Online Supplement to "Formal Contracts, Relational Contracts, and the Threat-Point Effect"

Hideshi Itoh* Hodaka Morita†

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In this supplemental material we analyze the general model in which both the value of the seller's product for the buyer and the alternative-use value of the product are affected not only by the seller's action but also by uncertainty.

1 Model with Uncertainty

The seller's action, equal to his private cost, is denoted by $a \in \mathcal{A} \subset \mathbb{R}$, which includes the least costly action $\underline{a} \geq 0$. Let $v(a, \theta)$ be the value of the seller's product for the buyer, where θ is the state of nature drawn from support $\Theta = [\underline{\theta}, \overline{\theta}]$ according to a cumulative distribution function $F(\cdot)$, and let $m(a, \theta)$ be the alternative-use value, which accrues to the seller. The buyer's outside payoff is independent of the seller's action and normalized to zero.

We assume $v(a, \theta)$ is strictly increasing in a and $v(a, \theta) > a$ for all a and θ. The alternative-use value $m(a, \theta)$ may be increasing or decreasing in a, while we assume that investment affects $v(a, \theta)$ at least as much as $m(a, \theta)$ at margins:

$$v(a, \theta) - v(a', \theta) \geq m(a, \theta) - m(a', \theta) \quad \text{for all } \theta \text{ and } a > a'. \tag{SM}$$

Let $q^*(a, \theta) \in \{0, 1\}$ denote the efficient trade: $q^*(a, \theta) = 1$ if $v(a, \theta) \geq m(a, \theta)$, and $q^*(a, \theta) = 0$, otherwise. Note that assumption (SM) implies $q^*(a, \theta)$ is increasing in a for all θ. The efficient action a^* is defined by $a^* = \arg\max_a \{E[v^+(a, \theta)] - a\}$ where

$$v^+(a, \theta) = \max\{v(a, \theta), m(a, \theta)\} = m(a, \theta) + q^*(a, \theta)[v(a, \theta) - m(a, \theta)],$$

and $E[x(\theta)] = \int x(\theta) dF(\theta)$. We assume a^* is unique, $a^* > \underline{a}$, and the probability of trade

*Graduate School of Commerce and Management, Hitotsubashi University.
†School of Economics, UNSW Business School, University of New South Wales.

being efficient is positive under a^*.

We assume that a, θ, v, and m are observable to both parties but unverifiable, while delivery of the product and transfer payments are verifiable.

At the beginning of each period the seller and the buyer can agree on a compensation plan, with the seller promising a particular action. The compensation plan consists of $\{f, \overline{p}, (b(a,\theta))_{a\in\mathcal{A},\theta\in\Theta}\}$, where f is paid from the buyer to the seller at the beginning of the period, $\overline{p}$ is a formal fixed-price contract contingent on the delivery of the product, and $b(a,\theta)$ is an additional informal payment made by the buyer when the seller's action is a and state θ realizes. In order to investigate the value of the formal fixed-price contract in resolution of the holdup problem, we compare it with the case of no formal contract, in which $\overline{p}$ is not specified in a compensation plan. Since f does not play any essential role in the results, we mostly ignore it in the parties' payoffs.

The timing in each period is as follows. First, the seller and the buyer can agree on a compensation plan. If $\overline{p}$ is specified in the plan, the agreement includes signatures by the seller and the buyer on the formal fixed-price contract. Second, the seller chooses an action. Third, the state of nature realizes. Fourth, after observing the seller's action and the state of nature, the buyer and the seller negotiate to an ex post efficient outcome if there is inefficiency. This applies either when trade turns out to be inefficient under a fixed-price contract, or when trade turns out to be efficient under no formal contract. In these cases, we assume that the transfer is determined by the generalized Nash bargaining solution, with $\alpha \in [0,1)$ being the seller's share of the surplus. Finally, the seller produces and sells the product to the buyer according to the compensation plan or at the negotiated price, or in the outside market.

2 Spot Transaction

No formal contract

First, suppose that no formal fixed-price contract is written at the beginning. If state θ satisfying $q^*(a,\theta) = 1$ realizes, the seller and the buyer negotiate trade and a price. The negotiated price $\rho_0(a,\theta)$ is defined by

$$\rho_0(a,\theta) = m(a,\theta) + \alpha[v(a,\theta) - m(a,\theta)].$$

The seller's payoff is then $\rho_0(a,\theta) - a$. On the other hand, if state θ satisfying $q^*(a,\theta) = 0$ realizes, there is no negotiation and trade does not occur. The seller's payoff is $m(a,\theta) - a$.

Define $\sigma_0^+(a,\theta)$ and $\rho_0^+(a,\theta)$ by

$$\begin{aligned}
\sigma_0^+(a,\theta) &= \alpha \max\{v(a,\theta) - m(a,\theta), 0\} = \alpha q^*(a,\theta)[v(a,\theta) - m(a,\theta)],\\
\rho_0^+(a,\theta) &= \max\{\rho(a,\theta), m(a,\theta)\} = m(a,\theta) + \sigma_0^+(a,\theta).
\end{aligned}$$

Then the seller chooses action a that maximizes $E[\rho_0^+(a,\theta)] - a$. Denote the optimal action under no contract by a^o:

$$a^o \in \arg\max_a \left\{E[\rho_0^+(a,\theta)] - a\right\} \tag{S1}$$

In this setup it is easy to show that the seller does not overinvest.

Proposition S1 If no formal fixed-price contract is written at the beginning, the seller does not overinvest under spot transaction: $a^* \geq a^o$.

Proof Suppose instead $a^* < a^o$. Since a^* is uniquely efficient,

$$E[v^+(a^*,\theta)] - a^* > E[v^+(a^o,\theta)] - a^o,$$

or

$$a^o - a^* > E[v^+(a^o,\theta)] - E[v^+(a^*,\theta)]$$

holds. On the other hand, since a^o is optimal under spot transaction,

$$a^o - a^* \leq E[\rho_0^+(a^o,\theta)] - E[\rho_0^+(a^*,\theta)].$$

By $\alpha < 1$, $a^o > a^*$, and (SM),

$$\rho_0^+(a^o,\theta) - \rho_0^+(a^*,\theta) \leq v(a^o,\theta) - v(a^*,\theta) \leq v^+(a^o,\theta) - v^+(a^*,\theta)$$

holds for all θ. Therefore

$$a^o - a^* \leq E[v^+(a^o,\theta)] - E[v^+(a^*,\theta)]$$

must hold, which is a contradiction. **Q.E.D.**

To make the analysis interesting, we hereafter assume $a^* > a^o$: There exists $a < a^*$ such that the following inequality holds:

$$a - a^* < E[\rho_0^+(a,\theta)] - E[\rho_0^+(a^*,\theta)].$$

The joint surplus from trade at action a is denoted by $\pi(a) \equiv E[v^+(a,\theta)] - a$, and the joint surplus at no trade by $\overline{\pi} \equiv \max_a\{E[m(a,\theta)] - a\}$. To simplify analysis (particularly under repeated transaction), we assume $q(a^o,\theta) = 0$, or equivalently $v(a^o,\theta) \leq m(a^o,\theta)$, for all θ: for all states trade is inefficient under action a^o. This implies $v^+(a^o,\theta) = m(a^o,\theta)$ for all θ, and hence $\overline{\pi} = \pi(a^o)$: the equilibrium outcome under spot transaction without formal contracting is that, in neither state the seller and the buyer trade.

Formal fixed-price contracts

Next, suppose that the buyer and the seller sign a formal fixed-price contract with price $\overline{p}$ at the beginning. If state θ satisfying $q^*(a,\theta) = 1$ realizes, there is no room for negotiation and the parties trade with this price. The seller's payoff is $\overline{p} - a$. If state θ satisfying $q^*(a,\theta) = 0$ realizes, however, they negotiate to cancel the contract and break off trade. The seller is paid $\rho_1(a,\theta)$, which is defined by

$$\rho_1(a,\theta) = \overline{p} + \alpha[m(a,\theta) - v(a,\theta)].$$

The seller's payoff is then $w + \rho_1(a,\theta) - a$. While there is renegotiation if $q(a,\theta) = 0$, the payment to the seller is *decreasing* in a by assumption (SM).

Define $\sigma_1^+(a,\theta)$ and $\rho_1^+(a,\theta)$ by

$$\begin{aligned}
\sigma_1^+(a,\theta) &= \alpha \max\{m(a,\theta) - v(a,\theta), 0\} = \alpha(1 - q^*(a,\theta))[m(a,\theta) - v(a,\theta)]. \\
\rho_1^+(a,\theta) &= \max\{\rho_1(a,\theta), m(a,\theta)\} = \overline{p} + \sigma_1^+(a,\theta).
\end{aligned}$$

The seller chooses a to maximize $E[\rho_1^+(a,\theta)] - a$, the solution of which is obviously $a = \underline{a}$.

In short, nothing changes with the introduction of uncertainty concerning the under-investment results under spot transaction, except for possible renegotiation of the initial contract contingent on the realization of the state of the nature.

General formal contracts

A general formal contract is written as $\{p(\eta_b,\eta_s), q(\eta_b,\eta_s)\}$, where $\eta_b = (a_b,\theta_b)$ and $\eta_s = (a_s,\theta_s)$ are the messages sent by the buyer and the seller, respectively. For all messages (η_b,η_s), the contract specifies trade decision $q(\eta_b,\eta_s) \in \{0,1\}$ and payment from the buyer to the seller $p(\eta_b,\eta_s)$. If the trade decision is inefficient for action and state $\eta = (a,\theta)$, the parties renegotiate the contract to the efficient trade decision $q^*(\eta)$. The ex post payoffs to the buyer and the seller under η, resulting from the contract and renegotiation, are, respectively, as follows:

$$
\begin{aligned}
u_B(\eta_b, \eta_s \mid \eta) = q(\eta_b, \eta_s)&\Big[v(\eta) - p(\eta_b, \eta_s) + (1-\alpha)(1-q^*(\eta))(m(\eta) - v(\eta))\Big] \\
&+ (1 - q(\eta_b, \eta_s))\Big[-p((\eta_b, \eta_s)) + (1-\alpha)q^*(\eta)(v(\eta) - m(\eta))\Big] \\
u_S(\eta_b, \eta_s \mid \eta) = q(\eta_b, \eta_s)&\Big[p(\eta_b, \eta_s) + \alpha(1-q^*(\eta))(m(\eta) - v(\eta))\Big] \\
&+ (1 - q(\eta_b, \eta_s))\Big[p(\eta_b, \eta_s) + m(\eta) + \alpha q^*(\eta)(v(\eta) - m(\eta))\Big]
\end{aligned}
$$

Note

$$
u_B(\eta_b, \eta_s \mid \eta) + u_S(\eta_b, \eta_s \mid \eta) = \max\{v(\eta), m(\eta)\} = q^*(\eta)v(\eta) + (1 - q^*(\eta))m(\eta)
$$

holds for all (η_b, η_s, η).

For each $\eta = (a, \theta)$, truth telling must form a Nash equilibrium:

$$
\begin{aligned}
u_S(\eta) \equiv u_S(\eta, \eta \mid \eta) \geq u_S(\eta, \hat{\eta} \mid \eta), \quad \forall \hat{\eta} \\
u_B(\eta) \equiv u_B(\eta, \eta \mid \eta) \geq u_B(\hat{\eta}, \eta \mid \eta), \quad \forall \hat{\eta}
\end{aligned}
$$

Using the zero-sum feature of the payoffs yields $u_B(\hat{\eta}) \geq u_B(\eta, \hat{\eta} \mid \hat{\eta})$ if and only if $u_S(\hat{\eta}) \leq u_S(\eta, \hat{\eta} \mid \hat{\eta})$. Thus we must have

$$
\begin{aligned}
u_S(\hat{\eta}) - u_S(\eta) &\leq u_S(\eta, \hat{\eta} \mid \hat{\eta}) - u_S(\eta, \hat{\eta} \mid \eta) \\
&= q(\eta, \hat{\eta})\Big[\sigma_1^+(\hat{\eta}) - \sigma_1^+(\eta)\Big] \\
&\quad + (1 - q(\eta, \hat{\eta}))\Big[\rho_0^+(\hat{\eta}) - \rho_0^+(\eta)\Big].
\end{aligned}
\tag{S2}
$$

Proposition S2 Suppose $\rho_0(a, \theta) = m(a, \theta) + \alpha[v(a, \theta) - m(a, \theta)]$ is either weakly increasing in a for all θ, or weakly decreasing in a for all θ. Then formal contracts are of no value under spot transactions.

Proof Suppose instead there is a formal contract $\{p(\eta_b, \eta_s), q(\eta_b, \eta_s)\}$ under which the seller's optimal choice is $\hat{a} > a^o$ satisfying $\pi(\hat{a}) > \overline{\pi} = \pi(a^o)$. Then by the seller's incentive compatibility constraints the following inequality must hold.

$$
E[u_S(\hat{a}, \theta)] - E[u_S(a^o, \theta)] \geq \hat{a} - a^o
$$

By specificity (SM) and $\hat{a} > a^o$, $\sigma_1^+(\hat{\eta}) \leq \sigma_1^+(\eta)$ holds for all θ where $\hat{\eta} = (\hat{a}, \theta)$ and $\eta = (a^o, \theta)$.

Hence by (S2),

$$u_S(\hat{\eta}) - u_S(\eta) \le (1 - q(\eta, \hat{\eta})) \left(\rho_0^+(\hat{\eta}) - \rho_0^+(\eta)\right)$$

for all θ. We thus obtain

$$E\Big[u_S(\hat{a}, \theta) - u_S(a^o, \theta))\Big] \le E\Big[(1 - q((a^o, \theta), (\hat{a}, \theta))) \left(\rho_0^+(\hat{a}, \theta) - \rho_0^+(a^o, \theta)\right)\Big]$$

Now suppose first $\rho_0(\hat{a}, \theta) \ge \rho_0(a^o, \theta)$ for all θ. Then by the seller's incentive compatibility constraints

$$\begin{aligned}
\hat{a} - a^o &\le E\Big[u_S(\hat{a}, \theta) - u_S(a^o, \theta))\Big] \\
&\le E\Big[(1 - q((a^o, \theta), (\hat{a}, \theta))) \left(\rho_0^+(\hat{a}, \theta) - \rho_0^+(a^o, \theta)\right)\Big] \\
&\le E\Big[\rho_0^+(\hat{a}, \theta) - \rho_0^+(a^o, \theta)\Big]
\end{aligned}$$

which contradicts $\hat{a} \neq \arg\max_a E\Big(\left[\rho_0^+(a, \theta)\right] - a\Big)$

Next suppose $\rho_0(\hat{a}, \theta) \le \rho_0(a^o, \theta)$ for all θ. Then

$$\hat{a} - a^o \le E\left[(1 - q((a^o, \theta), (\hat{a}, \theta))) \left(\rho_0^+(\hat{a}, \theta) - \rho_0^+(a^o, \theta)\right)\right] \le 0$$

which contradicts $\hat{a} > a^o$. **Q.E.D.**

Proposition S2 identifies a sufficient condition for the well-known result of Che and Hausch (1999) to hold when alternative-use value depends on a. Formal contracts cannot improve the seller's effort incentives from the no contract case if the effects of uncertainty θ is not too large to alter the sign of the effects of investment a on the negotiated price. For example, this condition holds if both $v(a, \theta)$ and $m(a, \theta)$ are additively separable in terms of a and θ: $v(a, \theta) = v(a) + x(\theta)$ and $m(a, \theta) = m(a) + y(\theta)$, with $\alpha v(a) + (1 - \alpha)m(a)$ being monotone in a. It also holds if θ does not affect the negotiated price because, either (as in our paper) there is no uncertainty in the value for the buyer and the alternative-use value, or, as in Edlin and Hermalin (2000), the parties can renegotiate only *before* uncertainty resolves.[1] Finally, the condition holds if, as in Che and Hausch (1999), the alternative-use value $m(a, \theta)$ does not depend on a.

If the condition is violated, formal contracts contingent on messages may be of value, although fixed-price contracts are not, as the following example shows.

[1]Under the assumption of this timing, Edlin and Hermalin (2000) show a result similar to Proposition S2 that an optimal second-best arrangement among general mechanisms where both parties send messages about the seller's action is for the buyer to transfer ownership to the seller ex ante and simply bargain for the asset after the seller has invested, which arrangement corresponds to our no contract case.

Example S1

Let $\theta \in \{\theta_L, \theta_H\}$ and $\beta = \Pr\{\theta = \theta_H\}$. The agent's action is either $a = 0$ (no investment) or $a = 1$ (investment). We use notations $v_a(\theta) = v(a, \theta)$ and $m_a(\theta) = m(a, \theta)$ for $a = 0, 1$. The marginal effects of the seller's action on these values are denoted by $\Delta_v(\theta) = v_1(\theta) - v_0(\theta)$ and $\Delta_m = m_1(\theta) - m_0(\theta)$.

The key feature of the example is that the negotiated price is increasing in investment in state θ_H while it is decreasing in state θ_L. We assume $\alpha = 0$ (the buyer's take-it-or-leave-it offer), and no-trade surplus satisfies the following conditions:

$$m_0 \equiv m_0(\theta_L) = m_0(\theta_H) \quad \text{and} \quad m_1(\theta_H) > m_0 > m_1(\theta_L).$$

The first assumption is for simplicity: state does not affect no-trade surplus under no investment. The second assumption is crucial. It implies $\Delta_m(\theta_H) > 0$ and $\Delta_m(\theta_L) < 0$: the threat-point effect is positive in state θ_H, but negative in state θ_L.

As for $v_a(\theta)$, we assume

$$v_1(\theta_H) > m_1(\theta_H), \quad m_0 \geq v_0(\theta_H), \quad \text{and} \quad v_1(\theta_L) > m_0 \geq v_0(\theta_L).$$

These assumptions imply $\Delta_v(\theta) > 0$ and $\Delta_v(\theta) > \Delta_m(\theta)$ for all θ, consistent with the assumptions of the model. And for all states trade is efficient when the seller invests while it is inefficient when he does not.

We assume

$$\beta\Delta_m(\theta_H) > 1 > \beta\Delta_m(\theta_H) + (1 - \beta)\Delta_m(\theta_L).$$

Since $\Delta_v(\theta) > \Delta_m(\theta)$, the first inequality implies the efficient action is $a = 1$. The second inequality implies underinvestment ($a = 0$) occurs if no contract is written. And the seller obviously chooses $a = 0$ under fixed-price contract.

Now consider the following form of formal contracts: $\{p_j^i, q_j^i\}$ where $i \in \{0, 1\}$ is the buyer's report concerning the seller's action, $j \in \{L, H\}$ is the seller's report concerning state, p_j^i is the payment by the buyer to the seller, and $q_j^i \in \{0, 1\}$ is the decision of trade (1) or no trade (0). Note fixed-price contract corresponds to $p_j^i \equiv \overline{p}$ and $q_j^i \equiv 1$. The trade decision and the payment are specified as follows:

$$q_L^0 = q_L^1 = 1, \quad p_L^0 = p_L^1 = m_0, \quad q_H^0 = q_H^1 = 0, \quad \text{and} \quad p_H^0 = p_H^1 = 0.$$

The idea is to utilize positive "market incentive" by not specifying trade when state is θ_H, while the formal contract specifying trade in state θ_L prevents the negotiated price from affecting the seller's incentive negatively.

We first show that this contract induces investment by the seller, provided that both the buyer and the seller report truthfully. If the seller invests, his expected payoff is $\beta m_1(\theta_H) + (1-\beta)m_0 - 1$. If the seller does not invest, his expected payoff is $\beta m_0 + (1-\beta)m_0 = m_0$. It is optimal for the seller to invest because of assumption $\beta\Delta_m(\theta_H) > 1$.

We next show that the buyer reports truthfully. Suppose the seller's investment is a, the true state is θ_j, and the seller reports the state truthfully. The buyer's payoff is

$$v_a(\theta_j)q_j^i - p_j^i + (1-q_j^i)(v_a(\theta_j) - m_a(\theta_j))$$

which does not depend on the buyer's report i by the construction of the formal contract. Hence she has no incentive to misreport.

The remaining task is to show that the seller reports truthfully. Suppose the seller's investment is a, the true state is θ_j, and the buyer reports a truthfully. If the seller reports θ_H, his payoff is $m_a(\theta_j) - a$. If the seller reports θ_L, his payoff is $m_0 - a$. The seller reports truthfully in state θ_H if $m_a(\theta_H) - a \geq m_0 - a$, which conditions are satisfied for $a = 0, 1$ because $m_1(\theta_H) > m_0(\theta_H) = m_0$. Finally, the seller reports truthfully in state θ_L if $m_a(\theta_L) - a \leq m_0 - a$ holds, which inequalities are satisfied because $m_1(\theta_L) < m_0(\theta_L) = m_0$.

3 Repeated Transactions

We now consider the case in which the seller and the buyer engage in infinitely repeated transactions, with the common discount factor $\delta \in (0, 1)$. Without loss of generality, we can focus on optimal stationary contracts under which in every period the parties agree on the same compensation plan maximizing the joint surplus, the seller chooses the same action on the equilibrium path, and the seller's incentives are provided via discretionary payments alone (MacLeod and Malcomson, 1989; Levin, 2003).

No formal contract

We first assume no formal contract is written, and obtain necessary and sufficient conditions under which there exists a self-enforcing stationary relational contract that implements a given action $\hat{a} > a^o$ satisfying $\pi(\hat{a}) > \overline{\pi}$.

At the beginning of each period, the buyer and the seller agree with the following relational contract:

- The buyer pays f to the seller at the beginning of the period.

- The seller chooses $\hat{a}$.
- If state θ satisfying $q^*(a, \theta) = 1$ realizes, the seller delivers the product and the buyer pays $b(a, \theta)$.
- If state θ satisfying $q^*(a, \theta) = 0$ realizes, the seller does not deliver the product (but sells in an outside opportunity) and the buyer pays $b(a, \theta)$.
- If either party breaches delivery or payment, they follow the generalized Nash bargaining solution, and from the next period on, they terminate the relationship.

The discretionary plan must satisfy the seller's incentive compatibility constraints:

$$E[b(\hat{a}, \theta) + (1 - q^*(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \geq E[b(a, \theta) + (1 - q^*(a, \theta))m(a, \theta)] - a \quad \text{for all } a \tag{S3}$$

We next derive the buyer's self-enforcing condition. If state θ satisfying $q^*(a, \theta) = 1$ realizes, the buyer's short-term gain from not paying $b(a, \theta)$ and negotiating to trade by price $\rho_0(a, \theta)$ instead is $b(a, \theta) - \rho(a, \theta)$. If state θ satisfying $q^*(a, \theta) = 0$ realizes, then the buyer's gain from deviation is $b(a, \theta)$ since trade is inefficient. The buyer's reneging temptation is hence written as $\max_{a,\theta}\{b(a, \theta) - q^*(a, \theta)\rho_0(a, \theta)\}$. The buyer will then lose her future per period gain $E[q^*(\hat{a}, \theta)v(\hat{a}, \theta)] - E[b(a, \theta)]$. The buyer therefore honors the agreement if and only if

$$\max_{a,\theta}\{b(a, \theta) - q^*(a, \theta)\rho_0(a, \theta)\} \leq \frac{\delta}{1 - \delta}\left(E[q^*(\hat{a}, \theta)v(\hat{a}, \theta)] - E[b(\hat{a}, \theta)]\right) \tag{S4}$$

The seller's self-enforcing condition is obtained in a similar fashion. The seller honors the agreement if and only if

$$-\min_{a,\theta}\{b(a, \theta) - q^*(a, \theta)\rho_0(a, \theta)\} \leq \frac{\delta}{1 - \delta}\left(E[b(\hat{a}, \theta) + (1 - q^*(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} - \overline{\pi}\right). \tag{S5}$$

Combining (S4) and (S5) yields a single necessary condition:

$$\max_{a,\theta}\{b(a, \theta) - q^*(a, \theta)\rho_0(a, \theta)\} - \min_{a,\theta}\{b(a, \theta) - q^*(a, \theta)\rho_0(a, \theta)\} \leq \frac{\delta}{1 - \delta}\left(\pi(\hat{a}) - \overline{\pi}\right). \tag{S6}$$

And (S3) and (S6) are also sufficient for investment $\hat{a}$ to be implemented: one can find an appropriate f such that (S4), (S5), and the parties' participation constraints are satisfied.

The next proposition provides the necessary and sufficient condition for compensation plan implementing $\hat{a}$ to exist. The condition looks identical to the one in Proposition A2 in the online appendix of the paper. Note $\Delta_0(a, a')$ there is modified here to the change in *expected* payments in terms of the seller's action under no formal contract: $\Delta_0(a, a') \equiv E[\rho_0^+(a, \theta) - \rho_0^+(a', \theta)]$.

Proposition S3 Suppose no formal fixed-price contract is written. The seller's action $\hat{a}$ satisfying $\pi(\hat{a}) > \overline{\pi}$ can be implemented by a relational contract if and only if (DE-NC S) holds.

$$\hat{a} - a^o - \Delta_0(\hat{a}, a^o) \leq \frac{\delta}{1-\delta}(\pi(\hat{a}) - \overline{\pi}) \tag{DE-NC S}$$

Proof

Necessity:

Suppose $\hat{a} > a^o$ can be implemented: There exists a compensation plan $(b(a, \theta))_{a \in \mathcal{A}, \theta \in \Theta}$ satisfying (S3) and (S6). The left-hand side of (S6) is rewritten as follows:

$$\begin{aligned}
&\max_{a,\theta}\{b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta)\} - \min_{a,\theta}\{b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta)\} \\
&\quad \geq \max_{\theta}\{b(\hat{a},\theta) - q^*(\hat{a},\theta)\rho_0(\hat{a},\theta)\} - \min_{\theta}\{b(a^o,\theta) - q^*(a^o,\theta)\rho_0(a^o,\theta)\} \\
&\quad \geq E[b(\hat{a},\theta) - q^*(\hat{a},\theta)\rho_0(\hat{a},\theta)] - E[b(a^o,\theta) - q^*(a^o,\theta)\rho_0(a^o,\theta)] \\
&\quad \geq \hat{a} - a^o - E[(1 - q^*(\hat{a},\theta))m(\hat{a},\theta)] + E[(1 - q^*(a^o,\theta))m(a^o,\theta)] \\
&\qquad\quad - E[q^*(\hat{a},\theta)\rho_0(\hat{a},\theta)] + E[q^*(a^o,\theta)\rho_0(a^o,\theta)] \\
&\quad = \hat{a} - a^o - E[\rho_0^+(\hat{a},\theta) - \rho_0^+(a^o,\theta)]
\end{aligned}$$

This is the left-hand side of (DE-NC S).

Sufficiency:

Supposing (DE-NC S), we construct a compensation plan that satisfies (S3) and (S6). Define $b(a, \theta)$ as follows:[2]

$$\begin{aligned}
b(\hat{a},\theta) &= q^*(\hat{a},\theta)\rho_0(\hat{a},\theta) - \Delta_0(\hat{a}, a^o) + \hat{a} - a^o \\
b(a,\theta) &= q^*(a,\theta)\rho_0(a,\theta), \quad \text{for all } a \neq \hat{a}
\end{aligned} \tag{S7}$$

(S3) is satisfied for $a = a^o$:

$$\begin{aligned}
&E[b(\hat{a},\theta)] + E[(1 - q^*(\hat{a},\theta))m(\hat{a},\theta)] - E[b(a^o,\theta)] - E[(1 - q^*(a^o,\theta))m(a^o,\theta)] \\
&\quad = E[\rho_0^+(\hat{a},\theta)] - \Delta_0(\hat{a}, a^o) + \hat{a} - a^o - E[\rho_0^+(a^o,\theta)] \\
&\quad = \hat{a} - a^o
\end{aligned}$$

[2]The fixed payment f is only used to guarantee that (S4), (S5), and the participation constraints are satisfied.

For $a \neq a^o$, (S3) holds because

$$
\begin{aligned}
&E[b(\hat{a},\theta)] + E[(1-q^*(\hat{a},\theta))m(\hat{a},\theta)] - E[b(a,\theta)] - E[(1-q^*(a,\theta))m(a,\theta)] \\
&\quad = \hat{a} - a^o + E[b(a^o,\theta)] + E[(1-q^*(a^o,\theta))m(a^o,\theta)] - E[b(a,\theta)] - E[(1-q^*(a,\theta))m(a,\theta)] \\
&\quad = \hat{a} - a^o + \Delta_0(a^o, a) \\
&\quad \geq \hat{a} - a^o + a^o - a = \hat{a} - a
\end{aligned}
$$

where the last inequality follows from the optimality of a^o under spot transaction with no formal contract:

$$\Delta_0(a^o, a) \geq a^o - a \quad \text{for all } a. \tag{S8}$$

We next show

$$\max_{a,\theta}\{b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta)\} = \max_{\theta}\{b(\hat{a},\theta) - q^*(\hat{a},\theta)\rho_0(\hat{a},\theta)\}. \tag{S9}$$

First, $b(\hat{a},\theta) - q^*(\hat{a},\theta)\rho_0(\hat{a},\theta) = \Delta_0(a^o,\hat{a}) + \hat{a} - a^o \geq 0$ holds by (S8). And for $a \neq \hat{a}$, $b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta) = 0$, and hence we obtain (S9). Similarly, we can show

$$\min_{a,\theta}\{b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta)\} = \min_{\theta}\{b(a^o,\theta) - q^*(a^o,\theta)\rho_0(a^o,\theta)\}.$$

Therefore

$$
\begin{aligned}
&\max_{a,\theta}\{b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta)\} - \min_{a,\theta}\{b(a,\theta) - q^*(a,\theta)\rho_0(a,\theta)\} \\
&\quad = \hat{a} - a^o - \Delta_0(\hat{a}, a^o)
\end{aligned}
$$

which completes the proof. **Q.E.D.**

Formal fixed-price contracts

At the beginning of each period, the buyer and the seller write a formal fixed-price contract that the seller delivers the product and the buyer pays $\overline{p}$. And they agree with the following relational contract:

- The buyer pays f to the seller at the beginning of the period.
- The seller chooses $\hat{a}$.
- If state θ satisfying $q^*(a,\theta) = 1$ realizes, the seller delivers the product and the buyer pays $b(a,\theta)$ in addition to $\overline{p}$.

- If If state θ satisfying $q^*(a,\theta) = 0$ realizes, they cancel the formal contract, the seller does not deliver the product (but sells in an outside opportunity) and the buyer pays $b(a,\theta)$.
- If either party breaches delivery or payment, they follow the generalized Nash bargaining solution, and from the next period on, they terminate the relationship.

As in the no formal contract case, without loss of generality we assume that incentives are provided via discretionary payments alone. The seller's incentive compatibility constraints are then given as follows.

$$\begin{aligned} &E[b(\hat{a},\theta) + q^*(\hat{a},\theta)\overline{p} + (1-q^*(\hat{a},\theta))m(\hat{a},\theta)] - \hat{a} \\ &\quad \geq E[b(a,\theta) + q^*(a,\theta)\overline{p} + (1-q^*(a,\theta))m(a,\theta)] - a \quad \text{for all } a \end{aligned} \tag{S10}$$

The buyer's reneging temptation is derived as follows. First, if state θ satisfying $q(a,\theta) = 1$ realizes, the buyer can refuse to pay $b(a,\theta)$ though she has to follow the formal contract and pay $\overline{p}$. Her short-term gain is $b(a,\theta)$. Next if state satisfying $q(a,\theta) = 0$ realizes, the buyer can refuse to cancel the formal contract and to pay $b(a,\theta)$, and instead negotiate to obtain

$$v(a,\theta) - \overline{p} + (1-\alpha)[m(a,\theta) - v(a,\theta)] = m(a,\theta) - \rho_1(a,\theta).$$

The buyer's reneging temptation is thus written as $\max_{a,\theta}\{b(a,\theta) + (1-q^*(a,\theta))(m(a,\theta) - \rho_1' a,\theta)\}$. The buyer honors the agreement if and only if

$$\begin{aligned} &\max_{a,\theta}\{b(a,\theta) + (1-q^*(a,\theta))(m(a,\theta) - \rho_1(a,\theta))\} \\ &\quad \leq \frac{\delta}{1-\delta}\left(E[q^*(\hat{a},\theta)v(\hat{a},\theta)] - \overline{p} - E[b(\hat{a},\theta)]\right). \end{aligned} \tag{S11}$$

Similarly, the seller honors the agreement if and only if

$$\begin{aligned} &-\min_{a,\theta}\{b(a,\theta) + (1-q^*(a,\theta))(m(a,\theta) - \rho_1(a,\theta))\} \\ &\quad \leq \frac{\delta}{1-\delta}\left(\overline{p} + E[b(\hat{a},\theta) + (1-q^*(\hat{a},\theta))m(\hat{a},\theta)] - \hat{a} - \overline{\pi}\right). \end{aligned} \tag{S12}$$

Combining these conditions yields

$$\begin{aligned}&\max_{a,\theta}\{b(a,\theta)+(1-q^*(a,\theta))(m(a,\theta)-\rho_1(a,\theta))\}\\&\quad-\min_{a,\theta}\{b(a,\theta)+(1-q^*(a,\theta))(m(a,\theta)-\rho_1(a,\theta))\}\\&\quad\leq\frac{\delta}{1-\delta}\left(\pi(\hat{a})-\overline{\pi}\right)\end{aligned} \tag{S13}$$

Define the change in *expected* payments in terms of the seller's action under a fixed-price contract by

$$\begin{aligned}\Delta_1(a,a')&=E\Big[\rho_1^+(a,\theta)-\rho_1^+(a',\theta)\Big]\\&=E\Big[\sigma_1^+(a,\theta)-\sigma_1^+(a',\theta)\Big]\\&=\alpha E\Big[(1-q^*(a,\theta))(m(a,\theta)-v(a,\theta))-(1-q^*(a',\theta))(m(a',\theta)-v(a',\theta))\Big]\end{aligned}$$

which is nonpositive if $a>a'$. The following proposition extends Proposition A3 in the online appendix of the paper.

Proposition S4 The seller's action $\hat{a}$ satisfying $\pi(\hat{a})>\overline{\pi}$ can be implemented by combining a formal fixed-price contract and a relational contract if and only if (DE-FP S) holds.

$$\hat{a}-\underline{a}-\Delta_1(\hat{a},\underline{a})\leq\frac{\delta}{1-\delta}\left(\pi(\hat{a})-\overline{\pi}\right) \tag{DE-FP S}$$

Proof

Necessity:

The left-hand side of (S13) is rewritten as follows:

$$\begin{aligned}
&\max_{a,\theta}\{b(a,\theta)+(1-q^*(a,\theta))(m(a,\theta)-\rho_1(a,\theta))\}\\
&\qquad\quad-\min_{a,\theta}\{b(a,\theta)+(1-q^*(a,\theta))(m(a,\theta)-\rho_1(a,\theta))\}\\
&\quad\geq \max_{\theta}\{b(\hat{a},\theta)+(1-q^*(\hat{a},\theta))(m(\hat{a},\theta)-\rho_1(\hat{a},\theta))\}\\
&\qquad\quad-\min_{\theta}\{b(\underline{a},\theta)+(1-q^*(\underline{a},\theta))(m(\underline{a},\theta)-\rho_1(\underline{a},\theta))\}\\
&\quad\geq E\Big[b(\hat{a},\theta)+(1-q^*(\hat{a},\theta))(m(\hat{a},\theta)-\rho_1(\hat{a},\theta))\Big]\\
&\qquad\quad-E\Big[b(\underline{a},\theta)+(1-q^*(\underline{a},\theta))(m(\underline{a},\theta)-\rho_1(\underline{a},\theta))\Big]\\
&\quad\geq \hat{a}-\underline{a}-E\Big[q^*(\hat{a},\theta)\overline{p}+(1-q^*(\hat{a},\theta))\rho_1(\hat{a},\theta)\Big]+E\Big[q^*(\underline{a},\theta)\overline{p}+(1-q^*(\underline{a},\theta))\rho_1(\underline{a},\theta)\Big]\\
&\quad=\hat{a}-\underline{a}-\Delta_1(\hat{a},\underline{a}).
\end{aligned}$$

This is the left-hand side of (DE-FP S).
Sufficiency:
To show the sufficiency part, define $b(a,\theta)$ as follows:

$$\begin{aligned}
b(\hat{a},\theta)&=\hat{a}-\underline{a}-(1-q^*(\hat{a},\theta))(m(\hat{a},\theta)-\rho_1(\hat{a},\theta))-E[\sigma_1^+(\hat{a},\theta)]\\
b(a,\theta)&=-(1-q^*(a,\theta))(m(a,\theta)-+\rho_1(a,\theta))-E[\sigma_1^+(a,\theta)]\quad\text{for all } a\neq\hat{a}
\end{aligned}$$

Then the incentive compatibility constraints (S10) are satisfied:

$$\begin{aligned}
&E[b(\hat{a},\theta)]+E[q^*(\hat{a},\theta)\overline{p}]+E[(1-q^*(\hat{a},\theta))m(\hat{a},\theta)]\\
&\quad-E[b(a,\theta)]-E[q^*(a,\theta)\overline{p}]-E[(1-q^*(a,\theta))m(a,\theta)]\\
&\quad=\hat{a}-\underline{a}\geq\hat{a}-a\quad\text{for all } a\neq\hat{a}
\end{aligned}$$

We next show the following:

$$\begin{aligned}
&\max_{a,\theta}\{b(a,\theta)+(1-q^*(a,\theta))(m(a,\theta)-\rho_1(a,\theta))\}=\hat{a}-\underline{a}-E[\sigma_1^+(\hat{a},\theta)]\\
&\min_{a,\theta}\{b(a,\theta)+(1-q^*(a,\theta))(m(a,\theta)-\rho_1(a,\theta))\}=-E[\sigma_1^+(\underline{a},\theta)]
\end{aligned}$$

First, for $a=\hat{a}$,

$$b(\hat{a},\theta)+(1-q^*(\hat{a},\theta)(m(\hat{a},\theta)-\rho(\hat{a},\theta))=\hat{a}-\underline{a}-E[\sigma_1^+(\hat{a},\theta)]$$

holds for all θ. Second, for $a \neq \hat{a}$,

$$b(a,\theta) + (1 - q^*(a,\theta))(m(a,\theta) - \rho_1(a,\theta)) = -E[\sigma_1^+(a,\theta)] \geq -E[\sigma_1^+(\underline{a},\theta)]$$

holds for all θ since $\underline{a}$ is the minimum level of investment and $\sigma_1^+(a,\theta)$ is decreasing in a. Finally, $\hat{a} - \underline{a} - E[\sigma_1^+(\hat{a},\theta)] + E[\sigma_1^+(\underline{a},\theta)] > 0$ is satisfied. Therefore,

$$\begin{aligned}
&\max_{a,\theta}\{b(a,\theta) + (1 - q^*(a,\theta))(m(a,\theta) - \rho_1(a,\theta))\} - \min_{a,\theta}\{b(a,\theta) + (1 - q^*(a,\theta))(m(a,\theta) - \rho_1(a,\theta))\} \\
&\quad = \hat{a} - \underline{a} - E[\sigma_1^+(\hat{a},\theta) - \sigma_1^+(\underline{a},\theta)] = \hat{a} - \underline{a} - \Delta_1(\hat{a},\underline{a})
\end{aligned}$$

holds, which completes the proof. **Q.E.D.**

Comparison

The following comparative result, which extends Proposition A4, is now immediate.

Proposition S5 Consider the implementation of $\hat{a}$ satisfying $\pi(\hat{a}) > \overline{\pi}$.

(a) Suppose $(a^o - \underline{a}) + \Delta_0(\hat{a}, a^o) - \Delta_1(\hat{a},\underline{a}) < 0$ holds. If $\hat{a}$ can be implemented under repeated transactions without any formal contract, the same action can be implemented under repeated transactions with an appropriate fixed-price contract. And there is a range of parameter values in which $\hat{a}$ can be implemented only if a formal fixed-price contract is written.

(b) Suppose $(a^o - \underline{a}) + \Delta_0(\hat{a}, a^o) - \Delta_1(\hat{a},\underline{a}) > 0$ holds. If $\hat{a}$ can be implemented under repeated transactions with a formal fixed-price contract, the same action can be implemented under repeated transactions without any formal contract. And there is a range of parameter values in which $\hat{a}$ can be implemented only if no formal fixed-price contract is written.

General formal contracts

We extend the analysis of relational contracts to general formal contracts. Consider formal (short-term) contracts $\{p(\eta_b,\eta_s), q(\eta_b,\eta_s)\}$ along with the relational contract that consists of the following promises:

- The buyer pays f to the seller at the beginning of each period.
- The seller chooses $\hat{a}$ and both the buyer and the seller report truthfully.

- If state θ satisfying $q^*(\eta) = 1$ realizes and the formal contract specifies trade $q(\eta, \eta) = 1$, then the seller delivers the product to the buyer, and the buyer pays $b(\eta)$ in addition to $p(\eta, \eta)$.
- If state θ satisfying $q^*(\eta) = 1$ realizes and the formal contract specifies no trade $q(\eta, \eta) = 0$, then the buyer and the seller cancel the formal contract to trade, and the buyer pays $b(\eta)$.
- If state θ satisfying $q^*(\eta) = 0$ realizes and the formal contract specifies trade $q(\eta, \eta) = 1$, then they cancel the formal contract to no trade, and the buyer pays $b(\eta)$.
- If state θ satisfying $q^*(\eta) = 0$ realizes and the formal contract specifies no trade $q(\eta, \eta) = 0$, then the seller sells the product in an outside opportunity, and the buyer pays $b(\eta)$.
- If either party breaches delivery or payment, they follow the generalized Nash bargaining solution, and from the next period on, they terminate the relationship.

The ex post payoffs to the buyer and the seller in state η when both report truthfully are respectively given as follows:

$$\begin{aligned} u_B(\eta) &= q^*(\eta)v(\eta) - b(\eta) - p(\eta, \eta)Q(\eta) \\ u_S(\eta) &= (1 - q^*(\eta))m(\eta) + b(\eta) + p(\eta, \eta)Q(\eta) \end{aligned}$$

where $Q(\eta) = q^*(\eta)q(\eta, \eta) + (1 - q^*(\eta))(1 - q(\eta, \eta))$. Note that $u_B(\eta) + u_S(\eta) = q^*(\eta)v(\eta) + (1 - q^*(\eta))m(\eta)$ holds for all η.

If the parties follow the informal promises, the seller's incentive compatibility constraints are given as follows.

$$\begin{aligned} &E\Big[(1 - q^*(\hat{a}, \theta))m(\hat{a}, \theta) + b(\hat{a}, \theta) + p((\hat{a}, \theta), (\hat{a}, \theta))Q(\hat{a}, \theta)\Big] - \hat{a} \\ &\quad \geq E\Big[(1 - q^*(a, \theta))m(a, \theta) + b(a, \theta) + p((a, \theta), (a, \theta))Q(a, \theta)\Big] - a \quad \text{for all } a \end{aligned} \tag{S14}$$

The buyer's payoff from deviating in state $\hat{\eta}$ by reporting η is written as

$$\begin{aligned} u_B(\eta, \hat{\eta} \mid \hat{\eta}) = {} & q^*(\hat{\eta})v(\hat{\eta}) + (1 - q^*(\hat{\eta}))q(\eta, \hat{\eta})m(\hat{\eta}) - p(\eta, \hat{\eta}) \\ & - (1 - q(\eta, \hat{\eta}))\rho_0^+(\hat{\eta}) - q(\eta, \hat{\eta})\sigma_1^+(\hat{\eta}), \end{aligned}$$

For example, suppose $\hat{\eta}$ satisfying $q^*(\hat{\eta}) = 1$ realizes and the buyer deviates by reporting η such as $q(\eta, \hat{\eta}) = 1$ and not paying $b(\hat{\eta})$. Her payoff is then $v(\hat{\eta}) - p(\eta, \hat{\eta})$. Note there is no negotiation after reneging in this case. As another case, suppose $\hat{\eta}$ satisfying $q^*(\hat{\eta}) = 0$

realizes and the buyer reports η such as $q(\eta, \hat{\eta}) = 1$ and does not pay $b(\hat{\eta})$ but negotiate to obtain $v(\hat{\eta}) - p(\eta, \hat{\eta}) + (1-\alpha)(m(\hat{\eta}) - v(\hat{\eta}))$. Her payoff is thus $m(\hat{\eta}) - p(\eta, \hat{\eta}) - \alpha(m(\hat{\eta}) - v(\hat{\eta})) = m(\hat{\eta}) - p(\eta, \hat{\eta}) - \sigma_1^+(\hat{\eta})$. One can check the other two cases similarly to obtain the buyer's payoff as above. The buyer's reneging temptation is thus $\max_{\eta,\eta'}[u_B(\eta', \eta \mid \eta) - u_B(\eta)]$.

Similarly, the seller's payoff from deviating in state η by reporting $\hat{\eta}$ is written as

$$\begin{aligned} u_S(\eta, \hat{\eta} \mid \eta) = (1 - q^*(\eta))(1 - q(\eta, \hat{\eta}))m(\eta) + p(\eta, \hat{\eta}) \\ + (1 - q(\eta, \hat{\eta}))\rho^+(\eta) + q(\eta, \hat{\eta})\sigma_1^+(\eta) \end{aligned}$$

The seller's reneging temptation is hence $-\min_{\eta,\eta'}[u_S(\eta', \eta \mid \eta') - u_S(\eta')]$.

The sum of these reneging temptations are rewritten as follows:

$$\begin{aligned} &\max_{\eta,\eta'}\Big[u_B(\eta', \eta \mid \eta) - u_B(\eta)\Big] - \min_{\eta,\eta'}\Big[u_S(\eta', \eta \mid \eta') - u_S(\eta')\Big] \\ &\quad \geq \max_{\theta}\Big[u_B((a,\theta),(\hat{a},\theta) \mid (\hat{a},\theta)) - u_B(\hat{a},\theta)\Big] - \min_{\theta}\Big[u_S((a,\theta),(\hat{a},\theta) \mid (a,\theta)) - u_S(a,\theta)\Big] \\ &\quad \geq E\big[u_B((a,\theta),(\hat{a},\theta)) \mid (\hat{a},\theta)) - u_B(\hat{a},\theta)\Big] - E\Big[u_S((a,\theta),(\hat{a},\theta) \mid (a,\theta)) - u_S(a,\theta)\Big] \\ &\quad \geq \hat{a} - a - E\Big[(1 - q((a,\theta),(\hat{a},\theta)))(\rho^+(\hat{a},\theta) - \rho^+(a,\theta)) + q((a,\theta),(\hat{a},\theta))(\sigma_1^+(\hat{a},\theta) - \sigma_1^+(a,\theta))\Big] \\ &\quad = \hat{a} - a - E\Big[(\rho_0^+(\hat{a},\theta) - \rho_0^+(a,\theta)) - q((a,\theta),(\hat{a},\theta))(\rho_0(\hat{a},\theta) - \rho_0(a,\theta))\Big] \end{aligned}$$

Now consider the implementation of action $\hat{a} > a^o$ and suppose $\rho_0(a,\theta)$ is increasing in a for all θ. By setting $a = a^o$, we obtain

$$\begin{aligned} &\max_{\eta,\eta'}[u_B(\eta', \eta) - u_B(\eta)] - \min_{\eta,\eta'}[u_S(\eta', \eta) - u_S(\eta)] \\ &\quad \geq \hat{a} - a^o - \Delta_0(\hat{a}, a^o) \end{aligned}$$

The right-hand side is attained by no contract. Hence the reneging temptation is minimized by not writing a formal contract.

Next suppose $\rho_0(a,\theta)$ is decreasing in a for all θ. By setting $a = \underline{a}$, we obtain

$$\begin{aligned} &\max_{\eta,\eta'}[u_B(\eta', \eta) - u_B(\eta)] - \min_{\eta,\eta'}[u_S(\eta', \eta) - u_S(\eta)] \\ &\quad \geq \hat{a} - \underline{a} - \Delta_1(\hat{a}, \underline{a}) \end{aligned}$$

The right-hand side is attained by a fixed-price contract. Hence the reneging temptation is minimized by writing a fixed-price contract. We have hence shown the following result.

Proposition S6 Suppose $\rho_0(a,\theta) = m(a,\theta) + \alpha[v(a,\theta) - m(a,\theta)]$ is either weakly increasing in a for all θ, or weakly decreasing in a for all θ. Then it is without loss of generality to confine attention to no contract or fixed-price contracts under repeated transactions.