Online Appendix to "Formal Contracts, Relational Contracts, and the Threat-Point Effect"

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A1 General Bargaining

In the main text, we presented Propositions 1–6 for the special case of the buyer's take-it-or-leave-it offer. Here we formally state and prove these propositions for the general case in which the seller and the buyer follow the generalized Nash bargaining solution with the seller's share $\alpha \in [0, 1)$ and the buyer's share $1 - \alpha$.

A1.1 Spot Transactions

Suppose that no formal fixed-price contract is written. If trade is efficient, i.e., $q^*(a) = 1$ (or, equivalently, $a \geq \overline{a}$), the seller and the buyer negotiate and agree to trade with price $\rho_0(a)$, which is defined by

$$\rho_0(a) = m(a) + \alpha[v(a) - m(a)].$$

The seller's payoff is then $\rho_0(a) - a$. On the other hand, if trade is inefficient, there is no negotiation and trade does not occur. The seller's payoff is $m(a) - a$. Define the seller's ex post payoff under no contract $\rho_0^+(a)$ by

$$\rho_0^+(a) = \max\{\rho_0(a), m(a)\} = m(a) + \alpha q^*(a)[v(a) - m(a)].$$

Note $\rho_0^+(\underline{a}) = m(\underline{a})$ and $\rho_0^+(a^*) = \rho_0(a^*)$ hold under the assumption $\underline{a} < \overline{a} < a^*$.

The seller chooses action $a^o \in \arg\max_a \left\{\rho_0^+(a) - a\right\}$. This condition is rewritten as

$$\rho_0^+(a^o) - \rho_0^+(a) \geq a^o - a \quad \text{for all } a \in \mathcal{A}. \tag{A1}$$

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The following proposition corresponds to Proposition 1 in the main text.

Proposition A1 If no formal contract is written at the beginning, the seller does not overinvest under a spot transaction: $a^* \geq a^o$ holds.

Proof Suppose instead $a^* < a^o$. Since a^o is optimal under a spot transaction,

$$a^o - a^* \leq \rho_0^+(a^o) - \rho_0^+(a^*)$$

holds. By $\alpha < 1$, $a^o > a^*$, and (M) in the main text, the right-hand side satisfies

$$\rho_0^+(a^o) - \rho_0^+(a^*) \leq v(a^o) - v(a^*) \leq v^+(a^o) - v^+(a^*).$$

Therefore, $a^o - a^* \leq v^+(a^o) - v^+(a^*)$ must hold, which is a contradiction because a^* is uniquely efficient and hence $v^+(a^*) - a^* > v^+(a^o) - a^o$, or $a^o - a^* > v^+(a^o) - v^+(a^*)$ holds. **Q.E.D.**

The seller does not overinvest, because of the holdup effect that the seller cannot capture the full marginal contribution ($\alpha < 1$), along with the insufficient threat-point effect due to assumption (M). Note that a^o is (weakly) increasing in α. For example, while $a^o = \underline{a}$ holds if the threat-point effect is negative and the holdup effect is sufficiently strong (α is close to zero), a^o will eventually become larger than $\underline{a}$ for sufficiently high α, and will approach a^* as α goes to one. As before, we confine our attention to the interesting case in which the holdup effect is so strong that trade is inefficient without a formal contract ($a^o < \overline{a} < a^*$), and hence the joint surplus at a^o, $\pi(a^o)$, becomes equal to $\overline{\pi} = m(a^o) - a^o$. These properties are satisfied, for example, if α is so small that $a^o = \underline{a}$ holds. In fact, under our assumption of $a^o < \overline{a}$, $\rho_0^+(a^o) = m(a^o)$ and hence $a^o = \underline{a}$ must hold if $m(\cdot)$ is decreasing ($a^o \geq \underline{a}$ if $m(\cdot)$ is increasing).

Next, suppose that the buyer and the seller sign a formal fixed-price contract $\overline{p}$ at the beginning. If trade is efficient, there is no room for negotiation and the parties trade with price $\overline{p}$. The seller's payoff is $\overline{p} - a$. If trade is inefficient, however, they negotiate to cancel the contract and the seller obtains $\rho_1(a)$ in addition to w, which is defined by

$$\rho_1(a) = \overline{p} + \alpha[m(a) - v(a)].$$

Note that it is *decreasing* in a by assumption (M). Define the seller's ex post payoff under the fixed-price contract $\rho_1^+(a)$ by

$$\rho_1^+(a) = \max\{\rho_1(a), \overline{p}\} = \overline{p} + \alpha(1 - q^*(a))[m(a) - v(a)].$$

The seller chooses a to maximize $\rho_1^+(a) - a$, the solution of which is obviously $a = \underline{a}$ as before, since $\rho_1^+(a)$ is decreasing in a. The outcome is no better than the case with no formal fixed-price contract.

A1.2 Repeated Transactions

No formal contract

Consider the relational contract without formal contracting in the main text. If the buyer follows the relational contract, the seller who chooses action a obtains payoff $f + b(a) + (1 - q^*(a))m(a) - a$. His incentive compatibility constraint is given as follows:

$$b(\hat{a}) + (1 - q^*(\hat{a}))m(\hat{a}) - \hat{a} \geq b(a) + (1 - q^*(a))m(a) - a \quad \text{for all } a. \tag{IC0 A}$$

The self-enforcing condition is a straightforward extension of the corresponding condition (DE0) in the main text:

$$\max_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} - \min_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} \leq \frac{\delta}{1-\delta} \Big[\pi(\hat{a}) - \overline{\pi} \Big]. \tag{DE0 A}$$

Define the seller's "marginal" ex post payoff under no contract by

$$\Delta_0(a, a') = \rho_0^+(a) - \rho_0^+(a').$$

The following proposition generalizes the corresponding Proposition 2 in the main text.

Proposition A2 Suppose no formal fixed-price contract is written. The seller's action $\hat{a} > a^o$ can be implemented by a relational contract if and only if (DE-NC A) holds:

$$\hat{a} - a^o - \Delta_0(\hat{a}, a^o) \leq \frac{\delta}{1-\delta} \Big[\pi(\hat{a}) - \overline{\pi} \Big] \tag{DE-NC A}$$

Proof As of in Proposition 2 in the main text, the seller's incentive compatibility constraints (IC0 A) and the self-enforcing condition (DE0 A) are necessary and sufficient for $\hat{a}$ to be implemented.

Necessity Suppose $\hat{a} > a^o$ can be implemented, i.e., there exists a compensation plan $b(\cdot)$ satisfying (IC0 A) and (DE0 A). The left-hand side of (DE0 A) is rewritten as follows:

$$\begin{aligned}
&\max_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} - \min_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} \\
&\geq \Big\{ b(\hat{a}) - q^*(\hat{a})\rho_0(\hat{a}) \Big\} - \Big\{ b(a^o) - q^*(a^o)\rho_0(a^o) \Big\} \\
&\geq \hat{a} - a^o - (1 - q^*(\hat{a}))m(\hat{a}) + (1 - q^*(a^o))m(a^o) - q^*(\hat{a})\rho_0(\hat{a}) + q^*(a^o)\rho_0(a^o) \\
&= \hat{a} - a^o - \left(\rho_0^+(\hat{a}) - \rho_0^+(a^o) \right).
\end{aligned}$$

This is the left-hand side of (DE-NC A).

Sufficiency Supposing (DE-NC A) holds, we construct a compensation plan that satisfies (IC0 A) and (DE0 A). Define $b(a)$ as follows:

$$\begin{aligned}
b(\hat{a}) &= \hat{a} + \overline{\pi} - (1 - q^*(\hat{a}))m(\hat{a}) \\
b(a) &= q^*(a)\rho_0(a), \quad \text{for all } a \neq \hat{a}.
\end{aligned} \tag{A2}$$

The right-hand side of (IC0 A) is then equal to $\rho_0^+(a) - a$ for $a \neq \hat{a}$, the maximum value of which is $\overline{\pi} = \rho_0^+(a^o) - a^o$. Since the left-hand side is equal to $\overline{\pi}$, $b(\cdot)$ satisfies (IC0 A).

We next show

$$\max_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} = b(\hat{a}) - q^*(\hat{a})\rho_0(\hat{a}). \tag{A3}$$

First, $b(\hat{a}) - q^*(\hat{a})\rho_0(\hat{a}) = -\Delta_0(a^o, \hat{a}) + \hat{a} - a^o \geq 0$ holds by (A1). And for $a \neq \hat{a}$, $b(a) - q^*(a)\rho_0(a) = 0$, and hence we obtain (A3). Similarly, we can show

$$\min_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} = b(a^o) - q^*(a^o)\rho_0(a^o).$$

Therefore

$$\begin{aligned}
&\max_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} - \min_a \Big\{ b(a) - q^*(a)\rho_0(a) \Big\} \\
&= \hat{a} - a^o - \Delta_0(\hat{a}, a^o)
\end{aligned}$$

which completes the proof. **Q.E.D.**

Note that the seller's ex post payoff without a formal contract is $\rho_0^+(a) = m(a) + \alpha q^*(a)[v(a) - m(a)]$, which is equal to $m(a)$ if $\alpha = 0$. Condition (DE-NC A) thus includes the necessary and sufficient condition (DE-NC) in the main text as a special case in which the buyer makes a take-it-or-leave-it offer.

Formal Fixed-Price Contract

Consider next the case in which at the beginning of each period, the buyer and the seller sign a formal fixed-price contract that the seller will deliver the product and that the buyer will pay $\overline{p}$. And they agree with the relational contract in the main text.

Define the seller's "marginal" ex post payoff under a fixed-price contract by $\Delta_1(a, a') = \rho_1^+(a) - \rho_1^+(a')$. The following proposition generalizes Proposition 3 in the main text.

Proposition A3 The seller's action $\hat{a} > a^o$ can be implemented by combining a formal fixed-price contract and a relational contract if and only if (DE-FP A) holds:

$$\hat{a} - \underline{a} - \Delta_1(\hat{a}, \underline{a}) \leq \frac{\delta}{1-\delta}\Big[\pi(\hat{a}) - \overline{\pi}\Big] \tag{DE-FP A}$$

Proof As in the main text, the seller's incentive compatibility constraints (IC1 A) and the self-enforcing condition (DE1 A) are necessary and sufficient for $\hat{a}$ to be implemented:

$$\begin{aligned} &b(\hat{a}) + m(\hat{a}) + q^*(\hat{a})(\overline{p} - m(\hat{a})) - \hat{a} \\ &\quad \geq b(a) + m(a) + q^*(a)(\overline{p} - m(a)) - a \quad \text{for all } a \end{aligned} \tag{IC1 A}$$

$$\begin{aligned} &\max_a \Big\{b(a) - (1 - q^*(a))(\rho_1(a) - m(a))\Big\} \\ &\quad - \min_a \Big\{b(a) - (1 - q^*(a))(\rho_1(a) - m(a))\Big\} \\ &\quad \leq \frac{\delta}{1-\delta}\Big[\pi(\hat{a}) - \overline{\pi}\Big]. \end{aligned} \tag{DE1 A}$$

Necessity The left-hand side of (DE1 A) is rewritten as follows:

$$\begin{aligned} &\max_a \Big\{b(a) - (1 - q^*(a))(\rho_1(a) - m(a))\Big\} \\ &\qquad - \min_a \Big\{b(a) - (1 - q^*(a))(\rho_1(a) - m(a))\Big\} \\ &\geq \Big\{b(\hat{a}) - (1 - q^*(\hat{a}))(\rho_1(\hat{a}) - m(\hat{a}))\Big\} \\ &\qquad - \Big\{b(\underline{a}) - (1 - q^*(\underline{a}))(\rho_1(\underline{a}) - m(\underline{a}))\Big\} \\ &\geq \hat{a} - \underline{a} - q^*(\hat{a})\overline{p} - (1 - q^*(\hat{a}))\rho_1(\hat{a}) + q^*(\underline{a})\overline{p} + (1 - q^*(\underline{a}))\rho_1(\underline{a}) \\ &= \hat{a} - \underline{a} - \Delta_1(\hat{a}, \underline{a}). \end{aligned}$$

This is the left-hand side of (DE-FP A).

Sufficiency To show the sufficiency part, define $b(a)$ as follows:

$$\begin{aligned} b(\hat{a}) &= \hat{a} - \underline{a} + (1 - q^*(\hat{a}))(\overline{p} - m(\hat{a})) \\ b(a) &= (1 - q^*(a))(\overline{p} - m(a)) \quad \text{for all } a \neq \hat{a}. \end{aligned}$$

Then the right-hand side of (IC1 A) becomes $\overline{p} - a$ for $a \neq \hat{a}$, which is maximized at $a = \underline{a}$. The left-hand side is equal to $\overline{p} - \underline{a}$, and hence $b(\cdot)$ satisfies (IC1 A).

We next show the following:

$$\begin{aligned} \max_a \Big\{ b(a) - (1 - q^*(a))(\rho_1(a) - m(a)) \Big\} &= \hat{a} - \underline{a} + \overline{p} - \rho_1^+(\hat{a}) \\ \min_a \Big\{ b(a) - (1 - q^*(a))(\rho_1(a) - m(a)) \Big\} &= \overline{p} - \rho_1^+(\underline{a}). \end{aligned}$$

First, for $a = \hat{a}$, $b(\hat{a}) - (1 - q^*(\hat{a})(\rho_1(\hat{a}) - m(\hat{a})) = \hat{a} - \underline{a} + \overline{p} - \rho_1^+(\hat{a})$ holds. Second, for $a \neq \hat{a}$, $b(a) - (1 - q^*(a))(\rho_1(a) - m(a)) = \overline{p} - \rho_1^+(a) \geq \overline{p} - \rho_1^+(\underline{a})$ holds since $\underline{a}$ is the minimum level of investment and $\rho_1^+(a)$ is decreasing in a. Finally, $\hat{a} - \underline{a} - \rho_1^+(\hat{a}) > -\rho_1^+(\underline{a})$ is satisfied. Therefore,

$$\begin{aligned} &\max_a \Big\{ b(a) + (1 - q^*(a))(m(a) - \rho_1(a)) \Big\} - \min_a \Big\{ b(a) + (1 - q^*(a))(m(a) - \rho_1(a)) \Big\} \\ &= \hat{a} - \underline{a} - \rho_1^+(\hat{a}) + \rho_1^+(\underline{a}) = \hat{a} - \underline{a} - \Delta_1(\hat{a}, \underline{a}) \end{aligned}$$

holds, which completes the proof. **Q.E.D.**

With a fixed-price contract $\overline{p}$, the seller's ex post payoff is $\rho_1^+(a) = \overline{p} + \alpha(1 - q^*(a))[m(a) - v(a)]$. If the buyer makes a take-it-or-leave-it offer ($\alpha = 0$), this ex post payoff is reduced to $\overline{p}$, and hence $\Delta_1(\hat{a}, \underline{a}) = 0$ holds. Condition (DE-FP A) is then reduced to the necessary and sufficient condition (DE-FP) in the main text.

When $\alpha > 0$ and trade is inefficient, the seller can obtain, in addition to the fixed price, some share of the surplus from canceling the formal contract. This additional part is decreasing in a, and hence provides the seller with a negative incentive to invest. This in turn raises the total reneging temptation represented by the left-hand side of (DE-FP A), since $\Delta_1(\hat{a}, \underline{a}) < 0$ holds for $\alpha > 0$ (and becomes more negative as α increases).

A1.3 Comparison

Propositions A2 and A3 immediately yield the following extension of Proposition 4, as discussed in Subsection III.C.

Proposition A4 Suppose the buyer and the seller follow the generalized Nash bargaining with the seller's share α, and consider the implementation of $\hat{a}$ satisfying $\pi(\hat{a}) > \overline{\pi}$.

(a) Suppose $(a^o - \underline{a}) + \Delta_0(\hat{a}, a^o) - \Delta_1(\hat{a}, \underline{a}) < 0$ holds. If $\hat{a}$ can be implemented under repeated transactions without any formal contract, the same action can be implemented under repeated transactions with an appropriate fixed-price contract. And there is a range of parameter values in which $\hat{a}$ can be implemented only if a formal fixed-price contract is written.

(b) Suppose $(a^o - \underline{a}) + \Delta_0(\hat{a}, a^o) - \Delta_1(\hat{a}, \underline{a}) > 0$ holds. If $\hat{a}$ can be implemented under repeated transactions with a formal fixed-price contract, the same action can be implemented under repeated transactions without any formal contract. And there is a range of parameter values in which $\hat{a}$ can be implemented only if no formal fixed-price contract is written.

Note $\Delta_0(\hat{a}, a^o)$ and $\Delta_1(\hat{a}, \underline{a})$ are rewritten as follows:

$$\begin{aligned}\Delta_0(\hat{a}, a^o) &= \rho_0^+(\hat{a}) - \rho^+(a^o) = m(\hat{a}) - m(a^o) + \alpha[v(\hat{a}) - m(\hat{a})];\\ \Delta_1(\hat{a}, \underline{a}) &= -\alpha[m(\underline{a}) - v(\underline{a})].\end{aligned}$$

Hence the condition for a formal fixed-price contract to be of value in Proposition A4 (a) becomes (9) in Subsection III.C:

$$a^o - \underline{a} + m(\hat{a}) - m(a^o) + \alpha\Big\{[v(\hat{a}) - m(\hat{a})] - [v(\underline{a}) - m(\underline{a})]\Big\} < 0.$$

A2 General Contracts

We prove Propositions 5 and 6 presented at the end of Section IV, under the assumption of the generalized Nash bargaining solution.

A2.1 Spot Transactions

A general formal contract is denoted by $\{p(a_b, a_s), q(a_b, a_s)\}$, where a_b and a_s are the messages sent by the buyer and the seller, respectively, concerning the seller's action. For all messages (a_b, a_s), the contract specifies trade decision $q(a_b, a_s) \in \{0, 1\}$ and payment from the buyer to the seller $p(a_b, a_s)$. If the trade decision is inefficient under the seller's action a, the parties renegotiate the contract to the efficient trade decision $q^*(a)$. The ex post payoffs to the buyer and the seller, when the seller chooses a and the buyer and the seller report (a_b, a_s), are, respectively, written as follows:

$$\begin{aligned}
u_B(a_b, a_s \mid a) &= q(a_b, a_s)\Big[v(a) - p(a_b, a_s) + (1-\alpha)(1-q^*(a))\,(m(a)-v(a))\Big] \\
&\quad + (1-q(a_b,a_s))\Big[-p(a_b,a_s) + (1-\alpha)q^*(a)\,(v(a)-m(a))\Big]; \\
u_S(a_b, a_s \mid a) &= q(a_b, a_s)\Big[p(a_b, a_s) + \alpha(1-q^*(a))\,(m(a)-v(a))\Big] \\
&\quad + (1-q(a_b,a_s))\Big[p(a_b,a_s) + m(a) + \alpha q^*(a)\,(v(a)-m(a))\Big].
\end{aligned}$$

Not writing a formal contract corresponds to $q(a_b, a_s) \equiv 0$ and $p(a_b, a_s) \equiv 0$, and a fixed-price contract $\overline{p}$ corresponds to $q(a_b, a_s) \equiv 1$ and $p(a_b, a_s) \equiv \overline{p}$.

We rewrite these payoffs using $v^+(a) = q^*(a)v(a) + (1-q^*(a))m(a)$ and $\rho_0^+(a) = m(a) + \alpha q^*(a)[v(a) - m(a)]$. Also define $\sigma_1^+(a)$ by

$$\sigma_1^+(a) = \alpha(1-q^*(a))[m(a) - v(a)] = \alpha \max\{m(a) - v(a), 0\}.$$

Note that under the fixed-price contract $\overline{p}$, $\rho_1^+(a) = \overline{p} + \sigma_1^+(a)$, and hence

$$\Delta_1(a, a') = \rho_1^+(a) - \rho_1^+(a') = \sigma_1^+(a) - \sigma_1^+(a')$$

holds. Then the ex post payoffs are rewritten as follows.

$$\begin{aligned}
u_B(a_b, a_s \mid a) &= q(a_b, a_s)\Big[v^+(a) - p(a_b, a_s) - \sigma_1^+(a)\Big] \\
&\quad + (1-q(a_b,a_s))\Big[v^+(a) - p(a_b,a_s) - \rho_0^+(a)\Big]; \\
u_S(a_b, a_s \mid a) &= q(a_b, a_s)\Big[p(a_b, a_s) + \sigma_1^+(a)\Big] \\
&\quad + (1-q(a_b,a_s))\Big[p(a_b,a_s) + \rho_0^+(a)\Big].
\end{aligned}$$

Note the following equation holds for all (a_b, a_s, a):

$$u_B(a_b, a_s \mid a) + u_S(a_b, a_s \mid a) = v^+(a)$$

For each a, truth telling must form a Nash equilibrium:

$$\begin{aligned}
u_S(a) \equiv u_S(a, a \mid a) &\geq u_S(a, \hat{a} \mid a), \quad \text{for all } \hat{a}; \\
u_B(a) \equiv u_B(a, a \mid a) &\geq u_B(\hat{a}, a \mid a), \quad \text{for all } \hat{a}.
\end{aligned}$$

Using the zero-sum feature of the payoffs yields $u_B(\hat{a}) \geq u_B(a, \hat{a} \mid \hat{a})$ if and only if $u_S(\hat{a}) \leq$

$u_S(a, \hat{a} \mid \hat{a})$. Thus we must have

$$\begin{aligned} u_S(\hat{a}) - u_S(a) &\leq u_S(a, \hat{a} \mid \hat{a}) - u_S(a, \hat{a} \mid a) \\ &= q(a, \hat{a})\Big[\sigma_1^+(\hat{a}) - \sigma_1^+(a)\Big] + (1 - q(a, \hat{a}))\Big[\rho_0^+(\hat{a}) - \rho_0^+(a)\Big]. \end{aligned} \tag{A4}$$

Proposition A5, corresponding to Proposition 5 in the main text, extends the well-known result of Che and Hausch (1999) to the case where the alternative-use value depends on a.[1]

Proposition A5 Under a spot transaction, no formal contract can induce the seller to choose $\hat{a} > a^o$.

Proof Suppose instead that there is a formal contract $\{p(a_b, a_s), q(a_b, a_s)\}$ under which the seller's optimal choice is $\hat{a} > a^o$ satisfying $\pi(\hat{a}) > \overline{\pi} = \pi(a^o)$. Then by the seller's incentive compatibility constraints, the following inequality must hold:

$$u_S(\hat{a}) - u_S(a^o) \geq \hat{a} - a^o.$$

Assumption (M) and $\hat{a} > a^o$ yield $\sigma_1^+(\hat{a}) \leq \sigma_1^+(a^o)$, and hence by (A4),

$$u_S(\hat{a}) - u_S(a) \leq (1 - q(a, \hat{a}))\Big[\rho_0^+(\hat{a}) - \rho_0^+(a)\Big]$$

holds for all a. We thus obtain

$$u_S(\hat{a}) - u_S(a^o) \leq (1 - q(a^o, \hat{a}))\Big[\rho_0^+(\hat{a}) - \rho_0^+(a^o)\Big].$$

Now suppose first $\rho_0(\hat{a}) \geq \rho_0(a^o)$. Then by the seller's incentive compatibility constraints,

$$\begin{aligned} \hat{a} - a^o &\leq u_S(\hat{a}) - u_S(a^o) \\ &\leq (1 - q(a^o, \hat{a}))\Big[\rho_0^+(\hat{a}) - \rho_0^+(a^o)\Big] \leq \rho_0^+(\hat{a}) - \rho_0^+(a^o), \end{aligned}$$

which contradicts $\hat{a} \neq \arg\max_a \left\{\rho_0^+(a) - a\right\}$.

Next suppose $\rho_0(\hat{a}) \leq \rho_0(a^o)$. Then

$$\hat{a} - a^o \leq (1 - q(a^o, \hat{a}))\Big[\rho_0^+(\hat{a}) - \rho_0^+(a^o)\Big] \leq 0,$$

[1] If $v(\cdot)$ and $m(\cdot)$ depend not only on the seller's action but also observable but unverifiable state of nature θ, the same result holds under an additional assumption that the negotiated price $\rho_0(a, \theta) = m(a, \theta) + \alpha[v(a, \theta) - m(a, \theta)]$ is either increasing in a for all θ, or decreasing in a for all θ. See the supplemental material available on a website for the proof and an example in which formal contracts contingent on messages are valuable if the effects of uncertainty θ are so large that $\rho_0(a, \theta)$ is increasing in a for some θ and is decreasing in a for other θ.

which contradicts $\hat{a} > a^o$. **Q.E.D.**

A2.2 Repeated Transactions

We next extend the analysis of no contract versus fixed-price contracts under repeated transactions to general formal contracts. Consider formal (short-term) contracts $\{p(a_b, a_s), q(a_b, a_s)\}$ along with the relational contract that consists of the following promises:

- The buyer pays f to the seller at the beginning of each period.
- The seller chooses $\hat{a}$ and both the buyer and the seller report truthfully.
- If action $a \geq \overline{a}$ is chosen and the formal contract specifies trade $q(a, a) = 1$, then the seller delivers the product to the buyer, and the buyer pays $b(a)$ in addition to $p(a, a)$.
- If action $a \geq \overline{a}$ is chosen and the formal contract specifies no trade $q(a, a) = 0$, then the buyer and the seller cancel the formal contract, and the buyer pays $b(a)$ for the product.
- If action $a < \overline{a}$ is chosen and the formal contract specifies trade $q(a, a) = 1$, then they cancel the formal contract to no trade. The seller sells the product in an outside opportunity, and the buyer pays $b(a)$.
- If action $a < \overline{a}$ is chosen and the formal contract specifies no trade $q(a, a) = 0$, then the seller sells the product in an outside opportunity, and the buyer pays $b(a)$.
- If either party breaches report, delivery, or payment, they follow the generalized Nash bargaining solution, and from the next period on, they terminate the relationship.

The ex post payoffs to the buyer and the seller when the seller chooses a and both report truthfully are respectively given as follows:

$$u_B(a) = q^*(a)v(a) - b(a) - p(a, a)Q(a)$$
$$u_S(a) = (1 - q^*(a))m(a) + b(a) + p(a, a)Q(a)$$

where $Q(a) = q^*(a)q(a, a) + (1 - q^*(a))(1 - q(a, a))$, which takes the value of 1 if the formal contract specifies the efficient trade decision and takes zero otherwise. Note that $u_B(a) + u_S(a) = v^+(a)$ holds for all a. If the parties follow the informal promises, the seller's incentive compatibility constraints are given as follows:

$$u_S(\hat{a}) - \hat{a} \geq u_S(a) - a \quad \text{for all } a. \tag{A5}$$

Suppose that the seller's action is $\hat{a}$, and the seller reports truthfully. The buyer's ex post payoff from deviating by reporting $a \neq \hat{a}$ is written as

$$u_B(a, \hat{a} \mid \hat{a}) = v^+(\hat{a}) - p(a, \hat{a}) - q(a, \hat{a})\sigma_1^+(\hat{a}) - (1 - q(a, \hat{a}))\rho_0^+(\hat{a}).$$

For example, suppose $\hat{a}$ satisfies $q^*(\hat{a}) = 1$ and the buyer deviates by reporting a such as $q(a, \hat{a}) = 1$ and not paying $b(\hat{a})$. Since $\sigma_1^+(\hat{a}) = 0$ holds for such $\hat{a}$, her payoff is then $v^+(\hat{a}) - p(a, \hat{a})$. Note there is no negotiation after reneging in this case. As another case, suppose $\hat{a}$ satisfies $q^*(\hat{a}) = 0$ and the buyer reports a such as $q(a, \hat{a}) = 1$ and does not pay $b(\hat{a})$ but negotiates to obtain $v(\hat{a}) - p(a, \hat{a}) + (1 - \alpha)[m(\hat{a}) - v(\hat{a})]$. Her payoff is then $m(\hat{a}) - p(a, \hat{a}) - \alpha[m(\hat{a}) - v(\hat{a})] = m(\hat{a}) - p(a, \hat{a}) - \sigma_1^+(\hat{a})$. One can check the other two cases similarly to obtain the buyer's payoff as above. The buyer's reneging temptation is thus

$$\max_{a,a'}\{u_B(a', a \mid a) - u_B(a)\}.$$

Similarly, suppose the seller's action is a and the buyer reports truthfully. The seller's payoff from deviating by reporting $\hat{a} \neq a$ is written as

$$u_S(a, \hat{a} \mid a) = p(a, \hat{a}) + q(a, \hat{a})\sigma_1^+(a) + (1 - q(a, \hat{a}))\rho_0^+(a).$$

The seller's reneging temptation is hence

$$\max_{a,a'}\{u_S(a, a' \mid a) - u_S(a)\} = -\min_{a,a'}\{u_S(a) - u_S(a, a' \mid a)\}.$$

The next proposition generalizes Proposition 6 in the main text.

Proposition A6 It is without loss of generality to confine attention to no contract or fixed-price contracts under repeated transactions.

Proof Suppose instead there exists a formal contract $\{p(a_b, a_s), q(a_b, a_s)\}$ which, along with an appropriate relational contract, can implement $\hat{a} > a^o$ with the total reneging temptation smaller than those under no contract or the fixed-price contract:

$$\begin{aligned} &\max_{a,a'}\Big\{u_B(a', a \mid a) - u_B(a)\Big\} - \min_{a,a'}\Big\{u_S(a') - u_S(a', a \mid a')\Big\} \\ &\quad < \min\Big\{\hat{a} - a^o - \Delta_0(\hat{a}, a^o), \quad \hat{a} - \underline{a} - \Delta_1(\hat{a}, \underline{a})\Big\}. \end{aligned} \tag{A6}$$

The left-hand side of (A6) is rewritten as follows:

$$
\begin{aligned}
&\max_{a,a'}\Big\{u_B(a',a \mid a) - u_B(a)\Big\} - \min_{a,a'}\Big\{u_S(a') - u_S(a',a \mid a')\Big\} \\
&\quad \geq \Big\{u_B(a^o,\hat{a} \mid \hat{a}) - u_B(\hat{a})\Big\} - \Big\{u_S(a^o) - u_S(a^o,\hat{a} \mid a)\Big\} \\
&\quad \geq \hat{a} - a^o - q(a^o,\hat{a})\Big[\sigma_1^+(\hat{a}) - \sigma_1^+(a^o)\Big] - (1 - q(a^o,\hat{a}))\Big[\rho_0^+(\hat{a}) - \rho_0^+(a^o)\Big] \\
&\quad = \hat{a} - a^o + q(a^o,\hat{a})\Big[\rho_0(\hat{a}) - \rho_0(a^o)\Big] - \Delta_0(\hat{a},a^o).
\end{aligned}
$$

First suppose $\rho_0(\hat{a}) \geq \rho_0(a^o)$ holds. Then

$$
\hat{a} - a^o + q(a^o,\hat{a})\Big[\rho_0(\hat{a}) - \rho_0(a^o)\Big] - \Delta_0(\hat{a},a^o) \geq \hat{a} - a^o - \Delta_0(\hat{a},a^o),
$$

which contradicts (A6).

Next, suppose $\rho_0(\hat{a}) < \rho_0(a^o)$ holds. This implies $m(\cdot)$ is decreasing, and hence $a^o = \underline{a}$. Then

$$
\begin{aligned}
&\hat{a} - a^o + q(a^o,\hat{a})\Big[\rho_0(\hat{a}) - \rho_0(a^o)\Big] - \Delta_0(\hat{a},a^o) \\
&\quad \geq \hat{a} - \underline{a} + \Big[\rho_0(\hat{a}) - \rho_0(\underline{a})\Big] - \Big[\rho_0^+(\hat{a}) - \rho_0^+(\underline{a})\Big] = \hat{a} - \underline{a} - \Delta_1(\hat{a},\underline{a}),
\end{aligned}
$$

which, again, contradicts (A6). **Q.E.D.**