# Organizing to Adapt and Compete

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## Online Appendix

## Appendix B: Strategic Communication

We divide the proofs for Section V into three appendices. Appendix B1 characterizes communication equilibria and provides the proof of Lemma 1 while Appendix B2 derives expressions for the quality of horizontal and vertical communication and provides the proof of Lemma 2. Appendix B3 uses the previous results to study the relative performance of Centralization, Divisional Centralization, and Decentralization when the own-division bias is vanishingly small and presents the proof of Proposition 3.

### Appendix B1 - Communication Equilibria

A communication equilibrium under each organizational structure is characterized by (i.) communication rules for the division managers, (ii.) decision rules for the decision makers and (iii.) belief functions for the message receivers. The communication rule for Manager $j = 1, 2$ specifies the probability of sending message $m_j \in M_j$ conditional on observing state $\theta_j$ and we denote it by $\mu_j\left(m_j \mid \theta_j\right)$, where the message space is $M_j = [-s, s]$. Under Centralization, the decision rules map messages $m = (m_1, m_2)$ into decisions $q_1 \in \mathbf{R}^+$ and $q_2 \in \mathbf{R}^+$, and we denote them by $q_1^C(m)$ and $q_2^C(m)$. Under Decentralization, the decision rule for Manager 1 maps the state $\theta_1$ and messages $m = (m_1, m_2)$ into decision $q_1 \in \mathbf{R}^+$ while the decision rule for Manager 2 maps the state $\theta_2$ and messages $m = (m_1, m_2)$ into decision $q_1 \in \mathbf{R}^+$, and we denote them by $q_1^D(m, \theta_1)$ and $q_2^D(m, \theta_2)$. Under Divisional Centralization, the decision rules map the state $\theta_1$ and message $m = m_2$ into

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decisions $q_1 \in \mathbf{R}^+$ and $q_2 \in \mathbf{R}^+$, and we denote them by $q_1^{DC}(m,\theta_1)$ and $q_2^{DC}(m,\theta_1)$. Finally, the belief functions are denoted by $g_j(\theta_j \mid m_j)$ for $j = 1,2$ and characterize the receiver's posterior probability of state $\theta_j$ conditional on receiving message $m_j$.

We focus on Perfect Bayesian Equilibria of the communication subgame which require that communication rules are optimal for the division managers given decision rules. Formally, whenever $\mu_j(m_j \mid \theta_j) > 0$ then $m_j \in \arg\max_{m \in M_j} \mathrm{E}\left[\lambda\pi_j^l + (1-\lambda)\pi_k^l \mid \theta_j\right]$ for $l \in \{C, D, DC\}$, where $\pi_j^l$ and $\pi_k^l$ are, respectively, the profits of Divisions $j$ and $k$, $j \neq k$, given that decisions are made according to $q_1^l(\cdot)$ and $q_2^l(\cdot)$. Perfect Bayesian Equilibria also require that the decision rules are optimal for the decision makers given the belief functions. Thus, under Centralization $q_1^C(\cdot)$ and $q_2^C(\cdot)$ solve $\max_{(q_1,q_2)} \mathrm{E}[\pi_1 + \pi_2 \mid m]$, under Decentralization $q_j^D(\cdot)$ solves $\max_{q_j} \mathrm{E}[\lambda\pi_j + (1-\lambda)\pi_k \mid m, \theta_j]$, and under Divisional Centralization $q_1^{DC}(\cdot)$ and $q_2^{DC}(\cdot)$ solve $\max_{(q_1,q_2)} \mathrm{E}[\lambda\pi_1 + (1-\lambda)\pi_2 \mid m, \theta_1]$ . Finally, Perfect Bayesian Equilibria require that the belief functions are derived from the communication rules using Bayes' rule whenever possible, that is, $g_j(\theta_j \mid m) = \mu_j(m_j \mid \theta_j) / \int_P \mu_j(m_j \mid \theta_j) \mathrm{d}\theta_j$, where $P = \left\{\theta_j : \mu_j(m_j \mid \theta_j) > 0\right\}$, $j = 1,2$.

PROPOSITION B1. *For* $\lambda \in (1/2, 1]$ and $t \neq 0$ *there exists an integer* $\bar{N}(\lambda,t)$, *such that for all* $N \leq \bar{N}(\lambda,t)$ *there exists at least one equilibrium* $(\mu_1(\cdot)\ \mu_2(\cdot)\ , q_1(\cdot), q_2(\cdot), g_1(\cdot), g_2(\cdot))$, *where*

*a.* $\mu_j(m_j \mid \theta_j)$ *is uniform, supported on* $[a_{j,i-1}, a_{j,i}]$ *if* $\theta_j \in (a_{j,i-1}, a_{j,i})$,

*b.* $g_j(\theta_j \mid m_j)$ *is uniform supported on* $[a_{j,i-1}, a_{j,i}]$ *if* $m_j \in (a_{j,i-1}, a_{j,i})$,

*c.* $a_{j,i+1} - a_{j,i} = a_{j,i} - a_{j,i-1} + 4b^s(a_{j,i})$ *for* $i = 1, ..., N_j - 1$

*with* $b^s = b^C$ *under Centralization as given by (17),*
$b^s = b^D$ *under Decentralization as given by (18),*
$b^s = b^{DC}$ *under Divisional Centralization as given by (19).*

*d.* $q_j(m) = q_j^C$, $j = 1,2$, *under Centralization, where* $q_j^C$ *are given by (23) and (24), and*
$q_j(m,\theta_1) = q_j^{DC}$, $j = 1,2$, *under Divisional Centralization, with* $q_j^D$ *as in (25) and (26), and*
$q_j(m,\theta_j) = q_j^D$, $j = 1,2$, *under Decentralization, where* $q_j^D$ *are given by (27) and (28).*

**Proof:** We first show that communication equilibria are interval equilibria. For the case of Centralization let $\mu_2(\cdot)$ be any communication rule for Manager 2. The expected utility of Manager 1 if the headquarter manager holds a posterior expectation $\nu_1$ over $\theta_1$ is given by

$$\mathrm{E}_{\theta_2}[U_1 \mid \theta_1, \nu_1] = \mathrm{E}_{\theta_2}\left[\lambda\pi_1\left(\widehat{q}_1^C, \widehat{q}_2^C, \theta_1\right) + (1-\lambda)\pi_2\left(\widehat{q}_1^C, \widehat{q}_2^C\right)\right], \tag{22}$$

with

$$\hat{q}_1^C \equiv \frac{b}{2\left(1-t^2\right)}(\mu - c + \nu_1 - t\mathrm{E}\left[\mu - c + \theta_2 \left|\mu_2(\cdot)\right.\right]), \text{ and} \tag{23}$$

$$\hat{q}_2^C \equiv \frac{b}{2\left(1-t^2\right)}(\mathrm{E}\left[\mu - c + \theta_2 \left|\mu_2(\cdot)\right.\right] - t\left(\mu - c + \nu_1\right)). \tag{24}$$

It can be shown that $\frac{\partial^2}{\partial\theta_1\partial\nu_1}\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1, \nu_1\right] > 0$ and $\frac{\partial^2}{\partial^2\theta_1}\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1, \nu_1\right] < 0$. This implies that for any two different posterior expectations of the headquarter manager, say $\underline{\nu}_1 < \overline{\nu}_1$, there is at most one type of Manager 1 that is indifferent between both. Now suppose that contrary to the assertion of interval equilibria there are two states $\theta_1^1 < \theta_1^2$ such that $\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^1, \overline{\nu}_1\right] \geq \mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^1, \underline{\nu}_1\right]$ and $\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^2, \underline{\nu}_1\right] > \mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^2, \overline{\nu}_1\right]$. But then $\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^2, \overline{\nu}_1\right] - \mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^2, \underline{\nu}_1\right] < \mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^1, \overline{\nu}_1\right] - \mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1^1, \underline{\nu}_1\right]$ which violates $\frac{\partial^2}{\partial\theta_1\partial\nu_1}\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1, \nu_1\right] > 0$ . The same argument can be applied to Manager 2 for any reporting strategy $\mu_1(\cdot)$ of Manager 1. Therefore all equilibria of the communication game under Centralization must be interval equilibria.

Now consider the case of Divisional Centralization. The expected utility of Manager 2 if Manager 1 holds a posterior expectation $\nu_2$ over $\theta_2$ is given by

$$\mathrm{E}_{\theta_1}\left[U_2 \mid \theta_2, \nu_2\right] = \mathrm{E}_{\theta_1}\left[(1-\lambda)\,\pi_1\left(\widehat{q}_1^{DC}, \widehat{q}_2^{DC}\right) + \lambda\pi_2\left(\widehat{q}_1^{DC}, \widehat{q}_2^{DC}, \theta_2\right)\right],$$

with

$$\hat{q}_1^{DC} \equiv \frac{b}{2(1-\frac{\lambda\tau^2}{1-\lambda})}\left(\mu - c + \theta_1 - \tau\left(\mu - c + \nu_2\right)\right), \text{ and} \tag{25}$$

$$\hat{q}_2^{DC} \equiv \frac{b}{2(1-\frac{\lambda\tau^2}{1-\lambda})}\left(\left(\mu - c + \nu_2\right) - \frac{\lambda}{1-\lambda}\tau\left(\mu - c + \theta_1\right)\right). \tag{26}$$

Again, it can be shown that $\frac{\partial^2}{\partial\theta_1\partial\nu_1}\mathrm{E}_{\theta_2}\left[U_2 \mid \theta_2, \nu_2\right] > 0$ and $\frac{\partial^2}{\partial^2\theta_1}\mathrm{E}_{\theta_1}\left[U_2 \mid \theta_2, \nu_2\right] < 0$. By the same argument used previously for Centralization, we can then conclude that all equilibria in this case are, again, interval equilibria.

For the case of Decentralization let $\mu_1(\cdot)$ and $\mu_2(\cdot)$ be communication rules of Manager 1 and Manager 2, respectively. Sequential rationality implies that, in equilibrium, decision rules must conform to

$$\hat{q}_1^D = \frac{b}{2}\left(\mu - c + \theta_1\right) - \tau\left(\mathrm{E}\left[q_2 \left|\nu_1, m_2\right.\right]\right), \text{ and} \tag{27}$$

$$\hat{q}_2^D = \frac{b}{2}\left(\mu - c + \theta_2\right) - \tau\left(\mathrm{E}\left[q_1 \left|\nu_1, m_2\right.\right]\right), \tag{28}$$

where $\nu_1$ denotes Manager 2's posterior expectation over $\theta_1$. It can readily be seen that

$$\frac{\partial^2}{\partial\theta_1\partial\nu_1}\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1, \nu_1\right] > 0$$

and the proof follows as in the preceding paragraph.

We now characterize all equilibria of the communication game. For Manager $j = 1, 2$, let $a_j$ be a partition of $[-s, s]$, any message $m_j \in (a_{j,i-1}, a_{j,i})$ be denoted by $m_{j,i}$, and $\overline{m}_{j,i}$ be the receiver's posterior belief of the expected value of $\theta_j$ after receiving message $m_{j,i}$.

*a. Centralization:* The expected utility of Manager 1 in state $a_{1,i}$ is given by

$$\begin{aligned} \mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i}\right] &= \frac{b}{4\left(1-t^2\right)}[(\mu-c)^2(1-\lambda)-\lambda\left(\overline{m}_{1,i}\right)^2 + \\ &\quad \overline{m}_{1,i}(\mu-c)\left(2\lambda\left(\mu-c+a_{1,i}\right)+(2\lambda-1)t\right)-2t\lambda(\mu-c)\left(\mu-c+a_{1,i}\right)]. \end{aligned}$$

In state $a_{1,i}$ Manager 1 must be indifferent between sending a message that induces a posterior $\overline{m}_{1,i}$ and a posterior $\overline{m}_{1,i+1}$ implying that $\mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i}\right] - \mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i+1}\right] = 0$. Given decision rules (23) and (24) , and letting $\overline{m}_{1,i} = \left(a_{1,i-1} + a_{1,i}\right)/2$ we have that $\mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i}\right] - \mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i+1}\right] = 0$ if and only if $a_{1,i} = \left(a_{1,i-1} + a_{1,i+1}\right)/\left(2 + 4b^C\right)$ where $b^C$ is given by (17). That is, communication equilibria under Centralization are equivalent to the constant-bias leading example in Crawford and Sobel (1982).

From (17), we have that $sign(b^C) = sign(t - \tau) = sign(t)$.

*b. Divisional Centralization:* Let $k = \lambda/(1-\lambda)$. The expected utility of Manager 2 in state $a_{2,i}$ is given by

$$\begin{aligned} \mathrm{E}_{\theta_1}\left[U_2 \mid a_{2,i}, \overline{m}_{2,i}\right] &= \frac{b(1-\lambda)}{4\left(1-k\tau^2\right)^2}[\left(2k(k-1)\tau^2+1-k^3\tau^2\right)\mathrm{E}_{\theta_1}\left[\left(\mu-c+\theta_1\right)^2\right] \\ &\quad ... + \left(\overline{m}_{2,i}\right)^2\left(2k\tau^2-k-\tau^2\right)+2k\overline{m}_{2,i}\left(\mu-c+\theta_2\right)\left(1-k\tau^2\right) + \\ &\quad ... + 2k(k-1)\tau\left(1-\tau^2\right)(\mu-c)\overline{m}_{2,i}-2k^2\tau(\mu-c)\left(\mu-c+\theta_2\right)\left(1-k\tau^2\right). \end{aligned}$$

In state $a_{2,i}$ Manager 1 must be indifferent between sending a message that induces a posterior $\overline{m}_{2,i}$ and a posterior $\overline{m}_{2,i+1}$ implying that $\mathrm{E}_{\theta_1}\left[U_2 \mid a_{2,i}, \overline{m}_{2,i}\right] - \mathrm{E}_{\theta_1}\left[U_2 \mid a_{2,i}, \overline{m}_{2,i+1}\right] = 0$. Given decision rules (25) and (26) , and letting $\overline{m}_{2,i} = \left(a_{2,i-1} + a_{2,i}\right)/2$ we have that $\mathrm{E}_{\theta_1}\left[U_2 \mid a_{2,i}, \overline{m}_{2,i}\right] -_{\theta_1} \left[U_2 \mid a_{2,i}, \overline{m}_{2,i+1}\right] = 0$ if and only if $a_{1,i} = \left(a_{1,i-1} + a_{1,i+1}\right)/\left(2 + 4b^{DC}\right)$ where $b^{DC}$ is given by (19).

We now show that $sign(b^{DC}) = sign(t)$. Rewrite $b^{DC}$ as $b^{DC}(\theta_2) = b_1^{DC}\theta_2 + b_2^{DC}$, where

$$\begin{aligned} b_1^{DC} &= \frac{(2\lambda-1)^2 t^2}{(1-\lambda)\left(-4\lambda^3+3t^2\lambda-t^2\right)}, \\ b_2^{DC} &= \frac{(2\lambda-1)\left(-2t+4t\lambda-4\lambda^2+t^2\right)}{2(1-\lambda)\left(-4\lambda^3+3t^2\lambda-t^2\right)}t(\mu-c). \end{aligned}$$

For all $\lambda \in [1/2, 1]$ and $t \in [-1, 1]$, we have that $\left(-4\lambda^3 + 3t^2\lambda - t^2\right) < 0$ and $\left(-2t + 4t\lambda - 4\lambda^2 + t^2\right) < 0$. This implies that $b_1^{DC} < 0$, and $sign(b_2^{DC}) = sign(t)$. That is, the communication bias increases (in absolute value) with the state if and only if $t < 0$. Therefore, to show that $sign(b^{DC}) = sign(t)$ we need only show that

$$
\begin{aligned}
b_1^{DC} s + b_2^{DC} &> 0 \text{ if } t > 0, \qquad (29)\\
-b_1^{DC} s + b_2^{DC} &< 0 \text{ if } t < 0. \qquad (30)
\end{aligned}
$$

The inequality (29) is equivalent to

$$\frac{(\mu - c)}{s} > \frac{2t\,(2\lambda - 1)}{\left(2t - 4t\lambda + 4\lambda^2 - t^2\right)} > 0.$$

The maximum of the intermediate term above for given $t > 0$ satisfies

$$\max_{\lambda > 1/2} \frac{2t\,(2\lambda - 1)}{\left(2t - 4t\lambda + 4\lambda^2 - t^2\right)} = \frac{t}{(1-t) + \sqrt{1 - t^2}} < \frac{1+t}{1-t} < \frac{(\mu - c)}{s},$$

where the last inequality follows from parameter restrictions that ensures positive quantities. This proves that (29) is satisfied. Finally, the inequality (30) is equivalent to

$$\frac{(\mu - c)}{s} > \frac{-2t\,(2\lambda - 1)}{\left(2t - 4t\lambda + 4\lambda^2 - t^2\right)} > 0.$$

By a similar reasoning as before we have that for given $t < 0$ satisfies

$$\max_{\lambda > 1/2} \frac{-2t\,(2\lambda - 1)}{\left(2t - 4t\lambda + 4\lambda^2 - t^2\right)} = \frac{-t}{(1-t) + \sqrt{1 - t^2}} < 1.$$

For $t < 0$ positive quantities is ensured as long as $\mu - c > s$. This proves that (30) is satisfied.

*c. Decentralization:* If Manager 1 observes state $\theta_1$ and sends message $m_{1,i}$ that induces a posterior belief $\overline{m}_{1,i}$ in Manager 2 his expected utility is given by

$$\mathrm{E}_{\theta_2}\left[U_1 \mid \theta_1, \overline{m}_{1,i}\right] = -\mathrm{E}_{\theta_2}\left[\lambda \pi_1\left(\widehat{q}_1^D, \widehat{q}_2^D, \theta_1\right) + (1-\lambda)\,\pi_1\left(\widehat{q}_1^D, \widehat{q}_2^D\right) \mid \theta_1, \overline{m}_{1,i}\right], \qquad (31)$$

where $q_1^D$ and $q_2^D$ are given by (27) and (28). In state $\theta_1 = a_{1,i}$ this can be written as $\mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i}\right]$ being equal to

$$
\begin{aligned}
&\frac{b}{4\left(1-t^2\right)}((1-\lambda)\left(1-\tau^2\right)^2 \mathrm{E}_{\theta_2}\left[(\mu - c + \theta_2)^2\right] + \tau^2\left(\lambda - \tau^2 + \lambda\tau^2\right)\mathrm{E}_{\theta_2}\left[m_2^2 | \mu_2(\cdot)\right]\\
&... + \left(\lambda + \lambda\tau^2 - 1\right)\tau^2\left(\overline{m}_{1,i}\right)^2 + \lambda\left(1 - \tau^2\right)^2\left(\mu - c + a_{1,i}\right)^2\\
&... + 2\lambda\tau^2\left(1 - \tau^2\right)\overline{m}_{1,i}\left(\mu - c + a_{1,i}\right) - 2\left(2\lambda - 1\right)\tau^3\overline{m}_{1,i}\mathrm{E}_{\theta_2}\left[m_2 | \mu_2(\cdot)\right]\\
&... - 2\lambda\tau\left(1-\tau^2\right)\mathrm{E}_{\theta_2}\left[(\mu - c + \theta_2)\right]\left(\mu - c + a_{1,i}\right)).
\end{aligned}
$$

In state $a_{1,i}$ Manager 1 must be indifferent between sending a message that induces a posterior $\overline{m}_{1,i}$ and a posterior $\overline{m}_{1,i+1}$ implying that $\mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i}\right] - \mathrm{E}_{\theta_2}\left[U_1 \mid a_{1,i}, \overline{m}_{1,i+1}\right] = 0$. If $(1 -$

$\lambda)/\lambda < \tau^2$ the above condition has no solution: essentially Manager 1 would like to induce the highest possible belief in Manager 2. If, however, $(1-\lambda)/\lambda > \tau^2$ then, given sequentially rational decision making (27) and (28) and letting $\overline{m}_{1,i} = (a_{1,i-1} + a_{1,i})/2$, we have that $\mathrm{E}_{\theta_2}[U_1 \mid a_{1,i}, \overline{m}_{1,i}] - \mathrm{E}_{\theta_2}[U_1 \mid a_{1,i}, \overline{m}_{1,i+1}] = 0$ if and only if $a_{1,i} = (a_{1,i-1} + a_{1,i+1}) / \left(2 + 4b^D(a_{1,i})\right)$, where $b^D(\theta_1)$ is given by (18).

We now show that $b^D(\theta_1)$ is always positive and increasing. The condition $(1-\lambda)/\lambda > \tau^2$ implies that the numerator of (18) is positive and thus $b^D(\theta_1)$ is increasing. Moreover, if $t < 0$, $(1-\tau)(\mu - c) > (\mu - c) > s$ where the last inequality follows from considering parameter values that induce positive quantities. Therefore $b^D(-s) > 0$. If $t > 0$ then the parameter restriction $(\mu - c)/s > (1+t)/(1-t)$ that guarantees positive quantities, and the fact that for $t > 0$ $(1+t)/(1-t) > 1/(1-t) \geq 1/(1-\tau)$, together imply that $b^D(-s) > 0$. This establishes that $b^D(\theta_1) > 0$ for all $\theta_1 \in [-s, s]$.

In summary, given the independence of Manager 1 and 2's private information, the multi-sender communication equilibrium decouples into two communication equilibria each of which is equivalent to a sender-receiver game in which the state-dependent bias of the sender satisfies (18). In particular, since the communication bias $b^D(\theta_1)$ is strictly positive, communication necessarily involves a finite number of intervals as shown in Crawford and Sobel (1982).

**Proof of Lemma 1:** Proposition B1 derives the expressions and properties for the communication bias under both Centralization, Divisional Centralization and Decentralization. ■

## Appendix B2 - Residual Variance of Communication

In this appendix we first derive closed form expressions for the residual variance under Centralization, Divisional Centralization, and Decentralization. We then prove that these residual variances possess some smoothness properties that enables us to characterize their behavior for $\lambda$ close to 1/2. We conclude by comparing the informativeness of vertical and horizontal communication.

### Vertical Communication

Under Centralization the communication bias is constant, as in the leading example in Crawford and Sobel (1982). Thus, for a given equilibrium with $n$ intervals the residual variance of communication $V_n^C$ satisfies

$$V_n^C = \frac{s^2}{3}\left(\frac{1}{n^2}\right) + \frac{1}{3}\left(b^C\right)^2\left(n^2 - 1\right). \tag{32}$$

The maximum number of intervals $N(b^C)$ satisfies $2N(b^C)(N(b^C)-1)|b^C| \leq 2s$ and is thus given by

$$N(b^C) = int\left(\frac{1}{2}\left(1+\sqrt{1+\frac{4s}{|b^C|}}\right)\right),$$

where $int(z)$ is the largest integer that does not exceed $z$. Therefore if $\left|b^C\right| < \frac{s}{2}$ the residual variance the communication under Centralization is

$$V^C = V^C_{N(b^C)} = \frac{s^2}{3}(\frac{1}{N(b^C)^2}) + \frac{1}{3}\left(b^C\right)^2\left(N(b^C)^2-1\right),$$

and $V^C = \frac{s^2}{3}$ if $\left|b^C\right| > \frac{s}{2}$.

**Horizontal Communication**

Under both Divisional Centralization and Decentralization the communication bias takes the form

$$b^s\left(\theta_i\right) = b_1^s\theta_i + b_2^s,$$

for $s \in \{DC, D\}$ with

$$b_1^D = (2\lambda-1)/\left(1-\lambda-\lambda\tau^2\right), \qquad b_2^D = (1-\tau)(\mu-c),$$
$$b_1^{DC} = (2\lambda-1)^2 t^2/(1-\lambda)\left(3t^2\lambda-4\lambda^3-t^2\right), \quad b_2^{DC} = \left(1-\lambda\frac{1-\tau^2}{t-\tau}\right)(\mu-c).$$

From Proposition B1, $b_1^D, b_2^D > 0$ while $b_1^{DC} < 0$ and $sign(b_2^{DC}) = sign(t)$. We now derive the residual variance for arbitrary $b_1, b_2 > 0$, to obtain (38) below. We will then consider the rate of change of both residual variances as the conflict vanishes.

Consider communication by Manager 1 and let $a_1 = \{a_{1,i}\}_{i=0}^{i=n}$ be a partition of $[-s,s]$ into $n$ intervals. If $a_1$ characterizes a communication equilibrium by Manager 1 to Manager 2 then it must satisfy the arbitrage condition

$$a_{1,i+1} - a_{1,i} = a_{1,i} - a_{1,i-1} + 4(b_1a_{1,i} + b_2), \tag{33}$$

with boundary conditions $a_{1,0} = -s$ and $a_{1,n} = s$. Solving this second order linear difference equation we obtain

$$a_{1,i} = Ax^i + B\frac{1}{x^i} - \frac{b_2}{b_1}, \tag{34}$$

where $x = 1 + 2b_1 + 2\sqrt{b_1\left(b_1+1\right)} > 1$ is the solution of the characteristic equation associated with (33) that exceeds 1 . Defining $r$ and $D$ as $r = x^n$ and $D = -b_2/sb_1$, the coefficients $A$ and $B$ are

$$\begin{aligned} A &= \left(-\frac{1}{1-r} - \frac{D}{1+r}\right)s, \text{ and} \\ B &= \left(\frac{1}{1-r} - \frac{D}{1+r}\right)sr, \end{aligned}$$

and the size of each interval is given by

$$a_{1,i} - a_{1,i-1} = \frac{(x-1)}{x^i}\left(Ax^{2i-1} - B\right). \tag{35}$$

**Maximum Number of Intervals.** Let $N(b_1, b_2)$ be the maximum number of intervals in a communication equilibrium. The solution to the second order difference equation characterizes a communication equilibrium as long as the solution (34) is monotonic, i.e. $a_{1,i} - a_{1,i-1} > 0$ is positive. Since $b_1 > 0$ and $b_2 > 0$, we have $A > 0$. Therefore it suffices that $Ax > B$ to guarantee that $Ax^{2i-1} > B$ for all $i$, $1 \le i \le N(b_1, b_2)$. From the definition of $A$ and $B$ we thus require

$$-r(D-1)(x+1) + r^2(D+1) + x(D+1) > 0.$$

The solution to this quadratic inequality is

$$r = x^n \le \frac{(D-1)}{2(D+1)}(x+1) + \frac{1}{2}\sqrt{\frac{(D-1)^2}{(D+1)^2}(x+1)^2 - 4x}. \tag{36}$$

It follows that $N(b_1, b_2)$ is given by

$$N(b_1, b_2) = int\left(\ln\left(\frac{(D-1)}{2(D+1)}(x+1) + \frac{1}{2}\sqrt{\frac{(D-1)^2}{(D+1)^2}(x+1)^2 - 4x}\right)/\ln x\right).$$

**Residual Variance** We next compute the residual variance of communication for a communication equilibrium with $n$ intervals, $n \ge 2$. The variance of the message $m_1$ to Manager 2 is

$$\begin{aligned}
\mathrm{E}_D\left[m_1^2\right] &= \sum_{i=1}^{n}\int_{a_{1,i-1}}^{a_{1,i}}\left(\frac{a_{1,i}+a_{1,i-1}}{2}\right)^2\frac{1}{2s}d\theta_1 = \frac{1}{8s}\sum_{i=1}^{n}(a_{1,i}-a_{1,i-1})(a_{1,i}+a_{1,i-1})^2 \\
&= \frac{1}{8s}\sum_{i=1}^{n}\left[\left(a_{1,i}^3 - a_{1,i-1}^3\right) + a_{1,i}a_{1,i-1}(a_{1,i}-a_{1,i-1})\right] \\
&= \frac{s^2}{4} + \frac{1}{8s}\sum_{i=1}^{n}a_{1,i}a_{1,i-1}(a_{1,i}-a_{1,i-1}).
\end{aligned}$$

And the residual variance on an $n-$partition equilibrium is

$$V_n^D = \mathrm{E}_D\left[(m_1-\theta_1)^2\right] = \frac{s^2}{12} - \frac{1}{8s}\sum_{i=1}^{n}a_{1,i}a_{1,i-1}(a_{1,i}-a_{1,i-1}).$$

We next compute $a_{1,i}a_{1,i-1}(a_{1,i}-a_{1,i-1})$. From (34) and the size of each interval (35) we have

$$\begin{aligned}
a_{1,i}a_{1,i-1}(a_{1,i}-a_{1,i-1}) &= (x-1)\Big[\frac{A^3}{x^2}x^{3i} - \frac{xB^3}{x^{3i}} + \frac{A^2(x+1)Ds}{x^2}x^{2i} - \frac{B^2(x+1)Ds}{x^{2i}} \\
&\quad + A\frac{(Ds)^2x + AB + ABx^2 - ABx}{x^2}x^i - B\frac{(Ds)^2x + AB + ABx^2 - ABx}{x^i x}\Big].
\end{aligned}$$

From the sum of a geometric series $\sum_{i=1}^{n} x^{ki} = x^k \frac{1-r^k}{1-x^k}$ we can simplify the summation of the previous terms to obtain

$$\begin{aligned}\sum_{i=1}^{N} a_i a_{i-1} \left(a_i - a_{i-1}\right) &= \left(r^3-1\right)\left(A^3 - \frac{B^3}{r^3}\right)\left(\frac{x}{x^2+x+1}\right) + \left(r^2-1\right)\left(A^2-\frac{B^2}{r^2}\right)(Ds) \\ &\quad + (r-1)\left(A-\frac{B}{r}\right)\left((Ds)^2 + AB\left(\frac{x^2-x+1}{x}\right)\right). \qquad (37)\end{aligned}$$

To further simplify this expression we first note that

$$\frac{x}{x^2+x+1} = \frac{1}{3+4b_1}$$

and

$$\frac{x^2-x+1}{x} = 1+4b_1,$$

which follows from $x$ being a solution to the characteristic equation of (33). Moreover, from the definitions of $A, B$ and $D$ we have

$$\begin{aligned}(r-1)\left(A-\left(\frac{B}{r}\right)\right) &= 2s, \\ \left(r^2-1\right)\left(A^2-\left(\frac{B}{r}\right)^2\right) &= -4Ds^2, \text{ and} \\ \left(r^3-1\right)\left(A^3-\left(\frac{B}{r}\right)^3\right) &= 2s\frac{\left(r^2+r+1\right)}{\left(r^2-1\right)^2}\left(\left(r^2+1\right)\left(3D^2+1\right)-2r\left(3D^2-1\right)\right)s^2,\end{aligned}$$

which, substituted into (37) yields

$$\begin{aligned}\sum_{i=1}^{N} a_i a_{i-1}\left(a_i - a_{i-1}\right) &= \frac{\left(r^2+r+1\right)\left(\left(r^2+1\right)\left(3D^2+1\right)-2r\left(3D^2-1\right)\right)}{\left(r^2-1\right)^2}\left(\frac{2}{4b_1+3}\right)s^3 \\ &\quad -2D^2s^3 - 2r\left(1+4b_1\right)\left(\left(\frac{1}{1-r}\right)^2-\left(\frac{D}{1+r}\right)^2\right)s^3.\end{aligned}$$

Substituting this expression into $V_n^s$ and after some simplifications we have

$$V_n^s = \frac{s^2}{12} - \left(\frac{1-4D^2b_1}{4\left(4b_1+3\right)} + 4\frac{b_1\left(b_1+1\right)}{\left(4b_1+3\right)}\frac{r\left(D^2-\left(\frac{1+r}{1-r}\right)^2\right)}{\left(r+1\right)^2}\right)s^2, \qquad (38)$$

where $r = x^n$. Therefore the residual variance of communication is given by $V^s = V^s_{N\left(b_1^s, b_2^s\right)}$, $s \in \{DC, C\}$. ■

**Absolute Continuity of Residual Variances**

The residual variance $V^s, s = \{C, D, DC\}$ , is continuous in the own-division bias $\lambda$, although non-differentiable whenever the number of intervals in the most informative communication equilibrium changes value. As the number of intervals tends to infinity as managers become more aligned with each other, the residual variance has an infinite number of points of discontinuity in every neighborhood of $\lambda = 1/2$. Nevertheless, the next lemma shows that $V^s$ retains certain smoothness properties that allows us to characterize its behavior in a neighborhood of $\lambda = 1/2$ through the function $\partial V^s/\partial\lambda$.

LEMMA B1. *The residual variance of communication $V^s, s = \{C, D, DC\}$ is an absolutely continuous function of $\lambda \in [1/2, 1]$ with a well-defined limit $\partial V^s/\partial\lambda$ as $\lambda$ tends to $1/2$. In particular,*

$$\begin{aligned}
\lim_{\lambda\to 1/2} \frac{\partial V^C}{\partial\lambda} &= \frac{4}{3}s\,(\mu - c)\,|t|, \\
\lim_{\lambda\to 1/2} \frac{\partial V^{DC}}{\partial\lambda} &= \frac{8}{3}s\,(\mu - c)\,|t|, \text{ and} \\
\lim_{\lambda\to 1/2} \frac{\partial V^D}{\partial\lambda} &= \frac{2}{9}s^2\left(3\frac{(\mu - c)\,(1-t)}{s} - 1\right)\frac{4}{1-t^2}.
\end{aligned}$$

**Proof:** The function $V^s$, $s = \{C, D, DC\}$ , is continuous and increasing in $\lambda$ and its derivative is defined except for a countable number of points. To establish absolute continuity of $V^s$ we need to further show that (i.) its derivative is integrable, and (ii.) $V^s$ maps sets of measure zero into sets of measure zero (Luzin N property; see Rudin 1986). Since the set of points of non-differentiability of $V^s$ is countable it follows readily that $V^s$ satisfies the Luzin N property (see Leoni 2009). We will now show that $\partial V^s/\partial\lambda$ is bounded in $[1/2, 1]$, whenever defined, and this will establish integrability.

First, the case of Centralization. Differentiating (32) we obtain

$$\left|\frac{\partial V_n^C}{\partial b^C}\right| = \frac{2}{3}|b^C|\left(n^2 - 1\right) \leq \frac{2}{3}s\frac{N(b^C) + 1}{N(b^C)},$$

where the last inequality follows from the definition of $N(b^C)$. Therefore we obtain the uniform bound

$$\sup_{\lambda,n}\left|\frac{\partial V_n^C}{\partial\lambda}\right| \leq \sup_{\lambda}\frac{2}{3}s\frac{N(b^C)+1}{N(b^C)}\sup_{\lambda}\frac{\partial b^C}{\partial\lambda} = \frac{2}{3}s\sup_{\lambda}\frac{\partial b^C}{\partial\lambda}.$$

We now show that $\partial V^C/\partial\lambda$ approaches a well-defined limit as $\lambda \longrightarrow 1/2$. From the previous bound we have

$$\lim_{|b^C|\to 0}\frac{\partial V^C}{\partial b^C} = \lim_{|b^C|\to 0}\frac{2}{3}s\frac{N(b^C)+1}{N(b^C)} = \frac{2}{3}s.$$

Given that $b^C = (2\lambda - 1)(\mu - c)\tau$ we readily have that

$$\lim_{\lambda \to \frac{1}{2}} \frac{\partial V^C}{\partial \lambda}_{\lambda = \frac{1}{2}} = \frac{2}{3} s \lim_{\lambda \to \frac{1}{2}} \frac{\partial \left| b^C \right|}{\partial \lambda} = \frac{4}{3} s (\mu - c) |t|. \tag{39}$$

Now we turn to the case of Divisional Centralization and Decentralization. Totally differentiating (38) for $n = N(b_1, b_2)$ we have

$$\frac{\partial V^s}{\partial \lambda} = \left[ \frac{\partial V^s}{\partial r} \frac{\partial r}{\partial x} \frac{\partial x}{\partial b_1} + \frac{\partial V^s}{\partial b_1} \right] \frac{\partial b_1}{\partial \lambda} + \frac{\partial V^s}{\partial D} \frac{\partial D}{\partial \lambda}, s = \{D, DC\}$$

where $r = x^{N(b_1, b_2)}$. Computing each element in the previous expression we have

$$\begin{aligned}
\frac{\partial V^s}{\partial r} \frac{\partial r}{\partial x} \frac{\partial x}{\partial b_1} &= N(b_1, b_2)\, r \left( 4 \frac{\left(D^2 - 1\right) \left(r^2 - 2r \frac{D+1}{D-1} + 1\right) \left(r^2 - 2r \frac{D-1}{D+1} + 1\right)}{(4b_1 + 3)} s^2 \right) \left( \frac{\sqrt{b_1 (b_1 + 1)}}{\left(r^2 - 1\right)^3} \right), \\
\frac{\partial V^s}{\partial b_1} &= s^2 \left( \frac{\left(3D^2 + 1\right)}{(4b_1 + 3)^2} - \frac{4\left(6b_1 + 4b_1^2 + 3\right)}{(4b_1 + 3)^2} \frac{r\left(D^2 - (\frac{1+r}{1-r})^2\right)}{(r+1)^2} \right), \text{ and} \\
\frac{\partial V^s}{\partial D} &= 2s^2 D b_1 \left( \frac{(r-1)^2 - 4rb_1}{(4b_1 + 3)(r+1)^2} \right).
\end{aligned} \tag{40}$$

To guarantee that $\partial V^s / \partial \lambda$ is bounded we now show that it approaches a finite limit as $\lambda \to 1/2$. From the definition of $r = x^{N(b_1, b_2)}$ in (36) we have

$$\lim_{\lambda \to 1/2} \frac{\frac{(D-1)}{2(D+1)}(x+1) + \frac{1}{2}\sqrt{\frac{(D-1)^2}{(D+1)^2}(x+1)^2 - 4x}}{r} = 1,$$

which implies

$$\lim_{\lambda \to 1/2} \left( r^2 - 2r \frac{D-1}{D+1} + 1 \right) = 0$$

and

$$\lim_{\lambda \to 1/2} \frac{r\left(D^2 - (\frac{1+r}{1-r})^2\right)}{(r+1)^2} = \frac{r\left(D^2 - \frac{\frac{4Dr}{D+1}}{\frac{-4r}{D+1}}\right)}{\frac{4Dr}{D+1}} = \frac{1}{4}(D+1)^2.$$

With these limits applied to (40) we have

$$\begin{aligned}
\lim_{\lambda \to \frac{1}{2}} \frac{\partial V^s}{\partial r} \frac{\partial r}{\partial x} \frac{\partial x}{\partial b_1} &= 0 \\
\lim_{\lambda \to \frac{1}{2}} \frac{\partial V^s}{\partial b_1} &= s^2 \left( \frac{3D^2 + 1}{9} - \frac{1}{3}(D+1)^2 \right) = \frac{2}{9} s^2 (-3D - 1) \\
\lim_{\lambda \to \frac{1}{2}} \frac{\partial V^s}{\partial D} &= 0.
\end{aligned}$$

We now consider the cases of Decentralization and Divisional Centralization. For Decentralization, we have that

$$\lim_{\lambda \to \frac{1}{2}} \frac{\partial b_1}{\partial \lambda} = \frac{4}{1-t^2}, \text{ and } \lim_{\lambda \to \frac{1}{2}} \frac{\partial D}{\partial \lambda} = -2t(\mu - c).$$

Therefore the limit of the total derivative as the own-division bias vanishes is

$$\lim_{\lambda \to 1/2} \frac{\partial V^D}{\partial \lambda} = \frac{8}{9} \frac{s^2}{1-t^2} \left( 3(1-t) \frac{(\mu - c)}{s} - 1 \right), \tag{41}$$

which is bounded for $|t| < 1$. This establishes that $\partial V^D / \partial \lambda$ is bounded and thus integrable.

We now turn to the case of Divisional Centralization. While in this case we have that

$$\lim_{\lambda \to \frac{1}{2}} \frac{\partial b_1}{\partial \lambda} = 0,$$

the derivative $\partial D / \partial \lambda$ becomes unbounded when $\lambda \to \frac{1}{2}$. As $\lim_{\lambda \to \frac{1}{2}} \frac{\partial V^s}{\partial D} = 0$, however, in this case we obtain

$$\lim_{\lambda \to 1/2} \frac{\partial V^{DC}}{\partial D} \frac{\partial D}{\partial \lambda} = \frac{8}{3} s \left(\mu - c\right) |t|$$

and

$$\lim_{\lambda \to 1/2} \frac{\partial V^{DC}}{\partial \lambda} = \frac{8}{3} s \left(\mu - c\right) |t| = 2 \lim_{\lambda \to 1/2} \frac{\partial V^C}{\partial \lambda}. \ \blacksquare$$

COROLLARY B1. *If* $\lim_{\lambda \to 1/2} \frac{\partial V^s}{\partial \lambda} > \lim_{\lambda \to 1/2} \frac{\partial V^l}{\partial \lambda}$ $s, l \in \{C, DC, D\}$ *then there exists an* $\varepsilon > 0$ *such that* $V^s(\lambda) > V^l(\lambda)$ *for all* $\lambda \in (1/2, 1/2 + \varepsilon)$. *Conversely, if* $\lim_{\lambda \to 1/2} \frac{\partial V^s}{\partial \lambda} < \lim_{\lambda \to 1/2} \frac{\partial V^l}{\partial \lambda}$ *then there exists an* $\varepsilon > 0$ *such that* $V^s(\lambda) < V^l(\lambda)$ *for all* $\lambda \in (1/2, 1/2 + \varepsilon)$.

**Proof:** As $V^l, l = \{C, DC, D\}$ are absolutely continuous the fundamental theorem of calculus holds (Rudin 1987) and we have that

$$\left[ V^s(\lambda) - V^l(\lambda) \right] - \left[ V^s(1/2) - V^l(1/2) \right] = \int_{1/2}^{\lambda} \frac{\partial}{\partial \lambda} \left( V^s - V^l \right) d\lambda.$$

As all structures achieve full revelation of information for $\lambda = 1/2$, if

$$\lim_{\lambda \to 1/2} \partial \left( V^s - V^l \right) / \partial \lambda > 0$$

it follows that there exists an $\varepsilon > 0$ such that

$$V^s(\lambda) > V^l(\lambda), \lambda \in (1/2, 1/2 + \varepsilon).$$

The last claim of the corollary follows from an equivalent argument. $\blacksquare$

**Informativeness of Vertical and Horizontal Communication**

**Proof of Lemma 2:** Proposition B1 derives the expressions for $b^C$, $b^{DC}$, and $b^D$. Consider first Part (i.) of the Lemma. We will show that, for any $\theta_i \in [-s,s]$, $\lambda \in (1/2,1]$ and $t<0$ we have $\left|b^C\right| < \left|b^D\right|$. That is, the point-wise communication bias under Decentralization is always larger than under Centralization when $t<0$. Then, it follows from Chen and Gordon (2013) that a smaller point-wise bias leads to more informative communication.

Given that, when $t<0$, $b^C$ is constant and negative and $b^D$ is positive and increasing, it follows that

$$\Delta = \min b^D - |b^C| = \frac{2\lambda-1}{1-\lambda-\lambda\tau^2}\left(\left(1-\lambda\tau-\lambda\tau^3\right)(\mu-c)-s\right).$$

Since $\left(1-\lambda\tau-\lambda\tau^3\right)>0$ for $-1\le t\le 0, 1/2\le\lambda\le 1$ then $\Delta\ge 0$ if and only if

$$\frac{\mu-c}{s}\ge\frac{1}{1-\lambda\tau-\lambda\tau^3}.$$

Since positive quantities requires $(\mu-c)/s>1$ when $t<0$ and

$$\sup_{\substack{-1<t\le 0\\ 1/2\le\lambda\le 1}}\frac{1}{1-\lambda\tau-\lambda\tau^3}=1,$$

then we readily have that $\Delta>0$.

We now turn to Part (ii.) of the Lemma. Part (ii.a) follows from Lemma B1 which states that

$$\lim_{\lambda\to 1/2}\frac{\partial V^{DC}}{\partial\lambda}=2\lim_{\lambda\to 1/2}\frac{\partial V^C}{\partial\lambda},$$

and corollary B1.

To prove Part (ii.b), first note that, *on average*, the absolute value of the communication bias is larger under Decentralization than under Centralization when $t>0$. As shown in Proposition B1, a necessary condition for informative horizontal communication is that $\frac{\lambda}{1-\lambda}\tau^2<1$. Therefore, whenever $b^D$ is well defined we have

$$\frac{1-\tau}{(1-\lambda)\left(1-\frac{\lambda}{1-\lambda}\tau^2\right)}>\frac{1-\tau}{(1-\lambda)\left(1-\tau^2\right)}>\frac{2}{1+\tau}>1>|\tau|,$$

implying

$$E\left[b^D\right]=(2\lambda-1)(\mu-c)\frac{1-\tau}{(1-\lambda)\left(1-\frac{\lambda}{1-\lambda}\tau^2\right)}>|b^C|.$$

However, unlike the case where $t<0$, we could have cases where the point-wise communication bias under Decentralization is smaller than under Centralization. To see this, note that when $t>0$, we have

$$\Delta=\min b^D(\theta_i)-|b^C|=\frac{2\lambda-1}{1-\lambda-\lambda\tau^2}\left(\left(-2\tau+\lambda\tau+\lambda\tau^3+1\right)(\mu-c)-s\right)$$

Since $\left(-2\tau + \lambda\tau + \lambda\tau^3 + 1\right) > 0$ for $0 \leq t \leq 1, 1/2 \leq \lambda \leq 1$ then $\Delta \geq 0$ if, and only if,

$$(\mu - c)/s \geq 1/\left(-2\tau + \lambda\tau + \lambda\tau^3 + 1\right).$$

We first note that

$$\max_{1/2 \leq \lambda \leq 1} \frac{1}{(-2\tau + \lambda\tau + \lambda\tau^3 + 1)} = \frac{2}{(t+2)(1-t)^2}.$$

Since the restriction to positive quantities requires $(\mu - c)/s > (1+t)/(1-t)$, and $(1+t)/(1-t) > 2/(t+2)(1-t)^2$ for $t < \sqrt{2} - 1$ it then follows that

$$\Delta \geq 0 \text{ if } 0 \leq t \leq \sqrt{2} - 1.$$

If, however, $t > \sqrt{2} - 1$, then for $\lambda$ close to $1/2$ we can have

$$(\mu - c)/s < 1/\left(-2\tau + \lambda\tau + \lambda\tau^3 + 1\right),$$

implying that $b^D(-s) < |b^C|$. Since $b^D(\theta_j)$ is increasing in $\theta_j$ the existence of a state $\bar{\theta}_j$ where $b^D(\theta_j) < b^C$ for $\theta_j < \bar{\theta}_j$ follows. We next show that this reversal may translate into horizontal communication being more informative than vertical communication for small own-division bias. Applying Corollary B1 we conclude that horizontal communication is more informative than vertical in a neighborhood of $\lambda = 1/2$ if, and only if,

$$\frac{1+t}{1-t} < \frac{\mu - c}{s} < \frac{2}{-9t + 3t^3 + 6},$$

where the first inequality follows from the parameter restriction that ensures positive quantities in equilibrium and the second from comparing the limits (39) and (41). For $t < 0.876\,49$, we have $\frac{1+t}{1-t} > \frac{2}{-9t+3t^3+6}$ implying that vertical communication is more informative than horizontal for $\lambda$ close to $1/2$. Define

$$t^* = \max\left\{0.876\,49, \min\left\{t : \frac{\mu - c}{s} < \frac{2}{-9t + 3t^3 + 6}\right\}\right\}.$$

If $t > t^*$ then we have that both $\frac{1+t}{1-t} < \frac{2}{-9t+3t^3+6}$ and $\frac{\mu-c}{s} < \frac{2}{-9t+3t^3+6}$, implying that there exists an $\varepsilon > 0$ such that for $\lambda \in (1/2, 1/2 + \varepsilon)$ horizontal communication is more informative than vertical. ■

### Appendix B3 - Relative Performance

We now turn to comparing the relative performance of Centralization, Divisional Centralization, and Decentralization for a vanishing small own-division bias. We start with a technical lemma that translates the smoothness properties of $V^s, s = \{C, DC, D\}$ derived in Lemma B1 to the comparison of profits under all organizational structures.

LEMMA B2. *The difference in performance* $\Pi^s - \Pi^D$, $s = \{C, DC\}$, *is an absolutely continuous function of* $\lambda$. *The limit* $\partial\left(\Pi^s - \Pi^D\right)/\partial\lambda$ , $s = \{C, DC\}$, *as* $\lambda$ *tends to* $1/2$ *exists and, if its positive, there exists an* $\varepsilon > 0$ *such that* $\Pi^s > \Pi^D$ *for* $\lambda \in (1/2, 1/2+\varepsilon)$, *while, if it is negative, then there exists an* $\varepsilon > 0$ *such that* $\Pi^s < \Pi^D$ *for* $\lambda \in (1/2, 1/2+\varepsilon)$.

**Proof:** Lemma B1 establishes that the residual variances of communication $V^C$, $V^{DC}$, and $V^D$ are absolutely continuous and the limit of $\partial V^s/\partial\lambda \quad s = \{C, DC, D\}$ exist as $\lambda \to 1/2$. As $\Pi^C$, $\Pi^{DC}$ and $\Pi^D$ are differentiable functions of $V^s$ it follows that $\Pi^s - \Pi^D$, $s = \{C, DC\}$ is absolutely continuous for $\lambda \in [1/2, 1]$ with a well defined limit as $\lambda \to 1/2$. By application of the fundamental theorem of calculus we have

$$\left[\Pi^s(\lambda) - \Pi^D(\lambda)\right] - \left[\Pi^s(1/2) - \Pi^D(1/2)\right] = \int_{1/2}^{\lambda} \partial\left(\Pi^s - \Pi^D\right)/\partial\lambda d\lambda.$$

Since both Centralization, Divisional Centralization and Decentralization achieve first best performance for $\lambda = 1/2$, if $\lim_{\lambda\to 1/2} \partial\left(\Pi^s - \Pi^D\right)/\partial\lambda > 0$ it follows that there exists an $\varepsilon > 0$ such that

$$\Pi^s(\lambda) > \Pi^D(\lambda), \lambda \in (1/2, 1/2+\varepsilon).$$

The case $\lim_{\lambda\to 1/2} \partial\left(\Pi^s - \Pi^D\right)/\partial\lambda < 0$ follows similarly from an equivalent argument. ■

The previous analysis showed that vertical communication is more informative than horizontal communication under Decentralization whenever $t < 0$. We now show that this communication advantage may be sufficient for the organization to move to a centralized structure even for a vanishing small conflict.

**Proof of Proposition 2:** First, we show that

$$\lim_{\lambda\to 1/2} \frac{\partial\left[\Pi^C - \Pi^{DC}\right]}{\partial\lambda} = 0. \tag{42}$$

That is, to a first order, there are no differences between Centralization and Divisional Centralization for a vanishingly small conflict of interest. In particular, this implies that

$$sign\left(\lim_{\lambda\to 1/2} \frac{\partial\left[\Pi^{DC} - \Pi^D\right]}{\partial\lambda}\right) = sign\left(\lim_{\lambda\to 1/2} \frac{\partial\left[\Pi^C - \Pi^D\right]}{\partial\lambda}\right)$$

and, by Lemma B2, it follows that whenever Decentralization dominates Centralization, it also dominates Divisional Centralization, and, conversely, whenever Decentralization is dominated by Centralization, it is also dominated by Divisional Centralization.

To prove (42), we have that the difference $\Pi^{DC} - \Pi^{C}$ in the performance of Centralization and Divisional Centralization is given by (21) and the limit of the rate of change of this difference is

$$\lim_{\lambda \to 1/2} \frac{\partial \left[\Pi^{DC} - \Pi^{D}\right]}{\partial \lambda} = \frac{b}{2(1-t^2)} \left( \frac{1}{2} \lim_{\lambda \to 1/2} \frac{\partial V^{DC}}{\partial \lambda} - \lim_{\lambda \to 1/2} \frac{\partial V^{C}}{\partial \lambda} \right).$$

Then (42) follows since Lemma B1 implies that $\lim_{\lambda \to 1/2} \partial V^{DC}/\partial \lambda = 2 \lim_{\lambda \to 1/2} \partial V^{C}/\partial \lambda$.

We now turn to the comparison between Centralization and Decentralization. The difference $\Pi^{C} - \Pi^{D}$ is given by (20) and the limit of the rate of change of this difference is

$$\lim_{\lambda \to 1/2} \frac{\partial \left[\Pi^{C} - \Pi^{D}\right]}{\partial \lambda} = \frac{b}{2(1-t^2)} \left( t^2 \lim_{\lambda \to 1/2} \frac{\partial V_i^{D}}{\partial \lambda} - \lim_{\lambda \to 1/2} \frac{\partial V^{C}}{\partial \lambda} \right).$$

From Lemma B2, Centralization dominates Decentralization for $\lambda$ close to 1/2 if $\lim_{\lambda \to 1/2} \partial \left(\Pi^{C} - \Pi^{D}\right)/\partial \lambda > 0$, which translates to

$$t^2 \lim_{\lambda \to 1/2} \frac{\partial V_i^{D}}{\partial \lambda} > \lim_{\lambda \to 1/2} \frac{\partial V_i^{C}}{\partial \lambda}.$$

Using the limits (39) and (41), the previous inequality translates into

$$\frac{8}{9} \frac{s^2 t^2}{1-t^2} \left( 3(1-t) \frac{(\mu - c)}{s} - 1 \right) > \frac{4}{3} s \left(\mu - c\right) |t|,$$

which can be written as

$$\frac{s}{\mu - c} < \frac{3(1-t)\left(2|t| - t - 1\right)}{2|t|}.$$

For $t > 0$ this condition is never satisfied. If $t < 0$ we can solve for $t$ to obtain that Centralization dominates Decentralization when $t < t^{**}$ with

$$t^{**} = \frac{1}{9} \left( \left(3 - \frac{s}{\mu - c}\right) - \sqrt{(\frac{s}{\mu - c})^2 - 6\frac{s}{\mu - c} + 36} \right). \blacksquare$$