

The Politics of Compromise

Alessandro Bonatti and Heikki Rantakari

Online Appendix

August 25, 2015

1 Additional Proofs

Proof of Lemma 1. A mechanism consists of (a) two functions $(x_{1,t}, x_{2,t})$, where $x_{i,t}$ denotes the project each agent i works on at time t , and (b) a measurable function $\zeta(\mathbf{x}, \boldsymbol{\tau}, t)$ that maps the public history of project developments $(\mathbf{x}, \boldsymbol{\tau})$ onto an adoption decision by probabilistically selecting (at most) one complete project to adopt at time t . The adoption rule is allowed to be time dependent, stochastic and ex post inefficient.

Now suppose that the first project x_i has been developed at time τ . If it is not adopted immediately, agent j faces a single-agent decision problem from time τ on. Each agent's continuation value is then well-defined under the rule ζ . Let $w_i(x, i, t)$ denote agent i 's continuation value if project x is developed by agent i at time t but it is not adopted. Finally, let $p_{i,t}$ denote the probability (as specified by ζ) that project $x_{i,t}$ is adopted immediately upon completion.

We solve a reduced-form problem in which we optimize directly over the agents' expected payoffs at the time the first project is developed. In particular, agent i 's expected payoff from developing the first project at time t is given by

$$v_i(x_{i,t})p_{i,t} + w_{i,t}(x_{i,t}, i, t)(1 - p_{i,t}). \quad (20)$$

Similarly, agent i 's expected payoff if agent j develops the first project at time t is given by

$$v_i(x_{j,t})p_{j,t} + w_{i,t}(x_{j,t}, j, t)(1 - p_{j,t}). \quad (21)$$

If the mechanism is symmetric, we can refer to the values in (20) and (21) as v_t and y_t ,

respectively. We then maximize over all paths v_t and y_t that satisfy the feasibility constraint

$$y_t \leq \phi(v_t). \quad (22)$$

Each agent's expected payoff is given by

$$V_0 = \int_0^\infty e^{-\int_0^t (r+2a_s^*) ds} (a_t^* (v_t + y_t) - c(a_t^*)) dt, \quad (23)$$

where a_t^* is the effort level generated by the functions v and y . An optimal mechanism maximizes (23) with respect to v_t and y_t , subject to the feasibility constraint and to the following restriction on the function a_t^* :

$$a_t^* = \arg \max_{\{a_t\}} \int_0^\infty e^{-\int_0^t (r+a_s+a_s^*) ds} (a_t v_t + a_t^* y_t - c(a_t)) dt.$$

Following the approach of Mason and Välimäki (2015), we write our maximization problem recursively, letting V_t denote each agent's continuation payoff. We normalize the cost parameter c to 1 so that $r = \rho$. We then obtain an optimal-control formulation of our original problem:

$$\begin{aligned} & \max_{\{v_t, y_t\}} V_0 \\ \text{s.t.} \quad & \dot{V}_t = \rho V_t + (a_t^*)^2 / 2 - a_t^* (v_t + y_t - 2V_t), \end{aligned} \quad (24)$$

$$a_t^* = v_t - V_t, \quad (25)$$

$$y_t \leq \phi(v_t).$$

Equation (24) is the law of motion of V_t , and (25) is the agents' first-order condition in the main text (5) written in terms of V_t . We can then write the Hamiltonian as

$$\mathcal{H}_t = \lambda_t [\rho V_t - (v_t - V_t) ((v_t - V_t) / 2 + y_t - V_t)] + \gamma_t (\phi(v_t) - y_t).$$

The necessary conditions for optimality are given by

$$\begin{aligned} \frac{\partial \mathcal{H}_t}{\partial v_t} &= \frac{\partial \mathcal{H}_t}{\partial y_t} = 0, \\ \dot{\lambda}_t &= -\frac{\partial \mathcal{H}_t}{\partial V_t}, \end{aligned}$$

in addition to the complementary slackness condition:

$$\gamma_t (\phi(v_t) - y_t) = 0. \quad (26)$$

The form of the transversality condition depends on whether a finite or infinite horizon is optimal.

In the infinite-horizon case, we use the transversality condition in Michel (1982):

$$\lim_{t \rightarrow \infty} \mathcal{H}_t = 0. \quad (27)$$

The first-order and complementary slackness conditions imply that $\dot{\mathcal{H}}_t = \dot{\lambda}_t \dot{V}_t + \lambda_t \ddot{V}_t$ is identically zero (i.e., the Hamiltonian for this problem is constant) along the optimal path. Conditions (26) and (27) then imply the Hamiltonian is nil. Because $\lambda \neq 0$, the optimal path V_t^* is constant, and the optimal controls v_t^* and y_t^* are therefore stationary. The constant value V^* as a function of the controls v and y is given by

$$V^*(v, y) = \frac{2v + y + \rho - \sqrt{(v - y - \rho)^2 + 6\rho v}}{3}. \quad (28)$$

Our original problem then reduces to maximizing (28) subject to (22). It is easy to verify that $V^*(v, y)$ is increasing in both of its arguments. Because $\phi(v)$ is strictly decreasing in v , constraint (22) binds. This establishes that the optimal mechanism involves immediate adoption of the first project. The objective $V^*(v, \phi(v))$ then coincides with (10) in the main text; hence, the optimal policy is given by $v_t^* = v_i(x_i^*)$ for all t .

Finally, in the case of a finite horizon, we have the terminal condition $V_T = 0$. It is immediate to verify that (22) binds in this case as well. Because the Hamiltonian is then strictly concave in v , the optimal policy $v(V)$ is given by the solution to the the first-order condition

$$v + \phi(v) - 2V + \phi'(v)(v - V) = 0. \quad (29)$$

The optimal policy $v(V)$ induces an autonomous differential equation for V_t . In particular, (29) and (24) imply that $v(V) > v_i(x_i^*)$ and $\dot{V}_t < 0$ for all $V < V(v_i(x_i^*), \phi(v_i(x_i^*)))$, which is defined in (10) in the main text. Because $V_T = 0$, the optimal path V_t is strictly decreasing. Thus, extending the horizon T improves the initial payoff V_0 , which means that setting a finite deadline is never optimal. ■

Proof of Proposition 3. (1.) The proof is analogous to part (1.) of Proposition 2. (2.) Suppose that firm 1 develops the first project and let the pivotal voter be $\hat{\theta} < 1/2$. For

$(x_1, x_2) \neq (1, 0)$, if the game continues, firm 2 will pursue either its favorite project (if x_1 is eliminated) or a project that leaves voter 1/2 just indifferent between the two alternatives (if a runoff is held). For γ sufficiently close to 1/2, the pivotal voter $\hat{\theta}$ is close to $\theta = 1/2$, which implies that the voter has a strict preference for immediately accepting the first developed project. Firm 1 takes advantage of this and choose a more selfish project. A positive degree of compromise arises when the pivotal voter's preference in favor of firm 2 is sufficiently strong that she prefers to wait for firm 2 to develop its fully selfish project as a competing project, rather than immediately accept firm 1's selfish project. This type is defined by

$$w(\Delta(\theta^*, 1)) = \chi(\rho) w(\Delta(\theta^*, 0)),$$

where

$$\chi(\rho) = 1 - \frac{1}{\sqrt{1 + \frac{2}{\rho}}} \quad (30)$$

is the expected delay until firm 2 completes its preferred project $x_2 = 0$ that is worth 1 to firm 2 and, thus, $v(\Delta(\theta^*, 0))$ to user θ^* . The minimum supermajority requirement is then given by $\underline{\gamma}(\rho) = 1 - F(\theta^*(\rho))$. From equation (30) it is then immediate that since $\chi(\rho)$ is strictly decreasing, $\theta^*(\rho)$ is also strictly decreasing, where $\chi(0) = 1$ and $\theta^*(0) = 1/2$. Conversely, as ρ grows without bound, we have $\chi \rightarrow 0$ and $\theta^* \rightarrow 0$, which implies that $\underline{\gamma} \rightarrow 1$. Symmetric calculations apply to the case in which firm 2 develops the first project.

(3.) The elimination of the first project following a negative vote allows the second firm to pursue a fully selfish project. The equilibrium level of compromise is given by

$$w(\Delta(\hat{\theta}, x_1)) = \chi(\rho) w(\Delta(\hat{\theta}, 0)).$$

Using the implicit-function theorem, it is immediate that $dx_1/d\hat{\theta} > 0$, so the level of compromise is increasing in the supermajority requirement and decreasing in ρ .

If the first project is set aside until the runoff, the second firm can no longer pursue its preferred project, but must persuade the median voter. Thus, the level of compromise is now given by

$$w(\Delta(\hat{\theta}, x_1)) = \chi(\rho) w(\Delta(\hat{\theta}, 1 - x_1)).$$

Again, the degree of compromise is decreasing in ρ . As we shift the supermajority requirement (captured by $\hat{\theta}$), we need to be slightly more careful, as the second firm's effort now depends on the level of compromise and affects the discount factor χ . Totally differentiating

the expression for the delay, we obtain

$$\begin{aligned} & \frac{dw(\Delta(\hat{\theta}, x_1))}{dx_1} - \chi \frac{dw(\Delta(\hat{\theta}, 1 - x_1))}{dx_1} - \frac{d\chi}{da_0} \frac{da_0}{dx_1} w(\Delta(\hat{\theta}, 1 - x_1)) \\ = & - \left[\frac{dw(\Delta(\hat{\theta}, x_1))}{d\hat{\theta}} - \chi \frac{dw(\Delta(\hat{\theta}, 1 - x_1))}{d\hat{\theta}} \right] \frac{d\hat{\theta}}{dx_1}. \end{aligned}$$

Because the payoffs depend only on the distance to the ideal point, we know that

$$\frac{dw(\Delta(\hat{\theta}, x_1))}{d\hat{\theta}} = - \frac{dw(\Delta(\hat{\theta}, 1 - x_1))}{dx_1}.$$

Because of the concavity of the payoff function, we know

$$\frac{dw(\Delta(\hat{\theta}, x_1))}{dx_1} > \frac{dw(\Delta(\hat{\theta}, 1 - x_1))}{dx_1}.$$

Finally, because $\chi \leq 1$, we obtain

$$1 - \frac{d\chi}{da_0} \frac{da_0}{dx_1} \underbrace{\frac{w(\Delta(\hat{\theta}, 1 - x_1))}{\frac{dw(\Delta(\hat{\theta}, x_1))}{dx_1} - \chi \frac{dw(\Delta(\hat{\theta}, 1 - x_1))}{dx_1}}_{>0} = \frac{d\hat{\theta}}{dx_1} > 0.$$

Therefore, the degree of compromise is increasing in γ .

(4.) We establish an upper bound on the degree of equilibrium compromise. This arises when the second firm prefers to stop its development efforts. Then, even if there exist more extreme voters who would prefer the second firm, the endorsement by the second firm effectively ends the game. Firm 2 will continue as long as

$$u(1) \geq w(\Delta(0, x_1)),$$

which together with (7) in the main text defines the maximum compromise project \bar{x}_1 . Thus, the condition

$$w(\Delta(\underline{\theta}(\rho), \bar{x}_1)) = \chi(\rho) w(\Delta(\underline{\theta}(\rho), 0))$$

defines the pivotal type $\underline{\theta}(\rho)$ and the bound on binding supermajority as $\bar{\gamma}(\rho) = F(1 - \underline{\theta}(\rho)) = 1 - F(\underline{\theta}(\rho)) < 1$. Because we know that equilibrium compromise is increasing in γ and that $x_1^* > \bar{x}_1$, there exists a unique $\gamma^*(\rho)$ that induces the second-best projects.

(5.) The upper bound on compromise is again given by the point at which firm 2 prefers to stop its development efforts, now simply under the requirement that the competing project

it may pursue is $x_2 = 1 - x_1$ instead of $x_2 = 0$. This threshold is given by

$$u(w(\Delta(0, 1 - x_1))) = w(\Delta(0, x_1)),$$

which together with (7) in the main text defines x_1^E . We then define $\tilde{\theta}(\rho)$ as the user who is indifferent between the two alternatives. The resulting supermajority requirement $\tilde{\gamma}(\rho) = F(1 - \tilde{\theta}(\rho)) = 1 - F(\tilde{\theta}(\rho)) < 1$ bounds the equilibrium degree of compromise. ■

2 General Cost Functions

For the next result, we maintain the following assumptions on the cost function and the discount rate.

Assumption 2 (Cost Function)

1. The cost function $c(a)$ is strictly concave for all $a > 0$.
2. $c'(0) = 0$ and $\lim_{a \rightarrow \infty} c'(a) > 1$.
3. $3c''(a) + (2a + r)c'''(a) \geq 0$ for all $a \geq 0$.

We now restate our main results in Propositions 1 and 2 for general cost functions.

Proposition 4 (General Cost) Consider a selection function $\xi(\mathbf{x}, \tau, \alpha)$.

1. If $\alpha = 1/2$, the agents develop the efficient-effort projects.
2. The equilibrium degree of compromise $\phi(v(\alpha))$ is decreasing in α .
3. For sufficiently low r , there exists $\alpha^*(r) < 1/2$ that induces the second-best projects.

Proof of Proposition 4. We begin by showing that, under Assumption 2, there exists a unique symmetric stationary equilibrium for any pair of symmetric projects $x_j = 1 - x_i$. Fix a pair of symmetric projects and let $v \triangleq v_i(x_i)$. Under symmetric stationary strategies, each agent's best response problem is given by

$$\max_a \frac{av + a^*\phi(v) - c(a)}{r + a + a^*}.$$

A necessary condition for a symmetric equilibrium is given by

$$D(a) \triangleq (r + 2a)(v - c'(a)) - a(v + \phi(v)) + c(a) = 0. \quad (31)$$

Note that $D(0) = rv > 0$ and that $\lim_{a \rightarrow \infty} D(a) < 0$ under Assumption 2. Furthermore, it is immediate to check that the agent's second-order condition is satisfied. Therefore, an equilibrium exists. Furthermore, $D(a) = 0$ has a unique solution a^* if

$$D'(a) = -c''(a)(r + 2a) + v - \phi(v) - c'(a) \quad (32)$$

is negative whenever $D(a) = 0$. Note that Assumption 2 states that (32) is strictly decreasing. Therefore $D''(a) < 0$ and $D(0) > 0$ imply that $D(a) = 0$ has a unique solution.

(1.) This result follows from the proof of Proposition 1 in Section III.A, which did not rely on quadratic costs.

(2.) The equilibrium projects $v(\alpha)$ satisfy the two conditions

$$\begin{aligned} \phi(v) &= u(w) \triangleq \max_a \left[\frac{aw - c(a)}{r + a} \right], \text{ and} \\ (1 - \alpha)w + \alpha\phi(w) &= (1 - \alpha)\phi(v) + \alpha v. \end{aligned}$$

Because the option value $u(w)$ is strictly increasing, the same steps as in the proof of Proposition 2 apply, and the degree of compromise $\phi(v(\alpha))$ is weakly decreasing in α .

(3.) We begin by comparing the first-best and the equilibrium effort levels as a function of v . The first-best effort is decreasing in v . To see this, note that the planner's payoff function

$$W(a, v) \triangleq \frac{a(v + \phi(v)) - c(a)}{r + 2a}$$

is strictly quasiconcave in a , and it is supermodular in a and in the sum of payoffs, $v + \phi(v)$, which is itself decreasing in v . Conversely, the symmetric equilibrium effort level is increasing in v . This follows by differentiating (31) with respect to v , which yields a positive cross-partial derivative of the agent's payoff. Therefore, there exists a unique pair of efficient-effort projects.

Quasiconcavity of the planner's payoff implies that, if effort levels are (weakly) above first-best, the equilibrium payoffs can be strictly improved by increasing the degree of compromise. We conclude that the ex post optimal selection function ($\alpha = 1/2$) induces insufficient compromise, and that the second-best projects must be attained by some $\alpha < 1/2$.

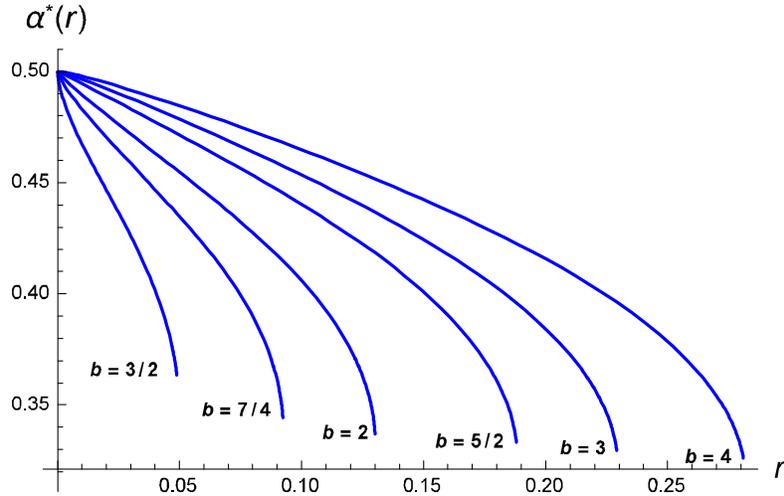
Likewise, any selection function that induces projects with $v < \phi(v)$, i.e., $x_1 < x_2$, can be improved by increasing the weight on the first proposal. The weight $\alpha = 0$ yields the development of the maximum-compromise projects \bar{x} . As $r \rightarrow 0$, the value of these projects approaches $1 = \phi(v(\bar{x})) > v(\bar{x})$, i.e. excessive compromise. Finally, because the equilibrium degree of compromise is continuous in α , there exists $\alpha \in (0, 1/2)$ that achieves

the second-best. ■

In Figure 5 below we illustrate the optimal weight α^* as a function of the discount rate r for several examples of power cost functions. In particular, we let $\phi(v) = \sqrt{1 - v^2}$, and we assume $c(a) = a^b/b$.

We then turn to the case of privately observed project development.

Figure 5: Optimal Weight α^* , Power Cost Functions $c(a) = a^b/b$



3 Disclosure

Proposition 5 (Unobservable Project Development) *Assume that $\rho \leq \bar{\rho}$, and suppose that project developments are unobservable. Under the optimal selection function $\alpha^*(\rho)$, agents pursue the second-best projects $x_i^*(\rho)$, and disclose their development immediately.*

Proof of Proposition 5. To establish the result, it suffices to rule out a deviation in which agent i develops a project worth $v_i(x_i) = w(v^*, \alpha^*)$, and then waits until the other agent's project is developed. This deviation is not profitable if

$$v^*(\rho) \geq w(v^*(\rho), \alpha^*(\rho)) \frac{a(v^*(\rho))}{\rho + a(v^*(\rho))}. \quad (33)$$

The left-hand side of (33) is the prize from developing the second-best project. The right-hand side is the expected discounted value of developing the best alternative project and waiting for the second development given the equilibrium effort level $a(v^*(\rho))$. Using the

expression for equilibrium effort and rewriting $w(v)$ as

$$w^*(v) = \phi(v) + \sqrt{2\rho^*(v)\phi(v)},$$

we can rewrite the above condition as

$$v - \frac{(\sqrt{2\rho^*(v)\phi(v)} + \phi(v))(\rho^*(v) - \sqrt{6v\rho^*(v) + (\rho^*(v) + \phi(v) - v)^2} + \phi(v) + v)}{2\phi(v) - \rho^*(v)} \geq 0.$$

Using the definition of $\rho^*(v)$, we obtain

$$v - \frac{2(v\phi'(v) + \phi(v) + v)}{\phi(v)(2\phi'(v) + 3) + v} \left(\sqrt{-\frac{\phi(v)(2\phi'(v) + 1)(v - \phi(v))^2}{(\phi'(v) + 2)(v\phi'(v) + \phi(v) + v)}} + \phi(v) \right) \geq 0. \quad (34)$$

Let z denote the square root term, which is equal to $\sqrt{2\rho^*(v)\phi(v)}$. Notice that the left-hand side of (34) is decreasing in z . Solving (34) as an equality, we obtain

$$\hat{z}(v) := \frac{v^2 + v\phi(v) - 2\phi(v)^2}{2(v\phi'(v) + \phi(v) + v)}.$$

Now consider the difference $\hat{z}(v)^2 - 2\rho^*(v)\phi(v)$. Multiplying through by the denominator in \hat{z} , and dividing by $(v - \phi(v))^2$, the sign of this difference is determined by

$$(2\phi(v) + v)^2 + \frac{4\phi(v)(2\phi'(v) + 1)(v\phi'(v) + \phi(v) + v)}{\phi'(v) + 2}. \quad (35)$$

The second term is negative and bounded above by

$$4\phi(v) \left(2 - \frac{3}{\phi'(v) + 2} \right),$$

which is increasing in $\phi'(v)$, and equal to $-4\phi(v)$ when $\phi'(v) = -1$. The entire expression is bounded from below by $v(v + 4\phi(v)) > 0$. Therefore, $\hat{z}(v)^2 - 2\rho^*(v)\phi(v) > 0$, which implies that condition (33) is satisfied. \blacksquare

4 Voting and Licensing

In this section, we extend the analysis of the voting model (Section IV) to allow firms to license their intellectual property to users. We consider the following setting. There is a continuum of small potential users of the standard, distributed $[0, 1]$ according to $F(\theta)$. The

net payoff that a user of type θ derives from standard x is given by $K - (\theta - x)^2 - P$, where P is the licensing fee for the standard. The potential users all have voting rights in the SSO, with the SSO requiring that at least a majority, i.e., $\gamma \geq 1/2$, votes in favor of a standard for it to be endorsed by the SSO.

There are two large firms that are championing (potentially) different standards. In particular, each firm has sufficient proprietary technology to construct a fully functioning standard by itself. There are thus no standard-essential patents. However, certain technologies work better than others. To normalize overall technology, we let x denote the type of standard that uses a fraction $(1 - x)$ of firm 2's technology and a fraction x of firm 1's technology. The standards 0 and 1 are then the standards constructed solely from each firm's own technology, while $x = 1/2$ denotes the technologically efficient standard per the preferences of the end users.

The issue of licensing fees has already received significant attention in the literature, in particular, how to avoid the abuse of ex post market power. Here, we are agnostic as to the exact price determination mechanism, but we implicitly rule out excessive use of market power in pricing the licenses after the standard has been established.

Thus, we let the licensing fee to depend on the characteristics of the standard, so the license fee becomes $P(x)$. The firms share the revenues obtained from the license fees proportionally to their technological contribution to the standard. Thus, firm 2 will receive $(1 - x)P(x)$ and firm 1 will receive $xP(x)$. We assume that each firm prefers a standard composed of only their own technology. For example, $xP(x)$ is strictly increasing.

Now, given this basic setup, the structure of the game is identical to the main analysis. First, each firm chooses which standard to work on until the completion of the first standard. At that point, the first firm is able to call a vote on whether the standard should be accepted. If the standard is rejected, the second firm can continue to develop its own standard until both standards are on the table. For simplicity, we restrict attention to a binary (majority) vote to determine which of the two standards is adopted in that case.

Voting on two proposals Suppose that both firms have successfully developed their proposals and that a standard is sufficiently valuable that either standard is preferred by all potential consumers over no standard. Then, user θ prefers the standard of firm 2 over the standard of firm 1 as long as

$$K - (\theta - x_2)^2 - P(x_2) \geq K - (\theta - x_1)^2 - P(x_1),$$

which rearranges to

$$\theta \leq \frac{x_1 + x_2}{2} + \frac{P(x_1) - P(x_2)}{2(x_1 - x_2)}.$$

Note that ex post market power may be detrimental to efficiency. In particular, several voters may prefer a less efficient standard if that also guarantees them a sufficiently lower price. In this case, the firms will have an incentive to pursue fully selfish standards as an indirect means of committing to lower prices. This perverse result will occur, for example, if the firms have sufficient market power to set the price at the minimum willingness to pay for all potential consumers. The framework thus confirms the importance of “fair and reasonable” licensing terms and the need to restrict the market power ex post through ex ante commitments. For the rest of this section, we will assume that such perverse incentives do not arise, which simply requires that the price P is not too sensitive to the efficiency of the standard.

Voting on a single proposal Now consider the game when the first proposal is on the table, and all the players expect the continuation game to follow the efficient choice. At this stage, we need to consider both the incentives of the second firm and those of the voters. We begin with the firms’ incentives. Suppose that firm 2 developed its proposal, and consider the incentives of firm 1. By accepting the proposal, the firm will receive $x_2 P(x_2)$. Alternatively, it can develop its own proposal x_1 , which will be accepted if $x_1 \leq 1 - x_2$. Thus, firm 1 prefers to develop a competing proposal if

$$x_2 P(x_2) < u((1 - x_2) P(1 - x_2)).$$

Using the expression for $u(w)$ in (7) in the main text, we obtain an equivalent condition for firm 1’s acquiescence,

$$4\Delta^2 P(\Delta) \leq \rho(1 - 2\Delta), \text{ where} \\ \Delta \triangleq 1/2 - x_2.$$

We now turn to the preferences of a voter of type θ . The value of immediate acceptance is given by

$$K - (\theta - x_2)^2 - P(x_2),$$

while the value of blocking the current alternative and waiting for the arrival of the new alternative is

$$\frac{a^* V}{a^* + r},$$

where

$$V = K - (\theta - 1 + x_2)^2 - P(1 - x_2) \text{ and} \\ ca^* = (1 - x_2) P(1 - x_2) - u((1 - x_2) P(1 - x_2))$$

is firm 1's optimal effort level when developing the second project. Simplifying, type θ will prefer immediate acceptance as long as

$$\frac{K - \left(\theta - \frac{1}{2} - \Delta\right)^2 - P(\Delta)}{\sqrt{1 + \frac{1}{\rho}(1 + 2\Delta)P(\Delta)}} \geq 2\Delta(2\theta - 1).$$

Two basic and intuitive results from Section IV still apply: simple majorities or small supermajority requirements induce no compromise because the preferences of the pivotal voters in the two possible rounds are too similar, and the more stringent the supermajority requirement, the greater the need for compromise (weakly). The following results follow directly from Proposition 3.

Corollary 1 (Voting and Licensing) *Fix a supermajority requirement $\gamma \geq 1/2$.*

1. *If γ is sufficiently low, the firms pursue their favorite projects $(x_1, x_2) = (1, 0)$.*
2. *If γ is sufficiently high, the degree of equilibrium compromise is increasing in γ .*

Unlike in the baseline model, extreme voter types do not have the same preferences as the firms, even accounting for the cost of development. We now consider whether there exists a voter who is willing to block a project even if the other firm is willing to accept it. If so, then there will exist a supermajority majority requirement that is equivalent to requiring acceptance by firm 1. Thus, the firm's indifference condition can be used to describe the relationship among the licensing fee of the standard, the amount of patience and the level of compromise as $4\Delta^2 P(\Delta) = \rho(1 - 2\Delta)$. Substituting into voter θ 's problem, we obtain

$$K - P(\Delta) - (\theta - 1/2 - \Delta)^2 \geq 2\theta - 1. \quad (36)$$

Therefore, for a fixed a pair of projects (summarized by Δ), we obtain the condition

$$2 \left(\sqrt{1 - 2\Delta + K - P(\Delta)} - (1 - \Delta) \right) < 1. \quad (37)$$

If (37) holds, then there exists a range $[\tilde{\theta}, 1]$ of voters who would prefer to wait for the second firm's proposal, but the second firm is unwilling to develop it due to the associated delay and development costs. Conversely, whenever K is sufficiently large, no solution exists to (36) holding with equality, and voters are always more willing to endorse projects $(1/2 + \Delta, 1/2 - \Delta)$ compared to the firms.

5 Fixed-Date Decisions

One may wonder whether a static procedure that creates actual competition among projects would be more conducive to compromise. One way to probabilistically induce a “horse race” is to postpone decisions until a given date. This creates a contest-like environment wherein agents trade-off the “probability of winning” with the value of having their project adopted in the absence of a competing proposal.

More formally, suppose that a project (if any) must be adopted on a fixed, non-renegotiable date T . If two projects have been developed by that date, the one yielding the higher total payoff is adopted. If symmetric projects are developed, each is adopted with equal probability. Players observe nothing until the deadline.¹

It is not hard to see that there cannot be a pure-strategy equilibrium in project selection: each agent has an incentive to either (a) undercut the other by choosing a more socially valuable project that is adopted with probability one or (b) deviate to his favorite project. By a similar logic, each player’s distribution cannot have atoms, and its support must include each agent’s most preferred project.

Thus, each agent is pursuing potentially different projects in equilibrium and must be indifferent among all of them, implying that the payoff for developing each project v in the support of $F(\cdot)$ must be constant. This in turn means that each agent’s effort level does not depend on the realization of his mixed strategy. It does, however, depend on calendar time, and it increases as the deadline T approaches. Proposition 6 summarizes our results.

Proposition 6 (Fixed-Date Decisions) *There exists a symmetric equilibrium with the following properties.*

1. *Each agent i randomizes over all projects x_i such that $v_i(x_i) \in [v_L, 1]$ according to a positive and continuous distribution.*
2. *The lower bound of the support v_L is increasing in r and c and decreasing in T .*
3. *Each agent i ’s effort level $a_{i,t}$ is deterministic and strictly increasing over time.*

The equilibrium is essentially unique in the sense that it pins down the distribution of each agent’s project choice at any time t . However, as nothing is observed until T , the agents have no reason to change their project over time. Randomization over projects can then occur at time 0, with each agent pursuing a fixed project throughout (this is certainly the case with switching costs).

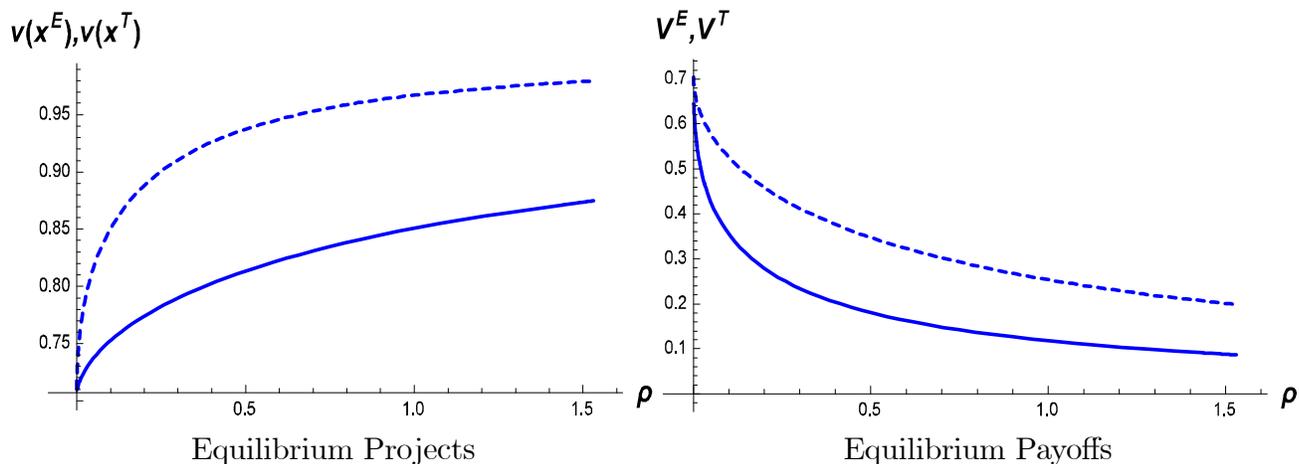
¹This allows for the best comparison of our model with a static game. While this violates our earlier assumption of observable project developments, it makes for the most interesting comparison with a static decision criterion. In particular, the only dynamics at play in this game concern the equilibrium effort.

The distribution function can be solved in closed form, and it is given by

$$F(v) = \frac{1 - e^{-\int_{v_L}^v \frac{1}{w-\phi(w)} dw}}{1 - e^{-\int_{v_L}^1 \frac{1}{w-\phi(w)} dw}}.$$

Random (inefficient) project choice is also a feature of the static model of competing policy proposals in Hirsch and Shotts (2015). Here, we can contrast static decision making with dynamic in Section III.A. The ex post optimal selection function yields the development of projects x_i^E . Instead, setting a fixed date for (efficient) adoption decisions does not guarantee (i.e., does not induce with probability one) any strictly positive degree of compromise. Consistent with intuition, numerical examples suggest the optimal deadline is decreasing in ρ . In Figure 6, we compare the project choices and equilibrium values in the dynamic and static decision-making environments.²

Figure 6: Optimal Decision Date (solid) vs. Dynamic Procedure (dashed)



Depending on the length of the deadline, the lower bound v_L may be above or below v^E . In this example, the equilibrium payoff is lower, for any T , than in the efficient-continuation selection function characterized in Proposition 1 that yields the efficient-effort projects. Overall, our analysis suggests that the power to generate alternatives and to commit to *dynamic* (vs. fixed-date) decision making is a necessary condition for generating efficient compromise through delegation. In other words, the threat of a competing project may be more effective than the (probabilistic) development of an actual alternative.

²For any payoff frontier $\phi(v)$, the discount rate, optimal deadline, and equilibrium payoff can be solved for in closed form as a function of v_L . Figure 6 parametrically plots the lower bound of the support (left) and the equilibrium payoff (right). The frontier is given by $\phi(v) = \sqrt{1-v^2}$.

Proof of Proposition 6. (1.) We look for a mixed-strategy equilibrium wherein players randomize over the projects they pursue. A similar logic to the previous claim suggests that each player's distribution over projects x cannot have atoms and that its support must include each agent's most preferred project. Thus, we characterize a mixed-strategy equilibrium in which players randomize over projects with values $[v_L, 1]$ according to an absolutely continuous $F(v)$. Let the expected payoff $\beta(v)$ associated with developing each project v in the support of $F(\cdot)$ be given by

$$\beta(v) = v - \int_{\beta}^v p(x)(v - \phi(x))f(x) dx, \quad (38)$$

where $p(v)$ denotes the probability of each agent succeeding in the development phase conditional on having chosen project v . Because each agent is randomizing, he must be indifferent among all projects in the support. Hence, the constant (undiscounted) prize β must be equal to the lowest value in the support, i.e., v_L , because the highest-compromise project is adopted with probability one. Each agent's HJB is given by

$$rV_t = \max_a \left[a \left(e^{-r(T-t)}\beta - V_t \right) - c(a) + \dot{V}_t \right].$$

The first-order condition for effort at time t is given by

$$ca_t = e^{-r(T-t)}\beta - V_t.$$

Because the expected payoffs β are constant in v , the optimal effort levels at each time t do not depend on the project chosen. Therefore, let p denote the probability of developing any project by the deadline

$$p := 1 - e^{-\int_0^T a_t dt}.$$

Each agent's value V_t then satisfies the following ODE and boundary condition

$$\begin{aligned} rV_t &= \frac{(e^{-r(T-t)}\beta - V_t)^2}{2c} + \dot{V}_t, \\ V_T &= p \int_{\beta}^1 \phi(x)f(x) dx =: w. \end{aligned}$$

We can then solve for V_t in closed form and obtain

$$V_t(w, \beta) = e^{-r(T-t)} \left(\beta - \frac{2\rho(\beta - w)}{(1 - e^{-r(T-t)})(\beta - w) + 2\rho} \right).$$

This implies that effort levels are given by

$$a_t = \frac{2r(\beta - w)}{e^{r(T-t)}(\beta - w + 2\rho) - (\beta - w)}, \quad (39)$$

and integrating over time, we have

$$p = 1 - \left(\frac{2e^{rT}\rho}{(e^{rT} - 1)(\beta - w) + 2\rho e^{rT}} \right)^2.$$

Now rewrite the prize β as

$$\beta = 1 - p + p \int_{\beta}^1 \phi(x) f(x) dx = 1 - p + w.$$

This means that

$$\beta - w = \left(\frac{2e^{rT}\rho}{(e^{rT} - 1)(\beta - w) + 2\rho e^{rT}} \right)^2,$$

which can be written in terms of $\beta - w$ as the following equation

$$\sqrt{\beta - w}(\beta - w) = \left(1 - \sqrt{\beta - w}\right) \frac{2e^{rT}\rho}{e^{rT} - 1}. \quad (40)$$

Let the solution to (40) be given by

$$\beta - w = B\left(\frac{2e^{rT}\rho}{e^{rT} - 1}\right), \quad (41)$$

and notice that the function $B(\cdot)$ is strictly increasing. Finally, to derive the equilibrium distribution $F(v)$, we differentiate $\beta(v)$ and set it equal to zero. We therefore solve the following initial-value problem

$$\begin{aligned} 1 - pF(v) - pf(v)(v - \phi(v)) &= 0, \\ F(\beta) &= 0. \end{aligned}$$

The solution is given by

$$F(v) = \frac{1 - e^{-\int_{\beta}^v \frac{1}{x - \phi(x)} dx}}{p},$$

where $p = 1 - (\beta - w)$ from above, and $\beta - w$ is solved as a function of parameters only in

(41). An equation for β is then given by

$$\beta - w = \beta - p \int_{\beta}^1 \phi(x) f(x) dx = \beta + p \int_{\beta}^1 \phi'(x) F(x) dx.$$

Using (41), the equilibrium β is therefore implicitly defined by the following equation

$$B \left(\frac{2e^{rT}\rho}{e^{rT} - 1} \right) = \beta - \phi(\beta) - \int_{\beta}^1 \phi'(v) e^{-\int_{\beta}^v \frac{1}{x-\phi(x)} dx} dv. \quad (42)$$

(2.) Differentiating the right-hand side of (42) with respect to β , we obtain

$$1 - \int_{\beta}^1 \frac{\phi'(v)}{\beta - \phi(\beta)} e^{-\int_{\beta}^v \frac{1}{x-\phi(x)} dx} dv > 0.$$

Because the left-hand side is strictly increasing in its argument, the equilibrium β is strictly increasing in ρ and strictly decreasing in T .

(3.) This follows immediately from the first-order condition (39). ■

References

- Hirsch, Alexander V., and Kenneth W. Shotts.** 2015. “Competitive Policy Development.” *American Economic Review*, 105(4): 1646–1664.
- Mason, Robin, and Juuso Välimäki.** 2015. “Getting it Done: Dynamic Incentives to Complete a Project.” *Journal of the European Economic Association*, 13(1): 62–97.
- Michel, Phillippe.** 1982. “On the transversality condition in infinite horizon optimal problems.” *Econometrica*, 50(4): 975–985.