Mechanisms with Ex-post Reports

An online appendix to "Becoming the Neighbor Bidder: Endogenous Winner's Curse in Dynamic Mechanisms" (2014)

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March, 2014

In these notes, I consider mechanisms á la Mezzetti (2004). Allocation and payments in each period are determined on different rounds of reporting, rather than being determined simultaneously. More precisely, first-round type reports determine the allocation, while second-round ex-post-utility reports, the payments. Such mechanisms restore efficiency in environments where standard, single-report mechanisms generically fail, lifting the winner's curse and eliminating the dynamic equilibrium externality.

Start at the second period. There is no efficiency conflict at history $h^{(0)}$, and therefore no need to resort to two-stage reporting. The same applies to history h^D, if double sourcing is allowed. Therefore, consider history $h^{(i)}$. Let agent i's type be the pair (v, w_{i2}), while agent $-i$'s type is w_{-i2}. The first-best allocation rule disregards v and allocates the second unit to i if $w_{i2} > w_{-i2}$, and to $-i$ if $w_{i2} < w_{-i2}$.

In Mezzetti's *generalized Groves mechanism*, the second-period winner reports her ex-post utility, and this report determines the payment due to the loser up to a function that depends on the opponent's type report. Let (v', w'_{i2}, w'_{-i2}) be the profile of type reports. If the second-period unit goes to i — that is to say, if $w'_{i2} > w'_{-i2}$ —, this agent is asked to report her ex-post utility, while $-i$ need not send any further messages (her utility from the allocation is known to be 0). If i reports payoff $\widetilde{u}_{i2}$, second-period ex-post payments, denoted by $\tau^*_{i2}, \tau^*_{-i2}$, are given by:

$$\begin{aligned}
\tau^*_{i2}(\widetilde{u}_{i2}; v', w'_{i2}, w'_{-i2}) &= \gamma_i(w'_{-i2}),\\
\tau^*_{-i2}(0; \widetilde{u}_{i2}, v', w'_{i2}, w'_{-i2}) &= -\widetilde{u}_{i2} + \gamma_{-i}(v', w'_{i2}).
\end{aligned}$$

If the second-period unit goes to agent $-i$, and she reports $\widetilde{u}_{-i2}$, we have:

$$\begin{aligned}
\tau^*_{i2}(0; \widetilde{u}_{-i2}, v', w'_{i2}, w'_{-i2}) &= -\widetilde{u}_{-i2} + \gamma_i(w'_{-i2}),\\
\tau^*_{-i2}(\widetilde{u}_{-i2}; v', w'_{i2}, w'_{-i2}) &= \gamma_{-i}(v', w'_{i2}).
\end{aligned}$$

In either case, the utility report of an agent does not affect her own payoff, only that of her opponent. Thus, there are no incentives to misreport.

The second-period allocation is determined in the first round of reports of the second period. Assume that types are v, w_{i2}, w_{-i2}. If agent i reports truthfully, agent $-i$'s ex-post payoff from reporting w'_{-i2} is:

$$\begin{aligned}
&s_{-i2}(w'_{-i2}; v, w_{i2}, w_{-i2}) :=\\
&(v + w_{-i2} - \gamma_{-i}(v, w_{i2}))I(w_{i2} < w'_{-i2}) + (v + w_{i2} - \gamma_{-i}(v, w_{i2}))I(w_{i2} > w'_{-i2})\\
&= v + w_{i2} - \gamma_{-i}(v, w_{i2}) + (w_{-i2} - w_{i2})I(w_{i2} < w'_{-i2}),
\end{aligned}$$

where I denotes the indicator function, taking the value 1 if the statement in the argument is true and 0 otherwise. Therefore, truthful reporting is a best response. Similarly, consider the incentive problem of agent i, assuming that $-i$ reports truthfully. We have:

$$\begin{aligned}
&s_{i2}(v', w'_{i2}; v, w_{i2}, w_{-i2}) :=\\
&(v + w_{i2} - \gamma_i(w_{-i2}))I(w'_{i2} > w_{-i2}) + (v + w_{-i2} - \gamma_i(w_{i2}))I(w'_{i2} < w_{-i2})\\
&= v + w_{-i2} - \gamma_i(w_{-i2}) + (w_{i2} - w_{-i2})I(w'_{i2} > w_{-i2}).
\end{aligned}$$

Again, truthful reporting is a best response. It follows that truthful reporting is an ex-post equilibrium.

Notice that v' does not enter i's payoff function: The allocation rule disregards this report, and payments are determined by the (truthful) second-round reports.

In the first period, there is no need to resort to two stages of reporting in a direct-revelation mechanism: Incentives are given for the neighbor to truthfully report v together with her second-period idiosyncratic-component signal.

In what follows, for the sake of concreteness, consider the case where $\gamma_i(w_{-i2}) = w_{-i2}$ and $\gamma_{-i}(v, w_{i2}) = v + w_{i2}$. In this case, we have:

$$\begin{aligned} s^*_{-i2}(v, w_{i2}, w_{-i2}) &:= \max\{w_{-i2} - w_{i2}, 0\}, \\ s^*_{i2}(v, w_{i2}, w_{-i2}) &:= v + \max\{w_{i2} - w_{-i2}, 0\}. \end{aligned}$$

Continuation payoffs upon winning and losing are, respectively,

$$\begin{aligned} \underline{S}_2 &:= E[s^*_{-i2}(V, W_{i2}, W_{-i2})] = E[\max\{W_{-i2} - W_{i2}, 0\}] = S_2^0, \\ \overline{S}_2 &:= E[s^*_{-i2}(V, W_{i2}, W_{-i2})] = E(V) + E[\max\{w_{i2} - w_{-i2}, 0\}] = E(V) + S_2^0. \end{aligned}$$

The informational value of winning, Δ^S, is equal to $E(V)$. The winner's curse is lifted: The continuation payoff of the non-neighbor is the same as under symmetric information. Therefore, the dynamic equilibrium externality vanishes: The continuation payoff in the event of losing is the same whether or not the opponent has also been excluded.

For this reason, there is no need to resort to deposits and contingent rebaits to implement this mechanism. Implementation details are presented in the next theorem. While they are not needed in a direct mechanism, the suggested auction features ex-post reports in both periods. This way, bids in the auction can be unidimensional.

Theorem 1. *The first-best allocation rule is implemented by the following sequential auction. The first-period auction is a standard second-price auction. Before the second auction, the winner reports the value of the unit to the* *auctioneer. The loser is charged an entry fee for the second-period auction, equal to the difference between the reported value and the winning bid. If this fee is not paid, the second-period auction is cancelled. Otherwise, in the second period, bidders (simultaneously) submit bids, and the highest bidder wins. Each bidder — even the loser — pays her opponent's bid. The winner reports the value of the unit, and the loser gets a bonus equal to the reported value.*

Proof. Since the first period always results in trade, the only type of history that is allowed for is $h^{(a)}, h^{(b)}$. Consider bidder i's problem of type w_{i1}, w_{i2}, if bidder $-i$ of type w_{-i1}, w_{-i2} adopts the following strategy:

- If $-i$ is awarded the second-period unit, report $u_{-i2} = v + w_{-i2}$.
- At history $h^{(-i)}$, regardless of v, bid w_{i2}.
- At history $h^{(i)}$, bid w_{-i2}.
- If $-i$ is awarded the first-period unit, report $u_{-i1} = v + w_{-i1}$.
- If i is awarded the first-period unit, pay the entry fee.
- In the first period, bid $w_{-i1} + (1 + \delta)E(V)$.

As discussed above, the second-period winner has no incentives to misreport the value of the second-period unit. At history $h^{(i)}$, agent i's ex-post payoff from bidding b in the second-period auction is:

$$\begin{aligned} s_{i2}(b; v, w_{i2}, w_{-i2}) :&= (v + w_{i2} - w_{-i2})I(b > w_{-i2}) + (u_{-i2} - w_{-i2})I(b < w_{-i2}) \\ &= v + (w_{i2} - w_{-i2})I(b > w_{-i2}). \end{aligned}$$

Bidding $b = w_{i2}$ is a best response. Next, consider history $h^{(-i)}$. If charged entry fee $e_1 = u_{-i1} - w_{-i1} = v$, agent i's payoff is:

$$\begin{aligned} &s_{i2}(b; e_1, v, w_{i2}, w_{-i2}) \\ &:= -e_1 + (v + w_{i2} - w_{-i2})I(b > w_{-i2}) + (v + w_{-i2} - w_{-i2})I(b < w_{-i2}) \\ &\ = (w_{i2} - w_{-i2})I(b > w_{-i2}); \end{aligned}$$

she is willing to pay the entry fee, and, once again, bidding $b = w_{i2}$ is a best response.

At the end of the first period, if the first-period unit is awarded to agent i, her report determines her opponent's entry fee. There is no gain to her from under-reporting this value, and over-reporting may only discourage $-i$ from entering the second-period auction and thus cause trade to shut down.

At the bidding stage of the first period, the problem is equivalent to one of independent and private values; bidding her valuation, $w_{i1} + (1+\delta)E(V)$, is a weakly-dominant strategy. $\square$

In equilibrium, the difference between the reported value and the winning bid is the same in both periods. The auctioneer could exploit this feature of the equilibrium in the design by imposing penalties for (off-equilibrium) bids and reports that are inconsistent. For the selected γ_i, γ_{-i} functions, there is no need for the auctioneer to do so: Leaving the utility reports unrestricted but requiring both bidders to participate does the trick.

As Mezzetti (2004) notes, the second-period winner is indifferent between reporting truthfully and lying in the second-stage reports. This leaves open the possibility of inefficient equilibria. Of course, the existence of inefficient or otherwise undesirable equilibria is a standard concern in mechanisms designed under partial implementation, and the mechanisms I propose are no exception — especially the mechanism in Theorem C1. However, in the second-best auction of Theorem 2 in my paper, agents have stronger incentives to behave as the desired equilibrium dictates.

To maximize revenues, a seller with full commitment power can hold the efficient second-period auction and capture the surplus created by means of entry fees. The proof of this result is very similar to the proof of Theorem 3 in the paper, so the details are omitted.

Theorem 2. *The following mechanism maximizes expected revenues. The first-period auction is a second-price or English auction with reserve price* $r^{\pi*} := \left(\phi^0\right)^{-1}\left(-E(V) + \delta\Delta^{\omega}\right)$*. After the bids are in and the winner is an-*

nounced, but before allocating the unit, the winner is charged a second-period entry fee of $e^1 := \delta\,(E(V) + S_2^0)$*, and the loser, or both bidders if the first unit goes unsold, of* $e^2 := \delta S_2^0$*. If both bidders pay the corresponding entry fee, the first-period unit is allocated and a second-price or English auction follows. If only one bidder pays the fee, she gets the first-period unit and a rebate of* $e - \delta E(U)$*, where* e *is the fee she paid; she also gets the second-period unit for free. If no bidder pays the fee, the first-period winner gets the first-period unit but the second-period unit is withheld.*

Full commitment power enables the seller to raise the maximum surplus and capture it via entry fees. With limited commitment power, the seller cannot credibly threaten to exclude bidders who don't pay the fees, and distortions to the allocation may be profitable to save on information rents. The general consideration of ex-post reports requires a characterization of incentive compatibility; Mezzetti (2004) focuses on efficiency.

References

Mezzetti, C. (2004). Mechanism design with interdependent valuations: Efficiency. *Econometrica* 72(5):1617–1626.