# OPTIMAL DESIGN OF TRADE AGREEMENTS IN THE PRESENCE OF RENEGOTIATION: ONLINE APPENDIX

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### Abstract

This online Appendix includes various proofs and extensions that were left out of our manuscript "Optimal Design of Trade Agreements in the Presence of Renegotiation" due to space constraints.

# Online Appendix A

Here we prove the following proposition, which formalizes an argument we made informally in the Conclusion:

**Proposition A1.** Consider menu contracts of the type  $\{(P, b^D), (FT, b^{FT})\}$ : (i) If the support of  $\gamma$  is sufficiently small, a property rule is optimal; (ii) If the support of  $\gamma$  is sufficiently large (on both sides of  $\gamma^*$ ), it is optimal to use a carrot  $(b^{FT} < 0)$  together with a liability rule, and in particular the optimal  $b^D$  satisfies  $0 < b^D < \gamma^* < \bar{b}^{prohib}$ .

**Proof:** In this extended setting, a contract is summarized by the pair  $(b^D, b^{FT})$ . For each contract and state of the world,  $(b^D, b^{FT}; \gamma)$ , there will be one of four possible equilibrium outcomes: (i) the importer chooses P without renegotiating; (ii) the importer's threat point is P but the governments renegotiate to policy FT; (iii) the importer chooses FT without renegotiating; (iv) the importer's threat point is FT but the governments renegotiate to policy P. The first step of the analysis is to characterize the mapping from  $(b^D, b^{FT}; \gamma)$  to these four possible outcomes. One way to proceed is to build on the graphical apparatus of Figure 1: we continue working within the  $(b^D, \gamma)$  space and think of  $b^{FT}$  as a parameter that shifts the key curves in this space.

As we will show later, it can never be optimal to set  $b^{FT} > 0$  or  $b^D < 0$ . Since proving this claim involves a tedious and taxonomic argument, we postpone this argument to a later part of this proof, and here we focus on the intuitive case where  $b^{FT} \le 0$  and  $b^D \ge 0$ .

Let us start by characterizing the locus of points where the importer is indifferent between the two threat points (P and FT), for a given  $b^{FT}$ . Clearly, the importer is indifferent between the two threat points when  $\gamma = S(b^D) - S(b^{FT})$ . This threat-point-indifference curve is depicted in

Figure A1. Note that introducing  $b^{FT} < 0$  in the contract shifts the threat-point-indifference curve upwards relative to Figure 1.

Next we ask: given  $b^{FT}$ , what are the regions of the  $(b^D, \gamma)$  space in which governments renegotiate the contract? Let us first derive the region in which the threat point is P but governments
renegotiate toward FT (which we continue to label  $FT_R$ ). It is immediate to verify that the threat
point is P iff  $\gamma > S(b^D) - S(b^{FT})$ , and that in this case governments will renegotiate to FT iff  $\gamma < S(b^D) - S(b^D - \gamma^*) = R(b^D)$ . Notice that this latter condition is exactly the same as in the
case of  $b^D$ -only contracts. Intuitively, conditional on the threat point being P the level of  $b^{FT}$  does
not affect the outcome. The  $R(b^D)$  curve is depicted in Figure A1, and is the same as the  $R(b^D)$ curve in Figure 1. Thus, the region  $FT_R$  is the region above the  $\gamma = S(b^D) - S(b^{FT})$  curve and
below the  $R(b^D)$  curve. Note for future reference that these curves intersect for  $b^D = \gamma^* + b^{FT}$ ,
and note also that if  $b^{FT}$  is sufficiently large and negative the  $FT_R$  region will be empty. In Figure
A1 we depict the case in which the  $FT_R$  region overlaps with the positive quadrant, which (as we
show below) must be the case at an optimal contract.

We next characterize the region where the threat point is FT but governments renegotiate toward P (which we continue to label  $P_R$ ). Clearly the threat point is FT iff  $\gamma < S(b^D) - S(b^{FT})$ , and it is easy to show that in this case governments will renegotiate toward P iff  $\gamma < S(b^{FT} + \gamma^*) - S(b^{FT})$ . It can be easily verified that  $\gamma = S(b^{FT} + \gamma^*) - S(b^{FT})$  is just the horizontal line that goes through the point of intersection between the  $\gamma = S(b^D) - S(b^{FT})$  curve and the  $R(b^D)$  curve. The  $P_R$  region is therefore the region that lies above this horizontal line and below the  $\gamma = S(b^D) - S(b^{FT})$  curve, as depicted in Figure A1.

Having characterized the mapping from  $(b^D, b^{FT}; \gamma)$  to the four possible outcomes, we can now turn to the characterization of the optimal contract.

We start by extending the result of Proposition 1, which is an intermediate step toward proving Proposition A1. We argue that it can never be strictly optimal to set  $b^D > \gamma^* + b^{FT}$ , and the optimal contract never induces renegotiation toward P, while it does induce renegotiation toward FT for an intermediate range of  $\gamma$ .

We will suppose by contradiction that it is strictly optimal to set  $b^D > \gamma^* + b^{FT}$  and will show that the initial contract can be (weakly) improved upon. We can write the expected joint surplus

$$E\Omega(b^{D}, b^{FT})|_{b^{D} \ge \gamma^{*} + b^{FT}} = V(FT) + \int_{S(b^{D}) - S(b^{FT})}^{\infty} [\gamma - \gamma^{*} - c(b^{D})] dH(\gamma)$$

$$+ \int_{S(b^{FT} + \gamma^{*}) - S(b^{FT})}^{S(b^{FT})} [\gamma - \gamma^{*} - c(b^{e}(b^{FT}; \gamma))] dH(\gamma) - \int_{0}^{S(b^{FT} + \gamma^{*}) - S(b^{FT})}^{S(b^{FT})} c(b^{FT}) dH(\gamma)$$

$$= \int_{S(b^{FT} + \gamma^{*}) - S(b^{FT})}^{S(b^{FT})} [\gamma - \gamma^{*} - c(b^{e}(b^{FT}; \gamma))] dH(\gamma) - \int_{0}^{S(b^{FT} + \gamma^{*}) - S(b^{FT})}^{S(b^{FT})} dH(\gamma)$$

where (with a slight abuse of notation)  $b^e(b^{FT}; \gamma)$  denotes the equilibrium transfer in region  $P_R$ ; note that  $b^e$  depends only on  $b^{FT}$  and not on  $b^D$ . To understand this expression, refer to Figure A1 and notice that if  $b^D > \gamma^* + b^{FT}$  there are three relevant intervals of  $\gamma$ : for  $\gamma > S(b^D) - S(b^{FT})$ , we are in region P and the joint surplus is  $V(FT) + \gamma - \gamma^* - c(b^D)$ ; for  $S(b^{FT} + \gamma^*) - S(b^{FT}) < \gamma < S(b^D) - S(b^{FT})$ , we are in region  $P_R$  and the joint surplus is  $V(FT) + \gamma - \gamma^* - c(b^e(b^{FT}; \gamma))$ ; and for  $\gamma < S(b^{FT} + \gamma^*) - S(b^{FT})$  we are in region FT and hence the joint surplus is  $V(FT) - c(b^{FT})$ .

We can now write down the partial derivatives of  $E\Omega$ :

$$\left. \frac{\partial E\Omega}{\partial b^D} \right|_{b^D \geq \gamma^* + b^{FT}} = -c'(b^D)[1 - H(S(b^D) - S(b^{FT}))] + (1 + c'(b^D))[c(b^D) - c(b^e(\cdot))]h(S(b^D) - S(b^{FT}))$$

and

$$\frac{\partial E\Omega}{\partial b^{FT}}\Big|_{b^{D} \ge \gamma^* + b^{FT}} = -(1 + c'(b^{FT}))[c(b^{D}) - c(b^{e}(\cdot))]h(S(b^{D}) - S(b^{FT})) \\
- \int_{S(b^{D}) - S(b^{FT})} \frac{dc(b^{e}(b^{FT}; \gamma))}{db^{FT}} dH(\gamma) - c'(b^{FT})H(S(b^{FT} + \gamma^*) - S(b^{FT}))$$
(2)

where we have used the fact that  $\Omega$  is continuous at the border between the FT region and the  $P_R$  region (i.e. at  $\gamma = S(b^{FT} + \gamma^*) - S(b^{FT})$ ).

We also note for future reference that, in analogy with the result of Lemma 2, one can show that  $b^e(b^{FT}; \gamma)$  is increasing in  $b^{FT}$ ; intuitively, a higher  $b^{FT}$  worsens the threat point for the importer and hence the importer gets a worse deal in the renegotiation. Finally, recall that in the  $P_R$  region  $\gamma^* + b^{FT} < b^e < S^{-1}(\gamma + S(b^{FT}))$ .

There are two cases to consider, depending on whether  $b^D > 0$  or  $b^D = 0$ .

Suppose first that  $b^D > 0$  at the initial contract. In this case we can improve over the initial contract by lowering  $b^D$  to  $\max\{0, \gamma^* + b^{FT}\}$ . From expression 1 it is clear that this will increase  $E\Omega$ , because it induces no change in the policy and (i) for states  $\gamma$  that lie above  $S(b^D) - S(b^{FT})$  before and after the change, the transfer  $b^D$  is reduced, and (ii) for states  $\gamma$  that lie below  $S(b^D) - S(b^{FT})$  before the change but above  $S(b^D) - S(b^{FT})$  after the change, the transfer goes from  $b^e$  to  $b^D$ , which is an improvement since  $b^e > \gamma^* + b^{FT}$ .

Next suppose  $b^D=0$  at the initial contract. In this case the initial contract can be dominated by increasing  $b^{FT}$  slightly toward zero. To see this, notice that (i) since we have supposed that  $b^D \geq \gamma^* + b^{FT}$ , we have  $b^{FT} < 0$  and hence  $c'(b^{FT}) < 0$ ; and (ii) given  $b^D = 0$ , in region  $P_R$  we have  $\gamma < -S(b^{FT})$ , and hence  $b^e < S^{-1}(\gamma + S(b^{FT})) < 0$ , and recalling that  $b^e$  is increasing in  $b^{FT}$ , this in turn implies  $\frac{dc(b^e(b^{FT};\gamma))}{db^{FT}} < 0$ . These two observations together imply that  $\frac{\partial E\Omega}{\partial b^{FT}}|_{b^D \geq \gamma^* + b^{FT}} > 0$  when evaluated at  $b^D = 0$ .

To summarize, we have just shown that the result of Proposition 1 extends to this more general class of contracts, in the sense that we can focus without loss of generality on contracts with  $b^D < \gamma^* + b^{FT}$ , and the optimal contract never induces renegotiation toward P, while it does induce renegotiation toward FT for an intermediate range of  $\gamma$ .

We can now turn to proving the claims made in Proposition A1. It is convenient to start with the case of large uncertainty (Proposition A1(ii)).

Large uncertainty.

For our purposes it suffices to focus on the case of full support, i.e.  $\gamma \in (0, \infty)$ . Recall that we are focusing on the case where  $b^D \geq 0$  and  $b^{FT} \leq 0$  (we show later that it can never be optimal to set  $b^D < 0$  or  $b^{FT} > 0$ ).

Let  $\tilde{b}^D$  denote the optimal value of  $b^D$  conditional on  $b^{FT} = 0$ . Given our results above and our focus on  $b^D \ge 0$ , it follows that  $0 \le \tilde{b}^D \le \gamma^*$ . We now argue that, starting from  $(b^{FT} = 0, b^D = \tilde{b}^D)$ , we can raise expected joint surplus by making  $b^{FT}$  slightly negative. Decreasing  $b^{FT}$  slightly has

no impact on the policy allocation, but it has two effects on the expected equilibrium transfer. First, for  $\gamma < S(b^D)$  there is now a small transfer  $b^{FT}$ , which introduces a cost, but this is a second order cost since c'(0) = 0. Second, for  $\gamma$  just above  $\gamma = S(b^D)$  the threat point switches from P to FT, so for these states, before the change governments renegotiate toward FT and the equilibrium transfer is nonnegligible, and after the change the importer chooses FT without renegotiating and the transfer is negligible (because  $b^{FT}$  is close to zero); this is a first-order beneficial effect. Note that, within the renegotiation region  $FT_R$ , decreasing  $b^{FT}$  has no impact on the threat point, hence it does not affect the equilibrium transfer.

To see this more formally, let us write down the expected joint surplus as a function of  $b^D$  and  $b^{FT}$ . As we argued above we can focus on the case  $b^D \leq \gamma^* + b^{FT}$ . We can then write the expected joint surplus as

$$E\Omega(b^{D}, b^{FT})|_{b^{D} \leq \gamma^{*} + b^{FT}} = V(FT) + \int_{R(b^{D})}^{\infty} [\gamma - \gamma^{*} - c(b^{D})] dH(\gamma)$$

$$- \int_{S(b^{D}) - S(b^{FT})}^{R(b^{D})} c(b^{e}(b^{D}; \gamma)) dH(\gamma) - \int_{0}^{S(b^{D}) - S(b^{FT})} c(b^{FT}) dH(\gamma)$$
(3)

Differentiating this expression with respect to  $b^{FT}$  and evaluating at  $(b^{FT}=0,b^D=\tilde{b}^D)$  we obtain

$$\left.\frac{\partial E\Omega}{\partial b^{FT}}\right|_{(b^{FT}=0,b^D=\tilde{b}^D)}=-h(S(\tilde{b}^D))c(b^e(\tilde{b}^D;S(\tilde{b}^D)))<0$$

where we have used the facts that  $S(b^{FT}) = b^{FT} + c(b^{FT})$  and c(0) = c'(0) = 0; it can easily be shown that  $b^e(\tilde{b}^D; S(\tilde{b}^D)) \neq 0$ ; and recall that we are assuming a large enough support of  $\gamma$ , hence  $h(S(\tilde{b}^D)) > 0$ . We can conclude that, when the support of  $\gamma$  is large enough,  $b^{FT} = 0$  cannot be optimal, and coupled with the fact that the optimal  $b^{FT}$  cannot be positive (as we next argue), this implies that the optimal  $b^{FT}$  is strictly negative.

We now rule out the possibility that  $b^D=0$  at an optimal contract. Given our results above, the only case we need to rule out is  $b^D=0 \le \gamma^* + b^{FT}$ . Letting  $\tilde{b}^{FT}$  denote the optimal value of  $b^{FT}$  conditional on  $b^D=0$ , we can write

$$\left.\frac{\partial E\Omega}{\partial b^D}\right|_{(b^D=0,b^{FT}=\tilde{b}^{FT})} = -\int\limits_{-S(\tilde{b}^{FT})}^{R(0)} \frac{dc(b^e(b^D;\gamma))}{db^D} dH(\gamma) + h(-S(\tilde{b}^{FT}))[c(b^e(0;-S(\tilde{b}^{FT}))-c(\tilde{b}^{FT})]$$

In this case it is immediate to establish that in the FT<sub>R</sub> region  $b^e \leq S^{-1}(-\gamma) < 0$ . It follows that at the lower border of the FT<sub>R</sub> region, where  $\gamma = -S(\tilde{b}^{FT})$ , it must be  $b^e \leq \tilde{b}^{FT} \leq 0$ . This implies  $c(b^e(0; -S(\tilde{b}^{FT})) \geq c(\tilde{b}^{FT})$ , so the second term of the expression above is nonnegative. Also recall that  $\frac{\partial b^e(b^D;\gamma)}{\partial b^D} > 0$ , hence  $\frac{dc(b^e(b^D;\gamma))}{db^D} < 0$ . We can conclude that  $\frac{\partial E\Omega}{\partial b^D}|_{(b^D=0,b^{FT}=\tilde{b}^{FT})} > 0$ , and hence  $b^D = 0$  cannot be optimal. This, together with the fact that the optimal  $b^D$  cannot be negative (as we argue below), implies that the optimal  $b^D$  is strictly positive.

We now return to our earlier claim that it cannot be optimal to set  $b^D < 0$  or  $b^{FT} > 0$  (recall that we just ruled out the possibilities  $b^D = 0$  and  $b^{FT} = 0$ , so we can focus on strict inequalities).

To establish this claim, we need to rule out several possibilities:

- (a)  $b^D < 0$  and  $b^{FT} < 0$ . We need to distinguish two subcases:
- $(a_i) \ b^D \ge \gamma^* + b^{FT}$ . In this case it is easy to show that the equilibrium transfer  $b^e$  in the renegotiation region  $(P_R)$  is negative. Our strategy to improve on the initial contract depends on whether  $b^D$  is higher or lower than  $b^e$  at the initial contract. If  $b^D < b^e$ , we can improve on the initial contract by increasing  $b^D$  slightly; to see this, refer to expression 1 and note that in this case  $c(b^D) > c(b^e)$  and  $c'(b^D) < 0$ , therefore  $\frac{\partial E\Omega}{\partial b^D}|_{b^D \ge \gamma^* + b^{FT}} > 0$ . If  $b^D > b^e$ , then we can improve on the initial contract by increasing  $b^{FT}$  slightly, because  $\frac{\partial E\Omega}{\partial b^{FT}}|_{b^D \ge \gamma^* + b^{FT}} > 0$ ; to see this, note that in this case  $c(b^D) c(b^e) < 0$ ,  $c'(b^{FT}) < 0$  and  $\frac{dc(b^e(b^{FT};\gamma))}{db^{FT}} < 0$  (and recall the assumption that  $1 + c'(\cdot) > 0$  for any transfer level).

 $(a_{ii}) \ b^D < \gamma^* + b^{FT}$ . Also in this case the equilibrium transfer  $b^e$  in the renegotiation region (FT<sub>R</sub>) is negative. Our strategy to improve on the initial contract depends on whether  $b^{FT}$  is higher or lower than  $b^e$  in absolute level. If  $|b^e| > |b^{FT}|$ , we can improve on the initial contract by increasing  $b^D$  slightly toward zero. This has three first-order beneficial effects: (i) it reduces the transfer (in absolute value) for states  $\gamma > R(b^D)$ , where the importer chooses P without renegotiating; (ii) it improves the threat point for the importer in the FT<sub>R</sub> region and hence it makes  $b^e$  less negative; (iii) for states just above  $\gamma = S(b^D) - S(b^{FT})$ , before the change governments renegotiate toward FT and after the change the importer chooses FT without renegotiating, thus the equilibrium transfer switches from  $b^e$  to  $b^{FT}$ ; since we are focusing on the case  $|b^e| > |b^{FT}|$ , also this effect is beneficial.

If  $|b^e| < |b^{FT}|$ , on the other hand, we can improve on the initial contract by increasing  $b^{FT}$  slightly toward zero. This has two beneficial first-order effects: (i) it reduces the transfer (in absolute value) for states  $\gamma < S(b^D) - S(b^{FT})$ , where the importer chooses FT without renegotiating, and (ii) for states just above  $\gamma = S(b^D) - S(b^{FT})$ , the equilibrium transfer switches from  $b^{FT}$  to  $b^e$ ; since we are focusing on the case  $|b^e| < |b^{FT}|$ , this effect is beneficial.

(b)  $b^D < 0$  and  $b^{FT} > 0$ .

It can be easily shown that we can lower  $b^{FT}$  to zero, and in fact we can make it slightly negative, without affecting the policy allocation or the equilibrium transfer for any  $\gamma$ . This takes us back to the previous case where  $b^D < 0$  and  $b^{FT} < 0$ , which we already ruled out.

(c)  $b^D > 0$  and  $b^{FT} > 0$ . Here our strategy to improve on the initial contract depends on whether  $b^D$  is higher or lower than  $\gamma^*$ . If  $b^D < \gamma^*$ , we can improve on the initial contract by increasing  $b^{FT}$  to zero. And if  $b^D < \gamma^*$ , the initial contract can be improved upon by increasing both  $b^D$  and  $b^{FT}$  toward zero in such a way that  $S(b^D) - S(b^{FT})$  is kept constant. We leave the proof of these claims to the reader.

Finally, the claim that an optimal contract entails  $b^D < \gamma^*$  follows from the fact that an optimal contract entails  $b^D < \gamma^* + b^{FT}$  and  $b^{FT} < 0$ .

Small uncertainty.

We can now turn to the case of small uncertainty (Proposition A1(i)).

The first observation is that a noncontingent allocation (where the same policy is chosen for all  $\gamma$  in the support) can be implemented at zero cost (i.e. with no transfers occurring in equilibrium) with a property rule. Thus, conditional on a noncontingent allocation being optimal, a property rule is optimal. We next show that if the support of  $\gamma$  is sufficiently small, a noncontingent allocation is indeed optimal.

Let the support of  $\gamma$  be given by  $(\gamma^* - \varepsilon, \gamma^* + \varepsilon)$ ; note that we are considering a symmetric support, but the argument is easily extended to the case of an asymmetric support. Consider a

contingent allocation with threshold  $\hat{\gamma}_{\varepsilon} \in (\gamma^* - \varepsilon, \gamma^* + \varepsilon)$  (we use the subscript  $\varepsilon$  because we need to allow this allocation to vary as we drive  $\varepsilon$  to zero). We have shown above that at an optimum it must be the case that for  $\gamma = \hat{\gamma}_{\varepsilon}$  the importer is indifferent between choosing P without renegotiating and renegotiating toward FT. In other words, it must be  $\hat{\gamma}_{\varepsilon} = R(b^D) = S(b^D) - S(b^D - \gamma^*)$ . This implies that for states  $\gamma$  just above  $\hat{\gamma}_{\varepsilon}$  the importer will pay a transfer  $b^D$  that is close to  $R^{-1}(\hat{\gamma}_{\varepsilon})$ ; clearly, this transfer does not become small as  $\varepsilon$  gores to zero. For states  $\gamma$  just below  $\hat{\gamma}_{\varepsilon}$  the governments will renegotiate and the equilibrium transfer may be lower, but this transfer is unrelated to  $\varepsilon$  and hence does not become small as the support shrinks.

Now consider replacing this contingent allocation with a noncontingent allocation where policy FT is chosen in all states (and no transfers are incurred). As  $\varepsilon \to 0$  this noncontingent allocation must dominate, because it implies a non-negligible savings in transfer costs for each state  $\gamma$ , while the associated loss in terms of policy efficiency is at most of magnitude  $\varepsilon$  for each state  $\gamma$ . Note that this argument holds even if the threshold  $\hat{\gamma}_{\epsilon}$  approaches one of the bounds of the support as  $\varepsilon \to 0$ . We have thus shown that if the support is sufficiently small, a noncontingent allocation must be optimal, and therefore a property rule is optimal. **QED** 

# Online Appendix B

In this Appendix we show that our key results survive in a setting with three discrete policy levels. In particular, we now allow that the Home government chooses a trade policy from the three options  $T \in \{FT, P_1, P_2\}$ : "Free Trade," or two possible levels of "Protection," with  $P_2 > P_1$ . For notational simplicity, we define  $P_0 \equiv FT$ . We think of each increment  $P_i - P_{i-1}$  as having the same magnitude for  $i = \{1, 2\}$ , and we let  $\gamma_i$  denote the incremental value to Home of switching from  $P_{i-1}$  to  $P_i$ , or

$$\gamma_i \equiv v(P_i) - v(P_{i-1}) \text{ for } i = \{1, 2\}.$$

Similarly,  $\gamma_i^*$  denotes the incremental cost to Foreign of a switch from  $P_{i-1}$  to  $P_i$  for  $i = \{1, 2\}$ , or

$$\gamma_i^* \equiv v^*(P_i) - v^*(P_{i-1}) \text{ for } i = \{1, 2\}.$$

As in our basic model of section 2, we abstract from the possibility of introducing a "carrot" in the contract, that is, we set the transfer associated with  $P_0$  equal to zero,  $b_0^D \equiv 0$ , but like our analysis in section 2 the analysis here could be extended to allow  $b_0^D \neq 0$ . In this setting, the exante contract now takes the form  $\{b_1^D, b_2^D\}$  where  $b_i^D \geq 0$  for  $i = \{1, 2\}$  represents the contractually specified damages to be paid by Home for the increment in protection  $P_i - P_{i-1}$ ; that is, under the contract  $\{b_1^D, b_2^D\}$ , if Home selects  $P_i$  for  $i = \{1, 2\}$  then its damage payments under the contract are given by  $\sum_{i=1}^{\tilde{\imath}} b_i^D$ .

To maintain tractability, we impose the following assumptions:

$$c(b) \equiv c \cdot |b|$$
 (Assumption B1)

$$\gamma_2 = \alpha \gamma_1$$
, with  $0 < \alpha \le \frac{\sigma c}{1 + \sigma (1 - c)}$  (Assumption B2)

$$\gamma_1^* = \gamma_2^* \equiv \gamma^* > 0$$
 (Assumption B3)

According to Assumptions B1-B3, transfer costs increase linearly in the magnitude of the ex-post transfer, Home payoffs are increasing and sufficiently concave in the level of Home protection,

and Foreign payoffs decline linearly in the level of Home protection. As will become clear below, with sufficient concavity we can avoid consideration of various "global" issues that would further complicate our analysis – such as the possibility of "large" renegotiations from  $P_2$  to  $P_0$  that might otherwise occur under the contracts we consider – and we can rely instead on a "local" analysis.

As before we refer to the outcome that maximizes joint surplus as the "first best" outcome. Under Assumptions B1-B3, the first best policy is  $P_0$  if  $\gamma_1 < \gamma^*$ ,  $P_1$  if  $\gamma_2 < \gamma^* < \gamma_1$ , and  $P_2$  if  $\gamma_2 > \gamma^*$  (and the first best level of the transfer is always b = 0). And again we assume that  $\gamma^*$  is known ex-ante, and that the  $\gamma_i$  are uncertain ex-ante and observed ex-post by the governments but not by the court/DSB. We have in mind that the  $\gamma_i$  are drawn from a joint distribution that satisfies Assumption B2 for any draw. We refer to a realization of  $(\gamma_1, \gamma_2)$  as a "state."

We establish below that the analogs to Propositions 1 and 2 from the main body of the paper extend to this setting. To accomplish this we first introduce the analogs of Figure 1 from the main body of the paper.

In particular, with  $\gamma_1$  on the vertical axis and  $b_1^D$  on the horizontal axis, Figure B1 depicts the "local" trade-offs involved in choosing between  $P_1$  and  $P_0$ . Similarly, with  $\gamma_2$  on the vertical axis and  $b_2^D$  on the horizontal axis, Figure B2 depicts the local trade-offs involved in choosing between  $P_2$  and  $P_1$ . For the consideration between  $P_1$  and  $P_0$ , Home's "local" threat point is  $P_1$  for  $\gamma_1 > S(b_1^D) \equiv (1+c)b_1^D$ , and it is  $P_0$  for  $\gamma_1 < S(b_1^D)$ . This is depicted in Figure B1. For the consideration between  $P_2$  and  $P_1$ , Home's local threat point is  $P_2$  for  $\gamma_2 > S(b_1^D + b_2^D) - S(b_1^D) = S(b_2^D) \equiv (1+c)b_2^D$ , and it is  $P_1$  for  $\gamma_2 < S(b_2^D)$ , where we have used Assumption B1 to characterize this local threat point as a function only of  $b_2^D$ . This is depicted in Figure B2.

Focusing first on Figure B1, which depicts the local trade-offs involved in choosing between  $P_1$  and  $P_0$ , there are two possibilities for "local" renegotiation: it may be that the threat point is  $P_0$  and renegotiation changes the policy to  $P_1$ , or vice-versa. Local renegotiation from  $P_0$  to  $P_1$  would occur if (i) the local threat point is  $P_0$ , so  $\gamma_1 < S(b_1^D)$ , and (ii) there exists a transfer  $b^e$  such that both governments are made better off by the policy change, which requires  $\gamma_1 > S(b^e)$  (for the importer) and  $b^e > \gamma^*$  (for the exporter); this in turn implies the condition  $\gamma_1 > S(\gamma^*)$ . Putting things together, local renegotiation from  $P_0$  to  $P_1$  would occur if  $S(\gamma^*) < \gamma_1 < S(b_1^D)$ ; we label this region by  $RN_{P_0 \to P_1}$ . Applying a similar logic, local renegotiation from  $P_1$  to  $P_0$  would occur if (i) the local threat point is  $P_1$ , so  $\gamma_1 > S(b_1^D)$ , and (ii) there exists a (possibly negative) transfer  $b^e$  such that both governments are made better off by the policy change, which requires  $S(b_1^D) - S(b^e) > \gamma_1$  (for the importer) and  $\gamma^* > b_1^D - b^e$  (for the exporter); this (it can be shown) implies the condition  $\gamma_1 < (1-c)\gamma^* + 2cb_1^D \equiv R_1(b_1^D)$ . Thus local renegotiation from  $P_1$  to  $P_0$  would occur if  $S(b_1^D) < \gamma_1 < R_1(b_1^D)$ ; we label this region by  $RN_{P_1 \to P_0}$ . Notice that Figure B1 is just a linear version of Figure 1 from the main body of the paper.

We focus next on Figure B2, which depicts the local trade-offs involved in choosing between  $P_2$  and  $P_1$ . Applying a similar logic as above, it is direct to verify that local renegotiation from  $P_1$  to  $P_2$  would occur if  $S(\gamma^*) < \gamma_2 < S(b_2^D)$ , a region that we label by  $RN_{P_1 \to P_2}$ , while local renegotiation from  $P_2$  to  $P_1$  would occur if  $S(b_1^D) < \gamma_2 < \min[(1-c)\gamma^* + 2c(b_1^D + b_2^D), S(\gamma^*)] \equiv R_2(b_1^D + b_2^D)$ , a region that we label by  $RN_{P_2 \to P_1}$ . Note that Figure B2 differs from Figure B1 in one important respect: the R curve now takes on the value of  $S(\gamma^*)$  for  $b_2^D$  in a left interval of  $\gamma^*$ , and when  $b_1^D \ge \gamma^*$  it becomes horizontal at  $S(\gamma^*)$ .

Figures B1 and B2 can be used for "local analysis" of the choice between the local alternatives  $P_1$  and  $P_0$  and the local alternatives  $P_2$  and  $P_1$ , but as we next establish our assumptions imply that certain global conclusions can also be drawn from local analysis:

**Lemma B1**: Under Assumptions B1-B3: (i) If renegotiation from  $P_0$  to  $P_1$  cannot achieve a Pareto improvement, neither can renegotiation from  $P_0$  to  $P_2$ ; (ii) If renegotiation from  $P_2$  to  $P_1$  cannot achieve a Pareto improvement, neither can renegotiation from  $P_2$  to  $P_0$ .

**Proof**: (i) As implied by our discussion above, when  $P_0$  is the threat point renegotiation to  $P_1$  cannot achieve a Pareto improvement if  $\gamma_1 < (1+c)\gamma^*$ . Since  $\gamma_2 < \gamma_1$  this implies  $\gamma_1 + \gamma_2 < 2(1+c)\gamma^*$ . This in turn implies (it is easy to show) that when  $P_0$  is the threat point, renegotiation to  $P_2$  cannot achieve a Pareto improvement either. (ii) As implied by our discussion above, when  $P_2$  is the threat point renegotiation to  $P_1$  cannot achieve a Pareto improvement if  $\gamma_2 > \min[(1-c)\gamma^* + 2c(b_1^D + b_2^D), (1+c)\gamma^*]$ . Since  $\gamma_2 < \gamma_1$  this implies in turn that  $\gamma_1 + \gamma_2 > \min[2(1-c)\gamma^* + 4c(b_1^D + b_2^D), 2(1+c)\gamma^*] \ge \min[2(1-c)\gamma^* + 2c(b_1^D + b_2^D), 2(1+c)\gamma^*] = 2(1-c)\gamma^* + 2c(b_1^D + b_2^D)$ , where the last equality follows (it is easy to show) from the fact that  $P_2$  is the threat point. With  $\gamma_1 + \gamma_2 > 2(1-c)\gamma^* + 2c(b_1^D + b_2^D)$ , and with a similar argument to that used above, it then follows that when  $P_2$  is the threat point renegotiation to  $P_0$  cannot achieve a Pareto improvement either. QED

According to Lemma B1, if local renegotiation cannot achieve a Pareto improvement, then global renegotiation cannot achieve a Pareto improvement either.

**Lemma B2**: Under Assumptions B1-B3, renegotiation from  $P_2$  to  $P_0$  cannot occur for any state if  $b_1^D > \gamma^*$ .

**Proof**: We start by noting that, if renegotiation from  $P_2$  to  $P_0$  is to occur, the renegotiated transfer must be negative (because otherwise Home would be better off choosing the contractual  $P_0$  option with no transfer); we let  $\hat{b}_0^e < 0$  denote the transfer associated with renegotiation from  $P_2$  to  $P_0$ . Also, for the exporter to be better off under the renegotiated outcome, it must be  $\hat{b}_0^e > b^D - 2\gamma^*$ , where  $b^D \equiv b_1^D + b_2^D$ . Therefore a necessary condition for there to be renegotiation from  $P_2$  to  $P_0$  is  $b^D < 2\gamma^*$ . And since we are considering a contract where  $b_1^D > \gamma^*$ , clearly  $b^D > \gamma^*$ . Thus if renegotiation from  $P_2$  to  $P_0$  is to occur when  $b_1^D > \gamma^*$ , we must have  $\gamma^* < b^D < 2\gamma^*$ .

Next we derive another necessary condition for there to be renegotiation from  $P_2$  to  $P_0$  in some states.

Notice that for  $P_2$  to be the threat point it must be that  $\gamma_1 > \frac{(1+c)b^D}{1+\alpha}$ . Letting  $\hat{b}_{2\to 1}^e$  denote the transfer associated with the Nash Bargaining solution when the threat point is  $P_2$  and renegotiation is to  $P_1$ , we have

$$\hat{b}_{2\to 1}^e = b^D - \frac{\sigma(1+c)\gamma^* + (1-\sigma)\alpha\gamma_1}{(1+c)}.$$

Plugging  $\hat{b}_{2\to 1}^e$  into the Nash Bargaining objective (labeled NB) when the bargain leads to  $P_1$  and simplifying then yields

$$NB_{P_1} = \sigma^{\sigma} \left(\frac{1-\sigma}{1+c}\right)^{1-\sigma} [(1+c)\gamma^* - \alpha\gamma_1].$$

With a similar calculation we can derive that  $\hat{b}_{2\to 0}^e$ , the transfer associated with renegotiation from  $P_2$  to  $P_0$ , is given by

$$\hat{b}_{2\to 0}^e = \frac{-(1-c)\sigma(2\gamma^* - b^D) + (1-\sigma)[(1+c)b^D - (1+\alpha)\gamma_1]}{(1-c)}.$$

Plugging  $\hat{b}_{2\to 0}^e$  into the Nash Bargaining objective when the bargain leads to  $P_0$  and simplifying

then yields

$$NB_{P_0} = \sigma^{\sigma} \left( \frac{1-\sigma}{1-c} \right)^{1-\sigma} [(1-c)2\gamma^* + 2cb^D - (1+\alpha)\gamma_1].$$

Now solving for  $\gamma_1$  that satisfies  $NB_{P_0} > NB_{P_1}$  yields

$$\gamma_1 < \frac{(1+c)^{1-\sigma}(1-c)2\gamma^* - (1+c)(1-c)^{1-\sigma}\gamma^* + 2c(1+c)^{1-\sigma}b^D}{(1+c)^{1-\sigma}(1+\alpha) - \alpha(1-c)^{1-\sigma}}.$$

Putting together this condition and the condition that  $P_2$  is the threat point, i.e.  $\gamma_1 > \frac{(1+c)b^D}{1+\alpha}$ , we conclude that a necessary condition for renegotiation from  $P_2$  to  $P_0$  to occur in any state is

$$\frac{(1+c)^{1-\sigma}(1-c)2\gamma^* - (1-c)^{1-\sigma}(1+c)\gamma^* + (1+c)^{1-\sigma}2cb^D}{(1+c)^{1-\sigma}(1+\alpha) - (1-c)^{1-\sigma}\alpha} > \frac{(1+c)b^D}{1+\alpha}.$$

This can be re-arranged as

$$[(1+\alpha)(1-c)^{\sigma}(1+c)^{-\sigma} - \alpha]b^{D} < (1+\alpha)[2(1-c)^{\sigma}(1+c)^{-\sigma} - 1]\gamma^{*}.$$

We now show that under Assumption B2 the inequality just above is violated for all  $b^D \in (\gamma^*, 2\gamma^*)$  and hence renegotiation from  $P_2$  to  $P_0$  can never occur. This follows from three key observations: (a) it can never be the case that the LHS is negative and the RHS is positive (this is easy to verify); (b) if both sides of the inequality are negative, it is straightforward to confirm that the condition is violated for  $b^D = 2\gamma^*$ , and therefore it is violated for all  $b^D \in (\gamma^*, 2\gamma^*)$ ; and (c) if both sides of the inequality are positive, one can plug in  $b^D = \gamma^*$  and verify that the condition is violated if  $\alpha < (1+c)^{\sigma} (1-c)^{-\sigma} - 1$ , which (it can be shown) is implied by our restriction  $\alpha \leq \frac{\sigma c}{1+\sigma(1-c)}$  in Assumption B2; thus under this parameter restriction the condition is violated for all  $b^D \in (\gamma^*, 2\gamma^*)$ . QED

Finally we will make use of the following:

**Lemma B3**: Under Assumptions B1-B3, if the threat point is  $P_0$ , then the policy outcome is as follows: if  $\gamma_1 < (1+c)\gamma^*$ , the policy remains  $P_0$ ; if  $\gamma_2 < (1+c)\gamma^* < \gamma_1$ , the policy is renegotiated to  $P_1$ ; if  $\gamma_2 > (1+c)\gamma^*$ , the policy is renegotiated to  $P_2$ .

**Proof**: This is easily shown graphically by considering the Pareto frontier in  $(\omega^*, \omega)$  space for a given state  $(\gamma_1, \gamma_2)$ . With  $\omega$  on the vertical axis and  $\omega^*$  on the horizontal axis, in what follows we refer to "point  $P_i$ " as the point in  $(\omega^*, \omega)$  space that is associated with policy  $P_i$  and zero transfer. The Pareto frontier is the outer envelope of three sub-frontiers, each associated with a protection level  $P_i$ . If the threat point is  $P_0$ , clearly any renegotiation will entail a positive transfer from Home to Foreign. This implies that the relevant part of the Pareto sub-frontier associated with policy  $P_1$  is the straight line emanating from point  $P_1$  and going South-East with slope -(1+c), and similarly for the Pareto sub-frontier associated with  $P_2$ . From graphical inspection it is then clear that if the threat point is  $P_0$  the relevant part of the Pareto frontier, i.e. the part that lies North-East of  $P_0$ , is given by the sub-frontier associated with policy  $P_1$  (resp.  $P_2$ ) if  $P_2 < (1+c)\gamma^* < \gamma_1$  (resp.  $P_2 > (1+c)\gamma^*$ ), while if  $P_1 < (1+c)\gamma^*$  point  $P_2$  is Pareto-undominated. The claim follows immediately. QED

We now establish that the analog of Proposition 1 in the main body of the paper holds in this extended setting. We state:

**Proposition B1.** Under Assumptions B1-B3, setting  $b_i^D > \gamma^*$  for any  $i \in \{1,2\}$  is weakly dominated. Furthermore, renegotiation can only reduce the level of protection.

We prove the first part of the Proposition by considering three cases in turn: (i) the case  $b_1^D \leq \gamma^*, b_2^D > \gamma^*;$  (ii) the case  $b_1^D > \gamma^*, b_2^D > \gamma^*;$  and (iii) the case  $b_1^D > \gamma^*, b_2^D \leq \gamma^*.$  (i) Suppose  $b_1^D \leq \gamma^*, b_2^D > \gamma^*.$  We show that we can (weakly) improve on this contract by

- lowering  $b_2^D$  to  $\gamma^*$ . There are a few possibilities to consider:
- $(i_a)$  First focus on states for which  $P_0$  is neither the threat point nor the renegotiated outcome, before or after the contract change. Then clearly the relevant choice is between  $P_1$  and  $P_2$ , and hence the only relevant figure is Figure B2, and we can apply the same argument as in the main text to show that, for these states, lowering  $b_2^D$  to  $\gamma^*$  can only increase joint surplus.
- $(i_b)$  Next focus on states for which  $P_0$  is the renegotiated outcome before or after the contract change. First note that there cannot be renegotiation from  $P_2$  to  $P_0$  before or after the contract change: this is because, by inspection of Figure B2, when  $b_2^D \geq \gamma^*$  there can never be renegotiation from  $P_2$  to  $P_1$ , and therefore by Lemma B1 there cannot be renegotiation from  $P_2$ to  $P_0$ . Next suppose there is renegotiation from  $P_1$  to  $P_0$  before the contract change. Clearly, the reduction in  $b_2^D$  can have an effect only if the threat point switches to  $P_2$ . But this can never happen: if there is renegotiation from  $P_1$  to  $P_0$  before the contract change, by inspection of Figure B1 it must be  $\gamma_1 < (1+c)\gamma^*$ ; and if the threat point is  $P_2$  after the change then  $\gamma_2 > (1+c)\gamma^*$ ; but this implies  $\gamma_2 > \gamma_1$ , which is impossible because  $\alpha \leq 1$ . Finally, suppose there is renegotiation from  $P_1$  to  $P_0$  after the contract change. This implies  $\gamma_1 > (1+c)b_1^D$ . Then the threat point must have been  $P_1$  also before the contract change (it cannot have been  $P_2$  because the threat level of protection can only move up with a reduction in  $b_2^D$ ; and it cannot have been  $P_0$  because then  $\gamma_1 < (1+c)b_1^D$ , a contradiction with the condition just above). But if the threat point was  $P_1$ before the contract change, from inspection of Figure B1 there must have been renegotiation from  $P_1$  to  $P_0$  also before the contract change, so the contract change is immaterial.
- $(i_c)$  What remains is to consider states for which  $P_0$  is the threat point before or after the contract change. If  $P_0$  is not the threat point initially, clearly it cannot be the threat point after the contract change, so we just need to consider states for which  $P_0$  is the threat point initially. Notice first that since  $b_1^D < \gamma^*$  and by inspection of Figure B1 (and applying Lemma B1), in this case there can never be renegotiation to a higher level of protection, so if the threat point is  $P_0$  the policy outcome is also  $P_0$ . Also note that the threat point cannot switch from  $P_0$  to  $P_1$ , because  $b_2^D$  does not affect the importer's choice between  $P_0$  and  $P_1$ . In principle the contract change could have an effect if it leads the threat point to switch from  $P_0$  to  $P_2$ , but this can never happen. To see this, note that for  $P_0$  to be the threat point before the contract change we must have  $\gamma_1 < (1+c)b_1^D < (1+c)\gamma^*$  and hence  $\gamma_2 < (1+c)\gamma^*$ , but this contradicts the possibility that  $P_2$  is the threat point after the contract change, because it implies that Home prefers  $P_1$  to  $P_2$  when  $b_2^D = \gamma^*$ .
- (ii) Suppose  $b_1^D > \gamma^*, b_2^D > \gamma^*$ . We show that we can increase joint surplus by setting  $b_1^D =$

Note first that, since  $b_1^D > \gamma^*$  and  $b_2^D > \gamma^*$  in the initial contract, by inspection of Figures B1 and B2 there cannot be renegotiation downwards for any state. And since  $b_1^D = b_2^D = \gamma^*$  in the new contract, there is no renegotiation for any state under the new contract. Moreover, it is easy

to show that the contract change cannot affect the policy outcome. It remains to ask: How does the contract change affect the equilibrium transfer? For states in which there was no renegotiation before the contract change, clearly the transfer can only get smaller. For states in which there was renegotiation (upwards) before the contract change, clearly the renegotiated transfer had to be at least equal to  $\gamma^*$  if renegotiation was up to  $P_1$ , and at least equal to  $2\gamma^*$  if renegotiation was up to  $P_2$  (this follows immediately from the fact that  $b_1^D > \gamma^*$  and  $b_2^D > \gamma^*$  in the initial contract, and renegotiation must make the exporter weakly better off than under the initial contract), and therefore the transfer cannot become larger as a result of the contract change.

(iii) Suppose  $b_1^D > \gamma^*, b_2^D \le \gamma^*$ . We show that we can increase joint surplus by setting  $b_1^D = \gamma^*$ . We start with a few preliminary observations.

First recall from Lemma B2 that given  $b_1^D > \gamma^*$  renegotiation from  $P_2$  to  $P_0$  cannot occur for any state. Also, from Figure B1 it is clear that renegotiation from  $P_1$  to  $P_0$  cannot occur either, so the only possible type of renegotiation downwards is from  $P_2$  to  $P_1$ . In this latter case, notice that the renegotiated transfer must be nonnegative; this follows because at the threat point the transfer is  $b^D > \gamma^*$  and renegotiation from  $P_2$  to  $P_1$  provides a gain of  $\gamma^*$  to the exporter, so for the renegotiation to make the exporter weakly better off, the transfer cannot be reduced by more than  $\gamma^*$  relative to  $b^D$ . Next note that renegotiation upwards can only occur from  $P_0$  to  $P_1$  or  $P_2$ , and only under the initial contract: renegotiation from  $P_1$  to  $P_2$  is ruled out by inspection of Figure B2 given  $b_2^D \leq \gamma^*$ ; and any type of renegotiation upwards is clearly ruled out under the new contract. Finally, recall that if  $b_1^D \geq \gamma^*$  the  $R_2$  curve in Figure B2 is horizontal.

To establish that joint surplus increases if we reduce  $b_1^D$  to  $\gamma^*$ , we need to consider several possibilities:

- (iii<sub>a</sub>) First focus on states for which  $P_2$  is neither the threat point nor the renegotiated outcome, before or after the contract change. Then clearly the relevant choice is between  $P_0$  and  $P_1$  and hence the only relevant figure is Figure B1, and we can apply the same argument as in the main text to show that, for these states, lowering  $b_1^D$  to  $\gamma^*$  can only increase joint surplus.
- (iii<sub>b</sub>) Next focus on states for which  $P_2$  is the renegotiated outcome before or after the contract change. Given our preliminary observations above, the only such case is the one in which there is renegotiation from  $P_0$  to  $P_2$  under the initial contract. In this case, after the contract change it must be that the threat point is  $P_2$  and there is no renegotiation. To see this, note first that for there to be renegotiation from  $P_0$  to  $P_2$ , by Lemma B3 it must be  $\gamma_2 > (1+c)\gamma^*$ , which implies  $\gamma_2 > (1+c)b_2^D$  (because  $b_2^D < \gamma^*$ ) and  $\gamma_1 + \gamma_2 > (1+c)2\gamma^*$  (because  $\gamma_1 > \gamma_2$ ). And since the new contract has  $b^D < 2\gamma^*$ , then  $P_2$  must be the new threat point. But then the contract change can only increase joint surplus, because the policy outcome is unaffected and the transfer must be weakly lower, by a now-familiar argument.
- (iii<sub>c</sub>) Next we focus on states for which  $P_2$  is the threat point before or after the contract change (and  $P_2$  is not the renegotiated outcome before or after the contract change, since we already considered these cases in (iii<sub>b</sub>)). Consider first the case where  $P_2$  is the threat point under the initial contract. Since a reduction in  $b_1^D$  makes  $P_0$  less attractive relative to higher levels of protection and does not affect the tradeoff between  $P_1$  and  $P_2$ , then  $P_2$  must remain the threat point after the contract change. By inspection of Figure B2 and recalling that renegotiation downwards can only be to  $P_1$ , the policy outcome is then unaffected: either it is  $P_2$  before and after the contract change, or there is renegotiation from  $P_2$  to  $P_1$  before and after the contract change. How is the transfer affected by the contract change? If the policy outcome is  $P_2$ , the transfer is reduced directly; and if the policy is renegotiated to  $P_1$ , it is easy to see (by inspection of the expression for  $\hat{b}_{2\rightarrow 1}^e$  in the

proof of Lemma B2) that the renegotiated transfer is increasing in  $b^D$ , and hence it decreases as a result of the reduction in  $b_1^D$ .

It remains to consider states for which the threat point switches from  $P_0$  to  $P_2$  as a result of the contract change (recall from our discussion above that the threat point cannot switch from  $P_1$  to  $P_2$ ). Here there are two possibilities to consider: (1) under the initial contract there is renegotiation from  $P_0$  to  $P_1$ ; and (2) under the initial contract there is no renegotiation. (The case in which under the initial contract there is renegotiation from  $P_0$  to  $P_2$  was already considered in (iii<sub>b</sub>)).

Starting from case (1), by Lemma B3 it must be  $\gamma_2 < (1+c)\gamma^* < \gamma_1$ , and since  $P_2$  is the threat point after the contract change, it must be  $\gamma_2 > (1+c)b_2^D$ . But then by inspection of Figure B2 there must be renegotiation from  $P_2$  to  $P_1$  after the contract change, hence the policy outcome is unaffected. How does the contract change affect the transfer? Note that the disagreement payoff of Home increases weakly, because  $b^D$  is reduced, and the disagreement payoff of Foreign decreases by the amount  $2\gamma^* - b^D$ , which is nonnegative because  $b^D < 2\gamma^*$  after the contract change. As a consequence, the renegotiated transfer decreases weakly.

Moving to case (2), we first argue that after the contract change there must be renegotiation from  $P_2$  to  $P_1$ . To see this, note that for the policy outcome to be  $P_0$  under the initial contract, it must be  $\gamma_2 < \gamma_1 < (1+c)\gamma^*$ , otherwise by Lemma B3 there would be renegotiation upwards. But this, together with the fact that the threat point is  $P_2$  after the contract change, by inspection of Figure B2 implies that there must be renegotiation from  $P_2$  to  $P_1$  after the contract change (and recall that renegotiation from  $P_2$  to  $P_0$  is ruled out by Lemma B2). In this case, the change in joint surplus implied by the contract change is given by  $\gamma_1 - \gamma^* - c\hat{b}_{2\rightarrow 1}^e$ , where  $\hat{b}_{2\rightarrow 1}^e = b^{D'} - \frac{\sigma(1+c)\gamma^* + (1-\sigma)\alpha\gamma_1}{(1+c)}$  is the transfer associated with renegotiation from  $P_2$  to  $P_1$  (derived in the proof of Lemma B2), and  $b^{D'}$  denotes the level of  $b^D$  after the contract change. Noting that  $\gamma_1 \in (\frac{1+c}{1+\alpha}b^{D'}, (1+c)\gamma^*)$  (the lower bound on  $\gamma_1$  follows because the threat point after the contract change is  $P_2$ , and the upper bound comes from the observation above) and that  $b^{D'} \in (\gamma^*, 2\gamma^*)$ , one can verify that under our Assumption B2 it must be that  $\gamma_1 - \gamma^* - c\hat{b}_{2\rightarrow 1}^e \ge 0$  for all  $\gamma_1 \in (\frac{1+c}{1+\alpha}b^{D'}, (1+c)\gamma^*)$  and  $b^{D'} \in (\gamma^*, 2\gamma^*)$ , thus ensuring that the contract change is weakly beneficial.

The second statement of Proposition B1 is straightforward to show. First, if  $b_1^D \leq \gamma^*$  and  $b_2^D \leq \gamma^*$  then it is clear from Figures B1 and B2 that renegotiation can only be liberalizing. And if it is weakly optimal to set  $b_i^D > \gamma^*$  for some i, then by a similar argument to that in the main text (see especially note 28), it must be that there is zero probability that the state  $(\gamma_1, \gamma_2)$  is such that the protection level is renegotiated upwards. QED

We note that, as in the case of binary policy, also in this extended setting renegotiation can occur in equilibrium only for intermediate states of the world, provided the damage payments  $b_i^D$  are positive. More specifically, if  $b_1^D > 0$  and  $b_2^D > 0$  then it is clear from Figures B1 and B2 that renegotiation can occur in equilibrium only for intermediate values of  $\gamma_1$  and  $\gamma_2$ .

We next establish that the analog of Proposition 2 in the main body of the paper holds in this extended setting. To this end, we first formalize what we mean in this extended setting by a "property rule" and a "liability rule." In line with the main body of the paper, we define a property rule as a contract that specifies the allowable levels of protection without providing a buy-out option; that is, each level of protection  $(P_0, P_1 \text{ or } P_2)$  is either allowed (with no compensation owed) or not allowed at all. Within this contract class, there is no loss of generality in focusing on the following three contracts: (a) a contract specifying that only  $P_0$  is allowed (a strict FT obligation); (b) a contract specifying that only  $P_0$  and  $P_1$  are allowed (i.e. protection is "capped,"

or "bound," at  $P_1$ ); and (c) the fully discretionary contract specifying that any level of protection  $(P_0, P_1 \text{ or } P_2)$  is allowed, which is equivalent to having no contract at all (the "empty" contract).

In analogy with the case of binary trade policy considered in the main text, each of the above property-rule contracts is outcome-equivalent to a contract that provides a buy-out option but where this option is prohibitively costly. In particular, contract (a) above corresponds to setting  $b_1^D \geq \frac{\bar{\gamma}_1}{1+c}$  (so that  $P_0$  is always preferred to  $P_1$  and a payment of  $b_1^D$ ) and  $b_1^D + b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c}$  (so that  $P_0$  is always preferred to  $P_2$  and a payment of  $b_1^D + b_2^D$ ); contract (b) above corresponds to setting  $b_1^D = 0$  (so that  $P_1$  is allowed with no compensation owed) and  $b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c}$  (so that  $P_2$  and a payment of  $b_2^D$  is never preferred); and contract (c) corresponds to setting  $b_1^D = b_2^D = 0$ .

By contrast, a liability rule is a contract that includes a non-prohibitive buy-out option, so that for some state  $(\gamma_1, \gamma_2)$  Home would "buy" protection  $(P_1 \text{ or } P_2)$  in exchange for a positive contractually-specified transfer. It is easy to verify that this corresponds to setting either  $0 < b_1^D < \frac{\bar{\gamma}_1}{1+c}$ , or  $b_2^D < \frac{(1+\alpha)\bar{\gamma}_1}{1+c} - b_1^D$ .

To establish the analog of Proposition 2 in our extended setting, we now also assume that the empty contract  $b_1^D = b_2^D = 0$  (which is equivalent to the absence of an ex-ante contract) is suboptimal.<sup>2</sup> We need this restriction only for establishing the large-uncertainty result stated below, where it serves to offset an artifact of our linear cost assumption (Assumption B1) which does not arise with the convex costs that we assume in the body of the paper, namely that a small transfer causes a first-order deadweight loss. We conjecture that with convex costs in our extended setting the empty contract would always be suboptimal in the presence of large uncertainty, just as in the case of binary policy.

We may now state the analog of Proposition 2 in the main body of the paper:

**Proposition B2.** Under Assumptions B1-B3: (i) If the support of each  $\gamma_i$  is sufficiently small, a property rule is optimal; (ii) If the support of each  $\gamma_i$  is sufficiently large, a liability rule is optimal. **Proof**:

- (i) Consider first the case of small uncertainty. We wish to establish that the optimal contract is a property rule. The argument proceeds along similar lines to that in the main body of the paper, but there are now six cases to consider:
- (i<sub>a</sub>) Each support lies below  $\gamma^*$ , that is,  $\gamma^* \geq \bar{\gamma}_1 \geq \bar{\gamma}_2$ , so that the first-best policy is  $P_0$  in every state of the world. It is intuitively clear and direct from Figures B1 and B2 that in this case the optimal contract is the strict-FT contract (that is,  $b_1^D \geq \frac{\bar{\gamma}_1}{1+c}$ ,  $b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D$ ). The key feature to notice from Figures B1 and B2 is that this contract will not be renegotiated when  $\gamma^* \geq \bar{\gamma}_1 \geq \bar{\gamma}_2$  this follows because from Figures B1 and B2 it is clear that local renegotiation cannot achieve a Pareto improvement in this case, and from Lemma B1 global renegotiation cannot therefore achieve a Pareto improvement either and the first best is therefore achieved.
- (i<sub>b</sub>) Each support lies above  $\gamma^*$ , that is,  $\underline{\gamma}_1 \geq \underline{\gamma}_2 \geq \gamma^*$ . In this case, the first best is always  $P_2$ , and an argument similar to the one just above establishes that the optimal contract would be the empty contract ( $b_1^D = b_2^D = 0$ ); but recall that we are assuming that the empty contract is suboptimal, so this case is assumed away.

<sup>&</sup>lt;sup>1</sup>It is easy to see that no property rule can strictly improve on the three property rules described above. This is an immediate implication of the fact that Home will always choose the highest level of protection among those allowed in the contract.

<sup>&</sup>lt;sup>2</sup> A sufficient condition for this to be the case is that  $\Pr(\gamma_2 > \gamma^*)$  is not too high, or that  $\alpha$  is sufficiently below 1 (with the upper bound on  $\alpha$  possibly being below the bound specified in Assumption B2). In this case, it is straighforward to show that the empty contract is strictly dominated by some non-empty contract (with  $b_i^D > 0$  for some i).

- (i<sub>c</sub>) The supports of  $\gamma_1$  and  $\gamma_2$  lie on the opposite sides of  $\gamma^*$ , that is,  $\underline{\gamma}_1 \geq \gamma^* \geq \overline{\gamma}_2$  so that the first-best policy is  $P_1$  in every state of the world. For this case it is intuitive and direct from Figures B1 and B2 that capping protection at  $P_1$  (that is,  $b_1^D = 0$ ,  $b_2^D \geq \frac{(1+\alpha)\overline{\gamma}_1}{1+c}$ ) is optimal, ensuring that  $P_1$  is always the policy outcome and no transfers are induced in equilibrium, thereby achieving the first best.
- $(i_d)$  A fourth case arises when  $\gamma^* \geq \bar{\gamma}_2$  and  $\bar{\gamma}_1 > \gamma^* > \underline{\gamma}_1$ . Here the support of  $\gamma_2$  lies below  $\gamma^*$ , so that in no state of the world would  $P_2$  be the first-best policy, while the support of  $\gamma_1$  includes  $\gamma^*$ . If the support of  $\gamma_1$  is nevertheless sufficiently small, we now argue that the optimal contract must be either the strict-FT contract  $(b_1^D \geq \frac{\bar{\gamma}_1}{1+c}, b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D)$  or the protection-cap contract  $(b_1^D = 0, b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c})$ . To see this, note from Figure B1 that, if the support of  $\gamma_1$  around  $\gamma^*$  is sufficiently small,  $b_1^D \geq \frac{\bar{\gamma}_1}{1+c}$  induces no renegotiation for any  $\gamma_1$ ; and similarly from Figure B2 with  $\gamma^* \geq \bar{\gamma}_2$ ,  $b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D$  induces no renegotiation for any  $\gamma_2$ . Hence the strict-FT contract induces zero transfers in equilibrium and results in the non-contingent policy outcome  $P_0$ . As Figures B1 and B2 make clear, a liability rule may achieve a more efficient policy allocation, since such a rule can induce  $P_0$  for low values of  $\gamma_1$  and  $P_1$  for sufficiently high values of  $\gamma_1$  either by setting  $0 < b_1^D < \frac{\bar{\gamma}_1}{1+c}$  in order to achieve  $P_1$  directly for high- $\gamma_1$  states (i.e., when  $\gamma_1 > R_1(b_1^D)$ ), or by setting  $b_1^D > \frac{\bar{\gamma}_1}{1+c}$  and  $b_2^D < \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D$  to achieve  $P_1$  indirectly in some high- $\gamma_1$  states by inducing renegotiation from  $P_2$  to  $P_1$  but the associated benefit is small because the support of  $\gamma_1$  around  $\gamma^*$  is small. And the cost of achieving this state-contingency is not small, because (it can easily be shown) the equilibrium transfers associated with it do not become negligible as the support shrinks. Thus a liability rule is dominated by the strict FT contract (and, by a similar argument, also by the protection-cap contract).
- $(i_e)$  A fifth case arises when  $\underline{\gamma}_1 \geq \gamma^*$  and  $\bar{\gamma}_2 > \gamma^* > \underline{\gamma}_2$ . Here the support of  $\gamma_1$  lies above  $\gamma^*$ , so that in no state of the world would  $P_0$  be the first-best policy, while the support of  $\gamma_2$  includes  $\gamma^*$ . If the support of  $\gamma_2$  is nevertheless sufficiently small, then proceeding as before (and recalling the assumption that the empty contract is suboptimal) we now argue that the optimal contract must be the protection-cap contract. To see this, note from Figure B2 that, if the support of  $\gamma_2$  around  $\gamma^*$  is sufficiently small,  $b_2^D \geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D$  will not induce renegotiation for any  $\gamma_2$ ; and similarly from Figure B1 with  $\gamma^* \leq \underline{\gamma}_1$ ,  $b_1^D = 0$  induces no renegotiation for any  $\gamma_1$ . Hence the protection-cap contract induces zero transfers in equilibrium and results in the non-contingent policy outcome  $P_1$ . As Figures B1 and B2 make clear, a liability rule may achieve a more efficient policy allocation by setting  $0 < b_2^D < \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D$  in order to induce  $P_0$  for low values of  $\gamma_2$  and achieve  $P_2$  for high- $\gamma_2$  states (i.e., when  $\gamma_2 > R_2(b_2^D)$ ), but the associated benefit is small because the support of  $\gamma_2$  around  $\gamma^*$  is small. And the cost of achieving this state-contingency is not small, because the equilibrium transfers associated with it (it can be shown) do not become negligible as the support shrinks. Thus a liability rule is dominated by the protection-cap contract.
- $(i_f)$  A final case in principle would arises if  $\bar{\gamma}_1 > \gamma^* > \underline{\gamma}_1$  and  $\bar{\gamma}_2 > \gamma^* > \underline{\gamma}_2$ , so that  $\gamma^*$  is included in the support of both  $\gamma_1$  and  $\gamma_2$ . But note that, since  $\alpha < 1$  by assumption, holding  $\alpha$  fixed if the support of  $\gamma_1$  (and hence of  $\gamma_2$ ) is sufficiently small this case cannot arise.<sup>3</sup>
- (ii) Consider next the case of large uncertainty. In particular, suppose  $\bar{\gamma}_i > S(\gamma^*)$  for i=1,2. We wish to establish that in this case the optimal contract is a liability rule, that is (as we highlighted above) either  $0 < b_1^D < \frac{\bar{\gamma}_1}{1+c}$  or  $b_2^D < \frac{(1+\alpha)\bar{\gamma}_1}{1+c} b_1^D$ . We prove this claim by ruling out the optimality

<sup>&</sup>lt;sup>3</sup>Making the support of  $\gamma_1$  small enough that case  $(i_f)$  cannot arise simplifies the proof, but this does not mean that case  $(i_f)$  can never arise when a property rule is optimal. It may well be that case  $(i_f)$  arises and the support of  $\gamma_1$  is sufficiently small that a property rule is optimal.

of each of the possible property rules for this case. Recalling the assumption that the empty contract is suboptimal, we need to rule out the optimality of the strict-FT contract and of the protection-cap contract. For this purpose it is sufficient to show that, for either  $b_1^D=0$  or  $b_1^D\geq \frac{\bar{\gamma}_1}{1+c}$ , setting  $b_2^D\geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c}-b_1^D$  cannot be optimal. But it is direct to see that this is implied by  $\bar{\gamma}_i>(1+c)\gamma^*$  (i=1,2), because in this case setting  $b_2^D\geq \frac{(1+\alpha)\bar{\gamma}_1}{1+c}-b_1^D$  with  $b_1^D=0$  or  $b_1^D\geq \frac{\bar{\gamma}_1}{1+c}$  would require that  $b_1^D>\gamma^*$  and/or  $b_2^D>\gamma^*$ , which is ruled out by Proposition B1 (and with large support the qualifier regarding weak domination described in note 28 of the body of the paper does not apply here). Hence, it follows that for sufficiently large uncertainty, the optimum is a liability rule. QED

Figure A1

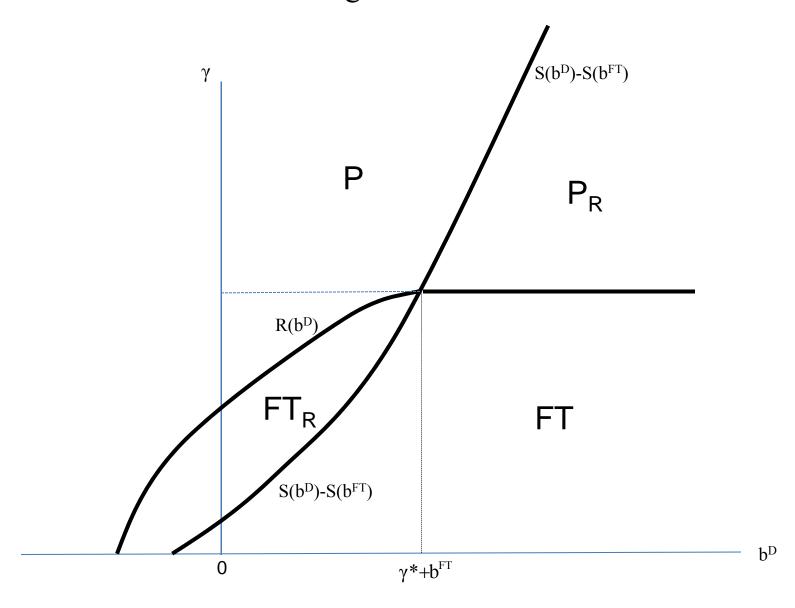


Figure B1

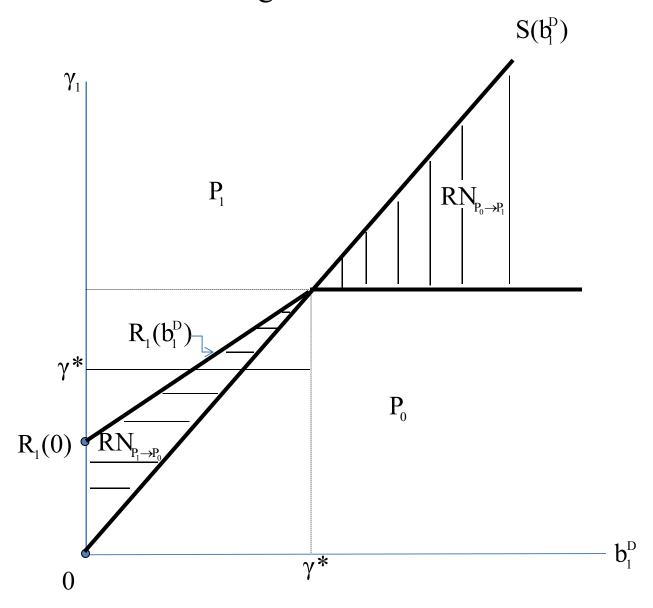


Figure B2

