

Appendix to "Balance Sheets and Exchange Rate Policy"

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A. Derivation of the risk premium

The purpose of this appendix is to sketch a justification for our specification of the risk premium (equations 10 and 11 in the main text). The argument outlined below closely follows Bernanke, Gertler, and Gilchrist (1999, henceforth BGG).¹

Consider the contracting problem between a single entrepreneur, indexed by j , and foreign lenders in any period t . At the time of contracting, j 's net worth ($P_t N_t^j$), the dollar interest rate (ρ_{t+1}), and prices in period t are known. For now, assume also that the period $t + 1$ rental rate on capital in dollars, R_{t+1}/S_{t+1} , is known. We shall discuss this assumption shortly.

Entrepreneurs and foreign creditors are risk neutral. Their joint problem is to choose a level of investment (K_{t+1}^j), a dollar loan (D_{t+1}^j), and a repayment schedule so as to maximize the expected return to the entrepreneur, such that creditors are paid at least their opportunity cost of funds, and subject to resource and information constraints. The latter are as follows. Investment in period t , K_{t+1}^j , yields $\omega_{t+1}^j K_{t+1}^j (R_{t+1}/S_{t+1})$ dollars next period, where ω_{t+1}^j is a random shock. The distribution of ω_{t+1}^j is public information and is such that ω_{t+1}^j is i.i.d. across j and t , and its expected value is one. Crucially, as in Townsend (1979) and Williamson (1987), we assume that the realization of ω_{t+1}^j cannot be observed by lenders unless they pay a proportional monitoring cost of $\zeta \omega_{t+1}^j K_{t+1}^j (R_{t+1}/S_{t+1})$; in contrast, ω_{t+1}^j is observed freely by the entrepreneur.

Under these conditions, it has been shown by Williamson (1987) that the optimal contract is a *standard debt contract*. Such a contract stipulates a fixed repayment, say of B_{t+1}^j dollars; if the entrepreneur cannot repay that amount, lenders monitor the outcome and seize the whole yield on the investment. Clearly, monitoring occurs only if the realization of ω_{t+1}^j is low enough. Letting $\bar{\omega}$ be such that $B_{t+1}^j = \bar{\omega} K_{t+1}^j (R_{t+1}/S_{t+1})$, monitoring occurs if and only if ω_{t+1}^j is below $\bar{\omega}$, an event interpretable as bankruptcy.

¹See Céspedes (2000) for a related analysis in an open economy.

The resulting problem is formally identical to that analyzed in appendix A of BGG; for convenience, we summarize here the implications that are key for our purposes. To provide the lender with an expected return of ρ_{t+1} , it must be the case that

$$\begin{aligned} & K_{t+1}^j(R_{t+1}/S_{t+1}) \left\{ [\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)] \right\} \quad (\text{A.1}) \\ & = (1 + \rho_{t+1})D_{t+1}^j = (1 + \rho_{t+1})(Q_t K_{t+1}^j - P_t N_t^j)/S_t \end{aligned}$$

where $H(\cdot)$ denotes the c.d.f. of ω_{t+1}^j . The first term of A.1 gives the expected dollar yield on investment. With probability $1 - H(\bar{\omega})$ there is no bankruptcy, and lenders are repaid $B_{t+1}^j = \bar{\omega}K_{t+1}^j(R_{t+1}/S_{t+1})$. With probability $H(\bar{\omega})$, the entrepreneur goes bankrupt, and lenders are repaid whatever is left after monitoring costs; this is the term $(1 - \zeta)K_{t+1}^j(R_{t+1}/S_{t+1}) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$. The first equality in A.1 gives the opportunity cost of the loan D_{t+1}^j . The second equality takes into account that the loan must equal the value of investment minus the entrepreneur's net worth.

The optimal contract maximizes the entrepreneur's utility

$$\left[\int_{\bar{\omega}}^{\infty} \omega_{t+1}^j dH(\omega_{t+1}^j) - \bar{\omega}(1 - H(\bar{\omega})) \right] R_{t+1} K_{t+1}^j \quad (\text{A.2})$$

subject to A.1. As in Williamson (1987), a key aspect of the contract is that it minimizes expected monitoring costs. Moreover, expected monitoring costs decrease with net worth, which should be intuitive.

To simplify, one can rewrite the constraint A.1 as

$$\kappa_{jt} - 1 = (1 + \eta_{t+1})\kappa_{jt} [\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)] \quad (\text{A.3})$$

where

$$\kappa_{jt} = Q_t K_{t+1}^j / P_t N_t^j \quad (\text{A.4})$$

is the ratio of the value of investment to net worth, and

$$1 + \eta_{t+1} = \frac{R_{t+1} S_t}{Q_t S_{t+1} (1 + \rho_{t+1})} \quad (\text{A.5})$$

is the risk (or "external finance") premium. Also, there is no change in the solution if the objective function A.2 is multiplied or divided by positive variables known

at t , so we can take the objective to be

$$\left\{ \int_{\bar{\omega}}^{\infty} \omega_{t+1}^j dH(\omega_{t+1}^j) - \bar{\omega}(1 - H(\bar{\omega})) \right\} \kappa_{jt} \quad (\text{A.6})$$

Rewritten in this way, the problem is to choose the investment/net worth ratio κ_{jt} and the cutoff $\bar{\omega}$ to maximize A.6 subject to A.3. Since the external finance premium $1 + \eta_{t+1}$ is a parameter of this problem, BGG show that, under suitable conditions,² the optimal cutoff $\bar{\omega}$ is an increasing function of $1 + \eta_{t+1}$, or, expressing that function in inverse form,

$$1 + \eta_{t+1} = \Delta(\bar{\omega}) \quad (\text{A.7})$$

where $\Delta(\cdot)$ is an increasing and differentiable function.³ Note that the optimal cutoff depends only on the external finance premium, reflecting the linearity of the problem; in particular, it is independent of j 's net worth.

The optimal investment/net worth ratio, κ_{jt} , turns out to be a function of $\bar{\omega}$:

$$\kappa_{jt} = \Psi(\bar{\omega}) \quad (\text{A.8})$$

where $\Psi(\cdot)$ is also increasing and differentiable.⁴ Since the cutoff $\bar{\omega}$ is independent of j , as we observed, κ_{jt} is the same for all j , and therefore the same as the *aggregate* ratio of investment to net worth:

$$\frac{Q_t K_{t+1}}{P_t N_t} = \Psi(\bar{\omega}) \quad (\text{A.9})$$

Combining A.7 and A.9, one obtains the risk premium as an increasing function of the value of aggregate investment relative to aggregate net worth.

$$1 + \eta_{t+1} = \Delta(\Psi^{-1}(\frac{Q_t K_{t+1}}{P_t N_t})) \equiv F\left(\frac{Q_t K_{t+1}}{P_t N_t}\right) \quad (\text{A.10})$$

where F is increasing and differentiable.

This completes the derivation of the optimal contract when R_{t+1}/S_{t+1} is known at the time of contracting. For most of the analysis of the paper, which deals

²Two conditions are that (i) $\bar{\omega}$ times the hazard rate of H be increasing in $\bar{\omega}$ and (ii) that $(1 + \eta_{t+1})(1 - \zeta) < 1$.

³The conditions of the previous footnote ensure that $\bar{\omega}(1 - H(\bar{\omega})) + (1 - \zeta) \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$ is maximized at some positive ω^* . BGG show that Δ is increasing and differentiable on $(0, \omega^*)$.

⁴Over $(0, \omega^*)$, where ω^* is given in the previous footnote.

with adjustment under perfect foresight, this is an appropriate assumption. If R_{t+1}/S_{t+1} is uncertain as of the time of contracting, matters can be considerably more complicated. In such cases, we assume that the cutoff $\bar{\omega}$ cannot depend on aggregate risk; this can be taken to be an approximation to the true optimal contract, or perhaps derived from more primitive assumptions on information and timing.⁵ Given that assumption, the preceding analysis survives intact, requiring only that R_{t+1}/S_{t+1} be replaced by its expectation at t .

Two additional details deserve comment. First, it is only a matter of accounting to show that the economy's net worth in any period t must equal aggregate capital income minus foreign debt repayment and monitoring costs, as given by equation 13 in the text. In particular, monitoring costs as a fraction of the return to capital are given by

$$\zeta \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j) \equiv \Phi(1 + \eta_{t+1}) \quad (\text{A.11})$$

since A.7 implies $\bar{\omega} = \Delta^{-1}(1 + \eta_{t+1})$.

Second, our assumption in the text is that entrepreneurs consume a fraction $(1 - \delta)$ of their net worth and reinvest the rest. This can be derived from more primitive assumptions; for instance, one can assume that an individual entrepreneur j "dies" in period $t + 1$ with probability $(1 - \delta)$, and that surviving entrepreneurs are patient enough so that they choose not to consume their wealth until death.⁶

B. Steady State and Linear Approximation

Here we sketch the proof of the existence and uniqueness of a non-stochastic steady state, describe our linear approximation to the equilibrium system, and discuss non-stochastic dynamics.

One can show that, in steady state, the Lagrange multiplier associated with A.3 must equal the inverse of $\delta(1 + \rho)$. The analysis of BGG shows that the

⁵To deal with aggregate risk, BGG assume that entrepreneurs bear all such risk, which is justified by their assumption that borrowers are risk neutral while lenders are risk averse. In contrast, we assume that both contracting sides are risk neutral.

⁶To keep the number of entrepreneurs constant, one can assume that each dead entrepreneur is replaced by a newborn one. A minor problem arises since new entrepreneurs must have some initial net worth to be able to borrow. This can be remedied by assuming that new entrepreneurs are born with an exogenous and arbitrarily small endowment, or that they have a small endowment of labor (as in Carlstrom and Fuerst 1998). The effects of choosing either assumption would be negligible, and so we ignore this issue in the text.

Lagrange multiplier is an increasing function of $\bar{\omega}$, which goes to one as $\bar{\omega}$ goes to zero and to infinity as $\bar{\omega}$ goes to ω^* , where $\omega^* > 0$ is defined in their footnote 2. So, provided that $0 < \delta(1 + \rho) < 1$ there is a unique, strictly positive, steady state solution for $\bar{\omega}$, with which one can pin down the values of η and $\frac{QK}{N}$ at the steady state.

Now we need to solve for the remaining variables, whose steady state levels are given by:

$$Y = AK^\alpha \quad (\text{B.1})$$

$$Q = S^{1-\gamma} \quad (\text{B.2})$$

$$\frac{\alpha Y}{QK} = (1 + \rho)(1 + \eta) \quad (\text{B.3})$$

$$N + SD = QK \quad (\text{B.4})$$

$$Y = \gamma[(1 - \alpha)Y + QK] + SX \quad (\text{B.5})$$

Using B.1 and B.2 into B.5 we obtain

$$(1 - \gamma(1 - \alpha))Y - \gamma \frac{S^{1-\gamma}}{A^{1/\alpha}} Y^{1/\alpha} - SX = 0 \quad (\text{B.6})$$

Now, using B.1 and B.2 into B.3 we obtain

$$S^{1-\gamma} Y^{\frac{1-\alpha}{\alpha}} = \alpha \frac{A^{1/\alpha}}{(1 + \rho)(1 + \eta)} \quad (\text{B.7})$$

which is a hyperbola in (Y, S) space. Using B.7 in B.6 we obtain

$$\left[1 - \gamma(1 - \alpha) - \frac{\alpha\gamma}{(1 + \rho)(1 + \eta)} \right] Y = SX \quad (\text{B.8})$$

Given that $1 - \gamma(1 - \alpha) - \frac{\alpha\gamma}{(1 + \rho)(1 + \eta)} = (1 - \gamma) + (\gamma\alpha - \frac{\alpha\gamma}{(1 + \rho)(1 + \eta)}) > 0$, this is a ray from the origin in (Y, S) space. The steady state values of Y and S must solve B.7 and B.8. Clearly, there is a unique positive solution. With this result in hand, it is simple to solve for the steady state values of K and Q using B.1 and B.2.

Log linearizing the equilibrium equations around the steady state just found, we obtain a system that includes, whether wages are flexible or sticky, equations 20, 21, and 22 in the text, the linearized version of the interest parity condition 10:

$${}_{t-1}y_{t+1} - (q_t - p_t) - k_{t+1} = \rho'_{t+1} + \eta'_{t+1} + {}_t(s_{t+1} - p_{t+1}) - (s_t - p_t) \quad (\text{B.9})$$

and the following equation for the risk premium:

$$\begin{aligned} \eta'_{t+1} &= \phi \eta'_t + \mu (q_t - p_t + k_{t+1} - y_t) \\ &\quad + \mu \delta (1 + \rho) \psi [(s_t - p_t - {}_{t-1}(s_t - p_t)) - (y_t - {}_{t-1}y_t)] \end{aligned} \quad (\text{B.10})$$

Here, $\lambda = \frac{\gamma QK}{\gamma QK + SX} = \frac{\alpha \gamma}{(1 - \gamma + \alpha \gamma)(1 + \rho)(1 + \eta)} < 1$, $\mu = \frac{F'(\cdot) QK}{F(\cdot) N}$, and $\phi =$

$\delta (1 + \rho) (1 - \mu \psi) + \mu \{ \delta (1 + \rho) (1 + \psi) \eta + \delta (1 + \rho) - 1 \} \left(\frac{v}{\varepsilon_\Delta} \right)$ where v is the elasticity of $\int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$ with respect to $\bar{\omega}$ and ε_Δ is the elasticity of the Δ function in A.10. Now equation 15 of the main text follows from B.10 (after using 20 in the main text) to eliminate the term $(k_{t+1} + q_t - p_t)$ and recalling that the real exchange rate e_t equals $s_t - p_t$.

The calculation of the saddle-path $z_t = \zeta \eta'_t$ is straightforward. Without uncertainty, equations 15 and 16 in the main text can be written in matrix form as

$$\begin{bmatrix} z_{t+1} \\ \eta'_{t+1} \end{bmatrix} = \Phi \begin{bmatrix} z_t \\ \eta'_t \end{bmatrix} \quad (\text{B.11})$$

We assume that the roots of Φ are real and that a saddle-path exists. Existence of a saddle-path requires the two roots to be on opposite sides of the unit circle; a sufficient condition is that $\phi < 1 + \mu$. The roots of Φ are real if $[(1 - \lambda)(1 + \mu) + (1 + \phi)\lambda]^2 > 4\phi\lambda$. Both sufficient conditions must hold, in particular, if μ is small enough – that is, if financial imperfections are not too stringent.

Standard techniques now yield the saddle-path coefficient $\zeta = \frac{\lambda}{\mu(1-\lambda)} \{ \xi_2 - \phi \}$, where ξ_2 is the *smaller* eigenvalue of Φ . The term in curly brackets is negative, and hence $\zeta < 0$ as stated in the text.

C. Optimality of Flexible Rates

Here we show the validity of an assertion in section IV of the main text. For all t , it is easy to show that taking logs in eq. 10 in the main text one obtains

$$\log K_{t+1} = \log \alpha + \gamma \log Y_t + \log_t Z_{t+1} - \gamma \log Z_t - \log(1 + \rho_{t+1}) - \log(1 + \eta_{t+1})$$

So

$$\log K_{t+1} = \gamma[\alpha \log K_t + (1 - \alpha) \log L_t] + \Lambda_t$$

where the term $\Lambda_t = \log \alpha + \log {}_tZ_{t+1} - \gamma \log Z_t - \log(1 + \rho_{t+1}) - \log(1 + \eta_{t+1})$ is independent of monetary or exchange rate policy. Using the previous expression lagged one period to replace $\log K_t$, and continuing back to period 0, we get

$$\begin{aligned} \log K_{t+1} &= \gamma(1 - \alpha) \log L_t + \gamma\alpha[\gamma[\alpha \log K_{t-1} + (1 - \alpha) \log L_{t-1}] + \Lambda_{t-1}] + \Lambda_t \\ &= \gamma(1 - \alpha)[\log L_t + \alpha \log L_{t-1}] + (\gamma\alpha)^2 \log K_{t-1} + \Lambda_t + \gamma\alpha\Lambda_{t-1} \\ &= \dots \\ &= \gamma(1 - \alpha)[\log L_t + \alpha \log L_{t-1} + \dots + \alpha^t \log L_0] + (\gamma\alpha)^{t+1} \log K_0 + \Upsilon_{t+1} \end{aligned}$$

where Υ_{t+1} is, again, independent of policy, as claimed.

References

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