Fertility and Childlessness in the United States - Online Appendix

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A Data - For Online Publication

A.1 Cohabitation

When dealing with never-married individuals we might be skeptical about whether these individuals are single or just unmarried with a partner (specially poor women who have many children). In the Census 1990, among never-married women aged 45-70, only 1.8% declared themselves as being with a partner. The percentages for mothers and childless were respectively 3.7% and 1.3%. These percentages vary, however, for different education levels: Table 1 provides the proportion of single mothers, aged 45-70, saying they are with an unmarried partner for each education level. This table shows that very few women who have not married are living with a partner. The highest percentages of cohabitation are seen for women aged 36-40, having achieved Grade 1-4, or with a doctoral degree: respectively 21.5% and 21.7% of never-married women in these education levels claim to be unmarried but have a partner.

Category	%	Category	%
1	3.8	7	3.6
2	4.8	8	2.8
3	4.0	9	3.8
4	3.6	10	3.0
5	3.8	11	2.8
6	4.0	12	8.2

Table 1: Percentage of Single Mothers, Aged 45-70, with an Unmarried Partner

According to Akerlof et al. (1996), more than 65% of white women experiencing an out-of-wedlock birth married before the first birthday of the child in 1975. Bumpass and Westoff

(1970) estimate that between 1960-64 and 1985-89, around 80% of white women who gave birth to a first child out-of-wedlock married within the ten years that followed. This percentage is lower for black women (a bit larger than 50%). Without focusing on first births, Bramlett and Mosher (2002) estimate that, in 1995, 42.7% of never married women between ages 40-44 had ever cohabited. Their definition for cohabitation was being unmarried but having a sexual relationship while sharing the same usual address. They also say that the probability of transition to marriage after one year of cohabitation is 30%, after 5 years, 70%, and after 10 years, 84%. Consequently, if cohabitation lasts, marriage is very likely to follow.

Rindfuss and VandenHeuvel (1990) compared cohabitants to both married and single individuals and concluded that cohabitants' attitudes were in many respects closer to those of single than of married people. In particular, cohabitants' fertility expectations are more similar to those of single than married people. Indeed, comparing the percentage of childless individuals who expect children, cohabitants are much closer to never-married than to married respondents (among women, 11% of cohabiting, 4% of single and 40% of married people expect to have a child within two years). Consequently, although cohabitation has many of the characteristics of a marriage (sharing a dwelling unit), cohabitants also share some of the characteristics of single people (fertility expectations). This puts cohabitants in a middle position between single and married people.

A.2 Evidence of Malthusian Mechanisms from the Census

According to Malthus, fertility increases with income. In this appendix, we first provide some evidence that Malthusian mechanisms can be detected both at the regional and state levels. We also show that in many US states, there is a reversal of the correlation between the completed fertility of mothers and childlessness when mean education increases. We use the same data as described in Section II for married women only.¹

A.2.1 Fertility vs. Education across Regions and States

Our theory allows for the possibility that, at low levels of education and non-labor income, a positive relationship between the fertility of mothers and education prevails. In Section IV, we estimate that 1.8% of the women in our sample live in a Malthusian regime of fertility (Regime [C]); such a low percentage explains why we cannot detect Malthusian segments at the 1990 aggregate level. In order to see Malthusian mechanisms in US data, we first looked at historical data, in which more women faced fertility constraints. US Census data display such a positive relationship for low-educated women born before 1880. Plotting the first seven deciles of Table A4 from Jones and Tertilt (2008) leads to Figure 1. For the first three generations, fertility is increasing in education when education is low.

¹We focus on married women only because there are too few observations at the state level to do the analysis for singles.

Figure 1: Children Ever Born, by Occupational Income Decile and Cohort

Another way to detect Malthusian mechanisms consists in using our dataset for the year 1990 and computing the empirical moments at a finer geographical level. Figure 2 shows the relationship between the completed fertility of mothers and their education for nine regions, defined in the sense of the Census regions and divisions. We see that regional fertility differentials appear for low levels of education, and that these differentials vanish when education increases. We find a Malthusian effect between the first and the second categories of education (from no school to grade 4) in three out of nine regions, namely in the South Atlantic, East South Central, and Pacific divisions. In the six other regions, such Malthusian segments cannot be detected.

Figure 2: Relationship between the completed fertility of mothers and education at the regional level. Left Panel: Pacific (black), South Central (gray), and South Atlantic (dashed black). Right Panel: Mountain (black), West South Central (gray), West North Central (dashed black), East North Central (dotted gray), Middle Atlantic (dotted black), and New England (dashed gray)

Since the regional level remains strongly aggregated, we also explore the same relationship between the fertility of mothers and education at the state level, in the sense of the Federal Information Processing Standards (FIPS) coding scheme used by IPUMS. The results are

shown in Table 2. In 31 of the 50 states and the District of Columbia, a positive relationship between the fertility of mothers and education can be detected between the first and the second level of education. This is the case for instance in Mississippi, Washington, North Carolina and Connecticut (Figure 3 left panel). Longer Malthusian segments only appear in states with a low number of observations (Montana, New Hampshire and the District of Columbia). Nevertheless, if we focus on the completed fertility of mothers aged between 45 and 59 only (Table 3), we find longer Malthusian segments in 7 states, namely Connecticut, the District of Columbia, Maryland, Montana, South Carolina, New Hampshire, and West Virginia. The low number of (unweighted) observations in the Malthusian segments of New Hampshire, Montana, the District of Columbia, and West Virginia should lead us to be cautious. However, the number of observations in the three other states is clearly sufficient to claim the existence of a longer Malthusian segment (Figure 3 right panel).² The reason why stronger Malthusian effects can be detected in this sub-population may lie in a bias due to aggregating people over age: the oldest women in our main sample (aged between 60 and 70) are those who survived old age. Thus, they are those with the highest quality of life and they may be much less concerned with poverty leading to Malthusian situations, whatever their age.³

Figure 3: Relationship between the completed fertility of mothers and education at the state level. Left Panel: Washington (black), North Carolina (gray), Kentucky (dotted black), and Arkansas (dashed gray). Right panel (45-59-year-old women): Connecticut (black), Maryland (gray), West Virginia (dotted black), and South Carolina (dashed gray).

A.2.2 Correlation Reversals

Here, we highlight that for various states the correlation between the completed fertility of mothers and the childlessness rates changes sign when education increases. Table 4 provides the childlessness rates for each American state (FIPS), by education category. The last two

²There are 25 observations in the first category of education in Maryland, but the number of observations in subsequent categories is high enough to affirm that there is a Malthusian segment.

³The same phenomenon can be detected at the regional level. Once we focus on women aged between 45 and 59, we find Malthusian evidence in 5 rather than 3 regions.

State	Education Category											
	1	2	3	4	5	6	7	8	9	10	11	12
Alabama	4.64	4.52	4.01	3.59	3.60	3.53	2.89	2.77	2.75	2.59	2.44	2.48
Alaska	6.94 ^a	6.78	5.79	4.48	4.36	3.33	3.72	2.91	3.27	2.86	2.69	2.32 ^a
Arizona	6.59	5.30	4.40	4.09	3.72	3.43	3.12	3.03	3.14	2.94	2.63	2.63 ^a
Arkansas	5.25	4.86	4.06	3.66	3.62	3.44	2.89	2.84	3.00	2.60	2.57	2.13 ^a
California	5.35	5.46	4.26	3.76	3.45	3.46	3.00	2.88	2.85	2.72	2.54	2.35
Colorado	5.15	5.53	4.16	3.97	3.69	3.67	3.10	3.02	3.00	2.77	2.55	2.61
Connecticut	3.00	3.41	3.33	3.15	3.29	3.25	3.00	2.89	2.90	2.83	2.63	2.24
Delaware	4.43 ^a	3.41 ^a	3.69	3.88	3.55	3.71	3.03	2.90	2.85	2.71	2.43	2.42 ^a
District of Columbia	3.84 ^a	4.15 ^a	3.88	4.62	3.76	3.76	3.24	2.82	2.53	2.75	2.43	2.02 ^a
Florida	3.78	3.60	3.41	3.36	3.32	3.32	2.89	2.82	2.89	2.71	2.57	2.46
Georgia	4.64	4.67	3.91	3.62	3.53	3.30	2.86	2.76	2.72	2.63	2.48	2.34
Hawaii	4.70	4.81	4.30	3.82	4.14	4.00	3.16	2.97	2.66	2.62	2.55	2.45 ^a
Idaho	4.78 ^a	5.21 ^a	4.32	3.98	3.73	3.86	3.52	3.51	3.57	3.42	2.91	3.10 ^a
Illinois	4.75	4.90	3.65	3.68	3.49	3.46	3.12	3.03	3.10	2.86	2.70	2.02
Indiana	4.17	4.27	3.99	3.82	3.53	3.67	3.12	3.04	3.04	3.02	2.62	2.34
Iowa	5.38 ^a	5.01 ^a	4.14	3.98	3.80	3.93	3.39	3.25	3.31	2.84	2.81	2.43 ^a
Kansas	4.22 ^a	4.58 ^a	3.96	3.77	3.56	3.55	3.21	3.10	3.00	2.87	2.76	2.12 ^a
Kentucky	4.02	4.99	3.94	3.48	3.45	3.19	3.00	2.90	2.94	2.64	2.43	2.47 ^a
Louisiana	5.01	4.87	4.42	3.95	3.90	3.95	3.32	3.02	2.95	2.99	2.82	2.16
Maine	3.32 ^a	4.90 ^a	3.86	3.51	3.66	3.71	3.27	3.13	3.11	2.86	2.68	1.52 ^a
Maryland	3.85	3.46	3.72	3.52	3.57	3.44	2.95	2.84	2.97	2.68	2.50	2.42
Massachusetts	4.17	3.32	3.49	3.42	3.53	3.45	3.26	3.09	3.08	2.98	2.75	2.30
Michigan	5.23	4.39	3.97	3.88	3.86	3.78	3.36	3.20	3.19	2.91	2.79	2.52
Minnesota	6.67	4.67	4.27	4.24	4.04	3.99	3.63	3.34	3.33	2.99	2.74	2.78
Mississippi	4.91	5.82	4.84	4.47	3.97	3.75	3.04	2.91	2.97	2.72	2.51	2.44 ^a
Missouri	4.06	4.69	3.71	3.65	3.44	3.57	3.06	2.94	3.13	2.89	2.55	2.19
Montana	2.80 ^a	3.63 ^a	4.60	4.11	3.68	3.87	3.42	3.30	3.48	3.07	2.77	2.35 ^a
Nebraska	3.21 ^a	4.49 ^a	3.85	3.76	3.71	3.55	3.40	3.34	3.59	2.93	2.90	2.67 ^a
Nevada	4.25 ^a	4.29	3.70	3.64	3.45	3.38	3.05	2.98	2.83	2.82	2.81	2.93 ^a
New Hampshire	3.03 ^a	3.03 ^a	3.60	3.58	3.46	3.44	3.23	3.13	3.21	2.84	2.83	2.58 ^a
New Jersey	3.52	3.61	3.20	3.20	3.16	3.16	2.90	2.88	2.91	2.70	2.56	2.26
New Mexico	6.25	5.84	4.97	4.48	4.09	4.23	3.33	3.08	3.19	2.76	2.63	2.36 ^a
New York	3.96	3.60	3.32	3.33	3.34	3.38	3.04	2.93	2.96	2.76	2.60	2.37
North Carolina	3.62	4.57	3.87	3.56	3.38	3.31	2.79	2.70	2.62	2.57	2.47	2.43
North Dakota	5.46 ^a	6.38 ^a	4.38	4.33	4.07	4.24	3.76	3.51	3.31	3.24	3.08	1.43 ^a
Ohio	4.29	3.84	3.84	3.70	3.60	3.49	3.15	3.04	3.08	2.90	2.68	2.51
Oklahoma	4.41	4.71	3.72	3.66	3.52	3.25	2.94	2.78	2.86	2.67	2.56	2.48 ^a
Oregon	4.91	6.49	3.78	3.75	3.49	3.61	3.08	3.01	2.91	2.78	2.60	3.27 ^a
Pennsylvania	3.72	4.41	3.63	3.36	3.35	3.31	3.01	2.93	2.93	2.74	2.68	2.52
Rhode Island	4.09	3.65	3.12	3.12	3.21	3.28	3.07	2.87	3.06	2.76	2.67	2.18 ^a
South Carolina	4.23	4.61	4.28	3.83	3.54	3.48	2.90	2.82	2.77	2.71	2.57	2.84 ^a
South Dakota	2.91 ^a	5.00 ^a	4.01	4.85	3.96	4.17	3.65	3.49	3.54	3.24	3.16	5.06 ^a
Tennessee	4.78	4.67	3.66	3.48	3.19	3.20	2.75	2.69	2.66	2.58	2.44	2.37
Texas	5.57	5.60	4.40	3.81	3.56	3.52	3.01	2.83	2.85	2.66	2.51	2.51
Utah	7.05 ^a	4.86 ^a	5.13	4.32	4.32	4.19	3.94	4.15	4.06	4.13	3.55	3.28 ^a
Vermont	3.29 ^a	5.86 ^a	4.04	3.88	3.68	3.33	3.38	3.27	2.98	2.95	2.69	3.16 ^a
Virginia	4.00	4.50	3.81	3.43	3.35	3.36	2.83	2.72	2.72	2.64	2.45	2.30
Washington	4.77	5.72	4.18	3.76	3.57	3.51	3.13	3.03	3.01	2.80	2.62	2.17
West Virginia	3.95 ^a	4.51	4.05	3.53	3.55	3.44	2.88	2.74	2.81	2.59	2.46	2.89 ^a
Wisconsin	6.05	4.12	4.30	4.06	3.95	3.96	3.52	3.44	3.39	3.05	2.74	2.75 ^a
Wyoming	10.00 ^a	6.53 ^a	4.11	3.90	3.89	4.01	3.38	3.12	3.13	2.93	2.94	2.00 ^a

^a indicates moments with fewer than 30 unweighted observations.

Table 2: The completed fertility of mothers by education category at the state level in 1990 for women aged between 45 and 70.

State	Education Category											
	1	2	3	4	5	6	7	8	9	10	11	12
Alabama	3.90	4.72	4.04	3.58	3.51	3.57	2.88	2.73	2.71	2.53	2.43	2.43 ^a
Alaska	7.24 ^a	5.68 ^a	5.48	4.23 ^a	4.86	3.42	3.57	2.83	3.22	2.76	2.63	2.38 ^a
Arizona	6.26	5.03	4.47	4.20	3.90	3.51	3.18	3.02	3.19	2.84	2.56	2.68 ^a
Arkansas	4.53	5.02	4.16	3.74	3.79	3.49	2.92	2.80	2.93	2.46	2.46	2.15 ^a
California	5.25	5.34	4.29	3.82	3.57	3.58	3.03	2.83	2.78	2.59	2.42	2.32
Colorado	4.61 ^a	5.14	4.16	4.21	3.72	3.69	3.06	2.95	2.91	2.64	2.41	2.63
Connecticut	2.86 ^a	3.36	3.46	3.22	3.50	3.49	3.01	2.85	2.87	2.68	2.55	2.27 ^a
Delaware	4.84 ^a	5.26 ^a	3.88	4.03	3.89	3.80	2.96	2.84	2.80	2.58	2.22	2.03 ^a
District of Columbia	3.03 ^a	4.06 ^a	4.39	4.41	4.34	4.21	3.29	2.59	2.28	2.62	2.39	1.83 ^a
Florida	3.63	3.57	3.55	3.49	3.43	3.50	2.93	2.79	2.81	2.63	2.45	2.36
Georgia	4.66	4.46	3.96	3.55	3.56	3.34	2.82	2.70	2.62	2.53	2.41	2.23
Hawaii	4.91	4.53	4.21	4.10	4.33	4.45	3.15	2.99	2.67	2.53	2.43	2.31 ^a
Idaho	4.60 ^a	4.69 ^a	4.47	3.95	3.84	3.99	3.43	3.48	3.51	3.37	2.98	2.94 ^a
Illinois	4.81	5.00	3.83	4.01	3.59	3.55	3.11	2.97	3.01	2.73	2.60	1.99
Indiana	3.93 ^a	4.93	4.05	3.94	3.58	3.69	3.13	3.00	3.04	2.88	2.52	2.30 ^a
Iowa	3.40 ^a	5.69 ^a	4.47	4.01	3.82	3.96	3.37	3.14	3.17	2.74	2.63	2.42 ^a
Kansas	4.80 ^a	5.24 ^a	4.04	3.87	3.75	3.66	3.23	3.11	2.94	2.76	2.60	2.19 ^a
Kentucky	4.19	4.37	3.87	3.47	3.45	3.28	2.98	2.85	2.86	2.51	2.35	2.20 ^a
Louisiana	5.02	4.73	4.50	3.92	3.89	4.01	3.34	3.00	2.87	2.86	2.77	2.28
Maine	4.43 ^a	5.64 ^a	3.78	3.59	3.75	3.64	3.26	3.09	3.13	2.71	2.51	1.44 ^a
Maryland	3.42 ^a	3.52	3.84	3.59	3.70	3.49	2.93	2.77	2.89	2.57	2.40	2.34
Massachusetts	3.85	3.38	3.48	3.64	3.82	3.64	3.27	3.04	2.99	2.87	2.60	2.23
Michigan	5.07	4.58	4.01	3.93	3.94	3.90	3.33	3.11	3.11	2.78	2.66	2.50
Minnesota	6.05 ^a	4.75 ^a	4.49	4.28	4.14	3.99	3.56	3.19	3.20	2.85	2.59	2.55 ^a
Mississippi	5.33	5.55	4.68	4.71	4.00	3.86	3.04	2.87	2.96	2.58	2.40	2.56 ^a
Missouri	3.13 ^a	4.39	3.84	3.74	3.47	3.60	3.04	2.95	3.10	2.79	2.44	2.13
Montana	2.20 ^a	3.73 ^a	4.42	4.52	3.66	3.93	3.37	3.14	3.27	2.87	2.63	2.35 ^a
Nebraska	3.62 ^a	4.54 ^a	4.08	3.96	3.49	3.63	3.38	3.31	3.53	2.83	2.80	2.53 ^a
Nevada	4.64 ^a	3.92 ^a	3.69	3.72	3.57	3.76	3.09	2.99	2.84	2.83	2.71	2.91 ^a
New Hampshire	3.00 ^a	3.34 ^a	3.84	3.84	3.54	3.38	3.16	3.13	3.15	2.69	2.81	2.01 ^a
New Jersey	3.65	3.47	3.29	3.51	3.47	3.33	2.92	2.82	2.83	2.61	2.47	2.19
New Mexico	6.12	5.65	4.84	4.58	4.27	4.41	3.35	3.04	3.17	2.58	2.54	2.12 ^a
New York	3.85	3.53	3.36	3.49	3.47	3.57	3.05	2.90	2.91	2.68	2.51	2.20
North Carolina	3.16	4.61	3.75	3.58	3.29	3.36	2.76	2.65	2.57	2.47	2.36	2.33
North Dakota	5.46 ^a	11.00 ^a	4.31	4.75 ^a	4.72	4.34	3.75	3.45	3.22	3.18	2.68	1.43 ^a
Ohio	3.97	3.61	3.97	3.75	3.71	3.57	3.13	3.01	3.01	2.78	2.58	2.44
Oklahoma	4.02 ^a	6.00 ^a	3.77	3.73	3.43	3.34	2.93	2.77	2.85	2.63	2.53	2.46 ^a
Oregon	4.54 ^a	6.37 ^a	3.92	4.03	3.62	3.62	3.08	2.92	2.88	2.62	2.47	3.49 ^a
Pennsylvania	3.90	4.46	3.86	3.56	3.53	3.47	3.01	2.89	2.86	2.66	2.61	2.45
Rhode Island	5.03 ^a	3.67	3.29	3.56	3.39	3.31	3.06	2.84	2.99	2.70	2.52	2.14 ^a
South Carolina	3.20	3.78	4.11	3.85	3.55	3.57	2.87	2.72	2.76	2.57	2.43	2.17 ^a
South Dakota	2.90 ^a		4.22	4.26	4.01	3.96	3.58	3.32	3.42	2.97	3.07	4.00 ^a
Tennessee	4.24	4.17	3.64	3.48	3.21	3.31	2.76	2.64	2.56	2.49	2.39	2.31
Texas	5.52	5.63	4.51	3.90	3.62	3.61	3.04	2.81	2.85	2.59	2.44	2.44
Utah	7.59 ^a	5.31 ^a	5.59	4.77	4.29	4.11	3.87	4.20	4.08	4.07	3.45	3.55 ^a
Vermont	2.76 ^a	9.00 ^a	3.96	4.03 ^a	3.44	3.29	3.32	3.33	2.82	2.79	2.55	3.22 ^a
Virginia	3.67	4.51	3.79	3.46	3.40	3.37	2.80	2.67	2.59	2.54	2.33	2.22
Washington	4.07	5.95	4.17	3.91	3.58	3.64	3.11	2.96	2.91	2.67	2.45	2.10
West Virginia	3.37 ^a	3.74 ^a	4.13	3.67	3.60	3.39	2.89	2.70	2.91	2.53	2.45	2.66 ^a
Wisconsin	5.79 ^a	3.89 ^a	4.33	4.00	4.02	4.03	3.48	3.33	3.24	2.86	2.67	2.72 ^a
Wyoming			4.25	3.85 ^a	3.79	3.95	3.35	3.04	3.00	2.85	2.84	2.00 ^a

^a indicates moments with fewer than 30 unweighted observations.

Table 3: The completed fertility of mothers by education category at the state level in 1990 for women aged between 45 and 70.

columns give the correlation between the completed fertility of mothers (from Table 2) and the childlessness rates, respectively when education is below or equal to grade 11 and when education is beyond grade 11.

Figure 4: The completed fertility of mothers and childlessness rates among married women: New Mexico (top left panel), Illinois (top right panel), Tennessee (bottom left panel), and Pennsylvania (bottom right panel).

The U-shaped relationship between childlessness rates and education also prevails at the state level. This establishes that the U-shaped relationship between childlessness rates and education at the aggregate level is not due to an aggregation artifact. A second remarkable regularity lies in the reversal of the correlation between childlessness rates and the completed fertility of mothers as education increases. Out of the 50 states and the District of Columbia, 37 have a positive correlation for low-education levels (below or equal to grade 11) and a negative one for high-education levels (beyond grade 11). Figure 4 shows this reversal for New Mexico, Illinois, Tennessee, and Pennsylvania.

A.3 Uneducated Single Mothers

Another question raised by the facts concerns the identity of those uneducated mothers who have many children. Table 5 gives some information. The column $(1 - \phi n)w^f + a^f$ reports the total income of the person. Earnings are in column $(1 - \phi n)w^f$. Not surprisingly, earnings decrease with the number of children, probably because hours worked drop as the number of

State	Education Category												Correlations	
	1	2	3	4	5	6	7	8	9	10	11	12	≤ 11	> 11
Alabama	11.4	12.5	6.8	6.1	7.5	5.5	7.1	7.8	5.8	9.3	10.3	1.9	0.924	0.016
Alaska	2.4	14.6	7.7	0.0	9.1	1.3	7.2	12.6	6.3	10.4	13.1	33.5	0.465	-0.799
Arizona	7.0	6.2	7.0	6.7	7.5	6.1	8.9	9.2	8.9	12.3	15.5	18.6	0.084	-0.960
Arkansas	11.3	9.7	6.3	7.0	5.8	6.6	7.4	8.5	7.1	6.5	13.1	22.2	0.940	-0.883
California	7.7	5.3	7.0	6.0	6.5	6.1	8.2	9.2	9.9	11.9	16.0	19.8	0.136	-0.992
Colorado	9.0	8.1	5.5	8.4	4.9	2.6	6.6	7.4	7.1	9.9	13.8	25.8	0.716	-0.772
Connecticut	11.3	6.6	9.4	9.2	8.8	5.5	8.5	8.5	7.9	9.5	14.3	26.5	-0.654	-0.983
Delaware	20.8 ^a	11.0	13.9	5.0	7.1	7.8	7.3	6.9	3.7	8.6	11.7	14.0	0.649	-0.837
D. of Columbia	20.6 ^a	17.3	20.8	11.3	11.5	5.1	13.8	15.9	10.6	21.3	19.3	33.4	-0.035	-0.699
Florida	11.7	12.9	10.9	10.6	8.9	7.2	9.3	10.3	9.4	12.0	14.7	21.8	0.724	-0.948
Georgia	9.5	13.2	7.4	6.6	6.3	6.1	7.4	8.2	7.2	8.5	14.0	12.9	0.901	-0.881
Hawaii	2.2	3.5	4.9	5.9	2.4	5.7	8.2	10.0	8.4	10.9	23.0	16.4	-0.669	-0.631
Idaho	13.4	16.3	6.3	6.3	5.5	4.6	3.9	4.7	7.4	6.0	12.7	0.0	0.957	-0.305
Illinois	11.2	6.7	8.0	7.4	6.7	6.3	7.4	8.2	6.8	9.9	15.8	18.2	0.490	-0.925
Indiana	11.6	8.8	7.0	6.9	6.4	6.4	7.1	8.2	7.1	8.0	10.8	16.4	0.751	-0.970
Iowa	16.2	0.0	8.1	6.7	4.8	2.2	5.9	6.8	3.8	9.1	11.3	16.2	0.435	-0.958
Kansas	9.2	0.0	7.0	5.1	5.5	5.9	6.3	6.1	5.1	6.5	10.5	12.2	-0.416	-0.875
Kentucky	14.4	7.6	6.7	7.2	6.3	4.6	7.8	9.1	7.2	11.0	9.6	28.7	0.346	-0.608
Louisiana	10.7	8.3	6.0	6.4	6.5	5.3	7.0	5.7	7.3	9.0	11.4	11.3	0.854	-0.713
Maine	0.0 ^a	2.5	9.8	3.8	3.0	4.8	5.8	8.4	8.4	8.6	11.9	7.0	0.093	0.013
Maryland	13.0	12.4	10.7	7.8	8.2	5.4	8.3	9.0	7.0	9.1	14.4	21.5	0.606	-0.898
Massachusetts	5.5	5.1	8.4	8.0	7.9	7.3	7.9	8.1	7.2	9.2	14.8	20.8	-0.350	-0.970
Michigan	7.2	7.5	7.5	6.6	6.0	5.4	6.3	7.7	5.8	8.7	12.3	17.2	0.542	-0.943
Minnesota	4.2	2.6	6.1	4.7	5.8	5.0	6.2	6.4	4.8	10.4	15.6	21.0	-0.370	-0.865
Mississippi	5.4	9.5	7.2	7.2	5.9	5.9	7.2	7.4	6.1	9.1	13.4	12.4	0.767	-0.951
Missouri	11.5	14.7	6.5	7.9	6.3	5.9	7.4	8.2	6.9	9.3	14.9	21.7	0.965	-0.993
Montana	0.0 ^a	0.0 ^a	5.0	3.4	5.7	4.1	5.0	6.6	4.8	8.7	11.9	12.2	0.677	-0.955
Nebraska	0.0 ^a	12.4	11.1	3.6	5.8	3.8	6.8	7.0	5.9	8.0	7.5	11.6	0.885	-0.851
Nevada	19.3	7.1	9.3	7.4	6.5	6.5	11.1	10.1	7.7	14.6	16.9	26.0	0.602	-0.100
New Hampshire	0.0 ^a	13.0 ^a	9.7	7.8	4.2	1.9	6.2	7.0	6.0	8.6	9.4	35.7	0.072	-0.810
New Jersey	12.5	5.6	9.4	9.8	8.7	6.0	8.5	9.5	9.0	9.5	13.6	20.4	0.038	-0.961
New Mexico	7.9	6.0	6.4	5.4	4.5	2.1	5.9	6.5	4.3	9.3	12.7	24.8	0.794	-0.901
New York	9.9	6.2	8.1	8.6	7.1	7.7	8.2	9.7	8.5	11.0	13.3	19.1	0.474	-0.977
North Carolina	6.5	7.4	7.6	6.2	6.3	6.4	7.5	8.8	7.9	10.9	12.2	23.2	0.757	-0.783
North Dakota	53.6 ^a	0.0 ^a	4.8	3.8	1.1	0.7	4.6	6.1	4.6	10.3	20.1	44.0	0.308	-0.969
Ohio	12.1	7.3	8.6	6.5	5.5	4.4	7.0	8.3	7.9	9.2	12.9	7.8	0.990	-0.451
Oklahoma	16.1	10.5	5.7	6.4	5.1	7.3	7.0	6.5	4.2	8.9	8.4	14.7	0.732	-0.815
Oregon	5.2	13.4	5.9	5.8	5.1	3.9	7.0	6.5	6.8	10.8	14.8	13.7	0.890	-0.252
Pennsylvania	9.7	8.6	10.3	6.6	7.1	6.8	7.8	8.4	7.3	9.0	13.7	20.1	0.506	-0.911
Rhode Island	6.1	4.8	11.7	7.5	8.4	6.4	8.5	9.1	5.4	9.1	9.6	41.5	-0.620	-0.907
South Carolina	7.4	13.0	7.3	6.7	6.4	6.3	7.5	8.5	8.3	10.5	12.6	16.2	0.793	-0.265
South Dakota	0.0 ^a	0.0 ^a	6.4	3.3	4.7	4.1	4.0	6.8	6.6	8.6	6.9	53.8 ^a	0.059	0.946
Tennessee	18.4	10.6	8.5	7.4	7.9	6.9	9.1	8.7	7.6	9.1	13.0	18.7	0.831	-0.889
Texas	7.1	5.6	6.3	7.3	5.5	5.9	7.0	7.3	6.8	9.1	13.4	15.1	0.181	-0.913
Utah	3.7 ^a	15.8 ^a	9.8	0.5	3.8	2.5	4.9	4.4	4.2	6.7	9.9	3.3	0.079	-0.090
Vermont	0.0 ^a	0.0 ^a	8.4	5.3	7.3	7.4	5.3	8.2	6.6	9.6	15.7	0.0	-0.440	-0.701
Virginia	7.2	9.7	8.6	9.3	7.2	7.0	8.3	9.5	8.5	10.7	13.6	18.7	0.511	-0.973
Washington	9.8	6.5	7.9	8.0	6.3	4.6	6.7	7.6	8.1	10.8	14.3	24.1	0.319	-0.993
West Virginia	14.9	8.5	7.1	7.4	6.1	3.6	8.3	9.3	6.2	8.8	10.8	42.7	0.438	0.380
Wisconsin	5.2	12.2	6.4	8.6	6.5	3.4	6.5	6.4	6.6	8.6	14.6	23.2	-0.253	-0.860
Wyoming	41.9 ^a	0.0 ^a	4.0	2.2	4.3	5.9	6.0	4.7	6.9	10.4	9.0	0.0	0.861	0.630

^a indicates moments with fewer than 30 unweighted observations.

Table 4: Percentage of childless married women by education category at the state level in 1990 for women aged between 45 and 70, and correlation between the completed fertility of mothers and childlessness rates among married women for low- and high-education groups.

children rises. More interestingly, the column a^f reports the difference between total income and earnings. We observe that fertility increases with a^f , which is a prediction of our model. Now, who are these women? Very few of them report to be with an unmarried partner (see Table 5). Less than half of them are black. A majority are head of their household.

n	nobs.	$(1 - \phi n)w^f + a^f$	$(1 - \phi n)w^f$	a^f	% with partner	% black	% head of household
1	17,531	7,302.87	5,014.92	2,287.95	6.7%	33.5%	35.0%
2	11,797	7,889.39	5,817.37	2,072.03	9.5%	39.1%	45.3%
3	7,108	6,923.74	4,553.75	2,369.99	11.1%	39.4%	52.5%
4	4,636	6,712.25	3,998.48	2,713.77	6.4%	32.1%	56.8%
5	2,882	6,239.59	3,556.02	2,683.57	5.0%	41.8%	61.1%
6+	5,446	5,844.52	2,576.98	3,267.54	6.1%	36.4%	64.5%

Table 5: Single Mothers with no Education

A.4 Five-year cohorts

The stylized facts highlighted in Section II can also be found if we consider each 5-years cohort separately. Figures 5 and 6 show the relationship between childlessness and education and the fertility of mothers and education for married and single women for each cohort. The only major difference between cohorts is in the childlessness rate of single women, with intermediate levels of education (Figure 6): older single women were much less likely to be mothers than younger single women.

Figure 5: Childlessness Rate and Completed Fertility of Mothers, by Education Category and by Age, Married Women

Figure 6: Childlessness Rate and Completed Fertility of Mothers, by Education Category and by Age, Single Women

A.5 Standard errors of the mean

Table 6 reports the standard errors of the mean for the completed fertility and the childlessness rates of both single and married mothers.

Education Category	Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
	Married	Single	Married	Single	Women	Men
1	0.70	1.61	7.49	22.05	0.941	0.849
2	0.55	2.35	6.30	21.54	0.717	0.524
3	0.23	1.29	2.06	11.58	0.231	0.194
4	0.32	2.27	2.62	16.76	0.297	0.286
5	0.25	2.03	1.98	15.69	0.221	0.246
6	0.25	2.29	2.09	16.26	0.231	0.271
7	0.09	0.60	0.57	06.21	0.074	0.089
8	0.15	0.83	0.86	09.38	0.126	0.114
9	0.27	1.60	1.55	16.15	0.225	0.226
10	0.23	0.62	1.07	15.56	0.199	0.144
11	0.35	0.47	1.41	22.01	0.346	0.175
12	1.47	1.34	5.08	48.75	1.398	0.403

Table 6: Standard Errors ($\times 10^{-3}$) from U.S. Census 1990

A.6 Race, Ethnicity and Place of Birth

A.6.1 Race and Ethnicity

In this first part of the appendix, we split the population into five groups, Whites, Blacks, Natives, Asians and Hispanics. We constructed each group from the two variables RACE and HISPAN. In order for an individual to be considered as White, Black, Native or Asian, he or she has to be from that particular race and “not Hispanic”. Table 7 gives the number of observations (unweighted) by education category for each group. Figures 7 and 8 show the childlessness rate and completed fertility of mothers, by education category, for married women and single women respectively. Figure 9 shows marriage rates for men and women for each group, by education category.

	Blacks	Whites	Natives	Asians	Hispanics
1	1,264	4,631	274	1,518	4,400
2	1,673	5,276	200	868	6,002
3	8,897	59,603	806	2,397	12,487
4	4,029	29,879	300	832	3,059
5	5,468	47,968	471	680	2,598
6	5,877	40,798	361	338	2,018
7	24,421	429,963	1,985	8,115	15,078
8	8,787	160,406	937	,2988	5,104
9	2,631	46,909	275	1,758	1,838
10	4,092	88,089	262	4,551	2,025
11	3,641	49,819	164	1,941	1,283
12	242	3,974	16	207	172
Total	71,022	967,315	6,051	26,193	56,064

Note: We do not know the race or ethnicity of 435 observations so the sum of the five groups is not equal to the sum of the observations of Table 2.

Table 7: Number of Observations by Education Category and by Race or Ethnicity

For all groups, we find a negative relationship between the fertility of married mothers and education. For single mothers, we have few observations for Asians and Natives so the relationship is not as clear. For other single mothers, the relationship holds for Blacks and Hispanics and is slightly hump-shaped for Whites. The fertility differential between single and married mothers is larger for white women than for others: married mothers have between 1.5 and 2 times more children than single mothers. Both the U-shaped relationship between childlessness and education and the hump-shaped relationship between marriage rates and education hold in general. In particular for Asian married women, childlessness always increases with education; childlessness of single Hispanic women is flat for the first education levels and then increases. Comparing across groups, we see that Black women

Figure 7: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Married Women

Figure 8: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Single Women

Figure 9: Marriage Rates for Female and Male, by Education Category, Single Women

are in general more likely to be childless if married, less likely to be childless if single and that the U-shaped relationship (Fact 2) is more pronounced for them than for other groups. Both single and married Black mothers have more children than White mothers. Differential fertility between groups decreases with education.

A.6.2 Place of birth

Some education groups in our sample could comprise large shares of immigrants who may have made their marriage and fertility decisions in another country. It could then be the case that some of our facts do not hold when withdrawing these persons. To assess this, we divide the population into two groups: those who were born in the U.S. and those who were born abroad. The place of birth is obtained thanks to the variable “Birthplace” (BPL) in IPUMS. The percentage of persons born in the U.S. in our sample is 90,96% (so only 9,04% were born abroad). Interestingly, as Table 8 indicates, the main group is not Hispanic (Central and Southern America) but European.

Continent	%
Africa	0.009
Asia	0.220
Central America and Caribbean	0.265
Europe	0.356
Southern America	0.047
Oceania	0.038
Other north America	0.066

Table 8: Distribution of foreign born persons by continent of birth

Removing foreign born individuals from the sample results in the moments shown in Table 9. The facts highlighted in Section II are robust to the deletion of these observations. In particular, we see that the relationship between childlessness rates and education for single women is even more U-shaped than in the benchmark case which includes foreign born persons.

Table 10 shows the facts for the individuals that were born abroad only. We can observe that the relationship between marriage rates of males and education is no longer hump-shaped but rather high and flat. Fertility of mothers still decreases with education. We also find that childlessness rates are monotonously increasing with education. We suspect that, this absence of U-shaped relationship between childlessness and education among foreign born persons, is due to a selection effect. Indeed, the persons who are included in the Census are those who succeeded in emigrating to the U.S.. They are probably in better health and have higher non-labor income than the persons without education born in the U.S.. Then, they are less concerned with social sterility.

Category	Educ shares. in %	Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
		Married	Single	Married	Single	Women	Men
1	0.63	0.106	0.837	4.727	3.566	0.533	0.591
2	0.78	0.089	0.648	4.886	3.915	0.785	0.838
3	6.75	0.078	0.655	3.977	3.517	0.907	0.906
4	3.39	0.070	0.570	3.670	3.435	0.934	0.926
5	5.25	0.064	0.587	3.538	3.364	0.945	0.930
6	4.64	0.058	0.523	3.500	3.447	0.949	0.936
7	43.53	0.073	0.788	3.090	2.537	0.948	0.938
8	16.19	0.081	0.842	2.974	2.129	0.944	0.948
9	4.64	0.072	0.831	3.007	2.244	0.947	0.948
10	8.79	0.100	0.936	2.806	1.939	0.917	0.935
11	5.04	0.137	0.961	2.619	1.924	0.837	0.932
12	0.37	0.192	0.964	2.429	1.693	0.739	0.925

Note: Weights are used to compute the moments. Education shares are unweighted.

Table 9: Facts from U.S. Census 1990 without foreign born persons

A.7 Disability

The U-shaped relationship between childlessness and education is highly affected when we consider the data without individuals who have any lasting physical or mental health condition that prevents or causes difficulty working, living independently or taking care of their own personal needs (respectively, variables DISABWRK, DIFFMOB and DIFFCARE of IPUMS). Note that these variables say nothing about the ability to reproduce.⁴ For married women the relationship between childlessness and education is not affected, but the relationship for single women becomes flat for the first education levels and only increases from Grade 11 on.

We cannot distinguish in the 1990 Census those who are born with a disability from those who have acquired one later in life. Among the former group of people, are those who did not receive an education due to their handicap, and have therefore not gotten married, nor did they have children. This group biases our estimates of social sterility upwards. However, this bias is expected to be small, as the persons born with a severe handicap in 1920-45 had a low chance to survive to 45 years and beyond and are therefore absent from our sample. For example, a person affected by the Down Syndrome, which is one of the most common disabilities, life expectancy was below 18 in the thirties (while it is nowadays around 50 years). In the second group, we have those who became disabled because of poverty. Either because they worked or lived in an unhealthy or dangerous environment or because they

⁴See <http://www.census.gov/hhes/www/disability/sipp/disab02/ds02t2.html> for information about disability types and their proportions in the U.S.

Category	Educ shares in %	Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
		Married	Single	Married	Single	Women	Men
1	5.6	0.077	0.430	4.976	4.270	0.875	0.912
2	5.9	0.058	0.441	4.693	3.643	0.898	0.933
3	14.9	0.078	0.517	3.670	3.356	0.919	0.935
4	3.3	0.094	0.479	3.420	3.309	0.925	0.927
5	3.4	0.095	0.546	3.126	3.095	0.933	0.926
6	1.8	0.105	0.698	2.949	2.640	0.942	0.937
7	32.7	0.116	0.803	2.796	2.093	0.928	0.939
8	11.9	0.111	0.795	2.724	2.318	0.932	0.950
9	5.8	0.110	0.917	2.617	1.979	0.909	0.944
10	8.9	0.100	0.936	2.806	1.939	0.917	0.935
11	5.1	0.140	0.931	2.487	1.822	0.870	0.943
12	0.8	0.186	0.905	2.317	1.892	0.836	0.940

Note: Weights are used to compute the facts. Education shares are unweighted.

Table 10: Facts from U.S. Census 1990 including only foreign born persons - Total number of observations (unweighted) 97135

had a bad medical treatment, or none, and remained single and childless. These two factors are likely to affect their health aged 45-70. This leads to an endogeneity problem in the relationship between disability and education: adults who lived in the worst conditions (the ones with the lowest education) are the most likely to suffer health problems when older. In other words, the lowly educated are more likely to have health problems when old than the highly educated. In this group, disabled women are, *de facto*, lowly educated and the constraint $c^f > \hat{c}$ reflects their incapacity to have children when they do not have a husband investing in them. Our position is that we should keep this group in the sample.

A.8 Historical Data

A.8.1 IPUMS Data

In the historical exercise we also use Census data, taken from IPUMS. Table 11 lists the 22 cohorts that were considered as well as the Census from which they were constructed. As the fertility of single women was not asked before 1970, the data is only for ever-married women.

Two remarks before analyzing completed fertility over time should be noticed. First, there was a change on how the number of children ever born was reported. First, for the 1940 Census, there was only one case for the number of children that a woman had had while in

FOR ALL WOMEN WHO ARE OR HAVE BEEN MARRIED		
Has this woman been married more than once? (Yes or No)	Age at first marriage	Number of children ever born (Do not include stillbirths)
48	49	50

38. If female and ever married (Mar, Wd, D, or Sep in item 12) —

How many children has she ever borne, not counting stillbirths?

_____ children, or ☐ None

P20. If this is a woman who has ever been married —

How many babies has she ever had, not counting stillbirths?

Do not count her stepchildren or adopted children.

..... OR None ☐

(Number)

25. If this is a girl or a woman —

How many babies has she ever had, not counting stillbirths?

Do not count her stepchildren or children she has adopted.

1 2 3 4 5 6 7 8

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

9 10 11 12 or more None

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

20. If this person is a female —

How many babies has she ever had, not counting stillbirths?

Do not count her stepchildren or children she has adopted.

None 1 2 3 4 5 6

☐ ☐ ☐ ☐ ☐ ☐

7 8 9 10 11 12 or more

☐ ☐ ☐ ☐ ☐ ☐

If this person is a female —

20. How many babies has she ever had, not counting stillbirths? Do not count her stepchildren or children she has adopted.

None 1 2 3 4 5 6 7 8 9 10 11 12 or more

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

Figure 10: Question on Children Ever Born in 1940, 50, 60, 70, 80 and 90 censuses

later Censuses a case for “none” was added to the questionnaire - see Figure 10. This could then lead many childless women in cohorts 1971-1900 to be counted as not respondents instead of as childless and hence explain partly the increase in childlessness rates for the following cohorts. This was already pointed out in Grabill and Glick (1959).

A second aspect concerns the high childlessness rates for the cohorts of women born at the beginning of last century. These women have been touched by the 1918 Spanish Influenza. Pregnant women were particularly vulnerable to the Spanish Influenza. If they were affected by the flu, they had high chances to either die or have a miscarriage (Bloom-Feshbach et al. (2011)).

Morgan (1991) also shows childlessness data using vital registration estimates. His estimates for the cohorts that were born at the beginning of the 1900s are lower than the average rates from the Census (see Figure 1 in Morgan (1991)). Our estimates of childlessness rates are close to those of Figure A2 in Jones and Tertilt (2008).

Cohort	Age	Census	Completed Fertility of Mothers	Childlessness Rate
1871-1875	65-69	1940	4.228	0.164
1876-1880	60-64	1940	3.998	0.171
1881-1885	55-59	1940	3.915	0.169
1886-1890	50-54	1940	3.758	0.158
1891-1895	45-49	1940	3.585	0.150
1896-1900	40-44	1940	3.353	0.156
1901-1905	45-49	1950	3.197	0.190
1906-1910	40-44	1950	2.883	0.203
1911-1915	45-49	1960	2.916	0.175
1916-1920	40-44	1960	2.985	0.132
1921-1925	45-49	1970	3.151	0.099
1926-1930	40-44	1970	3.372	0.081
1931-1935	45-49	1980	3.447	0.070
1936-1940	40-44	1980	3.236	0.066
1941-1945	45-49	1990	2.786	0.082
1946-1950	40-44	1990	2.471	0.102
1954-1958	40-44	1998	2.320	0.137
1956-1960	40-44	2000	2.367	0.134
1958-1962	40-44	2002	2.359	0.120
1960-1964	40-44	2004	2.357	0.132
1962-1966	40-44	2006	2.345	0.146
1964-1968	40-44	2008	2.297	0.120

Note: Data for cohorts born after 1954 are taken from Tables available at the Census webpage.

Table 11: Cohorts for the Historical Exercise and Completed Fertility of Mothers and Childlessness Rates for Married Women of Each Cohort

A.8.2 TFP

Data on TFP are taken from Chari et al. (2006) for the period 1901-2000 and from Conference Board (2013) for the period 2000-2010. We compute 30-years averages to represent the mean life-cycle wage (for example, the wage of the cohort born in 1871 is indexed on average TFP between 1901 and 1931). For the cohorts born in the 50s and 60s, we forecast future TFP assuming the same growth rate as the one observed in the twentieth century (average annual growth rate of 0.47%).

A.8.3 Changes in Education

To compute changes in education, we divided the sample of each cohort into the 12 categories of education. For cohorts generated from Censuses before 1900, we cannot distinguish between categories 11 and 12 so they were merged. The codes (from the variable EDUCD) that were used for each Census are provided in Table 12. Results are provided in Tables 13 and 14.

Education Level		Census			
		1940-1980	1990	2000-2006	2008
1	No school	2	2	2	2
2	Grade 1-4	14-17	13	10	14-17
3	Grade 5-8	22, 23, 25, 26	20	21, 24	22, 23, 25, 26
4	Grade 9	30	30	30	30
5	Grade 10	40	40	40	40
6	Grade 11	50	50	50	50
7	Grade 12	60	61, 62	61, 62	61, 63, 64
8	1 year of college	65, 70	71	65, 71	65, 71
9	2 years of college	80	82-83	81	81
10	Bachelor degree	90, 100	101	101	101
11	Master degree	110-113	114, 115	114, 115	114, 115
12	doctoral degree		116	116	116

Note: Data for years 2010-2008 is taken from the American Community Survey (ACS). As Census 1998 is not available in IPUMS, we used Census 2000 for the education shares of cohort 1958-1958.

Table 12: Codes for Education Shares for Each Cohort (from EDUCD variable)

Cohort	Age	Census	1	2	3	4	5	6	7	8	9	10	11	12
1871-1875	65-69	1940	0.062	0.120	0.559	0.036	0.038	0.017	0.097	0.013	0.022	0.030	0.005	
1876-1880	60-64	1940	0.054	0.112	0.539	0.044	0.044	0.022	0.111	0.016	0.019	0.032	0.006	
1881-1885	55-59	1940	0.050	0.108	0.530	0.053	0.049	0.021	0.116	0.014	0.021	0.032	0.007	
1886-1890	50-54	1940	0.048	0.106	0.512	0.055	0.051	0.024	0.123	0.016	0.024	0.034	0.008	
1891-1895	45-49	1940	0.041	0.093	0.490	0.057	0.061	0.027	0.134	0.019	0.029	0.039	0.008	
1890-1900	40-44	1940	0.026	0.079	0.472	0.061	0.069	0.032	0.155	0.021	0.033	0.043	0.009	
1901-1905	45-49	1950	0.013	0.070	0.406	0.061	0.072	0.041	0.187	0.036	0.040	0.055	0.019	
1906-1910	40-44	1950	0.008	0.050	0.347	0.065	0.081	0.053	0.230	0.039	0.040	0.064	0.021	
1911-1915	45-49	1960	0.009	0.033	0.306	0.073	0.086	0.056	0.271	0.044	0.038	0.061	0.021	
1916-1920	40-44	1960	0.006	0.027	0.233	0.069	0.089	0.063	0.347	0.047	0.037	0.063	0.019	
1921-1925	45-49	1970	0.007	0.021	0.162	0.058	0.079	0.080	0.406	0.053	0.042	0.065	0.026	
1926-1930	40-44	1970	0.007	0.016	0.132	0.059	0.083	0.075	0.414	0.058	0.046	0.080	0.029	
1931-1935	45-49	1980	0.007	0.015	0.097	0.046	0.066	0.058	0.435	0.075	0.057	0.094	0.049	
1936-1940	40-44	1980	0.006	0.012	0.075	0.044	0.061	0.053	0.437	0.086	0.061	0.108	0.059	
1941-1945	45-49	1990	0.009	0.008	0.040	0.024	0.037	0.034	0.388	0.191	0.066	0.121	0.075	0.007
1946-1946	40-44	1990	0.008	0.007	0.030	0.018	0.025	0.024	0.344	0.212	0.079	0.154	0.092	0.007
1954-1958	42-46	2000	0.011	0.005	0.026	0.014	0.018	0.018	0.305	0.226	0.091	0.184	0.094	0.007
1956-1960	40-44	2000	0.011	0.005	0.026	0.014	0.018	0.019	0.305	0.228	0.093	0.188	0.086	0.007
1958-1962	40-44	2002	0.006	0.007	0.025	0.013	0.018	0.020	0.311	0.076	0.232	0.197	0.086	0.008
1960-1964	40-44	2004	0.006	0.006	0.028	0.013	0.019	0.021	0.318	0.154	0.106	0.224	0.097	0.008
1962-1966	40-44	2006	0.007	0.006	0.029	0.016	0.018	0.019	0.307	0.148	0.110	0.228	0.103	0.009
1964-1968	40-44	2008	0.010	0.006	0.029	0.016	0.017	0.020	0.252	0.168	0.110	0.243	0.116	0.011

Note: Education categories 11 and 12 were merged before 1970.

Table 13: Proportion of Women in Each Education Category

Cohort	Age	Census	1	2	3	4	5	6	7	8	9	10	11	12
1871-1875	65-69	1940	0.078	0.163	0.550	0.029	0.030	0.012	0.064	0.013	0.016	0.033	0.014	
1876-1880	60-64	1940	0.063	0.152	0.545	0.033	0.036	0.013	0.079	0.013	0.016	0.033	0.017	
1881-1885	55-59	1940	0.059	0.145	0.539	0.040	0.041	0.018	0.080	0.013	0.016	0.036	0.015	
1886-1890	50-54	1940	0.054	0.136	0.526	0.043	0.048	0.019	0.085	0.014	0.019	0.041	0.015	
1891-1895	45-49	1940	0.041	0.118	0.515	0.051	0.055	0.023	0.100	0.015	0.022	0.042	0.017	
1890-1900	40-44	1940	0.027	0.097	0.499	0.056	0.066	0.030	0.114	0.018	0.024	0.048	0.019	
1901-1905	45-49	1950	0.014	0.086	0.445	0.061	0.070	0.043	0.135	0.027	0.028	0.059	0.031	
1906-1910	40-44	1950	0.014	0.086	0.445	0.061	0.070	0.043	0.135	0.027	0.028	0.059	0.031	
1911-1915	45-49	1960	0.012	0.071	0.378	0.063	0.081	0.054	0.172	0.033	0.036	0.063	0.038	
1916-1920	40-44	1960	0.008	0.043	0.256	0.067	0.082	0.065	0.269	0.048	0.042	0.071	0.049	
1921-1925	45-49	1970	0.007	0.031	0.186	0.059	0.078	0.070	0.294	0.052	0.054	0.096	0.072	
1926-1930	40-44	1970	0.007	0.026	0.171	0.064	0.082	0.062	0.289	0.054	0.053	0.109	0.084	
1931-1935	45-49	1980	0.006	0.023	0.125	0.043	0.053	0.044	0.326	0.064	0.070	0.128	0.118	
1936-1940	40-44	1980	0.006	0.015	0.093	0.042	0.050	0.041	0.343	0.075	0.076	0.129	0.130	
1941-1945	45-49	1990	0.010	0.010	0.050	0.026	0.034	0.028	0.301	0.192	0.058	0.150	0.117	0.024
1946-1946	40-44	1990	0.010	0.008	0.035	0.018	0.024	0.021	0.260	0.214	0.078	0.191	0.125	0.018
1954-1958	42-46	2000	0.012	0.005	0.030	0.016	0.021	0.022	0.306	0.219	0.072	0.179	0.102	0.015
1956-1960	40-44	2000	0.012	0.005	0.030	0.017	0.022	0.024	0.318	0.216	0.071	0.177	0.095	0.013
1958-1962	40-44	2002	0.007	0.008	0.030	0.016	0.022	0.025	0.332	0.062	0.204	0.189	0.092	0.014
1960-1964	40-44	2004	0.007	0.006	0.033	0.016	0.023	0.027	0.337	0.142	0.082	0.209	0.101	0.017
1962-1966	40-44	2006	0.009	0.008	0.036	0.019	0.023	0.025	0.344	0.134	0.081	0.202	0.102	0.016
1964-1968	40-44	2008	0.012	0.008	0.036	0.019	0.022	0.027	0.286	0.159	0.082	0.220	0.110	0.017

Note: Education categories 11 and 12 were merged before 1970.

Table 14: Proportion of Men in Each Education Category

A.8.4 Changes in Wages

For each cohort we run the following linear regression for single individuals with a positive income wage (INCWAGE variable in the Census):

$$\ln(\text{INCWAGE}) = \text{constant} + \ln(\gamma)i + \rho e \quad (1)$$

where $i = 0$ if the individual is a man and $i = 1$ if she is a woman. e are the years of schooling that we give to each category of education (Table 2). The estimated parameters $\ln(\gamma)$ and ρ denote the natural logarithm of the gender wage gap and the mincer coefficient respectively. Table 15 provides the estimated values for γ and ρ with the respective standard errors (SE) for each cohort. The values of the estimates that we use in Section IV.A are those of the cohort 1941-1945 (in bold).

Cohort	γ	SE	ρ	SE	Cohort	γ	SE	ρ	SE
1871-1875	0.988	(0.0076)	0.092	(0.0009)	1926-1930	0.755	(0.0020)	0.073	(0.0003)
1876-1880	0.928	(0.0053)	0.093	(0.0007)	1931-1935	0.816	(0.0025)	0.080	(0.0004)
1881-1885	0.927	(0.0041)	0.085	(0.0005)	1936-1940	0.787	(0.0022)	0.086	(0.0003)
1886-1890	0.891	(0.0033)	0.083	(0.0004)	1941-1945	0.869	(0.0019)	0.092	(0.0003)
1891-1895	0.859	(0.0028)	0.072	(0.0004)	1946-1950	0.904	(0.0013)	0.094	(0.0002)
1896-1900	0.805	(0.0023)	0.090	(0.0003)	1954-1958	0.893	(0.0011)	0.111	(0.0002)
1901-1905	0.787	(0.0025)	0.057	(0.0003)	1956-1960	0.877	(0.0015)	0.112	(0.0003)
1906-1910	0.843	(0.0021)	0.046	(0.0003)	1958-1962	0.843	(0.0010)	0.119	(0.0002)
1911-1915	0.767	(0.0023)	0.076	(0.0003)	1960-1964	0.841	(0.0011)	0.126	(0.0002)
1916-1920	0.772	(0.0019)	0.073	(0.0003)	1962-1966	0.848	(0.0009)	0.125	(0.0002)
1921-1925	0.789	(0.0022)	0.066	(0.0003)	1964-1968	0.836	(0.0009)	0.112	(0.0002)

Note: As Census 1998 is not available in IPUMS, we used Census 2000 to construct cohort 1954-1958.

Table 15: Estimates for the Gender Wage Gap and Returns to Schooling for Cohorts born between 1971-1968

A.9 Returns to Schooling

Individuals in the extreme education levels are relatively few. This could imply that Mincer equations linking economic returns to schooling works well for the middle of the distribution of wages, but is less adequate for the tails of the distribution. As the tails are important to reproduce the U-shaped childlessness pattern, we looked at this question in detail.

We estimate a Mincer equation by regressing the natural logarithm of the variable “wage and salary income (INCWAGE)” on a constant plus the years of education for our sample of women who are still working (UHRSWORK > 0). Results are provided in Table 16. We see that the regression coefficients are significant at the 1% level and close to 0.1.

Dependent Variable	Married		Single		All
	All	Childless	All	Childless	
Years of Education	0.097 (0.000137)	0.113 (0.000451)	0.111 (0.000310)	0.109 (0.000365)	0.100 (0.000126)
Constant	8.019	7.961	8.157	8.210	8.005

Note: The dependent variable is the logarithm of the variable “wage and salary income”. We only show results for women in our sample for whom the variable “usual hours worked per week” was positive.

Table 16: Univariate Estimates of the Returns to Schooling

Figure 11 plots the estimates of the variable “wage and salary income” together with the mean observations for different years of education, for married and single women respectively (childless and mothers). In both cases, the fit for highly educated women is good, so we do not expect biases for the right tail of the wage distribution. For low skilled women results are mixed. For married women, the exponential function clearly underestimate the earnings of the low education women. For singles, however, the fit remains good.

Figure 11: Earnings as a Function of Education. Married (left), Single (right)

As most women who are concerned by social sterility are single women, our estimate of this type of childlessness is robust to alternative specifications of the wage function that would track the observed wages even better. For married women, on the contrary, our specification is too pessimistic concerning their income, and we may overestimate the proportions of the women in the regimes of social sterility [S] and constrained fertility [C].

As fertility decisions affect the number of hours worked and consequently, earnings, we remove this endogeneity by reestimating the Mincer equation on a subsample, including only the childless women. Results are presented in Figure 12. We find the same result as above, that the exponential relationship between years of schooling and wages underestimates the wages for low educated married women, but does fine for single women.

Figure 12: Earnings as a Function of Education among Childless. Married (left), Single (right)

B Theory

In this Appendix, we solve the two stages of the game, which is the maximization problem of a single woman and of a couple. Before studying the precise conditions on wages and non-labor income under which the various constraints bind, we list the different possible “regimes”, describing different living arrangements.

B.1 Possible Regimes

Table 17 summarizes consumption and fertility choices in each regime, where we define for married couples:

$$a = a^f + a^m - \mu.$$

B.1.1 Single Women

A single woman maximizes (1) subject to (2), (3), $b^f(c^f, n) \leq 0$ and $0 \leq n \leq \underline{n}_M$. She can be in different regimes, depending on which constraint is binding.

Regime 0. [N] If a single woman is naturally sterile, she consumes her total income net of costs. Her consumption can be either smaller or greater than $\hat{c}$ as natural sterility is not correlated to education and earnings.

Regime I. [U] If no other constraint than the budget constraint is binding and the corner solution with no children does not dominate, the unconstrained fertility regime prevails. The equation of n_1 shows that the effect of the woman’s education on fertility is ambiguous. A higher w^f implies the usual income effect which raises fertility. But the increase in w^f also

Single Men

$$c^m = (1 - \delta^m)w^m + a^m - \mu$$

Single Women

$$\begin{aligned} [N] \quad c_0^f &= (1 - \delta^f)w^f + a^f - \mu \leq \hat{c} \\ [U] \quad c_1^f &= \frac{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu}{2} \\ [S] \quad c_{II}^f &= (1 - \delta^f)w^f + a^f - \mu < \hat{c} \\ [C] \quad c_{III}^f &= \hat{c} \\ [O] \quad c_V^f &= (1 - \delta^f)w^f + a^f - \mu \\ [M] \quad c_V^f &= a^f - \mu \end{aligned}$$

Couples

$$\begin{aligned} [U] \quad c_{VI}^f &= \frac{\hat{\theta}y}{2} > \hat{c} & c_{VI}^m &= \frac{(1 - \hat{\theta})y}{2} \\ [S] \quad c_{VII}^f &= \hat{\theta}(w^m + w^f + a) < \hat{c} & c_{VII}^m &= (1 - \hat{\theta})(w^m + w^f + a) \\ [C] \quad c_{VIII}^f &= \hat{c} & c_{VIII}^m &= (1 - \hat{\theta})[y - \hat{c}]/(2 - \hat{\theta}) \\ [O] \quad c_{IX}^f &= \hat{\theta}(w^m + w^f + a) & c_{IX}^m &= (1 - \hat{\theta})(w^m + w^f + a) \\ [C] \& [M] \quad c_X^f &= \hat{c} & c_X^m &= \left(\frac{2\alpha - 1}{\alpha}\right)w^m + a - \hat{c} \\ [M] \quad c_{XI}^f &= \hat{\theta}\left[\left(\frac{2\alpha - 1}{\alpha}\right)w^m + a\right] & c_{XI}^m &= (1 - \hat{\theta})\left[\left(\frac{2\alpha - 1}{\alpha}\right)w^m + a\right] \\ [N] \quad c_{XII}^f &= \hat{\theta}(w^m + w^f + a) \leq \hat{c} & c_{XII}^m &= (1 - \hat{\theta})(w^m + w^f + a) \end{aligned}$$

Note: $y = w^f(1 + \phi\alpha(\nu - \eta)) + w^m(1 + \phi(1 - \alpha)(\nu - \eta)) + a$, $\hat{\theta} = \theta(w^f, w^m)$

Table 17: Possible Regimes

$$\begin{aligned} n_0 &= 0 \\ n_I &= \frac{1}{2} \left[\frac{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu}{\phi w^f} \right] - \nu \\ n_{II} &= 0 \\ n_{III} &= \frac{w^f(1 - \delta^f - \phi\eta) + a^f - \hat{c} - \mu}{\phi w^f} \\ n_{IV} &= 0 \\ n_V &= \underline{n}_M \end{aligned}$$

$$\begin{aligned} n_{VI} &= \frac{y}{2\phi(\alpha w^f + (1 - \alpha)w^m)} - \nu > 0 \\ n_{VII} &= 0 \\ n_{VIII} &= \frac{1}{2 - \hat{\theta}} \left[\frac{y - \hat{c}}{\phi(\alpha w^f + (1 - \alpha)w^m)} \right] - \nu \\ n_{IX} &= 0 \\ n_X &= \bar{n}_M \\ n_{XI} &= \bar{n}_M \\ n_{XII} &= 0 \end{aligned}$$

raises the time cost to rearing children, which reduces fertility through a substitution effect. If $a^f > \mu$, the substitution effect dominates and n_t decreases with w^f , while the reverse is true if $a^f < \mu$.

Regime II. [S] If her income does not allow the consumption level required to have children to be reached, the single woman lives in poverty and is prevented from choosing to have children.

Regime III. [C] If her income is sufficiently high to escape Regime II but not high enough to have as many children as in Regime I, it may be optimal for the single woman to work and consume more in order to be able to procreate. A necessary condition for this regime to prevail is $a^f < \hat{c} + \mu$. Otherwise the woman would be able to consume more than $\hat{c}$ without working. $a^f < \hat{c} + \mu$ implies that n_{III} increases with the wage w^f . Regime III echoes Malthus's fertility theory (see e.g. Galor (2005)).

Regime IV. [O] When being childless yields the highest utility and the minimal consumption constraint (2) does not bind, childlessness prevails. This regime arises when the opportunity cost of having children is high and the single woman fully specializes in labor market activities.

Regime V. [M] When the opportunity cost of having children is low and the non-labor income is high, it may be optimal for the woman to fully specialize in the production of children.

B.1.2 Couples

The problem for a couple is

$$\max_{c^f, c^m, n} \theta(w^f, w^m) \ln c^f + (1 - \theta(w^f, w^m)) \ln c^m + \ln(\nu + n)$$

subject to (2), (3), (3) and $0 \leq n \leq \bar{n}_M$.⁵ Six regimes are possible when none of the spouses are naturally sterile. When one of them is naturally sterile, the couple lives Regime XII described hereafter.

⁵An alternative could be to add two participation constraints: $u^f(\text{married}) \geq u^f(\text{single})$ and $u^m(\text{married}) \geq u^m(\text{single})$. This implies that the partner with the highest negotiation power can have an interest in reducing his/her welfare in order to incite his/her match to accept marriage. However, because rationality is common knowledge, such a marriage contract would not be credible: as couples cannot divorce, the partner with the highest negotiation power has an incentive to deviate from the agreement.

Regime VI. [U] If no constraint is binding and the corner solution with no children does not dominate we have the unconstrained fertility regime. An increase in the wage of the husband has an ambiguous effect on fertility:

$$\frac{\partial n_{VI}}{\partial w^m} > 0 \Leftrightarrow w^f > \frac{1 - \alpha}{2\alpha - 1} a. \quad (2)$$

In most of the literature (Galor and Weil (1996), etc.) $\alpha = 1$, then an increase in the wage of the husband has a pure income effect and $\partial n_{VI} / \partial w^m > 0$. When $\alpha < 1$, the income effect always dominates the substitution effect in families with very low non-labor incomes ($a < 0$). In families with higher non-labor incomes ($a > 0$), the substitution effect dominates only when the wage of the woman is low relative to the non-labor income of the couple.

In the case of an increase in the wife's wage, we find:

$$\frac{\partial n_{VI}}{\partial w^f} < 0 \Leftrightarrow w^m > \frac{\alpha}{1 - 2\alpha} a.$$

As long as $\alpha > 1/2$, an increase in the woman's wage always reduces fertility when $a > 0$. In families with very low non-labor income ($a < 0$), the income effect dominates if the wage of the husband is also very low.⁶

Parameter α is related to the slope of the wage-fertility relationship. Higher α implies a more negative link between mother's education and fertility, together with a less negative link between father's education and fertility. We will come back to this result in the section on identification (Section IV.B). Finally, n_{VI} does not depend on the negotiation power $\theta(w^f, w^m)$ as there are no gender differences in preferences for children.

Regime VII. [S] When the total net income is too low to guarantee a sufficient consumption level to procreate, even if the husband would not consume, the couple is childless due to poverty.

Regime VIII. [C] When the bargaining power of the wife is too low to guarantee a sufficient consumption level allowing her to procreate in the interior regime, it can be optimal for the husband to give up some consumption in order to have children. This is the only regime in which fertility depends on the wife's bargaining power. This makes fertility subject to three effects: the usual income and substitution effects and a negotiation power effect. The latter arises because an increase in the wife's education gives her more bargaining power. As, in this regime, her consumption is fixed, the only way to obtain more utility is through increased fertility. The net effect on fertility is the result of these three forces. Notice that once w^f becomes sufficiently high, the couple will be able to have children in Regime VI. This is due to both the increased bargaining power of the wife allowing her to benefit more from the marriage surplus and to an income effect: as w^f increases, the total income of the couple increases allowing both spouses to consume more. This regime does not exist for a single woman as it relies on the additional income provided by the husband.

⁶Jones and Schoonbroodt (2010) provide a discussion on the possible impacts of women's wages on fertility.

Regime IX. [O] When choosing to be childless yields the highest utility, the couple chooses to have no children. In this case, both spouses specialize in labor market activities.

Regime X. [C] and [M] When it is optimal for the husband to give up some of his consumption for his wife to specialize entirely in procreation, the couple has the maximum number of children. Compared to Regime VIII, the optimal fertility rate of the couple does not depend on the negotiation power of the wife because she is not able to give birth to an additional child to increase her utility.

Regime XI. [M] When the couple is sufficiently rich (either from non-labor income or from husband education) for the wife to specialize in childrearing without requiring any sacrifice from the husband, the couple has the maximum fertility.

Regime XII. [N] When at least one the spouses is naturally sterile, the couple behaves like in Regimes VII and IX. The economic characteristics of couples living in Regime XII are heterogenous such that $c_{xII}^f \leq \hat{c}$.

B.2 Optimal Decisions of Single Women

Conditional on being single, we can determine the optimal fertility and consumption of a woman as a function of her wage and non-labor income. We impose the following assumptions⁷ on the parameters, they ensures that the problem is not degenerated in the sense that only corner solutions prevail:

Assumption 1

$$1 - \delta^f > \phi(\nu - \eta)(1 - \hat{c}), \quad \eta \geq \nu + \frac{\sqrt{1 - \delta^f}(\sqrt{1 - \delta^f} - 2\sqrt{\phi\nu})}{\phi}$$

We define non-labor income thresholds $\underline{a}$ and $\bar{a}$, and wage thresholds, $\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f), \mathcal{W}_3^f(a^f), \mathcal{W}_4^f(a^f)$ and $\mathcal{W}_5^f(a^f)$ as

Definition 1 (Wage thresholds for singles)

$$\begin{aligned} \mathcal{W}_0^f(a^f) &= \frac{\hat{c} + \mu - a^f}{1 - \phi\eta - \delta^f}, \quad \mathcal{W}_2^f(a^f) = \frac{2\hat{c} + \mu - a^f}{1 - \delta^f + \phi(\nu - \eta)}, \\ \mathcal{W}_3^f(a^f) &= \frac{a^f - \mu}{1 - \delta^f + \phi(\nu - \eta)}, \quad \mathcal{W}_5^f(a^f) = \frac{a^f - \mu}{\phi(\nu + \eta) + \delta^f - 1}, \end{aligned}$$

⁷Assumptions 1 and 2 (see below) hold under the values presented in Table 3.

$\mathcal{W}_1^f(a^f)$ is the smallest root, in w^f , of the quadratic equation $u(\hat{c}, n_{III}) = u(c_{IV}, 0)$:

$$((1 - \delta^f)w^f + a^f - \mu) \nu = \hat{c} \left(\frac{w^f(1 - \delta^f - \phi\eta) + a^f - \mu - \hat{c}}{\phi w^f} + \nu \right), \quad (3)$$

$\mathcal{W}_4^f(a^f)$ is the highest root of the quadratic equation in w^f :

$$\frac{(w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu)^2}{4\phi w^f} = \nu((1 - \delta^f)w^f + a^f - \mu).$$

$\mathcal{W}_0^f(a^f)$ is the value of w^f above which a single woman can have children and consume at least $\hat{c}$. $\mathcal{W}_2^f(a^f)$ is the value of w^f that solves $n_{III} = n_I$. $\mathcal{W}_3^f(a^f)$ is the value of w^f such that $n_I = \underline{n}_M$. $\mathcal{W}_4^f(a^f)$ is either the lowest or the unique value of w^f such that $u(c_{IV}^f, 0) = u(c_I^f, n_I)$. $\mathcal{W}_5^f(a^f)$ is the value of w^f that solves $n_I = 0$.

Definition 2 (Non-labor income thresholds for singles)

$$\underline{a} = \hat{c} \left(\frac{\phi(\nu + \eta) - 1 + \delta^f}{\phi\nu} \right) + \mu \quad , \quad \bar{a} = \hat{c} + \mu > \underline{a}$$

$\underline{a}$ is the value of a under which a single woman is always childless. $\bar{a}$ is the value of a^f such that $\hat{c} = c_V^f$.

The following proposition describes the optimal behavior of a single woman endowed with (w^f, a^f) .

Proposition 1 (Fertility of singles) *Under Assumption 1, the optimal choice of a single woman who is not naturally sterile, and who is endowed with non-labor income a^f and wage w^f , is given by:*

1. when $a^f < \underline{a}$:

- $\forall w^f < \mathcal{W}_0^f(a^f)$, $c^f = c_H^f$, $n = 0$,
- $\forall w^f \geq \mathcal{W}_0^f(a^f)$, $c^f = c_{IV}^f$, $n = 0$.

2. when $a^f \in [\underline{a}, \bar{a}]$:

- $\forall w^f < \mathcal{W}_0^f(a^f)$, $c^f = c_H^f$, $n = 0$,
- $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[$, $c^f = c_{IV}^f$, $n = 0$,
- $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[$, $c^f = \hat{c}$, $n = n_{III}$,
- $\forall w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_4^f(a^f)]$, $c^f = c_I^f$, $n = n_I$,
- $\forall w^f > \mathcal{W}_4^f(a^f)$, $c^f = c_{IV}^f$, $n = 0$.

3. when $a^f \geq \bar{a}$:

- $\forall w^f < \mathcal{W}_3^f(a^f), c^f = c_V^f, n = \underline{n}_M,$
- $\forall w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_4^f(a^f)], c^f = c_I^f, n = n_I,$
- $\forall w^f > \mathcal{W}_4^f(a^f), c^f = c_{IV}^f, n = 0.$

Proof. To prove Proposition 1, we need to show that, under Assumption 1:

1. when $a^f < \underline{a}$, $\mathcal{W}_5^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_0^f(a^f)$ and $\mathcal{W}_0^f(a^f)$ is greater than the highest root of Equation (3);
2. when $a^f \in [\underline{a}, \bar{a}[$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$;
3. when $a^f \geq \bar{a}$, $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

Proof and Implications of 1.

Straightforward computations indicate that when $a^f < \underline{a}$, $\mathcal{W}_5^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_0^f(a^f)$. When $w^f \leq \mathcal{W}_0^f(a^f)$, becoming a mother is not feasible either in Regime I or III.

Once $w^f > \mathcal{W}_0^f(a^f)$, a single woman can have children in Regime III. Comparing utility in Regime IV with utility in Regime III, $u(c_{IV}^f, 0) \leq u(\hat{c}, n_{III})$ if and only if,

$$((1 - \delta^f)w^f + a^f - \mu) \nu \leq \hat{c} \left(\frac{w^f(1 - \delta^f - \phi\eta) + a^f - \mu - \hat{c}}{\phi w^f} + \nu \right). \quad (4)$$

The left-hand side (hereafter LHS) is linear and increasing in w^f and the right-hand side (hereafter RHS) is increasing and concave in w^f (since $a^f - \mu < \hat{c}$), so Equation (4) holding with equality (i.e. Equation (3)) has at most two solutions. At $\mathcal{W}_0^f(a^f)$, the LHS of Equation (4) is higher than the RHS, implying that $u(c_{IV}^f, 0) > u(\hat{c}, n_{III})$. This implies that $\mathcal{W}_0^f(a^f)$ is either smaller than the lowest root or greater than the highest root of Equation (3). Under Assumption 1, at $\mathcal{W}_2^f(a^f)$, $u(\hat{c}, n_{III}) \geq u(c_{IV}^f, 0)$ when $a < \bar{a}$, meaning that $\mathcal{W}_2^f(a^f)$ is in between the roots of Equation (3). As $\mathcal{W}_0^f(a^f) > \mathcal{W}_2^f(a^f)$, $\mathcal{W}_0^f(a^f)$ is above the highest root of Equation (3). We can then conclude that once the woman can have children in Regime I, choosing to live in this regime is not optimal (so we do not have to compare other indirect utilities to $u(c_I^f, n_I)$).

This means that when $a^f < \underline{a}$, women are either socially sterile [S], when $w^f \leq \mathcal{W}_0^f(a^f)$, or childless due to the high opportunity cost of having children [O], when $w^f > \mathcal{W}_0^f(a^f)$: once women become able to procreate ($w^f > \mathcal{W}_0^f(a^f)$), becoming a mother is not optimal.

Proof and Implications of 2.

- First, we show that, for $a^f \in [\underline{a}, \bar{a}[$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

Considering Equation (4), when $w^f = \mathcal{W}_2^f(a^f)$, we have

$$\begin{aligned} \text{LHS}(\mathcal{W}_2^f(a^f)) &= \frac{2(1 - \delta^f)\hat{c} + (a^f - \mu)\phi(\nu - \eta)}{1 - \delta^f + \phi(\nu - \eta)} \nu \quad \text{and} \\ \text{RHS}(\mathcal{W}_2^f(a^f)) &= \frac{\hat{c}^2}{2\hat{c} - a^f + \mu} \frac{1 - \delta^f + \phi(\nu - \eta)}{\phi}. \end{aligned}$$

When $a^f \in [\underline{a}, \bar{a}]$, $\text{RHS}(\mathcal{W}_2^f(a^f)) > \text{LHS}(\mathcal{W}_2^f(a^f))$ is satisfied under Assumption 1. This ensures that $\mathcal{W}_2^f(a^f)$ is in between the two roots that solve Equation (3). As $\mathcal{W}_1^f(a^f)$ is the smallest root of Equation (3) (Definition 1), $\mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

$\mathcal{W}_0^f(a^f) < \mathcal{W}_2^f(a^f)$ when $a^f > \underline{a}$ and the Inequality (4) is not satisfied for $\mathcal{W}_0^f(a^f)$. So $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

We can then conclude that, under Assumption 1, $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[$, $u(c_{\text{IV}}^f, 0) > u(\hat{c}, n_{\text{III}})$ and that $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[$, $u(\hat{c}, n_{\text{III}}) \geq u(c_{\text{IV}}^f, 0)$.

- Second, we show that when $a \in [\underline{a}, \bar{a}]$, $\mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

At $\mathcal{W}_2^f(a^f)$, $u(\hat{c}, n_{\text{III}}) = u(c_1^f, n_1)$ and Regime I is feasible. So, $\forall w^f > \mathcal{W}_2^f(a^f)$, $u(c_1^f, n_1) > u(\hat{c}, n_{\text{III}})$. At $\mathcal{W}_2^f(a^f)$, $u(\hat{c}, n_{\text{III}}) \geq u(c_{\text{IV}}^f, 0)$. So, we can conclude that $u(c_1^f, n_1) \geq u(c_{\text{IV}}^f, 0)$ at $\mathcal{W}_2^f(a^f)$.

Regime I exists for $w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_5^f(a^f)[$. For all $w^f > \mathcal{W}_5^f(a^f)$, $u(c_1^f, n_1)$ is not defined since $\partial n_1 / \partial w^f < 0$, so n_1 would be negative. Then, for $w^f > \mathcal{W}_5^f(a^f)$, Regime IV prevails. Let us compare the utility in Regime IV with the utility in Regime I when $w^f \leq \mathcal{W}_5^f(a^f)$. $u(c_{\text{IV}}^f, 0) \geq u(c_1^f, n_1)$ if and only if,

$$\ln \frac{(1 - \delta^f)w^f + a^f - \mu}{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu} + \ln \nu \geq \ln \frac{w^f(1 - \delta^f + \phi(\nu - \eta)) + a^f - \mu}{\phi w^f} - 2 \ln 2. \quad (5)$$

Considering Equation (5), at $w^f = 0$, the LHS is equal to $\ln \nu$ and the RHS goes to $+\infty$. The limits of the LHS and the RHS at $+\infty$ are respectively,

$$\ln \frac{1}{1 + \phi(\nu - \eta)} + \ln \nu \quad \text{and} \quad \ln \frac{1 + \phi(\nu - \eta)}{\phi} - 2 \ln 2.$$

This implies that the RHS is above the LHS for low values of w^f but as we cannot rank the two limits, the RHS can be above or below the LHS for large values of w^f . As both sides of the inequality are strictly decreasing and convex with w^f , the LHS can be equal to the RHS for either one or two values of w^f . At $w^f = \mathcal{W}_5^f(a^f)$, the LHS is strictly larger than the RHS, implying that $u(c_{\text{IV}}^f, 0) > u(c_1^f, n_1)$. This implies that $\mathcal{W}_5^f(a^f)$ is either in between the roots of Equation (5) holding at equality or at the right of the only root. Since $u(c_1^f, n_1)$ is not defined for $w^f \geq \mathcal{W}_5^f(a^f)$, the relevant root of LHS=RHS, $\mathcal{W}_4^f(a^f)$, is strictly lower than $\mathcal{W}_5^f(a^f)$. This proves that $u(c_{\text{IV}}^f, 0) < u(c_1^f, n_1)$ for $w^f < \mathcal{W}_4^f(a^f)$, and that $u(c_{\text{IV}}^f, 0) \geq u(c_1^f, n_1)$ for $w^f \in [\mathcal{W}_4^f(a^f), \mathcal{W}_5^f(a^f)]$. Intuitively, because there is a fixed cost to becoming a parent, the optimal fertility is not continuous in w^f .

We showed that $\forall w^f < \mathcal{W}_4^f(a^f)$, $u(c_1^f, n_1) > u(c_{\text{IV}}^f, 0)$ and that at $\mathcal{W}_2^f(a^f)$, Regime I exists and $u(c_1^f, n_1) > u(c_{\text{IV}}^f, 0)$. This implies that $\mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f)$.

Regime V is not reachable when $a^f < \bar{a}$ since the consumption in Regime V is equal to $a^f - \mu$, which does not allow the woman to reach the minimal consumption level allowing her to procreate.

We have proved that, when $a \in [\underline{a}, \bar{a}]$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$ under Assumption 1. We can then conclude that when $a \in [\underline{a}, \bar{a}]$,

- $\forall w^f < \mathcal{W}_0^f(a^f), c^f = c_{\text{II}}^f, n = 0,$
- $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[, u(c_{\text{IV}}^f, 0) > u(\hat{c}, n_{\text{III}}) \text{ and } c^f = c_{\text{IV}}^f, n = 0,$
- $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[, u(\hat{c}, n_{\text{III}}) \geq u(c_{\text{IV}}^f, 0) \text{ and } c^f = \hat{c}, n = n_{\text{III}},$
- $\forall w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_4^f(a^f)[, u(c_1^f, n_1) > u(c_{\text{IV}}^f, 0) \text{ and } c^f = c_1^f, n = n_1,$
- $\forall w^f \geq \mathcal{W}_4^f(a^f), c^f = c_{\text{IV}}^f, n = 0.$

Proof and Implications of 3.

We are now in the case $a^f \geq \bar{a}$, where Regime V is feasible for all $w^f \geq 0$. Furthermore, Regimes II and III no longer exist since even with a wage equal to zero, a woman can consume more than $\hat{c}$. We then just need to compare the utilities in Regimes I, IV and V.

We can check that $n_1 > \underline{n}_M \iff w^f < \mathcal{W}_3^f(a^f)$. $\forall w^f < \mathcal{W}_3^f(a^f)$, $u(c_1^f, n_1)$ is not defined as n_1 would be above the possible maximum.

Let us now show that $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$. We already know that both the LHS and the RHS of Equation (5) are strictly decreasing and convex in w^f . We can check that at $\mathcal{W}_3^f(a^f)$ the LHS is lower than the RHS. It follows from Definition 1 that $\mathcal{W}_3^f(a^f) < \mathcal{W}_5^f(a^f)$. Since at $\mathcal{W}_4^f(a^f)$ the LHS is equal to the RHS, then $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

At $\mathcal{W}_3^f(a^f)$, we can show that $u(c_V^f, \underline{n}_M) = u(c_1^f, n_1) > u(c_{\text{IV}}^f, 0)$. As $u(c_1^f, n_1)$ is increasing in w^f and $u(c_V^f, \underline{n}_M)$ is unaffected by w^f , we have that $u(c_1^f, n_1) \geq u(c_V^f, \underline{n}_M)$ for $w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_5^f(a^f)]$. As $u(c_{\text{IV}}^f, 0)$ is increasing in w^f and $u(c_V^f, \underline{n}_M)$ is unaffected by w^f , we also have that $u(c_V^f, \underline{n}_M) > u(c_{\text{IV}}^f, 0) \forall w^f < \mathcal{W}_3^f(a^f)$.

Given that $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$, we can conclude that:

- when $w^f < \mathcal{W}_3^f(a^f)$, $c^f = c_V$ and $n = \underline{n}_M$,
- when $w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_4^f(a^f)[, u(c_{\text{IV}}^f, 0) \geq u(c_1^f, n_1) > u(c_V^f, \underline{n}_M)$, which implies that $c^f = c_1^f$ and $n = n_1$,
- when $w^f > \mathcal{W}_4^f(a^f)$, $u(c_{\text{IV}}^f, 0) > u(c_V^f, \underline{n}_M)$ and $c^f = c_V^f$ and $n = 0$.

■

The following paragraphs describe the main results of this proposition.

Figure 13 represents the relationship between the fertility of a single fecund woman and her wage, when $a^f \geq \underline{a}$. The left panel shows a non-monotonic relationship between wage and fertility. The net non-labor income ($a^f - \mu$) is still not large enough to cover alone the minimal amount of consumption needed to procreate. For a low w^f , Regime III prevails and an increase in the wage of the woman increases her fertility. As her consumption is fixed at level $\hat{c}$ in this regime, fertility is positively related to her wage until it is high enough to reach Regime I. In Regime I, n_1 decreases with w^f because the substitution effect dominates ($a^f \geq \underline{a} > \mu$).

Figure 13: Fertility Conditional on Being Single when $a^f \in [\underline{a}, \bar{a}]$ and when $a^f \geq \bar{a}$

In the right panel, non-labor income is high enough to ensure a consumption $\hat{c}$ for any fertility level, even in the absence of labor income. As a consequence, Regime III disappears, and Regime V becomes possible. Regime V is a corner solution where the wage of the woman is so small that it is optimal for her to specialize in childrearing and consume her net non-labor income $a^f - \mu$.

On the whole, Figure 13 shows that uneducated single women will either be socially sterile or in the maximum fertility case, this will depend on their non-labor income. Conversely, highly educated women will be childless (Regime IV) because the opportunity cost of having children (non-participation to the workforce) is high. This figure also shows another feature: the existence of a fixed cost of becoming a parent implies that the passage from childlessness to parenthood is not represented by a continuous function. The fixed cost makes it never optimal to have a very low number of children.

B.3 Optimal Decisions of Couples

We now describe the optimal decisions of couples and show that for some values of the model's parameters, the relationship between fertility and the wage of women is rightly described in Section III. Notice that all the cases described in this section exist for $w^m > 0$ but sufficiently small to ensure that all the thresholds on a are positive and for $\theta(w^f, w^m)$ fixed to $\hat{\theta}$. At the end of this subsection, we show that our proof still holds in the cases where w^m becomes high. Indeed, in these cases, situations of poverty disappear and things become much easier. We also evoke how our results are changed when θ is not fixed but depends on w^f and w^m .

Definitions for wage and non-labor income thresholds are given in Definitions 3 and 4 respectively. These thresholds are used to describe the optimal decisions of couples as a function of wages and non-labor incomes, formally stated in Proposition 2. Results are valid under Assumption 2.

Definition 3 (Wage thresholds for couples)

$$\begin{aligned}
\mathcal{W}_A^f(a, w^m) &= \frac{\hat{c} - a - (1 - \phi(1 - \alpha)\eta)w^m}{1 - \alpha\phi\eta}, \\
\mathcal{W}_B^f(a, w^m) &= \frac{\hat{c} - a + \left(\phi(1 - \alpha)((1 - \hat{\theta})\nu - \eta) - 1\right)w^m}{1 - \alpha\phi(\eta + \nu(1 - \hat{\theta}))}, \\
\mathcal{W}_C^f(a, w^m) &= \frac{\Xi_1 + \left(\Xi_1^2 + 4\hat{\theta}\alpha(1 + \phi\alpha(\nu - \eta))\Xi_2\right)^{\frac{1}{2}}}{2\hat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))}, \\
\mathcal{W}_D^f(a, w^m) &= \frac{\frac{2\hat{c}}{\hat{\theta}} - a - (1 + \phi(1 - \alpha)(\nu - \eta))w^m}{1 + \alpha\phi(\nu - \eta)}, \\
\mathcal{W}_E^f(a, w^m) &= \frac{a - \hat{c} + \left(1 - (1 - \hat{\theta})\phi(1 - \alpha)(\nu - \eta) - \frac{1-\alpha}{\alpha}(2 - \hat{\theta})\right)w^m}{(1 - \hat{\theta})(1 + \alpha\phi(\nu - \eta))}, \\
\mathcal{W}_F^f(a, w^m) &= \frac{a + \left(\frac{2-3\alpha}{\alpha} + \phi(1 - \alpha)(\nu - \eta)\right)w^m}{1 + \alpha\phi(\nu - \eta)}, \\
\mathcal{W}_G^f(a, w^m) &= \left(\frac{\hat{c}}{\hat{\theta}}\right)^{\hat{\theta}} \left(\frac{\frac{2\alpha-1}{\alpha}w^m + a - \hat{c}}{1 - \hat{\theta}}\right)^{1-\hat{\theta}} \frac{1 + \alpha\phi(\nu - \eta)}{\alpha\phi\nu} - a - w^m \\
\mathcal{W}_I^f(a, w^m) &= \frac{1 - \phi\alpha\eta}{\phi\alpha\nu}a + \left(\frac{(2\alpha - 1)(1 + \phi\alpha(\nu - \eta))}{\alpha^2\phi\nu} - 1\right)w^m, \\
\mathcal{W}_J^f(a, w^m) &= \frac{\Xi_3}{[1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu}, \\
\mathcal{W}_{\underline{J}}^f(a, w^m) &= \frac{-\Xi_3 + (\Xi_3^2 - 4([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu)\Xi_4)^{\frac{1}{2}}}{2([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu)}, \\
\mathcal{W}_{\bar{J}}^f(a, w^m) &= \frac{-\Xi_3 - (\Xi_3^2 - 4([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu)\Xi_4)^{\frac{1}{2}}}{2([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu)}
\end{aligned}$$

where

$$\begin{aligned}
\Xi_1 &\equiv (1 - \hat{\theta})(1 + \phi\alpha(\nu - \eta))(w^m + \alpha a) - 2\alpha(a - \hat{c} + (1 + \phi(1 - \alpha)(\nu - \eta))w^m), \\
\Xi_2 &\equiv (2 - \hat{\theta})(1 + \phi\alpha(\nu - \eta))(1 - \alpha)w^m(a + w^m) \\
&\quad - (a + \alpha w^m)(a - \hat{c} + (1 + \phi(1 - \alpha)(\nu - \eta))w^m), \\
\Xi_3 &\equiv 2(w^m[1 - \phi(\nu + \eta) + \phi\alpha(1 - \alpha)(\nu - \eta)^2] + a[1 - \phi\alpha(\nu + \eta)]), \\
\text{and } \Xi_4 &\equiv (a + (1 + \phi(1 - \alpha)[\nu - \eta])w^m)^2 - 4\phi\nu(1 - \alpha)(w^m + a)w^m.
\end{aligned}$$

We finally denote as $\mathcal{W}_{\underline{C}}^f(a, w^m) \leq \mathcal{W}_{\bar{C}}^f(a, w^m)$ the two potential roots of the equation

$$U(\hat{c}, c_{VIII}^m, n_{VIII}) = U(c_{IX}^f, c_{IX}^m, 0)$$

$\mathcal{W}_A^f(a, w^m)$ is such that the couple can afford positive fertility ($n \rightarrow 0$, $c^f = \hat{c}$ and $c^m = 0$). $\mathcal{W}_B^f(a, w^m)$ is such that $n_{VIII} = 0$. $\mathcal{W}_C^f(a, w^m)$ is the value of w^f that maximizes

$U(c_{\text{VIII}}^m, c_{\text{VIII}}^f, n_{\text{VIII}}) - U(c_{\text{IX}}^m, c_{\text{IX}}^f, 0)$. $\mathcal{W}_D^f(a, w^m)$ is such that $n_{\text{VIII}} = n_{\text{VI}}$. $\mathcal{W}_E^f(a, w^m)$ is such that $n_{\text{VIII}} = \bar{n}_M$. $\mathcal{W}_F^f$ is such that $n_{\text{VI}} = \bar{n}_M$. $\mathcal{W}_G^f$ is the unique root of $U(c_{\text{X}}^f, \hat{c}, \bar{n}_M) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0) = 0$. $\mathcal{W}_I^f(a, w^m)$ is such that $U(c_{\text{XI}}^m, c_{\text{XI}}^f, \bar{n}_M) = U(c_{\text{IX}}^m, c_{\text{IX}}^f, 0)$. Finally, $\mathcal{W}_J^f(a, w^m)$ is the unique maximum of $U(c_{\text{VI}}^m, c_{\text{VI}}^f, n_{\text{VI}}) - U(c_{\text{IX}}^m, c_{\text{IX}}^f, 0)$ while $\mathcal{W}_{\underline{J}}^f(a, w^m)$ and $\mathcal{W}_{\bar{J}}^f(a, w^m)$ are the lowest and the highest roots of the equation $U(c_{\text{VI}}^m, c_{\text{VI}}^f, n_{\text{VI}}) = U(c_{\text{IX}}^m, c_{\text{IX}}^f, 0)$ respectively.

For ease of notation, we will denote $\mathcal{W}_\kappa^f(a, w^m)$ (for $\kappa = A, B, \underline{C}, C, \bar{C}, D, E, F, G, I, \underline{J}, J, \bar{J}$) as $\mathcal{W}_\kappa^f$ and use the complete notation only when necessary.

Definition 4 (Non-labor income thresholds for couples)

A_0 is the value of a that solves $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) = U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ when $w^f = \mathcal{W}_G^f$.

A_1 is the value of a that solves $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VII}}) = U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ when $w^f = \mathcal{W}_D^f$.

$A_2 = \hat{c} - (1 - \phi(1 - \alpha)\eta)w^m$ is the value of a above which a couple can have children even if the wife has no labor income (or has the maximal number of children).

A_3 is the lowest value of a such that $\mathcal{W}_G^f = 0$.

A_4 is the value of a such that $\mathcal{W}_G^f = \mathcal{W}_E^f$.

$A_5 = \frac{2\hat{c}}{\theta} - \frac{2\alpha-1}{\alpha}w^m$. Above A_5 , even when her wage is zero and her negotiation power is minimal, a woman can consume more than $\hat{c}$ even in the interior regime.

Assumption 2 Parameters $\alpha, \phi, \nu, \eta, \underline{\theta}$, and $\hat{c}$ satisfy:

- $(1 + \alpha\phi(\nu - \eta))^2 - 4\phi\alpha\nu < 0$, this condition ensures that the problem is not degenerated, in the sense that the interior regime is chosen for some values of w^f and a ,
- when $w^f = \mathcal{W}_D^f$,

$$\frac{\partial U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}})}{\partial w^f} = \frac{\partial U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})}{\partial w^f} - \varepsilon$$

with $\varepsilon \rightarrow 0^+$, this assumption excludes a degenerated case where, over a range of w^f , couples first choose the Regime VIII then Regime IX and then Regime VI.

- when $w^f = \mathcal{W}_C^f$,

$$\frac{\partial [U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, n_{\text{IX}})]}{\partial a} > 0, \forall a < A_4.$$

This assumption ensures that Regime VIII will be chosen for some triplets of (w^f, w^m, a) .

- $\mathcal{W}_G^f(A_2, w^m) < \mathcal{W}_E^f(A_2, w^m)$; furthermore, A_5 lies in-between the two roots of equation $\mathcal{W}_G^f = \mathcal{W}_E^f$ solved with respect to a .
- $\forall a < A_4$; $\mathcal{W}_J^f < \mathcal{W}_D^f$. Thus assumption ensures that for high wages, Regime IX is preferred to Regime VI.

Proposition 2 Under Assumption 2, the optimal choice of a couple with non-labor income a and wages w^f and w^m is given by:

1. When $a \leq A_0$:

- if $w^f < \mathcal{W}_A^f$, $c^f = c_{VII}^f$, $c^m = c_{VII}^m$, $n = 0$
- if $w^f \geq \mathcal{W}_A^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

2. When $a \in]A_0, A_1]$:

- if $w^f < \mathcal{W}_A^f$, $c^f = c_{VII}^f$, $c^m = c_{VII}^m$, $n = 0$
- if $w^f \in [\mathcal{W}_A^f, \mathcal{W}_{\underline{C}}^f[$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$
- if $w^f \in [\mathcal{W}_{\underline{C}}^f, \mathcal{W}_{\underline{C}}^f]$, $c^f = c_{VIII}^f$, $c^m = c_{VIII}^m$, $n = n_{VIII}$
- if $w^f > \mathcal{W}_{\underline{C}}^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

3. When $a \in]A_1, A_2]$:

- if $w^f < \mathcal{W}_A^f$, $c^f = c_{VII}^f$, $c^m = c_{VII}^m$, $n = 0$
- if $w^f \in [\mathcal{W}_A^f, \mathcal{W}_{\underline{C}}^f[$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$
- if $w^f \in [\mathcal{W}_{\underline{C}}^f, \mathcal{W}_D^f[$, $c^f = c_{VIII}^f$, $c^m = c_{VIII}^m$, $n = n_{VIII}$
- if $w^f \in [\mathcal{W}_D^f, \mathcal{W}_{\underline{J}}^f]$, $c^f = c_{VI}^f$, $c^m = c_{VI}^m$, $n = n_{VI}$
- if $w^f > \mathcal{W}_{\underline{J}}^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

4. When $a \in]A_2, A_3]$:

- if $w^f < \mathcal{W}_{\underline{C}}^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$
- if $w^f \in [\mathcal{W}_{\underline{C}}^f, \mathcal{W}_D^f[$, $c^f = c_{VIII}^f$, $c^m = c_{VIII}^m$, $n = n_{VIII}$
- if $w^f \in [\mathcal{W}_D^f, \mathcal{W}_{\underline{J}}^f]$, $c^f = c_{VI}^f$, $c^m = c_{VI}^m$, $n = n_{VI}$
- if $w^f > \mathcal{W}_{\underline{J}}^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

5. When $a \in]A_3, A_4]$:

- if $w^f \leq \mathcal{W}_G^f$, $c^f = c_X^f$, $c^m = c_X^m$, $n = \bar{n}_M$
- if $w^f \in]\mathcal{W}_G^f, \max\{\mathcal{W}_E^f, \mathcal{W}_{\underline{C}}^f\}[$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$
- if $w^f \in [\max\{\mathcal{W}_E^f, \mathcal{W}_{\underline{C}}^f\}, \mathcal{W}_D^f[$, $c^f = c_{VIII}^f$, $c^m = c_{VIII}^m$, $n = n_{VIII}$
- if $w^f \in [\mathcal{W}_D^f, \mathcal{W}_{\underline{J}}^f]$, $c^f = c_{VI}^f$, $c^m = c_{VI}^m$, $n = n_{VI}$
- if $w^f > \mathcal{W}_{\underline{J}}^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

6. When $a \in]A_4, A_5]$:

- if $w^f \leq \mathcal{W}_E^f$, $c^f = c_X^f$, $c^m = c_X^m$, $n = \bar{n}_M$
- if $w^f \in]\mathcal{W}_E^f, \mathcal{W}_D^f[$, $c^f = c_{VIII}^f$, $c^m = c_{VIII}^m$, $n = n_{VIII}$
- if $w^f \in [\mathcal{W}_D^f, \mathcal{W}_{\underline{J}}^f]$, $c^f = c_{VI}^f$, $c^m = c_{VI}^m$, $n = n_{VI}$

- if $w^f > \mathcal{W}_J^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

7. When $a > A_5$:

- if $w^f \leq \mathcal{W}_F^f$, $c^f = c_{XI}^f$, $c^m = c_{XI}^m$, $n = \bar{n}_M$
- if $w^f \in]\mathcal{W}_F^f, \mathcal{W}_J^f]$, $c^f = c_{VI}^f$, $c^m = c_{VI}^m$, $n = n_{VI}$
- if $w^f > \mathcal{W}_J^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$

Proof. We use a two-step strategy. First, we highlight some general properties of the model. Second, we prove each part of the proposition. Notice that all the cases exist only when w^m is sufficiently close to zero. Successive increases of w^m will make cases of poverty disappear. For instance, one can easily check that for high values of w^m , $\mathcal{W}_A^f$, $\mathcal{W}_B^f$, A_2 and A_5 become negative making the study of fertility behaviors of couples much easier.

Step 1: Some General Properties of the Model

In what follows, we assume w^m small enough to ensure that $A_0 > 0$.

- *Regimes VII and IX are “continuous” in the sense that $U(c_{VII}^f, c_{VII}^m, 0) = U(c_{IX}^f, c_{IX}^m, 0)$, $\forall w^f > 0$.* A couple is able to have children once its total income is greater than $\hat{c}$ which is satisfied when $w^f > \mathcal{W}_A^f$. This implies that $\forall w^f < \mathcal{W}_A^f$, childlessness is due to poverty, while it is due to a high opportunity cost of having children otherwise.
- *Regimes VIII and VI are “continuous”.* Regime VIII exists as long as $n_{VIII} \geq 0$ which is satisfied $\forall w^f \geq \mathcal{W}_B^f$. Regime VI exists once $c_{VI}^f \geq \hat{c}$ which is satisfied $\forall w^f \geq \mathcal{W}_D^f$. We can verify that when $w^f = \mathcal{W}_D^f$, $c_{VI}^f = \hat{c}$ and $n_{VI} = n_{VIII}$, implying that $U(\hat{c}, c_{VIII}^m, n_{VIII}) = U(c_{VI}^f, c_{VI}^m, n_{VI})$. It also implies that Regime VIII is defined $\forall w^f \in [\mathcal{W}_B^f, \mathcal{W}_D^f]$ while Regime VI is defined $\forall w^f \geq \mathcal{W}_D^f$ ($\lim_{w^f \rightarrow \infty} n_{VI} > 0$).
- Equation $U(c_{VI}^f, c_{VI}^m, n_{VI}) = U(c_{IX}^f, c_{IX}^m, 0)$ can be written as

$$([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu)(w^f)^2 - \Xi_3 w^f + \Xi_4 = 0. \quad (6)$$

As $[1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu < 0$, Equation (6) has, at most, two positive roots and admits a unique maximum at $w_f = \mathcal{W}_J^f$, then $\mathcal{W}_J^f > 0$ if $\Xi_3 > 0$.

- When $a^f \leq A_0$:
 - $\forall w^f > 0$, $U(c_{VII}^f, c_{VII}^m, 0) = U(c_{IX}^f, c_{IX}^m, 0) > U(\hat{c}, c_{VIII}^m, n_{VIII})$ and,
 - when $w^f = \mathcal{W}_D^f$, $U(c_{VI}^f, c_{VI}^m, n_{VI}) = U(\hat{c}, c_{VIII}^m, n_{VIII}) < U(c_{IX}^f, c_{IX}^m, 0)$.

$U(\hat{c}, c_{VIII}^m, n_{VIII}) - U(c_{IX}^m, c_{IX}^f, 0)$ is positive when,

$$(1 + \phi\alpha(\nu - \eta))w^f + (1 + \phi(1 - \alpha)(\nu - \eta))w^m + a - \hat{c} \geq \left(\frac{\hat{\theta}}{\hat{c}}\right)^{\frac{\hat{\theta}}{2-\hat{\theta}}} (2 - \hat{\theta}) [\nu\phi\alpha(w^f + w^m + a)(\alpha w^f + (1 - \alpha)w^m)]^{\frac{1}{2-\hat{\theta}}} \quad (7)$$

Since the LHS is linearly increasing with respect to w^f and the RHS is increasing and convex with w^f , we know that $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ admits a unique maximum, denoted $\mathcal{W}_C$, and at most two positive roots, denoted $\mathcal{W}_{\underline{C}}^f < \mathcal{W}_{\overline{C}}^f$. Furthermore, we can check that Inequality (7) is not satisfied when $w^f = 0$ and $a < \hat{c} - (1 + \phi(1 - \alpha)(\nu - \eta))w^m$.

Under Assumption 2, when $w^f = \mathcal{W}_C^f$, $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ is increasing in a , for $a < A_4$. This means that the maximum value of $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ increases monotonically with a . It implies that there is a value, $a = A_0$, where A_0 solves

$$(1 + \phi\alpha(\nu - \eta))\mathcal{W}_C^f(A_0, w^m) + (1 + \phi(1 - \alpha)(\nu - \eta))w^m + A_0 - \hat{c} = \left(\frac{\hat{\theta}}{\hat{c}}\right)^{\frac{\hat{\theta}}{2-\hat{\theta}}} (2 - \hat{\theta}) \left[\nu\phi\alpha(\mathcal{W}_C^f(A_0, w^m) + w^m + A_0)(\alpha\mathcal{W}_C^f(A_0, w^m) + (1 - \alpha)w^m) \right]^{\frac{1}{2-\hat{\theta}}}. \quad (8)$$

This means that when $a^f \leq A_0$, $U(c_{\text{VII}}^f, c_{\text{VII}}^m, 0) = U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0) > U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}})$, $\forall w^f \geq 0$. As $w^f = \mathcal{W}_D^f$ implies that $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) = U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}})$, we also find that $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, n_{\text{IX}})$ for $w^f = \mathcal{W}_D^f$ and $a \leq A_0$.

- $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$, $\forall w^f > \mathcal{W}_D^f$ and $a \leq A_1$. As already shown, Equation (6) is quadratic with respect to w^f . It admits a unique maximum for $w^f = \mathcal{W}_I^f < \mathcal{W}_D^f$ when $a \leq A_1$. Then $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ is decreasing $\forall w^f > \mathcal{W}_D^f$. Since $\forall a \leq A_1$ and $w^f = \mathcal{W}_D^f$, $U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0) > U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) = U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})$, then $\forall a \leq A_1$ and $\forall w^f > \mathcal{W}_D^f$, $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$.
- *Regime X is never preferred to Regime IX when $a < A_3$.* This is due to $\mathcal{W}_G^f < 0$ for $a < A_3$.
- *There exists a unique $A_4 > 0$ such that $\forall a \in [A_3, A_4]$, $\mathcal{W}_G^f \leq \mathcal{W}_E^f$ while $\forall a \in]A_4, A_5]$, $\mathcal{W}_G^f > \mathcal{W}_E^f$.* Equation $\mathcal{W}_G^f = \mathcal{W}_E^f$ can be written as

$$\begin{aligned} & \left(\frac{\hat{c}}{\hat{\theta}}\right)^{\hat{\theta}} \left(\frac{\frac{2\alpha-1}{\alpha}w^m + a - \hat{c}}{1 - \hat{\theta}}\right)^{1-\hat{\theta}} \frac{1 + \alpha\phi(\nu - \eta)}{\alpha\phi\nu} \\ &= a + w^m - \frac{a - \hat{c} + \left(1 - (1 - \hat{\theta})\phi(1 - \alpha)(\nu - \eta) - \frac{1-\alpha}{\alpha}(2 - \hat{\theta})\right) w^m}{(1 - \hat{\theta})(1 + \alpha\phi(\nu - \eta))}, \end{aligned} \quad (9)$$

where the LHS is increasing and concave with respect to a while the RHS is linearly increasing with a . It implies that the equation has at most two roots and as the LHS is only defined for $a > \hat{c} - \frac{2\alpha-1}{\alpha}w^m$, which is positive for sufficiently low values of w^m , these roots are strictly positive. Furthermore, under Assumption 2, $\mathcal{W}_G^f(A_2, w^m) < \mathcal{W}_E^f(A_2, w^m)$ and A_5 lies in between the two roots of Equation (9). It directly implies that there exists a unique A_4 such that $\mathcal{W}_G^f < \mathcal{W}_E^f$ when $a \in [A_3, A_4[$ and $\mathcal{W}_G^f \geq \mathcal{W}_E^f$ $\forall a \in [A_4, A_5]$.

Step 2: Proof of Each Part of Proposition 2

We still assume w^m low enough for A_0 to be positive. This will be relaxed afterwards.

Step 2.1: Cases where $a < A_2$

Regimes X and XI are not reachable because $c_X^m < 0$ and $c_{XI}^f < \hat{c}$ for $a^f < A_2$. So we only need to compare the utilities $U_{VII}(a, w^f)$, $U_{VIII}(a, w^f)$, $U_{VI}(a, w^f)$ and $U_{IX}(a, w^f)$ to see the outcome of a marriage.

Case 1: $a < A_0$: Here, we show that, as in the first case of Proposition 1, once agents with very low non-labor income are able to be parents, the opportunity cost of childrearing is too high for them to be willing to. $\mathcal{W}_A^f$, defined in Definition 3, solves:

$$(1 - \phi\alpha\eta)\mathcal{W}_A^f + 1 - \phi(1 - \alpha)\eta w^m + a = \hat{c}.$$

We can check that $\mathcal{W}_A^f > 0 \iff a < A_2$ and that $A_0 < A_2$. For $w^f < \mathcal{W}_A^f$, the consumption of a woman who wants to have children would be lower than $\hat{c}$ even if her husband does not consume. Consequently, for $w^f < \mathcal{W}_A^f$, the couple is socially sterile. For $w^f \geq \mathcal{W}_A^f$, the couple can decide whether to have children or to be childless in Regime IX.

When $a < A_0$, $U(c_{VIII}^f, c_{VIII}^m, n_{VIII}) < U(c_{IX}^f, c_{IX}^m, 0), \forall w^f$. By continuity we know that $U(c_{VI}^f, c_{VI}^m, n_{VI})$ is lower than $U(c_{IX}^f, c_{IX}^m, 0)$ for $w^f = \mathcal{W}_D^f$. As shown in Step 1 of the proof, when $a < A_1$, $U(c_{VI}^f, c_{VI}^m, n_{VI}) - U(c_{IX}^f, c_{IX}^m, 0)$ decreases with w^f which directly implies that $\forall w^f > \mathcal{W}_D^f$, $U(c_{VI}^f, c_{VI}^m, n_{VI}) < U(c_{IX}^f, c_{IX}^m, 0)$.

We can then conclude that if $w^f < \mathcal{W}_A^f$, $c^f = c_{VII}^f$, $c^m = c_{VII}^m$, $n = 0$, while if $w^f \geq \mathcal{W}_A^f$, $c^f = c_{IX}^f$, $c^m = c_{IX}^m$, $n = 0$.

Case 2: $a \in [A_0, A_1]$: In this case, we will show that Malthusian mechanisms, where fertility is related positively to female's wages, can be at play among some poor couples.

As $A_1 < A_2$, $\mathcal{W}_A^f > 0$ and couples are socially sterile when $w^f < \mathcal{W}_A^f$. Furthermore,

$$\lim_{w^f \rightarrow \mathcal{W}_A^f} U(c_{VIII}^f, c_{VIII}^m, n_{VIII}) = -\infty$$

because $c_{VIII}^m \rightarrow 0$ when $w^f \rightarrow \mathcal{W}_A^f$. $U(c_{VIII}^f, c_{VIII}^m, n_{VIII})$ is then lower than $U(c_{IX}^f, c_{IX}^m, 0)$ in the neighborhood of $\mathcal{W}_A^f$.

When $a > A_0$, the difference $U(c_{VIII}^f, c_{VIII}^m, n_{VIII}) - U(c_{IX}^f, c_{IX}^m, 0)$ admits two positive roots, denoted $\mathcal{W}_C^f$ and $\mathcal{W}_D^f$, and is positive between them. As $a < A_1$, when $w^f \geq \mathcal{W}_D^f$, $U(c_{VIII}^f, c_{VIII}^m, n_{VIII}) = U(c_{VI}^f, c_{VI}^m, n_{VI}) < U(c_{IX}^f, c_{IX}^m, 0)$. This means that once the unconstrained fertility regime can be reached by the couple, having children is no longer optimal.

We conclude that for $a \in [A_0, A_1]$, couples are in Regime VII for $w^f < \mathcal{W}_A^f$, in Regime IX for $w^f \in]\mathcal{W}_A^f, \mathcal{W}_C^f]$, in Regime VIII for $w^f \in]\mathcal{W}_C^f, \mathcal{W}_D^f]$ and in Regime IX again for $w^f > \mathcal{W}_D^f$.

Case 3: $a \in [A_1, A_2]$: In this case, we can show that for intermediary values of non-labor incomes, a couple can either be in a Malthusian or a modern fertility pattern, where fertility decreases with the mothers' wage.

As $a \leq A_2$, $\mathcal{W}_A^f \geq 0$ and couples are socially sterile for $w^f < \mathcal{W}_A^f$. Using the same reasoning as for the previous case,

$$\lim_{w^f \rightarrow \mathcal{W}_A^f} U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) = -\infty$$

and $U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ in the neighborhood of $\mathcal{W}_A^f$.

Since $a > A_1$, $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})$ is higher than $U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ for $w^f = \mathcal{W}_D^f$. $\mathcal{W}_D^f$ then lies in between the roots of Equation (6), denoted by $\mathcal{W}_J^f$ and $\mathcal{W}_C^f$. Furthermore, as $U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) = U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})$ for $w^f = \mathcal{W}_D^f$, we know that $\mathcal{W}_D^f$ also lies in between $\mathcal{W}_C^f$ and $\mathcal{W}_J^f$.

We conclude that for $a \in]A_1, A_2]$, couples are in Regime VII when $w^f < \mathcal{W}_A^f$, in Regime IX when $w^f \in]\mathcal{W}_A^f, \mathcal{W}_C^f]$, in Regime VIII when $w^f \in]\mathcal{W}_C^f, \mathcal{W}_D^f]$, in Regime VI when $w^f \in]\mathcal{W}_D^f, \mathcal{W}_J^f]$ and in Regime IX again when $w^f > \mathcal{W}_J^f$.

Step 2.2: Cases where $a \geq A_2 = \hat{c}$

For $a > A_2$, Regime VII no longer exists: a couple can procreate for any $w^f \geq 0$. Regimes X and XI become reachable.

Case 4: $a \in]A_2, A_3[$:

We will show that in this case, having the maximum fertility is not yet optimal.

Here, $\mathcal{W}_G^f < 0$. As long as $w^f < \mathcal{W}_D^f$, only Regimes IX, VIII and X can be reached. We then have to compare Regimes VIII and IX for $w^f < \mathcal{W}_D^f$. As $a > A_0$, $U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) \geq U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ for $w^f \in [\mathcal{W}_C^f, \mathcal{W}_D^f]$. This implies that Regime IX prevails $\forall w^f \in [0, \mathcal{W}_C^f]$.

As shown above, if $a > A_1$, then $\mathcal{W}_D^f \in [\min\{\mathcal{W}_C^f, \mathcal{W}_J^f\}, \max\{\mathcal{W}_C^f, \mathcal{W}_J^f\}]$. This implies that Regime VIII prevails $\forall w^f \in [\mathcal{W}_C^f, \mathcal{W}_D^f[$ while Regime VI prevails $\forall w^f \in [\mathcal{W}_D^f, \mathcal{W}_J^f[$. Finally, Regime IX prevails when $w^f \geq \mathcal{W}_J^f$.

Case 5: $a \in]A_3, A_4[$:

For these values of a , having the maximum number of children is feasible only in Regime X. This behavior will be optimal only for very low values of w^f .

In line with previous cases and Step 1, under Assumption 2, we know that, for $a \in]A_3, A_4[$ $\mathcal{W}_G^f < \max\{\mathcal{W}_C^f, \mathcal{W}_E^f\} < \mathcal{W}_D^f < \mathcal{W}_J^f$. We can conclude that Regime X prevails when $w^f < \mathcal{W}_G^f$ while Regime IX prevails when $w^f \in]\mathcal{W}_G^f, \max\{\mathcal{W}_C^f, \mathcal{W}_E^f\}[$. When $w^f \in [\max\{\mathcal{W}_C^f, \mathcal{W}_E^f\}, \mathcal{W}_D^f[$, Regime VIII is chosen by the couple while Regime VI is preferred to others when $w^f \in [\mathcal{W}_D^f, \mathcal{W}_J^f]$. Finally, for $w^f > \mathcal{W}_J^f$, Regime IX prevails again.

Case 6: $a \in]A_4, A_5[$: The only difference between Case 6 and Case 5 lies in the fact that for $a \in]A_4, A_5[$, $\mathcal{W}_G^f > \mathcal{W}_E^f$. Therefore, we have that $\mathcal{W}_E^f < \mathcal{W}_C^f < \mathcal{W}_D^f < \mathcal{W}_J^f$. The results of Case 6 directly follow.

Case 7: $a > A_5$: When $a > A_5$, the constrained regimes [C] (Regime VIII and X) no longer exist. Indeed, a woman with a wage equal to zero (or equivalently, having the maximum

number of children) would consume more than $\hat{c}$. This implies that only Regimes VI, IX and XI potentially exist. Under Assumption 2, $\mathcal{W}_F^f < \mathcal{W}_I^f < \mathcal{W}_J^f$. One can easily check that $\mathcal{W}_F^f(a, 0) < \mathcal{W}_I^f(a, 0)$; intuitively, higher w^m will reinforce this inequality as higher w^m means a smaller opportunity cost for the wife to fully specialize in labor market activities.

For $w^f < \mathcal{W}_F^f$, $U(c_{XI}^f, c_{XI}^m, n_{XI}) > U(c_{IX}^f, c_{IX}^m, n_{IX})$. By continuity, we know that for $w^f = \mathcal{W}_F^f$, $U(c_{VI}^f, c_{VI}^m, n_{VI}) > U(c_{IX}^f, c_{IX}^m, n_{IX})$ (which is why we know that $\mathcal{W}_F^f < \mathcal{W}_J^f$). This implies that for $w^f \in [\mathcal{W}_F^f, \mathcal{W}_J^f]$, Regime VI prevails, while for $w^f > \mathcal{W}_J^f$, Regime IX prevails.

■

Relaxing assumptions on w^m and θ :

As explained at the beginning of the proof, all the seven cases we have studied exist for $\theta(w^f, w^m)$ fixed to $\hat{\theta}$ and when $w^m > 0$ but sufficiently small to ensure that all the thresholds on a are positive. Keeping $\theta(w^f, w^m) = \hat{\theta}$, successive increases of w^m will make some cases disappear. For instance, if a couple enjoy a $w^m > \frac{\hat{c}}{1-\phi\eta(1-\alpha)}$, A_2 becomes negative and this couple will never be concerned with social sterility. This is due to a simple income effect: the higher the income of the husband, the more the couple is protected against situations of childlessness or constrained fertility due to poverty. In the extreme case where $w^m \geq \frac{2\alpha}{(2\alpha-1)\underline{\theta}}\hat{c}$, only Regime VI exists and regimes of fertility with Malthusian properties disappear.

The present proof does not necessarily hold when the assumption on $\theta(w^f, w^m)$ fixed is relaxed. Indeed, equations involving θ (like Equations (7) to (9)) would not behave in the same way (for instance, their left or right hand sides would not be strictly concave or convex). This could induce the existence of more thresholds on a and w^f . Nevertheless, intuitively, we can say that for $\underline{\theta} \rightarrow 1$, our results hold as θ would vary only weakly around one half.

We propose in the next subsection to verify if the results of our proposition hold for the values of parameters provided in Table 3. We can see that despite the estimated value of $\underline{\theta}$ is not close to one, our proposition still holds. In a weak sense, this allows us to affirm that there exists a non empty set of parameters of the model for which our proposition holds even if θ is not constant.

Interpretations:

Figures 14, 15 and 16 and following paragraphs describe the main results of the proposition. For very low non-labor income ($a < A_0$) and, low w^m , a couple will be childless whatever the wife's wage. The wage allowing the couple to have children is, in such a case, higher than the wage for which they would choose to be childless (Regime IX).

For higher levels of non-labor income ($a > A_0$), having children becomes feasible and optimal for some wages. Figure 14 (left) shows that, when $a \in]A_0, A_1]$, a couple can have children in the constrained fertility regime (Regime VIII) but the unconstrained regime (Regime VI) is never optimal. In Regime VIII, the husband reduces his consumption in order to enable his wife to have children. As shown in Figure 14 (right), Regime VI becomes optimal for some wages and $a > A_1$. In Figure 14, the substitution effect is dominated in the constrained fertility regime and an increase in w^f increases fertility.

Figure 14: Fertility Conditional on Being Married when $a \in]A_0, A_1]$ and when $a \in]A_1, A_2]$

Figure 15: Fertility Conditional on Being Married when $a \in]A_2, A_3]$ and when $a \in]A_3, A_4]$

Figure 16: Fertility Conditional on Being Married when $a \in]A_4, A_5]$ and when $a > A_5$

$\mathcal{W}_A^f$ corresponds to the wage allowing couples to procreate. For a wage greater than but close to $\mathcal{W}_A^f$, the consumption of the husband is close to zero, implying that parenthood is not optimal. This explains why Regime IX always precedes Regime VIII in Figure 14 and Figure 15 (left).

It can be shown that $\mathcal{W}_D^f$ decreases with a . This implies that higher levels of non-labor income move the fertility hump in Figure 14 leftwards. This means that a couple with higher non-labor income can reach the unconstrained fertility regime (Regime VI) for smaller w^f .⁸

In Figures 15 and 16, $a > \hat{c} - (1 - \phi(1 - \alpha)\eta)w^m$. Social sterility disappears as a couple can always ensure a consumption level of $\hat{c}$ to the wife. It also implies that fertility decreases with w^f in Regime VIII because the substitution effect dominates. When $a \in]A_3, A_4]$ and w^f is low, a couple has to choose between two extreme cases: Regime X where the wife specializes in child rearing and both spouses have a low consumption level; and Regime IX where she specializes in labor market activities and is childless. In other words, the couple has to choose between having a maximum of children but lacking of consumption goods and consuming more but lacking of children.⁹ When w_f is very low, the surplus from consumption that could come from her labor market activity is also very low and the couple chooses to have the maximal number of children. When her wage becomes greater than $\mathcal{W}_G^f$, the surplus from consumption provides a higher marginal utility than the surplus from children. Then, the couple becomes childless in Regime IX. Finally, for higher wages, Regime VIII dominates the previous two.

Regimes X and XI, where the wife has the highest fertility appear in Figures 15 (right) and 16, for low wages. In Regime X, the wife consumes $\hat{c}$ and specializes in childrearing. This regime becomes feasible once $a > A_2$, but is optimal for some wages only when $a \geq A_3$ (when $a < A_3$ the consumption of the husband is too low). Once w^f becomes sufficiently high, the wife still consumes $\hat{c}$ but spends a part of her time on labor market activities. The additional income earned by the couple is allocated to the consumption of the husband. Finally, Figure 16 (right) exhibits the same fertility pattern as Figure 13 for a single woman and the same intuitions apply.

Poor couples will then be either childless or in a regime where the wife has the highest fertility. In all cases, couples in which the wife is highly educated choose to remain childless.

B.4 Fertility as a Function of w^f , a and w^m (married)

Figure 17 allows to see how the various 2d plots of the main text for married women are slices for a given level of a of a 3d picture. The parameters are set equal to those estimated in Subsection IV.B. Three different levels of w^m are shown.

⁸Using as a proxy for wealth the fact of owning or renting the dwelling, we observe that those who rent the housing unit have larger childlessness rates, for all levels of education. This is supported by Figure 14. We can also observe that the decreasing relationship between completed fertility of mothers and education moves leftwards. This is also in line with Figures 14, 15 and 16 as $\mathcal{W}_2^f$ decreases with a .

⁹When $w^f \in [0, \mathcal{W}_G^f[$, Regime VIII is always dominated by at least one of these two extreme regimes.

Figure 17: Fertility for different levels of mother's education (w^f) and non-labor income (a). Father with no school (left), Grade 12 (middle), Doctoral degree (right)

B.5 Marriage Decisions

We now turn to the first stage of the game and consider the decision to accept a marriage offer from a randomly drawn person of the opposite sex. Once this potential partner has been drawn, we know the vector (w^f, a^f, w^m, a^m) . To understand how potential spouses decide to accept or reject a marriage offer, we first analyze their behavior in a fictive situation where they are sure their partner is not naturally sterile. Then, we describe how the risk of natural sterility modifies these behaviors.

Behaviors without natural sterility

Notice that here, we are able to provide some elements to highlight this choice but we cannot perform a complete inspection as comparing indirect utilities in all the cases would not be analytically tractable.

Let us first consider the potential bride. From Propositions 1 and 2 we know the two relevant regimes she has to compare. When the potential bride has both a low wage and a low non-labor income, she could either be in Regime II or in Regime III if she remains single. If the relevant regime for the woman when single is Regime II, then she always prefers to be married with a husband allowing her to consume at least $\hat{c}$, as:

$$u(c_{II}^f, 0) < \min\{u(\hat{c}, n_{VIII}), u(c_{VII}^f, n_{VII}), u(c_{IX}^f, 0), u(\hat{c}, \bar{n}_M), u(c_{XI}^f, \bar{n}_M)\}. \quad (10)$$

She will reject a marriage offer only if she is in Regime VII if married and if $u(c_{II}^f, 0) > u(c_{VII}^f, 0)$, that is:

$$(1 - \delta^f)w^f + a^f - \mu > \theta(w^f, w^m)(w^m + w^f + a). \quad (11)$$

The only motive for a very poor woman to marry a man who does not provide enough income to allow her to procreate lies in the scale economies, in goods and in time, allowed by marriage. Moreover, an uneducated woman will accept to marry an uneducated man because similar education ensures that negotiation powers remain close to one half which allows the surplus from marriage to be shared equally.

If the relevant regime for a woman when single is Regime III, she always prefers to be a single mother who consumes $\hat{c}$ rather than a childless married woman who consumes less than $\hat{c}$ (i.e. $u(\hat{c}, n_{\text{III}}) > u(c_{\text{VII}}^f, 0)$). Hence, she would always reject marriage offers leading to Regime VII. Moreover, $u(\hat{c}, n_{\text{III}}) < \min\{u(\hat{c}, \bar{n}_{\text{M}}), u(c_{\text{XI}}^f, \bar{n}_{\text{M}})\}$, since in Regimes X and XI the woman would consume at least $\hat{c}$ and would have the maximum number of children. So she would always accept marriage offers leading to Regimes X and XI.

For higher w^f and a^f , Regimes I, IV or V are accessible as a single person. When a woman earns high wages, Regime IV does not dominate in only two circumstances: (a) when the additional income coming from the man is not high enough to incite the woman to become a mother but sufficiently close to her own income for the marriage surplus to be shared (Regime IX) and (b) when the man's wage is high enough to incite her to become a mother (Regime VI or XI).

We can conclude that, unlike highly educated women, lowly educated women will accept almost any match on the marriage market. This highlights the role of marriage as an institution protecting women against poverty, social sterility and even against living in a Malthusian like regime. Highly educated women do not need to be protected against social sterility by marriage. On the contrary, for highly educated women, being matched with highly educated men is the occasion to have children rather than being single and childless because marriage reduces the opportunity cost of motherhood.

As being single is not only a personal choice, let us now turn to the problem of the potential husband. A single man has the following indirect utility,

$$\bar{u}^m = \ln((1 - \delta^m)w^m + a^m - \mu) + \ln \nu.$$

Getting married is the only way for him to become a father but entails two costs: an opportunity cost due to the time spent with children¹⁰ and a potential decrease of his consumption. This implies that a man will agree to marry without having children only if it increases his own consumption, which arises when the potential bride has enough education. Regimes VII and IX can then dominate singleness for the man.

The opportunity to become a father is not always sufficient to incite the man to marry: if having children decreases his consumption too sharply, he will remain single (singleness can dominate Regimes VI, VIII, X and XI). This could apply to a man with a low income, matched with a woman who would be socially sterile as single: in order to have children in Regime VIII, he would then have to give up too much of his income and consume too little.

Once again, marriage is a protection against social sterility: two partners could have children by pooling their income and their time while they would have remained childless and single otherwise. Men who live in Regime VIII agree to reduce their consumption in order to enable their wives to have children. This is optimal if $\hat{c}$ is not too large and the man not too poor. Marriage can even prevent poor women becoming poor single mothers. This is the case when a highly educated man agrees to marry a poorly educated woman: even if her negotiation

¹⁰In the case where $(1 - \alpha)\phi n > \delta^m - \phi\eta(1 - \alpha)$, his time dedicated to labor market activities is smaller when married than when single.

power is minimal ($\theta = \frac{1}{2}\underline{\theta}$), she will consume more than $\hat{c}$ because consumption increases with w^m .

The parameter α is important for marital decisions. For high values of α , the model predicts higher marriage rates for highly educated men than for highly educated women as better educated men have more reasons to marry: they can have children at a low opportunity cost. For highly educated women the incentive to marry is now low and marrying a lowly educated man would only entail costs (they would remain childless whatever the marital status). Poor women, on the contrary, will marry more often as they are less often rejected by highly educated men who can have children without reducing their consumption too much. For low values of α , the opportunity cost of becoming a father is high and nothing ensures that highly educated men marry more often than highly educated women. The parameters δ^i 's also affect the marital choice. A large δ^m implies that educated men are more willing to accept a marriage offer, in line with the male marriage pattern stressed in Section II.

The model predicts a positive degree of assortativeness due to the parameter $\underline{\theta}$. A higher $\underline{\theta}$ implies that spouses' incomes are more equally shared. An individual with a much larger income than his or her potential spouse will then be more likely to reject the offer as he/she would have to give up too much of his/her own consumption.

To conclude, the model predicts lower female marriage rates for extreme social classes. Women with low levels of education are more likely to be socially sterile while highly educated women will probably want to remain childless. This makes women from extreme social classes less attractive to men. Marriage could however occur when wages and non-labor incomes are sufficiently close for them to share the marriage surplus.

Natural sterility

The possibility of being naturally sterile or being matched with a naturally sterile partner has an impact on the incentive to marry. As couples face a higher risk of being sterile than single women, natural sterility tends *ceteris paribus* to reduce the overall marriage rate. In particular, the number of marriages between highly educated men and poorly educated women are affected. Indeed, an incentive for highly educated men to marry a lowly educated woman lies in the possibility of becoming a father. Now, when they marry a poor woman, they have to reduce their consumption *and* to take the risk of remaining childless. Women with high education levels are less concerned by this phenomenon as they can have children on their own.

A high probability of sterility among men will discourage almost all types of women to accept a marriage offer. Indeed, for a men, being rich remains a sufficient condition to convince a very poor woman who would have been subject to social sterility to accept a marriage offer as it offers to this woman a possibility to have children and to increase her consumption (condition (10) remains valid). Nevertheless, for other women, the higher the probability of male sterility, the weaker the incentive to accept a marriage offer. Obviously, as the probability of natural sterility is the same among all men, rich ones still have an advantage on the marriage market.

C Simulation and Robustness

C.1 Implementation Notes

Minimization was first run using the algorithm PIKAIA and the results used as initial values in UOBYQA. PIKAIA is a genetic algorithm developed by Charbonneau (2002), it allows global extrema to be found in highly non-linear optimization problems where there exist a high number of local extrema. We used PIKAIA in a first step to identify the region in the parameter space where the global maximum lies. Once this region has been identified, we used a faster algorithm called UOBYQA (Powell (2002)) designed to find a maximum of a well-behaved problem. It was developed for optimization when first derivatives of the objective function were not available and takes account of the curvature of the objective function, by forming quadratic models by interpolation. We ran these two algorithms in FORTRAN 90.

C.2 Alternative Measures of Sterility

In a previous version of the paper (see Baudin et al. (2012)), we followed an alternative where natural sterility equals 3.7%. This number is obtained from Leridon (2008) who uses the Henry database, which consists of data for rural France from the 18th century, when fertility control was ineffective. Restricting the sample to couples where the husband and the wife were still living together at age 50, 3.7 % of women who married at age 20-24 remained childless. The main problem from this study is that living conditions and bad nutrition at that time might positively influence childlessness.

An alternative to that of the Hutterites and the historical Frenchmen is to take the value of the country that has the lowest childlessness rate for a given education level. We looked at childlessness rates in 31 developing countries where the Census included the variable “children surviving” (data was taken from IPUMS International). We only considered developing countries as they are more likely to endorse a culture where the role of the woman is a procreation one. The 31 censuses considered are: Bolivia (2001), Brazil (2000), Chile (2002), Colombia (2005), Costa Rica (2000), Ecuador (2001), Jamaica (2001), Mexico (2012), Nicaragua (2005), Panama (2000), Peru (2007), Uruguay (1996), Venezuela (2001), Ghana (2000), Guinea (1996), Kenya (1999), Morocco (2004), Mali (1998), Malawi (2008), Rwanda (2002), Senegal (2002), Sierra Leone (2004), Tanzania (2002), Uganda (2002), South Africa (2001), Indonesia (1995), Cambodia (2008), Nepal (2001), Thailand (2000), Vietnam (2009) and Palestine (West Bank and Gaza) (1997). Among these countries, Vietnamese married women with 9 years of schooling have the lowest rate of childlessness, 1.26% (2,626 married women were childless among 208,761 married women). However, this number might underestimate natural sterility. A first reason for this is that it may be affected by divorce rates caused by fecundity problems. Another possible argument is that sterile vietnamese couples adopt orphans and then answer to the number of children they have ever born as if these children were biologically theirs.

C.3 Non-labor Income

In the main text, we assume that non-labor income does not depend on the education of individuals. Here, we relax this hypothesis with the following expression for a^i :

$$a^i = \beta w^i + \tilde{a}^i \quad (12)$$

where β describes the marginal impact of education on non-labor income (bequests and ones education might have common determinants, such as the education of the parents, see Hertz et al. (2007)). $\tilde{a}^i$ is the part of non labor income that is uncorrelated to education and differs for each individual: each adult draws a different $\tilde{a}^i$ from a log-normal distribution, whose mean and variance will be estimated as before. We then estimate the same model adding one parameter to the estimation, β . Results are provided in Table 18. We see that the estimates are similar comparing both specifications and that the estimated value for β is zero. Hence, the three facts exposed in Section II do not allow to identify an effect of education on income other than the one coming through the labor income.

Parameter	Value with $\beta = 0$	Value with $\beta > 0$
σ_a	0.247	0.249
$\bar{m}_a$	1.001	1.003
ν	9.362	9.438
$\hat{c}$	0.399	0.400
μ	0.272	0.270
$\underline{\theta}$	0.864	0.862
α	0.524	0.527
ϕ	0.206	0.206
η	0.114	0.110
δ^m	0.256	0.261
δ^f	0.077	0.081
β		0.000

Table 18: Identified Parameters with $\beta > 0$

C.4 Computation of the Standard Errors

Two methods are used in the literature to obtain the standard errors of parameters estimated by the simulated method of moments: bootstrapping and the delta method. The static nature of the model makes the bootstrapping method preferable to the delta method which is usually faster but tends to underestimate the standard errors. We first draw 200 random new samples with replacement from the original data. Each new bootstrap sample is of equal size of the original one (1,127,080 observations) but the frequency of each observation

changes. For each of these new datasets we generate the 72 moments and estimate the corresponding parameters. We then compute the standard errors of these estimators. By doing so, the uncertainty surrounding the estimated parameters comes exclusively from the uncertainty around the estimated moments. The results are reported in Column (2) of Table 19, and in Table 3 of the main text.

	Model only (1)		Data only (2)		Both (3)	
	mean	s.e.	mean	s.e.	mean	s.e.
σ_a	0.259	0.010	0.265	0.012	0.254	0.010
$\bar{m}_a$	1.005	0.010	1.005	0.012	1.006	0.010
ν	9.315	0.132	9.241	0.146	9.306	0.134
$\hat{c}$	0.406	0.008	0.404	0.009	0.403	0.007
μ	0.256	0.015	0.257	0.013	0.262	0.011
$\underline{\theta}$	0.878	0.014	0.883	0.014	0.873	0.013
α	0.110	0.005	0.108	0.006	0.111	0.007
ϕ	0.524	0.004	0.527	0.005	0.524	0.005
η	0.209	0.003	0.210	0.003	0.209	0.003
δ^m	0.275	0.018	0.276	0.015	0.268	0.014
δ^f	0.095	0.016	0.096	0.013	0.088	0.013

Table 19: Mean and Standard Errors of Parameters

We expected the uncertainty coming from the randomness of the artificial population used to simulate the model to be minimal when drawing a large enough number of individuals (typically, it is 100,000 per education category). To check this expectation we also provide in Column (1) of Table 19 the parameters when the only uncertainty comes from the model: that is when we estimate the parameters 200 times using the same empirical moments but drawing different households from the distribution. Column (3) of Table 19 presents the parameters when we combine both uncertainties, using the empirical moments from the bootstrap samples and drawing different households from the distribution. The difference between Columns (2) and (3) is very small.

C.5 Identified Parameters for Subsamples

Identification by Race or Ethnicity. In the main text we have assumed an homogeneous marriage market. Here we assume instead that there are fragmented markets for each race separately (see Appendix A.6.1 for how the groups are constructed). We therefore reestimate the parameters for each race independently. The results are provided in Table 20.

Some parameters show remarkable constancy across races, such as the mean of the non labor income m_a , the rearing cost ϕ and η , the bargaining power parameter $\underline{\theta}$. There are strong variations in the δ^i 's across races. Black single men loose 34.9% of their time endowment

Parameter	All	Blacks	Whites	Natives	Asians	Hispanics
σ_a	0.247	0.552	0.222	0.290	0.226	0.395
$\bar{m}_a$	1.001	1.072	0.964	1.032	0.955	1.111
ν	9.362	7.986	8.447	9.330	9.422	8.932
$\hat{c}$	0.399	0.358	0.406	0.344	0.279	0.280
μ	0.272	0.359	0.294	0.299	0.289	0.256
$\underline{\theta}$	0.864	1.000	0.852	0.875	0.754	0.830
α	0.524	0.660	0.540	0.606	0.503	0.589
ϕ	0.206	0.201	0.229	0.191	0.202	0.214
η	0.114	0.074	0.082	0.092	0.119	0.085
δ^m	0.256	0.349	0.121	0.294	0.192	0.251
δ^f	0.077	0.079	0.065	0.190	0.060	0.119
[O]	8.1%	6.1%	7.3%	3.4%	10.4%	5.8%
[S]	2.4%	5.1%	5.8%	1.8%	2.3%	3.4%
[N]	2.3%	2.3%	2.3%	2.4%	2.3%	2.3%

Table 20: Identified Parameters by Race or Ethnicity

compared to married black men, while this loss equals 25.6% in the whole population. Asian single men only loose 19.2%. Black fathers are characterized by a lower involvement in childrearing. Non-labor income is less dispersed for Whites and more dispersed for Blacks and Hispanics.

The breakdown of childlessness into its components shows that blacks are relatively more affected by social sterility, while Asians are more childless due to a higher opportunity cost in than all the other groups.

Identification Removing Disabled. Since 83.5% of single childless women with no schooling are disabled, we also identify the parameters of the model after removing the disabled from the data. The results are shown in Table 21. We can check that the parameter $\hat{c}$ still plays a role and that its value is not significantly different from its benchmark estimate. The estimated social sterility drops with respect to the benchmark.

C.6 Alternative Specification for θ

In this appendix, we provide results under two alternative specifications for the bargaining power θ . The first fixes the bargaining power to 1/2, independently of relative education levels. The second makes bargaining depend on relative potential incomes, hence including non labor incomes a^f and a^m .

First, assume $\underline{\theta} = 1$ (equal negotiation power of spouses). Table 22 reports the implied

Parameter	“All”	Without “disabled”
σ_a	0.247	0.237
$\bar{m}_a$	1.001	1.025
ν	9.362	9.342
$\hat{c}$	0.399	0.390
μ	0.272	0.265
$\underline{\theta}$	0.864	0.802*
α	0.524	0.511*
ϕ	0.206	0.210
η	0.114	0.115
δ^m	0.256	0.248
δ^f	0.077	0.085
[O]	8.1%	8.0%
[S]	2.4%	2.1%
[N]	2.3%	2.3%

* indicates a significant difference from “All”.

Table 21: Identified Parameters Without the “Disabled”

parameter values. The objective $f(p)$ and the R^2 deteriorate. The cost of marriage drastically increases for a highly educated individual matched with someone with a low education: he/she will have to give up a large part of his/her income to ensure equal consumption in the household. As a result, highly educated will reject unions with lowly educated more often and the percentage of married people in the extreme education categories decreases. As heterogamous unions become less valuable, marriage rates also decrease on average. But this is partly compensated by a rise in the parameter δ^m . Finally, as poor women are less often married, they are also less protected against social sterility when $\underline{\theta} = 1$. Then the estimates of both types of childlessness increase.

Second, we include non-labor incomes into the bargaining power of each spouse as follows:

$$\theta(w^f, w^m, a^f, a^m) = \frac{1}{2}\underline{\theta} + (1 - \underline{\theta}) \frac{w^f + a^f}{w^f + w^m + a^f + a^m}$$

This specification leads to the results shown in Table 22. We see that most of the parameters are significantly different but that the match of the model with the data is still very good. Some moments are better matched with one specification of θ while others with the other.

C.7 Changes in the Time Allocation Parameter α

The model assumes that the share that mothers (α) and fathers ($1 - \alpha$) devote to childrearing is constant across education groups and over time. Bianchi et al. (2004) show that the ratio

Parameter	$\theta(w^f, w^m)$	$\theta = 1/2$	$\theta(w^f, w^m, a^f, a^m)$
$f(p)$	0.426	0.599	0.449
R^2			
childlessness single	0.847	0.652	0.733
childlessness married	0.934	0.872	0.952
fertility single	0.903	0.913	0.912
fertility married	0.960	0.954	0.950
marriage female	0.453	0.315	0.441
marriage male	0.659	0.732	0.791
σ_a	0.247	0.273*	0.254
$\bar{m}_a$	1.001	1.002	1.046*
ν	9.362	9.202	9.447
$\hat{c}$	0.399	0.393	0.437*
μ	0.272	0.276	0.246*
$\underline{\theta}$	0.864	1.000	0.955*
α	0.524	0.523	0.518
ϕ	0.206	0.206	0.214
η	0.114	0.112	0.100*
δ^m	0.256	0.304*	0.347*
δ^f	0.077	0.073*	0.121*
[O]	8.1%	8.6%	7.7%
[S]	2.4%	2.7%	2.2%
[N]	2.3%	2.3%	2.3%

* indicates a significant difference from the benchmark $\theta(w^f, w^m)$.

Table 22: Identified Parameters for Alternative Specifications of the Bargaining Power

of married mothers' to married fathers' time in child care declined between the mid-1960s and the late 1990s, which could be an indication that either social norms changed for all education categories, or that the increase in the education of women makes it optimal for couples to have the father more involved in the raising of children.

To have some insights on the role of α we carried out the following two experiments. First, fixing $\alpha = 1$ instead of identifying it from the cross sectional data, and re-estimating the rest of the parameters, we find that the quality of the match is lower: the model becomes unable to reproduce (a) a reasonable marriage rate (especially for highly educated women who have lost their incentive to marry), (b) childlessness rates for highly educated married women (for whom the cost of raising children becomes extremely high so they have more incentives to be childless in [O]), (c) the gap between the fertility of married mothers and that of single mothers, who now face the same opportunity cost. Hence, allowing $\alpha < 1$ is important to generate the nice features of the model.

Second, consider that couples set α optimally under the constraint $\alpha \in [1/2, 1]$. A full-fledged model with proper bargaining on α would be the topic of another paper; however, a simple benchmark would be to assume the following rule:

$$\alpha = \begin{cases} 1 & \text{if } w^f < w^m \\ 1/2 & \text{if } w^f \geq w^m. \end{cases}$$

With this specification the marriage rates are reasonable, the U-shaped relationship between childlessness and education of married women is preserved, but the model fails in reproducing the high fertility of poorly educated married mothers (by about one child for the two lowest education categories), as poor married mothers face almost the same incentives as poor single mothers. Otherwise, the simple *ad-hoc* rule above does rather well, which indicates that bargaining over α could be a promising extension. Finally, we also made α depend on the education of the mother, independently of the education of the father: $\alpha = 1 - w^f/2$. The properties of this simulation are similar to those in the previous case.

C.8 Assortative Matching

The model accounts for 13.8% of the variation in the assortativeness of matching. In reality, the assortativeness is higher because, first, people meet several possible matches (we would need a dynamic model to reproduce this), and second, individuals are more likely to meet others who have similar levels of education to their own.

With a model able to match the level of assortative matching, in particular among poor people, we expect a lower $\hat{c}$. In fact, 53% of women with no education marry a man with no education. This is much higher than in our model. Consequently, our estimated value of $\hat{c}$ is probably too high because we have more women with no education marrying men with higher levels of education than in reality. This means that in order to match observed childlessness, $\hat{c}$ must be high. A way to increase the degree of endogenous assortativeness consists in modifying the bargaining rule (4) by making it less sensitive to the wage ratio. This will

increase the rejection rate of matches with very different people. However, achieving more assortativeness will be at the expense of matching a reasonable marriage rate, as people only meet once in our static set-up. This is a limitation of our approach.

A complementary and exogenous way to generate the right degree of assortativeness is to assume, as in Fernández-Villaverde et al. (2010), that a fraction λ of the female population draws a possible match from her education category, while $1 - \lambda$ draws from the total population. For each chosen value of λ , we can estimate the remaining parameters and see whether changes are important.¹¹ We provide the results in Table 25. Our main results are robust to this change. For instance, when λ equals 0.1, the fit of the model does not change significantly although the marriage matrix becomes much more satisfying.

The number of marriages by education category from the data are given in Table 23. This table is constructed in the following way: first, we drop from the data all the individuals who are not married ($\text{MARST} > 1$) or who do not have an identified partner in the Census ($\text{SPLOC} = 0$). Then, we sort observations, first, by their serial number, corresponding to the household, and then by their sex, so that the man of the household comes before his wife in the data. We then generate a variable saying that the husband has a corresponding wife after him (the serial number for both has to be the same). The last step is to generate a variable with the education of the husband and another variable with the education of the wife. The ratio

$$\frac{\pi(i, j)}{\sum_j \pi(i, j)}$$

gives the proportion of men of type j having married a woman of type i , where $\pi(i, j)$ is the number of couples in which the woman has an education level i and the man an education level j . Dividing this number by the proportion of women of type i in the total population,

$$\frac{\sum_i \pi(i, j)}{\sum_{i,j} \pi(i, j)},$$

we obtain the data in Table 24. Each cell gives, for each married man, his increased chances marrying a woman in a given category of education, compared with a purely random matching framework. If there was no assortativeness, all the cells would equal 1. The first cell means that a man of category 1 has 57 times more chances of marrying a woman of education category 1 than in the case of pure random matching. We are able to compute similar statistics with the simulated data. These are provided at the bottom of Table 24. We see that the model generates some assortative matching endogenously, although lower than in the data. Regressing each cell in the top part of the table on the corresponding cell in the bottom part leads to an R^2 of 0.138 meaning that we are able to account for 13.8% of the variations in the assortativeness matching.

To assess the influence of having abstracted from exogenous assortative matching on the estimated parameters, we conducted the following exercise. A share, λ , of the population draws a potential spouse from his/her education category. The remaining share $1 - \lambda$ draws

¹¹In Fernández-Villaverde et al. (2010), a fraction of the population is randomly matched, the rest being matched following a Gale-Shapley algorithm, which generates perfect assortativeness.

his/her partner randomly from the whole population. For different values of λ , we reestimate the parameters, minimizing the same objective as before. Table 25 presents the results. The case $\lambda = 0$ refers to the benchmark case of Table 3. The parameters which seems to be the most sensitive to changing λ are α and $\underline{\theta}$: these are significantly different from the benchmark for all the different given values of λ .

Table 25 also provides the fit of the model with respect to each of the moments used for estimating the parameters. These R^2 are computed as follows:

$$R^2 = 1 - \frac{\sum_{k=1}^{12} (d_k - s_k)^2}{\sum_{k=1}^{12} (d_k - \bar{d}_k)^2}$$

where d_k and s_k are respectively the empirical and simulated moments for each education category k . Comparing the different R^2 , we see that the simulated moments are quite insensitive to the choice of λ . For childlessness rates and fertility of single women, the best specification is the benchmark, for the fertility of married women, it is $\lambda = 0.2$ or $\lambda = 0.3$, and for marriage rates, $\lambda = 0.4$.

The value of λ that minimizes the objective function $f(p)$ is 0.1. For $\lambda = 0.1$, we obtain the marriage matrix presented in Table 26. Regressing now each cell in the top part of Table 24 on the corresponding cell of Table 26 leads to an R^2 of 0.784. We conclude that introducing some exogenous assortative matching, captured by $\lambda > 0$, allows us to get much closer to the observed assortativeness, without modifying the estimated parameters or the ability of the model to reproduce the targeted empirical moments very much.

	1	2	3	4	5	6	7	8	9	10	11	12
1	71,179	13,086	17,869	3,842	2,971	1,751	13,915	4,122	1,630	1,431	988	332
2	14,465	68,397	53,512	7,801	5,114	3,638	18,566	5,054	1,398	1,543	778	104
3	26,113	98,004	472,453	79,705	72,697	43,995	229,332	58,307	14,805	14,369	7,674	1,004
4	6,114	23,070	135,612	88,745	53,731	34,170	151,668	38,420	7,745	8,634	3,924	560
5	6,353	22,146	159,403	69,339	132,115	62,352	282,405	75,275	16,301	18,667	7,760	1,026
6	4,745	15,159	125,973	58,541	83,086	94,623	256,444	73,902	14,284	17,530	7,061	1,066
7	23,125	49,898	565,508	264,819	380,715	300,185	3,455,022	1,261,526	283,977	595,232	251,575	28,998
8	4,840	8,097	92,315	48,220	71,447	57,596	672,543	837,035	159,921	525,637	300,613	44,805
9	1,337	2,726	23,806	12,216	17,571	15,488	186,588	169,093	98,724	188,244	125,071	16,869
10	1,245	2,095	17,462	8,040	12,546	11,090	170,890	209,703	56,760	548,646	445,586	86,506
11	916	1,519	11,343	5,403	8,345	7,326	96,804	105,051	28,313	180,051	292,392	87,314
12	146	63	575	231	374	234	3,409	3,883	1,541	9,598	17,766	22,918

Table 23: Marriages per Education Category. Men in Columns, Women in Rows

	1	2	3	4	5	6	7	8	9	10	11	12
Data												
1	57.233	5.553	1.377	0.767	0.456	0.357	0.324	0.187	0.307	0.088	0.087	0.147
2	8.584	21.421	3.043	1.149	0.580	0.548	0.319	0.169	0.194	0.070	0.051	0.034
3	2.499	4.950	4.332	1.893	1.329	1.069	0.636	0.315	0.332	0.105	0.081	0.053
4	1.185	2.359	2.518	4.268	1.989	1.681	0.852	0.421	0.352	0.127	0.084	0.060
5	0.797	1.466	1.916	2.159	3.166	1.986	1.027	0.534	0.479	0.178	0.107	0.071
6	0.675	1.138	1.717	2.067	2.258	3.418	1.058	0.594	0.476	0.190	0.110	0.084
7	0.332	0.378	0.777	0.943	1.043	1.093	1.437	1.023	0.955	0.650	0.397	0.229
8	0.184	0.162	0.335	0.454	0.517	0.554	0.739	1.794	1.421	1.517	1.253	0.936
9	0.167	0.180	0.285	0.378	0.419	0.491	0.675	1.192	2.886	1.788	1.715	1.160
10	0.085	0.075	0.114	0.136	0.163	0.192	0.338	0.808	0.906	2.846	3.337	3.248
11	0.119	0.104	0.141	0.174	0.207	0.241	0.364	0.770	0.861	1.779	4.170	6.242
12	0.257	0.059	0.097	0.101	0.126	0.105	0.174	0.387	0.636	1.287	3.441	22.248
Simulation												
1	1.123	1.117	1.056	1.036	1.025	1.010	1.008	0.997	0.992	0.960	0.946	0.831
2	1.126	1.115	1.058	1.039	1.028	1.016	1.009	0.996	0.992	0.955	0.942	0.852
3	1.128	1.112	1.060	1.037	1.024	1.015	1.005	0.991	0.984	0.962	0.951	0.896
4	1.115	1.095	1.047	1.027	1.018	1.012	1.002	0.993	0.985	0.970	0.964	0.931
5	1.092	1.078	1.037	1.025	1.015	1.011	1.001	0.994	0.987	0.977	0.972	0.945
6	1.072	1.053	1.025	1.018	1.010	1.010	1.001	0.996	0.988	0.984	0.980	0.967
7	1.023	1.019	1.012	1.010	1.004	1.004	1.000	0.998	0.993	0.993	0.991	0.993
8	0.984	0.988	0.995	0.997	1.001	1.000	1.000	1.001	1.000	1.003	1.003	1.014
9	0.933	0.948	0.975	0.985	0.992	0.990	1.000	1.004	1.007	1.015	1.018	1.037
10	0.822	0.853	0.920	0.942	0.967	0.975	0.996	1.015	1.030	1.049	1.061	1.101
11	0.770	0.794	0.883	0.916	0.953	0.962	0.993	1.019	1.047	1.073	1.091	1.142
12	0.594	0.627	0.723	0.795	0.846	0.901	0.966	1.030	1.106	1.198	1.255	1.383

Table 24: Assortative Matching in the Data and Generated by the Model with $\lambda = 0$, Men's Education in Columns, Women's in Rows

λ	0	0.1	0.2	0.3	0.4	0.5
$f(p)$	0.426	0.421	0.457	0.479	0.523	0.606
R^2						
childlessness single	0.847	0.837	0.843	0.805	0.777	0.757
childlessness married	0.934	0.931	0.925	0.924	0.918	0.913
fertility single	0.903	0.897	0.875	0.882	0.886	0.875
fertility married	0.960	0.970	0.973	0.973	0.940	0.871
marriage female	0.453	0.476	0.455	0.555	0.567	0.498
marriage male	0.659	0.676	0.655	0.605	0.694	0.422
σ_a	0.247	0.252	0.252	0.284*	0.281*	0.284*
$\bar{m}_a$	1.001	1.002	0.999	0.990*	1.003	1.037
ν	9.362	9.074*	9.300	8.633*	8.557*	8.374*
$\hat{c}$	0.399	0.386	0.360*	0.336*	0.332*	0.308*
μ	0.272	0.253	0.264	0.252	0.257	0.284
$\underline{\theta}$	0.864	0.908*	0.898*	0.920*	0.990*	0.950*
α	0.524	0.504*	0.500*	0.503*	0.500*	0.500*
ϕ	0.206	0.213	0.208	0.218*	0.221*	0.224*
η	0.114	0.120*	0.109	0.120*	0.116	0.109
δ^m	0.256	0.287*	0.262	0.254	0.278	0.218*
δ^f	0.077	0.090	0.064	0.070	0.076	0.048*
[O]	8.1%	8.3%	8.6%	8.7%	8.4%	8.5%
[S]	2.4%	2.2%	2.3%	2.7%	2.5%	3.0%
[N]	2.3%	2.3%	2.3%	2.3%	2.3%	2.3%

* indicates a significant difference from the case $\lambda = 0$.

Table 25: Identified Parameters for λ between 0 and 0.5

	1	2	3	4	5	6	7	8	9	10	11	12
1	10.123	1.02	0.964	0.943	0.927	0.913	0.91	0.895	0.897	0.84	0.831	0.708
2	1.026	7.223	0.97	0.939	0.928	0.914	0.907	0.892	0.894	0.842	0.841	0.73
3	1.021	1.009	2.087	0.932	0.919	0.909	0.903	0.889	0.894	0.858	0.856	0.774
4	1.002	0.988	0.947	3.727	0.912	0.908	0.902	0.892	0.896	0.869	0.872	0.811
5	0.985	0.968	0.935	0.921	3.068	0.909	0.901	0.894	0.898	0.875	0.879	0.826
6	0.951	0.938	0.925	0.912	0.902	3.722	0.901	0.896	0.899	0.884	0.888	0.847
7	0.908	0.912	0.909	0.905	0.897	0.902	1.218	0.898	0.904	0.892	0.898	0.874
8	0.863	0.874	0.891	0.893	0.893	0.895	0.899	1.495	0.911	0.901	0.91	0.893
9	0.819	0.833	0.869	0.88	0.886	0.884	0.897	0.902	3.331	0.913	0.925	0.913
10	0.717	0.733	0.809	0.827	0.856	0.864	0.89	0.909	0.934	1.791	0.963	0.97
11	0.643	0.677	0.771	0.799	0.838	0.85	0.882	0.909	0.952	0.964	2.198	1.004
12	0.461	0.506	0.597	0.662	0.721	0.765	0.829	0.896	0.98	1.061	1.123	8.754

Table 26: Simulated Marriage Matrix for $\lambda = 0.1$, Men's Education in Columns, Women's in Rows

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