW.

A4. The distribution of financial wealth in the UCS and SHIW survey

Since the UCS sample only includes banked individuals and there is an asset threshold for participation in the survey, it is no surprise that investors in the two samples are different particularly in terms of average assets holdings. While in the UCS sample people have on average euros 212,000 (median 56,000) in financial assets, the average in SHIW 2004 is only 23,400 (median 7,000). To make a more meaningful comparison between the two dataset we select SHIW and UCS households so as to include only those with positive investments other than in transaction accounts but exclude those with financial assets greater than 1 million euros, to try account for the oversampling of the wealthy in UCS.

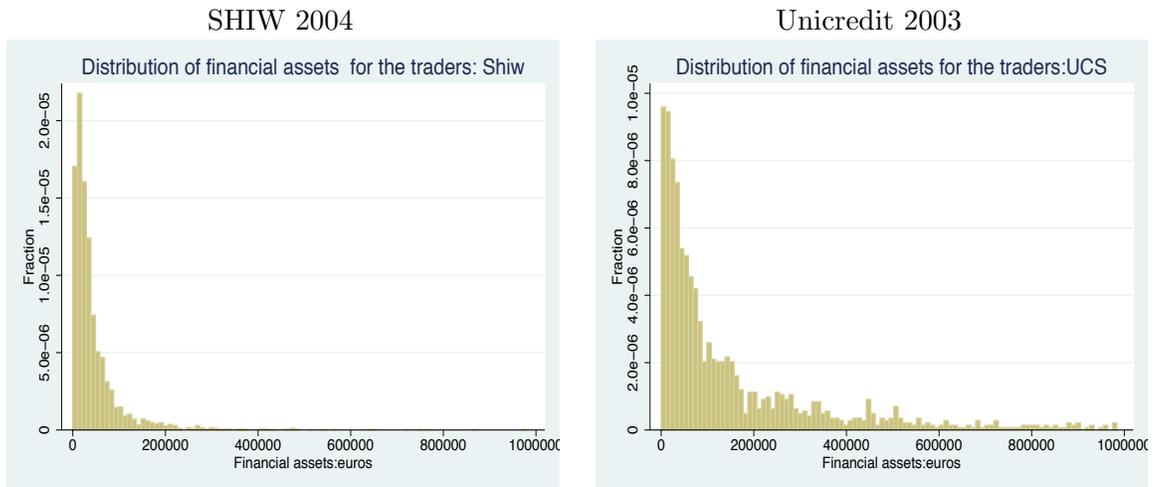


FIGURE A1. HOUSEHOLD DISTRIBUTION OF WEALTH IN SHIW AND UCS

Figure A1 shows the distribution of financial wealth in the 2003 UCS and the 2004 SHIW; once the comparison is limited to the sample of investors the shapes of the distribution become more similar, though large differences between the two dataset remain mainly because UCS is designed to oversample the financially wealthy, as can be seen from the thicker probability mass in the UCS distribution at higher levels of wealth.

A5. Definition of variables constructed from survey responses

Attention when buying durables . Response to the question in UCS 2007: “Before making a purchase involving a relatively large amount of money (such as a car, a washing machine or furniture), some people tend to visit several shops or

dealers in order to compare various prices and try to get a good balance in terms of price/quality ratio. How does this description fits your type?" Possible answers are: "Not at all", "Very little", "Somewhat.", "Close enough", "Very much". We code these answers with numbers between 1 and 5.

Cash holdings. Only available in SHIW which asks people: "What sum of money do you usually have in the house to meet normal household needs?"

Number of withdrawals. Only available in SHIW which asks people: "In 2004 how many cash withdrawals did you or other members of your household make directly at a bank or Post Office on average per month?"

Durable consumption purchases. Only available in SHIW which collects information on purchases of three categories of durables: precious objects, means of transport, and furniture, furnishings, household appliances and sundry articles, by asking: "During 2004 did you (or your household) buy ... (item...)? If "Yes, what is the total value of the objects bought (even if they were not paid for completely)?" Thus, SHIW collects separately whether a purchase took place and its value.

Financial diversification. The ratio of stocks held in mutual funds and other investment accounts to total stocks (direct plus indirect). The index ranges from 0 to 1.

Liquid assets. We use two measures of liquid assets, a narrow one corresponding roughly to M1, and a broader one similar to M2. The first measure is defined as the sum of average cash holdings and checking accounts; the broader measure adds to this savings accounts. Accounts are figures at the household level and figures are variables are defined in the same way in UCS and SHIW. Since UCS has no information on average cash holdings, we impute it from SHIW 2004. Imputation is done by first running a regression on the SHIW sample (retaining only those with a checking account) of average cash holdings (scaled by house value) on a number of variable observed also in UCS: a set of demographics, a fifth order polynomial in income (scaled by house value), a fourth order polynomial in age and interactions between demographics and house value and interactions between income and the polynomial in age and income and demographics. The regressions explains 64% of the cross sectional variability. The estimated coefficients are retrieved and used to predict cash holdings in UCS 2003.

Non durable consumption. From the SHIW question: "What was the monthly average spending of your household in 2004 on all consumer goods, in cash, by means of credit cards, cheques, ATM cards, etc?. Consider all spending, on both food and non-food consumption, and exclude only: purchases of precious objects, purchases of cars, purchases of household appliances and furniture, maintenance payments, extraordinary maintenance of your dwelling, rent for the dwelling, mortgage payments, life insurance premiums, contributions to private pension funds". Answers are multiplied by 12 to obtain an annual figure. Since non durable consumption is not available in UCS, it is imputed using SHIW information. This is done by first running a regression on the SHIW sample (retaining

only those with a checking account) of non durable consumption (scaled by house value) on a number of variable observed also in UCS: a set of demographics, a fifth order polynomial in income (scaled by house value), a fourth order polynomial in age, interactions between demographics and house value and interactions between income and the polynomial in age and income and demographics. The regression explains 91% of the cross sectional variability. The estimated coefficients are retrieved and used to predict cash holdings in UCS 2003.

Observing frequency. Response to the question: “How frequently do you check the value of your financial investment?” Coded as: every day; at least once a week; about every two weeks; about every month; about every three months; about every six months; about every year; less than once a year; never. The number of times (frequency) an investor observes her investments is computed from the responses as: every day=365; at least once a week=52; about every two weeks=26; about every month=12; about every three months=4; about every six months=2; about every year=1; less than once a year=0.36; never=0.10. To impute the last category we approximate “never” as meaning once every 10 years.

Stockholders. We construct two identifiers of stockholders: direct and total. Direct stockholders are investors owing stocks of single companies, either listed or unlisted. Total stockholders own stocks either directly or through a mutual fund or a managed investment account.

Risk aversion. Response to the question in UCS: “Which of the following statements comes closest to the amount of financial risk that you are willing to take when you make your financial investment?: (1) a very high return, with a very high risk of losing the money; (2) high return and high risk; (3) moderate return and moderate risk; (4) low return and no risk.”

Time spent in collecting financial information. Response to question: “How much time do you usually spend, in a week, to acquire information on how to invest your savings? (think about time reading newspapers, internet, talk to your financial advisor, etc.)”. Coded as: no time; less than 30 minutes; between 30 minutes and 1 hour; 1-2 hours; 2-4 hours; 4-7 hours; more than 7 hours. The hours per year indicator is constructed by coding: no time=0; less than 30 minutes= $0.25 \times 4 \times 12 = 12$ (assuming 15 minutes a week); between 30 minutes and 1 hour= $(45/60) \times 4 \times 12 = 36$ (assuming 45 minutes a week); 1-2 hours = $(90/60) \times 4 \times 12 = 72$ (assuming 90 minutes a week); 2-4 hours= $(180/60) \times 4 \times 12 = 144$ (assuming 3 hours a week); 4-7 hours= $(330/60) \times 4 \times 12 = 264$ (assuming 5 hours and 30 minutes a week); more than 7 hours $(450/60) \times 4 \times 12 = 360$ (assuming 7 hours and 30 minutes a week).

Trading. Response to question: “How often do you trade financial assets (sell or buy financial assets)?” Coded as: every day; at least once a week; about every two weeks; about every month; about every three months; about every six months; about every year; less than once a year; at maturity; never. The number of times (frequency) an investor trades her investments is computed from the responses as: every day=365; at least once a week=52; about every two weeks=26; about

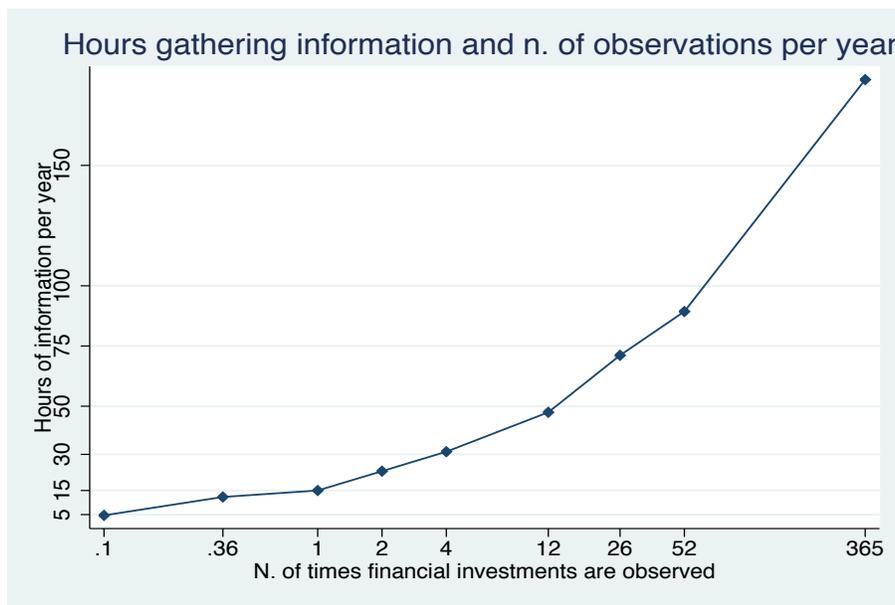


FIGURE A2. TIME SPENT GATHERING FINANCIAL INFORMATION

Source: UCS 2003 survey.

every month=12; about every three months=4; about every six months=2; about every year=1; less than once a year=0.36; at maturity =0.25; never=0.10. To impute the last two categories we assume that average maturity is 4 years and approximate “never” to mean once every 10 years.

APPENDIX B: INATTENTION, LIQUIDITY AND NON-DURABLE GOODS

This section reviews a class of models that use the rational inattention hypothesis to study consumption, savings, portfolio theory and liquidity. In these models the relevant decisions concern the rate of consumption - or savings - and the portfolio composition; the costs are those associated with keeping track of the information about financial variables. Examples of these models are Duffie and Sun (1990), Gabaix and Laibson (2001), Sims (2005), Reis (2006), Abel, Eberly and Panageas (2007, 2009) among others.

We describe a version of the rational inattention model of Abel, Eberly and Panageas (2009). Households maximize discounted expected utility derived from the consumption of non-durables c and period utility $u(c) = c^{1-\gamma}/(1-\gamma)$, with CRRA coefficient γ , and discount rate ρ . The purchases of non-durable goods are subject to a cash-in-advance constraint (CIA): agents must pay with resources drawn from a liquid asset account, with real value denoted by m . Non-negative liquid assets represent a broad monetary aggregate, such as M2, and have a low

real return r_L . The agent's source for the liquid asset is her financial wealth a , a fraction α of which is invested in risky assets and the remaining in risk-less bonds. The risk-less bond yields $r > r_L$, and the risky asset has continuously compounded normally distributed return, with instantaneous mean μ and variance σ^2 (the portfolio is assumed to be managed so that it stays continuously rebalanced with fraction α of risky asset). There is a fixed cost ϕ_o of observing the value of the agent's financial wealth and a fixed cost ϕ_T of changing α and transferring resources between the investment and the liquid assets accounts. From now on we call *observation* the agent's act of observing the value of her financial wealth, and *trade* the agent's act of adjusting the portfolio. We assume that the costs are fixed, in the sense that they are incurred regardless of the size of the adjustment, but they are proportional to the current value of the stock variable a . This specification, which is standard in the literature - see for example Grossman and Laroque (1990) and Abel, Eberly and Panageas (2007) - is adopted for convenience, since it preserves the homogeneity of the value function, and hence reduces the dimensionality of the problem. We remark that the models of Duffie and Sun (1990), Gabaix and Laibson (2001), Abel, Eberly and Panageas (2007) are obtained as a special case of this model by setting the trade cost $\phi_T = 0$.

We denote by $V(a, m)$ the value of the problem for an agent with current liquid assets m , who has just paid the observation cost discovering that her financial wealth is a . She must decide whether to pay the cost ϕ_T , choose α and transfer some resources to/from the liquid account, as well as τ , the length of the time period until the new observation of her financial assets. The budget constraint at the time of observation is

$$(B1) \quad a' + m' + a \phi_T I_{m' \neq m} = a + m$$

where $I_{\{ \cdot \}}$ is an indicator of transfers.

Then the value function solves:

$$V(a, m) = \max_{m', \alpha, \tau, c(\cdot)} \int_0^\tau e^{-\rho t} u(c(t)) dt + e^{-\rho \tau} \int_{-\infty}^{\infty} V(a'(1 - \phi_o)R(s, \tau, \alpha), m(\tau)) dN(s)$$

subject to the budget constraint (B1) at the time of the adjustment, and to the liquid asset-in-advance constraint between adjustment:

$$(B2) \quad dm(t)/dt = r_L m(t) - c(t) \quad , \quad m(0) = m', \quad m(t) \geq 0, \quad \text{for } t \in [0, \tau].$$

An interesting result common to the models of Duffie and Sun (1990), Gabaix and Laibson (2001), and Abel, Eberly and Panageas (2007, 2009) is that trades and observations coincide. In particular, consider an agent with $m = 0$, so at this time she must observe and trade. Then she would choose a transfer $m'(0) > 0$ from her financial asset to her liquid asset, a future observation date $\bar{\tau}$, a portfolio share α , and path of consumption between adjustments $c(\cdot)$, with the property

that $m(\bar{\tau}) = 0$, i.e. she is planning to run out of liquid asset just at the next observation date. In other words, starting from a state of zero liquid asset, the agent will observe at deterministic equally spaced intervals of length $\bar{\tau}$, and every time she observes the value of her financial assets she will transfer resources to her liquid account.

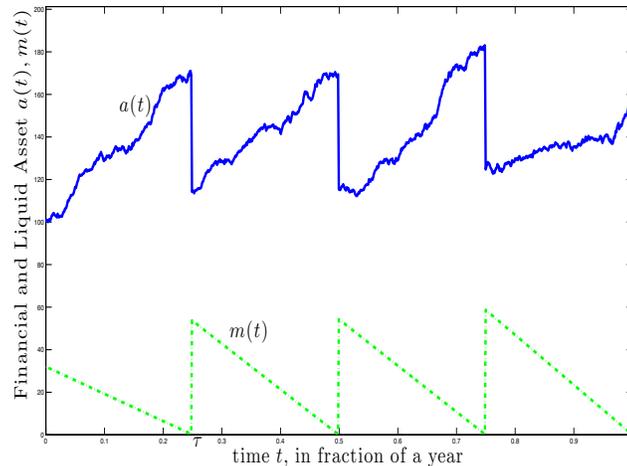


FIGURE B1. SIMULATED PATH OF FINANCIAL ASSET $a(t)$ AND LIQUID ASSETS $m(t)$

The nature of the optimal policy is illustrated in Figure B1, which displays the values for the financial asset and liquid asset in a simulation for an agent following the optimal policy. Notice that the liquid asset $m(t)$ follows a saw-tooth path familiar from the inventory models, such as Baumol-Tobin's classic problem, with withdrawals (and observations) at equally spaced time periods: $0, \bar{\tau}, 2\bar{\tau}$, etc. One difference compared to Baumol and Tobin is that the value of the withdrawal (the vertical jumps up in $m(t)$, and down for $a(t)$ in Figure B1), and hence the rate of consumption between successive withdrawals, depends on the level of the financial assets that the agent observes just before the withdrawal.²⁰ We stress that in this model following the optimal policy implies that every observation coincides with a trade. Also notice, for future reference, that the homogeneity of the problem implies that after an observation and trade, the agent sets the same ratio $m(0)/a(0) \equiv \bar{m}$.

The saw-tooth pattern of liquid asset holdings displayed in the figure makes clear that the model predicts a negative correlation between the average liquidity (scaled by non durable consumption) and the number of transactions (e.g. liquid-

²⁰In the example considered in the figure, following the model parametrization by Abel, Eberly and Panageas (2009), the time variation in the size of withdrawals is small because the variance of the portfolio return is small relative to its mean trend.

ity transfers), as in the standard inventory model of cash holdings.²¹ For instance, in the Baumol-Tobin deterministic model the average liquid balance equals half of the liquidity transfer. Together with the identity positing that the product of the number of transactions (n) times the average liquidity transfer ($2m$) is equal to the flow of expenditure over a given time period (c), gives us $M/c = 1/(2n)$. This implies that the log correlation between M/c and the frequency of transactions is -1 . This same prediction holds in this model and can be tested empirically, as we do in Section I.D.²²

Interestingly Abel, Eberly and Panageas (2009) show that the synchronization between observation and trading holds not only when the cost of transferring resources from financial asset to liquid asset is zero, i.e. when $\phi_T = 0$, but also for $\phi_T > 0$, provided that this cost is not too large. For the case when $\phi_T > 0$ this is a surprising result. To see that, notice that it implies that the following deviation is not optimal. Increase the amount of liquid asset withdrawn $m(0)$, keep the same consumption profile, pay the observation cost, and learn the value of the financial assets at the scheduled time $\bar{\tau}$. Note that by construction in the deviation $m(\bar{\tau}) > 0$. At this time consider following a sS type policy: if the ratio $m(\bar{\tau})/a(\bar{\tau})$ is similar to what it would have been after a withdrawal (given by \bar{m}), then do not trade. If the ratio is small enough, then pay the trade cost ϕ_T , trade, and set the ratio $m(0)/a(0)$ equal to \bar{m} . This deviation has the advantage of saving the fixed trading cost ϕ_T with a strictly positive probability (i.e. the probability that $m(\bar{\tau})/a(\bar{\tau})$ is large). It has the disadvantage that it increases the opportunity cost by holding more of the liquid asset. Abel, Eberly and Panageas (2009) show that indeed this deviation is not optimal, provided that ϕ_T is not too large. We will return to this when we present our model based on durable goods.

APPENDIX C: SPECIAL CASES

This appendix outlines special cases of the more general problem with both information and transactions costs described in the text.

C1. The frictionless case: $\phi_o = \phi_T = 0$

The first order conditions for the continuous time problem are:

$$\begin{aligned} \theta & : 0 = U'(\theta w)w + v'(w)[- \delta - \alpha\mu - (1 - \alpha)r]w - v''(w)(1 - \theta)\alpha^2\sigma^2w^2, \\ \alpha & : 0 = v'(w)[(1 - \theta)(\mu - r)]w + [v''(w)(1 - \theta)^2\sigma^2w^2]\alpha. \end{aligned}$$

²¹Unlike the classic currency management problem in Baumol and Tobin, the new models seem more appropriately applied to broader notions of liquidity, such as M1 or M2.

²²Alvarez and Lippi (2009) show that the negative correlation between m/c and n extends to stochastic inventory models.

We can rewrite the foc's as:

$$(C1) \quad \theta : \frac{U'(\theta w)}{v'(w)} + \frac{-v''(w)w}{v'(w)}(1-\theta)\alpha^2\sigma^2 = \delta + r + \alpha(\mu - r) ,$$

$$(C2) \quad \alpha : (1-\theta)\alpha = \frac{1}{-v''(w)w/v'(w)} \left[\frac{\mu - r}{\sigma^2} \right] .$$

We assume that the utility function is homogenous of degree $\gamma > 0$, $U(h) = \frac{h^{1-\gamma}}{1-\gamma}$. In this case the value function will be homogenous of degree γ too, so we can write it as:

$$(C3) \quad v(w) = v(1) w^{1-\gamma}$$

and foc becomes:

$$(C4) \quad \theta : \frac{\theta^{-\gamma}}{(1-\gamma)v(1)} + \gamma(1-\theta)\alpha^2\sigma^2 = \delta + r + \alpha(\mu - r) ,$$

$$(C5) \quad \alpha : \alpha(1-\theta) = \frac{1}{\gamma} \frac{\mu - r}{\sigma^2} .$$

and the Bellman equation

$$\rho v(1) = \frac{\theta^{1-\gamma}}{1-\gamma} + v(1)(1-\gamma) \left[-\theta\delta + (1-\theta)(r + \alpha(\mu - r)) - \frac{1}{2}\gamma\sigma^2\alpha^2(1-\theta)^2 \right]$$

The foc w.r.t. θ and the Bellman equation can be combined and simplify to:

$$\begin{aligned} \frac{\theta^{1-\gamma}}{v(1)(1-\gamma)} &= -\gamma(1-\theta)\theta\alpha^2\sigma^2 + \theta(\delta + r + \alpha(\mu - r)) , \\ \frac{\theta^{1-\gamma}}{v(1)(1-\gamma)} &= \rho - (1-\gamma) \left(-\theta\delta + (1-\theta)(r + \alpha(\mu - r)) - \frac{1}{2}\gamma\sigma^2(1-\theta)^2\alpha^2 \right) . \end{aligned}$$

Equating these two expressions we obtain:

$$(C6) \quad \begin{aligned} & -\gamma(1-\theta)\theta\alpha^2\sigma^2 + \theta[\delta + r + \alpha(\mu - r)] \\ & = \rho - (1-\gamma) \left[-\theta\delta + (1-\theta)(r + \alpha(\mu - r)) - \frac{1}{2}\gamma\sigma^2(1-\theta)^2\alpha^2 \right] . \end{aligned}$$

We are looking for a solution for this equation with $\theta > 0$. This will require some assumptions on parameters. Among these assumptions, we will include conditions that guarantee that the problem has a finite solution. We will consider two cases. In the first case α is exogenous, and the second where α is optimized over. In the first case this expression is quadratic equation in θ . In the second case, where we replace α by its solution found above, θ can be written as the zero of a higher

order polynomial.

In the case of α endogenous, substituting the first order conditions for $\alpha(1-\theta) = (\mu - r)/(\sigma^2\gamma)$ we obtain:

$$(C7) \quad \theta = \frac{1}{\gamma} \frac{\rho}{(\delta + r)} - \frac{1 - \gamma}{\gamma} \frac{\left[r + \frac{1}{2}(\mu - r)^2/(\gamma\sigma^2) \right]}{\delta + r}.$$

Few comments are in order. First, as $\delta \rightarrow \infty$, then $\theta \rightarrow 0$. Second, for $\gamma = 1$, the 'log' case, the expression for $\theta = \rho/(\delta + r)$. When $\gamma > 1$ this expression is always positive. When $\gamma < 1$, the expression for θ can be negative. It is non-negative if

$$(C8) \quad \rho \geq (1 - \gamma) \left[r + \frac{1}{2}(\mu - r)^2/(\gamma\sigma^2) \right].$$

This conditions ensures that the discount rate ρ is large enough so that utility is bounded from above.

Coming back to the expression for the value function, replacing the optimal value of α on the foc for θ derived above we obtain:

$$(C9) \quad v(1) = \frac{\theta^{-\gamma}}{(1 - \gamma)(r + \delta)},$$

thus, as long as θ as given in (C7) is non-negative, which is assured by condition (C8), the problem is well defined.

C2. The problem with observation cost only

This appendix derives the first order conditions that solve the problem with observation cost (and no transaction cost).

$$(C10) \quad v(1) = \frac{1 - e^{-(\rho + (1 - \gamma)\delta)\tau}}{(1 - \gamma)(\rho + (1 - \gamma)\delta)} \theta^{1 - \gamma} + e^{-\rho\tau} v(1) \int_{-\infty}^{\infty} \left[(1 - \theta) (1 - \phi_o) R(s, \tau, \alpha) + \theta e^{-\delta\tau} \right]^{1 - \gamma} dN(s)$$

Letting $\Omega \equiv (1 - \theta) (1 - \phi_o)$ for notation convenience, the foc with respect to τ gives

$$\frac{(\theta e^{-\delta\tau})^{1 - \gamma}}{1 - \gamma} = \rho v(1) \int_{-\infty}^{\infty} \left(\Omega R(s, \tau, \alpha) + \theta e^{-\delta\tau} \right)^{1 - \gamma} dN(s) + v(1) \int_{-\infty}^{\infty} (1 - \gamma) \left[\Omega R(s, \tau, \alpha) + \theta e^{-\delta\tau} \right]^{-\gamma} \left[\Omega \frac{\partial R(s, \tau, \alpha)}{\partial \tau} - \delta \theta e^{-\delta\tau} \right] dN(s)$$

where

$$\frac{\partial R(s, \tau, \alpha)}{\partial \tau} = \left[\alpha \mu + (1 - \alpha)r - \alpha^2 \frac{\sigma^2}{2} + \frac{\alpha \sigma s}{2\sqrt{\tau}} \right] R(s, \tau, \alpha)$$

Simple algebraic manipulation of the FOC for τ yields:

$$(C11) \quad \int_{-\infty}^{\infty} \left\{ \frac{\Omega \left(\rho R(s, \tau, \alpha) - (1 - \gamma) \frac{\partial R(s, \tau, \alpha)}{\partial \tau} \right) + \theta e^{-\delta \tau} (\rho + (1 - \gamma)\delta)}{[\Omega R(s, \tau, \alpha) + \theta e^{-\delta \tau}]^\gamma} \right\} dN(s) = \frac{(\theta e^{-\delta \tau})^{1-\gamma}}{(1 - \gamma) v(1)}$$

The foc with respect to θ gives:

$$(C12) \quad \frac{1 - e^{-(\rho + (1 - \gamma)\delta)\tau}}{v(1)(\rho + (1 - \gamma)\delta)e^{-\rho\tau}(1 - \gamma)} \theta^{-\gamma} = \int_{-\infty}^{\infty} \frac{(1 - \phi_o)R(s, \tau, \alpha) - e^{-\delta\tau}}{[(1 - \theta)(1 - \phi_o)R(s, \tau, \alpha) + \theta e^{-\delta\tau}]^\gamma} dN(s)$$

Rewrite eq (C10) as

$$\frac{1 - e^{-(\rho + (1 - \gamma)\delta)\tau}}{v(1)(\rho + (1 - \gamma)\delta)e^{-\rho\tau}(1 - \gamma)} \theta^{-\gamma} = \frac{e^{\rho\tau}}{\theta} - \frac{1}{\theta} \int_{-\infty}^{\infty} [(1 - \theta)(1 - \phi_o)R(s, \tau, \alpha) + \theta e^{-\delta\tau}]^{1-\gamma} dN(s)$$

Equating the last two equations gives

$$(C13) \quad e^{\rho\tau} = \int_{-\infty}^{\infty} \frac{(1 - \phi_o)R(s, \tau, \alpha)}{[(1 - \theta)(1 - \phi_o)R(s, \tau, \alpha) + \theta e^{-\delta\tau}]^\gamma} dN(s)$$

The foc with respect to α gives

$$(C14) \quad 0 = \int_{-\infty}^{\infty} \frac{\frac{\partial R(s, \tau, \alpha)}{\partial \alpha}}{[(1 - \theta)(1 - \phi_o)R(s, \tau, \alpha) + \theta e^{-\delta\tau}]^\gamma} dN(s)$$

where

$$\frac{\partial R(s, \tau, \alpha)}{\partial \alpha} = [(\mu - r - \alpha\sigma^2)\tau + \sigma s\sqrt{\tau}] R(s, \tau, \alpha)$$

Equations (C10), (C11), (C13) and (C14) give a system of four equations in the 4 unknowns $v(1)$, θ , τ and α .

C3. The Grossman - Laroque case: $\phi_o = 0$ and $\phi_T > 0$

In this appendix we provide more details on the model with adjustment cost only. One difference compared to the original GL model is that the portfolio share α can only be adjusted when the stock of durables is revised (so α becomes a state for the problem, which we omit for notation simplicity). The value function is:

$$\rho V(a, h) \geq \frac{h^{1-\gamma}}{1-\gamma} + V_a(a, h)a[r + \alpha(\mu - r)] - V_h(a, h)h\delta + \frac{1}{2}V_{aa}(a, h)a^2\alpha^2\sigma^2,$$

with equality in the inaction region. The possibility of adjustment gives

$$V(a, h) \geq \max_{a' \geq 0, \alpha} V(a', h(1 - \phi_T) + a - a') ,$$

with equality if adjustment of durables is optimal at that (a, h) . The maximization with respect to the post-adjustment value of a' gives the foc:

$$(C15) \quad V_a(a', h(1 - \phi_T) + a - a') = V_h(a', h(1 - \phi_T) + a - a') .$$

The maximization w.r.t. α gives:

$$\alpha = \frac{\mu - r}{\frac{-V_{aa}(a, h)a}{V_a(a, h)} \sigma^2} .$$

We can write all the conditions jointly as:

$$\rho V(a, h) = \max \left(\rho \max_{a' \geq 0, \alpha} V(a', h(1 - \phi_T) + a - a') , \right. \\ \left. \frac{h^{1-\gamma}}{1-\gamma} + V_a(a, h)a[r + \alpha(\mu - r)] - V_h(a, h)h\delta + \frac{1}{2}V_{aa}(a, h)a^2\alpha^2\sigma^2 \right)$$

Using the homogeneity of V , and $x = a/h$, rewrite this equation in the inaction region as

$$\rho V(x, 1) = \frac{1}{1-\gamma} + V_a(x, 1) x [r + \alpha(\mu - r)] - V_h(x, 1) \delta + \frac{1}{2}V_{aa}(x, 1) x^2\alpha^2\sigma^2$$

To write this PDE as an ODE, we use the homogeneity to express V_h in terms of V_a ,

$$V_h(x, 1) = (1 - \gamma)V(x, 1) - x V_a(x, 1)$$

so that, after replacing into the PDE and collecting terms, gives the ODE:

$$(\rho + \delta(1 - \gamma)) V(x, 1) = \frac{1}{1-\gamma} + V_a(x, 1) x [\delta + r + \alpha(\mu - r)] + \frac{1}{2}V_{aa}(x, 1) x^2\alpha^2\sigma^2$$

so that the solution for $V(x, 1)$ in the inaction range is

$$V(x, 1) = \frac{1}{(1-\gamma)(\rho + \delta(1-\gamma))} + \sum_{i=1}^2 A_i x^{\eta_i}$$

where the roots η_i solve the characteristic equation

$$0 = \rho + \delta(1 - \gamma) - \left(\delta + r + \alpha(\mu - r) - \frac{\alpha^2 \sigma^2}{2} \right) \eta - \frac{\alpha^2 \sigma^2}{2} \eta^2 .$$

The policy rule is characterized by three numbers: the optimal return point \hat{x} , and the 2 barriers (\underline{x}, \bar{x}) that delimit the inaction region: an agent with $x \in (\bar{x}, \underline{x})$ does not adjust. Outside this region, i.e. for $x \geq \bar{x}$ or $x \leq \underline{x}$, the value function is characterized by paying the fixed cost and adjusting so that the post-adjustment ratio is \hat{x} .

The closed form solution for the ODE, up to the two constant of integration A_1, A_2 in the inaction region allows us to then write a system of 5 equations and unknowns. The unknowns are $A_1, A_2, \underline{x}, \bar{x}, \hat{x}$. The five equations are the first order condition for the optimal return point equation (C16), a pair of value matching conditions at each boundary equations (C17), and a pair of smooth pasting conditions at each boundary equations (C18):

(C16)

$$V_a(\hat{x}, 1) = V_h(\hat{x}, 1) ,$$

(C17)

$$\left(\frac{1 + \bar{x} - \phi_T}{1 + \hat{x}} \right)^{1-\gamma} V(\hat{x}, 1) = V(\bar{x}, 1) , \quad \left(\frac{1 + \underline{x} - \phi_T}{1 + \hat{x}} \right)^{1-\gamma} V(\hat{x}, 1) = V(\underline{x}, 1) ,$$

(C18)

$$V_a(\underline{x}, 1)(1 - \phi_T) = V_h(\underline{x}, 1) , \quad V_a(\bar{x}, 1)(1 - \phi_T) = V_h(\bar{x}, 1) .$$

Equation (C16) follows from equation (C15). The value matching equations (C17) use that

$$V(\bar{a} - \tilde{\Delta}^*, \bar{h}(1 - \phi_T) + \tilde{\Delta}^*) = \max_{\tilde{\Delta}} V(\bar{a} - \tilde{\Delta}, \bar{h}(1 - \phi_T) + \tilde{\Delta}) = V(\bar{a}, \bar{h})$$

$$1 = \frac{V_a(\hat{a}, \hat{h})}{V_h(\hat{a}, \hat{h})} \text{ and } \frac{\bar{a} - \tilde{\Delta}^*}{\bar{h}(1 - \phi_T) + \tilde{\Delta}^*} = \frac{\hat{a}}{\hat{h}} \implies \Delta^* = \frac{\bar{x} - \hat{x}(1 - \phi_T)}{1 + \hat{x}}$$

where $\bar{x} = \bar{a}/\bar{h}$, $\hat{x} = \hat{a}/\hat{h}$ and $\Delta^* = \tilde{\Delta}^*/\bar{h}$, and the homogeneity of degree $(1 - \gamma)$ of $V(\cdot)$. The same derivation applies in the lower boundary \underline{x} . To derive the smooth pasting conditions equations (C18) consider pairs (a, h) such that $a + h(1 - \phi_T) = \bar{a} + \bar{h}(1 - \phi_T) = \hat{a} + \hat{h}$ and that $a/h > \bar{a}/\bar{h}$ for some fixed \bar{a}, \bar{h} so that after adjustment the agent will go to \hat{a}, \hat{h} . Notice that for any such pair (a, h) the agent will also pay the fixed cost and adjust to the same level \hat{a}, \hat{h} . Thus we have:

$$V(a, h) = V(\bar{a}, \bar{h}) = V(\hat{a}, \hat{h})$$

or using $a = \bar{a} + \bar{h}(1 - \phi_T) - h(1 - \phi_T)$:

$$V(\bar{a} + \bar{h}(1 - \phi_T) - h(1 - \phi_T), h) = V(\hat{a}, \hat{h})$$

and differentiating w.r.t. h we have

$$0 = -V_a(\bar{a} + \bar{h}(1 - \phi_T) - h(1 - \phi_T), h)(1 - \phi_T) + V_h(\bar{a} + \bar{h}(1 - \phi_T) - h(1 - \phi_T), h)$$

Taking the limit as $h \uparrow \bar{h}$ we have

$$V_a(\bar{a}, \bar{h})(1 - \phi_T) = V_h(\bar{a}, \bar{h})$$

and using homogeneity we obtain equation (C18). Repeating the same argument for values of $a/h < \underline{a}/\underline{h}$ we obtain the other smooth pasting condition.

For completeness we also describe the expected number of adjustment per unit of time, given the two barriers \bar{x} and \underline{x} and given the optimal return point \hat{x} . Let $T(a, h)$ be the expected time until an adjustment takes place, starting from state (a, h) . We have

$$0 = 1 + T_a(a, h)a(r + \alpha(\mu - r)) - T_h(a, h)h\delta + \frac{1}{2}T_{aa}(a, h)a^2\alpha^2\sigma^2$$

with boundaries $T(\bar{a}, \bar{h}) = T(\underline{a}, \underline{h}) = 0$. We are interested in $T(\hat{a}, \hat{h})$, which gives the expected time between successive adjustments. We notice that T is homogeneous of degree zero, and hence we can write:

$$0 = 1 + T_a(x, 1)x(r + \alpha(\mu - r) + \delta) + \frac{1}{2}T_{aa}(x, 1)x^2\alpha^2\sigma^2$$

with boundaries $T(\bar{x}, 1) = T(\underline{x}, 1) = 0$. The solution to this equation is

$$T(x) = B_0 + B_1x^\lambda + B_2 \log x$$

where

$$\lambda = 1 - \frac{r + \alpha(\mu - r) + \delta}{\alpha^2\sigma^2/2}, \quad B_2 = \frac{1}{\alpha^2\sigma^2/2 - (r + \alpha(\mu - r) + \delta)}.$$

The constants B_0 and B_1 are chosen to satisfy the terminal conditions, namely

$$0 = B_0 + B_1\underline{x}^\lambda + B_2 \log \underline{x}, \quad 0 = B_0 + B_1\bar{x}^\lambda + B_2 \log \bar{x}$$

or

$$B_0 = -B_2 \left[\frac{\log \bar{x} - \log \underline{x}}{\underline{x}^\lambda - \bar{x}^\lambda} \bar{x}^\lambda + \log \bar{x} \right], \quad B_1 = B_2 \frac{\log \bar{x} - \log \underline{x}}{\underline{x}^\lambda - \bar{x}^\lambda}.$$

APPENDIX D: THE BELLMAN EQUATION WHEN $\phi_o > 0, \phi_T > 0$

Here we use the homogeneity of the value function $V(a, h)$ to reduce the Bellman equation to a function of a single variable by setting $h = 1$.

$$\begin{aligned}
\bar{V}(a, 1, \alpha) &= \max_{\tau} \left\{ \int_0^{\tau} e^{-\rho t} U(e^{-t\delta}) dt \right. \\
&\quad \left. + e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left((1-\phi_o)e^{\delta\tau} R(s, \tau, \alpha) a, 1, \hat{\alpha}\right) dN(s) \right\} \\
&= \max_{\tau} \left\{ \frac{1 - e^{-(\rho+(1-\gamma)\delta)\tau}}{(1-\gamma)(\rho+(1-\gamma)\delta)} \right. \\
&\quad \left. + e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left((1-\phi_o)e^{\delta\tau} R(s, \tau, \alpha) a, 1, \hat{\alpha}\right) dN(s) \right\} \\
\hat{V}(a, 1) &= \max_{\tau, \alpha, 0 \leq a' \leq a+1-\phi_T} \left\{ \int_0^{\tau} e^{-\rho t} U([a+(1-\phi_T)-a']e^{-t\delta}) dt \right. \\
&\quad \left. + e^{-\rho\tau} \int_{-\infty}^{\infty} V\left(a'(1-\phi_o)R(s, \tau, \alpha), [a+(1-\phi_T)-a']e^{-\delta\tau}, \hat{\alpha}\right) dN(s) \right\} \\
&= \max_{\tau, \alpha, 0 \leq a' \leq a+1-\phi_T} [a+(1-\phi_T)-a']^{1-\gamma} \left\{ \int_0^{\tau} e^{-\rho t} U(e^{-t\delta}) dt \right. \\
&\quad \left. + e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left(\frac{a'}{[a+(1-\phi_T)-a']}(1-\phi_o)e^{\delta\tau} R(s, \tau, \alpha), 1, \hat{\alpha}\right) dN(s) \right\} \\
&= \max_{\tau, 0 \leq a' \leq a+1-\phi_T} [a+(1-\phi_T)-a']^{1-\gamma} \left\{ \frac{1 - e^{-(\rho+(1-\gamma)\delta)\tau}}{(1-\gamma)(\rho+(1-\gamma)\delta)} \right. \\
&\quad \left. + e^{-(\rho+(1-\gamma)\delta)\tau} \max_{\alpha} \int_{-\infty}^{\infty} V\left(\frac{a'}{[a+(1-\phi_T)-a']}(1-\phi_o)e^{\delta\tau} R(s, \tau, \alpha), 1, \hat{\alpha}\right) dN(s) \right\}
\end{aligned}$$

With these expressions it is easy to see that inaction is optimal around the optimal return point. Consider the feasible policy $a' = a$, we have that

$$\begin{aligned}
\hat{V}(a, 1) &= \max_{\tau, \alpha} \left\{ \int_0^{\tau} e^{-\rho t} U((1-\phi_T)e^{-t\delta}) dt \right. \\
&\quad \left. + e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left(ae^{\delta\tau} R(s, \tau, \alpha), (1-\phi_T), \hat{\alpha}\right) dN(s) \right\} < \bar{V}(a, 1, \hat{\alpha})
\end{aligned}$$

where the last inequality follows by the fact that utility is increasing in the stock of durables. This prove that the inaction region includes an interval that contains the target asset-to-durable ratio.

In the remainder of this section we given a detailed account of the relevant

results of the related applied mathematical literature used to characterize the form of the control and inaction sets. We also relate it to the analytical results in the stylized model in Alvarez et al 2011.

It is useful to first consider the related problem in the applied mathematical literature known as the “Stochastic Cash Balance Problem with Fixed Cost”. For our purposes the relevant references are :

- Edwin H. Neave’s paper Management Science, Vol 16, No 7, March 1970, pp 72-490, “The Stochastic Cash Balance Problem with Fixed Costs for Increases and Decreases”,
- Avner Bar-Ilan’s paper in International Economic Review, Vol. 31, No. 1 (Feb., 1990), pp. 229-234, “Trigger-Target Rules Need Not be Optimal with Fixed Adjustment Costs: A Simple Comment on Optimal Money Holding Under Uncertainty” ,
- Xin Chen and David Simchi-Levi’s paper in Probability in the Engineering and Informational Sciences, vol 23, 2009, pp 545562. “A New Approach for the stochastic cash balance problem with fixed cost”

In this problem the *one dimensional* state, when left uncontrolled, changes randomly, with iid increments. The problem is set in *discrete time*, so there are no meaningful distinction for the state to have continuous sample path. There is a period convex cost that the agent bears if no action is taken, and which depend on the state as of the beginning of the period. If an action is taking, the state can be reset to any desired value. Resetting the state involves paying a fixed cost, potentially different for increases ($K > 0$) and decreases ($Q > 0$), and in many cases also a proportional cost, also potentially different for increases and decreases (with per unit cost denoted by k and q respectively). The objective is to determine the optimal policy for each value of the state: i.e. whether an action should be taken, and if so where the state should be reseted after the action is taken.

In what follow we will specialize the case to $K = Q > 0$ so the fixed cost is the same for increases and decreases, and assume that there is no variable cost ($q = k = 0$). This special case is the one closer to the set-up in our paper. Following the notation in this literature, we denote x the state and $y(x)$ the optimal policy. The general solution is that (indeed for each iteration of the Bellman equation) one can find 6 numbers: $t \leq t^+ \leq T \leq U \leq u^- \leq u$ such that:

$$y(x) = \begin{cases} T & \text{if } x \leq t \\ \in \{x, T\} & \text{if } x \in (t, t^+) \\ x & \text{if } x \in [t^+, u^-] \\ \in \{x, U\} & \text{if } x \in (u^-, u) \\ U & \text{if } x \geq u \end{cases}$$

Note that if $q = k = 0$ then $U = T$. One can show that if $Q = K > 0$ then $t^+ < u^-$. Definitely in $[t^+, u^-]$ there is inaction. Also, with regularity conditions (such as letting the cost to grow unbounded if the state grows with no bounds) one has that $-\infty < T < U < +\infty$, so there is definitely control for large enough (absolute) value of the state. The issue is that for intermediate values, such $x \in (t, t^+)$ or $x \in (u^-, u)$ it is optimal to *either have control or inaction*. In other words, as x increases (or respectively as it decreases) there can be a sequence of intervals where the optimal policy alternates between control and inaction. Indeed one can construct examples of problems where this is the case (see for instance, the example in Neave 1970, section 3.1 or the example in Bar-Ilan 1990, section 3).

Notice that in continuous time problems where the state follows a diffusion this is not really a concern. The reason is that even though the results holds for discrete time approximation of the problem, the *ergodic set* will consist of the inaction set, $[t^+, u^-]$ in the notation above.

Now, what makes our problem more complicated is that even though it is set in continuous time, as long as there is a strictly positive observation cost, the agent observes the state at (endogenously) discretely sampled periods. Thus, for our purposes, the relevant problem is the discrete time described above. An additional complication is that in our problem the state is two dimensional. We do not emphasize the second, because -given the assumed homogeneity, the problem can be reduced to a one state.

While in general the inaction set can be complicated, Neave also show conditions under which it is not, i.e. where $t = t^+$ and $u^- = u$, or as he called where the policy is simple. In section 4 of Neave 1970, there is a Theorem (section 4.2 page 489). The conditions are that if a) the expected value of the period cost function is *quasi-convex* and *symmetric* and b) the distribution of the shocks to the state has *zero mean* and it is *symmetric*, and has a *quasi-concave density*. Under these assumptions the policies are "simple" so that the inaction set is an interval.

Indeed in a simpler related setup ("Optimal Price Setting with Observation and Menu Costs", by Alvarez, Lippi and Paciello, forthcoming in the QJE) we analyze a continuous time model with both fixed costs of observation and of transaction, and a one dimensional state (which follows a Brownian motion). We assume that the period return function is quadratic. For the case where the state has no drift in Proposition 3 of that paper we show a result similar to Neave's, so that the inaction set is an interval. Note that our result deals with the fact that due to the observation cost, the decisions are taken at state-dependent discretely sampled periods. While the set up of the two papers share that there are both observation and transaction cost, the set-up does not satisfies all the assumptions of Proposition 3. In particular the period return function is not symmetric in the state (which in this paper is the ratio of assets to durables) and the state has a non-zero drift. Nevertheless in every numerical computation we have solved we have found the inaction set to be an interval.

APPENDIX E: NO TRADE BETWEEN OBSERVATIONS

Notice that between observation dates, with $\sigma > 0$, the agent does not know her wealth. Moreover, there is a strictly positive probability that her wealth can be arbitrarily close to zero. This precludes the strategy where the agent increases her holding of durables between observations dates by withdrawing resources from her financial assets, since she may otherwise violate her budget constraint. Thus, we will only need to consider the case where the agent is allowed to decrease her durable holding, and increase her holding for financial assets in between observation dates. We will like to show that it is not optimal to decrease durable goods, and hence that the restriction on the policies is not binding. But to verify this in the case for $\sigma > 0$ is complicated, so we restrict attention to the case with $\sigma = 0$. We conjecture that the conclusion extends to the case of $\sigma > 0$.

In particular for $\sigma = 0$ we consider the following variational problem taking as given the interval τ , as well as the initial and final wealth $w(0), w(\tau)$, and where during the observation period the agent can sell durables at rate $x(t)$ and invest the proceeds in the financial assets which accumulate at rate r :

$$\max_{\{a(0), h(0), a(\tau), h(\tau), x(t)\}} \int_0^\tau u(h(t))e^{-\rho t} dt$$

subject to :

$$\begin{aligned} w(0) &= a(0) + h(0) , \quad w(\tau) = a(\tau)(1 - \phi_o) + h(\tau), \\ \dot{h}(t) &= -\delta h(t) - x(t) , \quad \dot{a}(t) = r a(t) + x(t), \\ a(0), h(0), a(\tau), h(\tau), x(t) &\geq 0. \end{aligned}$$

PROPOSITION 1: *If $\rho < r + \delta\gamma$, then the non-negativity constraint binds, and the optimal policy is $x(t) = 0$ for all $t \in [0, \tau]$.*

Note that in the case of $\phi_o = 0$ and $\sigma = 0$ (so $\mu = r$), the solution of the continuous time problem gives a value for the fraction of wealth in durables θ equal to $\theta = \frac{r - (r - \rho)/\gamma}{r + \delta}$. Thus the condition for $x(t) = 0$ given in the proposition is equivalent to the condition for $\theta < 1$. This is quite intuitive, for $x(t) = 0$ we require the return on the financial portfolio to be good relative to the discount rate and the depreciation rate. Moreover, if the parameters are such that $\theta < 1$, then when ϕ_o is small but positive, the agent will still choose $x(t) = 0$.

PROOF:

Let $e^{-\rho t}\lambda(t)$ be the multiplier of the law of motion of a at t , $e^{-\rho t}\mu(t)$ the multiplier of the law of motion of h at t , ν_0 the multiplier of the wealth at $t = 0$ constraint, and ν_1 the multiplier of the wealth constraint at $t = \tau$. The

Lagrangian is:

$$\int_0^\tau e^{-\rho t} \left[u(h(t)) + \lambda(t) (a(t)r + x(t) - \dot{a}(t)) + \mu(t) (-h(t)\delta - x(t) - \dot{h}(t)) \right] dt \\ + \nu_1[(1 - \phi)a(\tau) + h(\tau) - w(\tau)] + \nu_0[w(0) - a(0) - h(0)]$$

Integrating by parts we have:

$$\int_0^\tau e^{-\rho t} \left[u(h(t)) + \lambda(t) (a(t)r + x(t) - \rho a(t)) + \dot{\lambda}(t)a(t) \right. \\ \left. + \mu(t) (-h(t)\delta - x(t) - \rho h(t)) + \dot{\mu}(t)h(t) \right] dt \\ + \nu_1[(1 - \phi)a(\tau) + h(\tau) - w(\tau)] + \nu_0[w(0) - a(0) - h(0)] \\ - \lambda(\tau)e^{-\rho\tau}a(\tau) + \lambda(0)a(0) - \mu(\tau)e^{-\rho\tau}h(\tau) + \mu(0)h(0)$$

Then the current value Hamiltonian, with state state (h, a) and control x is:

$$H(a, h, y) = u(h) + \mu(-\delta h - x) + \lambda(ar + x)$$

where μ, λ are costates. The FOC for $t \in (0, \tau)$ include:

$$H_x = 0 : -\mu(t) + \lambda(t) \leq 0$$

with equality if $x(t) > 0$, and

$$\dot{\lambda} = \rho\lambda - H_a : \dot{\lambda}(t) = \rho\lambda(t) - \lambda(t)r \\ \dot{\mu} = \rho\mu - H_h : \dot{\mu}(t) = \rho\mu(t) - u'(h(t)) + \mu(t)\delta$$

While there are more first order conditions involving $a(0), h(0), a(\tau), h(\tau)$ the previous ones suffice for our current purpose. By way of contradiction, assume $x(t) > 0$ in an interval between $0 < t_0$ and $t_1 < \tau$. Then $\lambda = \mu$ and $\dot{\lambda}/\lambda = \rho - r$ in this interval:

$$u'(h(t)) = \lambda(\delta + r) = (\delta + r) \lambda(t_0) e^{(\rho-r)t}$$

for $t \in [t_0, t_1]$. By taking logs and differentiating w.r.t. time, using that $u(h) = h^{1-\gamma}/(1-\gamma)$ we obtain $\frac{\dot{h}(t)}{h(t)} = \frac{r-\rho}{\gamma}$. From the law of motion of h , if $x \geq 0$: $\frac{\dot{h}(t)}{h(t)} \leq -\delta$. Hence, if $\rho < r + \delta\gamma$ we arrive to a contradiction with $x(t) > 0$ on $[t_0, t_1]$.

APPENDIX F: UNICREDIT ADMINISTRATIVE PANEL DATA

In this appendix we use administrative panel data on assets stocks and net flows from investors to a) compute several measures of trading frequency and distinguish between types of assets trades; b) offer more direct evidence on the

timing of between assets liquidation and durable goods purchases.

F1. Data description

The Unicredit administrative data contain information on the stocks and on the net flows of 26 assets categories that investors have at Unicredit.²³ These data are available at monthly frequency for 35 months beginning in December 2006. There are two samples. The first is a sample of about 40,000 investors that were randomly drawn from the population of investors at Unicredit and that served as a reference sample from extracting the investors to be interviewed in the 2007 survey. We refer to this as the large sample. We do not have direct access to the administrative records for the large sample; calculations and estimates on this sample were kindly done at Unicredit. The second which we call the “survey sample” has the same administrative information for the investors that actually participated in the 2007 survey. We do have access to the survey-sample data which can additionally be matched with the information from the 2007 survey. A description of the merged data is in Guiso, Sapienza and Zingales (2010). Since some households left Unicredit after the interview the administrative data are available for 1,541 households instead on the 1,686 in the 2007 survey. Notice that both the large sample and the survey sample are balanced panel data. Since the administrative record registers both the stock of each asset category at the end of the period as well as the net trading flow into that category, we can directly identify trading decisions, which would not be possible if only assets valuation at the end of period were available. One of the 26 assets is the checking account. In what follows we distinguish assets into two categories: liquid assets, which we identify with the checking account, and investments the sum of the remaining 25 assets classes. We also experimented, with no change on the results, with a broader definition of liquid assets, and hence a narrower definition of financial assets.²⁴ For the following it is useful to establish some notation. Let denote the net flow of asset i owned by investor j in month t . Let the first asset be the checking account and let its net flow be $C_{jt} = f_{0jt}$. The net investments flow is the sum over the remaining assets $F_{jt} = \sum_{i=1}^{25} f_{ijt}$.

²³The list includes: checking accounts, time deposits, deposit certificates, stock mutual funds, money market mutual funds, bond mutual funds, other mutual funds, ETF, linked funds, Italian stocks, foreign stocks, unit linked insurance recurrent premium, unit linked insurance one shot premium, stock market index, life insurance recurrent premium, life insurance one shot premium, pension funds, T-bills short term, T-bonds, indexed T-bonds, other T-bills, managed accounts, own bank bonds, corporate bonds Italy, corporate bonds foreign, other bonds.

²⁴There are three other assets that could possibly be included in the liquid assets class: time deposits, deposit certificates, and money market mutual funds. Deposits certificates (pronti contro termini) are really repurchase agreements, with maturities between 1-3 months and 1 year, so for our purposes we do not think of them as liquid. In Italy money market mutual funds do not allow to transfer funds and write checks easily, as in the US; hence we do not think of them as liquid assets either. Time deposits in Italy have no pre-specified maturity, and allow investors to convert them into checking within a short period, typically around a week. For our purposes they can also be considered liquid assets, so we have also experimented defining liquid asset as the sum of both checking accounts and time deposits. We found the results to be essentially the same, so we do not systematically report the results with both definitions.

F2. Measuring trading frequency

We construct several measures of trading frequency from the survey sample. It is useful to start looking simply at the cross sectional distribution of net investment flows F_{jt} : a positive net flow in month t means that over that month the investor has purchased some investments (asset purchase), a negative net flow that he has sold investments (an asset sale). In the first case there must have been a transfer from the checking account i.e. from the liquid asset account; in the second a transfer to the checking account i.e. to the liquid asset account. Later we will return to the relation between investment sales/purchases and flows into/from liquid asset accounts. A zero value of F_{jt} means that the investors made no net trade in that month. Table F1 tabulates the distribution of net investment flows over the 35 months in the sample both in current value and scaled by the average total financial assets of each investor to remove some of the sample heterogeneity; Table F2 shows the conditional distribution of asset purchases (in absolute value) and sales. There are three main features. First, there is a lot of inaction. In more than 50% of the investor/month observations net investment flows are zero, which is consistent with infrequent portfolio adjustments. Second, both sales and purchases of assets tend to be lumpy; median asset sales is 7,200 euros and asset sales tends to be larger than purchases (median value 1,000 euros). On average, a liquidation accounts for 19% of the value of the investors assets while a purchase for about 5%. Third, asset liquidations are (almost twice) less frequent than purchases.

TABLE F1—DISTRIBUTION OF NET INVESTMENT FLOWS

Total Net Investment Flows (F_{jt})		
Percentile	Euro Values	Share of Average Assets
1th	-90,000	-0.63
5th	-10,787	-0.083
10th	-605	-0.004
25th	0	0
50th	0	0
75th	0	0
90th	1,512.8	0.011
95th	10,985	0.072
99th	75786	0.49
Mean	-98.9	-0.005
Sd	9.33	e+080.19

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1400 investors.

TABLE F2—DISTRIBUTION OF NET INVESTMENT FLOWS

Percentile	Net asset sales: $\max\{F_{jt}, 0\}$		Net asset purchases: $\max\{-F_{jt}, 0\}$	
	Euro values	Share of avg. asset	Euro values	Share of avg. assets
1th	2	0.00002	10	0.00005
5th	79	0.0005	100	0.0005
10th	245	0.001	100	0.0008
25th	1155.4	0.01	200	0.002
50th	7,323.7	0.05	1,000	0.008
75th	26,506	0.20	10,000	0.06
90th	75,046	0.55	37,8911.4	0.23
95th	120,430	0.83	10,985	0.46
99th	250,872	1.60	69,566	0.98
Mean	26,768.6	0.19	14,526.4	0.08
Sd	55,960	0.42	43,965	0.21
N obs	6,236	6,236	11,142	11,142

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1400 investors.

To obtain an estimate of the average number of trades due to net asset sales for each investor j in the sample we first define $L_{jt} = 1$ if in month t investor j has sold some investments, that is if the net investment flow $F_{jt} < 0$. We then compute the average annual number of trades with asset sales for household j as $N_j = \sum_{t=1}^{35} L_{jt} \times \frac{12}{35}$. We repeat this exercise but counting as asset sales those only net sales of asset in excess of 500 and 1000 euros. Table F3 shows summary statistics for the number of trades with asset sales in the whole sample as well as in the sample of stockholders. The latter are defined based on whether the investor owns stocks directly (direct stockholders) and directly or indirectly (total stockholders) in the first month of the sample. In the whole sample there is one asset sale per year in median and 1.4 in mean; stockholders sell assets roughly twice more frequently. If one focuses on trades that are likely to involve durable purchases (asset sales in excess of 1,000 euros) the median is around 0.7 per year and the mean is 1.

F3. Frequency of total trades

Total average number of trades in a year is computed as follows. We first define: *Assets purchase indicator*: $P_{jt} = 1$ if in month t , $f_{ijt} > 0$ for at least one financial asset $i > 0$ and 0 otherwise. *Assets sale indicator*: $S_{jt} = 1$ if in month t , $f_{ijt} < 0$ for at least one $i > 0$ and 0 otherwise. That is, we say there is an asset purchase (sale) if there is a net purchase (sale) for at least one of the 25

TABLE F3—SUMMARY STATISTICS FOR THE AVERAGE ANNUAL NUMBER OF ASSET SALES TRADES

	All Asset Sales N_{Sj}		Asset sales ≥ 500		Asset sales ≥ 1000	
	Median	Mean (sd)	Median	Mean (sd)	Median	Mean (sd)
Total sample	1.03	1.40 (1.29)	1.03	1.17 (1.11)	0.70	1.06 (1.03)
Stockholders (total)	1.71	1.81 (1.28)	1.37	1.53 (1.13)	1.02	1.40 (1.07)
Stockholders (direct)	1.71	1.97 (1.30)	1.37	1.69 (1.19)	1.37	1.55 (1.12)

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

investment categories. Notice that in the same month both indicators can take value 1 if the investor sells one type of investment and buys another. We then define $T_{jt} = 1$ if $(P_{jt} = 1 \text{ and } S_{jt} = 0)$ or $(P_{jt} = 0 \text{ and } S_{jt} = 1)$ or $(P_{jt} = 1 \text{ and } S_{jt} = 1)$. Implicit here is the idea that a trade is a trip to the bank/broker to buy or sell assets or to do both and in a trip one can buy and/or sell more than one asset. Finally we compute the average number of total trades per year as

$$N_{Tj} = \sum_{t=1}^{35} T_{jt} \times \frac{12}{35}$$

as well as the average number of trades that involve at least one asset sale and at least one asset purchase respectively:

$$N_{Pj} = \sum_{t=1}^{35} P_{jt} \times \frac{12}{35} \quad \text{and} \quad N_{Sj} = \sum_{t=1}^{35} S_{jt} \times \frac{12}{35}$$

Table F4 shows summary statistics for the total survey sample and the subsamples of direct and indirect stockholders. We report also a decomposition between trades that are sales of assets and trades that are purchases. On average the total number of trades is 4.5 (median 3.4) per year and there is significant heterogeneity in the sample (standard deviation 3.7). On average 2 trades a year involve assets sales and 3.6 involve assets purchases. Notice that the sum of trades involving assets liquidations and those involving assets sales do not sum to the total number of trades; this is because they also include rebalancing trades and they are thus counted twice, one as a sale and one as a purchase.²⁵ For comparison if we use

²⁵ The estimated trades frequency from the administrative data is positively correlated with the one reported in the survey by asking directly the investors and, as in the latter, stockholders trade significantly more frequently. However there are also important differences. First the average number of trades using the panel data measure is lower than the self reported one; second, the cross sectional distribution of trades frequency estimated from the panel is highly skewed but not as much as in the one from self

TABLE F4—SUMMARY STATISTICS FOR THE AVERAGE ANNUAL NUMBER OF TOTAL TRADES (N_{Tj})

Liquid Assets =	Median		Mean		Std Dev
	Checking	Broad	Checking	Broad	Checking
All trades (N_{Tj})	3.4	3.4	4.5	4.5	3.7
Of which asset Sales (N_{Sj})	1.4	1.4	2.0	2.0	2.0
Of which asset Purchases (N_{Pj})	2.4	2.4	3.6	3.5	3.6
Stockholders (N_{Tj}) (direct+indirect)	5.1	5.1	5.8	5.8	3.6
Stockholders (N_{Tj}) (direct)	5.8	5.5	6.0	6.0	3.4

Source: *Unicredit survey sample*, monthly administrative records (35 months) of 26 accounts for each of 1541 investors. Several of the statistics are computed for two definitions of Liquid Assets. The baseline case uses “Checking Account”, and the “Broad” case includes “Time deposits”. See discussion in footnote 24.

a broader definition of liquid assets that besides checking accounts includes time deposits (and hence it excludes them from financial assets) we obtain that the results are essentially the same.

F4. Rebalancing trades

Next we want to separate rebalancing trades to get a sense of their relative importance. We define two notions of rebalancing trades. The first is a broad notion: we define a rebalancing trade any month where there is net sale of one of the financial assets in the investments portfolio and at least one net purchase of

reported data. However these differences are to be expected. First, in the panel we cannot identify trades that occur at a frequency higher than the month, whereas in the survey some report that they trade daily, weekly or every two weeks; second, the panel-based measure is likely to underestimate the number of trades of those who trade more frequently because we use information on trades only at Unicredit and thus we miss trades at other banks. This is more of an issue for frequent traders who happen also to be wealthier and thus to have investments at more than one bank. Third, the concept of investments differs in the two measures. In the administrative data we include savings accounts which are likely to be excluded from the self reported definition since the question in the survey mentions T-bills, Bonds, Stocks, Mutual funds, Managed accounts etc. , but does not mention savings accounts explicitly. These differences can produce non-classical measurement error which can explain both the difference in mean number of trades and the imperfect correlation. When we adjust the survey-based estimates to make the two more comparable e.g. setting at 12 the number of trades per year (the maximum the panel-based estimate can take) in the survey when people report a larger number) or look at investors who only have accounts with Unicredit the correlation improves.

another asset class, or vice versa. Hence the number of rebalancing trades in each month is equal to the minimum between the number of net sales and the number of net purchases. For each investor this measure is then annualized. Table F5 shows summary statistics of this broad definition. In the whole sample the number of trades with some rebalancing is 1.13 per year (median 0.34). Stockholders rebalance almost twice more frequently than non-stockholders a very reasonable feature. The second indicator of rebalancing trades is narrow as it considers as rebalancing all trades that involve a simultaneous purchase and sales of two different investment classes and no net liquidation/purchase of overall investments. That is trades for which the value of sales matches exactly the value of purchases of assets during that month. The mean number of trades with rebalancing only is much smaller: 0.09 per year and around 0.14 for stockholders (Table F5).

TABLE F5—SUMMARY STATISTICS OF ESTIMATES OF THE ANNUAL NUMBER OF REBALANCING TRADES

	Number of trades with some rebalancing			Number of trades with only rebalancing		
	Median	Mean	Std dev	Median	Mean	Std dev
Whole sample	0.34	1.13	1.88	0.00	0.09	0.53
Stockholders (direct+indirect)	1.03	1.78	2.29	0.0	0.14	0.70
Stockholders (direct)	1.03	1.85	2.30	0.00	0.13	0.65

Source: *Unicredit survey sample*, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

To get a sense of the relative importance of rebalancing trades on total trades we have computed the ratio of rebalancing to total for each household. Table F6 shows summary statistics for the whole sample and from the sample of stockholders. Depending on whether we use the broad or narrow measure of rebalancing this ratio ranges between 2% and 20% of total trades. Thus overall rebalancing trades are only a small fraction of total trades. Looking among the largest trades, say those on the 95th percentile of the size distribution of trades, only half of these trades involve some rebalancing, and a substantial fraction (38%) never rebalances.

For comparison if we use a broader definition of liquid asset that includes time deposits, and hence excludes them from financial investments, we obtain that the results are essentially the same. With this alternative measure of liquid/financial asset we will have that the average ratio of trades with some rebalancing have a

TABLE F6—AVERAGE RATIO OF REBALANCING TRADES ON TOTAL FINANCIAL TRADES

	ratio of trades with some rebalancing			ratio of trades with only rebalancing		
	Median	Mean	Std dev	Median	Mean	Std dev
Whole sample	0.13	0.18	0.21	0	0.017	0.07
Stockholders (direct+indirect)	0.21	0.25	0.22	0	0.022	0.09
Stockholders (direct)	0.21	0.25	0.22	0	0.018	0.08

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

median (mean) of 0.34 (and 1.1), and those with rebalancing only have a median (and mean) of 0 (and 0.08).

F5. Evidence on assets sales and durable purchases

In this section we present two types of evidence on the durables goods / asset liquidations model. The first type of evidence shows that liquidations become more frequent and their size larger around the purchase of a house. The second exploits the time relation between investments liquidations and changes in the liquid assets (checking account).

To obtain direct evidence on the link between investments liquidations and durable purchases we rely on the large panel. Unicredit has agreed to add to each observation a flag for those households that over the 35 months covered in the sample have obtained a final approval for mortgage in a certain month. In Italy the closing of the house purchase takes place normally as soon as the final approval of the mortgage takes place. Hence we focus on investments liquidations in the months around a house purchase. We found that 875 households out of 40,000 in our sample have obtained a mortgage (and thus bought a house). We first focus on this group of investors and compute the fraction that liquidate investments in the same month they obtain the mortgage and at several months before and after obtaining the mortgage. Figure 4 shows the pattern of liquidations at various leads and lags over a 24 months window centered around the month of the purchase, month 0. The fraction of house purchasers who sell financial investments starts increasing around four months before the purchase and peaks exactly in the same month they settle for the payment. After the mortgage is obtained the fraction of asset sales drops and stays roughly constant. This pattern is fully

consistent with investors timing assets liquidations to meet the house payment. We briefly comment on the level of this fraction, as it relates to the prediction of our model. While it is clear that the fraction is much larger in the months before than right after a mortgage is obtained, not all the investors sell assets just before obtaining a final approval of a mortgage by Unicredit. Some reasons why this may be the case are the following: i) some mortgages may be obtain when investors refinance, although this is very rare in Italy, ii) some mortgages may be obtained when investors move to a smaller house, iii) some of the assets can be obtained from a different bank.

To show more formal evidence we run controlled probit regressions for the decision to sale investments and Tobit estimates for the amount of asset sold using the following specification

$$y_{it} = \sum_{j=-6}^{+6} \alpha_j DF_{it-j} + \beta w_{it_0} + \lambda S_{iT_0} + \delta' T + \eta_{it}$$

Where y_{it} is the outcome variable (a dummy = 1 if a sale of investments occurs in month t in the probit, or the amount of liquidated investments in the Tobit estimate), the variable DF_{it} is a dummy equal to 1 if a mortgage is obtained in month t ; to model the timing of asset sales we include up to 6 leads and lags in this indicator in our specification; the variable w_{it_0} is the value of the total assets of the investors at the start of the sample (denoted as t_0), the variable S_{iT_0} a dummy equal to 1 if the investors owns stock at the beginning of the sample, the T denote a vector of time dummies and η_{it} a regression error. Results of the estimates as shown in Table F7; the first column reports the probit estimates and the second column the Tobit estimates for the size of liquidations. Results are rather clear. Investors start selling investments around four months before the purchase of the house. The likelihood of a asset sale and its size both increase markedly as the month of the purchase approaches. In the month when the purchase of the house occurs the probability that the investor liquidates investments is 7 percentage points above the average probability of a liquidation in a generic month (which is 9.6 percent) and the size of liquidation is 34,400 euro above mean. Cumulatively over the month of purchase and the four months before the probability that an investors who purchases a house liquidates assets is 20.9 percentage points above the mean probability and the liquidation size is 103,000 euros above mean. At lags higher than 4 there is no statistically significant difference neither in the probability of liquidating investments nor on their size; the same is true after the purchase. As the estimates shows all the lead coefficients are not statistically different from zero. We find this pattern consistent with investors timing assets liquidations in view of the house purchase.

TABLE F7—TIMING OF ASSETS SALES AND HOUSE PURCHASES

Regressor	Probit estimates for asset sale decision		Tobit estimates of size of asset sold	
	Coefficient	Standard error	Coefficient	Standard error
Obtained mortgage:				
α_0 : current	0.070***	0.012	34413.8***	3889.2
α_1 : lag 1	0.053***	0.012	26985.6***	4075.6
α_2 : lag 2	0.029***	0.011	12338.9***	4437.1
α_3 : lag 3	0.027***	0.011	14200.6***	4457.9
α_4 : lag 4	0.030***	0.011	15696.9***	4489.7
α_5 : lag 5	0.007	0.010	1778.6	4982.8
α_6 : lag 6	0.017	0.011	5840.4	4949.9
α_{-1} : lead 1	-0.010	0.010	-6863.4	5103.3
α_{-2} : lead 2	0.003	0.010	3993.9	4868.3
α_{-3} : lead 3	-0.001	0.010	288.3	5015.2
α_{-4} : lead 4	0.002	0.010	-1205.9	5128.6
α_{-5} : lead 5	0.007	0.010	2277.3	5154.2
α_{-6} : lead 6	0.013	0.011	6592.5	5142.4
Other regressors:				
β : Investor total assets	1.51e-07***	1.24e-08	0.124***	0.006
λ : Stockholder	0.094***	0.003	42174.19***	2294.0
N. observations	31247		31247	
Pseudo R^2	0.07		0.0164	

Coefficients of the probit estimates are marginal effects. ***, **, denote significance at 1% or less and 5% respectively. Source: *Unicredit large sample*, monthly administrative records (35 months) of 26 accounts for each of 40,000 investors, out of which 875 have obtained a mortgage during one of the 35 months.

F6. Investments Sales and Liquid Asset

The information on flows of investments liquidation and purchases and on changes in the liquid asset (i.e. checking account) can be used to provide some additional evidence of the link between trades where investments are sold and durables purchased. For this we use the temporal patterns of asset sales and liquid asset changes in our panel data to show that the spending rate of liquid assets that comes from asset sales is at least twice as fast as the one consistent with a model with steady expenditure financed with cash and the observed frequency of asset liquidations.

We run a regression between C_{jt} -the net flow of euros in the checking account

in a given month- and the net investments flow F_{jt} distinguishing between the net flow of investment sales F_{jt}^S and investment purchases F_{jt}^P also in euro amounts during the same month, as well as with lags. Empirically four lags are sufficient to characterize the dynamics. We notice that by construction F_{jt}^S and F_{jt}^P are either zero or positive. So for instance $F_{jt}^S = 100$ means that over that month there is a net investments sale of 100 euros. Likewise if $F_{jt}^P = 100$ there is a net purchase of assets for 100 euros of value during the month. Thus F_{jt}^S and F_{jt}^P are never positive in the same month t for an investor j and they are zero if there are no trades with a net cash flow. The regression we run is

$$C_{jt} = \sum_{k=0}^4 \beta_k F_{jt-k}^S + \sum_{k=0}^4 \gamma_k F_{jt-k}^P + \delta W_{jt} + h_j + u_{jt}$$

where W_{jt} is investor j total financial assets, h_j is an investor j fixed effect and u_{jt} an error term. Estimated coefficients are shown in Table F8. To interpret these estimates remember that from Table F3 the annual number of assets sales for the median household is around 1. If assets sales were used mostly to finance a steady flow of consumption expenditures one would expect that since they occur once a year the euros obtained from the assets sold would be spent out at a rate of roughly 1/12 for each euro of assets sold. Hence in the first month one should see an increase in the cash account of about 0.92 cents per euro of investments liquidation and a negative effect of about 0.08 cents in the subsequent months. Instead we see a smaller increase in liquid assets over the same month of the sale of assets and a much faster decrease in the following two months. In the two month subsequent to one euro of asset sale, approximately 60 cents are spent (the sum of $(1-0.703)+0.23+0.16$). Instead the pattern that we document can be generated by a model where sales of investments are targeted to large disbursements, such as those involving durables goods purchases.

For comparison if we use a broader definition of liquid asset that includes time deposits (hence excludes them from financial investments) we obtain that the results are essentially the same. In particular the pattern of coefficients of the lags of investments sales on change in liquid asset account, i.e. of the coefficients β_k for $k = 0, 1, \dots, 4$ are 0.70, -0.22 , -0.16 , 0.002 and -0.03 respectively.

APPENDIX G: ACCOUNTING FOR BACKGROUND RISK IN TABLE 3

In this section we include controls for background risk on the regressions of liquid asset relative to consumption on trading frequency. We use the Unicredit sample. In this data set we have the following proxies for background risk: two occupational dummies for self employed and government employees and a measure of background risk using a self-reported measure of income risk. This indicator equals one if the respondent is unable to predict if his or her income will fall significantly, increase significantly, or remain unchanged in the 5 years following

TABLE F8—TEMPORAL PATTERN OF CHANGES IN THE LIQUID AND INVESTMENTS ASSETS

Change in liquid asset in a month		
Regressors	Coefficient	Standard Error
Flow of investment sales:		
β_0 : current	0.703***	0.0057
β_1 : lag 1	-0.23***	0.0062
β_2 : lag 2	-0.16***	0.0065
β_3 : lag 3	0.002	0.006
β_4 : lag 4	-0.03	0.0065
Flow of investment purchases:		
γ_0 : current	-0.65***	0.0065
γ_1 : lag 1	0.020***	0.007
γ_2 : lag 2	-0.076***	0.007
γ_3 : lag 3	0.056***	0.007
γ_4 : lag 4	-0.011**	0.006
Investor total assets:		
δ :	0.092***	0.0025
N. observations	31622	
R^2	0.47	

OLS regressions of the net flow into the checking account on the net flow of investments sales and purchases. Estimates include investors fixed effects. *** ** indicate significant at 1% or less and 5% respectively. Source: *Unicredit survey sample*, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

the interview. Running regression in the sample of sole self-employed or in the sample of sole government employees leads to a very similar estimate of the slope regression of log liquid assets on the number of trades.

TABLE G1—LIQUID ASSETS AND PORTFOLIO TRADES

	Table 3 last col. Whole sample	Table 3 last col. stockholders	Whole sample and controls for background risk	Stockholders and controls for background risk
	log liquid assets: log $M2$ /cons.	log liquid assets: log $M2$ /cons.	log liquid assets: log $M2$ /cons.	log liquid assets: log $M2$ /cons.
log N. of trades financial invest.	0.1266*** (0.0241)	0.1082*** (0.0289)	0.1155*** (0.0251)	0.0962*** (0.0301)
log household consumption	-0.5243*** (0.1404)	-0.6936*** (0.1574)	-0.5600*** (0.1419)	-0.6948*** (0.1598)
log household income	0.2882** (0.1191)	0.4457*** (0.1501)	0.2529** (0.1236)	0.3439** (0.1569)
Years of education	0.0017 (0.0104)	0.0061 (0.0132)	0.0155 (0.0117)	0.0239 (0.0147)
Age	0.0180*** (0.0031)	0.0200*** (0.0041)	0.0202*** (0.0034)	0.0228*** (0.0044)
Resident in the North	-0.3876*** (0.0850)	-0.4043*** (0.1080)	-0.3965*** (0.0886)	-0.3840*** (0.1120)
Male	0.0379 (0.0970)	0.1091 (0.1282)	-0.0235 (0.1072)	0.0843 (0.1431)
Married	-0.2410** (0.0944)	-0.1694 (0.1210)	-0.1867* (0.0982)	-0.1377 (0.1258)
Resident in a small city	0.1960** (0.0848)	0.1717 (0.1070)	0.2287*** (0.0882)	0.1705 (0.1107)
stockholder	0.1067 (0.0908)		0.1016 (0.0940)	
Self employed			0.1814 (0.1243)	0.2365 (0.1517)
Government employee			-0.1387 (0.1000)	-0.1397 (0.1255)
Income risk dummy			-0.1903* (0.0989)	-0.3201** (0.1273)
No. Observations	1363	875	1269	822
R^2	0.069	0.073	0.077	0.087

Source: The independent variable in each of the regressions is the log of the ratio of $M2$ to imputed durable consumption. *** ** indicate significant at 1% or less and 5% respectively, standard errors in parenthesis. Source: *Unicredit survey sample*, of monthly administrative records (for 35 months) of 26 accounts augmented by survey questions for 1541 investors.