The Dynamic Behavior of the Real Exchange Rate in Sticky Price Models: Comment

Online Appendix

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This note derives and log-linearizes the equilibrium conditions characterizing the solution to the models we use in the main text. Our model assumptions correspond exactly to Steinsson (2007). Our notation also closely matches Steinsson (2007) with minor differences. We point out these differences where they arise.

A Households

In this section we derive the equations determining the optimal decisions of the households in the world economy. We start by examining how spending and labor supply are allocated optimally over time and then solves for the optimal composition of spending within a given period.

A.1 Intertemporal decision problem

Household x located in home solves

$$\max \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t\right) - v\left(L_t\left(x\right), \xi_t\right) \right] \right\}, \tag{A1}$$

subject to

$$P_t C_t + \mathrm{E}_t \left[M_{t,t+1} B_{t+1} \right] = B_t + W_t\left(x\right) L_t\left(x\right) + \int_{N_H} \Phi_t\left(z\right) dz - T_t,$$

where C_t is consumption, P_t is the nominal price of consumption, B_{t+1} represents a portfolio of state contingent claims held by household x, $M_{t,t+1}$ is the state-price associated with this portfolio, $W_t\left(x\right)$ is the (possibly household specific) nominal wage rate of household x, $L_t\left(x\right)$ is labor supply, $\Phi_t\left(z\right)$ is profits received from home producer z, T_t is lump sum taxes and ξ_t

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is a country wide preference shock. The notation reflects that financial markets are complete so consumption and the portfolio of state contingent claims are the same for all domestic households. The first order conditions associated with this problem are

$$
\begin{aligned}
u_c(C_t) &= P_t \Lambda_t, && \text{(A2)}\\
M_{t,T}\Lambda_t &= \beta^{T-t}\Lambda_T, && \text{(A3)}\\
v_l(L_t(x), \xi_t) &= W_t(x)\Lambda_t, && \text{(A4)}
\end{aligned}
$$

where Λ_t is the lagrangian multiplier on the budget constraint. Equations (A2) and (A3) imply

$$M_{t,t+1} = \beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{P_t}{P_{t+1}}, \tag{A5}$$

which holds in all states of nature in period $t+1$. Defining the gross nominal interest rate as $I_t = \frac{1}{\mathrm{E}_t[M_{t,t+1}]}$ we get[1]

$$1 = \beta I_t \mathrm{E}_t \left[\frac{u_c(C_{t+1})}{u_c(C_t)} \frac{P_t}{P_{t+1}} \right]. \tag{A6}$$

This is the consumption Euler equation which is slightly different from the equation in Steinsson (2007). On page 8 Steinsson (2007) argues that

$$I_t = \mathrm{E}_t \left[\frac{1}{\beta} \frac{u_c(C_t)}{u_c(C_{t+1})} \frac{P_{t+1}}{P_t} \right] \tag{A7}$$

which should hold, he argues, if $I_t = \frac{1}{\mathrm{E}_t[M_{t,t+1}]}$. This is, however, not exactly true since the equation requires that $I_t = \mathrm{E}_t \left[\frac{1}{M_{t,t+1}} \right]$. To see this notice that equation (A5) implies

$$\frac{1}{M_{t,t+1}} = \frac{1}{\beta} \frac{u_c(C_t)}{u_c(C_{t+1})} \frac{P_{t+1}}{P_t}. \tag{A8}$$

Taking the period t expectation yields equation (A7) with the alternative definition of the nominal interest rate. This difference in consumption Euler equations is immaterial for the subsequent results, however, as they both reduce to the same log-linear expression. We will discuss this further below.

Using equations (A2) and (A4) we get the optimal labor supply relation

$$v_l(L_t(x), \xi_t) = u_c(C_t) \frac{W_t(x)}{P_t}. \tag{A9}$$

The notation indicates that the wage rate is household specific which will be the case under the assumption of heterogeneous labor markets. Under the homogeneous labor market assumption the wage rate is the same for all domestically located households. This implies that the labor supply will be identical across all households since the preference shock affects all households

[1] To see why this definition must hold, suppose that $B_{t+1}(j)$ is a state contingent claim that pays out one unit of home currency if state j arise in period $t+1$ and zero otherwise. $M_{t,t+1}(j)$ is the associated state price (nominal price in home currency divided by the probability of the state). The expectation over all states then equals the cost measured in period t home currency of acquiring a unit of home currency for certain in period $t+1$. This cost is the same as the inverse of the gross nominal interest rate.

identically.

Household x located in foreign solves

$$\max \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^*\right) - v\left(L_t^*\left(x\right), \xi_t^*\right) \right] \right\} \tag{A10}$$

subject to

$$P_t^* C_t^* + \frac{1}{\varepsilon_t} \mathrm{E}_t \left[M_{t,t+1} B_{t+1}^* \right] = \frac{B_t^*}{\varepsilon_t} + W_t^*\left(x\right) L_t^*\left(x\right) + \int_{N_F} \Phi_t^*\left(z\right) dz - T_t^*. \tag{A11}$$

An asterisk denotes a foreign variable and ε_t is the nominal exchange rate defined as the cost in home currency of a unit of foreign currency. The first order conditions are given as

$$\begin{aligned} u_c\left(C_t^*\right) &= P_t^* \Lambda_t^*, && \text{(A12)} \\ M_{t,T} \frac{\Lambda_t^*}{\varepsilon_t} &= \beta^{T-t} \frac{\Lambda_T^*}{\varepsilon_T}, && \text{(A13)} \\ v_l\left(L_t^*\left(x\right), \xi_t^*\right) &= W_t^*\left(x\right) \Lambda_t^*. && \text{(A14)} \end{aligned}$$

Using equation (A12) and (A13) gives

$$M_{t,t+1} = \beta \frac{u_c\left(C_{t+1}^*\right)}{u_c\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}}. \tag{A15}$$

Taking the period t expectation gives the consumption Euler equation

$$1 = \beta I_t \mathrm{E}_t \left[\frac{u_c\left(C_{t+1}^*\right)}{u_c\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}} \right]. \tag{A16}$$

Notice that we may incorporate a foreign nominal interest rate into the model by defining $I_t^* = \frac{1}{\mathrm{E}_t\left[M_{t,t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t}\right]}$. This implies

$$1 = \beta I_t^* \mathrm{E}_t \left[\frac{u_c\left(C_{t+1}^*\right)}{u_c\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \right]. \tag{A17}$$

Hence, we must have that

$$I_t \mathrm{E}_t \left[\frac{u_c\left(C_{t+1}^*\right)}{u_c\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}} \right] = I_t^* \mathrm{E}_t \left[\frac{u_c\left(C_{t+1}^*\right)}{u_c\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \right]. \tag{A18}$$

The log-linear version of this equation is the uncovered interest parity condition.

The optimal labor supply relation is given as

$$v_l\left(L_t^*\left(x\right), \xi_t^*\right) = u_c\left(C_t^*\right) \frac{W_t^*\left(x\right)}{P_t^*}. \tag{A19}$$

Using equation (A5) and (A15) we get

$$\frac{u_c\left(C_{t+1}\right)}{u_c\left(C_t\right)} \frac{P_t}{P_{t+1}} = \frac{u_c\left(C_{t+1}^*\right)}{u_c\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}}, \tag{A20}$$

which must hold in all states of nature in period $t+1$. This equation can be rewritten

$$Q_t = \frac{u_c\left(C_t^*\right)}{u_c\left(C_t\right)}, \tag{A21}$$

where we have defined $Q_t = \frac{\varepsilon_t P_t^*}{P_t}$ as the real exchange rate and assumed that $Q_0 = 1$.

A.2 Intratemporal decision problem

The previous analysis focused on how to allocate spending and work time optimally across time. We now analyze how households' choose spending optimally across different goods within a given period.

Aggregate consumption in home, C_t, is given by the following CES index

$$C_t = \left[\phi_{H,t}^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \phi_{F,t}^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{A22}$$

where $C_{H,t}$ and $C_{F,t}$ are bundles of home and foreign goods and where $\phi_{H,t}$ $\left(\phi_{F,t}\right)$ is a shock to the demand for home (foreign) goods. We assume that $\phi_{H,t} + \phi_{F,t} = 1$. Allocating spending in an optimal way requires

$$\min_{C_{H,t}, C_{F,t}} P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{A23}$$

subject to $C_t = \bar{C}$. Optimality requires

$$C_{H,t} = \phi_{H,t}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \tag{A24}$$

$$C_{F,t} = \phi_{F,t}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \tag{A25}$$

where

$$P_t = \left(\phi_{H,t}\left(P_{H,t}\right)^{1-\eta} + \phi_{F,t}\left(P_{F,t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{A26}$$

The subindices $C_{H,t}$ and $C_{F,t}$ are given as

$$C_{H,t} = \left(\int_{N_H} C_{H,t}(z)^{\frac{\theta_t-1}{\theta_t}} dz\right)^{\frac{\theta_t}{\theta_t-1}} \tag{A27}$$

$$C_{F,t} = \left(\int_{N_F} C_{F,t}(z)^{\frac{\theta_t^*-1}{\theta_t^*}} dz\right)^{\frac{\theta_t^*}{\theta_t^*-1}}. \tag{A28}$$

These equations reflect that the elasticity of substitution between different goods is production country specific. In other words, the elasticity of substitution is the same for goods produced within the same country regardless of where the good is sold. Allocating spending optimally requires

$$C_{H,t}(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\theta_t} C_{H,t}, \tag{A29}$$

$$C_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\theta_t^*} C_{F,t}, \tag{A30}$$

where

$$P_{H,t} = \left(\int_{N_H} P_{H,t}(z)^{1-\theta_t} dz\right)^{\frac{1}{1-\theta_t}}, \tag{A31}$$

$$P_{F,t} = \left(\int_{N_F} P_{F,t}(z)^{1-\theta_t^*} dz\right)^{\frac{1}{1-\theta_t^*}}. \tag{A32}$$

Using equations (A24), (A25), (A29) and (A30) we get that

$$C_{H,t}(z) = \phi_{H,t}\left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\theta_t}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \tag{A33}$$

$$C_{F,t}(z) = \phi_{F,t}\left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\theta_t^*}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t. \tag{A34}$$

The solution to the corresponding foreign problem gives rise to the demand functions

$$C_{F,t}^*(z) = \phi_{F,t}^*\left(\frac{P_{F,t}^*(z)}{P_{F,t}^*}\right)^{-\theta_t^*}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*, \tag{A35}$$

$$C_{H,t}^*(z) = \phi_{H,t}^*\left(\frac{P_{H,t}^*(z)}{P_{H,t}^*}\right)^{-\theta_t}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{A36}$$

The corresponding price indices are

$$P_t^* = \left(\phi_{F,t}^*\left(P_{F,t}^*\right)^{1-\eta} + \phi_{H,t}^*\left(P_{H,t}^*\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{A37}$$

and

$$P_{F,t}^* = \left(\int_{N_F} P_{F,t}^*(z)^{1-\theta_t^*} dz\right)^{\frac{1}{1-\theta_t^*}}, \tag{A38}$$

$$P_{H,t}^* = \left(\int_{N_H} P_{H,t}^*(z)^{1-\theta_t} dz\right)^{\frac{1}{1-\theta_t}}. \tag{A39}$$

A government sector in each country finances government spending through lump sum taxation. For convenience, government spending on goods follow demand functions identical to those used by the private sector.

B Firms

We assume that a continuum of goods producers exists in each country. Each producer uses a production function with decreasing returns to scale in labor and sells her particular good to households and governments in both home and foreign. The goods are sold under monopolistic competition and prices are staggered as in Calvo (1983). Moreover, we assume that producers employ local currency pricing and therefore sets two prices, one for each market.

Following Steinsson (2008), we consider two assumptions regarding labor markets. Under the first assumption each producer can only use the labor supply of a particular type of households in her production. Hence, labor markets are highly segmented. Under the second assumption, producers can use the labor supply of all households. In other words, the second assumption implies a country-wide labor market. We will consider each assumption in turn.

B.1 Heterogenous labor markets

Each producer z located in home has the following production function

$$\left(\frac{P_{H,t}(z)}{P_{H,T}}\right)^{-\theta_T}(C_{H,T}+G_{H,T})+\left(\frac{P^*_{H,t}(z)}{P^*_{H,T}}\right)^{-\theta_T}\left(C^*_{H,T}+G^*_{H,T}\right)=A_T f\left(L_T(x)\right), \quad \text{(B1)}$$

where the left hand side of the equation denotes total demand for producer z's good. We use a slightly different notation compared to Steinsson with respect to producer z prices. More specifically he denotes by $p_t(z)$ and $p_t^*(z)$ the home and foreign price of the good produced by home firm z. We use $P_{H,t}(z)$ and $P^*_{H,t}(z)$ to denote the same prices. The function f is increasing and concave. The notation reflects that each producer, under the heterogenous labor market assumption, can only use the labor supply of a particular type of household denoted by x. If the producer gets the opportunity to revise her prices in period t she solves

$$\begin{aligned}&\max_{P_{H,t}(z),P^*_{H,t}(z),\{L_k(x)\}_{k=t}^{\infty}} \mathrm{E}_t\sum_{T=t}^{\infty}\alpha^{T-t}M_{t,T} && \text{(B2)}\\ &\times\Bigg\{P_{H,t}(z)\left(\frac{P_{H,t}(z)}{P_{H,T}}\right)^{-\theta_T}(C_{H,T}+G_{H,T})\\ &+\varepsilon_T P^*_{H,t}(z)\left(\frac{P^*_{H,t}(z)}{P^*_{H,T}}\right)^{-\theta_T}\left(C^*_{H,T}+G^*_{H,T}\right)-W_{t,T}(x)L_{t,T}(x)\Bigg\}\end{aligned}$$

subject to equation (B1). The parameter α is the probability that a producer does not update her price in a particular period. It follows from the heterogeneous labor markets assumption that the nominal wage rate and labor demand in period T depends on when the producer has last updated her price. The first order conditions are

$$\mathrm{E}_t\sum_{T=t}^{\infty}\alpha^{T-t}M_{t,T}\begin{pmatrix}(1-\theta_T)P_{H,t}(z)^{-\theta_T}(P_{H,T})^{\theta_T}(C_{H,T}+G_{H,T})\\ +S_{t,T}\theta_T P_{H,t}(z)^{-\theta_T-1}(P_{H,T})^{\theta_T}(C_{H,T}+G_{H,T})\end{pmatrix} = 0, \quad \text{(B3)}$$

$$\mathrm{E}_t\sum_{T=t}^{\infty}\alpha^{T-t}M_{t,T}\begin{pmatrix}(1-\theta_T)\varepsilon_T P^*_{H,t}(z)^{-\theta_T}\left(P^*_{H,T}\right)^{\theta_T}\left(C^*_{H,T}+G^*_{H,T}\right)\\ +S_{t,T}\theta_T P^*_{H,t}(z)^{-\theta_T-1}\left(P^*_{H,T}\right)^{\theta_T}\left(C^*_{H,T}+G^*_{H,T}\right)\end{pmatrix} = 0, \quad \text{(B4)}$$

$$-W_{t,T}(x)+S_{t,T}A_T f_l\left(L_T(x)\right)=0, \quad \text{(B5)}$$

$\forall\ T \geq t$, where $S_{t,T}$ is the nominal marginal costs of production in period T for a producer that has changed price in period t. Using equation (A9) we can write equation (B5) as

$$\frac{S_{t,T}}{P_t} = \frac{v_l\left(L_T\left(x\right), \xi_T\right)}{A_T f_l\left(L_T\left(x\right)\right) u_c\left(C_T\right)}. \tag{B6}$$

Denoting total demand for producer z's good in period T by $D_{t,T}\left(z\right)$ the production function implies $L_{t,T}\left(x\right) = f^{-1}\left(D_{t,T}\left(z\right)/A_T\right)$. Notice that we use a different notation than Steinsson for the total demand for producer z's good in period T. He uses $y_T\left(z\right)$ whereas we use $D_{t,T}\left(z\right)$. With this definition we can write real marginal costs

$$\frac{S_{t,T}}{P_t} = \frac{v_l\left(f^{-1}\left(D_{t,T}\left(z\right)/A_T\right), \xi_T\right)}{A_T f_l\left(f^{-1}\left(D_{t,T}\left(z\right)/A_T\right)\right) u_c\left(C_T\right)}. \tag{B7}$$

Equation (B3) can be written as

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}\left(C_{H,T} + G_{H,T}\right)\left(P_{H,T}\right)^{\theta_T}\left(1-\theta_T\right) P_T \tag{B8}$$

$$\times \left(\frac{P_{H,t}\left(z\right)}{P_T} - \frac{S_{t,T}}{P_T}\frac{\theta_T}{\theta_T - 1}\right) = 0$$

or

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}\left(C_{H,T} + G_{H,T}\right)\left(P_{H,T}\right)^{\theta_T}\left(1-\theta_T\right) P_T \tag{B9}$$

$$\times \left(\frac{P_{H,t}\left(z\right)}{P_t}\frac{P_t}{P_{t+1}} \times \ldots \times \frac{P_{T-1}}{P_T} - \frac{S_{t,T}}{P_T}\frac{\theta_T}{\theta_T - 1}\right) = 0.$$

This equation can be written

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}\left(C_{H,T} + G_{H,T}\right)\left(P_{H,T}\right)^{\theta_T}\left(1-\theta_T\right) P_T \tag{B10}$$

$$\times \left(\frac{P_{H,t}\left(z\right)}{P_t}\prod_{k=t+1}^{T}\frac{1}{\Pi_k} - \frac{S_{t,T}}{P_T}\frac{\theta_T}{\theta_T - 1}\right) = 0,$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross home inflation rate. A similar manipulation of equation (B4) gives

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}\left(C^*_{H,T} + G^*_{H,T}\right)\left(P^*_{H,T}\right)^{\theta_T}\left(1-\theta_T\right) P_T \tag{B11}$$

$$\times \left(Q_T \frac{P^*_{H,t}\left(z\right)}{P^*_t}\prod_{k=t+1}^{T}\frac{1}{\Pi^*_k} - \frac{S_{t,T}}{P_T}\frac{\theta_T}{\theta_T - 1}\right) = 0.$$

The solution to the home producer's problem is characterized by equations (B1), (B7), (B10) and (B11). Similar equations characterize the solution to a generic foreign producer's problem.

In contrast to Steinsson (2008) we want to use output in our simulations. First, we want to investigate the volatility of the real exchange rate relative to output and second, we want to specify monetary policy rules that depend on output. In our models home nominal output in period t can be found by aggregating profit and wage income across all households located in home. Denoting home real output by Y_t we get

$$\begin{aligned} P_t Y_t &= \int_{N_H} \left(W_t(x) L_t(x) + \int_{N_H} \Phi_t(z)\, dz \right) dx \qquad \text{(B12)} \\ &= \int_{N_H} W_t(x) L_t(x)\, dx + \int_{N_H} \Phi_t(z)\, dz, \end{aligned}$$

where we have used that there are equally many households and producers in the economy. Profits $\Phi_t(z)$ are given as

$$\begin{aligned} \Phi_t(z) &= \phi_{H,t} P_{H,t}(z) \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\theta_t} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} (C_t + G_t) \qquad \text{(B13)} \\ &+ \varepsilon_t \phi^*_{H,t} P^*_{H,t}(z) \left(\frac{P^*_{H,t}(z)}{P^*_{H,t}} \right)^{-\theta_t} \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} (C^*_t + G^*_t) \\ &- W_t(z) L_t(z), \end{aligned}$$

where subscript t denotes the price charged in period t by firm z, which not necessarily has been updated in period t. Here $W_t(z) L_t(z)$ is the wage bill of producer z. Aggregating across producers yields

$$\begin{aligned} \int_{N_H} \Phi_t(z)\, dz &= \phi_{H,t} P_{H,t} \frac{\int_{N_H} (P_{H,t}(z))^{1-\theta_t}\, dz}{(P_{H,t})^{1-\theta_t}} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} (C_t + G_t) \qquad \text{(B14)} \\ &+ \varepsilon_t \phi^*_{H,t} P^*_{H,t} \frac{\int_{N_H} \left(P^*_{H,t}(z)\right)^{1-\theta_t}\, dz}{\left(P^*_{H,t}\right)^{1-\theta_t}} \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} (C^*_t + G^*_t) \\ &- \int_{N_H} W_t(z) L_t(z)\, dz. \end{aligned}$$

Hence, real output is given as

$$\begin{aligned} Y_t &= \phi_{H,t} \frac{\int_{N_H} (P_{H,t}(z))^{1-\theta_t}\, dz}{(P_{H,t})^{1-\theta_t}} \left(\frac{P_{H,t}}{P_t} \right)^{1-\eta} (C_t + G_t) \\ &+ \phi^*_{H,t} Q_t \frac{\int_{N_H} \left(P^*_{H,t}(z)\right)^{1-\theta_t}\, dz}{\left(P^*_{H,t}\right)^{1-\theta_t}} \left(\frac{P^*_{H,t}}{P^*_t} \right)^{1-\eta} (C^*_t + G^*_t). \qquad \text{(B15)} \end{aligned}$$

Similarly, foreign real output can be written

$$\begin{aligned} Y^*_t &= \phi^*_{F,t} \frac{\int_{N_F} \left(P^*_{F,t}(z)\right)^{1-\theta^*_t}\, dz}{\left(P^*_{F,t}\right)^{1-\theta^*_t}} \left(\frac{P^*_{F,t}}{P^*_t} \right)^{1-\eta} (C^*_t + G^*_t) \\ &+ \phi_{F,t} \frac{1}{Q_t} \frac{\int_{N_F} (P_{F,t}(z))^{1-\theta^*_t}\, dz}{(P_{F,t})^{1-\theta^*_t}} \left(\frac{P_{F,t}}{P_t} \right)^{1-\eta} (C_t + G_t). \qquad \text{(B16)} \end{aligned}$$

B.2 Homogeneous labor markets

Under the homogeneous labor market assumption

$$\left(\frac{P_{H,t}(z)}{P_{H,T}}\right)^{-\theta_T}(C_{H,T}+G_{H,T})+\left(\frac{P^*_{H,t}(z)}{P^*_{H,T}}\right)^{-\theta_T}(C^*_{H,T}+G^*_{H,T})$$
$$=A_T g(L_T(z),K_T(z)), \tag{B17}$$

where the function g has constant returns to scale in labor and capital, but decreasing returns to scale in labor. We assume, following Steinsson, that all producers are endowed with a non-depreciating stock of capital denoted by $\bar{K}(z)$. Each producer can use her capital stock in the production of her own good, or rent it out to other producers on country-wide capital markets. By renting out their capital stock, producers receives the capital rental rate from the renters.

Producer z located in home solves

$$\max_{P_{H,t}(z),P^*_{H,t}(z),\{L_k(z),K_k(z)\}_{k=t}^{\infty}} \mathrm{E}_t\sum_{T=t}^{\infty}\alpha^{T-t}M_{t,T}$$
$$\times\Bigg\{P_{H,t}(z)\left(\frac{P_{H,t}(z)}{P_{H,T}}\right)^{-\theta_T}(C_{H,T}+G_{H,T})+\varepsilon_T P^*_{H,t}(z)\left(\frac{P^*_{H,t}(z)}{P^*_{H,T}}\right)^{-\theta_T}(C^*_{H,T}+G^*_{H,T})$$
$$-W_T L_T(z)-R_T\left(K_T(z)-\bar{K}(z)\right)\Bigg\} \tag{B18}$$

subject to equation (B17). The first conditions can be written as

$$\mathrm{E}_t\sum_{T=t}^{\infty}\alpha^{T-t}M_{t,T}(C_{H,T}+G_{H,T})(P_{H,T})^{\theta_T}(1-\theta_T)P_T \tag{B19}$$
$$\times\left(\frac{P_{H,t}(z)}{P_t}\prod_{k=t+1}^{T}\frac{1}{\Pi_k}-\frac{S_T}{P_T}\frac{\theta_T}{\theta_T-1}\right)=0,$$

$$\mathrm{E}_t\sum_{T=t}^{\infty}\alpha^{T-t}M_{t,T}\left(C^*_{H,T}+G^*_{H,T}\right)\left(P^*_{H,T}\right)^{\theta_T}(1-\theta_T)P_T \tag{B20}$$
$$\times\left(Q_T\frac{P^*_{H,t}(z)}{P^*_t}\prod_{k=t+1}^{T}\frac{1}{\Pi^*_k}-\frac{S_T}{P_T}\frac{\theta_T}{\theta_T-1}\right)=0,$$

$$-W_T+S_T A_T g_L(L_T(z),K_T(z))=0, \tag{B21}$$

$$-R_T+S_T A_T g_K(L_T(z),K_T(z))=0. \tag{B22}$$

Notice that

$$\frac{W_T}{R_T}=\frac{g_L(L_T(z),K_T(z))}{g_K(L_T(z),K_T(z))}=\frac{g_L\left(\frac{L_T(z)}{K_T(z)},1\right)}{g_K\left(\frac{L_T(z)}{K_T(z)},1\right)}, \tag{B23}$$

where the second equation holds because $g(\cdot)$ has constant returns to scale. This equation implies that all producers will use the same capital labor ratio, so $\frac{L_T(z)}{K_T(z)}=\frac{L_T}{\bar{K}}$ where L_T is total labor demand in period T and $\bar{K} = \int_{N_H} K_T(z)\,dz$ is the aggregate capital stock. Using that

$$\frac{S_T}{P_T} = \frac{W_T/P_T}{A_T g_L\left(\frac{L_T}{\bar{K}}, 1\right)}, \tag{B24}$$

the implication is that marginal costs are the same across all producers within the same country. Also, notice that equation (B22) can be written as

$$R_T = S_T A_T g_K\left(\frac{L_T}{\bar{K}}, 1\right). \tag{B25}$$

This equation defines the rental rate of capital and has no implications for the equilibrium dynamics of the remaining variables. We are not interested in the rental rate of capital, so we will not use this equation any further.

Using equation (A9) we can write real marginal costs as

$$\frac{S_T}{P_T} = \frac{v_l(L_T, \xi_T)}{A_T g_L\left(L_T, \bar{K}\right) u_c(C_T)}. \tag{B26}$$

This equation shows that real marginal costs depend on aggregate labor demand in home, not labor demand of producer z as under the assumption of heterogenous labor markets. We can find an expression for total demand for home goods in period T by integrating equation (B17) across all producers in home. This gives

$$\begin{aligned} & \int_{N_H}\left(\frac{P_{H,T}(z)}{P_{H,T}}\right)^{-\theta_T}(C_{H,T}+G_{H,T}) + \left(\frac{P^*_{H,T}(z)}{P^*_{H,T}}\right)^{-\theta_T}\left(C^*_{H,T}+G^*_{H,T}\right)dz \\ = & \int_{N_H} A_T g\left(\frac{L_T(z)}{K_T(z)}, 1\right) K_T(z)\,dz \end{aligned} \tag{B27}$$

or

$$\begin{aligned} & \left(\frac{\int_{N_H} P_{H,T}(z)^{-\theta_T}\,dz}{(P_{H,T})^{-\theta_T}}\right)(C_{H,T}+G_{H,T}) \\ & + \left(\frac{\int_{N_H} P^*_{H,T}(z)^{-\theta_T}\,dz}{\left(P^*_{H,T}\right)^{-\theta_T}}\right)\left(C^*_{H,T}+G^*_{H,T}\right) = A_T g\left(L_T, \bar{K}\right). \end{aligned} \tag{B28}$$

The left hand side of this equation is total demand for home products which we denote by D_T, implying that $L_T = g^{-1}\left(D_T/A_T, \bar{K}\right)$. Hence, we may write marginal costs as

$$\frac{S_T}{P_T} = \frac{v_l\left(g^{-1}\left(D_T/A_T, \bar{K}\right), \xi_T\right)}{A_T g_L\left(g^{-1}\left(D_T/A_T, \bar{K}\right), \bar{K}\right) u_c(C_T)}. \tag{B29}$$

Under homogeneous labor markets nominal output is also given by equation (B12). However, in contrast to the heterogeneous labor market model we now need to account for the presence of two production factors. We get that the profits of home producers z are given by

$$\begin{aligned}\Phi_t(z) &= \phi_{H,t} P_{H,t}(z) \left(\frac{P_{H,t}(z)}{P_{H,t}}\right)^{-\theta_t} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (C_t + G_t) \\ &\quad + \phi^*_{H,t} \varepsilon_t P^*_{H,t}(z) \left(\frac{P^*_{H,t}(z)}{P^*_{H,t}}\right)^{-\theta_t} \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} (C^*_t + G^*_t) \\ &\quad - W_t(z) L_t(z) - R_t \left(K_t(z) - \bar{K}(z)\right).\end{aligned} \tag{B30}$$

Aggregating across producers yields

$$\begin{aligned}\int_{N_H} \Phi_t(z)\,dz &= \phi_{H,t} P_{H,t} \frac{\int_{N_H} (P_{H,t}(z))^{1-\theta_t}\,dz}{(P_{H,t})^{1-\theta_t}} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (C_t + G_t) \\ &\quad + \phi^*_{H,t} \varepsilon_t P^*_{H,t} \frac{\int_{N_H} \left(P^*_{H,t}(z)\right)^{1-\theta_t}\,dz}{\left(P^*_{H,t}\right)^{1-\theta_t}} \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} (C^*_t + G^*_t) \\ &\quad - \int_{N_H} W_t(z) L_t(z)\,dz - R_T \left(\int_{N_H} K_T(z)\,dz - \bar{K}\right),\end{aligned} \tag{B31}$$

where we have used that $\bar{K} = \int_{N_H} \bar{K}_T(z)\,dz$. Since $\int_{N_H} K_T(z)\,dz = \bar{K}$ the last term drops out so we have the same expression as under heterogeneous labor markets. Hence, real output in home and foreign are given by equations (B15) and (B16).

C Steady state

In this section we discuss the non-stochastic steady states of the models. We focus on symmetric, zero-inflation steady states where the growth rates of the real variables are zero. With symmetric we mean a steady state where all shocks and all real variables in Home and Foreign attain the same values, so e.g. $C = C^*$, $G = G^*, \theta = \theta^*$ and so on, where a variable without a time subscript denotes the steady state value. Furthermore, the labor supply of all households are identical. Notice that because of the optimal risk sharing condition, the symmetry assumption implies that $Q = 1$.

Steinsson focus on a steady state where $C = C^* = Y$ (Steinsson, 2007). In contrast, in our steady state $C + G = C^* + G^* = Y$. Furthermore, because of the symmetry assumption $Y = Y^*$. In the following we will discuss how to derive the steady state values of the various variables in the two models.

C.1 Heterogeneous labor markets

In the steady state we consider, real marginal costs are the same for all producers in Home and Foreign, which follows from the symmetry assumption. This, together with the fact that the steady state real exchange rate is unity, implies that all producers set the same relative prices

$$\frac{P_H(z)}{P} = \frac{P^*_H(z)}{P^*} = \frac{P^*_F(z)}{P^*} = \frac{P_F(z)}{P} = \frac{S}{P}\frac{\theta}{\theta - 1} = \frac{S^*}{P^*}\frac{\theta^*}{\theta^* - 1}. \tag{C1}$$

Equation (A31) then implies that $P_H/P = P_H(z)/P$. Similar conditions hold for the other indices P_F, P_F^* and P_H^* implying that we may write the resource constraints as

$$\phi_H \left(\frac{P_H}{P}\right)^{-\eta} (C+G) + \phi_H^* \left(\frac{P_H^*}{P^*}\right)^{-\eta} (C^* + G^*) = Af(L), \tag{C2}$$

$$\phi_F^* \left(\frac{P_F^*}{P^*}\right)^{-\eta} (C^* + G^*) + \phi_F \left(\frac{P_F}{P}\right)^{-\eta} (C+G) = A^* f(L^*), \tag{C3}$$

where we use $L(x) = L$ and $L^*(x) = L^*$. Furthermore, in a symmetric steady state $C = C^*, G = G^*, A = A^*$, and $L = L^*$ so the resource constraints imply

$$\phi_H \left(\frac{P_H}{P}\right)^{-\eta} + \phi_H^* \left(\frac{P_H^*}{P^*}\right)^{-\eta} = \phi_F^* \left(\frac{P_F^*}{P^*}\right)^{-\eta} + \phi_F \left(\frac{P_F}{P}\right)^{-\eta}. \tag{C4}$$

The Home and Foreign CPI price indices, equations (A26) and (A37) can be written

$$1 = \phi_H \left(\frac{P_H}{P}\right)^{1-\eta} + \phi_F \left(\frac{P_F}{P}\right)^{1-\eta}, \tag{C5}$$

$$1 = \phi_F^* \left(\frac{P_F^*}{P^*}\right)^{1-\eta} + \phi_H^* \left(\frac{P_H^*}{P^*}\right)^{1-\eta}. \tag{C6}$$

These equations imply that

$$\frac{P_H}{P} = \frac{P_F}{P} = \frac{P_F^*}{P^*} = \frac{P_H^*}{P^*} = 1 \tag{C7}$$

provided that $\eta \neq 1$ which is the relevant case in our analysis.[2] Using these conditions in equations (C4), (C5) and (C6) yields

$$\phi_H + \phi_H^* = \phi_F^* + \phi_F, \quad 1 = \phi_H + \phi_F, \quad 1 = \phi_F^* + \phi_H^* \tag{C8}$$

which in turn implies that $\phi_H^* = \phi_F$ and $\phi_H = \phi_F^*$. Hence, to have a symmetric steady state requires the same degree of home bias in Home and Foreign.[3]

Using the fact that relative prices are unity, and home and foreign private and public consumption are identical, equation (C2) implies

$$C = Af(L) - G \tag{C9}$$

[2] If $\eta = 1$, the appropriate CPI price index is Cobb-Douglas so in the steady state $1 = \left(\frac{P_H}{P}\right)^{\phi_H} \left(\frac{P_F}{P}\right)^{\phi_F}$ with a corresponding relation holding for the Foreign CPI. These relations also imply $\frac{P_H}{P} = \frac{P_F}{P} = \frac{P_F^*}{P^*} = \frac{P_H^*}{P^*} = 1$.

[3] In other words, it is not possible to have a symmetric steady state (as we have defined it) if the degrees of home bias in the two countries are not the same.

with a similar equation holding in Foreign. The optimal pricing relation (B10) implies

$$\frac{S}{P} = \frac{v_l(L,\xi)}{Af_l(L)\,u_c(C)} = \frac{\theta-1}{\theta}, \tag{C10}$$

which, using the previous expression for C, may be written

$$\frac{v_l(L,\xi)}{Af_l(L)\,u_c(Af(L)-G)} = \frac{\theta-1}{\theta}. \tag{C11}$$

As this equation implicitly defines L, it can be used to derive labor supply in the steady state, conditional on assumptions about functional forms and parameter values. Using this value of L, the steady state values of the remaining variables can be determined. In particular, we may use the fact that the definition of aggregate output in the two countries, equations (B15) and (B16) implies $Y = C + G = Af(L)$ and $Y^* = C^* + G^* = A^* f(L^*)$. We do not determine these steady state value here, however, as the log-linearized equations we will derive subsequently will not depend on these steady state values.

C.2 Homogeneous labor markets

The only difference between the homogeneous and heterogeneous labor market models in the steady state is that production is determined by different production functions. Using the same arguments as under the heterogeneous labor market the resource constraint in the homogeneous labor market model implies

$$C = Ag\left(L, \bar{K}\right) - G. \tag{C12}$$

where $\bar{K}$ is the aggregate stock of capital in the economy. The optimal pricing relation then implies

$$\frac{v_l(L,\xi)}{Ag_L\left(L,\bar{K}\right)u_c\left(Ag\left(L,\bar{K}\right)-G\right)} = \frac{\theta-1}{\theta} \tag{C13}$$

This equation may be solved to yield the steady state value of L conditional on assumptions about functional forms, parameter values and the steady state value of $\bar{K}$. Again, we will not do this, as the subsequent log-linearized equations do not depend on the steady state values of these parameters. What we will do, however, is to assume that two particular functions of the steady state values in the two models, denoted by ω and defined below, take the same value. This function defines the elasticity of marginal cost with respect to demand.

D Log-linearizing the models

In this section we log-linearize the equilibrium conditions of the two models around the zero inflation non-stochastic steady state. We use the linearized equations to determine the first-order equilibrium dynamics of the models.

D.1 Households

The equations characterizing the behavior of households are the two consumption Euler equations (A6), (A17) and the optimal risk sharing equation (A21). In log deviations from the steady state, these are given as

$$c_t = \mathrm{E}_t c_{t+1} - \sigma \left(i_t - \mathrm{E}_t \pi_{t+1}\right), \tag{D1}$$

$$c_t^* = \mathrm{E}_t c_{t+1}^* - \sigma \left(i_t^* - \mathrm{E}_t \pi_{t+1}^*\right), \tag{D2}$$

$$\sigma q_t = c_t - c_t^*, \tag{D3}$$

where a lower case variable denotes the log-deviation from steady state of the corresponding upper case variable. We have defined $\sigma = -\frac{u_c(C)}{u_{cc}(C)C}$. Notice that these equations are similar to those in Steinsson despite the differences in the original consumption Euler equations, equations (A6) and (A7). The difference in the original equations is related to Jensen's inequality, and is zero to a first order approximation. Therefore the log-linear consumption Euler equations are identical.

D.2 Firms

We now turn to linearizing the home price indices, equations (A26), (A31) and (A32). We may rewrite (A26) as

$$1 = \phi_{H,t} \left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} + \phi_{F,t} \left(\frac{P_{F,t}}{P_t}\right)^{1-\eta}. \tag{D4}$$

First, a remark on notation. A lower case variable indicates a log-deviation of the real equivalent of the particular upper case variable from steady state. For instance, $p_{H,t}$ is the log-deviation from steady state of $\frac{P_{H,t}}{P_t}$ whereas c_t is the log-deviation of C_t as previously noted. Using this definition, we can write the previous equation as

$$0 = \phi_H p_{H,t} + \phi_F p_{F,t}, \tag{D5}$$

where we have used that because $\phi_{H,t} + \phi_{F,t} = 1$, changes in relative demand have no impact on the price index up to the first order. We also use that all relative prices are unity in the steady state as discussed in section (C).

Because of the Calvo price setting assumption, in terms of log-deviations from steady state, equation (A31) can be written

$$\pi_{H,t} = \frac{1-\alpha}{\alpha} \left(p_{h,t} - p_{H,t}\right), \tag{D6}$$

where $p_{h,t}$ indicates the price set by producers changing price in period t and $p_{H,t}$ is the price index in period t. $\pi_{H,t}$ is the log-deviation from steady state of $\frac{P_{H,t}}{P_{H,t-1}}$. Similarly for equation (A32):

$$\pi_{F,t} = \frac{1-\alpha}{\alpha} \left(p_{f,t} - p_{F,t}\right). \tag{D7}$$

Steinsson argues that $\pi_t = \phi_H \pi_{H,t} + \phi_F \pi_{F,t}$. To see this, notice that

$$\begin{aligned}
\pi_t &= \log\left(\frac{P_t}{P_{t-1}}\right) \qquad\qquad \text{(D8)}\\
&= \log\left(\frac{P_{H,t}}{P_{H,t-1}}\frac{P_t}{P_{H,t}}\frac{P_{H,t-1}}{P_{t-1}}\right)\\
&= \pi_{H,t} - p_{H,t} + p_{H,t-1}.
\end{aligned}$$

and, similarly, that $\pi_t = \pi_{F,t} - p_{F,t} + p_{F,t-1}$. Combining these equations imply

$$\pi_t = \phi_H\left(\pi_{H,t} - p_{H,t} + p_{H,t-1}\right) + \phi_F\left(\pi_{F,t} - p_{F,t} + p_{F,t-1}\right). \qquad \text{(D9)}$$

Using that $0 = \phi_H p_{H,t} + \phi_F p_{F,t}$ we get

$$\pi_t = \phi_H \pi_{H,t} + \phi_F \pi_{F,t}. \qquad \text{(D10)}$$

Similar manipulations of the foreign indices give

$$\begin{aligned}
\pi^*_{F,t} &= \frac{1-\alpha}{\alpha}\left(p^*_{f,t} - p^*_{F,t}\right), \qquad \text{(D11)}\\
\pi^*_{H,t} &= \frac{1-\alpha}{\alpha}\left(p^*_{h,t} - p^*_{H,t}\right), \qquad \text{(D12)}\\
\pi^*_t &= \phi_H \pi^*_{F,t} + \phi_F \pi^*_{H,t}, \qquad \text{(D13)}
\end{aligned}$$

where we have used that $\phi^*_F = \phi_H$ and $\phi^*_H = \phi_F$ in the steady state.

D.2.1 Heterogenous labor markets

We now turn to the linearization of equations (B1), (B7), (B10) and (B11). Consider first equation (B1) which can be written

$$\begin{aligned}
D_{t,T} &= \phi_{H,T}\left(\frac{P_{H,t}(z)}{P_t}\frac{P_T}{P_{H,T}}\prod_{k=t+1}^{T}\frac{1}{\Pi_k}\right)^{-\theta_T}\left(\frac{P_{H,T}}{P_T}\right)^{-\eta}(C_T + G_T)\\
&\quad +\phi^*_{H,T}\left(\frac{P^*_{H,t}(z)}{P^*_t}\frac{P^*_T}{P^*_{H,T}}\prod_{k=t+1}^{T}\frac{1}{\Pi^*_k}\right)^{-\theta_T}\left(\frac{P^*_{H,T}}{P^*_T}\right)^{-\eta}(C^*_T + G^*_T),
\end{aligned} \qquad \text{(D14)}$$

where the left-hand side indicate total demand in period T for the good of a producer that last changed her price in period t. The log-linear version of this equation is

$$\begin{aligned}
&Y d_{t,T} = \qquad\qquad \text{(D15)}\\
&\phi_H (C+G)\left[\widehat{\phi}_{H,T} - \theta\left(p_{h,t} - p_{H,T} - \sum_{k=t+1}^{T}\pi_k\right) - \eta p_{H,T} + \frac{C}{C+G}c_T + \frac{G}{C+G}g_T\right]\\
&+\phi_F (C+G)\left[\widehat{\phi}^*_{H,T} - \theta\left(p^*_{h,t} - p^*_{H,T} - \sum_{k=t+1}^{T}\pi^*_k\right) - \eta p^*_{H,T} + \frac{C}{C+G}c^*_T + \frac{G}{C+G}g^*_T\right],
\end{aligned}$$

where Y, C and G are the steady state levels of output, consumption and public spending and $d_{t,T}$ is the log-linear version of $D_{t,T}$. We denote by $\widehat{\phi}_{H,T}$ and $\widehat{\phi}^*_{H,T}$ the log-linear versions of

$\phi_{H,T}$ and $\phi^*_{H,T}$. We have used that in a symmetric steady state $D = Y$, $C = C^*$, $G = G^*$, and $\phi^*_H = \phi_F$. We now use $Y = C + G$, denote with a superscript M weighted averages of home and foreign variables, such that $c^M_T = \phi_H c_T + \phi_F c^*_T$ and $c^{M*}_T = \phi_H c^*_T + \phi_F c_T$ and define $S_c = \frac{C}{C+G}$ to get

$$d_{t,T} = S_c c^M_T + (1 - S_c) g^M_T + \widehat{\phi}^M_{H,T} - \theta p^M_{h,t} + (\theta - \eta) p^M_{H,T} + \theta \sum_{k=t+1}^{T} \pi^M_k. \tag{D16}$$

Linearizing the foreign equivalent gives

$$d^*_{t,T} = S_c c^{M*}_T + (1 - S_c) g^{M*}_T + \widehat{\phi}^{M*}_{F,T} - \theta p^{M*}_{f,t} + (\theta - \eta) p^{M*}_{F,T} + \theta \sum_{k=t+1}^{T} \pi^{M*}_k, \tag{D17}$$

where $\widehat{\phi}^{M*}_{F,T} = \phi_H \widehat{\phi}^*_{F,T} + \phi_F \widehat{\phi}_{F,T}$, $p^{M*}_{f,t} = \phi_H p^*_{f,t} + \phi_F p_{f,t}$ and $p^{M*}_{F,T} = \phi_H p^*_{F,T} + \phi_F p_{F,T}$.

We now turn to linearizing the expression for real marginal costs, equation (B7):

$$\frac{S_{t,T}}{P_t} = \frac{v_l \left(f^{-1}\left(D_{t,T}/A_T\right), \xi_T\right)}{A_T f_l \left(f^{-1}\left(D_{t,T}/A_T\right)\right) u_c \left(C_T\right)}. \tag{D18}$$

We use $\left(h^{-1}\right)'(x) = \frac{1}{h'(h^{-1}(x))}$ to get

$$\begin{aligned}
s\, s_{t,T} \quad = \quad & \frac{v_{ll}(L,\xi)\frac{1}{f_l(L)}\frac{1}{A} A f_l(L) u_c(C) - v_l(L,\xi) A u_c(C) f_{ll}(L) \frac{1}{f_l(L)}\frac{1}{A}}{\left(A f_l(L) u_c(C)\right)^2} Y d_{t,T} \\
& + \frac{v_{ll}(L,\xi)\frac{1}{f_l(L)}\left(-\frac{Y}{A^2}\right) A f_l(L) u_c(C)}{\left(A f_l(L) u_c(C)\right)^2} A a_T \\
& - \frac{v_l(L,\xi)\left(f_l(L) u_c(C) + A u_c(C) f_{ll}(L) \frac{1}{f_l(L)}\left(-\frac{Y}{A^2}\right)\right)}{\left(A f_l(L) u_c(C)\right)^2} A a_T \\
& + \frac{v_{l\xi}(L,\xi)}{A f_l(L) u_c(C)} \xi_T - \frac{v_l(L,\xi)}{A f_l(L) u_c(C)} \frac{u_{cc}(C) C}{u_c(C)} c_T,
\end{aligned}$$

where s is steady state real marginal costs, $s_{t,T}$ is log-linearized expression for real marginal costs and $a_T = \log(A_T)$. This equation can be written

$$\begin{aligned}
s_{t,T} \quad = \quad & \left(\frac{v_{ll}(L,\xi) Y}{v_l(L,\xi) f_l(L) A} - \frac{f_{ll}(L) Y}{A \left(f_l(L)\right)^2}\right) d_{t,T} + \frac{1}{\sigma} c_T + \frac{v_{l\xi}(L,\xi)}{v_l(L,\xi)} \xi_T \\
& - \left(1 + \frac{v_{ll}(L,\xi) Y}{v_l(L,\xi) f_l(L) A} - \frac{f_{ll}(L) Y}{A \left(f_l(L)\right)^2}\right) a_T.
\end{aligned} \tag{D19}$$

Notice that ξ_T does not indicate a log-deviation from steady state but instead measures the absolute deviation from steady state.

This expression is comparable to the second to last equation on page 9 in Steinsson (2007) except for the last term in the parentheses. Steinsson has $\frac{\Psi_y Y}{\Psi A}$ where $\Psi = \frac{1}{f_l(f^{-1}(y/A))}$. It is straightforward to show that

$$\frac{\Psi_y Y}{\Psi A} = -\frac{f_{ll}(L) Y}{A^2 \left(f_l(L)\right)^2}. \tag{D20}$$

Hence, $\frac{\Psi_y Y}{\Psi A} \neq -\frac{f_{ll}(L)Y}{A(f_l(L))^2}$ in general. Correspondingly, we define

$$\omega = \frac{v_{ll}(L,\xi)Y}{v_l(L,\xi)Af_l(L)} - \frac{f_{ll}(L)Y}{A(f_l(L))^2}. \tag{D21}$$

Moreover, we define (as in Steinsson, 2007) $\widetilde{a}_T = (1+\omega)a_T - \frac{v_{l\xi}(L,\xi)}{v_l(L,\xi)}\xi_T$ so we have

$$s_{t,T} = \omega d_{t,T} + \frac{1}{\sigma}c_T - \widetilde{a}_T. \tag{D22}$$

Using the expression for $d_{t,T}$, equation (D16) we get

$$\begin{aligned} s_{t,T} \quad = \quad & \omega S_c c_T^M + \frac{1}{\sigma}c_T - \omega\theta p_{h,t}^M + \omega(\theta-\eta)p_{H,T}^M + \omega\theta\sum_{k=t+1}^{T}\pi_k^M - \widetilde{a}_T \\ & +\omega(1-S_c)g_T^M + \omega\widehat{\phi}_{H,T}^M. \end{aligned} \tag{D23}$$

Similarly, real marginal cost in foreign is given as

$$\begin{aligned} s_{t,T}^* \quad = \quad & \omega S_c c_T^{M*} + \frac{1}{\sigma}c_T^* - \omega\theta p_{f,t}^{M*} + \omega(\theta-\eta)p_{F,T}^{M*} + \omega\theta\sum_{k=t+1}^{T}\pi_k^{M*} - \widetilde{a}_T^* \\ & +\omega(1-S_c)g_T^{M*} + \omega\widehat{\phi}_{F,T}^{M*}. \end{aligned} \tag{D24}$$

These equations correspond to, but are slightly different from, equations (40)–(41) in Steinsson (2007). In section E we explain the reason for this difference.

We now turn to linearizing the optimal pricing relations equations (B10) and (B11). The log-linear version of equation (B10) is given as

$$\mathrm{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(p_{h,t} - \sum_{k=t+1}^{T}\pi_k - s_{t,T} + \frac{1}{\theta-1}\widehat{\theta}_T\right) = 0, \tag{D25}$$

where $\widehat{\theta}_T$ is the log-linear version of θ_T. This expression can be written

$$p_{h,t}\frac{1}{1-\alpha\beta} = \mathrm{E}_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left(s_{t,t+j} - \frac{1}{\theta-1}\widehat{\theta}_{t+j}\right) + \mathrm{E}_t\sum_{j=0}^{\infty}(\alpha\beta)^j\sum_{k=1}^{j}\pi_{t+k}. \tag{D26}$$

Consider the last term in this expression:

$$
\begin{aligned}
& \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \sum_{k=1}^{j} \pi_{t+k} \\
= \; & \mathrm{E}_t \left[\alpha\beta\pi_{t+1} + (\alpha\beta)^2 (\pi_{t+1} + \pi_{t+2}) + (\alpha\beta)^3 (\pi_{t+1} + \pi_{t+2} + \pi_{t+3}) + ... \right] \qquad \text{(D27)} \\
= \; & \mathrm{E}_t \left[\pi_{t+1} \sum_{k=1}^{\infty} (\alpha\beta)^k + \pi_{t+2} \sum_{k=2}^{\infty} (\alpha\beta)^k + \pi_{t+3} \sum_{k=3}^{\infty} (\alpha\beta)^k + ... \right] \\
= \; & \mathrm{E}_t \left[\pi_{t+1} \frac{\alpha\beta}{1-\alpha\beta} + \pi_{t+2} \frac{(\alpha\beta)^2}{1-\alpha\beta} + \pi_{t+3} \frac{(\alpha\beta)^3}{1-\alpha\beta} + ... \right] \\
= \; & \frac{1}{1-\alpha\beta} \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{t+j}.
\end{aligned}
$$

Hence, the expression for $p_{h,t}$ is

$$
p_{h,t} = (1-\alpha\beta) \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(s_{t,t+j} - \frac{1}{\theta-1} \widehat{\theta}_{t+j} \right) + \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{t+j}, \qquad \text{(D28)}
$$

which corresponds to equation (42) in Steinsson (2007).

Similar manipulations of equation (B11) and the foreign optimal price relations give

$$
p^*_{h,t} = (1-\alpha\beta) \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(s_{t,t+j} - \frac{1}{\theta-1} \widehat{\theta}_{t+j} - q_{t+j} \right) + \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi^*_{t+j}, \qquad \text{(D29)}
$$

$$
p^*_{f,t} = (1-\alpha\beta) \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(s^*_{t,t+j} - \frac{1}{\theta-1} \widehat{\theta}^*_{t+j} \right) + \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi^*_{t+j}, \qquad \text{(D30)}
$$

$$
p_{f,t} = (1-\alpha\beta) \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(s^*_{t,t+j} - \frac{1}{\theta-1} \widehat{\theta}^*_{t+j} + q_{t+j} \right) + \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{t+j}. \qquad \text{(D31)}
$$

Equation (D6) can be written

$$
\frac{1-\alpha}{\alpha} p_{h,t} = \pi_{H,t} + \frac{1-\alpha}{\alpha} p_{H,t}. \qquad \text{(D32)}
$$

Using this relation and combining (D23) and (D28) gives

$$
\begin{aligned}
& \pi_{H,t} + \frac{1-\alpha}{\alpha} p_{H,t} = \frac{1-\alpha}{\alpha} p_{h,t} \qquad \text{(D33)} \\
= \; & \kappa \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\omega S_c c^M_{t+j} + \frac{1}{\sigma} c_{t+j} - \omega\theta p^M_{h,t} + \omega (\theta - \eta) p^M_{H,t+j} + \omega\theta \sum_{k=t+1}^{t+j} \pi^M_k \right. \\
& \left. - \widetilde{a}_{t+j} + \omega (1 - S_c) g^M_{t+j} + \omega \widehat{\phi}^M_{H,t+j} - \frac{1}{\theta-1} \widehat{\theta}_{t+j} \right) + \frac{1-\alpha}{\alpha} \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{t+j},
\end{aligned}
$$

which corresponds to equation (46) on page 10 in Steinsson (2007) under the assumption that $S_c = 1$, although Steinsson writes $... + \phi^M_{H,t+j} ...$ instead of $... \omega\phi^M_{H,t+j} ...$ We have defined $\kappa =$

$\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$. Rewriting this equation gives

$$
\begin{aligned}
& (1+\omega\theta)\left(\pi_{H,t}+\frac{1-\alpha}{\alpha}p_{H,t}\right)-\omega\theta\phi_F\left(\pi^R_{H,t}+\frac{1-\alpha}{\alpha}p^R_{H,t}\right) \\
= \; & \kappa \mathrm{E}_t\sum_{j=0}^{\infty}(\alpha\beta)^j\Bigg[\left(\omega S_c+\sigma^{-1}\right)c^M_{t+j}+\phi_F q_{t+j}+\omega\left(\theta-\eta\right)p^M_{H,t+j}-\widetilde{a}_{t+j} \\
& +\omega\left(1-S_c\right)g^M_{t+j}+\omega\widehat{\phi}^M_{H,t+j}-\frac{1}{\theta-1}\widehat{\theta}_{t+j}\Bigg] \\
& +\frac{1-\alpha}{\alpha}\mathrm{E}_t\sum_{j=1}^{\infty}(\alpha\beta)^j\left(\left(1+\omega\theta\right)\pi_{t+j}-\omega\theta\phi_F\pi^R_{t+j}\right),
\end{aligned}
\tag{D34}
$$

where we have used that $\sigma^{-1}\left(c_{t+j}-c^*_{t+j}\right)=q_{t+j}$, that $p^M_{h,t}=\frac{\alpha}{1-\alpha}\pi^M_{H,t}+p^M_{H,t}$ and defined $\pi^R_{H,t}=\pi_{H,t}-\pi^*_{H,t}$ and other variables with a superscript R analoguously. This equation corresponds to the last equation on page 10 in Steinsson. Notice that this equation can be written in quasi-differenced form as

$$
\begin{aligned}
& (1+\omega\theta)\left(\pi_{H,t}+\frac{1-\alpha}{\alpha}p_{H,t}\right)-\omega\theta\phi_F\left(\pi^R_{H,t}+\frac{1-\alpha}{\alpha}p^R_{H,t}\right) \\
= \; & \kappa\Big[\left(\omega S_c+\sigma^{-1}\right)c^M_t+\phi_F q_t+\omega\left(\theta-\eta\right)p^M_{H,t}-\widetilde{a}_t+\omega\left(1-S_c\right)g^M_t \\
& +\omega\widehat{\phi}^M_{H,t}-\frac{1}{\theta-1}\widehat{\theta}_t\Big] \\
& +\alpha\beta\mathrm{E}_t\Bigg[\frac{1-\alpha}{\alpha}\left(\left(1+\omega\theta\right)\pi_{t+1}-\omega\theta\phi_F\pi^R_{t+1}\right) \\
& +\left(1+\omega\theta\right)\left(\pi_{H,t+1}+\frac{1-\alpha}{\alpha}p_{H,t+1}\right)\Bigg] \\
& -\alpha\beta\mathrm{E}_t\left[\omega\theta\phi_F\left(\pi^R_{H,t+1}+\frac{1-\alpha}{\alpha}p^R_{H,t+1}\right)\right].
\end{aligned}
\tag{D35}
$$

We now use that $\pi_{t+1}=\pi_{H,t+1}-p_{H,t+1}+p_{H,t}$ which holds because

$$
\begin{aligned}
\pi_{t+1} &= \log\left(\frac{P_{t+1}}{P_t}\right) \\
&= \log\left(\frac{P_{H,t+1}}{P_{H,t}}\frac{P_{t+1}}{P_{H,t+1}}\frac{P_{H,t}}{P_t}\right) \\
&= \pi_{H,t+1}-p_{H,t+1}+p_{H,t}.
\end{aligned}
\tag{D36}
$$

Using this relation gives

$$
\begin{aligned}
& \pi_{H,t}-\beta\mathrm{E}_t\pi_{H,t+1}+\kappa p_{H,t}-\phi_F\frac{\omega\theta}{1+\omega\theta}\left(\pi^R_{H,t}-\beta\mathrm{E}_t\pi^R_{H,t+1}+\kappa p^R_{H,t}\right) \\
= \; & \kappa\frac{\omega S_c+\sigma^{-1}}{1+\omega\theta}c^M_t+\kappa\frac{\omega\left(\theta-\eta\right)}{1+\omega\theta}p^M_{H,t}+\kappa\frac{\phi_F}{1+\omega\theta}q_t \\
& -\frac{\kappa}{1+\omega\theta}\left(\widetilde{a}_t-\omega\left(1-S_c\right)g^M_t-\omega\widehat{\phi}^M_{H,t}+\frac{1}{\theta-1}\widehat{\theta}_t\right).
\end{aligned}
\tag{D37}
$$

Similar manipulations of equation (D29) gives

$$\begin{aligned}
&\pi^*_{H,t} - \beta \mathrm{E}_t \pi^*_{H,t+1} + \kappa p^*_{H,t} + \phi_H \frac{\omega\theta}{1+\omega\theta} \left(\pi^R_{H,t} - \beta \mathrm{E}_t \pi^R_{H,t+1} + \kappa p^R_{H,t}\right) \\
= \;& \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} c^M_t + \kappa \frac{\omega(\theta - \eta)}{1+\omega\theta} p^M_{H,t} - \kappa \frac{\phi_H}{1+\omega\theta} q_t \\
&- \frac{\kappa}{1+\omega\theta}\left(\widetilde{a}_t - \omega(1-S_c) g^M_t - \omega \widehat{\phi}^M_{H,t} + \frac{1}{\theta-1}\widehat{\theta}_t\right).
\end{aligned} \tag{D38}$$

Using these two equations it is straightforward to show that

$$\pi^R_{H,t} = \beta \mathrm{E}_t \pi^R_{H,t+1} - \kappa p^R_{H,t} + \kappa q_t, \tag{D39}$$

$$\begin{aligned}
\pi^M_{H,t} =\;& \beta \mathrm{E}_t \pi^M_{H,t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} c^M_t - \kappa \frac{1+\omega\eta}{1+\omega\theta} p^M_{H,t} \\
&- \frac{\kappa}{1+\omega\theta}\left(\widetilde{a}_t - \omega(1-S_c) g^M_t - \omega \widehat{\phi}^M_{H,t} + \frac{1}{\theta-1}\widehat{\theta}_t\right).
\end{aligned} \tag{D40}$$

Notice that the last equation is different from the corresponding equation in Steinsson (2007) (the fifth equation on page 11). The equation in Steinsson adds $\kappa \frac{2\phi_H \phi_F}{1+\omega\theta} q_t$ to the right-hand side.

We now use that $\pi_{H,t} = \pi^M_{H,t} + \phi_F \pi^R_{H,t}$ and $\pi^*_{H,t} = \pi^M_{H,t} - \phi_H \pi^R_{H,t}$ to get

$$\begin{aligned}
\pi_{H,t} =\;& \beta \mathrm{E}_t \pi_{H,t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} c^M_t - \kappa \frac{1+\omega\eta}{1+\omega\theta} p^M_{H,t} - \kappa \phi_F p^R_{H,t} + \kappa \phi_F q_t \\
&- \frac{\kappa}{1+\omega\theta}\left(\widetilde{a}_t - \omega(1-S_c) g^M_t - \omega \widehat{\phi}^M_{H,t} + \frac{1}{\theta-1}\widehat{\theta}_t\right),
\end{aligned} \tag{D41}$$

$$\begin{aligned}
\pi^*_{H,t} =\;& \beta \mathrm{E}_t \pi^*_{H,t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} c^M_t - \kappa \frac{1+\omega\eta}{1+\omega\theta} p^M_{H,t} + \kappa \phi_H p^R_{H,t} - \kappa \phi_H q_t \\
&- \frac{\kappa}{1+\omega\theta}\left(\widetilde{a}_t - \omega(1-S_c) g^M_t - \omega \widehat{\phi}^M_{H,t} + \frac{1}{\theta-1}\widehat{\theta}_t\right),
\end{aligned} \tag{D42}$$

where we have used that $\phi^M_{H,t} = -\phi^{M*}_{F,t}$. Similar manipulations of the corresponding foreign variables yield

$$\begin{aligned}
\pi^*_{F,t} =\;& \beta \mathrm{E}_t \pi^*_{F,t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} c^{M*}_t - \kappa \frac{1+\omega\eta}{1+\omega\theta} p^{M*}_{F,t} - \kappa \phi_F p^{R*}_{F,t} - \kappa \phi_F q_t \\
&- \frac{\kappa}{1+\omega\theta}\left(\widetilde{a}^*_t - \omega(1-S_c) g^{M*}_t - \omega \widehat{\phi}^{M*}_{F,t} + \frac{1}{\theta-1}\widehat{\theta}^*_t\right),
\end{aligned} \tag{D43}$$

$$\begin{aligned}
\pi_{F,t} =\;& \beta \mathrm{E}_t \pi_{F,t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} c^{M*}_t - \kappa \frac{1+\omega\eta}{1+\omega\theta} p^{M*}_{F,t} + \kappa \phi_H p^{R*}_{F,t} + \kappa \phi_H q_t \\
&- \frac{\kappa}{1+\omega\theta}\left(\widetilde{a}^*_t - \omega(1-S_c) g^{M*}_t - \omega \widehat{\phi}^{M*}_{F,t} + \frac{1}{\theta-1}\widehat{\theta}^*_t\right),
\end{aligned} \tag{D44}$$

where $p^{M*}_{F,t} = \phi_H p^*_{F,t} + \phi_F p_{F,t}$ and $p^{R*}_{F,t} = p^*_{F,t} - p_{F,t}$. Using equations (D10) and (D13) gives

$$\begin{aligned}
\pi_t =\;& \beta \mathrm{E}_t \pi_{t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta} \left(\phi_H c^M_t + \phi_F c^{M*}_t\right) - \kappa \frac{1+\omega\eta}{1+\omega\theta} \left(\phi_H p^M_{H,t} + \phi_F p^{M*}_{F,t}\right) \\
&- \kappa \phi_H \phi_F \left(p^R_{H,t} - p^{R*}_{F,t}\right) + 2\kappa \phi_H \phi_F q_t \\
&- \frac{\kappa}{1+\omega\theta}\left[\widetilde{a}^M_t - \omega(1-S_c)\left(\phi_H g^M_t + \phi_F g^{M*}_t\right) - \omega(\phi_H - \phi_F)\widehat{\phi}^M_{H,t} + \frac{1}{\theta-1}\widehat{\theta}^M_t\right],
\end{aligned} \tag{D45}$$

which corresponds to the last equation on page 11 in Steinsson (2007), since $p^R_{F,t} = -p^{R*}_{F,t}$. This equation can be written as

$$\begin{aligned}
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta}\left(\phi_H c^M_t + \phi_F c^{M*}_t\right) - \kappa \phi_F \frac{(\phi_H - \phi_F)\,\omega\,(\theta-\eta)}{1+\omega\theta}\left(p_{F,t} - p^*_{H,t}\right) \\
&\quad +2\kappa\phi_H\phi_F q_t \qquad \text{(D46)} \\
&\quad -\frac{\kappa}{1+\omega\theta}\left[\widetilde{a}^M_t - \omega\,(1-S_c)\left(\phi_H g^M_t + \phi_F g^{M*}_t\right) - \omega\,(\phi_H - \phi_F)\,\widehat{\phi}^M_{H,t} + \frac{1}{\theta-1}\widehat{\theta}^M_t\right].
\end{aligned}$$

Similarly, we get

$$\begin{aligned}
\pi^*_t &= \beta \mathrm{E}_t \pi^*_{t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta}\left(\phi_H c^{M*}_t + \phi_F c^{M}_t\right) + \kappa \phi_F \frac{(\phi_H - \phi_F)\,\omega\,(\theta-\eta)}{1+\omega\theta}\left(p_{F,t} - p^*_{H,t}\right) \\
&\quad -2\kappa\phi_H\phi_F q_t \qquad \text{(D47)} \\
&\quad -\frac{\kappa}{1+\omega\theta}\left[\widetilde{a}^{M*}_t - \omega\,(1-S_c)\left(\phi_H g^{M*}_t + \phi_F g^{M}_t\right) - \omega\,(\phi_H - \phi_F)\,\widehat{\phi}^{M*}_{F,t} + \frac{1}{\theta-1}\widehat{\theta}^{M*}_t\right].
\end{aligned}$$

The calibration Steinsson uses has $\theta = \eta$. Hence, the variable $p_{F,t} - p^*_{H,t}$ is unimportant for the equilibrium dynamics. However, with $\theta \neq \eta$ we need an equation determining the evolution of $p_{F,t} - p^*_{H,t}$. Using the Phillips curve relationships for $\pi^*_{H,t}$ and $\pi_{F,t}$ and $\pi^*_{H,t} = \pi^*_t + p^*_{H,t} - p^*_{H,t-1}$ and $\pi_{F,t} = \pi_t + p_{F,t} - p_{F,t-1}$ gives

$$\begin{aligned}
&\pi^*_t - \pi_t - (1+\kappa+\beta)\left(p_{F,t} - p^*_{H,t}\right) + \left(p_{F,t-1} - p^*_{H,t-1}\right) \qquad \text{(D48)} \\
=\; &-\kappa\phi_F \frac{2\omega\,(\theta-\eta)}{1+\omega\theta}\left(p_{F,t} - p^*_{H,t}\right) + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta}\,(\phi_H - \phi_F)\,c^R_t - 2\kappa\phi_H q_t \\
&+\beta \mathrm{E}_t\left(\pi^*_{t+1} - \pi_{t+1}\right) - \beta \mathrm{E}_t\left(p_{F,t+1} - p^*_{H,t+1}\right) \\
&-\frac{\kappa}{1+\omega\theta}\left[\widetilde{a}^R_t - \omega\,(1-S_c)\left(g^M_t - g^{M*}_t\right) - \omega\left(\widehat{\phi}^M_{H,t} - \widehat{\phi}^{M*}_{F,t}\right) + \frac{1}{\theta-1}\widehat{\theta}^R_t\right],
\end{aligned}$$

which determines the evolution of $p_{F,t} - p^*_{H,t}$. Notice that $p_{F,t} - p^*_{H,t} = \tau_t + q_t$ where τ_t is the home terms of trade.

D.3 Homogeneous labor markets

We now want to linearize the equations of the homogeneous labor markets model. Consider first equation (B28). The log-linear version is given as

$$d_T = \widehat{\phi}^M_{H,T} - \eta p^M_{H,T} + S_c c^M_T + (1-S_c)\,g^M_T. \qquad \text{(D49)}$$

Notice that θ does not enter this equation. To see why consider the first fraction entering equation (B28)

$$\frac{\int_{N_H} P_{H,T}(z)^{-\theta_T}\,dz}{(P_{H,T})^{-\theta_T}} = \frac{\int_{N_H} P_{H,T}(z)^{-\theta_T}\,dz}{\left(\int_{N_H} P_{H,T}(z)^{1-\theta_T}\,dz\right)^{-\frac{\theta_T}{1-\theta_T}}}, \qquad \text{(D50)}$$

where the equality follows from equation (A31). The log-linear approximation of this fraction is given as

$$-\theta \int_{N_H} p_{h,T}(z)\, dz + \theta \int_{N_H} p_{h,T}(z)\, dz = 0, \tag{D51}$$

where $p_{h,T}(z)$ indicates the log-linear version of the real home price charged by home producer z in time T. This price might not have been changed in period T. Up to a linear approximation, this fraction does not affect aggregate demand for home goods. We explain the intuition for this result in the main text.

The log-linear version of (B29) is

$$s_T = \omega d_T + \frac{1}{\sigma} c_T - \widetilde{a}_T, \tag{D52}$$

where ω is defined as

$$\omega = \frac{v_{ll}(L,\xi)\, Y}{v_l(L,\xi)\, A g_l\left(L,\bar{K}\right)} - \frac{g_{ll}\left(L,\bar{K}\right) Y}{A\left(g_l\left(L,\bar{K}\right)\right)^2}, \tag{D53}$$

and $\widetilde{a}_T$ is defined as in the heterogeneous labor markets model. Similar equations determine aggregate demand and real marginal costs in foreign.

We now turn to linearizing the optimal pricing relations. The only difference between the homogeneous and heterogeneous labor market with respect to the pricing relations is that marginal costs are producer specific under heterogeneous labor markets. Under a homogeneous labor market marginal costs are the same for all producers within the same country. Hence, the log-linear optimal pricing functions under homogeneous labor markets are given by equations (D28), (D29), (D30) and (D31) under the restriction that $s_{t,t+j} = s_{t+j}$.

Combining equation (D49) and (D52) yields

$$s_T = \omega \widehat{\phi}^M_{H,T} - \omega \eta p^M_{H,T} + \omega S_c c^M_T + \omega \left(1 - S_c\right) g^M_T + \frac{1}{\sigma} c_T - \widetilde{a}_T. \tag{D54}$$

Inserting this into equation (D28) gives

$$\begin{aligned} p_{h,t} \;&=\; (1-\alpha\beta)\, \mathrm{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \Bigg(\omega \widehat{\phi}^M_{H,t+j} - \omega\eta p^M_{H,t+j} + \omega S_c c^M_{t+j} + \frac{1}{\sigma} c_{t+j} \\ &\quad - \widetilde{a}_{t+j} + \omega\left(1-S_c\right) g^M_{t+j} - \frac{1}{\theta-1}\widehat{\theta}_{t+j} \Bigg) + \mathrm{E}_t \sum_{j=1}^{\infty} (\alpha\beta)^j \pi_{t+j}, \end{aligned} \tag{D55}$$

which can be written

$$\begin{aligned} p_{h,t} \;&=\; (1-\alpha\beta)\Big[-\omega\eta p^M_{H,t} + \omega S_c c^M_t + \frac{1}{\sigma} c_t - \widetilde{a}_t + \omega\left(1-S_c\right) g^M_t + \omega \widehat{\phi}^M_{H,t} - \frac{1}{\theta-1}\widehat{\theta}_t \Big] \\ &\quad + \alpha\beta \mathrm{E}_t \left(\pi_{t+1} + p_{h,t+1}\right). \end{aligned} \tag{D56}$$

Now using equation (D6) and $\pi_{t+1} = \pi_{H,t+1} - p_{H,t+1} + p_{H,t}$ we get

$$\begin{aligned} \pi_{H,t} &= \beta \mathrm{E}_t \pi_{H,t+1} + \kappa \left(\omega S_c + \sigma^{-1}\right) c_t^M - \kappa \left(1 + \omega\eta\right) p_{H,t}^M - \kappa\phi_F p_{H,t}^R + \kappa\phi_F q_t \\ &\quad -\kappa \left(\widetilde{a}_t - \omega \left(1 - S_c\right) g_t^M - \omega \widehat{\phi}_{H,t}^M + \frac{1}{\theta - 1}\widehat{\theta}_t\right). \end{aligned} \tag{D57}$$

Similar manipulations of (D29) gives

$$\begin{aligned} \pi_{H,t}^* &= \beta \mathrm{E}_t \pi_{H,t+1}^* + \kappa \left(\omega S_c + \sigma^{-1}\right) c_t^M - \kappa \left(1 + \omega\eta\right) p_{H,t}^M + \kappa\phi_H p_{H,t}^R - \kappa\phi_H q_t \\ &\quad -\kappa \left(\widetilde{a}_t - \omega \left(1 - S_c\right) g_t^M - \omega \widehat{\phi}_{H,t}^M + \frac{1}{\theta - 1}\widehat{\theta}_t\right). \end{aligned} \tag{D58}$$

The foreign relations are given as

$$\begin{aligned} \pi_{F,t}^* &= \beta \mathrm{E}_t \pi_{F,t+1}^* + \kappa \left(\omega S_c + \sigma^{-1}\right) c_t^{M*} - \kappa \left(1 + \omega\eta\right) p_{F,t}^{M*} - \kappa\phi_F p_{F,t}^{R*} - \kappa\phi_F q_t \\ &\quad -\kappa \left(\widetilde{a}_t^* - \omega \left(1 - S_c\right) g_t^{M*} - \omega \widehat{\phi}_{F,t}^{M*} + \frac{1}{\theta - 1}\widehat{\theta}_t^*\right), \end{aligned} \tag{D59}$$

$$\begin{aligned} \pi_{F,t} &= \beta \mathrm{E}_t \pi_{F,t+1} + \kappa \left(\omega S_c + \sigma^{-1}\right) c_t^{M*} - \kappa \left(1 + \omega\eta\right) p_{F,t}^{M*} + \kappa\phi_H p_{F,t}^{R*} + \kappa\phi_H q_t \\ &\quad -\kappa \left(\widetilde{a}_t^* - \omega \left(1 - S_c\right) g_t^{M*} - \omega \widehat{\phi}_{F,t}^{M*} + \frac{1}{\theta - 1}\widehat{\theta}_t^*\right). \end{aligned} \tag{D60}$$

Using equations (D10) and (D13) we get

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa \left(\omega S_c + \sigma^{-1}\right) \left(\phi_H c_t^M + \phi_F c_t^{M*}\right) + \kappa\omega\eta\phi_F \left(\phi_H - \phi_F\right) \left(p_{F,t} - p_{H,t}^*\right) \\ &\quad +2\kappa\phi_H\phi_F q_t - \kappa\Big[\widetilde{a}_t^M - \omega \left(1 - S_c\right) \left(\phi_H g_t^M + \phi_F g_t^{M*}\right) - \omega \left(\phi_H - \phi_F\right) \widehat{\phi}_{H,t}^M + \frac{1}{\theta - 1}\widehat{\theta}_t^M\Big], \end{aligned} \tag{D61}$$

$$\begin{aligned} \pi_t^* &= \beta \mathrm{E}_t \pi_{t+1}^* + \kappa \left(\omega S_c + \sigma^{-1}\right) \left(\phi_H c_t^{M*} + \phi_F c_t^M\right) - \kappa\omega\eta\phi_F \left(\phi_H - \phi_F\right) \left(p_{F,t} - p_{H,t}^*\right) \\ &\quad -2\kappa\phi_H\phi_F q_t - \kappa\Big[\widetilde{a}_t^{M*} - \omega \left(1 - S_c\right) \left(\phi_H g_t^{M*} + \phi_F g_t^M\right) - \omega \left(\phi_H - \phi_F\right) \widehat{\phi}_{F,t}^{M*} + \frac{1}{\theta - 1}\widehat{\theta}_t^{M*}\Big]. \end{aligned} \tag{D62}$$

Finally we need an equation determining the evolution of $p_{F,t} - p_{H,t}^*$. Using the expressions for $\pi_{H,t}^*$ and $\pi_{F,t}$ gives

$$\pi_t^* - \pi_t - \left(1 + \kappa + \beta\right) \left(p_{F,t} - p_{H,t}^*\right) + \left(p_{F,t-1} - p_{H,t-1}^*\right) \tag{D63}$$

$$\begin{aligned} &= 2\kappa\omega\eta\phi_F \left(p_{F,t} - p_{H,t}^*\right) + \kappa \left(\omega S_c + \sigma^{-1}\right) \left(\phi_H - \phi_F\right) c_t^R - 2\kappa\phi_H q_t \\ &\quad +\beta \mathrm{E}_t \left(\pi_{t+1}^* - \pi_{t+1}\right) - \beta \mathrm{E}_t \left(p_{F,t+1} - p_{H,t+1}^*\right) \\ &\quad -\kappa \left[\widetilde{a}_t^R - \omega \left(1 - S_c\right) \left(\phi_H - \phi_F\right) g_t^R - \omega \left(\widehat{\phi}_{H,t}^M - \widehat{\phi}_{F,t}^{M*}\right) + \frac{1}{\theta - 1}\widehat{\theta}_t^R\right]. \end{aligned} \tag{D64}$$

D.4 Output and monetary policy

The log-linear version of equation (B15) is given as

$$y_t = \left(\eta - 1\right) \phi_F \left(p_{F,t} - p_{H,t}^*\right) + S_c c_t^M + \phi_F q_t + \widehat{\phi}_{H,t}^M + \left(1 - S_c\right) g_t^M, \tag{D65}$$

where we have denoted by y_t the log-linear expression for real output in home. In deriving this equation we use that the log-linear versions of

$$\frac{\int_{N_H} \left(P_{H,t}(z)\right)^{1-\theta_t} dz}{\left(P_{H,t}\right)^{1-\theta_t}} \text{ and } \frac{\int_{N_H} \left(P^*_{H,t}(z)\right)^{1-\theta_t} dz}{\left(P^*_{H,t}\right)^{1-\theta_t}} \tag{D66}$$

equal zero. Foreign real output can be written

$$y_t^* = -(\eta - 1)\phi_F\left(p_{F,t} - p^*_{H,t}\right) + S_c c_t^{M*} - \phi_F q_t + \widehat{\phi}^{M*}_{F,t} + (1 - S_c) g_t^{M*}. \tag{D67}$$

To close the models we specify equations for the nominal interest rates. These are written as rules for monetary policy. We employ two specifications. Under the first specification monetary policy is set as a function of domestic consumption and inflation as in Steinsson (2008):

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\psi_c c_t + (1 - \rho_i)\psi_\pi \pi_t + \varepsilon_t, \tag{D68}$$

$$i_t^* = \rho_i i^*_{t-1} + (1 - \rho_i)\psi_c c_t^* + (1 - \rho_i)\psi_\pi \pi_t^* + \varepsilon_t^*, \tag{D69}$$

where ε_t and ε_t^* are home and foreign monetary policy shocks, respectively. Under the second specification we instead assume that monetary policy depends on output instead of consumption:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\psi_c y_t + (1 - \rho_i)\psi_\pi \pi_t + \varepsilon_t, \tag{D70}$$

$$i_t^* = \rho_i i^*_{t-1} + (1 - \rho_i)\psi_c y_t^* + (1 - \rho_i)\psi_\pi \pi_t^* + \varepsilon_t^*. \tag{D71}$$

E The log-linear models

In this section we summarize the log-linearized equations of our models and compare them with the equations used by Steinsson (2008).

In all models the household sector is characterized by the equations

$$c_t = \mathrm{E}_t c_{t+1} - \sigma\left(i_t - \mathrm{E}_t \pi_{t+1}\right), \tag{E1}$$

$$c_t^* = \mathrm{E}_t c^*_{t+1} - \sigma\left(i_t^* - \mathrm{E}_t \pi^*_{t+1}\right), \tag{E2}$$

$$\sigma q_t = c_t - c_t^*. \tag{E3}$$

E.1 Our heterogeneous labor markets model

With heterogeneous labor markets, the supply side of the economy is characterized by the equations

$$\begin{aligned} \pi_t \quad = \quad & \beta \mathrm{E}_t \pi_{t+1} + \kappa \frac{\omega S_c + \sigma^{-1}}{1 + \omega\theta}\left(\phi_H c_t^M + \phi_F c_t^{M*}\right) \\ & - \kappa \phi_F \frac{(\phi_H - \phi_F)\,\omega\,(\theta - \eta)}{1 + \omega\theta}\left(p_{F,t} - p^*_{H,t}\right) + 2\kappa\phi_H\phi_F q_t - \eta_t, \end{aligned} \tag{E4}$$

$$\begin{aligned}\pi_t^* &= \beta \mathrm{E}_t \pi_{t+1}^* + \kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta}\left(\phi_H c_t^{M*} + \phi_F c_t^M\right) \\ &\quad + \kappa \phi_F \frac{(\phi_H - \phi_F)\,\omega\,(\theta-\eta)}{1+\omega\theta}\left(p_{F,t} - p_{H,t}^*\right) - 2\kappa\phi_H\phi_F q_t - \eta_t^*,\end{aligned} \tag{E5}$$

$$\begin{aligned}&\pi_t^* - \pi_t - (1+\kappa+\beta)\left(p_{F,t} - p_{H,t}^*\right) + \left(p_{F,t-1} - p_{H,t-1}^*\right) \\ = \; & -\kappa\phi_F \frac{2\omega(\theta-\eta)}{1+\omega\theta}\left(p_{F,t} - p_{H,t}^*\right) + \kappa\frac{\omega S_c + \sigma^{-1}}{1+\omega\theta}(\phi_H - \phi_F)\,c_t^R - 2\kappa\phi_H q_t \\ & + \beta\mathrm{E}_t\left(\pi_{t+1}^* - \pi_{t+1}\right) - \beta\mathrm{E}_t\left(p_{F,t+1} - p_{H,t+1}^*\right) - \frac{1}{\phi_H - \phi_F}\left(\eta_t - \eta_t^*\right).\end{aligned} \tag{E6}$$

In general, the shocks are given by

$$\eta_t = \frac{\kappa}{1+\omega\theta}\left[\widetilde{a}_t^M - \omega(1-S_c)\left(\phi_H g_t^M + \phi_F g_t^{M*}\right) - \omega(\phi_H - \phi_F)\,\widehat{\phi}_{H,t}^M + \frac{1}{\theta-1}\widehat{\theta}_t^M\right], \tag{E7}$$

$$\eta_t^* = \frac{\kappa}{1+\omega\theta}\left[\widetilde{a}_t^{M*} - \omega(1-S_c)\left(\phi_H g_t^{M*} + \phi_F g_t^M\right) - \omega(\phi_H - \phi_F)\,\widehat{\phi}_{F,t}^{M*} + \frac{1}{\theta-1}\widehat{\theta}_t^{M*}\right], \tag{E8}$$

but we will assume that only productivity shocks vary over time. In that case we have

$$\eta_t = \frac{\kappa}{1+\omega\theta}\widetilde{a}_t^M, \tag{E9}$$

$$\eta_t^* = \frac{\kappa}{1+\omega\theta}\widetilde{a}_t^{M*}. \tag{E10}$$

We also define

$$c_t^M = \phi_H c_t + \phi_F c_t^*, \quad c_t^{M*} = \phi_H c_t^* + \phi_F c_t, \quad c_t^R = c_t - c_t^*, \tag{E11}$$

$$\widetilde{a}_t^M = \phi_H \widetilde{a}_t + \phi_F \widetilde{a}_t^*, \quad \widetilde{a}_t^{M*} = \phi_H \widetilde{a}_t^* + \phi_F \widetilde{a}_t, \tag{E12}$$

$$\widetilde{a}_t = (1+\omega)\,a_t, \quad \widetilde{a}_t^* = (1+\omega)\,a_t^*, \tag{E13}$$

Finally, the monetary policy rules are given by

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\,\psi_c c_t + (1-\rho_i)\,\psi_\pi \pi_t + \varepsilon_t, \tag{E14}$$

$$i_t^* = \rho_i i_{t-1}^* + (1-\rho_i)\,\psi_c c_t^* + (1-\rho_i)\,\psi_\pi \pi_t^* + \varepsilon_t^*. \tag{E15}$$

As mentioned above, we also use versions of the models where real output enters the interest rate rule. Under the assumption that productivity shocks are the only real shocks impinging on the economy we have that output in the two economies is given as

$$y_t = (\eta - 1)\,\phi_F\left(p_{F,t} - p_{H,t}^*\right) + S_c c_t^M + \phi_F q_t, \tag{E16}$$

$$y_t^* = -(\eta - 1)\,\phi_F\left(p_{F,t} - p_{H,t}^*\right) + S_c c_t^{M*} - \phi_F q_t. \tag{E17}$$

These specifications underlie the results in the main text.

E.2 Steinsson's heterogeneous labor markets model

Steinsson (2008) uses a calibration of the model where $\theta = \eta$. In this situation, he argues that the model consists of equations (E1)–(E3), the Phillips curves

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa \frac{\omega + \sigma^{-1}}{1 + \omega\theta} \left(\phi_H c_t^M + \phi_F c_t^{M*} \right) + 2\kappa \phi_H \phi_F q_t - \eta_t, \tag{E18}$$

$$\pi_t^* = \beta \mathrm{E}_t \pi_{t+1}^* + \kappa \frac{\omega + \sigma^{-1}}{1 + \omega\theta} \left(\phi_H c_t^{M*} + \phi_F c_t^M \right) - 2\kappa \phi_H \phi_F q_t - \eta_t^*, \tag{E19}$$

and monetary policy rules given by equations (E14)–(E15).

The only difference between our model and Steinsson's is in the coefficient on $\phi_H c_t^M + \phi_F c_t^{M*}$ and the corresponding foreign variable in the Phillips curves. Steinsson has $\kappa \frac{\omega + \sigma^{-1}}{1+\omega\theta}$ whereas we have $\kappa \frac{\omega S_c + \sigma^{-1}}{1+\omega\theta}$. The difference reflects that the way Steinsson log-linearizes the resource constraint is erroneous. More specifically he argues that $Y_t = C_t + G_t$ implies the log-linear relation $y_t = c_t + g_t$. The correct version is $y_t = S_c c_t + (1 - S_c) g_t$ where $S_c = C/(C+G)$ is the steady state ratio of consumption to output. The implication is that if $S_c = 1$ our and Steinsson's heterogeneous labor market models are identical. With $S_c < 1$ the two models will differ. As shown in the main text, however, these differences are quantitatively unimportant.

E.3 Our homogeneous labor markets model

The only difference between our homogeneous and heterogeneous labor market models is the specification of the Phillips curves and the equation determining the evolution of $p_{F,t} - p_{H,t}^*$. Hence, the model consists of equations (E1)–(E3) and (E14)–(E15) in addition to

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa \left(\omega S_c + \sigma^{-1} \right) \left(\phi_H c_t^M + \phi_F c_t^{M*} \right) \\ &\quad + \kappa \omega \eta \phi_F \left(\phi_H - \phi_F \right) \left(p_{F,t} - p_{H,t}^* \right) + 2\kappa \phi_H \phi_F q_t - \widetilde{\eta}_t, \end{aligned} \tag{E20}$$

$$\begin{aligned} \pi_t^* &= \beta \mathrm{E}_t \pi_{t+1}^* + \kappa \left(\omega S_c + \sigma^{-1} \right) \left(\phi_H c_t^{M*} + \phi_F c_t^M \right) \\ &\quad - \kappa \omega \eta \phi_F \left(\phi_H - \phi_F \right) \left(p_{F,t} - p_{H,t}^* \right) - 2\kappa \phi_H \phi_F q_t - \widetilde{\eta}_t^*, \end{aligned} \tag{E21}$$

$$\begin{aligned} & \pi_t^* - \pi_t - (1 + \kappa + \beta) \left(p_{F,t} - p_{H,t}^* \right) + \left(p_{F,t-1} - p_{H,t-1}^* \right) \\ = \; & 2\kappa \omega \eta \phi_F \left(p_{F,t} - p_{H,t}^* \right) + \kappa \left(\omega S_c + \sigma^{-1} \right) \left(\phi_H - \phi_F \right) c_t^R - 2\kappa \phi_H q_t \\ & + \beta \mathrm{E}_t \left(\pi_{t+1}^* - \pi_{t+1} \right) - \beta \mathrm{E}_t \left(p_{F,t+1} - p_{H,t+1}^* \right) - \frac{1}{\phi_H - \phi_F} \left(\widetilde{\eta}_t - \widetilde{\eta}_t^* \right), \end{aligned} \tag{E22}$$

where

$$\widetilde{\eta}_t = \kappa \widetilde{a}_t^M, \tag{E23}$$
$$\widetilde{\eta}_t^* = \kappa \widetilde{a}_t^{M*}, \tag{E24}$$

under the assumption that productivity shocks are the only real shocks impinging on the economy.

E.4 Steinsson's homogeneous labor markets model

Steinsson argues that the homogeneous labor market model is identical to the heterogeneous labor markets model except for the slope of the two Phillips curves. His model consists of equations (E1)–(E3) and (E14)–(E15) and the equations

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa \left(\omega + \sigma^{-1}\right) \left(\phi_H c_t^M + \phi_F c_t^{M*}\right) + 2\kappa\phi_H\phi_F q_t - \widetilde{\eta}_t, \tag{E25}$$

$$\pi_t^* = \beta \mathrm{E}_t \pi_{t+1}^* + \kappa \left(\omega + \sigma^{-1}\right) \left(\phi_H c_t^{M*} + \phi_F c_t^{M}\right) - 2\kappa\phi_H\phi_F q_t - \widetilde{\eta}_t^*. \tag{E26}$$

There are two differences between our model and his. First, we again have a different coefficient on $\phi_H c_t^M + \phi_F c_t^{M*}$ and $\phi_H c_t^{M*} + \phi_F c_t^M$ in the Phillips curves. (As in the model with heterogeneous labor markets, this difference is quantitatively unimportant.) Second, he does not include the variable $p_{H,t}^* - p_{F,t}$ in his equations. This variable cancels out under heterogeneous labor markets when $\theta = \eta$, but this is not the case under homogeneous labor markets. The intuition is provided in the main text. This second difference has a quantitatively important effect on the results.

F Programming errors

As mentioned in the main text, we identified three errors in the part of Steinsson's Matlab code that computes the AR(5) based statistics.[4] The first error is present in the Matlab script "model_quant_estimation.m" (line 158). The main part of this script consists of a loop that simulates data from the model, estimates the AR(5) on the simulated data using a median unbiased estimator and passes the estimated coefficients to a script ("ar_lives.m" to be described below) that computes the AR(5) based statistics using the impulse response to the estimated AR(5). The error is that the script does not pass all the coefficients of the estimated AR(5) to "ar_lives.m". The script estimates the model

$$q_t = \mu + \alpha_1 q_{t-1} + \sum_{j=1}^{4} \psi_j \left(q_{t-j} - q_{t-1-j}\right) + \varepsilon_t, \tag{F1}$$

giving the estimates $\widehat{\mu}, \widehat{\alpha}_1, \widehat{\psi}_1, ..., \widehat{\psi}_4$. The impulse response functions are generated using the model

$$q_t = \left(\widehat{\alpha}_1 + \widehat{\psi}_1\right) q_{t-1} - \widehat{\psi}_1 q_{t-2} + \varepsilon_t, \tag{F2}$$

whereas the correct model is

$$q_t = \left(\widehat{\alpha}_1 + \widehat{\psi}_1\right) q_{t-1} + \left(\widehat{\psi}_2 - \widehat{\psi}_1\right) q_{t-2} + \left(\widehat{\psi}_3 - \widehat{\psi}_2\right) q_{t-3} + \left(\widehat{\psi}_4 - \widehat{\psi}_3\right) q_{t-4} - \widehat{\psi}_5 q_{t-5} + \varepsilon_t. \tag{F3}$$

The effect of this error on the estimated up-lives, half-lives and quarter-lives is ambiguous.

The second and third errors are in the script "ar_lives.m," which computes the impulse reponse function from the AR(5) and derives the up-life, half-life, and quarter-life based on the

[4] The Matlab programs are available on http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.1.519.

impulse response function. The second error is that Steinsson takes the absolute value of the impulse response when computing the up-life, half-life and quarter-life (various places in lines 50–69). To see the effect of this error suppose an impulse response falls to 0.25 in period t and -0.25 in $t+j$ whereafter it increases to zero. The true quarter-life is t but Steinsson's code will indicate $t+j$ as the estimated quarter-life. The third error relates to the way the up-life is estimated and implies that all up-lives are one quarter too long (line 57).

Table F1 reports the behavior of the real exchange rate in various versions of alternative versions of the homogeneous and heterogeneous labor market models using alternative Matlab codes. The simulations all use Steinsson's calibration of the models. The first rows of each panel report results using Steinsson's models and original Matlab code. The results in these rows match the results in Table 2 in the main text labelled "Reproduction of Steinsson's results". The second rows use the corrected version of Steinsson's model, sets $S_c = 1$ so there is no government spending in the steady, and use his original Matlab code. The results in the third rows are based on the same model and Matlab code as in the second rows except that we set $S_c = 0.75$. Hence, the differences between the first, second, and third rows are the marginal effect of correcting the model errors we have identified. As discussed in the main text, these errors are most important for the behavior of the real exchange rate in the homogeneous labor market model. Using the correct model rather than Steinsson's reduces all four measures of persistence as well as the two measures of volatility. The effect of using the correct model in the heterogeneous labor market and of changing the value of S_c has very little effect on the various statistics.

The models underlying the results in rows three, four, five, and six are our corrected models where the differences between the rows are explained by differences in Matlab programs. Rows four report results where we have corrected the first programming error mentioned above, namely the fact that Steinsson does not use all the estimated parameters in the AR(5) when deriving the impulse response function. This error has ambiguous effects on the estimated persistence of the real exchange rate. In the homogeneous labor market model, removing the error reduces the half-life but increases the up-life divided by the half-life and the quarter-life minus the half-life. For the heterogeneous labor market model, removing the error increases the half-life and up-life divided by the half-life but decreases the quarter-life minus half-life. Of course nothing happens with the HP-filtered or growth rate based statistics, as the programming error only relates to the computation of the impulse response function.

The results reported in rows five are derived using Steinsson's Matlab code where we have corrected both the first and second error. The second error is that Steinsson uses the absolute value of the impulse response function when computing the up-, half- and quarter-life. Comparing rows four and five shows that the second error has very little effects on the estimated behavior of the real exchange rate. Removing the error has no consequence for the half-lives or the up-lives divided by the half-lives but tends to reduce the quarter-lives minus the half-lives somewhat. This latter effect is most pronounced in the heterogeneous labor market model. Removing the third programming error, that the up-lives are estimated to be one quarter too long, reduces the up-lives divided by the half-lives.

To summarize, table F1 shows that for the homogeneous labor market model, the main reason for the discrepancy between our results and Steinsson's is the model errors. For the heterogeneous labor market model, the first programming error is the most important reason between our results and Steinsson's.

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Table F1: Effects of model and programming errors on behavior of real exchange rate in benchmark models

	HL	UL/HL	QL−HL	$\rho_{1,hp}$	$\frac{\mathrm{Std}(q_{t,hp})}{\mathrm{Std}(c_{t,hp})}$	$\frac{\mathrm{Std}(q_{t,hp})}{\mathrm{Std}(y_{t,hp})}$	$\frac{\mathrm{Std}(\Delta q_t)}{\mathrm{Std}(\Delta c_t)}$	$\frac{\mathrm{Std}(\Delta q_t)}{\mathrm{Std}(\Delta y_t)}$
(a) Homogeneous labor market								
Steinsson's model and Matlab code	3.45	0.41	2.27	0.82	6.86	–	6.86	–
Our model, $S_c = 1$ and Steinsson's Matlab code	1.92	0.38	1.37	0.75	2.51	0.92	2.85	1.09
Our model and Steinsson's Matlab code	1.92	0.38	1.37	0.75	2.37	0.96	2.70	1.14
Our model, Steinsson's Matlab code with error 1 corrected	1.85	0.39	1.50	0.75	2.37	0.96	2.70	1.14
Our model, Steinsson's Matlab code with error 1 + 2 corrected	1.85	0.39	1.49	0.75	2.37	0.96	2.70	1.14
Our model, Steinsson's Matlab code with error 1 + 2 + 3 corrected	1.85	0.26	1.49	0.75	2.37	0.96	2.70	1.14
(b) Heterogeneous labor market								
Steinsson's model and Matlab code	4.39	0.40	2.80	0.84	4.23	–	4.23	–
Our model, $S_c = 1$ and Steinsson's Matlab code	4.39	0.40	2.80	0.84	4.23	0.97	4.23	1.22
Our model and Steinsson's Matlab code	4.40	0.40	2.81	0.84	4.17	1.00	4.17	1.27
Our model, Steinsson's Matlab code with error 1 corrected	4.49	0.58	2.09	0.84	4.17	1.00	4.17	1.27
Our model, Steinsson's Matlab code with error 1 + 2 corrected	4.49	0.58	1.96	0.84	4.17	1.00	4.17	1.27
Our model, Steinsson's Matlab code with error 1 + 2 + 3 corrected	4.49	0.52	1.96	0.84	4.17	1.00	4.17	1.27

This table reports estimates of real exchange rate persistence and volatility in various versions of the benchmark models and using alternative Matlab codes. HL denotes half-life (measured in years), UL/HL denotes up-life divided by half-life, QL−HL denotes quarter-life minus half-life. HL, UL, and QL are median unbiased estimates. $\rho_{1,hp}$ denotes the autocorrelation of the HP-filtered real exchange rate; $\frac{\mathrm{Std}(q_{t,hp})}{\mathrm{Std}(c_{t,hp})}$ denotes the standard deviation of the HP-filtered real exchange rate relative to the standard deviation of HP-filtered consumption; $\frac{\mathrm{Std}(q_{t,hp})}{\mathrm{Std}(y_{t,hp})}$ is the relative standard deviation of the HP-filtered real exchange rate with respect to HP-filtered output; $\frac{\mathrm{Std}(\Delta q_t)}{\mathrm{Std}(\Delta c_t)}$ denotes the standard deviation of the growth rate of the real exchange rate relative to the growth rate of consumption; $\frac{\mathrm{Std}(\Delta q_t)}{\mathrm{Std}(\Delta y_t)}$ denotes the standard deviation of the growth rate of the real exchange rate relative to the growth rate of output. All statistics are medians across 1,000 simulations. As discussed in section F, we identified three errors in Steinsson's Matlab code. We denote as error 1 the fact that Steinsson does not use all AR(5) coefficients when computing the up-, half-, and quarter-lives. We denote as error 2 the fact that Steinsson uses the absolute value of the impulse response function when computing these lives. Error 3 is that Steinsson's up-lives are one quarter too long. The first row of each panel reports our reproduction of Steinsson's results using his models and calibration and Matlab code. The second and third rows report results using our models, Steinsson's calibration and Matlab code where we set $S_c = 1$ (second row) and $S_c = 0.75$ (third row). The latter is our benchmark calibration. The fourth, fifth, and sixth rows report results using our models, Steinsson's calibration and Matlab code where we corrected error 1 (fourth row) errors 1 and 2 (fifth row) and all three errors (sixth row).