TRACING VALUE-ADDED AND DOUBLE COUNTING IN GROSS EXPORTS

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Online Appendices

Appendix A: Derivation details for major equations in the two country, one sector model

Based on the property of inverse matrix, we have:

$$\begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix} \tag{A1}$$

Therefore, the following identities hold:

$$(1-a_{11})b_{11}-1=a_{12}b_{21}, b_{11}(1-a_{11})-1=b_{12}a_{21}$$

$$a_{21}b_{11}=(1-a_{22})b_{21},\ b_{11}a_{12}=b_{12}(1-a_{22})$$

$$(1-a_{22})b_{21}=a_{21}b_{11}, b_{22}a_{21}=b_{21}(1-a_{11})$$

$$a_{21}b_{12}=(1-a_{22})b_{22}-1, b_{22}(1-a_{22})-1=b_{21}a_{12}$$

Therefore,

$$b_{11}=(1-a_{11})^{-1}+[b_{11}(1-a_{11})-1](1-a_{11})^{-1}=(1-a_{11})^{-1}+b_{12}a_{21}(1-a_{11})^{-1} \tag{A2}$$

Given (A2), we have

$$b_{11}y_{11}-(1-a_{11})^{-1}y_{11}=b_{12}a_{21}(1-a_{11})^{-1}y_{11} \tag{A3}$$

$$b_{22}=(1-a_{22})^{-1}+b_{21}a_{12}(1-a_{22})^{-1}$$

Derivation of equation (11)

Using the relationship between gross output x and final demand y specified in equation (5), we have

$$y_2=y_{21}+y_{22}=(1-a_{22})x_2-a_{21}x_1 \tag{A4}$$

Also using $b_{11}a_{12}=b_{12}(1-a_{22})$,

$$\begin{aligned} v_1b_{11}a_{12}x_2 &= v_1b_{12}(1-a_{22})x_2=v_1b_{12}(y_2+a_{21}x_1)=v_1b_{12}a_{21}x_1+v_1b_{12}(y_{21}+y_{22}) \\ &= v_1b_{12}y_{22}+v_1b_{12}y_{21}+v_1b_{12}a_{21}x_1 \end{aligned} \tag{A5}$$

Derivation of equations (19) and (20)

Based on equation (6):

$$x_1 = b_{11}y_{11} + b_{12}y_{21} + b_{11}y_{12} + b_{12}y_{22} \tag{A6}$$

From the gross exports identity, we have:

$$x_1 = (1-a_{11})^{-1}(e_{12} + y_{11}), \qquad (1-a_{11})^{-1}e_{12} = x_1 - (1-a_{11})^{-1}y_{11} \tag{A7}$$

Combining (A6) and (A7), we can easily show that

$$\begin{aligned} &v_1 b_{12} a_{21}(1-a_{11})^{-1}e_{12} = v_1 b_{12} a_{21}[x_1 - (1-a_{11})^{-1}y_{11}] = \\ &v_1 b_{12} a_{21}[b_{11}y_{11} + b_{12}y_{21} + b_{11}y_{12} + b_{12}y_{22} - (1-a_{11})^{-1}y_{11}] \\ &= v_1 b_{12} a_{21}[b_{11}y_{12} + b_{12}y_{22} + b_{12}y_{21} + b_{12}a_{21}(1-a_{11})^{-1}y_{11}] \end{aligned} \tag{A8}$$

which is the first pure double counted term in Country 1's gross exports accounting equation (13) that is expressed as function of both countries' final demand.

Also based on equation (6):

$$x_2 = b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}y_{22} \tag{A9}$$

Also from gross exports identity,

$$x_2 = (1-a_{22})^{-1}(e_{21} + y_{22}), \qquad (1-a_{22})^{-1}e_{21} = x_2 - (1-a_{22})^{-1}y_{22} \tag{A10}$$

Combining (A9) and (A10), we can show that the second pure double counted term in equation (13) can be expressed as:

$$\begin{aligned} &v_2 b_{21} a_{12}(1-a_{22})^{-1}e_{21} = v_2 b_{21} a_{12}[x_2 - (1-a_{22})^{-1}y_{11}] = \\ &v_2 b_{21} a_{12}[b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{22}y_{22} - (1-a_{22})^{-1}y_{22}] \\ &= v_2 b_{21} a_{12}[b_{21}y_{11} + b_{22}y_{21} + b_{21}y_{12} + b_{21}a_{12}(1-a_{22})^{-1}y_{22}] \end{aligned} \tag{A11}$$

An alternative way to decompose the two pure double counted terms: Derivation of equation (21):

Based on (A1), (A7) and (A9),

$$
\begin{aligned}
& v_1 b_{12} a_{21}(1-a_{11})^{-1} e_{12} + v_2 b_{21} a_{12}(1-a_{22})^{-1} e_{21} \\
& = (1-a_{11}-a_{21}) b_{12} a_{21}(1-a_{11})^{-1} e_{12} + (1-a_{12}-a_{22}) b_{21} a_{12}(1-a_{22})^{-1} e_{21} \\
& = [(1-a_{11}) b_{12} a_{21} - a_{21} b_{12} a_{21}](1-a_{11})^{-1} e_{12} + [(1-a_{12}) b_{21} a_{12} - a_{12} b_{21} a_{12}](1-a_{22})^{-1} e_{21} \\
& = [a_{12} b_{22} a_{21} - a_{21} b_{12} a_{21}](1-a_{11})^{-1} e_{12} + [a_{21} b_{11} a_{12} - a_{12} b_{21} a_{12}](1-a_{22})^{-1} e_{21} \\
& = [a_{12} b_{21}(1-a_{11}) - a_{21} b_{12} a_{21}](1-a_{11})^{-1} e_{12} + [a_{21} b_{12}(1-a_{22}) - a_{12} b_{21} a_{12}](1-a_{22})^{-1} e_{21} \\
& = a_{12} b_{21} e_{12} - a_{21} b_{12} a_{21}(1-a_{11})^{-1} e_{12} + a_{21} b_{12} e_{21} - a_{12} b_{21} a_{12}(1-a_{22})^{-1} e_{21} \\
& = a_{12} b_{21}[e_{12} - a_{12}(1-a_{22})^{-1} e_{21}] + a_{21} b_{12}[e_{21} - a_{21}(1-a_{11})^{-1} e_{12}] \\
& = a_{12} b_{21}\{y_{12} + a_{12}[x_2 - (1-a_{22})^{-1} e_{21}]\} + a_{21} b_{12}\{y_{21} + a_{21}[x_1 - (1-a_{11})^{-1} e_{12}]\} \\
& = a_{12}[b_{21} y_{12} + b_{21} a_{12}(1-a_{22})^{-1} y_{22}] + a_{21}[b_{12} y_{21} + b_{12} a_{21}(1-a)^{-1} y_{11}]
\end{aligned}
\tag{A12}
$$

Derivation of equation (18)

$$
\begin{aligned}
& e_{12} + e_{21} - GDP_1 - GDP_2 = e_{12} + e_{21} - v_1 x_1 - v_2 x_2 \\
& = v_1[b_{11} y_{12} + b_{12} y_{22}] + 2v_1[b_{12} y_{21} + b_{12} a_{21}(1-a_{11})^{-1} y_{11}] + v_2[b_{21} y_{11} + b_{22} y_{21}] \\
& + 2v_2[b_{21} y_{12} + b_{21} a_{12}(1-a_{22})^{-1} y_{22}] + 2v_1 b_{12} a_{21}(1-a_{11})^{-1} e_{12} + 2v_2 b_{21} a_{12}(1-a_{22})^{-1} e_{21} \\
& - v_1(b_{11} y_{12} + b_{12} y_{22} + b_{12} y_{21} + b_{11} y_{11}) - v_2(b_{21} y_{11} + b_{22} y_{21} + b_{21} y_{12} + b_{22} y_{22}) \\
& = 2v_1[b_{12} y_{21} + b_{12} a_{21}(1-a_{11})^{-1} y_{11}] + 2v_2[b_{21} y_{12} + b_{21} a_{12}(1-a_{22})^{-1} y_{22}] \\
& + 2v_1 b_{12} a_{21}(1-a_{11})^{-1} e_{12} + 2v_2 b_{21} a_{12}(1-a_{22})^{-1} e_{21} - v_1(b_{12} y_{21} + b_{11} y_{11}) - v_2(+b_{21} y_{12} + b_{22} y_{22}) \\
& = v_1[b_{12} y_{21} + b_{12} a_{21}(1-a_{11})^{-1} y_{11}] + v_2[b_{21} y_{12} + b_{21} a_{12}(1-a_{22})^{-1} y_{22}] \\
& + 2v_1 b_{12} a_{21}(1-a_{11})^{-1} e_{12} + 2v_2 b_{21} a_{12}(1-a_{22})^{-1} e_{21} - (1-a_{11})^{-1} y_{11} - (1-a_{22})^{-1} y_{22}
\end{aligned}
\tag{A13}
$$

Appendix B

The derivation of gross exports accounting equation in G country N sector Model

B.1. The G-country, N-sector ICIO Model

Assume a world with G-countries, in which each country produces goods in N differentiated tradable sectors. Goods in each sector can be consumed directly or used as intermediate inputs, and each country exports both intermediate and final goods to all other countries.

All gross output produced by country s must be used as an intermediate good or a final good at home or abroad, or

$$X_s = \sum_r^G (A_{sr} X_r + Y_{sr}), \qquad \text{r,s} = 1,2\dots \text{G} \tag{B1}$$

Where X_s is the N×1 gross output vector of country s, Y_{sr} is the N×1 final demand vector that gives demand in country r for final goods produced in s, and A_{sr} is the N×N IO coefficient matrix, giving intermediate use in r of goods produced in s.

The G-country, N-sector production and trade system can be written as an ICIO model in block matrix notation

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1G} \\ A_{21} & A_{22} & \cdots & A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ A_{G1} & A_{G2} & \cdots & A_{GG} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} + \begin{bmatrix} Y_{11} + Y_{12} + \cdots + Y_{1G} \\ Y_{21} + Y_{22} + \cdots + Y_{2G} \\ \cdots\cdots \\ Y_{G1} + Y_{G2} + \cdots + Y_{GG} \end{bmatrix}, \tag{B2}$$

and rearranging,

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_G \end{bmatrix} = \begin{bmatrix} I - A_{11} & -A_{12} & \cdots & -A_{1G} \\ -A_{21} & I - A_{22} & \cdots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \cdots & I - A_{GG} \end{bmatrix}^{-1} \begin{bmatrix} \sum_r^G Y_{1r} \\ \sum_r^G Y_{2r} \\ \vdots \\ \sum_r^G Y_{Gr} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_G \end{bmatrix} \tag{B3}$$

where B_{sr} denotes the N×N block Leontief inverse matrix, which is the total requirement matrix that gives the amount of gross output in producing country s required for a one-unit increase in final demand in destination country r. Y_s is a N×1 vector that gives the global use of s' final goods.

B.2. Value-added share by source matrix

Let V_s be the $1 \times$N direct value-added coefficient vector. Each element of V_s gives the ratio of direct domestic value added in total output for country s. This is equal to one minus the intermediate input share from all countries (including domestically produced intermediates):

$$V_s = u(I - \sum_r^G A_{rs}), \tag{B4}$$

Define V, the G$\times$GN matrix of direct domestic value added for all countries,

$$V = \begin{bmatrix} V_1 & 0 & \cdots & 0 \\ 0 & V_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_G \end{bmatrix}. \tag{B5}$$

Multiplying these direct value-added shares with the Leontief inverse matrices produces the G$\times$GN value-added share (VB) matrix as equation (27) in the main text, it has the property:

$$\sum_s^G V_s B_{sr} = u. \tag{B6}$$

B.3. Decomposition of gross exports

Let E_{sr} be the N$\times$1 vector of gross bilateral exports from s to r.

$$E_{sr} \equiv A_{sr} X_r + Y_{sr} \qquad \textit{for} \qquad s \neq r \tag{B7}$$

A country's gross exports to the world equal

$$E_{s*} = \sum_{r \neq s}^G E_{sr} = \sum_{r \neq s}^G (A_{sr} X_r + Y_{sr}) \tag{B8}$$

From equation (29) in the main text we know that

$$\sum_{r=1}^G \sum_{g=1}^G B_{sg} Y_{gr} = \sum_{r=1}^G X_{sr} = X_s \tag{B9}$$

Therefore, following identity holds

$$V_s X_s \equiv V_s \sum_{r=1}^G \sum_{g=1}^G B_{sg} Y_{gr} \tag{B10}$$

Multiplying both sides of (B8) by (B6), we have

$$uE_{s*}(V_s B_{ss} + \sum_{t \neq s}^G V_t B_{ts}) E_{s*} = V_s B_{ss} \sum_{r \neq s}^G (A_{sr} X_r + Y_{sr}) + \sum_{t \neq s}^G V_t B_{ts} \sum_{r \neq s}^G (A_{sr} X_r + Y_{sr}) \tag{B11}$$

Now we add and subtract VT_{s*}, defined by equation (32) in the main text, to the first term on RHS of (B11). This gives

$$V_s B_{ss} E_{s*} = VT_{s*} + V_s B_{ss} \sum_{r \neq s}^{G} (A_{sr} X_r + Y_{sr}) - V_s \sum_{r \neq s}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} \tag{B12}$$

Recall that $X_s = \sum_{r=1}^{G} (A_{sr} X_r + Y_{sr})$ as defined in (B1), inserting it together with equation (B9) into (B12) gives

$$V_s B_{ss} E_{s*} = VT_{s*} + V_s B_{ss} (X_s - A_{ss} X_s - Y_{ss}) - V_s (X_s - \sum_{g=1}^{G} B_{sg} Y_{gs}) \tag{B13}$$

Where $X_s - A_{ss} X_s - Y_{ss}$ equals the difference between country s' gross output and gross output sold in domestic market, i.e. what country s' gross exports to the world market; $X_s - \sum_{g=1}^{G} B_{sg} Y_{gs}$ equals the difference between country s' gross output and its gross output finally consumed at domestic market . By rearranging terms,

$$V_s B_{ss} E_{s*} = VT_{s*} + V_s [B_{ss} (I - A_{ss}) - I] X_s + V_s [\sum_{g=1}^{G} B_{sg} Y_{gs} - B_{ss} Y_{ss}] \tag{B14}$$

Substitute $B_{ss}(I - A_{ss}) - I$ in equation (B14) by $\sum_{r \neq s}^{G} B_{sr} A_{rs}$ (the property of inverse matrix, see equation (B19) bellow) we have

$$V_s B_{ss} E_{s*} = V_s \sum_{r \neq s}^{G} \sum_{g=1}^{G} B_{sg} Y_{gr} + V_s \sum_{r \neq s}^{G} B_{sr} Y_{rs} + V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s \tag{B15}$$

Insert (B15) into (B11) and rearrange terms, we obtain equation (34) in the main text.

B.4. Further partition of equation (34)

The term that measures double counting by intermediate goods trade in equation (34) ($V_s \sum_{r \neq s}^{G} B_{sr} A_{rs} X_s$) can be further split into two parts: one is part of the home country's domestic value-added that is first exported but finally returns home in its intermediate imports to produce final goods and consumed at home, the other is a pure double counting portion due to two way intermediate trade.

Using the relation $X_s = Y_{ss} + A_{ss}X_s + E_{s*}$, it is easy to show that

$$X_s - (I - A_{ss})^{-1}Y_{ss} = (I - A_{ss})^{-1}E_{s*}. \tag{B16}$$

$(I - A_{ss})^{-1}Y_{ss}$ is the gross output needed to sustain final goods that is both produced and consumed in country s, using domestically produced intermediate goods; deduct it from country s' total gross output, what left is the gross output needed to sustain country s' production of its gross exports. Therefore, the left hand side of equation (B16) has straightforward economic meanings. We can further show that

$$(I - A_{ss})^{-1}Y_{ss} = B_{ss}Y_{ss} - \sum_{r \neq s}^{G} B_{sr}A_{rs}(I - A_{ss})^{-1}Y_{ss} \tag{B17}$$

the last term in RHS of (B17) is the final gross output needed to sustain final goods that is both produced and consumed in country s, but using intermediate goods that was originated in country s but shipped to other countries for processing before being re-imported by the source country in its intermediate goods imports (gross output sold indirectly in domestic market). Given (B17), it easy to see

$$V_s \sum_{r \neq s}^{G} B_{sr}A_{rs}X_s = V_s \sum_{r \neq s}^{G} B_{sr}A_{rs}(I - A_{ss})^{-1}Y_{ss} + V_s \sum_{r \neq s}^{G} B_{sr}A_{rs}(I - A_{ss})^{-1}E_{s*} \tag{B18}$$

Equation (B17) can be proven by using the property of inverse matrix:

$$\begin{bmatrix} B_{11} & B_{12} & \dots & B_{1G} \\ B_{21} & B_{22} & \dots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \dots & B_{GG} \end{bmatrix} \begin{bmatrix} I - A_{11} & -A_{12} & \dots & -A_{1G} \\ -A_{21} & I - A_{22} & \dots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \dots & I - A_{GG} \end{bmatrix} \begin{bmatrix} I & 0 & \dots & 0 \\ 0 & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & I \end{bmatrix}$$

we therefore have

$$B_{ss}(I - A_{ss}) - I = \sum_{r \neq s}^{G} B_{sr}A_{rs} \tag{B19}$$

Using (B19), we have

$$(I - A_{ss})^{-1}Y_{ss} + [B_{ss}(I - A_{ss}) - I](I - A_{ss})^{-1}Y_{ss} = B_{ss}Y_{ss} \tag{B20}$$

This is also the proof of equation (40) in the main text.

Appendix C Computation details for numerical examples 1 and 2

ICIO table underline numerical example 1

<table>
<tr><td colspan="2" rowspan="2">Input \ Output</td><td colspan="2">Intermediate Use</td><td colspan="2">Final Use</td></tr>
<tr><td>USA</td><td>CHN</td><td>USA</td><td>CHN</td></tr>
<tr><td rowspan="2">Intermediate Input</td><td>USA</td><td>100</td><td>50</td><td>30</td><td>20</td></tr>
<tr><td>CHN</td><td>0</td><td>50</td><td>70</td><td>80</td></tr>
<tr><td colspan="2">Value Added</td><td>100</td><td>100</td><td></td><td></td></tr>
<tr><td colspan="2">Total Input</td><td>200</td><td>200</td><td></td><td></td></tr>
</table>

Computation details:

The Gross exports decomposition matrix

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} = \begin{bmatrix} 60+46.69 & 40+53.3 \\ 0+93.33 & 0+106.67 \end{bmatrix} = \begin{bmatrix} 106.7 & 93.3 \\ 93.3 & 106.7 \end{bmatrix}$$

The value-added production and trade matrix

$$VBY = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 106.7 & 93.3 \\ 93.3 & 106.7 \end{bmatrix} = \begin{bmatrix} 53.3 & 46.7 \\ 46.7 & 53.3 \end{bmatrix}$$

Decomposition results based on equations (13) and (14)

	Terms in equation (13) &(14)	USA	CHN
Value-added exports	v1	20	46.7
	v2	26.7	0
Return home value	v3	23.3	0
	v4	0	0
	v5	0	0
Foreign Value	v6	0	23.3
	v7	0	0

	v8	0	0
Gross Exports	E	70	70
VAX ratio		0.67	0.67

Since there is no foreign value-added in USA's production, the 30 unit of domestic final demand are 100% its own value-added, just as its exports, so its GDP is equal to 100. For CHN, the value-added in its exports and domestic final consumption also sum to 100. Both countries have identical VAX ratios, but the reasons why value added exports smaller than the gross exports are different; For USA, due to some of its own value added that is initially exported returns home after being used as an intermediate input in CHN to produce final goods exports back to USA ; For CHN, due to its production for exports uses FV: intermediate goods from the USA which embeds USA's value added; The return home VA (23.3) is a true value added for USA 's national account, part of its GDP, but it is a double counting in official trade statistics and from China's point of view.

ICIO table underline numerical example 2 and computation details

The Input-Output (ICIO) Table for Case 1

Note: The unit is $100K unless otherwise noted.

Input \ Output	Intermediate use						Final use						Total Output
	C1	C2	C3	C4	C5	USA	C1	C2	C3	C4	C5	USA	
C1	0	**1**	0	0	0	0	0	0	0	0	0	0	**1**
C2	0	0	**2**	0	0	0	0	0	0	0	0	0	**2**
C3	0	0	0	**3**	0	0	0	0	0	0	0	0	**3**
C4	0	0	0	0	**4**	0	0	0	0	0	0	0	**4**
C5	0	0	0	0	0	**5**	0	0	0	0	0	0	**5**
USA	0	0	0	0	0	0	0	0	0	0	0	**15**	**15**
Value added	**1**	**1**	**1**	**1**	**1**	**10**							
Total input	**1**	**2**	**3**	**4**	**5**	**15**							

Computation details for case 1

Direct input coefficient matrix: $A=\begin{bmatrix} 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4/5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Direct value-added share: $A^v=\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 & 2/3 \end{bmatrix}$

Final demand matrix: $Y=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix}$

Leontief Inverse: $B=\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 & 1/15 \\ 0 & 1 & 2/3 & 2/4 & 2/5 & 2/15 \\ 0 & 0 & 1 & 3/4 & 3/5 & 3/15 \\ 0 & 0 & 0 & 1 & 4/5 & 4/15 \\ 0 & 0 & 0 & 0 & 1 & 5/15 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Gross exports: $E=\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 0 \end{bmatrix}$

As an illustration, we list how our approach computes each of the terms (v1 through v9) for Country 5 (the country that exports to USA, not USA itself):

Term 1 (v1 in Country 5's gross exports):

$$V_5 \sum_{r\neq 5}^{G} B_{55} Y_{5r} = 1/5 * 1 * (0+0+0+0+0) = 0$$

Term 2:

$$V_5 \sum_{r\neq 5}^{G} B_{5r} Y_{rr} = 1/5 * \begin{bmatrix} 0 & 0 & 0 & 0 & 5/15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 15 \end{bmatrix} = 1$$

Term 3:

$$V_5 \sum_{r\neq 5}^{G} \sum_{t\neq 5,r}^{G} B_{5r} Y_{rt} = V_5 \sum_{r\neq 5}^{G} B_{5r} \sum_{t\neq 5,r}^{G} Y_{rt} = 1/5 * \begin{bmatrix} 0 & 0 & 0 & 0 & 5/15 \end{bmatrix} \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \end{bmatrix} = 0$$

Term 4:

$$V_5 \sum_{r\neq 5}^{G} B_{5r} Y_{r5} = 1/5 * \begin{bmatrix} 0 & 0 & 0 & 0 & 5/15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

Term 5:

$$V_5 \sum_{r \neq 5}^{G} B_{5r} A_{r5} (I - A_{55})^{-1} Y_{55} = 1/5 * \begin{bmatrix} 0 & 0 & 0 & 0 & 5/15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4/5 \\ 0 \end{bmatrix} * (1-0)^{-1} * 0 = 0$$

Term 6:

$$V_5 \sum_{r \neq 5}^{G} B_{5r} A_{r5} (I - A_{55})^{-1} E_{5*} = 1/5 * \begin{bmatrix} 0 & 0 & 0 & 0 & 5/15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4/5 \\ 0 \end{bmatrix} * (1-0)^{-1} * 5 = 0$$

Term 7:

$$\sum_{t \neq 5}^{G} \sum_{r \neq 5}^{G} V_t B_{t5} Y_{5r} = \sum_{t \neq 5}^{G} V_t B_{t5} \sum_{r \neq 5}^{G} Y_{5r} = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/15 \end{bmatrix} \begin{bmatrix} 1/5 \\ 2/5 \\ 3/5 \\ 4/5 \\ 0 \end{bmatrix} * (0+0+0+0+0) = 0$$

Term 8:

$$\sum_{t \neq 5}^{G} \sum_{r \neq 5}^{G} V_t B_{t5} A_{5r} (I - A_{rr})^{-1} Y_{rr} = \sum_{t \neq 5}^{G} V_t B_{t5} \sum_{r \neq 5}^{G} A_{5r} (I - A_{rr})^{-1} Y_{rr}$$

$$= \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/15 \end{bmatrix} \begin{bmatrix} 1/5 \\ 2/5 \\ 3/5 \\ 4/5 \\ 0 \end{bmatrix} * (0*1*0 + 0*1*0 + 0*1*0 + 0*1*0 + 1/3*1*15)$$

$$= 4/5 * 5 = 4$$

Term 9:

$$\sum_{t \neq 5}^{G} \sum_{r \neq 5}^{G} V_t B_{t5} A_{5r} (I - A_{rr})^{-1} E_{r*} = \sum_{t \neq 5}^{G} V_t B_{t5} \sum_{r \neq 5}^{G} A_{5r} (I - A_{rr})^{-1} E_{r*}$$

$$= \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/15 \end{bmatrix} \begin{bmatrix} 1/5 \\ 2/5 \\ 3/5 \\ 4/5 \\ 0 \end{bmatrix} * (0*1*1 + 0*1*2 + 0*1*3 + 0*1*4 + 1/3*1*0)$$

$$= 4/5 * 0 = 0$$

Similarly, we can decompose other countries' gross exports, and obtain the results reported in the upper panel of table 1.

The ICIO Table for Case 2:

Input \ Output	Intermediate use						Final use						Total Output
	USA	C1	C2	C3	C4	C5	USA	C1	C2	C3	C4	C5	
USA	0	**10**	0	0	0	0	0	0	0	0	0	0	**10**
C1	0	0	**11**	0	0	0	0	0	0	0	0	0	**11**
C2	0	0	0	**12**	0	0	0	0	0	0	0	0	**12**
C3	0	0	0	0	**13**	0	0	0	0	0	0	0	**13**
C4	0	0	0	0	0	**14**	0	0	0	0	0	0	**14**
C5	0	0	0	0	0	0	15	0	0	0	0	**0**	**15**
Value added	**10**	**1**	**1**	**1**	**1**	**1**							
Total input	**10**	**11**	**12**	**13**	**14**	**15**							

Computation details for case 2

Direct input coefficient matrix: $A=\begin{bmatrix} 0 & 10/11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 11/12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12/13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13/14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14/15 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Direct value-added share: $A^v=\begin{bmatrix}1 & 1/11 & 1/12 & 1/13 & 1/14 & 1/15\end{bmatrix}$

Final demand matrix: $Y=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 15 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\text{Leontief Inverse: } B=\begin{bmatrix} 1 & 10/11 & 10/12 & 10/13 & 10/14 & 10/15 \\ 0 & 1 & 11/12 & 11/13 & 11/14 & 11/15 \\ 0 & 0 & 1 & 12/13 & 12/14 & 12/15 \\ 0 & 0 & 0 & 1 & 13/14 & 13/15 \\ 0 & 0 & 0 & 0 & 1 & 14/15 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Gross exports: } E=\begin{bmatrix} 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{bmatrix}$$

Again, we list how our approach computes each of the terms (v1 through v9) for Country 5 (the country that exports final goods to USA, the 6th in the trading sequence):

Term 1 (i.e., v1 in Country 5's gross exports):

$$V_6 \sum_{r \neq 6}^{G} B_{66} Y_{6r} = 1/15 * 1 * (15 + 0 + 0 + 0 + 0) = 1$$

Term 2:

$$V_6 \sum_{r \neq 6}^{G} B_{6r} Y_{rr} = 1/15 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

Term 3:

$$V_6 \sum_{r \neq 6}^{G} \sum_{t \neq 6,r}^{G} B_{6r} Y_{rt} = V_6 \sum_{r \neq 6}^{G} B_{6r} \sum_{t \neq 6,r}^{G} Y_{rt} = 1/15 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 15+0+0+0 \end{bmatrix} = 0$$

Term 4:

$$V_6 \sum_{r \neq 6}^{G} B_{6r} Y_{r6} = 1/15 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

Term 5:

$$V_6 \sum_{r\neq 6}^{G} B_{6r} A_{r6} (I - A_{66})^{-1} Y_{66} = 1/15 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 14/15 \\ 0 \end{bmatrix} * (1-0)^{-1} * 0 = 0$$

Term 6:

$$V_6 \sum_{r\neq 6}^{G} B_{6r} A_{r6} (I - A_{66})^{-1} E_{6*} = 1/15 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 14/15 \\ 0 \end{bmatrix} * (1-0)^{-1} * 15 = 0$$

Term 7:

$$\sum_{t\neq 6}^{G} \sum_{r\neq 6}^{G} V_t B_{t6} Y_{6r} = \sum_{t\neq 6}^{G} V_t B_{t6} \sum_{r\neq 6}^{G} Y_{6r} = \begin{bmatrix} 1 & 1/11 & 1/12 & 1/13 & 1/14 \end{bmatrix} \begin{bmatrix} 10/15 \\ 11/15 \\ 12/15 \\ 13/15 \\ 14/15 \end{bmatrix} * (15 + 0 + 0 + 0 + 0) = 14$$

Term 8:

$$\sum_{t\neq 6}^{G} \sum_{r\neq 6}^{G} V_t B_{t6} A_{6r} (I - A_{rr})^{-1} Y_{rr} = \sum_{t\neq 6}^{G} V_t B_{t6} \sum_{r\neq 6}^{G} A_{6r} (I - A_{rr})^{-1} Y_{rr}$$

$$= \begin{bmatrix} 1 & 1/11 & 1/12 & 1/13 & 1/14 \end{bmatrix} \begin{bmatrix} 10/15 \\ 11/15 \\ 12/15 \\ 13/15 \\ 14/15 \end{bmatrix} * (0*1*0 + 0*1*0 + 0*1*0 + 0*1*0 + 0*1*0) = 0$$

Term 9:

$$\sum_{t\neq 6}^{G} \sum_{r\neq 6}^{G} V_t B_{t6} A_{6r} (I - A_{rr})^{-1} E_{r*} = \sum_{t\neq 6}^{G} V_t B_{t6} \sum_{r\neq 6}^{G} A_{6r} (I - A_{rr})^{-1} E_{r*}$$

$$= \begin{bmatrix} 1 & 1/11 & 1/12 & 1/13 & 1/14 \end{bmatrix} \begin{bmatrix} 10/15 \\ 11/15 \\ 12/15 \\ 13/15 \\ 14/15 \end{bmatrix} * (0*1*10 + 0*1*11 + 0*1*12 + 0*1*13 + 0*1*14) = 0$$

Similarly, we can decompose other countries' gross exports, and obtain the results reported at the lower panel of table 1

Appendix D: Detailed numerical example of a two country supply chain[1]

We now consider an example in which both countries export (and import) intermediate goods in an inter-country supply chain. This example will show our accounting equation can decompose a country's gross exports into various value-added and double counted components in a way that is consistent with one’s intuition. We will also illustrate why and how our estimate of VS1* in such a case differs from Daudin et al, why and how our estimate of the share of domestic value-added (GDP) in exports differs from Johnson and Noguera's value-added to gross exports ratio, and why and how our estimate of foreign value-added (GDP) in exports differs from HIY's VS measure but our foreign content in exports generalizes it.

Suppose the world production and trade take place in five stages (in a year) as summarized by Table D1. In Stage 1, perhaps a design stage, Country 1 uses labor to produce a unit of Stage-1 output. This is exported to Country 2 as an input to Stage-2 production. In Stage 2, Country 2 adds a unit of labor to produce 2 units of Stage-2 output which are shipped back to country 1 as an input to Stage-3 production. Country 1 adds another unit of labor to produce 3 units of Stage-3 output which are then exported to country 2 as an input to Stage-4 production. In Stage 4, country 2 adds a unit of labor to produce 4 units of Stage-4 output which are shipped back to country 1 as an input to Stage-5 production. The Stage-5 output is the final good. 3 units of the final good are exported to country 2, and 2 units are absorbed domestically in country 1.

Suppose each unit of intermediate and final goods is worth \$1. The total output in country 1 is \$9, in country 2 is \$6, the total value added (labor inputs) in the two countries is \$3 and \$2 respectively. The total exports from 1 to 2 and from 2 to 1 are \$7 and \$6, respectively; and the exports of final goods from 1 to 2 and 2 to 1 are \$3 and \$0 respectively.

For this simple example of an international supply chain, we can decompose both countries’ gross exports into value-added and double counted components by intuition without using any equations. The intuitive decomposition is summarized in Table D2. We will then verify that our exports decomposition formula produces exactly the same results.

[1] We are grateful to Peter Dixon and Maureen Rimmer for helping us to develop this instructive example.

Table D1 A two country supply-chain

	Country 1				Country 2			
	Final Demand	Labor input	Imported input	Output	Imported input	Labor input	Output	Final Demand
Stage 1 in Stage 1 out		$1		 $1				
Stage 2 in Stage 2 out					$1	$1	 $2	
Stage 3 in Stage 3 out		$1	$2	 $3				
Stage 4 in Stage 4 out					$3	$1	 $4	
Stage 5 in Stage 5 out		$1	$4	 $5				
	$2							$3
Total	$2	$3	$6	$9	$4	$2	$6	$3

We proceed as follows: Starting from the last stage (Stage 5), each country contributes $2 of value-added with their (previously produced) intermediate inputs, and Country 1 contributes an additional $1 of labor input to produce a total of 5 units of the final good. We assume labor is homogenous across countries. Since 2 units of the final good stay in Country 1 and 3 units are consumed in Country 2, all the value-added embodied in intermediate inputs that are eventually absorbed by each country should be split as 40% for country 1 and 60% for country 2, in proportion to the units of the final good consumed by the two countries. Therefore, the total value added exports from Country 1 to 2 are 0.6*$3=$1.8 (which is recorded in the cell in row "total" and column 2a of Table D2). Similarly, Country 2's exports of value added to 1 are 0.4*$2=$0.8 (which is recorded in the cell in row "total" and column 2b). Out of Country 1's $7 of gross exports, the total amount of double counting, or the difference between its gross exports and its value-added exports is $5.2 (=$7-$1.8). This is recorded in the cell in row "total" and column 7a. Similarly, out of Country 2's $6 of gross exports, the total amount of double counting is $6-$0.8=$5.2, which is recorded in the cell in row "total" and column (7b).

The beauty of this simple example is that we can work out the structure of the double counted values by intuition. Given what happens in Stage 5, we can split a country's value added in production in each of the earlier stages into the sum of value-added exports in that stage (that is ultimately absorbed abroad) and the value added that is exported in that stage but returns home next stage as part of its imports from the foreign country. Then the amount of exports in each of the first 4 stages that are double counted can be computed as each stage's gross output minus value added exports in that stage. In Stage 1, Country 1's domestic value added is \$1 (recorded in the cell (S1, 1a)). Since we know by Stage 5, 40% of the final good stays in Country 1, and 60% is exported to Country 2, we can split the \$1 of domestic value added into \$0.6 of Country 1's exports of value added (recorded in the cell (S1, 2a)) and \$0.4 of the domestic value added that returns home in the next stage and eventually consumed at home in Stage 5 (recorded in (S1, 3a)). Out of Country 1's gross exports of \$1 in Stage 1, the total double counted amount is the difference between its gross exports and value added exports, or \$1-\$0.6=\$0.4, as recorded in (S1, 7a). In Stage 2, Country 2 uses \$1 of intermediate good from Country 1 as an input together with its additional \$1 of labor to produce \$2 exports. Its domestic value added is \$1 (recorded in (S2, 1b)). Again, since we know the split of the final good consumption in the two countries in Stage 5, we can split Country 2's domestic value added into \$0.4 of its exports of value added (recorded in (S2, 2b)) and \$0.6 of domestic value added that will return home in Stage 3 and eventually consumed at home in Stage 5(recorded in S2, 4b)). Recall that out of \$1 of intermediate good that Country 2 imports from Country 1, \$0.4 will go back to Country 1 and be consumed there eventually. This is recorded in (S2, 5b), which is numerically identical to (S1, 3a). The remaining \$0.6 is double counted intermediate goods, and is recorded in (S2, 6b). This can also be verified in the following way. Since we know Country 2's gross exports in Stage 2 is \$2 but its value added exports are only \$0.4, the total amount of double counting in this stage's gross exports must be the difference between the two, or \$1.6 as recorded in (S2, 7b).Therefore, the "pure double counted" portion of foreign intermediate good has to equal \$1.6 (S2, 7b) -\$0.6 (S2, 3b) - \$0.4 (S2, 5b), which equals to \$0.6, as recorded in (S2, 6b). This amount represents the part of Country 1's Stage 1 intermediate good exports that cross borders more than twice before it can be embed in the final goods for consumption.

In Stage 3, Country 1 uses \$2 of imported intermediate goods from Country 2 as an input with its additional \$1 of labor to produce \$3 exports. Country 1's domestic value added is \$1 (S3, 1a). Again, because 60% of the final good will be eventually absorbed in the foreign country, the \$1 of domestic value added can be split into \$0.6 of Country 1's exports of value added (S3, 2a) and \$0.4 of the domestic value added that is exported in Stage 3 but will return in Stage 4 and eventually consumed there in Stage 5(S3, 3a). Furthermore, the Stage 3 production does use imported intermediate good from the previous stage. The amount of foreign value added embedded in its intermediate good imported from Country 2 that is not pure double counting should be the same as Country 2's domestic value added that is sent to Country 1 in Stage 2 but returns home and will be eventually absorbed there. We know that amount is $0.6 (S2, 3b). Therefore, the amount of foreign value added that is used in Country 1's Stage 3 production for exports and that will be eventually absorbed in Country 2 should be the same as $0.6 in (S3, 5a).

Because the value of Country 1's stage 1 exports ($1) is already counted three times by the time Stage 3 exports take place, we record that amount as a pure double counting item in (S3, 4a). Since we know out of $3 of Country 1's gross exports in Stage 3, only $0.6 is exports of value added that will eventually be absorbed abroad, $3-$0.6=$2.4 represents the total amount of double counting in this stage's gross exports, and is recorded in (S3, 7a). Out of the $1 foreign value added from Stage 2, since the amount that will go back to the foreign country and is absorbed there is 0.6 (S3, 5a), the amount of pure double counting must be $1-$0.6=$0.4, as recorded in (S3, 6a).

One way to check the sensibility of our reasoning is to compare the total amount of double counting in Stage-3 gross exports with the sum of the double counted components. Out of Country 1's $3 of gross exports in Stage 3, we know the total amount of double counting is $2.4 (recorded in (S3, 7a)). We can check that the sum of the double counted components in Country 1's exports in this stage (the sum of (S3, 3a), (S3, 4a), (S3, 5a), and (S3, 6a)) is also $2.4.

We now move to Stage 4, when Country 2 combines $1 of domestic value (recorded in (S4, 1b)) with $3 of intermediate goods imported from Country 1 in the previous stage, and exports $4 of intermediate goods in gross terms to Country 1. Given that 40% of the final good will be absorbed in Country 1 by stage 5, we can split Country 2's $1 domestic value added in this stage into $0.4 which is Country 2's value added exports (S4, 2b), and $0.6 which is the amount of its domestic value added that will return home in Stage 5 and be absorbed at home (S4, 3b). Country 2's gross exports in this stage also contain 40% of County 1's value added from the previous stage, recorded as $0.4 in (S4, 5b).

Table D2 Intuitive accounting for the gross export flows in the two country supply chain

	From Country 1's Viewpoint							From Country 2's Viewpoint						
	Domestic Value-added In exports	A Value-added exports	Previous Intermediate exports returning home		Foreign intermediate imports		Total double counted intermediate in exports	Domestic Value-added in exports	Value-added exports	Previous Intermediate exports returning home		Foreign intermediate imports		Total double counted interme-diate in exports
			DV	Pure Double counting	FV	Pure Double counting				DV	Pure Double counting	FV	Pure Double counting	
	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)	(7a)	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)	(7b)
Stage 1 (S1): Country 1 exports and Country 2 imports	1	0.6	0.4	0	0	0	0.4							
Stage 2 (S2): Country 2 exports and Country 1 imports								1	0.4	0.6	0	0.4	0.6	1.6
Stage 3 (S3): Country 1 exports and Country 2 imports	1	0.6	0.4	1	0.6	0.4	2.4							
Stage 4 (S4): Country 2 exports and Country 1 imports								1	0.4	0.6	1	0.4	1.6	3.6
Stage 5 (S5): Country 1 exports and Country 2 imports	0.6 [a]	0.6	0	1.2	0.6	0.6	2.4							
Total	**2.6**	**1.8**	**0.8**	**2.2**	**1.2**	**1**	**5.2**	**2**	**0.8**	**1.2**	**1**	**0.8**	**2.2**	**5.2**
Terms in Table D4 that correspond to the previous row	DV_1	v1+v2	v3+v4	v5	v6+v7	v8	Sum of v3 to v8	DV_2	v1+v2	v3+v4	v5	v6+v7	v8	Sum of v3 to v8

Note: a. In stage 5, because Country 1 exports 3 units of the final goods and keeps 2 units at home, 40% of Country 1's domestic value added (or $0.4) in that stage stays home, and 60% of it (or $0.6) is its exports of value added to Country 2. The last row shows the concordance between the second to the last row of this table and the decomposition results reported in Table D4 that are derived from our gross exports accounting equations.
The gross exports accounting equations (13) and (14) provide the final decomposition results as the total row in the table, not the intermediate iteration in each the stage.

By symmetry, the pure double counting amount in (S4, 4b) must be the same as (S3, 4a), which is \$1. Let us next work out the pure double counting term in (S4, 6b). First, out of Country 2's \$4 gross exports in Stage 4, only \$0.4 is value added exports, we know the total amount of double counting must be \$3.6, which is recorded in (S4, 7b). Second, we also know \$3.6 of the total amount of double counting must be equal to the sum of the double counted components, or the sum of (S4, 3b), (S4, 4b), (S4, 5b) and (S4, 6b). This implies that (S4, 6b) should be \$1.6. The economic meaning of (S4, 6b) is repeated double counting of the intermediate goods that have been double counted in previous rounds of trade.

We now go to Stage 5. Because this is the final stage in which the final good is produced by Country 1 but distributed 40% and 60% in Countries 1 and 2, respectively, we record the values somewhat differently from the earlier stages (when the entire production was exported). While Country 1's domestic value added in the production is \$1 in this stage, only 60% of the final good is exported. So we record the amount of domestic value-added in Country 1's exports as \$0.6 (S5, 1a). The amount of Country 1's value added exports (that is absorbed in Country 2) is also \$0.6, as recorded in (S5, 2a).

Since Stage 5 production uses imported intermediate good from the previous stage, it embeds foreign value added from Stage 4. The amount of foreign value added from Stage 4 that is used in Country 1's Stage 5 production and eventually absorbed in the foreign country is proportional to the amount of the final good that is exported from Country 1 to 2. This means (S5, 5a) is \$0.6. This of course is the same value as in (S4, 3b).

To determine the value in (S5, 4a), we note that the total value added from Country 1 in the first and the 3rd stages are \$1. Both values are counted as part of Country 2's intermediate exports in Stage 4. Since only 60% of the final good are exported, the pure double counting associated with the domestically produced intermediate goods in the previous stages is \$2*0.6 = \$1.2.

To determine the value of (S5, 6a), we first note that the total amount of double counting in Stage 5 exports is the difference between the value of gross exports in that stage (\$3) and the value added exports in that stage (\$0.6), which is \$2.4, as recorded in (S5, 7a). The value in (S5, 6a) would simply be the difference between \$2.4 and the sum of the values in (S5, 2a), (S5, 4a), and (S5, 5a), which yields \$0.6. The amount in (S5, 6a) represents the value that is originally created in Country 2 but has been counted multiple times beyond the value added of Country 2 already assigned to Countries 1 and 2.

We can check the sensibility of the discussion by summing over the values across the five stages. For example, when we sum up the values over all stages in Column (2a), we obtain 1.8, which is exactly the amount of Country 1's value added exports that we intuitively think should be. Summing up the values in Column (7a) across the five stages yields \$5.2, which is the same as what we obtain intuitively earlier.

Separately, we can apply our decomposition formula and generate the measurements of the same set of economic concepts. To do so, we note that the five stages in this example are best represented by 5 sectors (e.g., car windows, paint on a car, rubber tires on a car and a whole car are considered in separate sectors), because an input output table is built on the assumption that all goods within a sector are homogenous, i.e the input-output and direct value-added coefficients are the same for all products within a sector. In our example, because different stages of the production have different direct value-added and IO coefficients, we have to treat the five stages as five different sectors.

The inter-country supply chain data in table D1 can be summarized by the following Input -output IO table:

Table D3 IO table constructed from two-country Supply Chain data

Input		Output	Intermediate use					Final use		Total output
			Country 1			Country 2		Country 1	Country 2	
			Sector 1	Sector 2	Sector 3	Sector 1	Sector 2			
Intermediate input	Country 1	Sector 1				1				1
		Sector 2					3			3
		Sector 3						2	3	5
	Country 2	Sector 1		2						2
		Sector 2			4					4
Value-added			1	1	1	1	1			
Total Input			1	3	5	2	4			

Note: an input -output table is built on homogenous products assumption, i.e input-output and direct value-added coefficients are the same for each product/industry. In order to use an input-output model, each stage of production has to be treated as a distinct product/industry because different stages of the production has different direct value-added and IO coefficients.

From table D3, we can obtain following matrixes:

$$A=\begin{bmatrix}0 & 0 & 0 & 1/2 & 0\\ 0 & 0 & 0 & 0 & 3/4\\ 0 & 0 & 0 & 0 & 0\\ 0 & 2/3 & 0 & 0 & 0\\ 0 & 0 & 4/5 & 0 & 0\end{bmatrix}$$

In A， $a_{11}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$， $a_{12}=\begin{bmatrix}1/2 & 0\\ 0 & 3/4\\ 0 & 0\end{bmatrix}$， $a_{21}=\begin{bmatrix}0 & 2/3 & 0\\ 0 & 0 & 4/5\end{bmatrix}$， $a_{22}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Value-added coefficient：

$v_1=\begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}$, $v_2=\begin{bmatrix}1/2 & 1/4\end{bmatrix}$

Final goods and exports

$y_{11}=\begin{bmatrix}0\\ 0\\ 2\end{bmatrix}$, $y_{12}=\begin{bmatrix}0\\ 0\\ 3\end{bmatrix}$, $y_{21}=\begin{bmatrix}0\\ 0\end{bmatrix}$, $y_{22}=\begin{bmatrix}0\\ 0\end{bmatrix}$, $e_{12}=\begin{bmatrix}1\\ 3\\ 3\end{bmatrix}$, $e_{21}=\begin{bmatrix}2\\ 4\end{bmatrix}$

Leontief inverse B = (I – A) $^{-1}$

$$B=\begin{bmatrix}1 & 1/3 & 1/5 & 1/2 & 1/4\\ 0 & 1 & 3/5 & 0 & 3/4\\ 0 & 0 & 1 & 0 & 0\\ 0 & 2/3 & 2/5 & 1 & 1/2\\ 0 & 0 & 4/5 & 0 & 1\end{bmatrix}$$

In B， $b_{11}=\begin{bmatrix}1 & 1/3 & 1/5\\ 0 & 1 & 3/5\\ 0 & 0 & 1\end{bmatrix}$， $b_{12}=\begin{bmatrix}1/2 & 1/4\\ 0 & 3/4\\ 0 & 0\end{bmatrix}$， $b_{21}=\begin{bmatrix}0 & 2/3 & 2/5\\ 0 & 0 & 4/5\end{bmatrix}$， $b_{22}=\begin{bmatrix}1 & 1/2\\ 0 & 1\end{bmatrix}$

And $(1-a_{11})^{-1}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$， $(1-a_{22})^{-1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Equations (13) and (14) can be converted to a 5-sector version easily by defining each of their terms in a matrix with proper dimensions. The formula in Equations (13) then allows us to decompose Country 1's gross exports as follows:

$$v1=v_1 b_{11} y_{12} = \begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1 & 1/3 & 1/5\\ 0 & 1 & 3/5\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\3\end{bmatrix} = \begin{bmatrix}1 & 2/3 & 3/5\end{bmatrix}\begin{bmatrix}0\\0\\3\end{bmatrix} = 1.8$$

$$v2=v_1 b_{12} y_{22} = \begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\ 0 & 3/4\\ 0 & 0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} = 0$$

$$v3=v_1 b_{12} y_{21} = \begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\ 0 & 3/4\\ 0 & 0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} = 0$$

$$v4 = v_1 b_{12} a_{21}(1-a_{11})^{-1} y_{11} = \begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\ 0 & 3/4\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 2/3 & 0\\ 0 & 0 & 4/5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\2\end{bmatrix}$$

$$= \begin{bmatrix}0 & 1/3 & 2/5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\2\end{bmatrix} = 0.8$$

$$v5 = v_1 b_{12} a_{21}(1-a_{11})^{-1} e_{12} = \begin{bmatrix}1 & 1/3 & 1/5\end{bmatrix}\begin{bmatrix}1/2 & 1/4\\ 0 & 3/4\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 2/3 & 0\\ 0 & 0 & 4/5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\3\\3\end{bmatrix}$$

$$= \begin{bmatrix}0 & 1/3 & 2/5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\3\\3\end{bmatrix} = 2.2$$

$$v6=v_2 b_{21} y_{12} = \begin{bmatrix}1/2 & 1/4\end{bmatrix}\begin{bmatrix}0 & 2/3 & 2/5\\ 0 & 0 & 4/5\end{bmatrix}\begin{bmatrix}0\\0\\3\end{bmatrix} = \begin{bmatrix}0 & 1/3 & 2/5\end{bmatrix}\begin{bmatrix}0\\0\\3\end{bmatrix} = 1.2$$

$$v7=v_2 b_{21} a_{12}(1-a_{22})^{-1} y_{22} = \begin{bmatrix}1/2 & 1/4\end{bmatrix}\begin{bmatrix}0 & 2/3 & 2/5\\ 0 & 0 & 4/5\end{bmatrix}\begin{bmatrix}1/2 & 0\\ 0 & 3/4\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} = 0$$

$$v8 = v_2 b_{21} a_{12} (1 - a_{22})^{-1} e_{21} = \begin{bmatrix} 1/2 & 1/4 \end{bmatrix} \begin{bmatrix} 0 & 2/3 & 2/5 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 3/4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1/3 & 2/5 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 3/4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 1$$

Similarly, we can also decompose country 2's gross exports into the 8 terms specified in equation (14). We summarize all the computation in table D4. It can be checked easily that the numbers in Table D4 generated by our formula match exactly with the corresponding ones that one can intuitively work out in Table D2. In particular, Country 1’s value added exports (that are absorbed abroad) from our formula in Table D4 are \$1.8, exactly as that in Table D2. In comparison, the total domestic value added in Country 1’s exports (that does not exclude exported value added that returns home but does exclude the pure double counted term) is \$2.6. This example confirms our theoretical discussion that value-added exports are generally smaller than domestic value-added (GDP) in exports and domestic content in exports. If one is interested in the share of domestic value added in a country’s exports, then the VAX ratio is not the right metric.

From Table D4, the VS measure produced by our decomposition formula (13) is 2.2. The intuitive discussion in connection with Table D2 illustrates why we argue that the VS measure is not a 'net' concept and is not equal to foreign value added in a country’s gross exports. The fundamental reason is that the VS measure has to include some pure double counted terms. (Again, these pure double counting terms would disappear if we use the HIY assumption that at least one of the countries does not export intermediate good.)

The more intermediate trade crosses border, the larger these double counted foreign intermediates imports are. With two-way intermediate trade, the part of foreign GDP that is embodied in the home country's gross exports will always be smaller than the VS measure. Relative to the original VS measure, our generalized measure includes double counted intermediate exports produced by the foreign country that may cross border several times (v8). The numerical results also show HIY's convention that a country's gross exports is equal to domestic content plus vertical specialization is also maintained by our accounting equation (as long as one defines domestic content and vertical specialization appropriately).

Finally, this example also shows that if one only considers returning domestic value-added in final goods, while excluding domestic content returning home via intermediate goods imports, such as Daudin et al (2011), then one would under-estimate VS1*. In this example, if one applies Daudin et al's narrow definition of VS1*, it would be zero as indicated by v3 in Table D4. If one also includes returning domestic value added in intermediate good and a pure double counting term, VS1* would become $3 instead. Our redefined measure of VS1* is more complete.

Table D4 Gross exports decomposition based on our accounting equation

Terms in accounting equation	E12	E21
v1= $v_1 b_{11} y_{12}$	1.8	0
v2= $v_1 b_{12} y_{22}$	0	0.8
v3= $v_1 b_{12} y_{21}$	0	1.2
v4= $v_1 b_{12} a_{21} (1-a_{11})^{-1} y_{11}$	0.8	0
v5= $v_1 b_{12} a_{21} (1-a_{11})^{-1} e_{12}$	2.2	1
v6= $v_2 b_{21} y_{12}$	1.2	0
v7= $v_2 b_{21} a_{12} (1-a_{22})^{-1} y_{22}$	0	0.8
v8= $v_2 b_{21} a_{12} (1-a_{22})^{-1} e_{21}$	1	2.2
E=Gross exports (sum v1 to v8)	7	6
VT=Value-added exports (sum of v1 and v2)	1.8	0.8
DV=Domestic value-added in gross exports (sum of v1 to v4)	2.6	2
FV=Foreign value-added in gross exports (v6+v7)	1.2	0.8
DC=Domestic content in gross exports (sum of v1 to v5)	4.8	3.0
Double counted home country's intermediate exports	2.2	1
Double counted foreign country's intermediate exports	1	2.2
VS=Vertical specialization(sum v6 to v8) = $v_1 b_{12}$	2.2	3
VS1* measure defined in this paper (sum v3 to v5)	3	2.2
VS1* measure defined in Daudin, et al. (v3 only)	0	1.2
Johnson & Noguera's VAX ratio	0.257	0.133
Share of domestic value-added in gross exports	0.371	0.333
Share of domestic content in gross exports = $v_1 b_{11}$	0.686	0.5

Source: Authors' estimates

Appendix E Detailed results of magnification of trade costs by multi-stage production

As discussed in section 4.2 in the main text, our gross export accounting method provides an ideal way to re-examine the magnification effect of trade cost by multi-stage production. In Table E1, we first report standard tariffs (on a country's exports) in columns (1a). These are trade-weighted tariff rate applied by a country's trading partners (in ad-valorem equivalent). Column (2a) reports the share of imported content in final goods exports. These imported intermediate inputs are used to produce final goods exports, and so incur multiple tariffs charges. These tariff rates on the imported inputs (as a share of f.o.b. export value) are presented in columns (3a); they are trade-weighted average tariffs for intermediate inputs from the other 25 countries/regions in our database that are used in the exporting country to produce final goods exports. The sum of the two tariffs is reported in Column (4a).

Columns (5a) reports our illustrative calculation of the first order magnification effect of using imported intermediate inputs to produce exports. It represents the magnification effect if tariffs were the only factor that augments the trading costs. For instance, one additional stage of production increases trade costs of Vietnam's merchandise production by 80% of its standard tariff.

Although the number is already quite high for a number of countries, these values still represent only the lower bound of the true multi-stage tariff charge. First, in this illustration, we only consider two stages of production, while in the real world, these inputs may have already crossed multiple borders before reaching the final exporter. Second, we ignore transport costs in this example, but transport costs are also magnified as intermediate goods cross multiple borders.

The second magnification force occurs because tariffs are applied to gross export values instead of the value added in the direct exporting country. Table E1 also reports the magnification ratio of the "effective" tariff rate to the standard tariff rate. Column (6a) reports the effective tariff rate, which equals the standard tariff rate in column (2a) divided by the domestic content share (which is 1 minus column (2a)) and weighted by trade. Column (7a) reports the implied magnification ratio due to the presence of vertical specialization. These effects are generally larger than the tariff magnification factor reported in column (5a).

Generally speaking, tariffs play a large role in the magnification of trade costs in the presence of GVCs for emerging market economies, while they play a smaller role for most developed countries. The fact that the domestic value added share in emerging economies' merchandise exports is usually lower than that in developed countries tends to amplify the effective trade cost for developing countries. As an implication, reducing tariffs and nontariff barriers in manufacturing sectors globally is fully consistent with the interest of emerging market economies because it lowers the cost of GVC participation for developing countries. Lowering "own" tariffs on intermediate inputs for domestic manufacturing production would significantly reduce the magnification effects as demonstrated in column (5), while lowering such tariffs in other countries would significantly reduce the effective rate of protection, as seen in columns (6) and (7), due to the lower domestic value-added share in most developing countries' manufacturing exports.

To see if the end-use classifications and the proportionality assumption produce different results, we go through the same set of calculation but using the proportionality assumption to construct our data set. All the estimates in Columns (1b), (2b), …,(7b) are the direct counterparts to Columns (1a), (2a), …, (7a). In Column 8, we report the difference in terms of % of each country's gross exports for the magnification factor computed using the two different databases. For Indonesia, Malaysia, and China, the BEC method produces a larger magnification effect. In comparison, for Canada, India, and Mexico, the BEC method produces a smaller magnification effect. In general, which method we use makes a difference.

Table E1 Magnification of trade costs on final goods exports from vertical specialization, 2004

Country or region	Database produced by BEC classification							Database produced by proportion assumption							100* difference	
	Standard Tariff (1a)	Foreign content share (VS) (2a)	Tariff on imported inputs (3a)	two stage tariffs 1a+3a (4a)	Magnifi-cation factor 4a/1a (5a)	Effective tariff rate (6a)	Magnifi-cation ratio 6a/1a (7a)	Standard Tariff (1b)	Foreign content share (VS) (2b)	Tariff on imported inputs (3b)	two stage tariffs 1b+3b (4b)	Magnifi-cation factor 4b/1b (5b)	Effective tariff rate (6b)	Magnifi-cation ratio 6b/1b (7b)	Magnifi-cation factor 5a-5b (8)	Magnifi-cation ratio 7a-7b (9)
Advanced economies																
Aus-New Zealand	15.55	0.13	0.34	15.89	1.02	27.00	1.74	13.48	0.15	0.55	14.03	1.04	26.02	1.93	**-1.9**	**-19.4**
Canada	1.60	0.38	0.24	1.84	1.15	7.05	4.41	1.36	0.38	0.30	1.66	1.22	7.52	5.53	**-7.1**	**-112.3**
Western EU	6.16	0.12	0.24	6.40	1.04	12.09	1.96	6.06	0.13	0.24	6.30	1.04	12.22	2.02	**-0.1**	**-5.4**
Japan	6.22	0.12	0.05	6.27	1.01	11.19	1.80	6.36	0.12	0.06	6.42	1.01	11.42	1.80	**-0.1**	**0.3**
USA	4.38	0.13	0.17	4.55	1.04	9.19	2.10	4.05	0.15	0.21	4.26	1.05	9.26	2.29	**-1.3**	**-18.8**
Asian NICs																
Hong Kong	10.16	0.42	0.00	10.16	1.00	27.91	2.75	10.02	0.40	0.00	10.02	1.00	26.09	2.60	**0.0**	**14.3**
Korea	6.05	0.32	1.46	7.51	1.24	17.32	2.86	6.34	0.35	1.74	8.08	1.27	19.62	3.09	**-3.3**	**-23.2**
Taiwan	4.76	0.42	1.40	6.16	1.29	20.08	4.22	4.45	0.43	1.40	5.85	1.31	19.56	4.40	**-2.0**	**-17.7**
Singapore	3.60	0.70	0.00	3.60	1.00	30.05	8.35	3.22	0.72	0.00	3.22	1.00	30.75	9.55	**0.0**	**-120.2**
Emerging Asia																
China	6.17	0.29	1.91	8.08	1.31	21.42	3.47	6.44	0.29	1.97	8.41	1.31	21.86	3.39	**0.4**	**7.7**
Indonesia	7.53	0.30	1.34	8.87	1.18	24.39	3.24	9.44	0.27	1.28	10.72	1.14	26.65	2.82	**4.2**	**41.6**
Malaysia	3.55	0.46	2.11	5.66	1.59	20.93	5.90	4.38	0.45	2.50	6.88	1.57	23.04	5.26	**2.4**	**63.6**
Philippines	5.57	0.39	1.07	6.64	1.19	22.47	4.03	3.50	0.42	0.94	4.44	1.27	16.52	4.72	**-7.6**	**-68.6**
Thailand	8.16	0.40	4.23	12.39	1.52	36.54	4.48	7.67	0.41	4.36	12.03	1.57	35.05	4.57	**-5.0**	**-9.2**
Vietnam	10.71	0.43	8.62	19.33	1.80	55.10	5.14	10.29	0.45	9.17	19.46	1.89	54.52	5.30	**-8.6**	**-15.4**
India	7.82	0.18	2.98	10.80	1.38	22.08	2.82	6.93	0.19	3.10	10.03	1.45	19.82	2.86	**-6.6**	**-3.6**
Other emerging economies																
Brazil	12.27	0.13	1.22	13.49	1.10	22.77	1.86	11.82	0.13	1.12	12.94	1.09	25.07	2.12	**0.5**	**-26.5**
EU accession	2.41	0.34	0.55	2.96	1.23	12.67	5.26	2.18	0.36	0.57	2.75	1.26	12.24	5.61	**-3.3**	**-35.7**
Mexico	0.88	0.31	1.00	1.88	2.14	6.36	7.23	0.67	0.30	1.02	1.69	2.52	5.73	8.55	**-38.6**	**-132.5**
Russian	5.36	0.18	1.61	6.97	1.30	17.23	3.21	3.64	0.16	1.34	4.98	1.37	14.86	4.08	**-6.8**	**-86.8**
South Africa	7.15	0.20	1.11	8.26	1.16	22.11	3.09	6.75	0.22	1.18	7.93	1.17	20.94	3.10	**-2.0**	**-1.0**

Source: Authors' estimates.

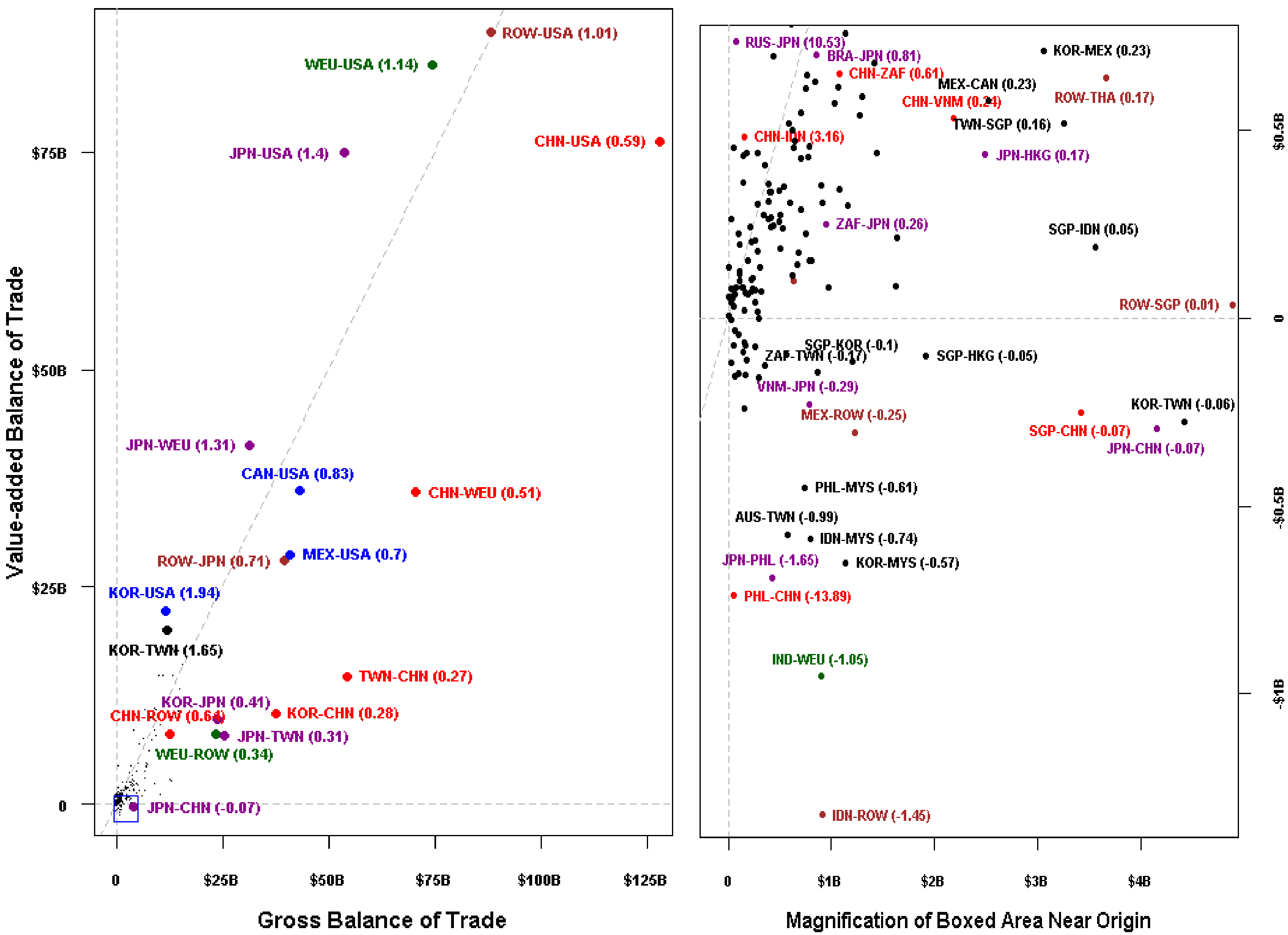
Appendix F The difference between bilateral trade imbalance in gross and value-added terms

Figure F1 provides a scatter plot of the trade balance in value added terms against the trade balance in standard trade statistics for all bilateral country pairs in our ICIO database. Without loss of generality, the two countries in any pair are always ordered in such a way that the trade balance in gross terms is non-negative. A negative value-added to gross BOT ratio indicates there is a sign change between BOT measured in gross and value-added terms. All observations that lie below the 45 degree line have their bilateral trade imbalance smaller in value-added terms than those in gross terms, and vice visa for observations that lie above the 45 degree line.

Zooming in near the origin shows that the trade balances of a number of country pairs even have opposite signs measured in value-added and gross terms. For example, Japan's trade balance vis- à-vis China is switched from a surplus in gross trade terms to a deficit in value added terms. This is consistent with the notion that a significant part of Japan's exports to China are components used by China-based firms for exports to the United States, the European Union and other markets. This further illustrates potentially misleading nature of gross bilateral trade imbalances.[2]

[2]Figure F1 also shows that the Korea-China-U.S. triple trade relationship is similar to the Japan-China-U.S. one.

Figure F1: Gross and VA Balance of Trade, 2004



Note: The first country labeled in each pair is the surplus country while the second runs a deficit. Numbers in parentheses are the ratio of value-added to gross surplus.