Online Appendix

"Wage Rigidity and Disinflation in Emerging Countries"

Notes on the Econometric Model*

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Abstract

This appendix outlines in detail the model estimated in Brazil and Uruguay.

1 Introduction

This short note is divided into two parts. In Section 2 we derive the likelihood function for the benchmark model presented in the paper. Section 3 presents the model with symmetric real wage rigidity discussed in Section IV.B of the paper.

2 Benchmark model

2.1 The Likelihood Function

Each individual wage change observation belongs to one of three regimes: p^R agents are subject to DRWR, p^N to DNWR, and $(1-p^N-p^R)$ belong to a regime in which wages are fully flexible. Wage observations in each period contain no error with probability q, and error with probability $1-q$.

Hence, the likelihood is:

$$L = p^R L_R + p^N L_N + \left(1 - p^N - p^R\right) L_F$$

*This note draws heavily on Goette (2002).

, where

$$L_R = \left[q^2 \left(L_{RU0} + L_{RC0}\right) + 2q(1-q)(L_{RU1} + L_{RC1}) + (1-q)^2(L_{RU2} + L_{RC2})\right]$$

$$L_N = \left[q^2(L_{NU0} + L_{NC0}) + 2q(1-q)(L_{NU1} + L_{NC1}) + (1-q)^2(L_{NU2} + L_{NC2})\right]$$

$$L_F = [q^2 L_{FU0} + 2q(1-q)L_{FU1} + (1-q)^2 L_{FU2}]$$

, in which we adopt the following notation for subscripts: N, R and F denote nominal, real and flexible agents, respectively; and U and C refer to unconstrained and constrained agents, respectively. The process of measurement error in log wage levels is represented by $m \sim N\left(0, \sigma_m\right)$. Since we are looking at changes in log wages within two consecutive years $(t, t+1)$, we have three possibilities for measurement error: there is no error in t or $t+1$, denoted by subscript 0, error occurs in one of the two periods (subscript 1), and error occurs in the two periods (subscript 2). Hence, L_{NU2} stands for the contribution to the likelihood of an agent who is downward nominal rigid but unconstrained, and whose wage is measured with error in the two periods. Next, we treat each type of agent separately and derive their contribution to the likelihood.

2.1.1 Downward Real Wage Rigidity

RU0: Real Types, unconstrained, no error

- **Unobserved Regime** The nominal wage change is RU if $xb+e > r \rightarrow e-r > -xb$, where $e \sim N\left(0, \sigma_e\right)$ and $r \sim N\left(\mu_r, \sigma_r\right)$.Therefore, $E(e-r) = -\mu_r$, $V(e-r) = \sigma_e^2 + \sigma_r^2$.

- **Observed nominal wage change** The observed wage change in RU0 is $dy = xb + e$. Therefore, the conditioning variable is $e = dy - xb$ with marginal density $\frac{1}{\sigma_e}\phi\left(\frac{dy-xb}{\sigma_e}\right)$, and $Cov(e-r, e) = \sigma_e^2$.

- **Conditional Distribution.** The conditional distribution of interest is $e-r|e = dy-xb$. It follows that:[1]

$$E(e-r|e = dy-xb) = -\mu_r + (dy-xb)$$

$$V(e-r|e = dy-xb) = \sigma_r^2$$

The conditional probability of RU0 is:

[1]Note that if X and Y have a joint normal distribution, then $X|Y = y$ is normal with

$$E(X|Y = y) = \mu_X + \sum_{XY}\sum_{YY}^{-1}(y - \mu_Y)$$

$$Cov(X|Y = y) = \sum_{XX} - \sum_{XY}\sum_{YY}^{-1}\sum_{YX}$$

$$\begin{aligned} Pr(e-r &> -xb|e=dy-xb) = \\ &= \int_{-xb}^{+\infty} \frac{1}{\sigma_r}\phi\left(\frac{s+\mu_r-(dy-xb)}{\sigma_r}\right) ds = \\ &= 1-\Phi\left(\frac{-xb+\mu_r-(dy-xb)}{\sigma_r}\right) \end{aligned}$$

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain:

$$L_{RU0} = 1-\Phi\left(\frac{-xb+\mu_r-(dy-xb)}{\sigma_r}\right)\frac{1}{\sigma_e}\phi\left(\frac{dy-xb}{\sigma_e}\right)$$

RU1: Real types, unconstrained, error in one period

- **Unobserved Regime** The nominal wage change is RU if $xb+e>r \rightarrow e-r>-xb$. Therefore, $E(e-r)=-\mu_r$, $V(e-r)=\sigma_e^2+\sigma_r^2$.

- **Observed nominal wage change** The observed wage change in RU1 is $dy = xb + e + m$. Therefore, the conditioning variable is $e+m = dy-xb$ with marginal density $\frac{1}{\sqrt{\sigma_e^2+\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+\sigma_m^2}}\right)$, and $Cov(e-r, e+m)=\sigma_e^2$.

- **Conditional Distribution.** The conditional distribution of interest is $e-r|e+m = dy-xb$. It follows that:

$$E(e-r|e+m=dy-xb) = -\mu_r + \frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy-xb)$$

$$V(e-r|e+m=dy-xb) = \sigma_e^2+\sigma_r^2-\frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}$$

The conditional probability of RU1 is:

$$\begin{aligned} Pr(e-r &> -xb|e+m=dy-xb) = \\ &= \int_{-xb}^{+\infty} \frac{1}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\phi\left(\frac{s+\mu_r-\frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\right) ds = \\ &= 1-\Phi\left(\frac{-xb+\mu_r-\frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\right) \end{aligned}$$

- **Likelihood Contribution to the Regime**. Putting the conditional and marginal probability together, we obtain:

$$L_{RU1} = \left[1-\Phi\left(\frac{-xb+\mu_r-\frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\right)\right]\frac{1}{\sqrt{\sigma_e^2+\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+\sigma_m^2}}\right)$$

RU2: Real types, unconstrained, error in both periods

- **Unobserved Regime** The nominal wage change is RU if $xb + e > r \rightarrow e - r > -xb$. Therefore, $E(e-r) = -\mu_r$, $V(e-r) = \sigma_e^2 + \sigma_r^2$.

- **Observed nominal wage change.** The observed wage change in RU2 is $dy = xb + e + m'$, where $m' = m_2 - m_1$. Therefore, the conditioning variable is $e + m' = dy - xb$, and $Cov(e - r, e + m') = \sigma_e^2$. The only difference with respect to RU1 is the definition of the variance, $V(m') = 2\sigma_m^2$, because m_1 and m_2 are independent. The marginal density is $\frac{1}{\sqrt{\sigma_e^2 + 2\sigma_m^2}} \phi\left(\frac{dy - xb}{\sqrt{\sigma 2e + 2\sigma 2m}}\right)$.

- **Conditional Distribution.** The conditional distribution of interest is $e - r|e + m\prime = dy - xb$. It follows that:

$$E(e - r|e + m' = dy - xb) = -\mu_r + \frac{\sigma_e^2}{\sigma_e^2 + 2\sigma_m^2}(dy - xb)$$

$$V(e - r|e + m' = dy - xb) = \sigma_e^2 + \sigma_r^2 - \frac{\sigma_e^4}{\sigma_e^2 + 2\sigma_m^2}$$

The conditional probability of RU2 is:

$$\begin{aligned} Pr(e - r &> -xb|e + m' = dy - xb) = \\ &= \int_{-xb}^{+\infty} \frac{1}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_e^4}{\sigma_e^2 + 2\sigma_m^2}}} \phi\left(\frac{s + \mu_r - \frac{\sigma_e^2}{\sigma_e^2 + 2\sigma_m^2}(dy - xb)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_e^4}{\sigma_e^2 + 2\sigma_m^2}}}\right) ds = \\ &= 1 - \Phi\left(\frac{-xb + \mu_r - \frac{\sigma_e^2}{\sigma_e^2 + 2\sigma_m^2}(dy - xb)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_e^4}{\sigma_e^2 + 2\sigma_m^2}}}\right) \end{aligned}$$

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain:

$$L_{RU2} = \left[1 - \Phi\left(\frac{-xb + \mu_r - \frac{\sigma_e^2}{\sigma_e^2 + 2\sigma_m^2}(dy - xb)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_e^4}{\sigma_e^2 + 2\sigma_m^2}}}\right)\right] \frac{1}{\sqrt{\sigma_e^2 + 2\sigma_m^2}} \phi\left(\frac{dy - xb}{\sqrt{\sigma_e^2 + 2\sigma_m^2}}\right)$$

RC0: Real Types, constrained, no error

- **Unobserved Regime** The nominal wage change is RC if $xb + e < r \rightarrow e - r < -xb$. Therefore, $E(e-r) = -\mu_r$, $V(e-r) = \sigma_e^2 + \sigma_r^2$.

- **Observed nominal wage change** The observed wage change in RC0 is $dy = r$. Therefore, the conditioning variable is $r = dy$ with $Cov(e - r, r) = -\sigma_r^2$ and marginal density $\frac{1}{\sigma_r} \phi\left(\frac{dy - \mu_r}{\sigma_r}\right)$.

- **Conditional Distribution.** The conditional distribution of interest is $e-r|r=dy$. It follows that:

$$E(e-r|r=dy) = -dy$$
$$V(e-r|r=dy) = \sigma_e^2$$

The conditional probability of RC0 is

$$\begin{aligned}
Pr(e-r &< -xb|r=dy) = \\
&= \int_{-\infty}^{-xb} \frac{1}{\sigma_e}\phi\left(\frac{s+dy}{\sigma_e}\right) ds = \\
&= \Phi\left(\frac{-xb+dy}{\sigma_e}\right)
\end{aligned}$$

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain

$$L_{RC0} = \Phi\left(\frac{-xb+dy}{\sigma_e}\right)\frac{1}{\sigma_r}\phi(\frac{dy-\mu_r}{\sigma_r})$$

RC1: Real Types, constrained, error in one period

- **Unobserved Regime** The nominal wage change is RU if $xb+e<r \rightarrow e-r<-xb$. Therefore, $E(e-r) = -\mu_r, V(e-r) = \sigma_e^2 + \sigma_r^2$
- **Observed nominal wage change** The observed wage change in RC1 is $dy = r+m$. Therefore, the conditioning variable is $r+m = dy$ with $Cov(e-r, r+m) = -\sigma_r^2$ and marginal density $\frac{1}{\sqrt{\sigma_m^2+\sigma_r^2}}\phi\left(\frac{dy-\mu_r}{\sqrt{\sigma_m^2+\sigma_r^2}}\right)$.
- **Conditional Distribution.** The conditional distribution of interest is $e-r|r+m=dy$. It follows that:

$$E(e-r|r+m=dy) = -\mu_r - \frac{\sigma_r^2}{\sigma_r^2+\sigma_m^2}(dy-\mu_r)$$
$$V(X|Y=dy-xb) = \sigma_e^2+\sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2+\sigma_m^2}$$

The conditional probability of RC1 is:

$$\begin{aligned}
Pr(e-r &< -xb|r+m=dy) = \\
&= \int_{-\infty}^{-xb} \frac{1}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_r^4}{\sigma_r^2+\sigma_m^2}}}\phi\left(\frac{s+\mu_r+\frac{\sigma_r^2}{\sigma_r^2+\sigma_m^2}(dy-\mu_r)}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_r^4}{\sigma_r^2+\sigma_m^2}}}\right) ds = \\
&= \Phi\left(\frac{-xb+\mu_r+\frac{\sigma_r^2}{\sigma_r^2+\sigma_m^2}(dy-\mu_r)}{\sqrt{\sigma_e^2+\sigma_r^2-\frac{\sigma_r^4}{\sigma_r^2+\sigma_m^2}}}\right)
\end{aligned}$$

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain

$$L_{RC1} = \Phi\left(\frac{-xb + \mu_r + \frac{\sigma_r^2}{\sigma_r^2+\sigma_m^2}(dy - \mu_r)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2+\sigma_m^2}}}\right) \frac{1}{\sqrt{\sigma_r^2 + \sigma_m^2}} \phi\left(\frac{dy - \mu_r}{\sigma_r^2 + \sigma_m^2}\right)$$

RC2: Real Types, constrained, error in both periods

- **Unobserved Regime** The nominal wage change is RU if $xb + e < r \rightarrow e - r < -xb$. Therefore, $E(e - r) = -\mu_r$, $V(e - r) = \sigma_e^2 + \sigma_r^2$

- **Observed nominal wage change** The observed wage change in RC2 is $dy = r + m'$, where $m' = m_2 - m_1$. Therefore, the conditioning variable is $r + m' = dy$, with $Cov(e - r, r + m') = -\sigma_r^2$. The only difference with respect to RC1 is the definition of the variance, $V(m') = 2\sigma_m^2$, because m_1 and m_2 are independent. The marginal density is $\frac{1}{\sqrt{\sigma_r^2+2\sigma_m^2}}\phi\left(\frac{dy-\mu_r}{\sqrt{\sigma_r^2+2\sigma_m^2}}\right)$.

- **Conditional Distribution.** The conditional distribution of interest is $e - r|r + m' = dy$. It follows that:

$$E(e - r|r + m' = dy) = -\mu_r - \frac{\sigma_r^2}{\sigma_r^2 + 2\sigma_m^2}(dy - \mu_r)$$
$$V(X|Y = dy - xb) = \sigma_e^2 + \sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2 + 2\sigma_m^2}$$

  The conditional probability of RC1 is:

$$\begin{aligned} Pr(e - r &< -xb|r + m' = dy) = \\ &= \int_{-\infty}^{-xb} \frac{1}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2+2\sigma_m^2}}} \phi\left(\frac{s + \mu_r + \frac{\sigma_r^2}{\sigma_r^2+2\sigma_m^2}(dy - \mu_r)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2+2\sigma_m^2}}}\right) ds = \\ &= \Phi\left(\frac{-xb + \mu_r + \frac{\sigma_r^2}{\sigma_r^2+2\sigma_m^2}(dy - \mu_r)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2+2\sigma_m^2}}}\right) \end{aligned}$$

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain

$$L_{RC2} = \Phi\left(\frac{-xb + \mu_r + \frac{\sigma_r^2}{\sigma_r^2+2\sigma_m^2}(dy - \mu_r)}{\sqrt{\sigma_e^2 + \sigma_r^2 - \frac{\sigma_r^4}{\sigma_r^2+2\sigma_m^2}}}\right) \frac{1}{\sqrt{\sigma_r^2 + 2\sigma_m^2}} \phi\left(\frac{dy - \mu_r}{\sigma_r^2 + 2\sigma_m^2}\right)$$

2.1.2 Downward Nominal Wage Rigidity

The nominal types can be treated as a special case of the real types with a degenerate distribution of r and $\mu_r = 0$.

NU0: Nominal Types, unconstrained, no error

- **Unobserved Regime** The nominal wage change is NU if $xb+e > 0 \rightarrow e > -xb$. Therefore, $E(e) = 0$, $V(e) = \sigma_e^2$.

- **Observed nominal wage change** The observed wage change in NU0 is $dy = xb + e$. Therefore, the conditioning variable is $e = dy - xb$ with $Cov(e,e) = \sigma_e^2$ and marginal density $\frac{1}{\sigma_e}\phi\left(\frac{dy-xb}{\sigma_e}\right)$

- **Conditional Distribution.** The conditional distribution is trivial. Given that there is no error and $dy > 0$, that probability is 1, and 0 otherwise.

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain:

$$L_{NU0} = \frac{1}{\sigma_e}\phi\left(\frac{dy - xb}{\sigma_e}\right), \qquad \text{if } dy > 0 \text{ and } 0 \text{ otherwise}$$

NU1: Nominal Types, unconstrained, error in one period

- **Unobserved Regime** The nominal wage change is NU if $xb+e > 0 \rightarrow e > -xb$. Therefore, $E(e) = 0$, $V(e) = \sigma_e^2$.

- **Observed nominal wage change** The observed wage change in NU1 is $dy = xb+e+m$. Therefore, the conditioning variable is $e + m = dy - xb$, with $Cov(e, e + m) = \sigma_e^2$ and marginal density $\frac{1}{\sqrt{\sigma_e^2+\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+\sigma_m^2}}\right)$

- **Conditional Distribution.** The conditional distribution of interest is $e-r|e+m = dy-xb$. It follows that:

$$E(e|e + m = dy - xb) = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_m^2}(dy - xb)$$

$$V(e|e + m = dy - xb) = \sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2 + \sigma_m^2}$$

The conditional probability of NU1 is:

$$\begin{aligned} Pr(e &> -xb|e + m = dy - xb) = \\ &= \int_{-xb}^{+\infty} \frac{1}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\phi\left(\frac{s - \frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy - xb)}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\right) ds = \\ &= 1 - \Phi\left(\frac{-xb - \frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy - xb)}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\right) \end{aligned}$$

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain:

$$L_{NU1} = \left[1 - \Phi\left(\frac{-xb - \frac{\sigma_e^2}{\sigma_e^2+\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+\sigma_m^2}}}\right)\right] \frac{1}{\sqrt{\sigma_e^2+\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+\sigma_m^2}}\right)$$

NU2: Nominal Types, unconstrained, error in both periods

- **Unobserved Regime** The nominal wage change is NU if $xb+e > 0 \rightarrow e > -xb$. Therefore, $E(e) = 0$, $V(e-r) = \sigma_e^2$.

- **Observed nominal wage change** The observed wage change in NU2 is $dy = xb+e+m'$, where $m' = m_2 - m_1$. Therefore, the conditioning variable is $e + m' = dy - xb$ and $Cov(e, e+m') = \sigma_e^2$. The only difference to NU1 is the definition of the variance $V(m') = 2\sigma_m^2$, because m_1 and m_2 are independent. The marginal density is $\frac{1}{\sqrt{\sigma_e^2+2\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+2\sigma_m^2}}\right)$

- **Conditional Distribution.** The conditional distribution of interest is $e - r|e + m' = dy - xb$. It follows that:

$$E(e|e+m' = dy-xb) = \frac{\sigma_e^2}{\sigma_e^2+2\sigma_m^2}(dy-xb)$$

$$V(e|e+m' = dy-xb) = \sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+2\sigma_m^2}$$

  The conditional probability of NU2 is:

$$\begin{aligned} Pr(e &> -xb|e+m' = dy-xb) = \\ &= \int_{-xb}^{+\infty} \frac{1}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+2\sigma_m^2}}}\phi\left(\frac{s - \frac{\sigma_e^2}{\sigma_e^2+2\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+2\sigma_m^2}}}\right) ds = \\ &= 1 - \Phi\left(\frac{-xb - \frac{\sigma_e^2}{\sigma_e^2+2\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+2\sigma_m^2}}}\right) \end{aligned}$$

- **Likelihood Contribution to the Regime**. Putting the conditional and marginal probability together, we obtain:

$$L_{NU2} = \left[1 - \Phi\left(\frac{-xb - \frac{\sigma_e^2}{\sigma_e^2+2\sigma_m^2}(dy-xb)}{\sqrt{\sigma_e^2 - \frac{\sigma_e^4}{\sigma_e^2+2\sigma_m^2}}}\right)\right] \frac{1}{\sqrt{\sigma_e^2+2\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+2\sigma_m^2}}\right)$$

NC0: Nominal Types, constrained, no error

- **Unobserved Regime** The nominal wage change is NC if $xb+e < 0 \rightarrow e < -xb$.Therefore, $E(e) = 0$, $V(e) = \sigma_e^2$

- **Observed nominal wage change** The observed wage change in NC0 is $dy = 0$. Therefore, the conditioning variable is $dy = 0$.

- **Conditional Distribution of Interest** The conditional distribution is trivial. Either $dy = 0$ or 0 otherwise.

- **Likelihood Contribution to the Regime.** Putting the conditional and marginal probability together, we obtain:

$$L_{NC0} = \Phi\left(\frac{-xb}{\sigma_e}\right) \quad \text{if} \quad dy = 0 \text{ and } 0 \text{ otherwise}$$

NC1: Nominal Types, constrained, error in one period

- **Unobserved Regime** The nominal wage change is NC if $xb+e < 0 \rightarrow e < -xb$.Therefore, $E(e) = 0, V(e) = \sigma_e^2$

- **Observed nominal wage change** The observed wage change in NC1 is $dy = m$. Therefore, the conditioning variable is $m = dy$, with $Cov(e, m) = 0$ (i.e., they are independent) and marginal density $\frac{1}{\sigma_m}\phi\left(\frac{dy}{\sigma_m}\right)$.

- **Conditional Distribution.** The conditional and the unconditional distribution of e are equal, since e is independent of m.

- **Likelihood Contribution to the Regime** The conditional probability of NC1 is:

$$Pr(e < -xb | m = dy) = \int_{-\infty}^{-xb} \frac{1}{\sigma_e}\phi\left(\frac{s}{\sigma_e}\right) ds = \Phi\left(\frac{-xb}{\sigma_e}\right)$$

  Putting the conditional and marginal probability together, we obtain:

$$L_{NC1} = \Phi\left(\frac{-xb}{\sigma_e}\right)\frac{1}{\sigma_m}\phi\left(\frac{dy}{\sigma_m}\right)$$

NC2: Nominal Types, constrained, measurement error both periods

- **Unobserved Regime** The nominal wage change is NC if $xb+e < 0 \rightarrow e < -xb$.Therefore, $E(e) = 0, V(e) = \sigma_e^2$

- **Observed nominal wage change** The observed wage change in NC2 is $dy = m'$, where $m' = m_1 - m_2$. The only difference with respect to NC1 is the definition of the variance,. $V(m') = 2\sigma_m^2$, because m_1 and m_2 are independent. The marginal density is $\frac{1}{\sqrt{2}\sigma_m}\phi\left(\frac{dy}{\sqrt{2}\sigma_m}\right)$.

- **Conditional Distribution.** The conditional distribution of interest of e is equal to the unconditional distribution, since e is independent of m'.

- **Likelihood Contribution to the Regime** The conditional probability of NC2 is:

$$Pr(e < -xb|m' = dy) = \int_{-\infty}^{-xb} \frac{1}{\sigma_e}\phi\left(\frac{s}{\sigma_e}\right) ds = \Phi\left(\frac{-xb}{\sigma_e}\right)$$

Putting the conditional and marginal probability together, we obtain:

$$L_{NC2} = \Phi\left(\frac{-xb}{\sigma_e}\right)\frac{1}{\sqrt{2}\sigma_m}\phi\left(\frac{dy}{\sqrt{2}\sigma_m}\right)$$

2.1.3 The Flexible Types

Flexible agents are always unconstrained. Hence, we only need to obtain the densities of the observations if they contain no error, error in one period, or error in both periods.

FU0:Flexible types, no error The observed nominal wage change is $dy = xb + e \rightarrow e = dy - xb$. The density of an observation is $\frac{1}{\sigma_e}\phi\left(\frac{dy-xb}{\sigma_e}\right)$. Hence:

$$L_{FU0} = \frac{1}{\sigma_e}\phi\left(\frac{dy - xb}{\sigma_e}\right)$$

FU1:Flexible types, error in one period The observed nominal wage change is $dy = xb + e + m \rightarrow e + m = dy - xb$. The density of an observation is $\frac{1}{\sqrt{\sigma_e^2+\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+\sigma_m^2}}\right)$. Hence:

$$L_{FU1} = \frac{1}{\sqrt{\sigma_e^2 + \sigma_m^2}}\phi\left(\frac{dy - xb}{\sqrt{\sigma_e^2 + \sigma_m^2}}\right)$$

FU2:Flexible types, error in both periods The observed nominal wage change is $dy = xb + e + m' \rightarrow e + m' = dy - xb$, where $m' = m_2 - m_1$. The density of an observation is $\frac{1}{\sqrt{\sigma_e^2+2\sigma_m^2}}\phi\left(\frac{dy-xb}{\sqrt{\sigma_e^2+2\sigma_m^2}}\right)$. Hence:

$$L_{FU2} = \frac{1}{\sqrt{\sigma_e^2 + 2\sigma_m^2}}\phi\left(\frac{dy - xb}{\sqrt{\sigma_e^2 + 2\sigma_m^2}}\right)$$

3 Symmetric Real Rigidity

3.1 The Likelihood Function

If real wage rigidity is symmetric, real rigid agents are always constrained. Using the same notation as before, but denoting with a prime the new likelihood function we obtain:

$$L^{'} = p^{R}L^{'}_{R} + p^{N}L^{'}_{N} + \left(1 - p^{N} - p^{R}\right)L^{'}_{F}$$

, where

$$L^{'}_{R} = \left[q^{2}L^{'}_{RC0} + 2q(1-q)L^{'}_{RC1} + (1-q)^{2}L^{'}_{RC2}\right]$$

$$L^{'}_{N} = \left[q^{2}(L^{'}_{NU0} + L^{'}_{NC0}) + 2q(1-q)(L^{'}_{NU1} + L^{'}_{NC1}) + (1-q)^{2}(L^{'}_{NU2} + L^{'}_{NC2})\right]$$

$$L^{'}_{F} = [q^{2}L^{'}_{FU0} + 2q(1-q)L^{'}_{FU1} + (1-q)^{2}L^{'}_{FU2}]$$

The expressions driving the contribution to the likelihood from downward nominal rigid agents and flexible agents are the same as in the benchmark model described in Section 2. Thus, $L^{'}_{N} = L_{N}$ and $L^{'}_{F} = L_{F}$. We only need to derive new expressions for real rigid agents.

3.1.1 Real Rigid Types

RC0: Real Types, constrained, no error The observed wage change in the absence of error is $dy = r$. Therefore, the conditioning variable is $r = dy$, with $Cov(e - r, r) = -\sigma_{r}^{2}$ and the marginal density is $\frac{1}{\sigma_r}\phi\left(\frac{dy-\mu_r}{\sigma_r}\right)$. Hence,

$$L^{'}_{RC0} = \frac{1}{\sigma_r}\phi(\frac{dy - \mu_r}{\sigma_r})$$

RC1: Real Types, constrained, error in one period The observed nominal wage change is $dy = r + m \rightarrow e + m = dy$.Therefore, the conditioning variable is $r + m = dy$, with $Cov(e - r, r + m) = -\sigma_{r}^{2}$, and marginal density $\frac{1}{\sqrt{\sigma_m^2+\sigma_r^2}}\phi\left(\frac{dy-\mu_r}{\sqrt{\sigma_m^2+\sigma_r^2}}\right)$. Hence,

$$L^{'}_{RC1} = \frac{1}{\sqrt{\sigma_r^2 + \sigma_m^2}}\phi\left(\frac{dy - \mu_r}{\sigma_r^2 + \sigma_m^2}\right)$$

RC2: Real Types, constrained, error in both periods The observed wage change in RC2 is $dy = r + m'$, where $m' = m_2 - m_1$. Therefore, the conditioning variable is $r + m' = dy$, with $Cov(e - r, r + m') = -\sigma_{r}^{2}$. The only difference with respect to RC1 is the definition of the variance, $V(m') = 2\sigma_{m}^{2}$, because m_1 and m_2 are independent. The marginal density is

$\frac{1}{\sqrt{\sigma_r^2+2\sigma_m^2}}\phi\left(\frac{dy-\mu_r}{\sqrt{\sigma_r^2+2\sigma_m^2}}\right)$. Hence,

$$L'_{RC2} = \frac{1}{\sqrt{\sigma_r^2 + 2\sigma_m^2}}\phi\left(\frac{dy - \mu_r}{\sigma_r^2 + 2\sigma_m^2}\right)$$

References

[1] Goette, L. (2002) Notes on the Analytic Model. Unpublished manuscript.