

The Labor Market for Directors and Externalities in Corporate Governance

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ABSTRACT

This paper studies how directors' reputational concerns in the labor market affect board structure, corporate governance, and firm value. In our setting, directors affect their firms' governance, and governance, in turn, affects firms' demand for new directors. Whether the labor market rewards a shareholder-friendly or management-friendly reputation is thus endogenous and depends on the aggregate quality of corporate governance. We show that directors' desire to be invited to other boards creates strategic complementarity of governance across firms. As a result, an equilibrium with strong aggregate governance can co-exist with a weak governance equilibrium, suggesting that countries or industries with similar characteristics can have very different governance systems. We also show that directors' reputational concerns amplify the governance system: strong systems become stronger and weak systems become weaker. We derive implications for regulations restricting multiple directorships, transparency, shareholder activism, and board size.

Keywords: board of directors, corporate governance, reputation, externalities, strategic complementarity, transparency

JEL Classification Numbers: D62, D71, D82, D83, G34, G38, J20

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Introduction

Why do corporate boards look the way they are? Are boards structured optimally to maximize shareholder value, and how do board regulations affect their composition? To a large extent, the structure of corporate boards is governed by the labor market for directors. On the demand side, firms decide which directors to invite based on directors' reputation and on the preferences of those controlling the nomination process. On the supply side, directors seek to develop their reputation in order to gain more board seats and thereby obtain prestige, power, compensation, and access to valuable networks. Thus, directors' reputation plays an important role, affecting both directors' actions and the structure of corporate boards (Fama and Jensen (1983)).

A number of recent institutional and regulatory changes to the director selection process have affected the labor market for directors and the value of reputation. Examples include a shift from plurality to majority voting, proxy access proposals, restrictions on the number of directorships, and increased boardroom transparency. These rules and practices also vary substantially across countries. However, the effect of these factors is not well understood, and some of the recent changes are highly debated.¹ This paper sheds light on these issues by developing a theory of the labor market for directors and studying how directors' reputational concerns affect board structure, directors' behavior, and ultimately firm value.

We emphasize that the effect of directors' reputational concerns is far from obvious. The key to our argument is that directors care about two conflicting types of reputation, and which type of reputation is rewarded more in the labor market depends on the aggregate quality of corporate governance. If governance is strong and boards of other firms protect the interests of their shareholders, then building a reputation for being shareholder-friendly can help in obtaining more directorships. Conversely, if governance is weak and boards of other firms are captured by their managers, who want to maintain power, then having a management-friendly reputation can be more useful in getting additional board seats. The empirical evidence is consistent with the importance of both types of reputation. Some papers, such as Coles and Hoi (2003) and Fich and Shivdasani (2007), find that directors who demonstrate shareholder-friendly behavior and monitor the management are more likely to gain additional directorships. Others, such as Helland (2006) and Marshall (2011), find that shareholder-friendly actions actually hurt directors' chances of being invited to other boards. Moreover, Zajac and Westphal

¹See, for example, "The Proxy Access Debate," *New York Times*, October 9, 2009. Details on the director selection process and on the recent changes mentioned above are provided in the section "Institutional background."

(1996), Eminet and Guedri (2010), and Bouwman (2011) find evidence that is directly consistent with the existence of conflicting reputational concerns: firms controlled by shareholders (managers) are more likely to invite directors who have demonstrated shareholder-friendliness (management-friendliness) in their previous board positions.²

To study how these conflicting reputational concerns affect directors' behavior and firm value, we develop a model with three key components. First, being a board member allows a director to affect corporate governance in his firm and thereby change the allocation of control between management and shareholders. Second, whether a director is shareholder-friendly or management-friendly is the director's private information, and by allocating control to either managers or shareholders, directors can affect the market's perception of their shareholder-friendliness. Shareholder-friendliness in our model can be interpreted as either having stronger preferences towards shareholder control or as having lower monitoring costs. In particular, directors with more knowledge and expertise can be considered more shareholder-friendly because such directors find it easier to effectively evaluate and monitor the CEO and to confront him when needed (see Section 1.1 for more discussion). Third, the allocation of control in a given firm determines, among other things, which type of directors it is looking for. In particular, firms that are controlled by shareholders (management) have a demand for shareholder-friendly (management-friendly) directors.

In our setting, the aggregate quality of corporate governance, the structure of the board, and, importantly, the type of reputation that directors need to develop to gain more directorships, are all endogenously determined in equilibrium. To the best of our knowledge, this is the first paper in the literature on reputational concerns in which the type of reputation that is rewarded is endogenous.³

Our main result shows that directors' concerns about their reputation in the labor market create strategic complementarity of corporate governance across firms. In particular, stronger governance in one firm leads to stronger governance in other firms, and vice versa.⁴ Intuitively, when corporate governance of most other firms in the market is weak, the decision of whom to invite to the boards of these firms is controlled by managers. Thus, to increase their chances of obtaining additional directorships, directors have incentives to build a reputation for being management-friendly. This type of reputation can be established by giving more control to the

²Section 5 provides a review of these and other relevant papers in the empirical literature. See also Adams, Hermalin and Weisbach (2010) for a discussion of this reputational trade-off.

³See the section "Related literature" for an overview of relevant papers.

⁴In what follows, we use the term "strong corporate governance" for firms in which shareholders have control, and "weak corporate governance" for firms in which the management has control.

managers of their firms and not interfering with their decisions. Conversely, when corporate governance in most other firms is strong, directors will strengthen corporate governance of their firms to build a reputation for being shareholder-friendly.

Our paper thus identifies a novel channel of strategic complementarities between firms, which works through directors' reputational concerns in the labor market. Strategic complementarities arise due to the dual role that directors' actions have on the supply and demand in the market for directors: in addition to affecting directors' own reputation (supply), they also affect the type of reputation that is rewarded in the market (demand).⁵

Strategic complementarity of governance across firms implies that there can be multiple equilibria, characterized by the aggregate quality of corporate governance. In particular, we show that when directors' reputational concerns are sufficiently important, an equilibrium in which aggregate governance is strong and the labor market rewards directors for being shareholder-friendly co-exists with a weak governance equilibrium, in which a management-friendly reputation is rewarded. The existence of multiple equilibria implies that countries with similar legal and institutional characteristics can have very different corporate governance systems. This result is consistent with Doidge, Karolyi, and Stulz (2007), who find large cross-country differences in corporate governance, even after controlling for country characteristics such as economic development, financial development, the rule of law, and the index of shareholder rights (La Porta et al. (1998)).

Directors' reputational concerns, which are the driving force of governance externalities in our model, can be affected by regulations that restrict the number of directorships a person can hold or that change the value of a given directorship. These regulations vary across countries. For example, while the US does not limit the number of board seats a single director can hold, many European and Asian countries impose such a limit, and this limit is different for different countries.⁶ We show that a policy that increases directors' reputational concerns, such as an increase in the allowed number of directorships, can be a double-edged sword, whose effect crucially depends on the aggregate quality of corporate governance. Specifically, when directors become more concerned about their reputation in the labor market, corporate governance be-

⁵The existing literature on labor markets points out other channels of strategic complementarities between firms: strategic complementarities can arise when both workers and firms make their investment or entry decisions and there are search frictions (e.g., Acemoglu (1996), Laing, Palivos and Wang (1995)), or when there are increasing returns to scale in either the matching technology (e.g., Diamond (1982)) or the production function (e.g., Benhabib and Farmer (1994)).

⁶See ECGI corporate governance codes at http://www.ecgi.org/codes/all_codes.php and the White Paper on Corporate Governance in Asia (OECD 2003). Other policies that can affect directors' reputational concerns include term limits and age limits on directors, which are imposed in some countries.

comes even stronger in systems with strong governance, where a shareholder-friendly reputation is rewarded. However, in systems where managers are in control and directors are rewarded for being management-friendly, stronger reputational concerns weaken corporate governance even further. In other words, directors' reputational concerns amplify the existing aggregate level of corporate governance. This result suggests that policies restricting the number of directorships are more likely to be beneficial in countries with weak governance systems.

Our paper has implications with respect to other dimensions of corporate governance. First, due to strategic complementarity, a small regulatory change, such as a marginal increase in the required percentage of independent directors, can have a very significant effect on the aggregate level of corporate governance. Intuitively, when the percentage of independent directors on a board increases, directors find it easier to object the management and promote strong governance. Moreover, the anticipation that other corporate boards will similarly increase their accountability to shareholders increases the relative value of a shareholder-friendly reputation in the labor market. This reinforces directors' incentives to promote strong governance in their firms and magnifies the initial effect.

Second, we show that increased transparency of board decision-making strengthens corporate governance if aggregate governance is already strong, but weakens governance even further if aggregate governance is weak. For example, the 2004 Securities and Exchange Commission (SEC) disclosure law, which increased transparency by requiring companies to disclose if one of their directors leaves the board due to a disagreement, could have had adverse unintended consequences.⁷ Intuitively, if aggregate governance is weak and a management-friendly reputation is rewarded in the labor market, directors may be more reluctant to oppose the management when they know that their actions will be disclosed to other market participants. Thus, increasing boardroom transparency with the goal of strengthening a weak governance system is likely to achieve the opposite outcome.

Finally, we study the effects of board size and shareholder activism on corporate governance. In particular, we consider how collective decision-making by directors and the ability of shareholders to intervene and exercise control affect the equilibrium. We highlight that due to externalities in the labor market for directors, board size and shareholder activism affect corporate governance not only within firms, but also across firms.

⁷Prior to the SEC ruling, disclosure was only required if the director leaving the firm requested his resignation letter to be made public. The new ruling requires all such departures to be disclosed in the firm's 8-K filing within four business days after the event, even if the director did not provide any written correspondence or request that the matter is made public. In China, a somewhat similar 2004 law requires firms to disclose the names of those independent directors who vote in dissent during the meeting (see Jiang, Wan, and Zhao (2012)).

The model yields new testable predictions for the director labor market and peer effects in corporate governance. Importantly, these effects operate through directors' reputational concerns. Section 5 describes these implications and discusses them in the context of the existing empirical evidence.

The paper proceeds as follows. The remainder of the section discusses the related literature and the institutional background. Section 1 introduces the model, and Section 2 presents the analysis. Section 3 provides the comparative statics and implications for shareholder welfare. Section 4 discusses the following extensions: the effect of transparency, shareholder activism, multiple directors on the board, a general number of firms, the value of shareholder control, and heterogeneous outside candidates. Section 5 offers testable predictions and describes the related empirical literature. Section 6 contains some concluding remarks and discusses other potential applications of our framework. All proofs are delegated to the Appendix.

Related literature

Our paper is related to the literature on reputational concerns, where agents distort their decisions to convince the market that their quality is high (e.g., Holmstrom (1999)). As in our paper, reputational concerns in these models can lead to strategic interactions between agents or between the agent and the market.⁸ Dewatripont, Jewitt, and Tirole (1999) study a model of career concerns with multi-tasking, where, as is standard in the literature, the agent has incentives to demonstrate high talent. They show that when talent and effort are complements, multiple equilibria can exist. In contrast to their paper and the existing literature, our model features two conflicting types of reputation - shareholder-friendly and management-friendly, so the reason for multiplicity of equilibria in our model is different. In Bar-Isaac and Deb (2013) and Bouvard and Levy (2012), the agent also cares about reputation with two different audiences, but differently from these papers, the type of reputation that is rewarded in our model is endogenously determined in equilibrium.

In the context of directors' reputational concerns, our paper is related to Song and Thakor (2006), Ruiz-Verdu and Singh (2011), and Levit (2012). These papers focus on the effect of reputational concerns on the interaction between the board and the manager in a single firm. In contrast, our paper studies how directors' reputational concerns affect all firms in the economy

⁸For example, reputational concerns can create herding behavior (e.g., Scharfstein and Stein (1990) and Zwiebel (1995)). Relatedly, Ordóñez (2012) shows that reputational concerns in credit markets lead to strategic complementarities in risk-taking between borrowers. Differently from our paper, strategic complementarities in his model are created by lenders' learning from aggregate performance.

and emphasizes the existence of externalities in corporate governance.

The literature has pointed out that governance externalities can arise from competition for managers (Acharya and Volpin (2010), Acharya, Gaborro, and Volpin (2012), and Dicks (2012)), the takeover market (Burkart and Raff (2012)), and the quality of reported earnings (Nielsen (2006) and Cheng (2011)). Our paper identifies a novel channel of governance externalities working through directors' reputational concerns, and studies its implications for board regulation. To our knowledge, this is the first paper to model the labor market for directors and its effect on equilibrium board structures. Section 5 discusses several empirical predictions that distinguish our mechanism from other potential reasons for governance externalities.

The paper is also related to the literature that studies how the structure of the board affects board decisions. Adams and Ferreira (2007), Harris and Raviv (2008), Chakraborty and Yilmaz (2011), and Levit (2012) examine what the optimal board structure should be, taking into account the trade-off between board independence and effective communication. In our paper, the structure of the board is determined endogenously in equilibrium through the labor market for directors. The endogenous nature of board structure relates our paper to Hermalin and Weisbach (1998), where the manager and the existing board bargain over the degree of independence of newly appointed directors. In their paper, it is the manager's past performance that determines whether more or less independent directors will be hired, while in our paper, these are directors' decisions over the firm's corporate governance that determine the demand for new directors. Finally, by analyzing the effect of board size and collective decision-making within the board, we contribute to the literature that studies interactions between individual board members (e.g., Warther (1998), Harris and Raviv (2008), and Malenko (2013)).

Institutional background

In this section, we describe the director election process in public US firms, and why firms differ with respect to shareholders' ability to elect directors.⁹ The election process starts with the current board of directors selecting director nominees to be presented for shareholder approval at the annual meeting. Since 2003, director nominees in NYSE and Nasdaq listed firms must be approved by independent directors. The board's nominees are included in the proxy statement, which is distributed to shareholders prior to the annual meeting. Generally, it is

⁹The election process in other countries has some differences. For example, in some European countries, shareholders owning a substantial share percentage have the right to put nominees on the ballot, and majority voting is the default rule in director elections.

hard for shareholders to nominate their own candidates for board seats. Shareholders seeking to nominate alternative candidates have to engage in a proxy fight, which involves distributing their own proxy materials and soliciting shareholder votes for the dissident nominees.

Recently, however, the system has been moving toward greater shareholder power over director elections. In August 2010, the SEC adopted the “proxy access” rule, which allowed large, long-term shareholders to include their nominees in the firm’s proxy statement instead of distributing their own proxy materials. Although the proxy access rule was later invalidated due to a lawsuit by business groups, an increasing number of shareholder proposals demanding proxy access have been recently submitted to companies for a shareholder vote. Shareholder voting rules have also been changing towards more shareholder power. In the past, most firms used plurality voting, under which the nominees who receive the highest number of affirmative votes are elected irrespective of how small the number of affirmative votes actually is. However, in recent years, firms have been increasingly switching to majority voting, under which a director has to receive a majority of the votes cast to be elected and otherwise, has to tender his resignation to the board for its consideration. According to the 2012 Georgeson’s Corporate Governance Review, 90% of S&P500 firms have moved from plurality to majority voting.

Overall, the extent of shareholder control over director elections depends on a number of governance characteristics, which vary across firms. First, the more independent the board is, the easier it is for directors to recommend candidates who are not supported by the manager. Second, the extent of shareholder control depends on whether the firm has adopted majority voting for director elections. Finally, it depends on how easy it is for shareholders to nominate their own candidates, i.e., whether the board is staggered, whether the firm has granted proxy access to large shareholders, and whether shareholders are allowed to call special meetings.

1 Setup

There are two identical firms in the economy, and the board of each firm consists of one director.¹⁰ The game has two stages - the allocation of control stage, followed by the director labor market stage.

At the first stage, each director decides whether to exert effort to transfer control of the firm from the manager (who has control by default) to the shareholders. For example, the director

¹⁰In Sections 4.3 and 4.4, respectively, we analyze how the number of directors on the board and the number of firms affect the equilibrium. In unreported analysis, we analyze the case of asymmetric firms and obtain similar results.

can push for the separation of the CEO and board chairman positions, for a higher proportion of independent directors on the board, or for board declassification. Other shareholder-friendly actions include adopting majority voting for director elections, providing proxy access to shareholders, and implementing nonbinding shareholder proposals.

The effort decisions are binary and are made simultaneously by the directors. Let $e_i \in \{0, 1\}$ be the effort decision of director in firm i and let $\chi_i \in \{0, 1\}$ be the variable that captures who has control of the firm. Here $\chi_i = 1$ stands for shareholder control in firm i , and $\chi_i = 0$ stands for management control. If the director exerts effort, the shareholders of firm i get control for sure. Otherwise, the manager of firm i maintains control. Thus, $\chi_i = e_i$. The allocation of control $\chi = (\chi_i, \chi_j)$, and hence directors' effort decisions as well, are publicly observable.¹¹

We assume that directors differ in their shareholder-friendliness. A shareholder-friendly director has a higher relative benefit from transferring control to shareholders than a management-friendly director. In particular, the type of the director of firm i is θ_i , where θ_i is distributed according to a continuous distribution function F with mean $\mathbb{E}[\theta]$, median μ , bounded density, and full support on \mathbb{R} . The direct utility of a director of type θ_i from the allocation of control in firm i is $v(\chi_i, \theta_i)$, and his cost of transferring control from the manager to the shareholders is $c(\theta_i) \in \mathbb{R}$. Certain directors can have lower costs of effort compared to others due to higher expertise or lower aversion to confrontation. Alternatively, directors can differ in their direct utility from the allocation of control due to different objectives or differences in opinion. In Section 1.1, we discuss in detail different interpretations of directors' types.

It is useful to define the net relative benefit from transferring control to shareholders:

$$\Delta(\theta) \equiv v(1, \theta) - v(0, \theta) - c(\theta).$$

We assume that $\Delta(\theta)$ is continuously differentiable and satisfies $\frac{\partial \Delta(\theta)}{\partial \theta} > 0$, $\lim_{\theta \rightarrow \infty} \Delta(\theta) = \infty$, and $\lim_{\theta \rightarrow -\infty} \Delta(\theta) = -\infty$. This implies that the relative net benefit from transferring control to shareholders is increasing in the director's type θ . Hence, high θ_i stands for shareholder-friendliness and low θ_i stands for management-friendliness. Types are independent across directors, and the type of each director is the director's private information.

At the second stage, each director can be hit by a shock, in which case he resigns from his firm and his firm has to appoint a new director. For example, directors may have to resign

¹¹In Section 4.1, we analyze an extension in which the director's effort decision does not uniquely determine the allocation of control. In this context, we study how the transparency of directors' effort decisions affects the equilibrium. In Section 4.2, we consider an extension in which an activist shareholder can intervene and affect the allocation of control in the firm.

due to health issues, family reasons, retirement, or because they have been appointed to an executive position. In particular, let $\xi_i \in \{0, 1\}$ be a random variable with $\Pr[\xi_i = 1] = \lambda$. If $\xi_i = 1$, director of firm i resigns from the board and no longer participates in the labor market for directors. Let u_{resign} be the director's utility upon resignation. If $\xi_i = 0$, the director remains on the board of firm i and can also be appointed to the board of firm j if that firm needs a new director. The shocks ξ_i are independent from the allocations of control χ_i , from directors' types θ_i , and are independent across firms.

If a director resigns, his firm searches for a new director. The firm can hire either the director of its peer firm if that director was not hit by a resignation shock, or an outside candidate, who is not serving on any board. We assume that the type of outside candidates is unknown and its expected value equals $\mathbb{E}[\theta]$, the prior.

While the allocation of control in firm i does not affect whether or not the firm needs a new director, it does affect who makes the director appointment decision. Specifically, if the manager has control ($\chi_i = 0$), then the manager makes the appointment decision, and if shareholders have control ($\chi_i = 1$), then shareholders make the appointment decision. We assume that shareholders have a preference for more shareholder-friendly directors and the manager has a preference for more management-friendly directors. In particular, if $\theta_{i,new} \in \mathbb{R}$ is the type of the director that firm i hires, then the additional payoff of the shareholders (the manager) of firm i is increasing (decreasing) in $\theta_{i,new}$. We do not rule out the possibility that directors' compensation can partly align their interests with those of the controlling party. However, provision of incentives through contracts is costly (first-best outcomes are not attainable) and hence, a firm will prefer to hire a director of the same type even if it can incentivize directors of the opposite type through contracts.

Note that we implicitly assume that the controlling party only cares about the new director's shareholder-friendliness and deliberately abstract from the effect of other relevant factors, such as directors' experience and expertise. However, our results will continue to hold in a setting where the party making the appointment decisions also cares about directors' expertise, as long as expertise and shareholder-friendliness are not perfectly correlated across directors. In addition, it is often argued that given the dual role of the board as both a monitor and advisor, even shareholders may prefer to have some management-friendly directors on the board (e.g., Adams and Ferreira (2007), Harris and Raviv (2008)). We allow for this possibility in the extension in Section 4.5.

Let π_i denote the reputation of director i , defined as the expected type of the director

at the beginning of the second stage conditional on all available information. The director's reputation will be endogenously determined by his effort strategy $e_i(\theta_i)$ and allocation of control χ_i . Directors' reputation will, in turn, determine firms' hiring decisions. Denote by $h_i(\chi_i, \pi_j) : \{0, 1\} \times \mathbb{R} \rightarrow \{0, 1\}$ the hiring decision of firm i based on the allocation of control χ_i , reputation π_j of director j , and given that director i has resigned ($\xi_i = 1$). In particular, if the firm hires the outside candidate, then $h_i = 0$, and if it hires the director of firm j , then $h_i = 1$.

The director gets an additional utility $\alpha > 0$ if he is hired by another firm. Parameter α measures the strength of directors' reputational concerns. While directors' financial compensation might be affected by the demand and supply of directors in the labor market, a large component of directors' utility from board seats is non-pecuniary. Indeed, when asked about their personal benefits from serving on the board, directors list prestige, valuable connections, power, and the opportunity to develop new areas of expertise as being more important than financial compensation (see, e.g., director surveys by Lorsch and MacIver (1989) and Burke (1997)). Matveyev (2013) estimates that along with compensation, directors also care about the size of the firm, its networking opportunities, and its stock performance. In addition, Matveyev (2013) notes that all outside directors within a firm usually get the same pay, and almost all within-firm variation in compensation comes from the fees directors get from serving on special committees. For these reasons, and for simplicity, we abstract from the effects of the labor market on α and take it as a given parameter. In the Supplementary Appendix, we analyze a setup in which a director's utility from an additional board seat depends on his type and the type of the firm he is invited to. We show that our results are robust to this assumption.

Since a director can only be hired by the other firm if he is not hit by a resignation shock but the other director is, the utility of director i is given by

$$u_i = v(\chi_i, \theta_i) - c(\theta_i)e_i + \alpha\xi_j(1 - \xi_i)h_j(\chi_j, \pi_i) + \xi_i u_{resign}. \quad (1)$$

All agents are risk neutral, and preferences are common knowledge.

1.1 Discussion: Directors' types

A key assumption of the model is that directors differ in the degree of their shareholder-friendliness. In particular, Δ , the relative net benefit from transferring control to shareholders, varies across directors and is persistent across firms for a given director. In this section, we

discuss several interpretations of shareholder-friendliness.

One reason why some directors are more shareholder-friendly than others is that they are more effective monitors, that is, their costs of promoting strong governance, $c(\theta)$, are lower. There are several interpretations of these costs. First, the costs can represent directors' personality traits. Directors who are more averse to confrontation and boardroom conflict are less likely to challenge the manager since it will inevitably lead to tension in the boardroom. Second, higher costs of effort could reflect a director's lack of knowledge and experience of how to evaluate and monitor the CEO. Incapable directors will have to invest more time and effort to be effective when confronting the manager. Finally, the costs of effort can represent the difference between the director's punishment from the manager for opposing him and the director's punishment from the shareholders for not monitoring the manager. This interpretation suggests that the costs of effort can in fact be negative (which is allowed in the model).

Another reason why directors can differ in their shareholder-friendliness is that they can have different direct utility from the allocation of control, $v(\chi, \theta)$. First, directors can disagree on the objective of the board. There is a long-standing debate between corporate governance experts on what the objective of the board should be.¹² Some argue that the board is the agent of shareholders and as such has the duty to promote shareholder value by allocating control to them. Others argue that the board represents the corporation as a whole, including employees, management, creditors, customers, and suppliers. According to this view, the board has the duty to promote the value of all stakeholders by allocating control to the management. Under this interpretation, directors with a high θ feel strongly about promoting shareholder value, and directors with a low θ feel strongly about promoting stakeholder value.

Even if directors agree on the objective of the board, whether it is to maximize shareholders' or all stakeholders' value, they can disagree on *how* to achieve this objective. For example, even if directors agree on shareholder value maximization, some directors may believe that shareholders' interests are best served by giving control to shareholders, while others may believe that the best way to enhance shareholder value is to give control to the manager. This could be because the success of the firm depends on managerial initiative and firm specific investments that the manager has incentives to take only if he has control (e.g., Grossman and Hart (1986) and Hart and Moore (1990)). Alternatively, this could be because the manager has expertise and private information that he will not communicate to the board unless he has control (e.g., Adams and Ferreira (2007) and Harris and Raviv (2008)). Either way, directors

¹²See, e.g., Bebchuk (2005) and "Bebchuk vs. Lipton" by A. Bernstein, *Directorship*, Vol. 33, Issue 6, p.20.

can fundamentally disagree on how to enhance shareholder value, and high θ would correspond to the belief that shareholder control is the most efficient way to achieve this objective.¹³ In Section 4.5, we build on this interpretation and analyze the case in which the relative value from shareholder control can differ across firms.

Regardless of the interpretation, both the cost of effort and the utility from the allocation of control depend on directors' personalities, backgrounds, and beliefs, and can therefore vary substantially across directors and not be easily observable by outsiders. At least to some extent, these characteristics are director-specific.¹⁴ In our analysis, we do not take a stand on what the most relevant interpretation is and think of a director's type θ as aggregating these different characteristics. The assumptions on the functional form of $\Delta(\theta)$ capture the above interpretations in a reduced form way.

2 Analysis

The solution concept is the Perfect Bayesian Equilibrium (PBE), which is characterized by directors' effort decisions $e_i^*(\theta_i)$, $e_j^*(\theta_j)$, beliefs about the type of directors $\pi_i^*(\chi_i)$, and firms' hiring decisions $h_i^*(\chi_i, \pi_j)$. A formal definition of a PBE is provided in the Appendix.

Let τ_j denote the ex-ante probability that shareholders of firm j obtain control. Then τ_j is determined by the effort strategy $e_j(\theta_j)$ and is given by

$$\tau_j = \Pr [\theta \in \{\theta : e_j(\theta) = 1\}]. \quad (2)$$

Let $\pi_i^*(\chi_i) \equiv \mathbb{E}[\theta|\chi_i]$ be the equilibrium inference about the type of director i upon outcome χ_i . Consider director i 's utility from having a reputation π_i . If $\chi_j = 1$, i.e., if shareholders of firm j get control, then firm j will hire director i if and only if his reputation is above the reputation of the outside candidate $\mathbb{E}[\theta]$. Similarly, if $\chi_j = 0$, i.e., the manager of firm j retains control, then firm j will hire director i if and only if his reputation is below the reputation of the

¹³Similarly, directors can agree on stakeholder value maximization but disagree on how to achieve this objective. For example, some directors may believe that since shareholders are the residual claimants on the firm's assets, allocating control to shareholders will guarantee that the value of all stakeholders is maximized. Other directors may be concerned that shareholders are myopic and can thus take advantage of other stakeholders to make a short-run profit. These directors will prefer to give more control to the management.

¹⁴It is not necessary for our results that θ_i is perfectly transferable across firms. Directors may adapt their behavior and conform to the existing corporate governance system of the firms in which they serve as board members. As long as there is some level of persistence of directors' characteristics, our results continue to hold. Consistent with the assumption that θ_i is transferable across firms, Bouwman (2011) finds that a firm's governance practices move in the direction of governance practices of the other firms its directors serve at.

outside candidate.¹⁵ In other words, $h_j(1, \pi_i) = 1$ if and only if $\pi_i \geq \mathbb{E}[\theta]$, and $h_j(0, \pi_i) = 1$ if and only if $\pi_i < \mathbb{E}[\theta]$. Given that, the expected direct benefit of director i from obtaining reputation π_i is given by $\alpha\lambda(1 - \lambda)\Gamma(\pi_i, \tau_j)$, where

$$\Gamma(\pi, \tau) = \tau \cdot 1\{\pi \geq \mathbb{E}[\theta]\} + (1 - \tau) \cdot 1\{\pi < \mathbb{E}[\theta]\}. \quad (3)$$

Note that for any $\underline{\pi}, \bar{\pi}$ such that $\bar{\pi} \geq \mathbb{E}[\theta] > \underline{\pi}$, $\Gamma(\bar{\pi}, \tau) > \Gamma(\underline{\pi}, \tau)$ if and only if $\tau > 0.5$. Intuitively, whether a director wants to have a shareholder-friendly or a management-friendly reputation depends on the allocation of control in other firms. If managers (shareholders) are the main decision-makers in other firms, i.e., τ is small (large), then the director is more likely to be invited to other boards if he is known for being management-friendly (shareholder-friendly). Another property of $\Gamma(\pi, \tau)$ is that a director benefits from there being more shareholder-controlled firms ($\frac{\partial \Gamma}{\partial \tau} > 0$) if and only if he has a shareholder-friendly reputation ($\pi \geq \mathbb{E}[\theta]$).

Let $U_i(e_i, \theta_i)$ be the expected utility of director i given his type θ_i and effort e_i , and taking as given beliefs $\pi_i^*(\cdot)$ and the probability of shareholder control in the other firm $\tau_j = \tau_j(e_j(\cdot))$. Then $U_i(e_i, \theta_i)$ is given by

$$U_i(e_i, \theta_i) = v(e_i, \theta_i) - c(\theta_i)e_i + \alpha\lambda(1 - \lambda)\Gamma(\pi_i^*(e_i), \tau_j) + \lambda u_{resign}. \quad (4)$$

Using (4), the following lemma shows that in any equilibrium, the director follows a threshold strategy and exerts effort only if his preference for shareholder control is sufficiently strong.¹⁶

Lemma 1 *In any equilibrium, there exists θ_i^* such that $e_i(\theta_i) = 1$ if and only if $\theta_i > \theta_i^*$.*

Given Lemma 1 and the properties of conditional expectations, it follows that

$$\pi_i^*(0) = \mathbb{E}[\theta | \theta \leq \theta_i^*] < \mathbb{E}[\theta] < \mathbb{E}[\theta | \theta > \theta_i^*] = \pi_i^*(1) \quad (5)$$

¹⁵Without loss of generality, we assume that if a director's reputation equals $\mathbb{E}[\theta]$, then shareholders will hire the director, and the manager will hire the outside candidate.

¹⁶In this sense, all equilibria of the game are semi-pooling: all types above (below) the threshold follow the same strategy, but types above the threshold follow a different strategy than types below the threshold. This result is due to the assumption that $\Delta(\cdot)$ is unbounded. If $\Delta(\cdot)$ were bounded, there could also exist pooling equilibria, in which all types follow the same strategy of exerting (or not exerting) effort, i.e., the threshold θ_i^* in Lemma 1 could be infinite. Since our results would continue to hold in this setting as well, we make the assumption that $\Delta(\cdot)$ is unbounded for simplicity.

and

$$\tau_j^* = \Pr[\theta > \theta_j^*] = 1 - F(\theta_j^*). \quad (6)$$

Consider the best response function $\beta_i(\theta_j)$ of director i , taking as given that director j exerts effort when his type exceeds the threshold θ_j . The best response function defines the threshold $\beta_i(\theta_j)$ such that types $\theta_i > \beta_i(\theta_j)$ exert effort. From (4) and the proof of Lemma 1, it follows that $\beta_i(\theta_j) = \beta(\theta_j)$, where

$$\beta(\theta) = \Delta^{-1}(\alpha\lambda(1-\lambda)(2F(\theta)-1)). \quad (7)$$

Since $\Delta(\theta)$ is strictly increasing, continuous, and takes all values on $(-\infty, +\infty)$, its inverse $\Delta^{-1}(\cdot)$ is a well defined, strictly increasing, and continuous function. Note that $\beta_i(\theta) = \beta_j(\theta)$, i.e., the directors' best response functions are identical. Note also that $\beta(\theta)$ strictly increases in θ and takes all values in the interval $[\Delta^{-1}(-\alpha\lambda(1-\lambda)), \Delta^{-1}(\alpha\lambda(1-\lambda))]$.

Since the best response threshold of a director is increasing in the threshold of his peer, the game exhibits *strategic complementarity*. Intuitively, if the director of firm j is more likely to exert effort (θ_j decreases), then shareholders of firm j are more likely to get control and have the power to appoint directors to their board. Therefore, director i 's relative reward for having a shareholder-friendly reputation is higher. This effect increases the incentives of director i to exert effort, decreasing the threshold θ_i^* .

The following lemma characterizes the set of equilibria of the model using the properties of $\beta(\theta)$ and the symmetry of the best response functions.

Lemma 2 *An equilibrium always exists, and any equilibrium is symmetric.*

Since all equilibria of the game are symmetric, i.e., $\theta_i^* = \theta_j^* = \theta^*$, any equilibrium θ^* is the solution of $\beta(\theta^*) = \theta^*$. Define $\tau_j^* = \tau_i^* = 1 - F(\theta^*)$ by τ^* , $\pi_i^*(1) = \pi_j^*(1)$ by $\pi^*(1)$, and $\pi_i^*(0) = \pi_j^*(0)$ by $\pi^*(0)$. Given (5) and the properties of $\Gamma(\pi, \tau)$ discussed above, $\Gamma(\pi^*(1), \tau^*) > \Gamma(\pi^*(0), \tau^*)$ if and only if $\tau^* > 0.5$ or, equivalently, $\theta^* < \mu$. In other words, a shareholder-friendly reputation generates a higher payoff than a management-friendly reputation if and only if there is a higher than 50% chance that the other firm will be controlled by shareholders. Based on this argument, the next definition classifies potential equilibria into two types.

Definition 1 *An equilibrium is called shareholder-friendly if $\tau^* > 0.5$ and is called management-*

friendly if $\tau^* < 0.5$.

Due to strategic complementarity, our model can have multiple equilibria. Moreover, the next proposition shows that when reputational concerns are sufficiently important, there always exist at least one shareholder-friendly and at least one management-friendly equilibrium. Thus, equilibria with strong and weak governance can co-exist for a given set of parameters, suggesting that countries or industries with similar characteristics can have different corporate governance systems as an equilibrium outcome.

Proposition 1 *There exist $\bar{\alpha}$ and $\underline{\alpha}$, $\bar{\alpha} \geq \underline{\alpha} > 0$, such that:*

- (i) *If $\alpha > \bar{\alpha}$, there exist at least one shareholder-friendly equilibrium and at least one management-friendly equilibrium.*
- (ii) *If $\alpha < \bar{\alpha}$, all equilibria are of the same type. In particular, all equilibria are management-friendly if $\Delta(\mu) < 0$ and shareholder-friendly if $\Delta(\mu) > 0$.*
- (iii) *If $\alpha < \underline{\alpha}$, the equilibrium is unique.*

The intuition behind Proposition 1 is as follows. In our model, strategic complementarity between firms' corporate governance decisions arises due to directors' reputational concerns, represented by parameter α . When α increases, reputation becomes more important for directors, and the complementarity in their decisions becomes stronger. For this reason, multiple equilibria are more likely to exist when reputational concerns are significant. Figure 1 illustrates this effect by plotting the best response function of directors for $c(\theta) = 2.75$, $\lambda = 0.5$, a standard normal distribution of types, and the utility function $v(\chi, \theta) = (\theta + 1)\chi$.

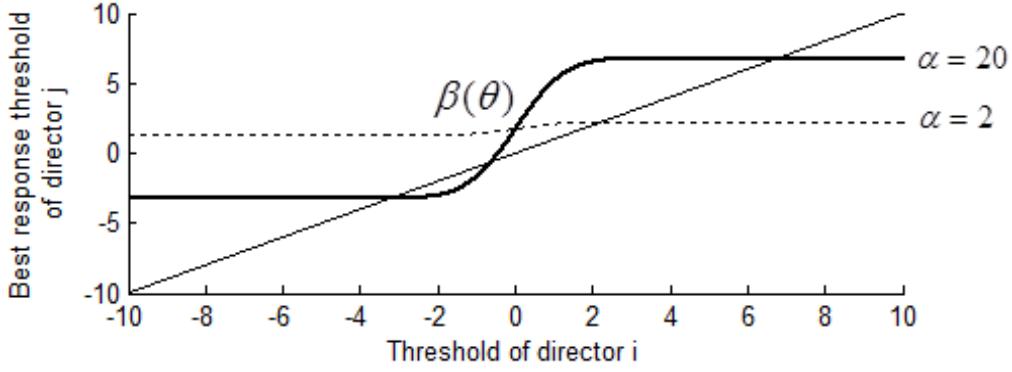


Figure 1: Best response function $\beta(\theta)$ for $c(\theta) = 2.75$, $\lambda = 0.5$, $v(\chi, \theta) = (\theta + 1)\chi$, and standard normal F

First, when $\alpha = 0$, directors do not care about additional board seats and hence make their effort decisions independently of other directors' strategies. Hence, the best response function $\beta(\theta)$ for $\alpha = 0$ is a constant function with value $\Delta^{-1}(0)$. Therefore, a unique equilibrium exists, and this equilibrium is management-friendly since directors' cost of effort $c(\theta) = 2.75$ is sufficiently large relative to $\mu = 0$. When α becomes positive, strategic complementarity arises, and the best response function $\beta(\theta)$ becomes strictly increasing. The dashed line in Figure 1 represents $\beta(\theta)$ for $\alpha = 2$. Although $\beta(\theta)$ is increasing, externalities between firms are not strong enough, and hence the game still has a unique, management-friendly, equilibrium (θ^* around 2.24, which corresponds to τ^* around 0.01). However, as Proposition 1 shows, when α increases further, strong externalities between firms give rise to multiple equilibria, some of which are shareholder-friendly. In particular, when $\alpha = 20$ (the bold line in Figure 1), the graph of the best response function crosses the 45-degree line in three points, corresponding to the three equilibria of the game. Two of them (θ^* around -3.25 and -0.64) are shareholder-friendly, and the third one (θ^* around 6.75) is management-friendly.

When multiple equilibria exist, some of them can be unstable. In what follows, we define stability and classify equilibria accordingly. The definition is based on local tatônnement stability, which requires that a dynamic adjustment process, in which directors take turns playing a best response to each others' current strategies, converges to the equilibrium from any strategy pair in the neighborhood of the equilibrium. Before introducing the formal definition of stability, we make the following assumption for the remainder of the paper.

Assumption 1 $\Delta(\mu) \neq 0$.

As the proof of Proposition 2 shows, this assumption ensures that for almost all $\alpha \in \mathbb{R}$, $\frac{\partial \beta(\theta)}{\partial \theta}|_{\theta=\theta^*} \neq 1$ in any equilibrium θ^* . In other words, whenever the best response function and the 45-degree line intersect, they are not tangent to each other. This assumption guarantees the existence of stable equilibria and is essentially a regularity condition. Under this assumption, local tatônnement stability is equivalent to the following definition.

Definition 2 *An equilibrium θ^* is called “stable” if $\frac{\partial \beta(\theta)}{\partial \theta}|_{\theta=\theta^*} < 1$.*

Thus, stability requires that the best response function of director i to the strategy of director j crosses the 45-degree line from above.

Proposition 2 *A stable equilibrium exists for almost all $\alpha \in \mathbb{R}$.*

Proposition 2 implies that if an equilibrium is unique, then it is stable. For example, the case $\alpha = 2$ in Figure 1 corresponds to a unique stable equilibrium. The case $\alpha = 20$ features three equilibria. The two boundary equilibria are stable, and the interior equilibrium is unstable.¹⁷

3 Shareholder welfare and comparative statics

In our setting, shareholder welfare can be measured by the equilibrium quality of corporate governance, defined as the probability that shareholders get control: $\tau^* = 1 - F(\theta^*)$. For example, one can think of shareholders of firm i as being infinitely shareholder-friendly ($\theta = +\infty$) and getting utility $v(\chi_i, +\infty)$ from the allocation of control χ_i . Hence, all equilibria can be ranked by shareholder welfare, where equilibria with a lower θ^* correspond to greater shareholder welfare. Since the best response function $\beta(\cdot)$ is bounded and increasing, by Tarski’s Fixed Point Theorem, $\beta(\cdot)$ has the least and the greatest fixed points (equilibria). We denote these two equilibria by $\underline{\theta}^*$ and $\bar{\theta}^*$ respectively and call them “the most shareholder-friendly” and “the least shareholder-friendly” equilibria of the game.¹⁸

Generally, as discussed in Section 1.1, allocating control to shareholders might not be the most effective way to maximize shareholder welfare. Managerial initiative and firm-specific investments, as well as efficient communication between the manager and the board, which are

¹⁷In the proof of Proposition 1, we show that all statements of the proposition are valid for stable equilibria.

¹⁸“The most shareholder-friendly” equilibrium can be management-friendly if all equilibria of the game are management-friendly, and vice versa.

important for good performance, may require delegating control to the manager. To incorporate this possibility, in Section 4.5, we analyze an extension in which the optimal allocation of control between the manager and shareholders can differ across firms.

In the remainder of this section, we study the comparative statics of the equilibrium probability of shareholder control. We focus on local comparative statics, when the equilibrium continues to exist upon a small change of parameters. All stable equilibria have this property.

Proposition 3 *Consider any stable equilibrium. Then:*

- (i) τ^* decreases with c .¹⁹
- (ii) If $F(\kappa_2, \cdot)$ first-order stochastically dominates $F(\kappa_1, \cdot)$ for $\kappa_2 > \kappa_1$, τ^* increases with κ .
- (iii) τ^* increases with α if and only if the equilibrium is shareholder-friendly.
- (iv) If the distribution of types is symmetric and $F(\sigma_2, \cdot)$ is more risky than $F(\sigma_1, \cdot)$ for $\sigma_2 > \sigma_1$ in the sense of a simple mean-preserving spread, then τ^* increases with σ if and only if the equilibrium is management-friendly.

These comparative statics results only hold in stable equilibria. Indeed, the proof of Proposition 3 is based on the fact that for any parameter p ,

$$\frac{\partial \theta^*}{\partial p} = M(\theta^*) \times \frac{\partial \beta(\theta)}{\partial p}|_{\theta=\theta^*},$$

where the multiplier

$$M(\theta^*) \equiv \frac{1}{1 - \frac{\partial \beta(\theta)}{\partial \theta}|_{\theta=\theta^*}}$$

is strictly positive in any stable equilibrium. The same logic implies the opposite comparative statics in non-stable equilibria, in which $\frac{\partial \beta(\theta)}{\partial \theta}|_{\theta=\theta^*} > 1$. This result is standard in the literature on games with strategic complementarities (see, for example, Vives (2005)).

Note also that since $\frac{\partial \beta(\theta)}{\partial \theta}|_{\theta=\theta^*} > 0$, the multiplier $M(\theta^*)$ is strictly greater than one in any stable equilibrium. Hence, small changes in parameters are amplified due to strategic complementarity of directors' effort decisions. To see the intuition, consider a uniform decrease in directors' cost of effort by c , for example, due to a new regulation. The direct effect of this

¹⁹In the subsequent discussion, we refer to changes in c as a uniform shift of $c(\theta)$. In other words, we analyze the comparative statics with respect to a parameter c such that $c(\theta) = c + c_0(\theta)$ for some function $c_0(\cdot)$.

change on a director's incentives to exert effort is represented by the term $\frac{\partial \beta(\theta)}{\partial c}$. However, since the director in the other firm is also more likely to exert effort, the other firm is now more likely to be controlled by shareholders. This second, indirect, effect increases the value of a shareholder-friendly reputation in the labor market and magnifies the director's incentives to exert effort. In the extreme case, when $\frac{\partial \beta(\theta)}{\partial \theta}|_{\theta=\theta^*} \rightarrow 1$, then $M(\theta^*) \rightarrow \infty$. This means that due to strategic complementarity, even a very small shock to the parameters of the model has a very significant effect on the equilibrium outcome.

The intuition for the comparative statics of κ is similar to the one of c . Note, however, that a higher likelihood of shareholder-friendly directors (higher κ) leads to a higher probability of shareholder control for two reasons. First, keeping directors' threshold strategy θ^* fixed, it is more likely that each individual director's type will be above the threshold. This effect is amplified by the decrease in directors' equilibrium threshold θ^* : knowing that the other firm is now more likely to be controlled by shareholders, each director has stronger incentives to exert effort and thereby build a shareholder-friendly reputation.

The third statement of Proposition 3 shows that directors' reputational concerns (α) increase the probability of shareholder control only if the equilibrium is shareholder-friendly ($\tau^* > 0.5$). This is because in a management-friendly equilibrium, managers of other firms, rather than shareholders, make the appointment decisions, and hence having a shareholder-friendly reputation hurts directors' chances of being invited to other boards. In this sense, directors' reputational concerns amplify corporate governance: as α increases, strong governance systems become stronger and weak systems become weaker. This suggests that a regulation that increases the value of reputation in the director labor market (e.g., increasing the allowed number of directorships a person can hold) strengthens corporate governance only if the existing quality of corporate governance is sufficiently strong.

Parameter σ captures the uncertainty about directors' types. For example, it reflects the quality of firms' disclosure about directors' education, employment, and prior history. The proposition shows that an increase in the uncertainty about directors' types attenuates the effect of reputational concerns on corporate governance: strong systems become weaker and vice versa. The reason is that higher uncertainty about the other director's type makes it harder for a director to predict whether the other firm will be controlled by managers or by shareholders. If the equilibrium is shareholder-friendly, this increased uncertainty decreases the director's relative reward for creating a shareholder-friendly reputation and thus reduces his incentives to exert effort. The opposite is true in a management-friendly equilibrium.

4 Extensions

In this section, we discuss several extensions of the basic model. The formal setups, results, and proofs for these extensions are provided in the Supplementary Appendix.

4.1 Transparency

While the board's decision-making process is generally opaque, some recent regulations have increased boardroom transparency. For example, as discussed in the introduction, the 2004 SEC law requires firms to disclose any director departure that is due to a disagreement, and the 2004 law in China requires firms to disclose the names of directors who vote in dissent. Our setting allows us to study whether increased transparency of board decision-making is beneficial for corporate governance.

To study the effect of transparency, we extend the basic model and assume that if a director exerts effort, shareholders get control only with some probability. Hence, the director's effort decision cannot be directly inferred from the publicly observable allocation of control. The extent to which the director's decision is observable depends on the degree of transparency. Specifically, the director's effort is publicly revealed prior to the second stage with probability η , which is a measure of transparency. Other assumptions of the model are unchanged.

We show that higher transparency increases the probability of shareholder control if and only if the equilibrium is shareholder-friendly. Intuitively, when directors' behavior is more likely to be publicly observed, they are more eager to take shareholder-friendly actions only if a shareholder-friendly reputation is rewarded in the market. Thus, the effect of regulations increasing boardroom transparency crucially depends on the aggregate corporate governance.

4.2 Shareholder activism

In practice, the allocation of control in the firm can be affected not only by its directors, but also by its shareholders. Activist investors can intervene if they are concerned that the board will not be monitoring the manager effectively. For example, an activist can launch a proxy fight and place its representatives on the board, ensuring that some directors will always take shareholder-friendly actions. To incorporate this possibility, we augment the basic model with an additional stage of shareholder activism before directors' effort decisions. In each firm, there is an activist investor who can incur a cost to intervene and transfer control to shareholders. If

the activist does not intervene, the allocation of control is determined by directors' decisions, as in the basic model. The activists' intervention decisions are publicly observable and made simultaneously. Their costs of intervention are their private information.

In equilibrium, each activist intervenes if and only if her cost does not exceed a certain threshold. Shareholder activism acts as a substitute for board monitoring: the activist is more likely to intervene when directors' effort to improve governance is lower. The comparative statics of the extended model is similar to that in the basic model. Specifically, an increase in c (κ) always decreases (increases) shareholder welfare net of intervention costs, while the effect of directors' reputational concerns depends on whether the system is in a shareholder-friendly or in a management-friendly equilibrium.

Interestingly, we show that there are externalities in shareholder activism across firms. In particular, the activists' intervention decisions are strategic substitutes: a higher probability of intervention in one firm leads to a lower probability of intervention in the other firm. The intuition is the following. If the activist in firm j is more likely to intervene and transfer control to shareholders, firm j is more likely to have a demand for shareholder-friendly directors. Hence, the director of firm i has stronger incentives to establish a shareholder-friendly reputation and thus is more likely to transfer control to shareholders in his firm. This implies that shareholder activism in firm i is needed less. Note that strategic substitutability of activists' intervention decisions is due to directors' reputational concerns and is not present when $\alpha = 0$.

4.3 Board size

Corporate boards typically consist of multiple directors and operate by collective decision-making. We therefore extend the model and assume that each board consists of $K \geq 2$ directors. The board's decision-making process is modeled as follows. All directors simultaneously decide whether to exert effort. If at least $T \in \{1, \dots, K\}$ directors exert effort (e.g., vote for a proposal that restricts the manager's power), then shareholders obtain control. Otherwise, the management retains control. Individual effort decisions are not observable, but the allocation of control in each firm is observable and thus affects directors' reputation. At the second stage, we abstract from the mechanical effect of board size on the supply and demand for directors in order to focus on the effect of board size on collective decision-making.²⁰

²⁰Specifically, we assume that at the second stage, with probability λK , exactly one director in each firm is hit by a resignation shock. Each of K directors has an equal chance of being hit by the shock. We also assume that if several directors have the same reputation, they are equally likely to be invited to the other board. In this case, conditional on $\chi_i = \chi_j$, the probability that a given director of firm i is invited to board j equals λ

Generally, a director’s actions are affected both by the board size and voting rule of his own firm (due to free-riding among directors) and by the board size and voting rule of the other firm (due to externalities in the labor market). Since the focus of our paper is on externalities between firms, we shut down the free-riding channel by assuming that directors’ effort is costless. When effort is costless, the only reason why board size affects directors’ behavior is due to externalities between firms.

We show that the extended model exhibits strategic complementarity as well. Moreover, the effect of board size on directors’ incentives to promote strong governance depends on the voting rule. For example, if unanimity is required to give control to shareholders ($T = K$), a larger board size decreases directors’ incentives to vote for shareholder control. This is because under unanimity, a larger board size at other firms implies that those boards are less likely to transfer control to shareholders, which decreases the value of a shareholder-friendly reputation in the labor market. When the voting rule is a simple majority rule, the effect of board size depends on the aggregate level of corporate governance. Specifically, a larger board size increases directors’ incentives to vote for shareholder control if and only if the equilibrium is shareholder-friendly. Intuitively, under a simple majority rule, the equilibrium is shareholder-friendly (management-friendly) if the probability that each director exerts effort is greater (smaller) than 50% and hence, when the number of directors on the board increases, the likelihood that at least half will exert effort increases (decreases). This result implies that under a simple majority rule, board size amplifies corporate governance in the sense that weak governance systems become weaker and strong governance systems become stronger as board size increases.

4.4 Multiple firms

We also consider the extension of the model to $N \geq 2$ firms. Given that the labor market for directors is somewhat segmented both by industry and by geographical location (see the discussion in Section 5), we think of N as representing the number of firms in the relevant segment and hence not being very large. As in the basic model, the director of each firm is hit by a resignation shock with probability λ , and the shocks are independent across directors.

Note that based on the allocation of control across firms after the first stage, the market is divided into two sets: firms controlled by shareholders search among directors with a shareholder-friendly reputation, and firms controlled by managers search among directors with a management-friendly reputation. We assume that the labor market allocation is efficient in

for any K , which allows us to abstract from the effect of K on the supply and demand for directors.

the following sense. In equilibrium, no firm that is controlled by shareholders (management) hires an outside candidate if a director serving on one of the boards has a shareholder-friendly (management-friendly) reputation and has the capacity to serve on another board. If such a situation occurred, both the firm and the director would be better off matching with each other. We also assume that if there is excess supply of directors in any segment of the market, all directors in this segment have equal probability of being invited to the boards of other firms. Finally, we assume that directors are not limited in the number of boards on which they can serve.²¹ We focus on symmetric equilibria.

We show that the results of the basic model continue to hold for any number of firms. In particular, for any N , the extended model exhibits strategic complementarity as well. Moreover, the comparative statics results of the extended model are similar to those in Proposition 3.²²

4.5 Value of shareholder control

If managers have high expertise or if they need to be given incentives to make firm-specific investments, shareholders may be better off delegating control to them despite potential conflicts of interest (e.g., Grossman and Hart (1986), Hart and Moore (1990), Adams and Ferreira (2007), Harris and Raviv (2008)). The relative value from management control can differ across firms. To capture this cross-sectional heterogeneity, we extend the model and assume that in each firm, shareholder control is optimal only with probability $\psi \in (0, 1]$. With probability $1 - \psi$, shareholder value is maximized if control is delegated to the manager. The optimal allocation of control in each firm is independent across firms and is privately observed by the firm's director. We also assume that when management control is optimal, the director's relative benefit from transferring control to shareholders is negative and is smaller than when shareholder control is optimal, but that it still increases with the director's shareholder-friendliness. Finally, we assume that if management control is optimal for a given firm, this firm demands management-friendly directors not only if it is controlled by managers, but also if it is controlled

²¹In practice, the number of board seats that a director can hold could be limited by the director's time constraints or by regulation (for example, many countries impose a limit on the number of directorships). If there were a limit on the number of directorships, then raising this limit would increase the value of reputation. Since the effect of reputational concerns on directors' incentives to exert effort depends on whether the equilibrium is shareholder- or management-friendly, the effect of increasing the allowed number of directorships would depend on the nature of equilibrium as well. To see this, note that in a two-firm case, changing α from zero to a positive number is equivalent to increasing the cap on the number of directorships from one to two.

²²In unreported analysis, we show that the effect of N on the equilibrium level of corporate governance is non-monotonic and is generally ambiguous. For example, the direction of the effect depends on whether or not $\Delta(\cdot)$ is bounded.

by shareholders. This allows us to capture the notion that management-friendly directors may be optimal for shareholders given the dual (advisory and monitoring) role of the board.²³

We show that the extended model exhibits strategic complementarity as well. Importantly, equilibria of this game feature two potential types of inefficiency. The first type of inefficiency is similar to the basic model and arises when directors do not transfer control to shareholders even though shareholder control is optimal. In addition, an equilibrium can feature another type of inefficiency, when directors allocate control to shareholders even though management control is optimal. The only reason why directors do this is to signal their shareholder-friendliness to other firms (which is beneficial if a shareholder-friendly reputation is rewarded). Hence, the second type of inefficiency does not arise if $\alpha = 0$ or if the equilibrium is management-friendly.

We measure shareholder welfare by the ex-ante probability that control is allocated efficiently. When the only source of inefficiency is excessive management control, then, similar to the basic model, a decrease in the costs of effort and an increase in κ always increase shareholder welfare, while an increase in directors' reputational concerns increases shareholder welfare if and only if the equilibrium is shareholder-friendly. However, when the equilibrium features excessive shareholder control as well, the effect of these parameters is ambiguous and depends on ψ . For example, shareholder welfare can actually increase with directors' costs of effort if ψ is small. Intuitively, since directors transfer control to shareholders even when it is inefficient to do so, higher costs of effort serve as a commitment to delegate control to managers, which can benefit shareholders. Since excessive shareholder control arises in shareholder-friendly, but not in manager-friendly equilibria, this result implies that the effect of regulations affecting c or κ depends on the aggregate level of corporate governance.

4.6 Outside candidates

In the basic model, we assume that the type of outside candidates is unknown and thus their expected type (reputation) is $\mathbb{E}[\theta]$. Some outside candidates, however, could have partly revealed the degree of their shareholder-friendliness through their professional activities and hence can have a reputation $\varphi \neq \mathbb{E}[\theta]$. For example, CEOs of other firms are a large source of potential outside candidates. Arguably, CEOs are intrinsically more management-friendly because they are managers themselves. In this case, $\varphi_{CEOs} < \mathbb{E}[\theta]$. We therefore, consider an extension of

²³In unreported analysis, we also consider an alternative specification, in which the firm demands shareholder-friendly directors whenever shareholders obtain control, even if management control is optimal. The results are qualitatively similar.

the basic model in which the set of potential outside candidates includes both directors with reputation $\mathbb{E}[\theta]$ and directors with reputation $\varphi \in \mathbb{R}$.

We show that if the outside candidates are sufficiently shareholder-friendly (management-friendly), corporate governance is actually weaker (stronger) compared to the basic model. To see the intuition, suppose, for example, that φ is relatively high. In this case, a director who obtains a shareholder-friendly reputation by transferring control to shareholders will never be invited to another board. Indeed, a shareholder-controlled firm prefers to hire an outside candidate with reputation φ since he is even more shareholder-friendly than the director. A management-friendly firm prefers to hire an outside candidate with reputation $\mathbb{E}[\theta]$ since he is more management-friendly than the director. In contrast, a director with a management-friendly reputation will be preferred by management-controlled firms over both types of outside candidates. Thus, compared to the basic model, the relative benefit from creating a shareholder-friendly reputation is lower. This effect reduces directors' incentives to promote strong corporate governance, and equilibria become less shareholder-friendly.

5 Empirical predictions

In this section, we discuss our paper in the context of the existing empirical literature and offer new testable predictions. We are not aware of other theories that have these predictions.

The premise of our paper is that directors have to trade off two conflicting types of reputation - one for being shareholder-friendly and one for being management-friendly. Consistent with the existence of this trade-off, the literature has found mixed results with respect to whether the labor market rewards directors for imposing discipline on the management. Consistent with the view that a shareholder-friendly reputation is rewarded, a number of papers find that directors are held accountable for failing to monitor the management effectively.²⁴ Conversely, consistent with the view that a management-friendly reputation is rewarded, Helland (2006) finds that directors of firms charged with fraud, experience an increase in the number of outside directorships, and Marshall (2011) shows that directors who resign from the board over a disagreement, experience a loss in board seats over the five year period following the dispute.²⁵

²⁴Coles and Hoi (2003) show that directors who rejected the Pennsylvania Senate Bill 1310 antitakeover provisions were three times as likely to gain additional directorships than those who retained the provisions. Fich and Shivdasani (2007) find that following a financial fraud lawsuit, directors are likely to lose board seats at other firms, particularly those with strong governance. See also Harford (2003), Yermack (2004), Srinivasan (2005), Ertimur, Ferri, and Stubben (2010), and Jiang, Wan, and Zhao (2012) in the context of Chinese firms.

²⁵Relatedly, Ertimur, Ferri, and Maber (2011) find no evidence that directors of firms involved in option

Most of the existing literature looks at the aggregate number of board seats gained by directors. In contrast, our paper emphasizes that whether directors' shareholder-friendly actions will be rewarded in the labor market crucially depends on the balance of power at other firms. Formally, the first implication is the following.

Prediction 1 *Directors who demonstrate their shareholder-friendliness are more (less) likely to be subsequently appointed to boards of firms with stronger (weaker) corporate governance.*

Shareholder-friendly directors can be identified as those who vote against the management (Jiang, Wan, and Zhao (2012)) or leave the board due to a disagreement (Marshall (2011)). Alternatively, one can look at firms where a director holds a board seat and measure the observable changes in these firms' corporate governance during the director's tenure (e.g., removal of antitakeover defenses or CEO-chairman separation).

Zajac and Westphal (1996), Eminet and Guedri (2010), and Bouwman (2011) find evidence consistent with the first prediction. For example, Zajac and Westphal (1996) show that directors on boards that have recently increased the ratio of outside directors, separated the CEO and chairman positions, or decreased executive compensation, have fewer subsequent appointments to firms with low board control but more appointments to firms with high board control.²⁶

The main result of our paper is the existence of corporate governance externalities between firms. While governance externalities between firms can be due to several reasons, the unique feature of our model is that externalities arise due to directors' reputational concerns in the labor market. Thus, another empirical implication, which helps distinguish our mechanism from other potential mechanisms, is the following.

Prediction 2 *A positive exogenous shock to the corporate governance of one firm should positively affect corporate governance of other firms, and this spillover effect should be stronger for firms whose directors have stronger reputational concerns.*

Since governance externalities arise through the labor market for directors, they are likely backdating incur reputational penalties at other firms.

²⁶Bouwman (2011) shows that a firm is more likely to select an individual as its director if this individual is a director at firms whose governance practices are similar to the firm's existing governance practices. In the context of French firms, Eminet and Guedri (2010) find that directors who implement governance reforms that increase (decrease) control over management are more likely to be appointed to boards with (without) nominating committees and boards with nominating committees dominated by non-executive (executive) directors.

to be stronger across firms in the same segment of the labor market, such as firms in the same geographic area and firms in the same industry. Indeed, the market for directors is somewhat segmented both by geographic location (e.g., Knyazeva, Knyazeva, and Masulis (2013)) and by industry, since firms look for candidates with relevant expertise and industry knowledge (e.g., Kini et al. (2011)). Thus, the empirical predictions of this section are likely to be stronger if a firm's peer group is defined as firms in related industries or firms in close geographic proximity.

The model also has implications for the effect of directors' reputational concerns on their incentives to take shareholder-friendly actions. Our analysis emphasizes that the effect of reputational concerns depends on corporate governance at other firms. In particular:

Prediction 3 *Directors with stronger reputational concerns are more (less) likely to take shareholder-friendly actions if corporate governance of other firms is stronger (weaker).*

While several papers (e.g., Marshall (2011) and Jiang, Wan, and Zhao (2012)) study the effect of directors' age and tenure (as proxies for reputational concerns) on the likelihood that they take shareholder-friendly actions, these papers do not look at the interaction of directors' reputational concerns with other firms' corporate governance practices.

Another factor that can affect directors' incentives to take shareholder-friendly actions is the degree of transparency about their actions. In 2004, the SEC adopted a law requiring firms to publicly disclose if one of their directors leaves the board due to a disagreement. A similar 2004 law in China requires firms to disclose if one of their independent directors votes in dissent. The analysis in Section 4.1 provides implications for the effect of these laws on directors' behavior and shows that the effect of transparency depends on whether a shareholder-friendly reputation is rewarded in the labor market. In particular:

Prediction 4 *Greater boardroom transparency increases (decreases) directors' incentives to take shareholder-friendly actions if corporate governance of other firms is strong (weak).*

Besides increased transparency about directors' actions, there has also been increased transparency about directors' qualifications. In 2009, the SEC approved a new rule named "Proxy Disclosure Enhancements." This rule requires firms to disclose in proxy statements, for each director and nominee, the specific experience, qualifications, attributes, or skills that qualify that person to serve as a director or committee member. Effectively, this rule reduces the

uncertainty about the ability of directors to be effective monitors. In our model, this can be interpreted as a decrease in σ , the uncertainty about directors' types. According to Proposition 3 part (iv), the effect of this rule depends on the quality of corporate governance, and it can have adverse unintended consequences when the corporate governance system is weak. In particular:

Prediction 5 *Enhanced disclosure about directors' qualifications increases (decreases) directors' incentives to take shareholder-friendly actions if corporate governance of other firms is strong (weak).*

Finally, the model has implications for policies limiting the number of board seats a single director can hold. Although US laws do not restrict the number of directorships, US firms have been increasingly adopting such restrictions voluntarily. According to the 2012 Spencer Stuart Board Index, 74% of S&P 500 firms now limit the number of directorships for their board members, compared to only 27% in 2006. Since restricting the number of directorships decreases directors' reputational concerns, our analysis implies that such a policy induces directors to take more shareholder-friendly actions if the equilibrium is management-friendly (governance is weak), but induces them to take more management-friendly actions otherwise. Hence, another prediction of the model concerns the market reaction to the adoption of such policies:

Prediction 6 *If a company adopts a restriction on the number of board seats for its directors, the stock price reaction is more likely to be positive if the company's peer firms have weak corporate governance.*

Note also that whether a firm has adopted a restriction on the number of directorships can serve as another proxy for the strength of its directors' reputational concerns in Predictions 2 and 3 (in addition to proxies such as age and tenure, which are currently used in the literature).

6 Conclusion

This paper develops a model of the labor market for directors and studies how directors' reputational concerns affect corporate governance, the structure of the board, and shareholder value. Importantly, whether directors would like to build a reputation for being shareholder-friendly

or management-friendly, is endogenous and depends on the allocation of control between shareholders and managers in other firms. In particular, the labor market only rewards directors for being shareholder-friendly if corporate governance in most firms is strong.

Our main result is that directors' reputational concerns create corporate governance externalities between firms. Stronger governance in one firm leads to stronger governance in other firms and vice versa, and this spillover effect is stronger when directors' concerns about reputation are stronger. As a result, an equilibrium with strong aggregate governance can co-exist with an equilibrium with weak aggregate governance, suggesting that countries with similar characteristics can have different governance systems. We also show that when directors' reputation in the labor market becomes more important for them, strong governance systems become stronger but weak systems become even weaker. This implies that the effect of certain regulations, such as restricting the number of board seats an individual can hold or increasing transparency of board decision-making, crucially depends on the existing state of corporate governance. Our analysis provides new empirical predictions about director appointments and peer effects in corporate governance.

While the focus of our paper is on the labor market for corporate directors, our framework can be applied to other settings where an agent's decisions affect both his own reputation and the type of reputation that is valued at his workplace. Examples include the CEO's choice of corporate culture (e.g., the level of employee friendliness), an employee's adoption of a new technology, or an academic's choice of research agenda.

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Appendix

This Appendix contains the proofs of the main results. The Supplementary Appendix contains the analysis of the extensions of the model.

Solution concept. A Perfect Bayesian Equilibrium is a set of directors' effort strategies $e_i^*(\theta_i)$, $e_j^*(\theta_j)$, beliefs about the type of directors $\pi_i^*(\chi_i)$, and firms' hiring strategies $h_i^*(\chi_i, \pi_j)$ such that the following conditions are satisfied:

1. *The effort decision of director of firm i maximizes his expected utility, where the beliefs about the director's type $\pi_i^*(\chi_i)$, the effort strategy of the other director $e_j^*(\theta_j)$, and firm j's hiring strategy $h_j^*(\chi_j, \pi_i)$ are taken as given.*
2. *Whenever possible, beliefs about directors' types are consistent with Bayes' rule, where directors' effort strategies $e_i^*(\theta_i)$, $e_j^*(\theta_j)$ are taken as given.*
3. *The hiring decision of the controlling party of firm i maximizes its expected utility, where beliefs $\pi_j^*(\chi_j)$ and the directors' effort strategies $e_i^*(\theta_i)$, $e_j^*(\theta_j)$ are taken as given.*

Proof of Lemma 1. Director i maximizes (4) taking $e_j(\theta)$ and $\pi_i^*(\chi_j)$ as given. From (4) it follows that $U_i(1, \theta_i) > U_i(0, \theta_i)$ if and only if

$$\Delta(\theta_i) > \alpha\lambda(1 - \lambda)[\Gamma(\pi_i^*(0), \tau_j) - \Gamma(\pi_i^*(1), \tau_j)],$$

where

$$\begin{aligned} & \Gamma(\pi_i^*(0), \tau_j) - \Gamma(\pi_i^*(1), \tau_j) \\ &= (2\tau_j - 1) \cdot [1\{\pi_i^*(0) \geq \mathbb{E}[\theta]\} - 1\{\pi_i^*(1) \geq \mathbb{E}[\theta]\}], \end{aligned}$$

which is independent of θ_i . Since $\lim_{\theta \rightarrow \infty} \Delta(\theta_i) = \infty$, $\lim_{\theta \rightarrow -\infty} \Delta(\theta_i) = -\infty$, and $\frac{\partial \Delta(\theta_i)}{\partial \theta} > 0$, then for any strategy $e_j(\theta)$ and beliefs $\pi_i^*(\chi)$, there exists θ_i^* such that $U_i(1, \theta_i) > U_i(0, \theta_i)$, and hence $e_i(\theta_i) = 1$, if and only if $\theta_i > \theta_i^*$. ■

Proof of Lemma 2. A symmetric equilibrium exists if the equation $\beta(\theta^*) = \theta^*$ has a solution. Since $\beta(\theta^*)$ is bounded and continuous, by the intermediate value theorem, a solution (not necessarily unique) always exists. Suppose, next, that there exists some asymmetric equilibrium in which $\theta_i^* > \theta_j^*$. In equilibrium, $\theta_i^* = \beta(\theta_j^*)$ and $\theta_j^* = \beta(\theta_i^*)$. Therefore, $\beta(\theta_j^*) > \beta(\theta_i^*)$. Since β is strictly increasing, this inequality implies $\theta_j^* > \theta_i^*$, which is a contradiction. ■

Proof of Proposition 1. (i) To prove that both a management-friendly and a shareholder-friendly equilibrium exist for a given α , we need to prove that the function $\Psi(\theta, \alpha) = \beta(\theta, \alpha) - \theta$ has at least one root on $(\mu, +\infty)$ and at least one root on $(-\infty, \mu)$, where $\beta(\theta, \alpha)$ is given by (7). Since $\beta(\theta, \alpha)$ is bounded on $(-\infty, +\infty)$, $\lim_{\theta \rightarrow -\infty} \Psi(\theta, \alpha) = +\infty$ and $\lim_{\theta \rightarrow +\infty} \Psi(\theta, \alpha) = -\infty$. Hence, by the intermediate value theorem, both types of equilibria exist if there exist $\theta_1 < \mu$ and $\theta_2 > \mu$ such that $\Psi(\theta_1, \alpha) < 0 < \Psi(\theta_2, \alpha)$. We next show that this condition is satisfied for a large enough α . Fix any $\theta_1 < \mu$ and $\theta_2 > \mu$. By (7), $\lim_{\alpha \rightarrow \infty} \Psi(\theta_1, \alpha) = -\infty$ and $\lim_{\alpha \rightarrow \infty} \Psi(\theta_2, \alpha) = +\infty$. Hence, there exists $\hat{\alpha}$ such that for any $\alpha \geq \hat{\alpha}$, $\Psi(\theta_1, \alpha) < 0 < \Psi(\theta_2, \alpha)$. Hence, for any $\alpha \geq \hat{\alpha}$ there exists at least one shareholder-friendly and at least one management-friendly equilibrium.²⁷ Consider the set $\bar{A} = \{\hat{\alpha} \geq 0 : \text{for any } \alpha \geq \hat{\alpha}, \text{ there exists at least one shareholder-friendly and at least one management-friendly equilibrium}\}$. The above arguments prove that this set is non-empty. Then $\bar{\alpha}$ in the statement of the proposition is defined as $\inf\{A\}$.

(ii) First, we prove that for any $\alpha_0 < \bar{\alpha}$, all equilibria must be of the same type. Suppose, on the contrary, that both types of equilibria exist for some $\alpha_0 < \bar{\alpha}$, i.e., there exist $\theta_1 < \mu$ and $\theta_2 > \mu$ such that $\Psi(\theta_i, \alpha_0) = 0$. We next show that in this case, both types of equilibria exist

²⁷The same arguments imply that if $\Delta(\mu) \neq 0$, then for almost all $\alpha \geq \hat{\alpha}$, there exists at least one *stable* shareholder-friendly and at least one *stable* management-friendly equilibrium. Indeed, since $\Psi(\theta_2, \alpha) > 0$ and $\lim_{\theta \rightarrow +\infty} \Psi(\theta, \alpha) = -\infty$, there exists at least one point on $(\theta_2, +\infty)$ where $\Psi(\theta, \alpha) = 0$ and $\Psi(\theta, \alpha)$ crosses the x-axis from above, i.e., at least one stable management-friendly equilibrium. The same argument holds for a stable shareholder-friendly equilibrium. The condition $\Delta(\mu) \neq 0$ allows to rule out situations when $\Psi(\theta, \alpha)$ is tangent to the x-axis, as in the proof of Proposition 2.

for any $\alpha > \alpha_0$ as well, which contradicts the definition of $\bar{\alpha}$ as $\inf\{A\}$. Indeed, for any $\alpha > \alpha_0$, $\Psi(\theta_1, \alpha) < \Psi(\theta_1, \alpha_0) = 0$ and $\Psi(\theta_2, \alpha) > \Psi(\theta_2, \alpha_0) = 0$. Since $\lim_{\theta \rightarrow -\infty} \Psi(\theta, \alpha) = +\infty$ and $\lim_{\theta \rightarrow +\infty} \Psi(\theta, \alpha) = -\infty$, then by the intermediate value theorem, there exist $\theta'_1 \in (-\infty, \theta_1)$ and $\theta'_2 \in (\theta_2, +\infty)$ such that $\Psi(\theta'_i, \alpha) = 0$. These are the shareholder-friendly and management-friendly equilibria for $\alpha > \alpha_0$.

Next, suppose $\Delta^{-1}(0) - \mu > (<) 0$. Consider any $\alpha_0 < \bar{\alpha}$. Since $\Psi(\mu, \alpha_0) = \Delta^{-1}(0) - \mu > (<) 0$ and since $\lim_{\theta \rightarrow +\infty} \Psi(\theta, \alpha_0) = -\infty$ ($\lim_{\theta \rightarrow -\infty} \Psi(\theta, \alpha_0) = +\infty$), there exists $\hat{\theta} > (<) \mu$ such that $\Psi(\hat{\theta}, \alpha_0) = 0$, i.e., there exists at least one management-friendly (shareholder-friendly) equilibrium. Since, as shown above, all equilibria are of the same type, then all equilibria must be management-friendly (shareholder-friendly).

(iii) Note that $\frac{\partial \beta(\theta, \alpha)}{\partial \theta} = 2\alpha\lambda(1-\lambda)f(\theta) \cdot d(\alpha\lambda(1-\lambda)(2F(\theta)-1))$, where $d(\cdot) = (\Delta^{-1})'$. Since $F(\theta) \in [0, 1]$ and $d(\cdot)$ is continuous, there exists \bar{d} such that $|d(\alpha\lambda(1-\lambda)(2F(\theta)-1))| < \bar{d}$ for all θ and all $\alpha < 1$. Since, by assumption, $f(\cdot)$ is bounded, there exists \bar{f} such that $f(\theta) < \bar{f}$ for all $\theta \in \mathbb{R}$. Then, $\frac{\partial \beta(\theta, \alpha)}{\partial \theta} < 2\alpha\lambda(1-\lambda)\bar{f}\bar{d}$ and hence there exists $\underline{\alpha} > 0$ such that for any $\alpha < \underline{\alpha}$, $\frac{\partial \beta(\theta, \alpha)}{\partial \theta} < 0.5$.

We next prove that for any $\alpha < \underline{\alpha}$, the equilibrium is unique. Suppose this is not true, and $\theta_1 < \theta_2$ are two equilibria for some $\alpha < \underline{\alpha}$. Then $\Psi(\theta_1, \alpha) = \Psi(\theta_2, \alpha) = 0$, and since $\Psi(\theta, \alpha)$ is continuous and differentiable, by the mean value theorem, there exists $\hat{\theta} \in (\theta_1, \theta_2)$ such that $\frac{\partial \Psi(\hat{\theta}, \alpha)}{\partial \theta} = 0 \Leftrightarrow \frac{\partial \beta(\hat{\theta}, \alpha)}{\partial \theta} = 1$. This contradicts the fact that $\frac{\partial \beta(\theta, \alpha)}{\partial \theta} < 0.5$ for all θ . ■

Proof of Proposition 2. First, the assumption $\Delta(\mu) \neq 0$ ensures that $\theta^* = \mu$ can never be an equilibrium. Indeed, if it were an equilibrium, (7) would imply $\mu = \beta(\mu) = \Delta^{-1}(0)$, which would contradict $\Delta(\mu) \neq 0$.

We start by proving the supplementary result that if $\Delta(\mu) \neq 0$, then for all $\alpha \in \mathbb{R}$ except for a set of Lebesgue measure zero, $\frac{\partial \beta(\theta, \alpha)}{\partial \theta}|_{\theta=\theta^*} \neq 1$ in any equilibrium θ^* . To prove this, consider the function $\Psi(\theta, \alpha) = \beta(\theta, \alpha) - \theta$. In any equilibrium θ^* , $\Psi(\theta^*, \alpha) = 0$. By (7),

$$\frac{\partial \Psi(\theta, \alpha)}{\partial \alpha}|_{\theta=\theta^*} = \lambda(1-\lambda)(2F(\theta^*)-1) \times d(\alpha\lambda(1-\lambda)(2F(\theta^*)-1)),$$

where $d(\cdot) = (\Delta^{-1})'$. By assumption, $\Delta'(\theta) > 0$ and hence $(\Delta^{-1})' > 0$. Moreover, since $\theta^* \neq \mu$, as shown above, then $2F(\theta^*) - 1 \neq 0$. Thus, in any equilibrium, $\frac{\partial \Psi(\theta, \alpha)}{\partial \alpha}|_{\theta=\theta^*} \neq 0$ and hence the Jacobian $d\Psi(\theta, \alpha) = \left(\frac{\partial \Psi(\theta, \alpha)}{\partial \theta}, \frac{\partial \Psi(\theta, \alpha)}{\partial \alpha}\right)$ has rank 1. By the Transversality Theorem (e.g., Proposition 17.D.3 in Mas-Colell, Whinston and Green (1995)), $\frac{\partial \Psi(\theta, \alpha)}{\partial \theta}|_{\theta=\theta^*} \neq 0 \Leftrightarrow \frac{\partial \beta(\theta, \alpha)}{\partial \theta}|_{\theta=\theta^*} \neq 1$

in any equilibrium θ^* for all $\alpha \in \mathbb{R}$ except for a set of Lebesgue measure zero.

Next, consider any α for which $\frac{\partial\beta(\theta,\alpha)}{\partial\theta}|_{\theta=\bar{\theta}^*} \neq 1$ in all equilibria. Since $\beta(\cdot)$ is bounded and increasing, by Tarski's Fixed Point Theorem, $\beta(\cdot)$ has the least and the greatest fixed points (equilibria). Denote the greatest equilibrium by $\bar{\theta}^*$ and note that it must be stable. Indeed, since $\frac{\partial\beta(\theta,\alpha)}{\partial\theta}|_{\theta=\bar{\theta}^*} \neq 1$, then $\bar{\theta}^*$ is unstable only if $\frac{\partial\beta(\theta,\alpha)}{\partial\theta}|_{\theta=\bar{\theta}^*} > 1$. In this case, there exists $\hat{\theta} > \bar{\theta}^*$ such that $\beta(\hat{\theta}, \alpha) - \hat{\theta} > 0$. Since $\beta(\cdot)$ is bounded, then $\lim_{\theta \rightarrow \infty} \beta(\theta, \alpha) - \theta = -\infty$. Since the function $\beta(\theta, \alpha) - \theta$ is continuous, the intermediate value theorem implies that there exists $\theta^{**} > \hat{\theta}$ such that $\beta(\theta^{**}, \alpha) - \theta^{**} = 0$. Since $\theta^{**} > \bar{\theta}^*$, this contradicts the fact that $\bar{\theta}^*$ is the greatest equilibrium, which completes the proof. ■

Proof of Proposition 3. Consider any parameter p and any equilibrium θ^* . Using the implicit function theorem for the equality $\beta(\theta^*) - \theta^* = 0$, we get $\frac{\partial\theta^*}{\partial p} = \frac{\frac{\partial\beta(\theta)}{\partial p}|_{\theta=\theta^*}}{1 - \frac{\partial\beta(\theta)}{\partial\theta}|_{\theta=\theta^*}}$. Since $\frac{\partial\beta(\theta)}{\partial\theta}|_{\theta=\theta^*} < 1$ in a stable equilibrium, then for any stable equilibrium θ^* ,

$$\operatorname{sgn}\left(\frac{\partial\theta^*}{\partial p}\right) = \operatorname{sgn}\left(\frac{\partial\beta(\theta)}{\partial p}|_{\theta=\theta^*}\right). \quad (\text{A1})$$

The opposite equality is satisfied for any non-stable equilibrium.

From (7) and the fact that $\Delta^{-1}(\cdot)$ is a strictly increasing function, it follows that $\frac{\partial\beta(\theta)}{\partial\alpha} > 0 \Leftrightarrow \tau^* > 0.5$. In addition, when $c(\theta) = c + c_0(\theta)$, $\Delta^{-1}(\cdot)$ is a strictly increasing function of c , and hence $\frac{\partial\beta(\theta)}{\partial c} > 0$. The comparative statics of τ^* with respect to c and α then follows from (A1), the fact that $\tau^* = 1 - F(\theta^*)$, and that $F(\cdot)$ does not depend on c and α .

By definition of FOSD, $\frac{\partial F(\kappa, \theta)}{\partial \kappa} < 0$ for any θ and hence $\frac{\partial\beta(\theta)}{\partial \kappa} < 0$, which implies that θ^* decreases with κ . Since $F(\kappa, \theta)$ decreases with κ for a given θ , then $\tau^* = 1 - F(\kappa, \theta^*)$ increases with κ . The implicit assumption in this analysis is that the distribution of outside candidates changes with κ as well, so that the expectations of the two distributions remain equal.

Finally, according to the definition of Diamond and Stiglitz (1974), the difference between two distributions is a simple mean-preserving spread if (a) the means of the distributions are the same, and (b) there exists a single crossing point $\hat{\theta}$ such that $F(\sigma_2, \theta) \geq F(\sigma_1, \theta)$ if and only if $\theta \leq \hat{\theta}$. If the distributions are symmetric, their medians equal their means and hence $F(\sigma_2, \mu) = F(\sigma_1, \mu) = 0.5$, where μ is the mean of both distributions. Thus, μ is the single crossing point of the distributions. It follows that $\frac{\partial\beta(\theta)}{\partial\sigma} > 0 \Leftrightarrow \frac{\partial F(\sigma, \theta)}{\partial\sigma} > 0 \Leftrightarrow \theta < \mu \Leftrightarrow \tau^* > 0.5$. Hence, θ^* increases with σ if and only if $\tau^* > 0.5$. Since the effect of σ on $F(\sigma, \theta)$ for a fixed θ is in the same direction, then $\tau^* = 1 - F(\sigma, \theta^*)$ increases with σ if and only if $\tau^* < 0.5$. ■

Supplementary Appendix

The Supplementary Appendix contains all the assumptions, formal results, and proofs of the extensions discussed in Section 4 of the paper. It also contains an extension of the model that allows for contingent reward from additional board seats.

1 Transparency

Suppose that if the director does not exert effort, the manager maintains control for sure, and if the director exerts effort, shareholders get control with probability $\rho \in (0, 1]$. The basic model corresponds to $\rho = 1$. The allocation of control (χ_i, χ_j) is publicly observable. Before the labor market stage, a public signal s_i of each director's effort decision is revealed. The signals are independent across firms and take the following form. With probability η the signal perfectly reveals the director's effort decision, that is, $s_i = e_i$. With probability $1 - \eta$ the signal is pure noise and does not reveal any information about e_i . That is, $s_i = \phi$, where ϕ is pure noise. Whether the signal of firm i is informative or pure noise is publicly known. Thus, higher η corresponds to greater transparency about directors' actions. At the time of making the effort decisions, directors do not know whether their effort will be revealed or not.

In the proof of Proposition B.1 below, we show that the best response function for this modified setup is given by

$$\beta(\theta) = \Lambda_\rho^{-1} \left(\alpha \lambda (1 - \lambda) (\rho + \eta(1 - \rho)) \left(2F(\theta) - 1 + \frac{1 - \rho}{\rho} \right) \right) \quad (\text{B1})$$

where $\Lambda_\rho \equiv \Delta(\theta_i) - \frac{1-\rho}{\rho} c(\theta_i)$.²⁸ Note that for any $\rho < 1$, $\beta(\theta)$ is increasing in η if and only if $\rho(1 - F(\theta)) < 0.5$. Since the probability of shareholder control is $\tau^* = \rho(1 - F(\theta^*))$, this implies, similar to the proof of Proposition 3, that in any stable equilibrium, greater transparency (higher η) increases the probability of shareholder control τ^* if and only if the equilibrium is shareholder-friendly ($\tau^* > 0.5$).²⁹ We conclude with the following result.

²⁸We make an additional assumption that $c(\theta)$ is weakly decreasing with θ . This guarantees that Λ_ρ is monotonic.

²⁹The comparative statics of θ^* with respect to ρ implies that $\frac{\partial \theta^*}{\partial \rho} < 0$ when $c(\theta) \geq 0$ for all θ . Intuitively, the higher is ρ , the more effective is the directors' effort and hence the higher is the net benefit from exerting effort. To show this, it is easy to see that $\frac{\partial \beta(\theta)}{\partial \rho}|_{\theta=\theta^*}$ is negative when $c(\theta) \geq 0$ for all θ .

Proposition B.1 *In any stable equilibrium, higher transparency increases the probability of shareholder control if and only if the equilibrium is shareholder-friendly.*

Proof. Let $\pi(\chi_i, s_i)$ be the equilibrium reputation of director i conditional on all public information. Note that $\pi_i^*(1, 1) = \pi_i^*(0, 1)$. Since shareholder control is obtained only if the director exerts effort, $\pi_i^*(1, s_i) = \pi_i^*(1, 1)$ for any s_i . Then, the expected utility $U_i(e_i, \theta_i)$ of director i , given his type θ_i and effort e_i , and taking as given beliefs $\pi_i^*(\cdot, \cdot)$ and the probability of shareholder control in the other firm τ_j , can be written as

$$U_i(0, \theta_i) = v(0, \theta_i) + \alpha\lambda(1 - \lambda) \left[\begin{array}{l} \eta\Gamma(\pi_i^*(0, 0), \tau_j) \\ + (1 - \eta)\Gamma(\pi_i^*(0, \phi), \tau_j) \end{array} \right] + \lambda u_{resign}$$

and

$$U_i(1, \theta_i) = \left[\begin{array}{l} \rho v(1, \theta_i) \\ + (1 - \rho)v(0, \theta_i) \end{array} \right] + \alpha\lambda(1 - \lambda) \left[\begin{array}{l} (\rho + \eta(1 - \rho))\Gamma(\pi_i^*(1, 1), \tau_j) \\ + (1 - \eta)(1 - \rho)\Gamma(\pi_i^*(0, \phi), \tau_j) \end{array} \right] - c(\theta_i) + \lambda u_{resign}.$$

It follows that $U_i(1, \theta_i) > U_i(0, \theta_i)$ if and only if

$$\rho\Delta(\theta_i) - (1 - \rho)c(\theta_i) > \alpha\lambda(1 - \lambda) \left[\begin{array}{l} \eta\Gamma(\pi_i^*(0, 0), \tau_j) + \rho(1 - \eta)\Gamma(\pi_i^*(0, \phi), \tau_j) \\ - (\rho + \eta(1 - \rho))\Gamma(\pi_i^*(1, 1), \tau_j) \end{array} \right]. \quad (\text{B2})$$

Since the right-hand side does not depend on θ_i , and since $c(\theta_i)$ is decreasing, Lemma 1 continues to hold and hence $\pi_i^*(1, 1) = \mathbb{E}[\theta|\theta > \theta_i^*]$, $\pi_i^*(0, 0) = \mathbb{E}[\theta|\theta \leq \theta_i^*]$, and

$$\pi_i^*(0, \phi) = \frac{\Pr[\theta \leq \theta_i^*]\mathbb{E}[\theta|\theta \leq \theta_i^*] + \Pr[\theta > \theta_i^*](1 - \rho)\mathbb{E}[\theta|\theta > \theta_i^*]}{\Pr[\theta \leq \theta_i^*] + \Pr[\theta > \theta_i^*](1 - \rho)}.$$

The function $\pi_i^*(0, \phi)$ is decreasing in ρ and takes values in the interval $(\mathbb{E}[\theta|\theta \leq \theta_i^*], \mathbb{E}[\theta])$ for $\rho \in (0, 1)$. It is immediate to see that Lemma 2 continues to hold. According to (3), the reputation term in inequality (B2) equals $[\rho + \eta(1 - \rho)](1 - 2\tau^*)$. This implies that the best response function is given by (B1). ■

2 Shareholder activism

We extend the model and assume that in each firm there is an activist investor who can incur a cost g_i to intervene and transfer control to shareholders before directors make their effort decisions. If the activist intervenes, shareholders get control regardless of directors' actions

at the next stage.³⁰ The activists' intervention decisions are publicly observable and made simultaneously. The cost g_i is distributed according to a distribution function $G(\cdot)$ on $[0, +\infty)$ and is privately observed by the activist. Suppose that the activist's utility from shareholder control is $v(\chi_i)$. Without loss of generality, we can set $v(0) = 0$ and $v(1) = v > 0$.

Let ω_i be the probability of shareholder intervention in firm i . Consider firm i and the activist's decision whether to intervene taking as given the intervention strategy of the activist in firm j . If the activist intervenes, then $\chi_i = 1$ and the activist's utility is $v - g_i$. If the activist does not intervene, her utility depends on the director's decision whether to exert effort, which, in turn, depends on whether the activist in firm j transferred control to shareholders. Consider two cases. First, if the activist in firm j does not intervene, the subgame is equivalent to the game in the basic model and hence $\Pr(e_i = 1) = \Pr(\theta_i > \theta_i^*) = \tau^*$, the equilibrium probability that directors exert effort in the basic model. Second, if the activist in firm j intervenes, then $\chi_j = 1$ regardless of directors' decisions at the next stage. In this case, director i understands that he will be invited to the board of firm j if and only if he transfers control to shareholders. Hence, the director exerts effort if and only if

$$v(1, \theta_i) - c(\theta_i) + \alpha\lambda(1 - \lambda) > v(0, \theta_i) \Leftrightarrow \theta_i > \Delta^{-1}(-\alpha\lambda(1 - \lambda))$$

Denote

$$\hat{\tau} = 1 - F(\Delta^{-1}(-\alpha\lambda(1 - \lambda)))$$

the probability of shareholder control in firm i in this subgame. It follows that the activist in firm i intervenes if and only if

$$v - g_i > (1 - \omega_j)\tau^*v + \omega_j\hat{\tau}v \Leftrightarrow g_i < v[1 - (1 - \omega_j)\tau^* - \omega_j\hat{\tau}]$$

Hence, the activist follows a threshold strategy and intervenes only if her costs do not exceed a certain threshold. Consider the best response function $\gamma_i(g_j)$ of activist i , taking as given that activist j intervenes if and only if her cost is below g_j . Then $\gamma_i(g_j) = \gamma(g)$, where

$$\gamma(g) = v[1 - (1 - G(g))\tau^* - G(g)\hat{\tau}]. \quad (\text{B3})$$

The function $\gamma(g)$ is decreasing: $\frac{\partial\gamma}{\partial g} = vG'(g)(\tau^* - \hat{\tau}) < 0$. Indeed, $\hat{\tau} > \tau^*$ because a director's incentives to establish a shareholder-friendly reputation are higher when he knows for sure that the other firm is controlled by shareholders, compared to the case when there is uncertainty.

³⁰The assumption that upon intervention shareholders get control for sure, regardless of directors' actions, is made for simplicity. The results would be similar if shareholder intervention led to a positive probability of shareholder control in cases when directors do not transfer control to shareholders.

Also, $\gamma(0) = v[1 - \tau^*]$, $\lim_{g \rightarrow \infty} \gamma(g) = v[1 - \hat{\tau}]$ and hence, for a given τ^* , there exists a unique equilibrium at the shareholder activism stage. This equilibrium is symmetric and characterized by some threshold g^* and probability of shareholder activism $\omega^* = G(g^*)$.

Consider the comparative statics of the extended model. If g^* is the equilibrium intervention threshold, then shareholder welfare net of intervention costs (i.e., the activist's welfare) equals

$$W(g^*) = v \Pr(\chi_i = 1) - \int_0^{g^*} x dG(x),$$

where $\Pr(\chi_i = 1) = \omega^* + (1 - \omega^*)[\tau^*(1 - \omega^*) + \hat{\tau}\omega^*]$ and $\omega^* = G(g^*)$. Using (B3) and the fact that $\gamma(g^*) = g^*$, we can rewrite $\Pr(\chi_i = 1)$ as $1 - \frac{g^*}{v} + G(g^*)\frac{g^*}{v}$. Hence,

$$W'(g^*) = \frac{\partial \Pr(\chi_i = 1)}{\partial g^*} v - g^* G'(g^*) = G(g^*) - 1 < 0.$$

Thus, the activist's welfare is lower when there is more intervention in equilibrium (g^* is higher). The comparative statics of g^* with respect to c , κ and α coincides with the comparative statics of $\gamma(g)$. Since τ^* and $\hat{\tau}$ decrease with c and increase with κ , then g^* increases with c and decreases with κ . Intuitively, shareholder activism is a substitute for the board of directors: when the board is ineffective (director's costs of effort are high or the likelihood of shareholder-friendly directors is low), there is more shareholder activism. The effect of α on g^* depends on whether the equilibrium is shareholder-friendly or management-friendly. If the equilibrium is shareholder-friendly, then both τ^* and $\hat{\tau}$ increase with α and hence g^* decreases with α . However, in a management-friendly equilibrium, τ^* decreases with α , while $\hat{\tau}$ increases with α , and hence the effect of α on g^* depends on the equilibrium level of g^* .

Taken together, these arguments imply that shareholder welfare net of intervention costs decreases with c , increases with κ , and can either increase or decrease with α depending on whether the equilibrium is shareholder-friendly or management-friendly.

3 Board size

Suppose that the board of each firm consists of $K \geq 2$ directors. Directors' types are independent across and within firms and are drawn from the same distribution F . At the first stage, all directors simultaneously decide whether to exert effort. If at least $T \in \{1, \dots, K\}$ directors exert effort, then shareholders obtain control, and otherwise, the management retains control. Individual effort decisions are not observable and hence do not directly impact directors' reputation in the labor market, but the allocation of control in each firm is observable.

At the second stage, for each firm, there is a probability $\delta \equiv \lambda K \in (0, 1)$ that one of its

directors will be hit by a resignation shock, will no longer participate in the labor market, and the firm will have to search for a new director. Each of K directors has an equal chance of being hit by the shock. Thus, for each director, the unconditional probability of having to resign is λ . We also assume that if several directors have the same reputation, they are equally likely to be invited to the board of the other firm. This setup coincides with the basic model when $K = 1$. Then, conditional on $\chi_i = \chi_j$, the probability that a director of firm i is invited to the board of firm j is $\delta \left[(1 - \delta) \frac{1}{K} + \delta \frac{K-1}{K} \frac{1}{K-1} \right] = \lambda$ for any $K \geq 2$. Thus, the expected value of reputation π is equal to $\alpha \lambda \Gamma(\pi, \tau)$ for any $K \geq 2$.

Consider any director's decision whether to exert effort to transfer control to shareholders. Let p^* be the equilibrium probability that exactly $T - 1$ of other $K - 1$ directors exert effort. In this event, the director's decision is pivotal for the outcome, that is, shareholders get control if and only if the director exerts effort. In all other cases, the director's decision has no impact on the outcome. Since the cost of effort is incurred whether or not the director is pivotal, the cost of effort effectively increases to $\frac{c(\theta)}{p^*}$. Following similar arguments to those in Lemma 1, it can be shown that in any equilibrium directors follow threshold strategies. Under symmetric threshold strategies θ^* , the probability of being pivotal is $p^* = p(\theta^*) \equiv C_{T-1}^{K-1} (1 - F(\theta^*))^{T-1} F(\theta^*)^{K-T}$.

Note that board size affects directors' incentives through two channels. The first channel is due to externalities between firms and is discussed in detail below. The second channel is due to the free-riding problem within the board, which is reflected in the higher effective cost of effort $\frac{c(\theta)}{p(\theta^*)}$: a director's incentives to exert effort depend on both the size of his board and the voting rule his board uses. Since the focus of our paper is on externalities between firms, we shut down the free-riding channel in subsequent analysis by assuming that $c(\theta) = 0$.

Consider symmetric threshold strategies. Then θ^* is an equilibrium if and only if it satisfies $\theta^* = \beta_{K,T}(\theta^*)$, where

$$\beta_{K,T}(\theta) = \Delta^{-1}(\alpha \lambda (1 - 2\tau(\theta)))$$

is the best response function of each director and

$$\tau(\theta) = \sum_{t=T}^K C_t^K (1 - F(\theta))^t F(\theta)^{K-t}$$

is the likelihood that control is obtained by shareholders of the other firm. As in the basic model, we call an equilibrium θ^* shareholder-friendly if $\tau(\theta^*) > 0.5$.

The expression for $\beta_{K,T}(\theta)$ shows that even though $c(\theta) = 0$, the number of directors at the other firm affects the best response function of each director due to externalities between firms. This is captured by the dependence of $\tau(\theta)$ on K . Since $\tau(\theta)$ decreases with θ (see the proof of Proposition B.2), the best response function is increasing in θ . Hence, the extended

model exhibits strategic complementarity as well. The next result analyzes the effect of board size K and voting requirement T on directors' incentives to exert effort and on the equilibrium probability of shareholder control $\tau^* \equiv \tau(\theta^*)$ in stable equilibria.³¹

Proposition B.2 *Suppose $c(\theta) = 0$ and consider any stable equilibrium. Then:*

- (i) *If T is fixed, then τ^* increases with K .*
- (ii) *If $T = K$ (unanimity rule), then τ^* decreases with K .*
- (iii) *If K is odd and $T = \frac{K+1}{2}$ (majority rule), then τ^* increases with K if and only if the equilibrium is shareholder-friendly.*
- (iv) *If K is fixed, then τ^* decreases with T .*

Proof. First, note that $\tau(\theta)$ decreases with θ . Indeed, let $B_{K,p}(x)$ be the cumulative density function of a Binomial distribution with parameters (K,p) . Then, by the properties of the Binomial distribution, $B_{K,p}(x)$ is first-order stochastically increasing as p increases, i.e., $B_{K,p_2}(x) < B_{K,p_1}(x)$ for $p_2 > p_1$. Note that $\tau(\theta) = 1 - B_{K,1-F(\theta)}(T-1)$. Since $F(\theta)$ increases with θ , $\tau(\theta)$ decreases with θ .

Since θ^* is a stable equilibrium, then, similar to the basic model, the comparative statics of θ^* with respect to K or T coincides with the comparative statics of $\beta_{K,T}(\theta)$ with respect to this parameter. In addition, since $\tau(\theta)$ decreases with θ and the comparative statics of θ^* with respect to K or T is the opposite of the comparative statics of $\tau(\theta)$ with respect to this parameter, then the comparative statics of $\tau^* = \tau(\theta^*)$ with respect to K or T is the opposite of the comparative statics of $\beta_{K,T}(\theta)$ with respect to this parameter.

Consider part (iv) and part (i). Recall $\tau(\theta) = 1 - B_{K,1-F(\theta)}(T-1)$. By the properties of the Binomial distribution, $\tau(\theta)$ decreases with T and hence $\beta_{K,T}(\theta)$ increases with T . Moreover, $B_{K_2,p}(x) < B_{K_1,p}(x)$ for $K_2 > K_1$. Hence, $\tau(\theta)$ increases with K and $\beta_{K,T}(\theta)$ decreases with K for a fixed T .

Consider part (ii). When $T = K$ then $\tau(\theta) = (1 - F(\theta))^K$, which decreases with K for any θ . Hence, $\beta_{K,T}(\theta)$ increases with K .

Consider part (iii). We show that $\beta_{K,T}(\theta)$ increases with K if and only if $\tau(\theta) < 0.5$. Consider the function

$$g(p, K) = \sum_{t=\frac{K+1}{2}}^K C_t^K p^t (1-p)^{K-t}$$

³¹The comparative statics results hold for any stable equilibrium that continues to exist with a change in K and T . However, since a change in K and T changes the best response function in a discrete way, a given equilibrium might disappear.

Note that

$$1 - g(p, K) = \sum_{t=0}^{\frac{K-1}{2}} C_t^K p^t (1-p)^{K-t} \stackrel{s=K-t}{=} \sum_{s=K}^{\frac{K+1}{2}} C_{K-s}^K p^{K-s} (1-p)^s = \sum_{t=\frac{K+1}{2}}^K C_t^K p^{K-t} (1-p)^t$$

and hence

$$g(p, K) > 0.5 \Leftrightarrow \sum_{t=\frac{K+1}{2}}^K C_t^K \left[p^t (1-p)^{K-t} - p^{K-t} (1-p)^t \right] > 0$$

For any $t > \frac{K}{2}$,

$$p^t (1-p)^{K-t} > p^{K-t} (1-p)^t \Leftrightarrow \left(\frac{p}{1-p} \right)^{2t-K} > 1 \Leftrightarrow p > 0.5$$

and hence $g(p, K) > 0.5 \Leftrightarrow p > 0.5$. Since the function $g(p, K)$ increases with K if and only if $p > 0.5$, it follows that $g(p, K)$ increases with K if and only if $g(p, K) > 0.5$. Since $\tau(\theta) = g(1-F(\theta), K)$, then $\tau(\theta)$ increases with K if and only if $\tau(\theta) > 0.5$ and hence $\beta_{K,T}(\theta)$ increases with K if and only if $\tau(\theta) < 0.5$. ■

4 Multiple firms

Consider the extension of the model to $N \geq 2$ firms described in Section 4.4. We search for symmetric equilibria, which are characterized by a threshold θ and $\tau = 1 - F(\theta)$, the probability that shareholders obtain control. Consider a director who was not hit by a resignation shock. Let k be the number of shareholder-controlled firms out of the other $N - 1$ firms. Among these firms, let d_{SH} be the number of firms that were hit by a resignation shock and thus have demand for a new (shareholder-friendly) director. Similarly, let d_M be the number of firms with demand for a management-friendly director out of the other $N - 1 - k$ management-controlled firms. Given k , d_{SH} , and d_M , the expected number of directorships that a director with reputation π will obtain is $\frac{d_{SH}}{k+1-d_{SH}}$ if $\pi \geq \mathbb{E}[\theta]$ and $\frac{d_M}{N-k-d_M}$ if $\pi < \mathbb{E}[\theta]$. Taking the expectation over possible realizations of k , d_{SH} , and d_M , we derive the expected value of reputation.

Lemma B.1 *Given τ , the expected value of reputation is $\alpha\lambda \left[1 - ((1-\tau) + \lambda\tau)^{N-1} \right]$ if $\pi \geq \mathbb{E}[\theta]$ and $\alpha\lambda \left[1 - ((1-\tau)\lambda + \tau)^{N-1} \right]$ otherwise.*

Proof. Explicitly, the expected value of reputation is given by

$$\Upsilon(\pi) \equiv \alpha(1-\lambda) \sum_{k=0}^{N-1} \left[\frac{C_k^{N-1} \tau^k (1-\tau)^{N-1-k} \times \sum_{i=0}^k \sum_{j=0}^{N-1-k}}{\left[C_i^k \lambda^i (1-\lambda)^{k-i} \times C_j^{N-1-k} \lambda^j (1-\lambda)^{N-1-k-j} \times \Lambda(k, i, j, \pi) \right]} \right]$$

where

$$\Lambda(k, i, j, \pi) = \begin{cases} \frac{i}{k+1-i} & \text{if } \pi \geq \mathbb{E}[\theta] \\ \frac{j}{N-k-j} & \text{if } \pi < \mathbb{E}[\theta] \end{cases}$$

We start with the auxiliary result that

$$\sum_{j=0}^Y C_j^Y \lambda^j (1-\lambda)^{Y-j} \times \frac{j}{Y+1-j} = \frac{\lambda}{1-\lambda} [1 - \lambda^Y]$$

To see why this is the case, note that since $C_j^Y \times \frac{j}{Y+1-j} = C_{j-1}^Y$ and since at $j = 0$ the expression in the summation that is in the left hand side is zero, then

$$\sum_{j=0}^Y C_j^Y \lambda^j (1-\lambda)^{Y-j} \times \frac{j}{Y+1-j} = \sum_{j=1}^Y C_{j-1}^Y \lambda^j (1-\lambda)^{Y-j}$$

Let $h = j - 1$, then

$$\begin{aligned} \sum_{j=1}^Y C_{j-1}^Y \lambda^j (1-\lambda)^{Y-j} &= \sum_{h=0}^{Y-1} C_h^Y \lambda^{h+1} (1-\lambda)^{Y-h-1} \\ &= \frac{\lambda}{1-\lambda} \left[\sum_{h=0}^Y C_h^Y \lambda^h (1-\lambda)^{Y-h} - \lambda^Y \right] \end{aligned}$$

Using $\sum_{h=0}^Y C_h^Y \lambda^h (1-\lambda)^{Y-h} = 1$ concludes this result.

Next, if $\pi < \mathbb{E}[\theta]$ then

$$\begin{aligned} \Upsilon(\pi) &= \alpha(1-\lambda) \sum_{k=0}^{N-1} \left[C_k^{N-1} \tau^k (1-\tau)^{N-1-k} \times \frac{\lambda}{1-\lambda} [1 - \lambda^{N-1-k}] \right] \\ &= \alpha\lambda \left(1 - ((1-\tau)\lambda + \tau)^{N-1} \right) \end{aligned}$$

where for the first equality we used the fact that $\sum_{i=0}^k C_i^k \lambda^i (1-\lambda)^{k-i} = 1$ and the auxiliary result proved above, and for the second equality we used the identity $\sum_{k=0}^n C_k^n \times x^k = (1+x)^n$. Similarly, if $\pi \geq \mathbb{E}[\theta]$ then

$$\begin{aligned} \Upsilon(\pi) &= \alpha(1-\lambda) \sum_{k=0}^{N-1} \left[C_k^{N-1} \tau^k (1-\tau)^{N-1-k} \times \frac{\lambda}{1-\lambda} [1 - \lambda^k] \right] \\ &= \alpha\lambda \left(1 - ((1-\tau) + \lambda\tau)^{N-1} \right), \end{aligned}$$

which completes the proof. ■

It follows from Lemma B.1 that, similar to the basic model, a reputation for being shareholder-friendly generates a higher payoff than a reputation for being management-friendly if and only if $\tau > 0.5$. Therefore, Definition 1 of shareholder-friendly and management-friendly equilibria can be extended to any number of firms.

By Lemma B.1, an equilibrium with threshold θ^* exists if and only if $\theta^* = \beta_N(\theta^*)$, where

$$\beta_N(\theta) = \Delta^{-1} \left(\alpha \lambda \left[(\lambda + (1 - \lambda) F(\theta))^{N-1} - (1 - (1 - \lambda) F(\theta))^{N-1} \right] \right).$$

The best response function $\beta_N(\theta)$ coincides with (7) for $N = 2$ and is strictly increasing in θ for any $N \geq 2$. Hence, the extended model exhibits strategic complementarity as well. Moreover, using the arguments in the proof of Proposition 3, it is immediate to see that the comparative statics of θ^* (and hence of τ^*) with respect to α, κ, c and σ is the same as in the basic model.

5 Value of shareholder control

Suppose there is a random variable $\delta_i \in \{SH, M\}$ such that shareholders of firm i are better off having control if $\delta_i = SH$ and are better off delegating control to the manager if $\delta_i = M$. We assume that δ_i are independent across firms, and that δ_i is director i 's private information. The prior probability that $\delta_i = SH$ is equal to $\psi \in (0, 1]$. Directors' relative value from shareholder control, $v_{\delta_i}(1, \theta) - v_{\delta_i}(0, \theta)$, depends on δ_i . Hence, the relative benefit from transferring control to the shareholders for a director of type θ , $\Delta_{\delta_i}(\theta) \equiv v_{\delta_i}(1, \theta) - v_{\delta_i}(0, \theta) - c(\theta)$, also depends on δ_i . Specifically, we assume that $\Delta_{SH}(\theta) \geq \Delta_M(\theta)$, that $\Delta_{SH}(\theta)$ has the same properties as $\Delta(\theta)$ in the basic model, and that $\frac{\partial \Delta_M(\theta)}{\partial \theta} > 0$, $\lim_{\theta \rightarrow \infty} \Delta_M(\theta) = \Delta_0 \leq 0$, and $\lim_{\theta \rightarrow -\infty} \Delta_M(\theta) = -\infty$. We also assume that if $\delta_i = M$, then regardless of the allocation of control, the firm demands management-friendly directors.

Similar to the basic model, directors follow threshold strategies, which are now conditional on the realized δ_i . In particular, any symmetric equilibrium is characterized by two thresholds $(\theta_{SH}^*, \theta_M^*)$, such that director i with signal δ_i and type θ_i allocates control to shareholders if and only if $\theta_i > \theta_{\delta_i}^*$. Proposition B.3 shows that while θ_{SH}^* is always finite, the threshold θ_M^* can be both finite and infinite. The ex-ante probability that a firm demands shareholder-friendly directors is given by $\tau(\theta_{SH}^*) = \psi(1 - F(\theta_{SH}^*))$. We call an equilibrium shareholder-friendly if $\tau(\theta_{SH}^*) > 0.5$, i.e., if a shareholder-friendly reputation is rewarded in the labor market.

Proposition B.3

(i) A symmetric equilibrium always exists, and in any symmetric equilibrium $\theta_{SH}^* < \infty$. If

$\alpha = 0$ or if the equilibrium is management-friendly, θ_M^* is infinite.

- (ii) In any stable equilibrium, the comparative statics of θ_{SH}^* with respect to c , κ and α is the same as in Proposition 3. In addition, θ_{SH}^* decreases in ψ . If θ_M^* is finite, the comparative statics of θ_M^* is similar.

Proof. Consider part (i). It is immediate to see that the extended model exhibits strategic complementarity as well. The equilibrium θ_{SH}^* is determined by the condition

$$\Delta_{SH}(\theta_{SH}^*) = \alpha\lambda(1-\lambda)(1-2\tau(\theta_{SH}^*)).$$

Since the right-hand side is bounded and the left-hand side takes all values in $(-\infty, \infty)$, this equation has at least one solution. For a given θ_{SH}^* , the threshold θ_M^* is unique and is given by $\theta_M^* = \infty$ if $\Delta_{SH}(\theta_{SH}^*) \geq \Delta_0$ and by $\theta_M^* = \Delta_M^{-1}(\Delta_{SH}(\theta_{SH}^*))$ otherwise. This proves that when $\alpha = 0$ or when $\tau(\theta_{SH}^*) \leq 0.5$, then $\Delta_{SH}(\theta_{SH}^*) \geq 0 \geq \Delta_0$ and hence $\theta_M^* = \infty$. Note also that since $\psi > 0$, there is no need to specify off-equilibrium events. These arguments imply that a symmetric equilibrium always exists.

Consider part (ii). To study the comparative statics in the extended setup, we impose the following stability requirement, which is similar to the one in the basic model. We define an equilibrium as stable if the best response function $\beta_{SH}(\theta_{SH}) = \Delta_{SH}^{-1}(\alpha\lambda(1-\lambda)(1-2\tau(\theta_{SH}^*)))$ crosses the 45-degree line from above. It follows that in any stable equilibrium, the comparative statics of θ_{SH}^* with respect to α , c and κ is the same as in the basic model.

Note also that θ_{SH}^* decreases in ψ in any stable equilibrium. Intuitively, if shareholder control is more beneficial for the other firm, it is more likely that the board of the other firm will transfer control to shareholders. This increases the value of a shareholder-friendly reputation and gives a director stronger incentives to exert effort. Since $\theta_M^* = \Delta_M^{-1}(\Delta_{SH}(\theta_{SH}^*))$ when $\theta_M^* < \infty$, the comparative statics of θ_M^* is the same as that of θ_{SH}^* . ■

Shareholder welfare can be measured by the ex-ante probability that control is allocated efficiently,

$$W = \psi(1 - F(\theta_{SH}^*)) + (1 - \psi)F(\theta_M^*). \quad (\text{B4})$$

All else equal, W increases with θ_M^* and decreases with θ_{SH}^* . Intuitively, control is inefficiently allocated to shareholders when $\delta_i = M$ and $\theta_i > \theta_M^*$ and inefficiently allocated to managers when $\delta_i = SH$ and $\theta_i < \theta_{SH}^*$.

The equation (B4) shows that the effect of parameters on shareholder welfare can be different depending on whether $\theta_M^* = \infty$ or $\theta_M^* < \infty$. Indeed, consider the comparative statics of W with respect to a parameter $p \in \{c, \alpha, \kappa\}$. If $\theta_M^* = \infty$, the comparative statics of W is the same

as in the basic model. However, when $\theta_M^* < \infty$,

$$\frac{\partial W}{\partial p} > 0 \Leftrightarrow \frac{\partial \theta_M^*}{\partial p} > \frac{\partial \theta_{SH}^*}{\partial p} \frac{\psi}{1-\psi} \frac{f(\theta_{SH}^*)}{f(\theta_M^*)}.$$

Since the equilibrium is stable, the signs of $\frac{\partial \theta_M^*}{\partial p}$ and $\frac{\partial \theta_{SH}^*}{\partial p}$ are identical and coincide with those of $\frac{\partial \theta^*}{\partial p}$ in the basic model. Hence, when $\theta_M^* < \infty$, the effect of these parameters is ambiguous and depends on ψ . For example, W can actually increase with costs of effort if ψ is small.

6 Outside candidates

Suppose firms can choose between outside candidates with reputation $\mathbb{E}[\theta]$ and those with reputation $\varphi \in \mathbb{R}$. As in the basic model, any equilibrium can be characterized by a threshold θ^* . For any φ , denote the most and the least shareholder-friendly stable equilibria by $\underline{\theta}^*(\varphi)$ and $\bar{\theta}^*(\varphi)$, respectively. The formal definition of these equilibria is given in Section 3.

Proposition B.4 *Let φ and $\mathbb{E}[\theta]$ be the expected types of outside candidates. There exist $\underline{\varphi}$ and $\bar{\varphi}$, $\underline{\varphi} < \mathbb{E}[\theta] < \bar{\varphi}$, such that:*

- (i) *If $\varphi > \bar{\varphi}$, then $\underline{\theta}^*(\varphi) > \underline{\theta}^*(\mathbb{E}[\theta])$ and $\bar{\theta}^*(\varphi) \geq \bar{\theta}^*(\mathbb{E}[\theta])$.*
- (ii) *If $\varphi < \underline{\varphi}$, then $\underline{\theta}^*(\varphi) \leq \underline{\theta}^*(\mathbb{E}[\theta])$ and $\bar{\theta}^*(\varphi) < \bar{\theta}^*(\mathbb{E}[\theta])$.*
- (iii) *If $\underline{\varphi} \leq \varphi \leq \bar{\varphi}$, then $\underline{\theta}^*(\varphi) = \underline{\theta}^*(\mathbb{E}[\theta])$ and $\bar{\theta}^*(\varphi) = \bar{\theta}^*(\mathbb{E}[\theta])$.*

Proof. Suppose that θ^* is an equilibrium of the model for a given φ . Let us find the best response function around this equilibrium. There are three possible cases.

First, suppose $\mathbb{E}[\theta|\theta < \theta^*] \leq \varphi \leq \mathbb{E}[\theta|\theta > \theta^*]$. In this case, as in the basic model, a director with a shareholder-friendly (management-friendly) reputation is hired if and only if the other firm is controlled by shareholders (managers), and hence the best response function is $\beta(\theta)$, given by (7).

Second, suppose $\varphi > \mathbb{E}[\theta|\theta > \theta^*]$. Note that a director with reputation $\mathbb{E}[\theta|\theta > \theta^*]$ will never be invited to the board of another firm: a firm controlled by shareholders will always hire the outside candidate with reputation φ , and a firm controlled by the management will always hire the outside candidate with reputation $\mathbb{E}[\theta]$. On the other hand, a director with reputation $\mathbb{E}[\theta|\theta < \theta^*]$ will always be hired by a management-controlled firm. Using (4), one can show that the best response function of a director changes from $\beta(\theta)$ to

$$\beta^H(\theta) = \Delta^{-1}(\alpha\lambda(1-\lambda)F(\theta))$$

Finally, suppose $\varphi < \mathbb{E}[\theta | \theta < \theta^*]$. Using similar arguments, a director with reputation $\mathbb{E}[\theta | \theta < \theta^*]$ will never obtain another directorship, while a director with reputation $\mathbb{E}[\theta | \theta > \theta^*]$ will be hired by a shareholder-controlled firm. Hence, the best response function of a director changes from $\beta(\theta)$ to

$$\beta^L(\theta) = \Delta^{-1}(-\alpha\lambda(1-\lambda)(1-F(\theta)))$$

Let $\bar{\theta}^H$ and $\underline{\theta}^H$ be the largest and smallest fixed points of $\beta^H(\theta)$, respectively. Similarly, let $\bar{\theta}^L$ and $\underline{\theta}^L$ be the largest and smallest fixed points of $\beta^L(\theta)$, respectively. Note that $\beta^H(\theta) > \beta(\theta) > \beta^L(\theta)$ for any θ , and all of these functions are continuous and bounded from above and below. Hence, $\bar{\theta}^H > \bar{\theta}^*(\mathbb{E}[\theta]) > \bar{\theta}^L$ and $\underline{\theta}^H > \underline{\theta}^*(\mathbb{E}[\theta]) > \underline{\theta}^L$.

Define $\bar{\varphi} \equiv \mathbb{E}[\theta | \theta > \underline{\theta}^H]$ and $\underline{\varphi} \equiv \mathbb{E}[\theta | \theta < \bar{\theta}^L]$. We next show that these $\bar{\varphi}$ and $\underline{\varphi}$ satisfy the statements of the lemma.

Consider part (i). Suppose $\varphi > \bar{\varphi}$. First, suppose on the contrary $\underline{\theta}^*(\varphi) \leq \underline{\theta}^*(\mathbb{E}[\theta])$. Note that $\bar{\varphi} = \mathbb{E}[\theta | \theta > \underline{\theta}^H] > \mathbb{E}[\theta | \theta > \underline{\theta}^*(\mathbb{E}[\theta])]$ and that by assumption $\mathbb{E}[\theta | \theta > \underline{\theta}^*(\mathbb{E}[\theta])] \geq \mathbb{E}[\theta | \theta > \underline{\theta}^*(\varphi)]$. Therefore, $\varphi > \mathbb{E}[\theta | \theta > \underline{\theta}^*(\varphi)]$. As shown above, the best response function is then given by $\beta^H(\cdot)$ and since $\underline{\theta}^*(\varphi)$ is the smallest fixed point of $\beta^H(\cdot)$, then $\underline{\theta}^*(\varphi) = \underline{\theta}^H$. However, since $\underline{\theta}^H > \underline{\theta}^*(\mathbb{E}[\theta])$, we get a contradiction with the assumption that $\underline{\theta}^*(\varphi) \leq \underline{\theta}^*(\mathbb{E}[\theta])$. Second, suppose on the contrary $\bar{\theta}^*(\varphi) < \bar{\theta}^*(\mathbb{E}[\theta])$. This implies that the best response function cannot be $\beta^H(\cdot)$ or $\beta(\cdot)$. Otherwise, since $\bar{\theta}^*(\varphi)$ is the largest fixed point of the best response function, we get $\bar{\theta}^*(\varphi) = \bar{\theta}^H$ or $\bar{\theta}^*(\varphi) = \bar{\theta}^*(\mathbb{E}[\theta])$, respectively, and both would contradict $\bar{\theta}^*(\varphi) < \bar{\theta}^*(\mathbb{E}[\theta])$. Hence, the best response function must be $\beta^L(\cdot)$, which requires that $\varphi < \mathbb{E}[\theta | \theta < \bar{\theta}^*(\varphi)] < \mathbb{E}[\theta]$. This contradicts the fact that $\varphi > \bar{\varphi} > \mathbb{E}[\theta]$.

Consider part (ii). Suppose $\varphi < \underline{\varphi}$. First, suppose on the contrary $\bar{\theta}^*(\varphi) \geq \bar{\theta}^*(\mathbb{E}[\theta])$. Note that $\underline{\varphi} = \mathbb{E}[\theta | \theta < \bar{\theta}^L] < \mathbb{E}[\theta | \theta < \bar{\theta}^*(\mathbb{E}[\theta])]$ and that by assumption $\mathbb{E}[\theta | \theta < \bar{\theta}^*(\mathbb{E}[\theta])] \leq \mathbb{E}[\theta | \theta < \bar{\theta}^*(\varphi)]$. Therefore, $\varphi < \mathbb{E}[\theta | \theta < \bar{\theta}^*(\varphi)]$. As shown above, the best response function is then given by $\beta^L(\cdot)$ and hence $\bar{\theta}^*(\varphi) = \bar{\theta}^L$. However, since $\bar{\theta}^L < \bar{\theta}^*(\mathbb{E}[\theta])$, we get a contradiction with the assumption that $\bar{\theta}^*(\varphi) \geq \bar{\theta}^*(\mathbb{E}[\theta])$. Second, suppose on the contrary $\underline{\theta}^*(\varphi) > \underline{\theta}^*(\mathbb{E}[\theta])$. Similar to the previous case, this implies that the best response function cannot be $\beta^L(\cdot)$ or $\beta(\cdot)$. Hence, the best response function must be $\beta^H(\cdot)$, which requires that $\varphi > \mathbb{E}[\theta | \theta > \underline{\theta}^*(\varphi)] > \mathbb{E}[\theta]$. This contradicts the fact that $\varphi < \underline{\varphi} < \mathbb{E}[\theta]$.

Consider part (iii). Suppose $\varphi \in [\underline{\varphi}, \bar{\varphi}]$ and consider four possibilities why the statement could be violated. First, suppose on the contrary $\bar{\theta}^*(\varphi) > \bar{\theta}^*(\mathbb{E}[\theta])$. By the same logic as above, since $\bar{\theta}^L < \bar{\theta}^*(\mathbb{E}[\theta]) < \bar{\theta}^*(\varphi)$, the best response function must be $\beta^H(\theta)$, which requires that $\varphi > \mathbb{E}[\theta | \theta > \bar{\theta}^*(\varphi)]$. Since $\bar{\theta}^*(\varphi) = \bar{\theta}^H$, $\varphi > \mathbb{E}[\theta | \theta > \bar{\theta}^H] > \mathbb{E}[\theta | \theta > \underline{\theta}^H] = \bar{\varphi}$, which is a contradiction. Second, suppose on the contrary $\bar{\theta}^*(\varphi) < \bar{\theta}^*(\mathbb{E}[\theta])$. Since $\bar{\theta}^H > \bar{\theta}^*(\mathbb{E}[\theta]) > \bar{\theta}^*(\varphi)$, the best response function must be $\beta^L(\cdot)$, which requires $\varphi < \mathbb{E}[\theta | \theta < \bar{\theta}^*(\varphi)] = \mathbb{E}[\theta | \theta <$

$\bar{\theta}^L] = \varphi$, which is a contradiction. Third, suppose on the contrary $\underline{\theta}^*(\varphi) < \underline{\theta}^*(\mathbb{E}[\theta])$. Since $\underline{\theta}^H > \underline{\theta}^*(\mathbb{E}[\theta]) > \underline{\theta}^*(\varphi)$, the best response function must be $\beta^L(\cdot)$, which requires $\varphi < \mathbb{E}[\theta|\theta < \underline{\theta}^*(\varphi)] = \mathbb{E}[\theta|\theta < \underline{\theta}^L]$. Since $\mathbb{E}[\theta|\theta < \underline{\theta}^L] < \mathbb{E}[\theta|\theta < \bar{\theta}^L] = \varphi$, we get a contradiction. Finally, suppose on the contrary $\underline{\theta}^*(\varphi) > \underline{\theta}^*(\mathbb{E}[\theta])$. Since $\underline{\theta}^L < \underline{\theta}^*(\mathbb{E}[\theta]) < \underline{\theta}^*(\varphi)$, the best response function must be $\beta^H(\cdot)$, which requires $\varphi > \mathbb{E}[\theta|\theta > \underline{\theta}^*(\varphi)] = \mathbb{E}[\theta|\theta > \underline{\theta}^H] = \bar{\varphi}$, which is a contradiction. ■

7 Contingent reward from additional board seats

In the basic model, a director's utility from an additional board seat (α) is fixed and does not depend either on his type or on the allocation of control in the firm he is invited to. In this section, we relax this assumption and assume that if a director of type θ_i is hired by a firm with control χ_j , he gets utility

$$\alpha(\chi_j, \theta_i) = \alpha_0 + \alpha_1(\chi_j, \theta_i),$$

where $\alpha_0 > 0$, $\alpha_1(\chi_j, \theta_i) > -\alpha_0$, $\alpha_1(1, \theta_i)$ increases in θ_i and $\alpha_1(0, \theta_i)$ decreases in θ_i . Intuitively, the function $\alpha(\chi_j, \theta_i)$ captures the notion that a shareholder-friendly director values an appointment to a shareholder-controlled board more than a management-friendly director, and vice versa. In addition, this function captures the idea that among two potential directors, a shareholder-controlled (management-controlled) firm may offer a higher compensation to the director who is relatively more shareholder-friendly (management-friendly).

Following the arguments in the basic model, a director of type θ_i finds it optimal to exert effort to transfer control to shareholders if and only if

$$\Delta(\theta_i) \geq \alpha_0 \lambda (1 - \lambda) (1 - 2\tau_j) + \lambda (1 - \lambda) [\alpha_1(0, \theta_i) (1 - \tau_j) - \alpha_1(1, \theta_i) \tau_j].$$

The left-hand side increases in θ_i , and the right-hand side decreases in θ_i . Hence, similarly to the basic model, directors follow threshold strategies. By the implicit function theorem and because $\tau_j = 1 - F(\theta_j)$,

$$\frac{\partial \theta_i^*}{\partial \theta_j^*} = \frac{f(\theta_j) [2\alpha_0 + \alpha_1(0, \theta_i) + \alpha_1(1, \theta_i)]}{\frac{1}{\lambda(1-\lambda)} \frac{\partial \Delta(\theta_i)}{\partial \theta_i}|_{\theta_i=\theta_i^*} - \frac{\partial \alpha_1(0, \theta_i)}{\partial \theta_i}|_{\theta_i=\theta_i^*} F(\theta_j) + \frac{\partial \alpha_1(1, \theta_i)}{\partial \theta_i}|_{\theta_i=\theta_i^*} (1 - F(\theta_j))} > 0.$$

Therefore, the best response function is strictly increasing and hence the game exhibits strategic complementarity as well. Since the best response functions are symmetric, any equilibrium is

symmetric. The comparative statics of θ^* with respect to α_0 in an equilibrium θ^* shows that

$$\frac{\partial\theta^*}{\partial\alpha_0} = \frac{2F(\theta^*) - 1}{\left(\frac{1}{\lambda(1-\lambda)}\frac{\partial\Delta(\theta)}{\partial\theta}|_{\theta=\theta^*} - \frac{\partial\alpha_1(0,\theta)}{\partial\theta}|_{\theta=\theta^*}F(\theta^*) + \frac{\partial\alpha_1(1,\theta)}{\partial\theta}|_{\theta=\theta^*}(1-F(\theta^*))\right)\left(1 - \frac{\partial\theta_i}{\partial\theta_j}|_{\theta=\theta^*}\right)}.$$

In a stable equilibrium, $\frac{\partial\theta_i}{\partial\theta_j}|_{\theta=\theta^*} < 1$ and hence $\frac{\partial\theta^*}{\partial\alpha_0} > 0 \Leftrightarrow F(\theta^*) > \frac{1}{2} \Leftrightarrow \tau^* < 0.5$. Thus, similarly to the basic model, the probability of shareholders getting control increases with directors' reputational concerns if and only if the equilibrium is shareholder-friendly. The comparative statics with respect to other parameters is similar.