

**Online Appendix for the paper “Optimal Life Cycle
Unemployment Insurance”**

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APPENDIX

Section A describes the data, Section B extends the analysis of the model in Section I and derives the extended redistribution formula discussed in Section I.C, Section C discusses computational details, Section D discusses the properties of the models in terms of earning losses upon displacement, Section E discusses further details about the robustness section (Section VI).

DATA APPENDIX

We describe the data from SIPP, CPS, PSID, Mathematica, and SCF used in the paper.

A1. The SIPP

The data from the Survey of Income and Program Participation (SIPP) is obtained from Chetty (2008) and Johnson and Mommaerts (2011) and they are constructed starting from the 1985, 1986, 1987, 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels of SIPP. Interviews were conducted every four months for a period of two to four years. As in Chetty (2008), the analysis on unemployment duration uses data that span the beginning of 1985 to the middle of 2000. The original sample is restricted to those male workers between 18 and 65 years of age with an unemployment spell, with at least three months of work history, that reported non-zero unemployment duration, that actively search for a job, that were not temporary layoffs and that received UI benefits in the first month of the unemployment spell, from all states but Maine, Vermont, Iowa, North Dakota, South Dakota, Alaska, Idaho, Montana and Wyoming since SIPP does not provide a unique identifiers for these small states. The final sample covers 4560 unemployment spells, see the Appendix in Chetty (2008) for further details. The analysis on wage losses span the years 1996 to 2007, covering the 2001 recession but not the 2007-2009 recession. As in Johnson and Mommaerts (2011), we do not use earlier panels because they lack detailed information on why respondents separate from their jobs, which we use to separately identify quits from layoffs. The original sample is restricted to those male white workers between 18 and 65 years of age who work full time (at least thirty hours per week in their main job), who are displaced and who have collected UI benefits at some point during their unemployment spell. We exclude self-employed workers. Respondents enter the sample when first displaced from a job at which they had been working for at least two months and they remain in the sample until they first find a new full time job that lasts for at least one month. To focus on displacement for exogenous reasons, we classify respondents as displaced if they report separating from their employer because of layoff, slack work, employer bankruptcy, or because the employer sold the business. We include in the sample only the first unemployment spell for each worker. Below we describe in more details the variables used in the paper.

Age This is equal to the age of workers measured in years, which corresponds to variable TAGE in the survey.

Age specific average benefits level These are calculated exactly as in the CPS analysis below, see Section A.A2.

Average benefits This is the average benefits in each state and year provided by the Department of Labor.

Marital status This is a dummy variable which is equal to one if the individual is currently married. This is constructed using variable EMS which can take the following values: 1. Married, spouse present; 2. Married, spouse absent; 3. Widowed; 4. Divorced; 5. Separated; 6. Never married. An individual is classified as married if he reports EMS=1 or EMS=2.

Individual UI benefits We use the imputation by Chetty (2008) who is based on estimating a first-stage equation for earnings using OLS on the full sample of individuals who report a job loss at some point during the sample period. He regresses nominal log wages in the year before job loss on years of education, age at job loss, years of tenure on the last job, a dummy for left-censoring of this job tenure variable, industry, occupation, month, and year dummies, and the unemployment rate in the relevant state/year. Using the coefficient estimates, he predicts log wages for each job loser, and recover the predicted wage in levels. He then uses the predicted wage to simulate the claimant unemployment benefit using the UI benefit calculator by Cullen and Gruber (2000).

Earnings losses Earnings losses upon displacement are calculated as the percentage differences between the value of monthly labor earnings in the job before displacement and the value of monthly earnings in the new job after reemployment.

Monthly labor earnings This is constructed using variable TPMSUM in the survey that reports total earnings from job before deductions received in the month of the survey.

Previous job tenure This variable is constructed using variable TSJDATE in the survey, which reports the year/month when the current job has started.

Unemployment duration SIPP reports the employment status of individuals for every week that they are in the sample. Weekly employment status (ES) can take the following values: 1. With a job this week; 2. With a job, absent without pay, no time on layoff this week; 3. With a job, absent without pay, spent time on layoff this week; 4. Looking for a job this week; 5. Without a job, not looking for a job, not on layoff. A job separation is defined as a change in ES from 1 or 2 to 3, 4, or 5. As in Cullen and Gruber (2000), unemployment duration is obtained by summing the number of consecutive weeks with $ES \geq 3$, starting at the date of job separation and stopping when the individual finds a job that lasts for at least one month (i.e., reports a string of four consecutive ES=1 or ES=2). Individuals are defined as being on temporary layoff if, at any point in the spell, they report ES = 3. They are defined as searching if they report ES = 4.

Job finding probability at unemployment duration This is constructed using the weekly employment status variable ES described above and corresponds to a transition from ES=4 to a string of four consecutive ES=1 or ES=2.

Year This is a dummy that identifies the year of the interview.

Years of education This is simply constructed using variable HIGRADE in the survey, which reports the highest grade or year of school attended.

Worker's wealth Asset data are generally collected only once in each panel, so pre-unemployment asset data is available for approximately half of the observations. Total net wealth is defined as values of stocks, bonds, savings and current accounts plus the value of properties, business and vehicle equities net of secured and unsecured debt. The values of assets and liabilities are constructed using answers to several questions that report the individually reported estimate of the average amount that husband and wife jointly hold in a specific asset or liability over the four months period preceding the date of the interview.

Worker's net liquid wealth is measured at the time of job loss as total wealth minus home, business and vehicle equity net of unsecured debt (which is equal to the amount owed by the household, excluding mortgages and vehicle loans). The variable is again constructed using answers to several questions that report the individually reported estimate of the average amount that husband and wife jointly hold in a specific asset or liability over the four months period preceding the date of the interview.

Displaced workers Wage losses upon re-employment are used as a calibration target, see Table 4. These are calculated by focusing on a sample of displaced workers. Displaced workers are identified using the variable ERSEND which report the "Main reason why the individual stopped working for employer". The variable ERSEND can take the following values: 1. On Layoff; 2. Retirement or old age; 3. Childcare problems; 4. Other family/personal obligations; 5. Own illness; 6. Own injury; 7. School/Training; 8. Discharged/fired; 9. Employer bankrupt; 10. Employer sold business; 11. Job was temporary and ended; 12. Quit to take another job; 13. Slack work or business conditions; 14. Unsatisfactory work arrangements (hours, pay, etc); 15. Quit for some other reason. Individuals are classified as displaced if they stopped working for the employer and they report ERSEND=1, 8, 9, or 10.

Hours This is constructed using the variable EJBHRS that report "How many hours per week the worker usually works at all activities at his main job".

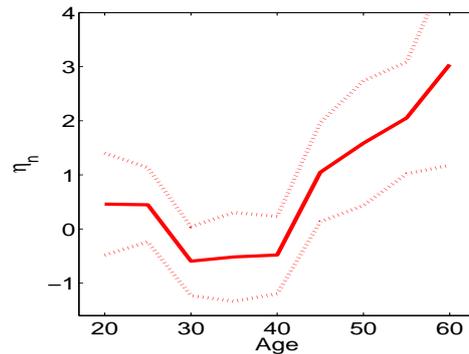
Table A1 gives summary statistics for the core sample and for two different age groups: workers of age from 20 to 40 years and from 40 to 60 years. Monetary values are in 1990

TABLE A1—SUMMARY STATISTICS, SIPP SAMPLE

Variable	All	20-40 years	41-60 years
Mean Annual wage	20711.27	18699.21	24665.52
Median Annual Wage	17780.48	16512.12	21745.91
Years of education	12.1	12.1	12.2
Weekly indiv UI benefits	165.72	153.30	187.87
Mean unemp. duration	20.45	18.90	22.68
Median unemp. duration	15.00	14.00	17.00
Mean liquid assets	22545.31	14897.08	34086.46
Mean net liquid assets	18583.77	11044.50	29902.03
Mean total wealth	62705.52	44955.76	90015.60
Percent with Mortgage	.45	.40	.54
Quartile of net liquid assets:			
Q ₁	-2177.89	-2231.28	-2081.14
Q ₂	-54.92	-63.28	-42.43
Q ₃	919.15	911.81	948.12
Q ₄	19897.43	12427.25	31077.48
Observations	4560	2873	1536

dollars converted using the CPI index. The median UI benefits recipient is a high school

graduate and has pre-UI gross annual earnings of 20,711. The group of workers with 20 to 40 years of age has 2873 observations, the group of workers with 41 to 60 years of age has 1522 observations. Mean unemployment duration is 20 weeks for the whole sample. Unemployment spells for workers with more than 40 years of age are 4 weeks longer than the analogous spells for workers with less than 40 years of age. The individually imputed UI benefit level (in 1990 dollars) is around 20% higher for the old than for the young. This difference is explained by the fact that mean pre-unemployment wage is 30% higher for the old. The resulting average replacement rate is therefore lower for the old workers than for the young.



(a) Micro-evidence, SIPP

Notes: Elasticity of unemployment to benefits for different age groups of workers. Estimates are based on model (13) using SIPP data and the age-specific average measure of benefits. Other details are as for panel (a) in Figure 1.

FIGURE A1. ELASTICITY OF UNEMPLOYMENT TO BENEFITS BY AGE GROUP: AGE-SPECIFIC AVERAGE MEASURE OF UI BENEFITS, SIPP

A2. Aggregate US states data using CPS

Aggregate US states data are calculated using monthly data from the Current Population Survey databases freely available from Integrated Public Use Microdata Series (IPUMS), see Ruggles et al. (2010) for description of IPUMS. Information on labor force, employment, unemployment, and other demographic and labor force characteristics is available in every month, while earnings data are available in the March survey. We restrict the sample to male workers with 16 to 64 years of age. Similar to SIPP the sample period is 1984-2000. For each state, semester and age group we aggregate individual data using CPS provided weights. Below we describe more in detail the variables used in the analysis.

Pre-unemployment wage: It is imputed at the individual level after running a conventional wage regression in each state and year using the March CPS survey. The dependent variable in the wage regression is weekly logged wages and the independent variables are a quadratic polynomial in age, four educational dummies (for high school dropouts, high school degree, some college and complete college), two race dummies and a marital status dummy. Using the estimated coefficients we impute wage to every

unemployed worker. Weekly wages are computed using the variable EARNWEEK in IPUMS which reports information on “usual labor earnings per week at the current job, before deductions”.

Age specific average benefits level We impute benefits at the individual level using the UI benefit calculator by Cullen and Gruber (2000). As an input we use the individual imputation for pre-unemployment wages calculated above. In three states, the UI benefits calculator requires previous job tenure as an input. For these states, we impute tenure using a quadratic function of age which we estimate using the Mathematica data below. Individual benefits are then aggregated to obtain a measure of average benefits for each age group in a given semester and given state. Average benefits are always adjusted to guarantee equality with the unconditional average benefits measure in each state and year as reported by the US Department of Labor. In all regressions, the age groups considered for constructing the age specific average measure of benefits always coincide with the age groups for which we calculate the age-specific elasticity of unemployment to benefits.

Educational composition We construct five educational dummies using information obtained from the variable EDUC in IPUMS which corresponds to answers to the following question: “What is the highest level of school (name/you) (have/has) completed or the highest degree (name/you) (have/has) received?”. We use this information to construct the following five educational groups: (i) less than 5 grade; (ii) 5th grade to 12th grade without diploma; (iii) High school diploma; (iv) some college or vocational program but no degree; (v) Bachelor’s degree or more.

Race composition We construct a dummy variable that is one if worker is white, as inferred from the answer to “I am going to read you a list of five race categories. Please choose one or more races that (NAME/you) (considers yourself/consider NAME/considers himself/considers herself) to be: White; Black or African American; American Indian or Alaska Native; Asian; OR Native Hawaiian or Other Pacific Islander”. Starting from this information we calculate the proportion of white unemployed workers in each state, period and age group.

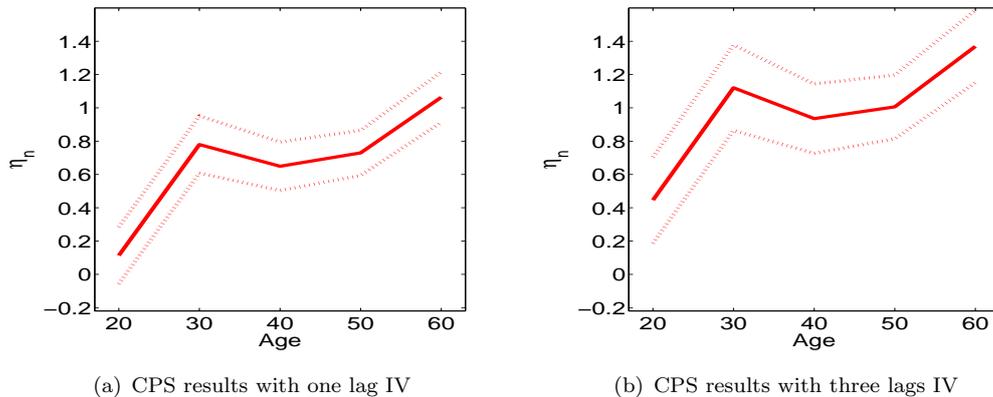
Married dummy We construct a dummy variable that is one if worker is married, as inferred from the answer to “(Are/Is)(name/you) now married, widowed, divorced, separated or never married?”. We use this information to calculate the proportion of married unemployed workers in each state, period, and age group.

Unemployment over population ratio We identify a worker as unemployed if he is non-employed and he is actively searching for a job using the variable EMPSTAT in IPUMS. The relevant question to assess his/her state is the following: “By (the week before last/last week), I mean the week beginning on Sunday, (date), and ending on Saturday, (date), did (name/you) do ANY work for (pay/either pay or profit)? 1. Yes 2. No 3. Retired 4. Disabled 5. Unable to work. A worker is actively searching for a job if he answers positively to the following question: “Have you been doing anything to find work during the last 4 weeks?”.

Unemployment duration Given the current employment status of the worker (see above), the unemployment duration of the current unemployment spell is calculated using the

variable DURUNEMP in IPUMS which reports the number of consecutive weeks for which the unemployed worker has remained jobless. The reported number is capped at 98 weeks. For each age group, the average unemployment duration in Section III.C is calculated over the sample period 1990-2010 using a sample of male workers unemployed at the time of the survey, and with positive unemployment duration. The average is weighted using CPS provided weights.

Wage profile The life cycle profile of wages is used as calibration targets, see Table 4 and Figure 4. These profiles are calculated using CPS data over the period 1990 to 2010. We use a sample of male workers with 20 to 64 years of age, who are employed and have received positive labor income in the week of the interview. We regress the log of “usual labor earnings per week at the current job, before deductions” (variable EARNWEEK) deflated using the CPI index and divided by hours worked in the previous week, on a full set of yearly age dummies and on dummies for four educational groups (high school dropouts, completed high school, some college and college degree or more), marital status (equal to one if married), race (equal to one if white), being US native and for state and year. Observations are weighted using the personal weight provided by CPS. We exponentiate the estimated dummy coefficients for age to calculate relative wages by age. To construct the educational dummies we again use the variable EDUC. Hours worked in previous week are calculated using variable HRSWORK.



Notes: Estimates of β_n in (14) when benefits are instrumented with its own lagged value. Panel (a) uses one lag as IV, Panel (b) three lags. Other details are as in Figure 1.

FIGURE A2. ELASTICITY OF UNEMPLOYMENT TO BENEFITS BY AGE, CPS RESULTS WITH IV

A3. The PSID

The Panel Study of Income Dynamics (PSID) started in 1968 collecting information on a sample of roughly 5,000 households. Of these, about 3,000 were representative of the US population as a whole (the core sample), and about 2,000 were low-income families (the Census Bureau’s Survey of Economic Opportunities, or SEO sample). We use the core and SEO samples in our analysis, dropping the Latino and Immigrant samples. Our sample focuses on male individuals with 21 to 65 years of age who at least at some point over the sample period 1978-1992 were heads of household.

Age To minimize measurement error we construct the variable age by using information on year of birth from the individual files and define age as the difference between the survey year and year of birth. For those heads with no information on year of birth, we utilize the first record on self-reported age available in the survey and then construct a consistent age series.

Education Up to 1991 (1990 data) the relevant question was coded according to: 1 if 0-5 grades completed, 2 for 6-8 grades, 3 for 9-11 finished grades, 4 for 12 grades (high school), 5 for 12 grades plus non-academic training, 6 for college dropout, 7 for college degree with no advanced degree, 8 for college and advanced degree and 9 to not available. Starting from 1992, the education variable corresponds to actual grade of school completed with figures in the 0-16 range. The education variable is recoded so as to belong to one of three following categories: high school graduate, college dropout, and bachelor degree or college and advanced/professional degree.

Employment (unemployment) status dummies The employment status is inferred using the question “Are you (head) working now, looking for work, retired, keeping house, a student or what?”. The household head is unemployed if at the interview date he is without a job and is actively looking for work.

Family size and Number of kids They are constructed using answers to: “Is someone other than this year’s head/wife included in the family unit? [Which is his/her] relationship to head?”.

Food consumption Food consumption is reported directly from PSID. Food consumption is the average weekly expenditures on food at home per capita in the household. Since interviews are usually conducted around March, it has been argued that people report their food expenditure for an average week around that period, rather than for the previous calendar year as is the case for family income. For robustness exercises we used both food consumption at home and out of home which is constructed using answers to the questions: “Did you (or anyone in your family) receive government food-stamps last month? In addition to what you bought with foodstamps, did you (or anyone in your family) spend any money on food that you use at home? How much? Do you have any food delivered to your door which is not included in that? About how much do you (or anyone else in your family) spend eating out, not counting meals at work or at school?”. We drop food consumption data that belong to either the bottom or the top percentile of the distribution of food expenditures per capita in the household.

Total consumption expenditures in non durables goods This is the imputation by Hryshko, Luengo-Prado and Sorensen (2010) for total consumption expenditures in non durable goods using CEX data, which extend the sample selection criteria by Blundell, Pistaferri and Preston (2008).³¹ Relative to Blundell, Pistaferri and Preston (2008), the imputation by Hryshko, Luengo-Prado and Sorensen (2010) uses data on regional price indexes rather than US city averages, disregards information on prices of transportation and alcohol, it drops the numbers of kids as independent regressor, it adds dummy variables to account for marital status. The table below reproduced from Hryshko, Luengo-Prado

³¹Hryshko, Luengo-Prado and Sorensen (2010) also expand the sample period to 2002, but our sample is 1978-1992 as in Blundell, Pistaferri and Preston (2008).

and Sorensen (2010) reports the variables and the coefficients used in their imputation. The definition of nondurable consumption is the same as in Attanasio and Weber (1995): it is the sum of food (defined above), alcohol, tobacco, and expenditure on other nondurable goods, such as services, heating fuel, public and private transport (including gasoline), personal care, and semidurables, defined as clothing and footwear. This definition excludes expenditure on various durables, housing (furniture, appliances, etc.), health, and education. It corresponds to the average weekly expenditures at home per capita in the household. We drop data on total consumption expenditures in non durables when the imputation is based on food consumption data that belong to either the bottom or the top percentile of the distribution of food expenditures per capita in the household.

Race It is the recoding of the answer to “And, are you white, black, American Indian, Aleut, Eskimo, Asian, Pacific Islander, or another race?” so as to get three groups: white, black and other.

Region This variable can take four values corresponding to the four regions Northeast, Midwest, South and West where the household head resides.

Wages They are constructed using information on labor income and yearly hours. Labor income corresponds to total annual labor income from all jobs. Self-employed income is split between labor and capital income. In this case only the labor part is added. Yearly hours correspond to total annual hours worked for money, from family files. It refers to all possible jobs of the worker. Hourly wage are then obtained by dividing labor income by yearly hours. Observations where the resulting hourly wage falls below half of the minimum wage in the corresponding year are dropped. Wages are expressed in 1982-84 dollars by using the CPI price index.

Time dummies These are dummies for the year of the survey.

A4. *Mathematica data*

Mathematica conducted two surveys on behalf of the Department of Labor: (i) the Pennsylvania Reemployment Bonus Demonstration, a sample of 5,678 job losers in Pennsylvania in 1991; (ii) the Study of Unemployment Insurance Exhaustees, a sample of 3,907 workers covered by UI benefits in 1998 in 25 states of the US. The datasets are publicly available through the Upjohn Institute. The information in the two datasets is similar. They contain information on prior wages, weeks of UI paid, as well as demographic characteristics, household income, job characteristics (tenure, occupation, industry), and receipt of severance pay. Since Pennsylvania is not included in the Exhaustees study, there is only one year of data for each state in the sample. We use the sub-sample of the two data sets provided by Chetty (2008), who focuses on prime-age unemployed male workers and make exclusions analogous to those applied to SIPP. In particular all observations with missing data on severance payments, years of job tenure, reported survey durations, or variables used to predict net liquid wealth are excluded. We also exclude temporary layoffs (individuals who expected a recall). The final sample comprises 2441 spells, 18% of them for workers who have received some severance payment. Below we describe more in details the variables used in the analysis:

TABLE A2—IV REGRESSION OF FOOD ON NONDURABLE EXPENDITURES, CEX: 1980-2002

Log nondurable cons.	0.730*** (15.84)	Log nondurable cons. x HS	0.023 (1.03)
Log nondurable cons. x 1980	0.122*** (9.45)	Log nondurable cons. x coll.	0.089*** (3.72)
Log nondurable cons. x 1981	0.103*** (9.09)	Log regional food CPI	0.643*** (3.88)
Log nondurable cons. x 1982	0.094*** (8.87)	Log regional fuel-util. CPI	-0.113*** (-2.75)
Log nondurable cons. x 1983	0.089*** (8.78)	White	0.047*** (6.91)
Log nondurable cons. x 1984	0.083*** (8.77)	Family size	0.055*** (17.34)
Log nondurable cons. x 1985	0.081*** (8.97)	High school	-0.252 (-1.22)
Log nondurable cons. x 1986	0.076*** (8.95)	College	-0.924*** (-4.10)
Log nondurable cons. x 1987	0.070*** (9.03)	Male head	0.082*** (15.41)
Log nondurable cons. x 1988	0.067*** (9.55)	Married	-0.030** (-2.42)
Log nondurable cons. x 1989	0.061*** (10.07)	Age	0.012*** (4.49)
Log nondurable cons. x 1990	0.051*** (10.05)	Age sq./100	-0.011*** (-4.13)
Log nondurable cons. x 1991	0.043*** (9.51)	Born 1924-1932	-0.017* (-1.67)
Log nondurable cons. x 1992	0.041*** (9.56)	Born 1933-1941	-0.012 (-0.90)
Log nondurable cons. x 1993	0.038*** (9.52)	Born 1942-1950	-0.004 (-0.24)
Log nondurable cons. x 1994	0.034*** (9.64)	Born 1951-1959	0.001 (0.06)
Log nondurable cons. x 1995	0.030*** (9.51)	Born 1960-1968	0.019 (0.80)
Log nondurable cons. x 1996	0.023*** (8.92)	Born 1969-1978	0.029 (1.02)
Log nondurable cons. x 1997	0.020*** (9.47)	Northeast	-0.013** (-2.33)
Log nondurable cons. x 1998	0.017*** (9.64)	Midwest	-0.061*** (-11.48)
Log nondurable cons. x 1999	0.013*** (8.57)	South	-0.037*** (-7.06)
Log nondurable cons. x 2000	0.011*** (9.59)	Constant	0.085 (0.24)
Log nondurable cons. x 2001	0.006*** (7.58)	Adj. R sq.	0.721
N	40,630	F	1264.1

Notes: t-statistics in parentheses. Instruments for log nondurable consumption (and its interaction with year and education dummies) are the averages of log head's wages specific to cohort, education, and head's sex in a given year (and their interactions with year and education dummies). *** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

Unemployment duration The unemployment duration is computed as the difference in weeks between the date when worker finds the job and the date when the worker lost his job, as inferred from the answers to the following questions: “What was the last date that you worked on that job before you applied for unemployment insurance benefits on (claims date)?”.

Severance pay This is a dichotomic variable that equals one if the worker has received a severance pay from the employer at the time of job displacement, according to the answer to the question “When that job ended, did you receive severance pay, a buyout or some other payment?”.

Tenure spline This is spline in the number of years spent working at the firm where the worker was laid off, as inferred from the answers to “Now, I’d like to ask you about the job you had just before you filed for unemployment benefits: When did you first start working for that employer?”; and “what was the last date that you worked on that job before you applied for unemployment insurance benefits?”.

Wage spline A spline of log wages in the pre-unemployment job. Wages are calculated using the question “How much did you usually make, before taxes and other deductions, per week when that job ended? Please include tips, commissions, and regular overtime”.

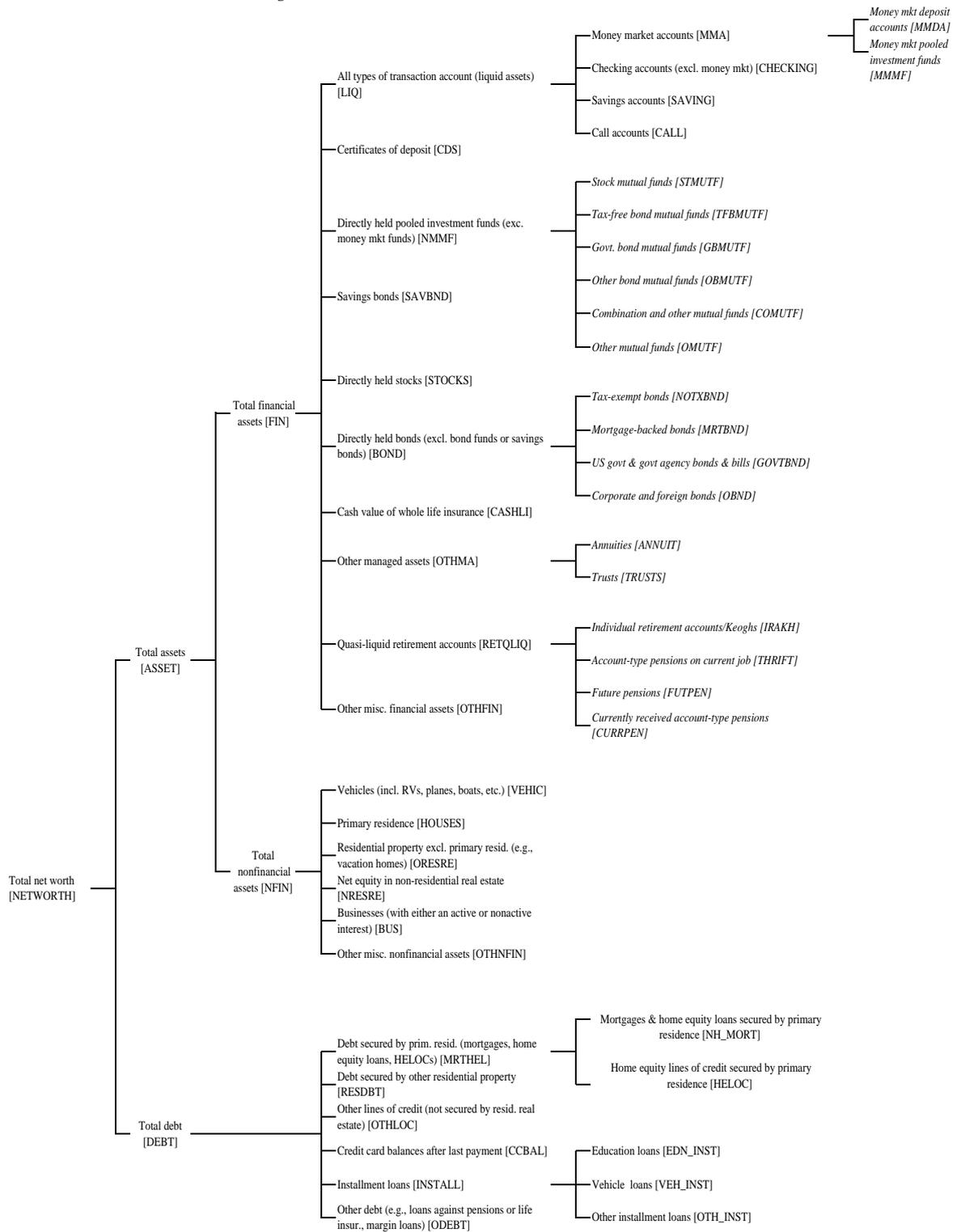
Unemployment benefits The log of the weekly UI benefit as reported in the Mathematica data. This information comes directly from the administrative data.

Educational dummies A series of dichotomic variables for dropout and college graduate workers, as inferred from the answer to the question “What is the highest diploma or degree you have received?”.

A5. SCF

The Survey of Consumer Finances is a triennial statistical survey of the balance sheet, pension, income and other demographic characteristics of families in the United States. Data are collected by the National Opinion Research Center at the University of Chicago under the sponsorship of the Federal Reserve Board. The SCF is intended to provide accurate descriptions of the current financial situations of US households, the data are primarily cross-section in nature, although some panel data have also been collected in recent years. Here we use the 2007 cross-section available at <http://www.federalreserve.gov/econresdata/scf/>. Since a large share of wealth is held by a relatively small share of the population, the SCF uses a dual sample frame, with a standard representative sample supplemented by a special sample of high-income taxpayers (see below). To calculate statistics we use sample weights as provided by SCF and we include all implicates. Our sample consists of household heads of 20 to 65 years of age who have been employed at least at some point of their life. The survey collects very detailed information on assets and liabilities, and the composition of household wealth which we use to construct the variable *Household’s net worth* following the criteria detailed in Figure A3.

Household’s average quarterly income in previous calendar year. It includes wages, self-employment and business income, taxable and tax-exempt interest, dividends, realized



Notes: Names in brackets refer to variables in the SCF Bulletin extract data.

FIGURE A3. CALCULATION OF NET WORTH IN SCF

capital gains, food stamps and other support programs provided by the government, pension income and withdrawals from retirement accounts, Social Security income, alimony and other support payments, and miscellaneous sources of income.” In calculating averages we drop households whose income is greater than 5 millions, which represents less than 1% of the households in the survey. The resulting average is divided by four to obtain a measure of households’ quarterly income.

DERIVATION OF \tilde{q}_n IN THE MODEL OF SECTION I

In this Appendix we extend the model of Section I in three dimensions:

- 1) We allow for the possibility that young workers in period one could save.
- 2) We assume that the benefit levels set by the government can affect the present value of the tax revenue available for the UI program. This is a natural equilibrium outcome when, as in the quantitative analysis, the UI program is financed through labor income taxes since benefits affect tax revenue through their effects on employment as well as on workers' human capital (which is accumulated on the job and/or lost upon unemployment). As a result the government budget constraint is as follows:

$$(B1) \quad T(b_y, b_o) \geq \beta_y \mu_y b_y + \beta_o \mu_o b_o.$$

- 3) We recognize that the optimal choice of benefits might be subject to some feasibility constraints, that impose that benefits can not fall below a minimum level \bar{b}_n so that the constraint in (9) has to be satisfied. In the quantitative analysis of Section III this minimum level is set to zero, so that $\bar{b}_n = 0$.

B1. Notation

Since young workers can save, outcomes when young affect workers decisions when old. Generally worker's choices are now contingent on the entire worker's history at the time when choices are made. In particular, this means that workers choose assets in periods $i = 2, 3, 4$ contingent on whether in period 2 the worker was unemployed, $j = u$, or employed, $j = e$. Similarly assets decisions in period $i = 5, 6$ are contingent on workers employment state, $j = u, e$ in period 2 as well on worker's employment state $z = u, e$ in period 5. Let a_i^j denote asset level in period $i = 3, 4, 5$, which is chosen in period $i - 1$ conditional on period two employment state $j = u, e$. Similarly let a_i^{jz} denote asset level in period $i = 6$, which is chosen in period five, conditional on period two employment state $j = u, e$ and period five employment state $z = u, e$. Clearly since all workers are initially identical, assets levels in period two, a_2 , are the same for all workers. We can then denote by \mathbf{a}_i , $i = 3, 4, 5, 6$ the vector of history dependent choice at $i - 1$ for next period asset values, so that

$$\mathbf{a}_i = (a_i^u, a_i^e), \quad \forall i = 3, 4, 5, \quad \text{while} \quad \mathbf{a}_6 = (a_6^{uu}, a_6^{ue}, a_6^{eu}, a_6^{ee})$$

With a slight abuse of notation we can also collect into a single vector all choices for next period assets $\mathbf{a} = (a_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5, \mathbf{a}_6)$. By applying the same logic to the choices for the unemployment probabilities in period two and five, we have that all workers choose the same unemployment probability μ_y in period two, while the choice for the unemployment probability in period five is μ_o^j , which is contingent on employment state in period 2, $j = u, e$. We can then define by

$$\boldsymbol{\mu} = (\mu_y, \mu_o^u, \mu_o^e)$$

the vector of choices of within-period unemployment probabilities. Let $p^j(\mu_y)$ denote the probability of being in state $j = u, e$ in period 2 and by $p^{jz}(\boldsymbol{\mu})$, the probability of being in state $j = u, e$ in period 2 and in state $z = u, e$ in period 5. Clearly we have $p^u(\mu_y) = \mu_y$ and $p^e(\mu_y) = 1 - \mu_y$, which allow to express $p^{jz}(\boldsymbol{\mu})$ as equal to

$$p^{jz}(\boldsymbol{\mu}) = p^j(\mu_y)p^z(\mu_o^j).$$

This makes explicit that the choice for the unemployment probability in period 5 is conditioned on the employment state in period two.

Since wage income is deterministic in age, consumption when employed in any period $i = 1, 2, 3, 4, 5, 6$ can be expressed as function of current period assets and next period assets so that

$$c_i^e(a_i, a_{i+1}) = w_i + \frac{a_i}{\beta} - a_{i+1}$$

where $w_4 = w_5 = w_6 = \bar{w}$. In period 2 and 5, the worker could be unemployed and his consumption is given by

$$c_2^u(a_2, a_3) = b_y + \frac{a_2}{\beta} - a_3, \quad \text{and} \quad c_5^u(a_5, a_6) = b_o + \frac{a_5}{\beta} - a_6,$$

respectively.

B2. Worker's problem

The utility at birth of workers is equal to the sum of the discounted utilities obtained by the worker in the first two periods of his life V^f , $i = 1, 2$, in the second two periods V^s , $i = 3, 4$, and the last two periods V^l , $i = 5, 6$, which are given by

$$\begin{aligned} V^f(\mu_y, a_2, \mathbf{a}_3) &= u(w_1 - a_2) + \beta \left[\psi(\mu_y) + \sum_{j=u, e} p^j(\mu_y) u(c_2^j(a_2^j, a_3^j)) \right], \\ V^s(\mu_y, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5) &= \beta^2 \sum_{j=u, e} p^j(\mu_y) \left[u(c_3^e(a_3^j, a_4^j)) + \beta u(c_4^e(a_4^j, a_5^j)) \right] \\ V^l(\boldsymbol{\mu}, \mathbf{a}_5, \mathbf{a}_6) &= \beta^4 \sum_{j=u, e} \sum_{z=u, e} p^{jz}(\boldsymbol{\mu}) \left[\psi(\mu_o^j) + u(c_5^z(a_5^j, a_6^{jz})) + \beta u(c_6^e(a_6^{jz})) \right] \end{aligned}$$

The utility at birth of workers can then be expressed as equal to

$$(B2) \quad U(b_y, b_o) = \max_{\boldsymbol{\mu}, \mathbf{a} \geq 0} \left\{ V^f(\mu_y, a_2, \mathbf{a}_3) + V^s(\mu_y, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5) + V^l(\boldsymbol{\mu}, \mathbf{a}_5, \mathbf{a}_6) \right\}$$

which fully characterizes the worker's problem.

B3. Government's problem

The government chooses b_y and b_o to maximize worker's utility at birth in (B2). In solving the problem the government takes into account that worker's unemployment probabilities are given by

$$\boldsymbol{\mu} = \arg \max \left\{ \max_{\mathbf{a} \geq 0} \left[V^f(\mu_y, a_2, \mathbf{a}_3) + V^s(\mu_y, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5) + V^l(\boldsymbol{\mu}, \mathbf{a}_5, \mathbf{a}_6) \right] \right\}$$

which defines $\boldsymbol{\mu}$ as a function of b_y and b_o , so we have $\mu_y = \mu_y(b_y, b_o)$, $\mu_o^u = \mu_o^u(b_y, b_o)$, and $\mu_o^e = \mu_o^e(b_y, b_o)$, which allow to define the unemployment level in period five as equal to

$$\mu_o(b_y, b_o) = \mu_y(b_y, b_o)\mu_o^u(b_y, b_o) + [1 - \mu_y(b_y, b_o)]\mu_o^e(b_y, b_o).$$

The Lagrangian of the government problem can then be written as

$$(B3) \quad \begin{aligned} L(b_y, b_o, \lambda, \omega_y, \omega_o) &= U(b_y, b_o) + \lambda [T(b_y, b_o) - \beta_y \mu_y(b_y, b_o)b_y - \beta_o \mu_o(b_y, b_o)b_o] \\ &+ \beta_y \omega_y (b_y - \bar{b}_y) + \beta_o \omega_o (b_o - \bar{b}_o) \end{aligned}$$

which incorporates both the government budget constraint in (B1) and the positive benefits constraint in (9). The constraint imposed by workers' choices for search effort are embodied in the construction of the functions $\mu_y(b_y, b_o)$, $\mu_o^u(b_y, b_o)$, and $\mu_o^e(b_y, b_o)$. After using the general envelope theorem, we obtain that it is optimal to increase b_n if

$$(B4) \quad \beta_n \mu_n E[u'(c_{un})] + \beta_n \omega_n > \lambda \beta_n \mu_n + \lambda \sum_{i=y,o} \beta_i \frac{\partial \mu_i}{\partial b_n} b_i - \lambda \frac{\partial T}{\partial b_n}$$

where $E[u'(c_{un})]$ denotes the expected marginal utility of consumption of an unemployed worker of age $n = y, o$ (period 2 or 5) which for $n = y$ is simply equal to

$$E[u'(c_{uy})] = u'(c_2^u(a_2, a_3^u))$$

since all unemployed young workers share the same history and consume the same. But for $n = o$, the expected marginal utility of consumption of an unemployed worker is equal to

$$E[u'(c_{uo})] = \frac{\mu_o^u u'(c_5^u(a_5^u, a_6^{uu})) + \mu_o^e u'(c_5^e(a_5^e, a_6^{eu}))}{\mu_y \mu_o^u + (1 - \mu_y) \mu_o^e}.$$

which recognizes that the consumption level of unemployed old workers depends on their employment status in period two, which could affect their asset position in period five. In the expression (B4) the second term in the left hand side and the last two terms in the right hand side of the inequality are novel relative to the first order condition (6) obtained in the simplified version of the model. By rearranging, we obtain that (B4) is equivalent to

$$(B5) \quad \tilde{\varrho}_n \equiv \frac{E[u'(c_{un})] + \frac{\omega_n}{\mu_n}}{1 + \tilde{\eta}_n - \frac{\partial T}{\partial b_n} \cdot \frac{1}{\beta_n \mu_n}} > \lambda$$

where

$$(B6) \quad \tilde{\eta}_n \equiv \sum_{i=y,o} \frac{\partial \mu_i}{\partial b_n} \cdot \frac{\beta_i b_i}{\beta_n \mu_n}$$

is the modified elasticity of age-group j unemployment to benefits as in (11). Equation (B5) corresponds to the expression for $\tilde{\varrho}_n$ in (10). Generally there are welfare gains from increasing transfers to young unemployed workers at the expense of the old whenever

$$(B7) \quad \tilde{\varrho}_y > \tilde{\varrho}_o.$$

This is the logic behind our modified redistribution formula $\tilde{\varrho}$.

B4. Back to the simple formula

Notice that when $a_2 = \mathbf{a}_3 = \mathbf{a}_4 = 0$, we also have that $a_5^u = a_5^e$ and $a_6^{uz} = a_6^{ez} \forall z = u, e$ which means that assets choices are independent of the employment state of workers in period two. In this case the employment choices of old workers become independent on the past history of workers $\mu_o^u = \mu_o^e$, and the cross elasticity in (B6) coincides with the simple elasticity studied in the main text. The simple formula of the main text is obtained when we also have $\omega_n = 0, \forall n = y, 0$ and the tax revenue allocated to the UI program is independent of the UI benefits set by the government, so that $\frac{\partial T}{\partial b_n} = 0 \forall n = y, o$.

COMPUTATIONAL DETAILS

Here we discuss first how we solve the quantitative model of Section III. Then we discuss how we calculate derivatives and redistribution formulas. Finally we turn to computational details about solving the optimal unemployment insurance program when search effort is observable.

C1. Solving the baseline economy

We discretize the state space of value functions. For points within the grid we interpolate using quadratic methods. The value functions for different n are constructed by backward induction, starting from $n = \bar{n}_w$ and then back until $n = 1$. In solving the model we impose (23) by using a recursive formulation. Let

$$C^r = \frac{1 - \beta^{\bar{n}_r}}{1 - \beta} \pi,$$

denote the present value of the cost to the government of providing retirement pensions to a worker who has just retired. Let $C^e(n, e, a)$ denote the cost to the government of providing the transfers net of taxes dictated by the government policy to an employed worker of age n , human capital e and with assets a . Similarly let $C^u(n, e, a)$ and $C^*(n, e, a)$ denote the analogous cost to the government when providing transfers to an unemployed worker with no human capital loss, and to an unemployed worker who, at some time during the unemployment spell, has experienced a loss in human capital, respectively. In calculating these cost functions the choice of workers for next period assets a' and the unemployment probability μ are taken as given. This is why all the state variables of the worker's problem enter the definition of the government cost function. Let $a^e(n, e, a)$, $a^u(n, e, a)$ and $a^*(n, e, a)$ denote the level of next period assets chosen, as function of age n , human capital e and current assets a , by a worker who is currently employed, unemployed with no loss of human capital and unemployed with human capital loss, respectively. Similarly let $\mu^u(n, e, a)$ and $\mu^*(n, e, a)$ denote the within-period unemployment probability chosen, as a function of state variables, by a worker who is unemployed with no loss of human capital and unemployed with human capital loss, respectively. Notice that for unemployed workers e refers to worker's human capital at displacement. These policy functions allow us to define explicitly the form of the government cost functions. The cost to the government of providing the transfers net of taxes dictated by the government policy to an employed worker of age n , human capital e and with assets a satisfies

$$\begin{aligned} C^e(n, e, a) &= -\tau_n w(e) + \beta(1 - \delta_n) C^e(n + 1, e + 1, a^e(n, e, a)) \\ &\quad + \beta \delta_n \mu^u(n + 1, e + 1, a^e(n, e, a)) C^u(n + 1, e + 1, a^e(n, e, a)) \\ &\quad + \beta \delta_n [1 - \mu^u(n + 1, e + 1, a^e(n, e, a))] C^e(n + 1, e + 1, a^e(n, e, a)) \end{aligned}$$

The analogous cost when providing transfers to an unemployed worker collecting UI benefits with no human capital loss satisfies

$$\begin{aligned} C^u(n, e, a) &= \rho_n w(e) + \beta(1 - \gamma) \mu^u(n + 1, e, a^u(n, e, a)) C^u(n + 1, e, a^u(n, e, a)) \\ &\quad + \beta(1 - \gamma) [1 - \mu^u(n + 1, e, a^u(n, e, a))] C^e(n + 1, e, a^u(n, e, a)) \\ &\quad + \beta \gamma \mu^*(n + 1, e, a^u(n, e, a)) C^*(n + 1, e, a^u(n, e, a)) \\ &\quad + \beta \gamma [1 - \mu^*(n + 1, e, a^u(n, e, a))] C^e(n + 1, \kappa(n + 1, e), a^u(n, e, a)) \end{aligned}$$

Finally the cost of the transfers to an unemployed worker collecting UI benefits who, at some time during the unemployment spell, has experienced a loss in human capital is equal to

$$\begin{aligned} C^*(n, e, a) &= \rho_n w(e) + \beta \mu^*(n + 1, e, a^*(n, e, a)) C^*(n + 1, e, a^*(n, e, a)) \\ &\quad + \beta [1 - \mu^*(n + 1, e, a^*(n, e, a))] C^e(n + 1, \kappa(n + 1, e), a^*(n, e, a)) \end{aligned}$$

where these expressions are defined for $n \leq \bar{n}_w$ with the convention that

$$C^e(\bar{n}_w + 1, e, a) = C^u(\bar{n}_w + 1, e, a) = C^*(\bar{n}_w + 1, e, a) = C^r$$

At birth, $n = 1$, workers have to search for a job, they have no experience and no assets so imposing (23) is equivalent to imposing the requirement that

$$C^u(1, 0, 0) = 0.$$

C2. Calibration of the functions $\psi(\mu)$

The second derivative of the function ψ has to be negative $\psi'' \leq 0$ and it plays a key role in determining the value of the elasticity of unemployment with respect to benefits. So we decided to model its profile directly and to impose explicitly the constraint that the second derivative is always negative, $\psi'' \leq 0$. To achieve this the function $\psi(\mu)$ is parameterized in terms of six values. The value of the second derivative of ψ at $\mu = 0$, the analogous value at $\mu = 1$ and its maximum value (minimum absolute value) which is reached at an endogenously determined μ^* . All these values are constrained to be negative. In addition, the function $\psi(\mu)$ is characterized by its value at $\mu = 1$ and the value of its first derivative at $\mu = 1$. To sum-up the function ψ is fully characterized by the following six values: $\psi(1)$, $\psi'(1) \geq 0$, $\psi''(1) \leq 0$, $\psi''(0) \leq 0$, $\psi''(\mu^*) \leq 0$ and finally by the value for μ^* at which ψ'' reaches its maximum. The profile of the second derivatives of ψ'' is a linear spline through the knots at $\psi''(1) \leq 0$, $\psi''(0) \leq 0$, and $\psi''(\mu^*) \leq 0$. We impose the constraint that ψ'' can never be greater than $\psi''(\mu^*)$. By integrating the profile of second derivatives and given an initial condition for the first derivative—which is provided by the value of $\psi'(1) \geq 0$ —we obtain a characterization of the entire profile of ψ' . Given the profile of ψ' and an initial condition for ψ at $\mu = 1$, we finally obtain the entire profile of the function ψ , which is plotted in the main text. It is easy to check that the resulting ψ function is perfectly approximated by a cubic spline evaluated at five different values where the intermediate knot μ^* is endogenously determined.

C3. Variables definition

We discuss the construction of some variables in the model.

MASS OF WORKERS COLLECTING UI BENEFITS

As discussed in the text, the fraction of workers of age n who are collecting UI benefits is equal to

$$(C1) \quad \mu_n = \int_{R^+} \chi^u(n, de)$$

MASS OF WORKERS SEARCHING FOR A JOB

Let $\chi^s(n, e)$ denote the measure of workers of age n who are searching for a job and who were displaced with human capital e . Given $\chi^e(n, e)$ and $\chi^u(n, e)$ we have that

$$\chi^s(n, e) = \chi^u(n-1, e) + \delta_{n-1} \chi^e(n-1, e-1)$$

The right hand side is the sum of two terms that measure the two inflows into the pool of searchers of age n . The first term measures the mass of workers of age $n-1$ who have collected benefits in a period and who will search for a job in the next period when they are one year older. The second term takes into account that the pool of workers searching for a job is also augmented by the fraction δ_{n-1} of the workers of age $n-1$ and human capital $e-1$ who are currently employed and lose their job. These workers will have to search for a new job in the next period when they have age n and human capital e . The mass of workers of age n searching for a job can then be defined as equal to

$$\sigma_n = \int_{R^+} \chi^s(n, de).$$

whose interpretation is analogous to that in (C1).

JOB FINDING RATE

The job finding rate of workers of age n is denoted by f_n and it is simply equal to the ratio between the mass of workers of age n who find a job in a period and the pool of searchers of age n in that period:

$$(C2) \quad f_n = \frac{\sigma_n - \mu_n}{\sigma_n}.$$

UNEMPLOYMENT RATE

In standard surveys workers are classified as unemployed if, at the interview date, workers are without a job and they are actively searching for a job. In terms of our model, this number is different depending on whether the interview were to be performed at the beginning of, during or at the end of the model period, which corresponds to one quarter in our calibration. To account for this problem we assume that the unemployment rate of workers of age n is equal to $u_n = 0.5\sigma_n + 0.5\mu_n$. This expression can be justified by assuming that workers interviews occur randomly over the period and that

workers find jobs at a constant uniform linear probability in the period. Let f_n denote the model probability of finding a job in a period for a given age group n . Assume that the probability of finding a job in the first i th fraction of the period is if_n where $i \in [0, 1]$.³² Then the probability that a worker of age n is classified as unemployed in the period is equal to

$$1 - \int_0^1 i f_n di = 1 - 0.5f_n$$

Given these considerations we can then define the unemployment of workers of age n as equal to

$$u_n = (1 - 0.5f_n) \sigma_n = 0.5\sigma_n + 0.5\mu_n$$

where the last equality uses the definition of the age specific job finding rate in (C2).

UNEMPLOYMENT DURATION

One period in the model corresponds to one quarter. Unemployment duration in the CPS data is measured in weeks. To convert the unemployment duration of the model into weeks we proceed as follows. Let f_n denote the probability of finding a job in a period for a given age group n . Remember that one period in the model correspond to one quarter. Now assume that the quarter is divided into 13 weeks and that workers are randomly interviewed in any of these 13 weeks to infer the duration of their current unemployment spell. Also assume that the probability that a worker finds a job in any week of the quarter is

$$\bar{f}_n = 1 - (1 - f_n)^{\frac{1}{13}}$$

This guarantees that the probability of remaining unemployed in all the 13 consecutive weeks of the quarter is $(1 - \bar{f}_n)^{13} = 1 - f_n$. Then, if workers have an unemployment duration in the model equal to $j = 0, 1, 2, 3, \dots$, their unemployment duration in weeks is calculated as equal to $I + 13j$ where

$$I = \frac{\sum_{i=1}^{12} i (1 - \bar{f}_n)^i}{\sum_{i=0}^{12} (1 - \bar{f}_n)^i},$$

which measures the average duration in weeks of workers who have been just displaced from their previous job and start searching for a new one in the current model period. After remembering that

$$\sum_{i=0}^T (1 - \bar{f}_n)^i = \frac{1 - (1 - \bar{f}_n)^{T+1}}{\bar{f}_n}$$

and that

$$\sum_{i=1}^T i (1 - \bar{f}_n)^{i-1} = \frac{1 - (1 + \bar{f}_n T) (1 - \bar{f}_n)^T}{(\bar{f}_n)^2}$$

³²Notice that the probability if_n can also be justified by taking a Taylor expansion, around $f_n = 0$, of the expression for the geometric probability of finding a job before i , which is equal to $1 - (1 - f_n)^i$.

we can express I as equal to

$$I = \frac{(1 - \bar{f}_n) - (1 + 12 \times \bar{f}_n) (1 - \bar{f}_n)^{13}}{\bar{f}_n - \bar{f}_n (1 - \bar{f}_n)^{13}}$$

This is the expression for I used in the computer code.

C4. Age profiles of separation rates and wage losses upon re-employment in baseline calibration

Figure A4 plots the age profiles in the baseline calibration of separation rates, δ_n , panel (a) and wage losses upon re-employment $\bar{\kappa}_n - 1$, panel (b).

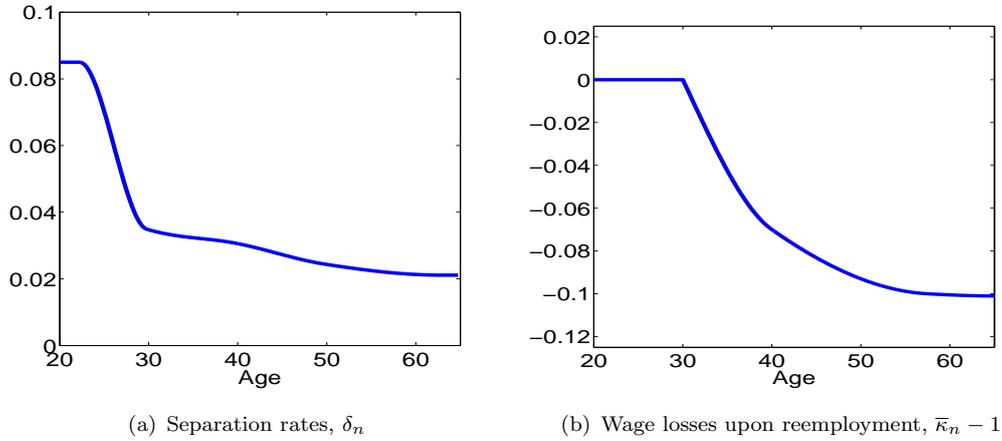


FIGURE A4. SOME FUNCTIONS IN THE BASELINE CALIBRATION OF LABORATORY ECONOMY

C5. Calculating elasticities and redistribution formulas in the quantitative analysis

Let $\boldsymbol{\rho} = \{\rho_1, \dots, \rho_{\bar{n}_w}\}$ denote the vector containing the entire age profile of UI replacement rates in a baseline economy. To calculate elasticities and redistribution formulas at age n of the corresponding baseline economy, we consider changes in replacement rates at p consecutive quarters starting from age n . For every n we then consider two economies one with lower and one with higher replacement rates at age n , $\boldsymbol{\rho}_n^i = \{\rho_1, \dots, \rho_{n-1}, \vartheta_n^i, \vartheta_{n+1}^i, \dots, \vartheta_{n+p-1}^i, \rho_{n+p}, \dots, \rho_{\bar{n}_w}\}$, $i = l, h$ where $\vartheta_{n+j}^l = \rho_{n+j} - \frac{\epsilon}{2}$ and $\vartheta_{n+j}^h = \rho_{n+j} + \frac{\epsilon}{2}$, $\forall j = 0, 1, \dots, p-1$. In the paper we work with $\epsilon = 0.02$ and $p = 4$, which corresponds to a change in benefits for an age group of one year. We consider one year changes in benefits both to increase sample size and to reduce the likelihood that the change in policy affects workers' search effort decisions though effects on unemployment duration dependence in benefits, which is an issue somewhat unrelated to age-dependent policies. To avoid this problem we could have indexed the level of replacement rates, rather than to current age, to the age at which the worker is displaced. But this specification would require having an additional state variable in the worker problem, which would involve additional computational costs. We checked that results are little affected when choosing $p = 3$ or $p = 5$ rather than $p = 4$.

CALCULATING η_n

Given the policy change for age group n the elasticity of unemployment is calculated as equal to

$$(C3) \quad \eta_n = \frac{\sum_{i=0}^{p-1} d\mu_{n+i} \rho_{n+i}}{\epsilon \sum_{i=0}^{p-1} \mu_{n+i}^0},$$

In the expression μ_{n+i}^0 denotes the measure of workers of age $n+i$ who are collecting UI benefits before the policy change while $d\mu_{n+i}$ denotes the difference between the level of workers of age $n+i$ who are collecting UI benefits in the economy with high benefits ρ_n^h and the analogous level in the economy with low benefits ρ_n^l . Notice that for completeness $d\mu_{n+i}$ should be indexed to the policy change considered. Yet to simplify notation we did not make this dependence explicit. Conceptually in the expression (C3), the exogenous change in benefits is equal to $db_n = w_n^0 d\rho_n = w_n^0 \epsilon$, where w_n^0 denotes the average wage at displacement of the workers of age n who are collecting UI benefits before the policy change. When $p = 1$, (C3) reads as $\eta_n = \frac{d\mu_n \rho_n}{\epsilon \mu_n^0}$ which is simply the elasticity of unemployment with respect to replacement rates. So in calculating the unemployment elasticity, the denominator includes just the really exogenous component of the change in government expenditures for the UI program, omitting any induced effect due to either changes in unemployment or in workers' human capital. Notice that (C3) takes into account that benefits are changed for p consecutive quarters. The expression in (C3) can be interpreted as a simple weighted average of the relevant unemployment elasticities with respect to UI replacement rates evaluated at n up to $n+p-1$, with weights equal to the relative unemployment of the age group, $\frac{\mu_{n+i}^0}{\sum_{i=0}^{p-1} \mu_{n+i}^0}$. Finally

notice that for the purpose of calculating unemployment elasticities unemployment is defined as the mass of workers who are collecting UI benefits, which is consistent with the empirical analysis.

CALCULATING \tilde{q}_n

Consider the model in Section B and let

$$D(b_y, b_o) = \sum_{i=y,o} \beta_i \mu_i(b_y, b_o) b_i - T(b_y, b_o)$$

denote the government budget deficit, where T denotes the expected present value of the tax revenue collected by the government to finance the UI program. Notice that the denominator of the modified formula in (10) is equal to the ratio of the derivative of the government budget deficit with respect to the age specific level of benefits and the level of the group specific unemployment rate—i.e. it is equal to $\frac{\partial D}{\partial b_n} \cdot \frac{1}{\mu_n}$. When b_n marginal increases, the budget deficit increases because of i) the increase in the transfers received by the already unemployed workers targeted by the policy change (equal to μ_n), ii) the increase in transfers due to the increase in unemployment of all different age groups

(which is measured by the term $\sum_{i=y,0} \frac{\partial \mu_i}{\partial b_n} \beta_i b_i$), and iii) the reduction in tax revenue (which is measured by the term $-\frac{dT}{db_j}$). This leads to an alternative expression for \tilde{q}_n (10) as equal to

$$(C4) \quad \tilde{q}_n = \frac{E[u'(c_{un})] + \frac{\omega_n}{\mu_n}}{\frac{1}{\beta_n \mu_n} \cdot \frac{\partial D}{\partial b_n}}$$

This is the expression we use to calculate \tilde{q}_n in the quantitative analysis of Section III where the government deficit is defined as equal to

$$(C5) \quad D(\boldsymbol{\rho}) = \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de) + \sum_{n=\bar{n}_w+1}^{\bar{n}_r} \beta^n \pi \chi^r(n) - \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \tau_n w(e) \chi^e(n, de)$$

The feasibility constraint in the quantitative analysis is $\rho_n \geq 0$. To calculate ω_n we notice that, if the constraint in (9) is not binding for age group n , we have that $\omega_n = 0$, while, if the constraint is binding, ω_n can be calculated by measuring by how much welfare would fall if we were to increase the value of the constraint \bar{b}_n . If we denote by W the expected welfare of workers at birth under a given choice of $\boldsymbol{\rho}$, we should have that

$$\frac{dW}{d\bar{b}_n} = -\beta^n \omega_n$$

which is zero when the constraint is not binding. Notice that the present value Lagrange multiplier of the feasibility constraint is $\beta^n \omega_n$, exactly as in (B3). The expected marginal utility of consumption of unemployed workers of age n is calculated as equal to the average expected marginal utility of all age groups affected by the policy change, $\frac{1}{p} \sum_{i=0}^{p-1} E[u'(c_{un+i})]$. Overall the modified formula in the quantitative analysis is calculated as equal to

$$(C6) \quad \tilde{q}_n = \frac{\frac{1}{p} \sum_{i=0}^{p-1} \left\{ E[u'(c_{un+i})] + \frac{\omega_{n+i}}{\mu_{n+i}} \right\}}{\frac{dD}{\epsilon \sum_{i=0}^{p-1} \beta^{n+i} \mu_{n+i}^0 w_{n+i}^0}},$$

which, exactly as in (C3), takes into account that benefits are changed for p consecutive quarters. In the expression above w_n^0 denotes again the average wage at displacement of the workers of age n who are collecting UI benefits before the policy change, while μ_n^0 is the mass of workers of age n who are collecting UI benefits before the policy change. As in (C3), the exogenous change in benefits is equal to $db_n = w_n^0 d\rho_n = w_n^0 \epsilon$. The denominator of (C6) corresponds to the denominator in (C4) which is the derivative of government deficit with respect to changes in benefits. The term dD denotes the difference between the government deficit in the economy with high benefits levels $\boldsymbol{\rho}_n^h$ and the analogous value in the economy with low benefit levels $\boldsymbol{\rho}_n^l$. In principle dD should have been indexed to the policy change considered. Yet to simplify notation we did not make this dependence explicit. Finally notice that in the denominator of the derivative of government deficit with respect to benefits, we focus just on the really exogenous change in government expenditures for the UI program, omitting any induced effects

due to either changes in unemployment or in workers' human capital.

CALCULATING THE CROSS ELASTICITY $\tilde{\eta}_n$

Let

$$B(b_y, b_o) = \sum_{i=y,o} \beta_i \mu_i(b_y, b_o) b_i$$

denote the total amount of government money spent for unemployment insurance. It is easy to check that in the model in Section B

$$(C7) \quad \frac{\partial B(b_y, b_o)}{\partial b_n} = \beta_n \mu_n (1 + \tilde{\eta}_n)$$

To calculate $\tilde{\eta}_n$ in (B6) we first calculate the changes in the money spent for unemployment insurance B and then obtain the elasticity $\tilde{\eta}_n$ by using the fact that (C7) implies that

$$(C8) \quad \tilde{\eta}_n = \frac{1}{\beta_n \mu_n} \cdot \frac{\partial B(b_y, b_o)}{\partial b_n} - 1$$

This is the expression we use to calculate $\tilde{\eta}_n$ in the quantitative analysis of Section III where the total amount of government money spent for unemployment insurance is defined as equal to

$$(C9) \quad B(\rho) = \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de).$$

Given (C7) and the policy changes discussed in Section C.C5, we then calculate $\tilde{\eta}_n$ as equal to

$$(C10) \quad \tilde{\eta}_n = \frac{dB}{\epsilon \sum_{i=0}^{p-1} \beta^{n+i} \mu_{n+i}^0 w_{n+i}^0} - 1$$

where dB is the difference in UI expenditures between the economy with high benefit levels ρ_n^h and the economy with low benefits ρ_n^l . Notice that for completeness dB should have been indexed to the policy change considered. Yet to simplify notation we did not make this dependence explicit. The logic of calculating the derivative as in (C10) is analogous to the logic used to obtain (C3) or (C6).

C6. Solving the optimal UI problem with observable search effort

Here we discuss how we solve the optimal life cycle unemployment insurance problem of Section IV where search effort is observable, which corresponds to the first best problem. To solve for this problem we search for the value of consumption c^* that makes $\Upsilon(1, 0, c^*)$ in (25) equal to zero:

$$(C11) \quad \frac{1 - \beta^{\bar{n}_w + \bar{n}_r}}{1 - \beta} c^* - Y(1, 0, c^*) = 0$$

where the function $Y(n, e, c)$ denotes the expected present value of income generated by a worker of age n searching for a new job who had human capital e at the start of the current job search spell and who has not experienced any loss in his human capital. Here c denotes the constant consumption flow granted to the worker in each remaining period of his life. Let $\bar{\mu}(n, e, c)$ denote the within-period unemployment probability function under the optimal policy of an unemployed worker of age n , who had human capital e at the start of the current job search spell, who is granted consumption flow c for each remaining period of his life and who has not experienced any loss in human capital during the current job search spell. Also let $\bar{\mu}^*(n, e, c)$ denote the within-period unemployment probability function analogous to $\bar{\mu}(n, e, c)$ but for an unemployed worker who has already experienced a loss in human capital. The functions $\bar{\mu}(n, e, c)$ and $\bar{\mu}^*(n, e, c)$ are characterized below by (C18) and by (C19), respectively. Given these functions we can express Y recursively as equal to:

$$(C12) \quad Y(n, e, c) = \bar{\mu}(n, e, c)\beta[(1 - \gamma)Y(n + 1, e, c) + \gamma Y^*(n + 1, e, c)] \\ + [1 - \bar{\mu}(n, e, c)]Y^e(n, e, c)$$

where

$$(C13) \quad Y^e(n, e, c) = w(e) + \beta[(1 - \delta_n)Y^e(n + 1, e + 1, c) + \delta_n Y(n + 1, e + 1, c)]$$

is the expected present value of income generated by an employed worker of age n with human capital e who is granted consumption level c for all the remaining periods of his life, while

$$(C14) \quad Y^*(n, e, c) = \bar{\mu}^*(c, e, n)\beta Y^*(n + 1, e, c) + [1 - \bar{\mu}^*(n, e, c)]Y^e(n, \kappa(n, e), c)$$

is the expected present value of income analogous to $Y(n, e, c)$ but for an unemployed worker who has already experienced a loss in human capital. The expression in (C14) incorporates the assumption that, after experiencing a loss in human capital, the worker is reemployed with human capital $\kappa(n, e)$ where e refers to worker's human capital at the start of the current job search spell.

Consider a worker of age n searching for a job who had human capital e at the start of the current job search spell, who is granted consumption flow c for each remaining period of his life and who has not experienced any loss in human capital during the current job search spell. Under the optimal policy, this worker generates a value measured in utils equal to

$$(C15) \quad S(n, e, c) = \max_{\mu \in [0,1]} \{ \psi(\mu) + \mu\beta[(1 - \gamma)S(n + 1, e, c) + \gamma S^*(n + 1, e, c)] \\ + (1 - \mu)S^e(n, e, c) \}$$

where

$$(C16) \quad S^e(n, e, c) = u'(c)w(e) + \beta[(1 - \delta_n)S^e(n + 1, e + 1, c) + \delta_n S(n + 1, e + 1, c)]$$

is the expected present value, again measured in utils, of the income produced by an

employed worker of age n with human capital e who is granted consumption level c for all the remaining periods of his life. The present utility value of searching for a worker who has already experienced a loss in human capital is instead equal to

$$(C17) \quad S^*(n, e, c) = \max_{\mu \in [0,1]} \{ \psi(\mu) + \mu\beta S^*(n+1, e, c) + (1-\mu) S^e(n, \kappa(n, e), c) \}$$

which is analogous to $S(n, e, c)$ but for the case when the unemployed worker has already experienced a loss in human capital. For any n , e , and c the within-period unemployment probability functions under the optimal policy $\bar{\mu}$ and $\bar{\mu}^*$ are implicitly defined by (C15) and (C17), respectively. So we have

$$(C18) \quad \bar{\mu}(n, e, c) = \arg_{\mu \in [0,1]} \max \{ \psi(\mu) + \mu\beta [(1-\gamma)S(n+1, e, c) + \gamma S^*(n+1, e, c)] + (1-\mu)S^e(n, e, c) \}$$

and

$$(C19) \quad \bar{\mu}^*(n, e, c) = \arg_{\mu \in [0,1]} \max \{ \psi(\mu) + \mu\beta S^*(n+1, e, c) + (1-\mu)S^e(n, \kappa(n, e), c) \}.$$

For any given value of c , we can use (C15), (C16) and (C17) to solve for S , S^e , and S^* backward starting from $n = \bar{n}_w$ after using the terminal conditions $S(\bar{n}_w + 1, e, c) = S^e(\bar{n}_w + 1, e, c) = S^*(\bar{n}_w + 1, e, c) = 0$. The within-period unemployment probability functions $\bar{\mu}$ and $\bar{\mu}^*$ are then obtained by using (C18) and (C19), respectively. Given $\bar{\mu}$ and $\bar{\mu}^*$ and after using the terminal conditions $Y(\bar{n}_w + 1, e, c) = Y^e(\bar{n}_w + 1, e, c) = Y^*(\bar{n}_w + 1, e, c) = 0$ we can use (C12), (C13) and (C14) to calculate Y , Y^e , and Y^* which can be used to solve for the value of c^* that satisfies (C11).

C7. Consumption equivalent gains relative to baseline calibration

All consumption equivalent gains are calculated relative to the baseline calibration discussed in Section III.C. Let's start defining the relevant policy rules under the baseline calibration for consumption, next period asset levels and within-period unemployment probabilities. Policy rules are always specified as a function of worker's age n , worker's human capital e and worker's current assets a . For unemployed workers e refers to human capital at the start of the current job search spell. More specifically, let $\hat{c}^e(n, e, a)$, $\hat{c}^u(n, e, a)$ and $\hat{c}^*(n, e, a)$ denote the consumption level chosen in the baseline calibration by a worker who is currently employed, unemployed with no loss of human capital and unemployed with human capital loss, respectively. Also let $\hat{a}^e(n, e, a)$, $\hat{a}^u(n, e, a)$ and $\hat{a}^*(n, e, a)$ denote the next period asset level chosen by a worker who is currently employed, unemployed with no loss of human capital and unemployed with human capital loss, respectively. Finally let $\hat{\mu}^u(n, e, a)$ and $\hat{\mu}^*(n, e, a)$ denote the within-period unemployment probability chosen in the baseline calibration by a worker who is unemployed with no loss of human capital and unemployed with human capital loss, respectively. Now suppose that in every possible state consumption level choices are multiplied by a factor θ . We can then calculate the utility value from consumption obtained by the worker in each possible state after this multiplicative change in consumption levels. When the worker is employed at age n , with human capital e and asset level a the

utility value he obtains from consumption becomes equal to

$$(C20) \quad L^e(n, e, a, \theta) = \theta^{1-\sigma} u(\widehat{c}^e(n, e, a)) + \beta(1-\delta_n)L(n+1, e+1, \widehat{a}^e(n, e, a), \theta) + \beta\delta_n L^j(n+1, e+1, \widehat{a}^e(n, e, a), \theta)$$

which uses the fact that the utility function is CARA and for simplicity we assumed that $\sigma \neq 1$. So the scaling factor in consumption θ can be taken out of the utility function as a multiplicative factor. The last term incorporates the fact that with probability δ_n a worker of age n has to search for a new job whose utility value from consumption is given by

$$(C21) \quad L^j(n, e, a, \theta) = \widehat{\mu}^u(n, e, a) L^u(n, e, a, \theta) + [1 - \widehat{\mu}^u(n, e, a)] L^e(n, e, a, \theta)$$

where

$$(C22) \quad L^u(n, e, a, \theta) = \theta^{1-\sigma} u(\widehat{c}^u(n, e, a)) + \beta(1-\gamma)L^j(n+1, e, \widehat{a}^u(n, e, a), \theta) + \beta\gamma L^{j^*}(n+1, e, \widehat{a}^u(n, e, a), \theta)$$

denotes the utility value from consumption when being unemployed without having experienced a loss in human capital. The function L^{j^*} in (C22) denotes the utility value from consumption when searching for a job after having experienced a loss in human capital, which satisfies the following relation

$$(C23) \quad L^{j^*}(n, e, a, \theta) = \widehat{\mu}^*(n, e, a) L^*(n, e, a, \theta) + [1 - \widehat{\mu}^*(n, e, a)] L^e(n, \kappa(n, e), a, \theta).$$

In the expression above L^* denotes the utility value from consumption when being unemployed after a loss in human capital, which satisfies

$$(C24) \quad L^*(n, e, a, \theta) = \theta^{1-\sigma} u(\widehat{c}^*(n, e, a)) + \beta L^{j^*}(n+1, e, \widehat{a}^*(n, e, a), \theta).$$

In writing (C20), (C22) and (C24) we adopted the convention that

$$L(\bar{n}_w + 1, e, a, \theta) = L(\bar{n}_w + 1, e, a, \theta) = L^*(\bar{n}_w + 1, e, a, \theta) = \theta^{1-\sigma} \cdot \frac{1 - \beta^{\bar{n}_r}}{1 - \beta} u(c^r(a))$$

The utility value from consumption at birth is equal to $L^j(1, 0, 0, \theta)$, which satisfies

$$L^j(1, 0, 0, \theta) = \theta^{1-\sigma} L^j(1, 0, 0, 1).$$

Notice that $L^j(1, 0, 0, 1)$ is the utility value from consumption at birth in the baseline calibration. Now consider a policy reform that yields a welfare gains relative to the welfare in the baseline calibration equal to $\Delta W \equiv \Delta J(1, 0, 0)$. The consumption equivalent gain change is calculated as equal to $\theta(\Delta W) - 1$ where $\theta(\Delta W)$ is given by

$$\theta(\Delta W) = \left[\frac{\Delta W}{L^j(1, 0, 0, 1)} + 1 \right]^{\frac{1}{1-\sigma}}.$$

C8. Baseline economy under the natural borrowing limit

We analyze our economy under the assumption that workers face a natural borrowing limit, equal to the present value of the income that the worker would obtain if he were to shirk in every working period until retirement. This implies that the assets of a worker of age n with human capital e should be greater than

$$l(n, e) = - \left[\frac{1 - \beta^{\bar{n}_w - n + 1}}{1 - \beta} \rho w(e) + \beta^{\bar{n}_w - n + 1} \frac{1 - \beta^{\bar{n}_r}}{1 - \beta} \pi \right].$$

To understand the term in square brackets notice that if a worker of age n who has human capital e shirks forever, he obtains UI benefits $\rho w(e)$ in each of the $\bar{n}_w - n + 1$ remaining periods of his working life and then he will obtain retirement pensions π that he will start receiving in $\bar{n}_w - n + 1$ periods. Under this assumption the key equations of the model in Section III should be modified to incorporate the new borrowing limit. The value of being an employed worker of age $n \leq \bar{n}_w$ with human capital e and assets a will now be given by:

$$(C25) \quad V(n, e, a) = \max_{a' \geq l(n+1, e+1)} u(c^e(e, a, a')) + \beta [(1 - \delta_n)V(n+1, e+1, a') + \delta_n J(n+1, e+1, a')]$$

which takes into account that next period assets should be greater than $l(n+1, e+1)$. Remember that $c^e(e, a, a') = (1 - \tau_n)w(e) + (1 + r)a - a'$. The value of searching is again given by

$$J(n, e, a) = \max_{\mu \in [0, 1]} \psi(\mu) + \mu U(n, e, a) + (1 - \mu)V(n, e, a)$$

The value of remaining unemployed at the end of the period should now satisfy

$$(C26) \quad U(n, e, a) = \max_{a' \geq l(n+1, e)} u(c^u(e, a, a')) + \beta (1 - \gamma) J(n+1, e, a') + \beta \gamma J^*(n+1, e, a')$$

where $c^u(e, a, a') = \rho_n w(e) + (1 + r)a - a'$ denotes again current period consumption when unemployed. The function J^* denotes the value of searching after experiencing a loss in human capital, which remains untouched and again given by

$$J^*(n, e, a) = \max_{\mu \in [0, 1]} \psi(\mu) + \mu U^*(n, e, a) + (1 - \mu)V(n, \kappa_n e, a)$$

The value of being unemployed after a loss in human capital satisfies

$$(C27) \quad U^*(n, e, a) = \max_{a' \geq l(n+1, e)} u(c^*(e, a, a')) + \beta J^*(n+1, e, a')$$

where again $c^*(n, e, a, a') = \rho_n w(e) + (1 + r)a - a'$ and the borrowing limit is as in (C26). We also still adopt the convention that

$$V(\bar{n}_w + 1, e, a) = U(\bar{n}_w + 1, e, a) = U^*(\bar{n}_w + 1, e, a) = \frac{1 - \beta^{\bar{n}_r}}{1 - \beta} u(c^r(a))$$

where $c^r(a) = \pi + \frac{ra}{1-\beta^{\bar{n}_r}}$. The government budget constraint is also untouched and reads as follows

$$(C28) \quad \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de) + \sum_{n=\bar{n}_w+1}^{\bar{n}_r} \beta^n \pi \chi^r(n) = \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \tau_n w(e) \chi^e(n, de)$$

where integrals are conventionally defined Lebesgue integrals. Here again $\chi^e(n, e)$ denotes the measure of employed workers of age n and experience e , $\chi^u(n, e)$ denotes the measure of unemployed workers of age n who were displaced with human capital e and finally $\chi^r(n) = \int \chi^e(\bar{n}_w, de) + \int \chi^u(\bar{n}_w, de) = \chi^r$ denotes the measure of retired workers of age n , which is constant and independent of age.

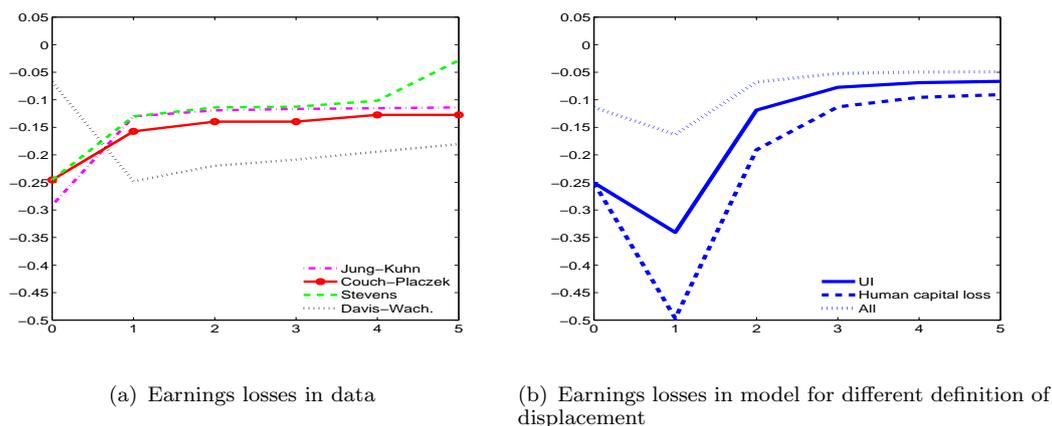
WAGE AND EARNINGS LOSSES IN THE LITERATURE AND IN THE MODEL

In this section we first briefly review estimates from the empirical literature on earnings losses upon displacement. We then compare estimates with measures of earnings losses in the model. We focus on two parametrizations of the model: one is the baseline calibration of Section III.C of the paper, the other is the re-calibrated version of the model discussed in Section VI.E where we double the magnitude of wage losses during unemployment. We then discuss robustness to alternative characterizations of earnings losses in the model.

D1. Earnings losses in the literature

We compare yearly earnings losses upon displacement in the model with estimates from the literature as obtained by Davis and von Wachter (2011), Stevens (1997), Couch and Placzek (2010) and also reported in Jung and Kuhn (2012). In Figure A5 we plot the profile of earnings losses at different years after worker displacement as estimated by the literature. Displacement occurs in period zero. The black dotted line corresponds to the estimates by Davis and von Wachter (2011). Davis and von Wachter (2011) focus on a sample of male employees with at least 3 years of job tenure prior to displacement and employed in firms with at least 50 employees. A worker is classified as displaced in year t if he separates from the employer in t and the employer experiences a mass layoff between $t - 1$ and t roughly defined as an employment contraction of at least 30% in the period. They measure yearly labor earnings of displaced workers and compare these earnings to the earnings of workers with identical characteristics in terms of age, tenure and firm size before displacement but who did not separate from their main employers in year t , $t + 1$, and $t + 2$. Davis and von Wachter (2011) just provide measure of income losses in thousands of dollars. To obtain a measure of earnings losses in percentage terms, we infer from Table 1 of their paper that the average annual income of the control group in their sample is \$ 49,806, which allows us to obtain a measure of losses as a proportion of labor income of the control group.³³ Davis and von Wachter (2011) provide measures of earnings losses in recessions and in expansions. We focus on the measure during expansion years, which are similar to the average effects in their sample. The red solid line in Figure A5 corresponds to the profile of earnings losses after displacement estimated by Couch and Placzek (2010), who mimic the analysis in Jacobson, LaLonde and Sullivan (1993), using Connecticut data over the period 1993-2004. Workers are defined as displaced, exactly as in Davis and von Wachter (2011). Earnings of displaced workers are compared to the earnings of continuously employed workers. Since earnings are at the quarterly frequency, we obtain a measure of yearly losses by cumulating losses in the year. The profile in Figure A5, is obtained using column FE of the table at page 27 of the Online Appendix of the paper. The green dashed line in Figure A5 corresponds to the profile of earnings losses estimated by Stevens (1997) using a sample of household heads on PSID data over the 1968-1988

³³To calculate the analogous annual earnings for workers of different age, we use our estimated age profile of labor income from CPS. For example this profile says that workers of 21 to 30 years of age earn around 72 % of the average earnings in the economy, which suggests setting an annual income of \$ 35,930 for the control group of displaced workers of this age. For other age groups we proceed analogously.



Notes: Panel (a) plots the profile of yearly earnings losses in the model and in several papers in the literature. The black dotted line corresponds to the estimates by Davis and von Wachter (2011), the red solid line to those by Couch and Placzek (2010), and the green dashed line to those by Stevens (1997). The purple dash dotted line is the profile of earnings losses generated by the model of Jung and Kuhn (2012). Panel (b) shows the profile of yearly earning losses for alternative definitions of worker displacement in the model. The blue solid line corresponds to the annual earnings losses of workers who collect UI benefits. The blue dotted line corresponds to the annual earnings losses of workers who separate from their job independently of whether they collect UI benefits in a period. The blue dashed line is the profile of earning losses for all workers who in the model lose their job and experience some loss in their human capital during their first unemployment spell.

FIGURE A5. PROFILE OF YEARLY EARNINGS LOSSES AFTER DISPLACEMENT

period. Differently from the other studies Stevens (1997) identifies worker displacement without relying on mass-layoffs events. A worker is classified as displaced if he leaves the current employer either because a plant or a business is closed or because the worker reports being laid off or fired. As discussed at page 170 of the paper, and given the nature of PSID, the timing of displacement is not precisely identified and a displacement attributed to year t could have in practice occurred in year $t - 1$. Losses are measured relative to a group of continuously employed workers. The green dashed line in Figure A5 corresponds to column 4 of Table 4 at page 175 of the paper by Stevens (1997). Jung and Kuhn (2012) proposes a search model to match the empirical evidence on earnings losses upon re-employment. For the sake of comparison, we plot as a purple dash dotted line the profile of earnings losses generated by the model of Jung and Kuhn (2012) as reported in Figure 6 of their paper.

By comparing empirical evidence, one can observe large variation in the reported estimates. In particular the long-term earnings losses by Davis and von Wachter (2011) are almost twice as large as the estimates in Couch and Placzek (2010) and Stevens (1997), which are remarkably similar between each other.

D2. Earnings losses in the model

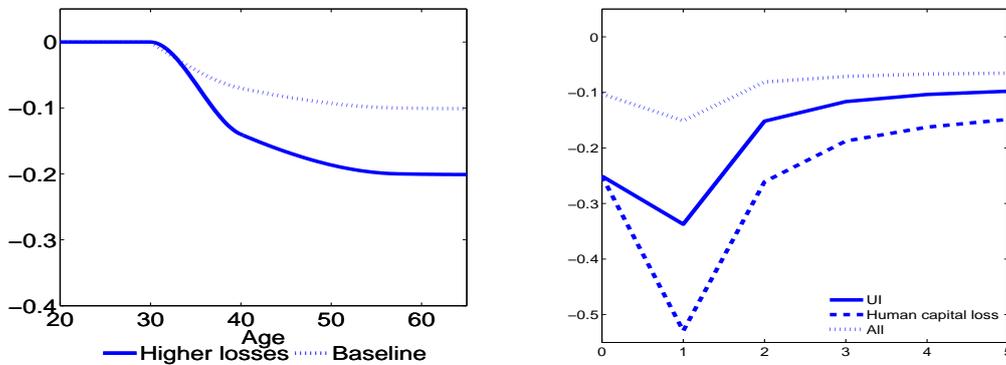
We now construct a measure of earnings losses after displacement in the model. We start defining workers as displaced at t if they were employed in $t - 1$ and collect UI benefits in t . This is a natural definition of displacement in the model because as discussed in Couch and Placzek (2010) and Jacobson, LaLonde and Sullivan (1993) there are no long term losses among separators who do not collect UI benefits—independently of whether the worker loses his job during a mass layoff event. We also consider two alternative

definitions of displacement: one where workers are defined as displaced whenever they separate from their job, independently of whether they collect UI benefits in a period; another where workers are defined as displaced if they lose their job and experience some loss in their human capital during their first unemployment spell. To measure earnings losses in the model we start simulating the economy and focus on workers who are continuously employed in the same job for at least 12 quarters.³⁴ Some of these workers get displaced in the next period. For simplicity we assume that workers separate from their job in the fourth quarter of the year. This only affects the earnings losses in the year of displacement but not in following years. This assumption about displacement implies that the annual earnings losses in the year of displacement for workers who collect UI benefits is exactly equal to 25 percent. We later study robustness of results to alternative assumptions about the timing of displacement in the year. For each displaced workers we then simulate 10000 labor market histories until retirement. For each subsequent year after displacement we then averaged out labor earnings across different histories. To be sure, our income definition excludes income from unemployment insurance or capital income, as in the empirical literature. Earnings losses in a year after displacement are calculated as percentage differences in labor income relative to a group of workers who are continuously employed until retirement. We later analyze robustness to alternative control groups. To characterize earnings losses by age, we later perform the same exercise for 4 different age groups depending on whether displaced workers have 25, 35, 45 or 55 years of age. The blue solid line in panel (b) of Figure A5 is the profile of earnings losses in the model for each year after displacement for an average worker in the baseline calibration of the model discussed in Section III.C of the paper. The blue dotted line corresponds to the annual earnings losses of workers who separate from their job, independently of whether they collect UI benefits in a period. The blue dashed line is the profile of earning losses for all workers who in the model lose their job and experience some loss in their human capital during their first unemployment spell.

Overall the model tends to under-predict somewhat earnings losses upon displacement as estimated by the literature. At short duration after displacement, differences between data and model are small and they can go either way depending on the definition of displacement. But when looking at long-term losses (more than three years after displacement), the losses generated by the model are around one half of those in the data. But there are strong reasons to believe that the mass-layoff literature (Couch and Placzek 2010, Davis and von Wachter, 2011) tends to systematically over-estimate earnings losses upon displacement. This is first because firms might undergo a mass layoff because of a poor skill quality of their workers—who also experience large average earnings losses—and secondly (and most importantly) because mass layoffs can at least be partly anticipated by workers in the firm and better workers—who have lower earnings losses upon displacement—quit the firm before the mass-layoff event. Autor et al. (2013) (see their Table 8) provide compelling evidence about this latter effect: around one third of the skilled workers in the firm (defined as workers whose pre-displacement wages are in the top tercile of the wage distribution of their age cohort) leave the firm within two years before the mass-layoff and these workers experience virtually no earn-

³⁴Jung and Kuhn (2012) focus on a sample of workers with at least 6 years of tenure (24 quarters).

ings losses. It is also useful to observe that Jacobson, LaLonde and Sullivan (1993) find that workers who separate from their job and who are not part of a mass-lay off, experience initial losses of 26 percent and no losses six years after displacement. In Couch and Placzek (2010) these long term losses are around 7 percent and similar to the model. This is why we think that keeping our calibration as baseline might be appropriate. Moreover the results in the paper are unaffected by the magnitude of wage losses during unemployment. Following the suggestion by one Referee, in Section VI.E of the paper we study the robustness of the results in the paper once we double the magnitude of wage losses due to human capital depreciation during unemployment. The profile of the optimal age-dependent UI replacement remains virtually unchanged, see Figure 14 in the paper. In this Online Appendix (see Section E.E3) we also perform several other robustness exercises, which confirm that the age profile of the optimal UI replacement rate is almost unaffected by changes in either the level or the age profile of human capital losses during unemployment. Here we just briefly describe the property of the model in terms of earnings losses once we double wage losses during unemployment. The new profile of wage losses, $\bar{\kappa}_n - 1$, corresponds to the blue solid line of panel (a) of Figure A6. The economy is recalibrated to match the targets of Section III.C in the paper, see Section E.E3 of this Online Appendix for further details. Panel (b) of Figure A6 is analogous to panel (a) of Figure A5, but for the economy with larger human capital losses during unemployment. The new re-calibrated model now matches better the profile of long term earnings losses upon displacement in the data, although long terms losses are still smaller than those found by Davis and von Wachter (2011).

(a) Wage losses: $\bar{\kappa}_n - 1$

(b) Earnings losses in model with higher wage losses for different definition of displacement

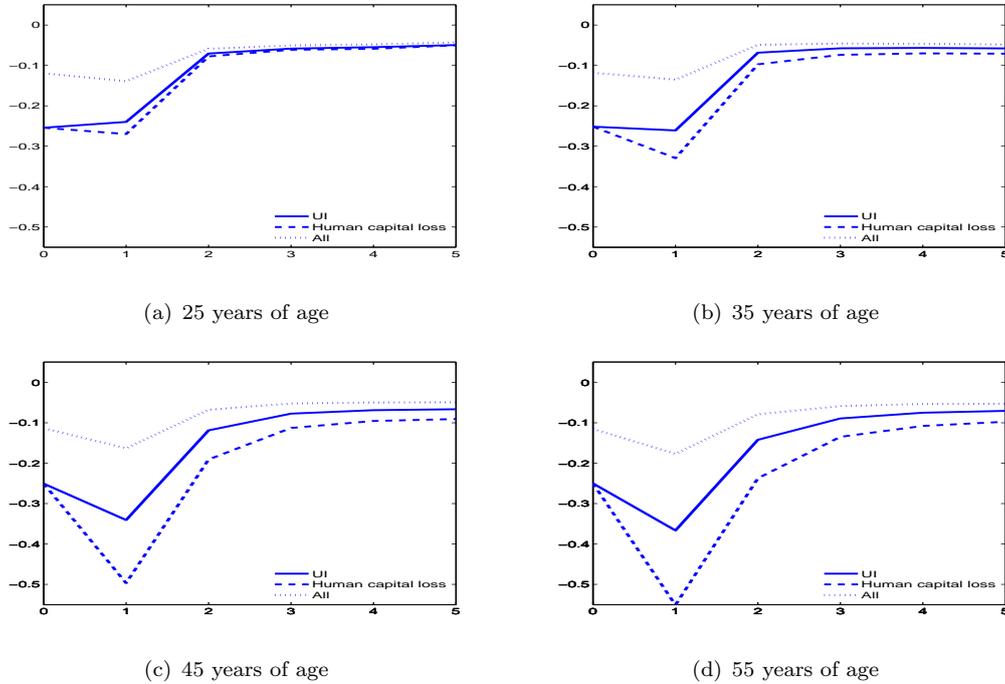
Notes: Panel (a) plots the age profile of wage losses during unemployment in the baseline calibration as a blue dotted line and in the economy where we double wage losses due to human capital depreciation during unemployment, as a blue solid line. Panel (b) is analogous to panel (a) of Figure A5, but where now the lines correspond to the calibration of the model with higher human capital losses during unemployment.

FIGURE A6. PROFILE OF EARNINGS LOSSES AFTER DISPLACEMENT IN RE-CALIBRATED MODEL AFTER DOUBLING WAGE LOSSES DURING UNEMPLOYMENT

D3. Earnings losses by worker age at displacement

We now characterize the profile of earnings losses upon displacement according to the age of the worker at displacement. As previously anticipated we focus on four different

age groups of workers: panel (a) of Figure A7 shows the profile for worker of 25 years of age at displacement, panel (b) focuses on workers of 35 years of age, panel (c) on workers of 45 years of age, and panel (d) on workers of 55 years of age. Each panel is analogous



Notes: Profile of earnings losses after displacement in the model for four different age groups. The parametrization corresponds to the baseline calibration of Section III.C in the paper. In each panel we consider three alternative definitions of displacement. The blue solid line corresponds to workers who collect UI benefits at some time during their first unemployment spell; the blue dotted line corresponds to losses of workers who separate from their job independently of whether they collect UI benefits in their first unemployment spell; the blue dashed line are losses for all workers who experience some human capital loss during their first unemployment spell.

FIGURE A7. PROFILE OF EARNINGS LOSSES AFTER DISPLACEMENT FOR WORKERS OF DIFFERENT AGE

to panel (b) of Figure A5, where we plot the profile of earnings losses for three groups of workers who differ in their definition of displacement: the dotted line corresponds to all workers who separate from their job, the solid line corresponds to workers who collect UI benefits, the dashed line corresponds to workers who separate from their job and lose some human capital during their first unemployment spell. Earnings losses increase substantially by age. To better evaluate, how the model reproduces the age variation in the data of earnings losses, Table A3 reports the present discounted value of earnings losses experienced by workers at displacement. Panel A measures losses from year of displacement until retirement, panel B focuses on losses in the next five years after displacement, Panel C focuses on losses in the next three years after displacement. Losses are measured as one minus the ratio of the present discounted value of earnings of displaced workers over the present value of earnings of the control group, which in the model are continuously employed workers. To calculate present values we use a 5 per cent discount rate, which is as Davis and von Wachter (2011). We present results for the previously mentioned four different age groups of workers. For each age group we consider the three alternative definitions of displacement: all workers who separated

from their job (which corresponds to 'All' in the table), workers who separate and collect UI benefits in their first unemployment spell ('UI' in the table) and workers who experience a human capital loss in their first unemployment spell ('HKLoss' in the table). For the sake of comparison we report in column three an analogous calculation by Davis and von Wachter (2011) based on their empirical estimates. We present results for

TABLE A3—ONE MINUS RATIO OF THE PDV OF EARNINGS OF DISPLACED WORKERS OVER THE PDV OF EARNINGS OF CONTROL GROUP

Worker age		Data	Baseline calibration			Re-calibrated model		
Data	Model	All	UI	HKLoss	All	UI	HKLoss	All
A) Life time PDV losses								
21-30	25	-7.8	-7.4	-7.8	-5.6	-7.6	-7.9	-5.7
31-40	35	-6.5	-8.3	-9.6	-5.9	-9.3	-11.2	-6.6
41-50	45	-15.1	-10.6	-13.8	-6.5	-13.0	-18.4	-7.7
51-60	55	-23.1	-14.3	-19.1	-8.7	-17.7	-25.7	-9.9
B) 5 years PDV losses								
21-30	25	-13.4	-13.6	-14.5	-8.3	-13.6	-14.4	-8.3
31-40	35	-12.8	-14.3	-16.8	-8.0	-14.6	-17.7	-7.9
41-50	45	-18.2	-17.6	-23.5	-9.1	-19.6	-28.2	-9.6
51-60	55	-25.3	-19.0	-26.2	-9.9	-22.3	-33.3	-10.7
C) 3 years PDV losses								
21-30	25	-14.5	-18.8	-20.0	-10.6	-18.8	-19.8	-10.5
31-40	35	-12.7	-19.5	-22.7	-10.1	-19.3	-22.9	-9.6
41-50	45	-17.2	-23.8	-31.3	-11.5	-24.7	-34.6	-11.2
51-60	55	-24.1	-25.4	-34.6	-12.4	-27.7	-40.2	-12.4

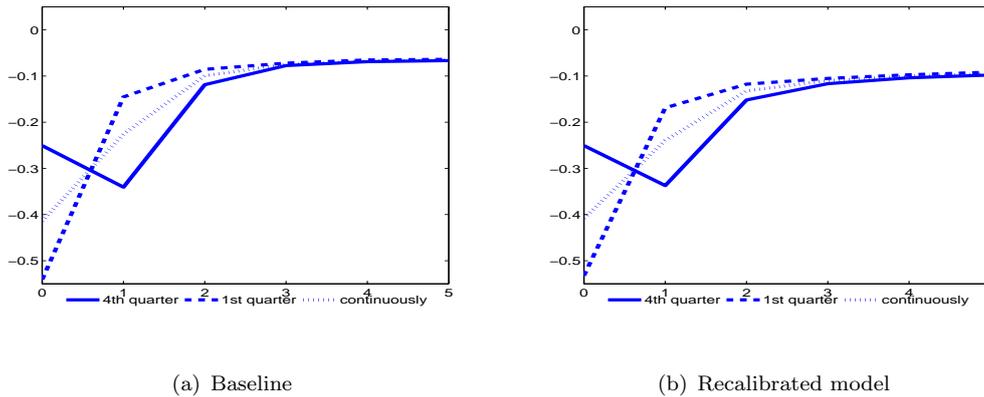
Note: The table reports the value of one minus the ratio of the Present Discounted Value (PDV) of earnings of displaced workers over the PDV of earnings of control group in the model and in the data for different age groups and different time horizons. Losses are calculated for three alternative definitions of displacement: all workers who separated from their job ('All' in table), workers who separate and collect UI benefits in their first unemployment spell ('UI' in table) and workers who experience a human capital loss in their first unemployment spell ('HKLoss' in table). Data correspond to the values from Davis and von Wachter (2011). For an average worker in the population the PDV of earnings losses in Jung and Kuhn (2012), Couch and Placzek (2010) and Stevens (1997) are approximately equal to 0.158, 0.164, and 0.144 respectively. The Re-calibrated model corresponds to the model specification where we doubled wage losses due to human capital depreciation during unemployment as in Figure A6.

the baseline calibration of Section III.C and for the Re-calibrated model of Section VI.E where we doubled wage losses due to human capital depreciation during unemployment as in Figure A6. Overall the model matches more closely the losses in present value at short horizons than at long term horizons. At a time horizon of three years, the model matches age variation in earnings losses reasonably accurately when focusing on the sample of workers who collect UI benefits. At longer time horizons, the model fit is not terrible, especially when focusing on the parametrization with higher wages losses during unemployment. Still allowing for alternative sources of wage losses upon displacement (like accumulation of job specific human capital) could clearly improve the model fit in terms of long term earnings losses upon displacement.

D4. Sensitivity and robustness of earnings losses results

In this Section we study robustness of results to alternative assumptions about (i) the timing of displacement within a year and (ii) the definition of the control group. In both cases we present results both for the baseline calibration of Section III.C and for the recalibrated model with larger wage losses during unemployment of Section VI.E.

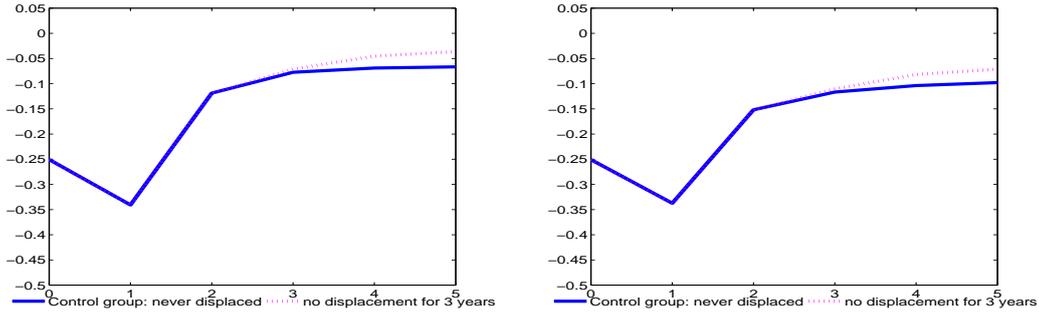
To save space, here we define workers as displaced if they separate from their job and collect UI benefits. Figure A8 presents the profile of earnings losses upon displacement when we assume that in the year of displacement the worker is always displaced in the first quarter of the year (dashed line), that the worker is displaced in the four quarter of the year (solid line) and that the worker has an equal probability of being displaced in the four quarters of the year (dotted line). As expected, alternative assumptions about the timing of displacement affect the measure of earnings losses mainly in the year of displacement. The effects on subsequent years are instead small.



Notes: Profile of yearly earnings losses in the model for alternative assumptions about the timing of displacement in the year of displacement. Panel (a) deals with baseline calibration, panel (b) with the parametrization with higher wage losses during unemployment. The blue solid line corresponds to earnings losses for workers who collect UI benefits in the last quarter of the year, the blue dashed line corresponds to losses when assuming that all workers first collect UI benefits in the first quarter of the year, the blue dotted line corresponds to losses when assuming that workers have an equal probability of collecting UI benefits in all quarters in the year.

FIGURE A8. EARNINGS LOSSES FROM YEAR OF DISPLACEMENT: CHANGING THE TIMING OF SEPARATION

Figure A9 presents the profile of earning losses after displacement when we consider as a control group workers who are employed at t , $t + 1$ and $t + 2$ where t is the year of displacement of the workers we are measuring losses. This profile corresponds to the red dotted line in the figure. For the sake of comparison we also report the previously reported profile, as a blue solid line, that was obtained using continuously employed workers as a control group. Overall one can see that the choice of the control group affects little the measure of earning losses in the model.



(a) Baseline

(b) Recalibrated model

Notes: Age profile of yearly earnings losses in the model for two alternative control groups. The blue solid line corresponds to earnings losses relative to continuously employed. The red dotted line are losses relative to workers who are employed at t , $t+1$ and $t+2$ where t is the year of displacement of the workers we are measuring losses. Panel (a) deals with baseline calibration, panel (b) with the parametrization with higher human capital losses during unemployment of Section VI.

FIGURE A9. EARNINGS LOSSES FROM YEAR OF DISPLACEMENT: CHANGING THE CONTROL GROUP

FURTHER DETAILS ON ROBUSTNESS

Here we provide further details on the robustness exercises discussed in Section VI.

E1. Age-dependent government budget constraints

In this section we better characterize the economy of Section with age-dependent government budget constraints. The population is divided into N mutually exclusive age groups with maximum age difference k within the group, so that $Nk = \bar{n}_w$. The set of age levels for the i th age group, $i = 1, 2, \dots, N$, is given by $\Gamma_i = \{(i-1)k+1, (i-1)k+2, \dots, ik\}$. Income taxes are the sum of two rates, one used to finance UI benefits expenditures for the specific age group, the other to finance expenditures for retirement pensions, so that $\tau_n = \bar{\tau}_n + \hat{\tau}_0$. Here $\hat{\tau}_0$ is the constant over-time income tax-rate set to finance retirement pensions:

$$(E1) \quad \sum_{n=\bar{n}_w+1}^{\bar{n}_r} \beta^n \pi \chi^r(n) = \hat{\tau}_0 \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} w(e) \chi^e(n, de)$$

The age-dependent component of income tax rates $\bar{\tau}_n$ is constant within its corresponding age group so that $\bar{\tau}_n = \hat{\tau}_i \forall n \in \Gamma_i$, where $\hat{\tau}_i$ satisfies the following group- i specific budget constraint:

$$(E2) \quad \sum_{n \in \Gamma_i} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de) = \hat{\tau}_i \sum_{n \in \Gamma_i} \beta^n \int_{R^+} w(e) \chi^e(n, de), \quad \forall i = 1, \dots, N$$

This constraint implies that an increase in benefits for a given age group i can not be financed by increasing tax revenue for another age group. We then search for the age profile of UI replacement rate $\rho \geq 0$ which maximizes worker's wealth at birth W_s subject to the same constraints as before but where now the tax rates $\hat{\tau}_i$'s $i = 0, 1, \dots, N$ satisfy the $N+1$ government budget constraints in (E1) and (E2). In solving the problem we consider age groups of five years, $k = 20$. The resulting optimal age-dependent

replacement rate under the age specific government budget constraints specified above corresponds to the solid line in Figure 13.

E2. Severance payments

We better characterize the economy with age-dependent severance payments in Section VI.D. Upon job displacement workers receive a government transfer equal to $\varsigma_n w(e)$ where worker's age n and worker's human capital e here refer to the last period before job displacement occurs. All the other model assumptions remain as in Section III. The value of being employed in the economy with severance payments becomes equal to

$$(E3) \quad \begin{aligned} V(n, e, a) &= \max_{a' \geq l} u(c^e(n, e, a, a')) + \beta(1 - \delta_n)V(n + 1, e + 1, a') \\ &+ \beta\delta_n J(n + 1, e + 1, a' + \varsigma_n w(e)) \end{aligned}$$

where the last term incorporates the fact that upon displacement the worker receives a severance pay of $\varsigma_n w(e)$, which increases his wealth at the start of the current job search spell. Nothing else changes relative to (18) and all the other value functions remain as in Section III, so equations (19)-(22) are left untouched. Of course, the government budget constraint in (23) has to be amended to include severance payments transfers. This yields the following constraint:

$$(E4) \quad \begin{aligned} &\sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \rho_n w(e) \chi^u(n, de) + \beta \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \varsigma_n w(e) \delta_n \chi^e(n, de) \\ &= \sum_{n=1}^{\bar{n}_w} \beta^n \int_{R^+} \tau_n w(e) \chi^e(n, de) - \sum_{n=\bar{n}_w+1}^{\bar{n}_r} \beta^n \pi \chi^r(n). \end{aligned}$$

The second term in the first row takes into account that a fraction δ_n of the mass of employed workers of age n and experience e , $\chi^e(n, e)$, will be displaced next period and will receive severance payments $\varsigma_n w(e)$.

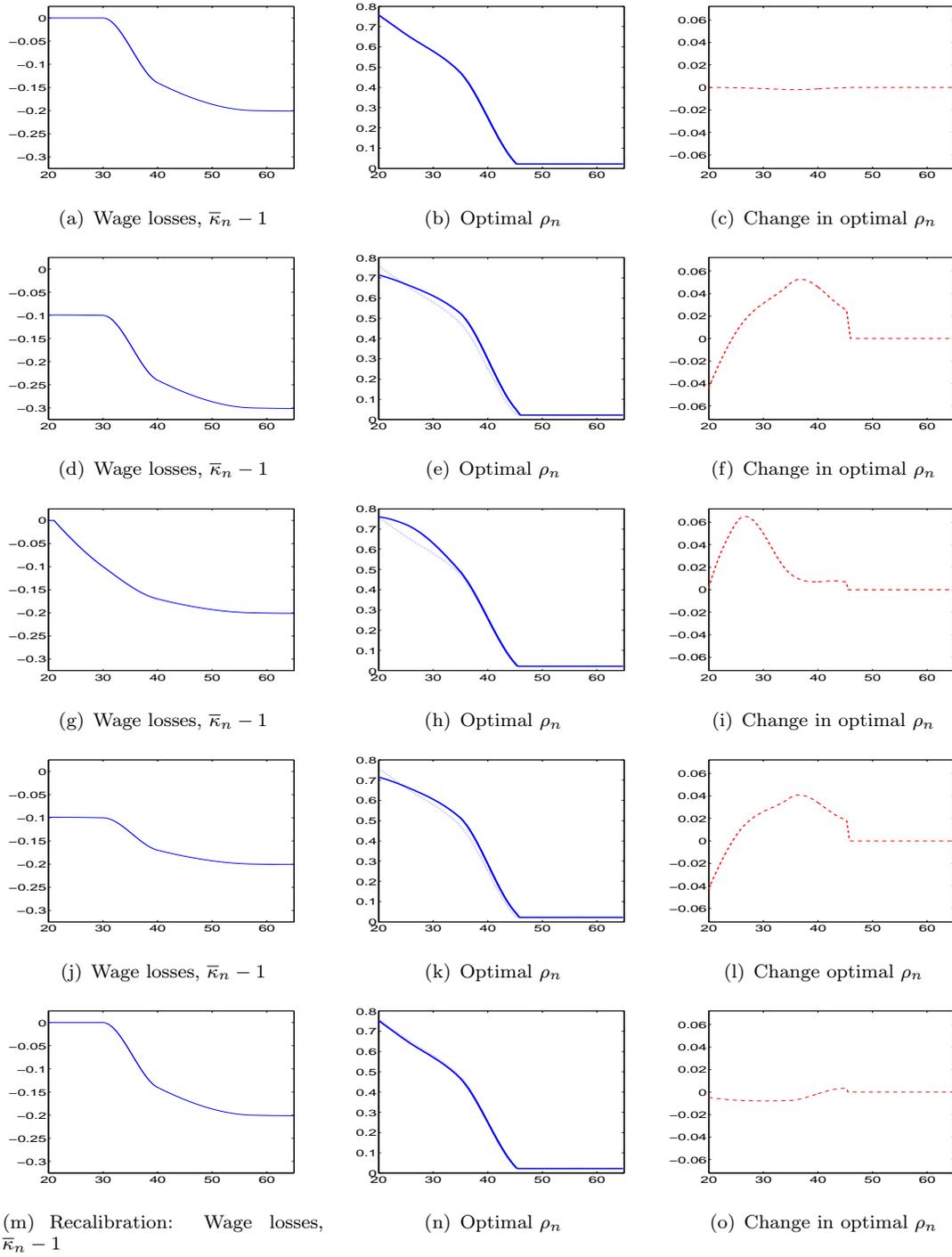
We study whether severance payments can improve welfare relative to the economy with optimal age-dependent benefits and taxes. So we keep their age profile as given. To be sure, let ρ_n^* and τ_n^* denote the optimal age profile of benefits and taxes as reported in Figure 10. Here we assume that $\rho_n = \rho_n^*$ and $\tau_n = \tau_n^* + \bar{\tau}$, where $\bar{\tau}$ is needed to keep the government budget constraint in (E4) satisfied. We then search for the vectors of severance payments $\varsigma = \{\varsigma_1, \dots, \varsigma_{\bar{n}_w}\}$ and the value of the tax rate $\bar{\tau}$ that maximize worker's utility at birth $W_s \equiv J(1, 0, 0)$ subject to the new government budget constraint in (E4), workers optimal choices as implied by (19)-(21) and (E3), and the feasibility constraint $\rho \geq 0$.³⁵ Exactly as in Section V we assume that ς_n is a cubic spline at the previously defined ten age knots and search for the value at the knots that maximize W_s . When we restrict severance payments to be independent of age $\varsigma_n = \varsigma, \forall n$, we find that the optimal constant over age severance payment is $\varsigma = 1.4$. This economy yields welfare gains equivalent to a 3.3 percent increase in life time consumption relative to the baseline economy. This is 0.1 percent higher than in the economy with optimal age-

³⁵In the economy we also impose the restriction that $\varsigma_{\bar{n}_w} = 0$, since this transfer would just have the nature of a retirement pension.

dependent benefits and taxes. When we allow severance payment to vary with age, we virtually find no additional gains (up to the fourth order). We believe that age variation in severance payments yields small welfare gains because severance payments discourage workers from accumulating precautionary savings and thereby are more distortionary than a combination of UI benefits and subsidies to job creation. Since UI benefits together with labor income tax rates can mimic reasonably well the effects of a subsidy to job creation, age variation in severance payments can play little role in improving welfare in our economy.

E3. Optimal UI replacement rates and the age profile of wage losses during unemployment

In Section VI.E of the paper we have seen that the optimal age-dependent UI replacement rate ρ_n remains virtually unchanged once we double the level of wage losses due to human capital depreciation during unemployment $\bar{\kappa}_n - 1$. We now show that this is a general property of the model. Figure A10 shows the profile of the optimal age-dependent UI replacement rate ρ_n for different parametrizations of the level of wage losses during unemployment. The first panel in each row corresponds to the new profile of wage losses upon unemployment $\bar{\kappa}_n - 1$, the second panel shows as a solid line the optimal age-dependent UI replacement rate ρ_n , the third panel plots the difference between the new profile of ρ_n and the profile of ρ_n obtained in the baseline calibration of the model of Section III.C in the paper—which for convenience is plotted as a dotted line in the second panel of each row. To simplify the exercise, economies are not recalibrated to match the targets of Section III.C, except for the parametrization of the economy in the last row, which corresponds to the robustness exercise of Section VI, where all economies are recalibrated to match the targets of Section III.C. There is some evidence that when wage losses increase across age groups the profile of the optimal age-dependent ρ_n increases for workers of 30 to 45 years of age, while it decreases for the very young. There is also some evidence that when wage losses increase more steeply with age, UI replacement rates tend to increase somewhat for the young. But overall all changes are quantitatively small. In practice this happens because the age profile of the extended redistribution formula \tilde{q} changes little in response to changes in the profile of $\bar{\kappa}_n - 1$. For example, with higher losses the marginal utility of consumption of the unemployed goes up which pushes up the value of the numerator of \tilde{q} . But with higher losses tax effects also become more important, so $-\frac{\partial T}{\partial b_n}$ goes up, which on balance leaves the overall profile of \tilde{q} unchanged.



Notes: Optimal age-dependent UI replacement rate ρ_n for different parametrizations of the level of wage losses during unemployment. The first panel in each row corresponds to the new profile of wage losses upon unemployment, the second panel plots as a solid line the optimal age-dependent UI replacement rate ρ_n obtained under the corresponding parametrization, the third panel is the difference between the new profile of ρ_n and the profile obtained in the baseline calibration of the model of Section III.C in the paper, which corresponds to the dotted line in the second panel of each row. No economy is recalibrated to match the targets of Section III.C, except for the parametrization in the last row.

FIGURE A10. DIFFERENT PROFILES OF WAGE LOSSES AND THE OPTIMAL AGE-DEPENDENT UI REPLACEMENT RATE ρ_n