Emerging Market Business Cycles Revisited: Learning about the Trend^{*}

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Abstract

Motivated by the observation that agents in emerging markets face a greater degree of uncertainty when formulating their expectations, we build an equilibrium business cycle model in which the agents cannot perfectly distinguish between the permanent and transitory components of TFP shocks and learn about those components using the Kalman filter. Calibrated to Mexico, the model predicts a higher variability of consumption relative to output and a strongly negative correlation between the trade balance and output for a wide range of variability and persistence of trend growth shocks vis-à-vis the transitory shocks. The inherent gradual learning implies a more realistic behavior of labor supply and output in response to persistent trend growth shocks.

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1 Introduction

This paper underscores learning about the "nature" of shocks in explaining salient features of emerging market economy (EME) business cycles—the higher variability of consumption relative to output and the negative correlation between cyclical components of the trade balance and output. To do so, it builds a small open economy model in which the representative agent observes all the past and current total factor productivity (TFP) shocks and knows the stochastic properties of the distributions of trend growth and transitory components, but does not observe the realizations of the individual components. Using the available information, she forms expectations about trend growth (or permanent) and transitory (or cycle) components of TFP shocks using the Kalman filter.¹

Two key mechanisms in the imperfect information model help explain the regularities in the data. First, under perfect information, in response to a positive and persistent trend growth shock, the agent reduces her labor supply due to the wealth effect while increasing her investment. When the persistence of the trend growth shock is higher than a threshold (around 0.2 in our calibration), the decline in labor supply leads to a fall in output even after capital starts to accumulate. This leads the model to generate low correlations of output with consumption and investment. Under imperfect information, when a positive, persistent trend growth shock hits, the agent only gradually realizes that the economy was hit by such a shock. This, in turn, contains the fall in hours worked, preventing a decline in output.

Second, under imperfect information, when the signals are modeled as *trend plus cycle*, the beliefs about the contemporaneous trend growth shock relative to the cycle shock can be higher even when the variability of the trend growth shock is lower than that of the cycle shock. To further elaborate, define TFP as $A_t \equiv e^{z_t} \Gamma_t^{\alpha, 2}$ Γ_t represents the cumulative product of growth shocks defined by $\Gamma_t = e^{g_t} \Gamma_{t-1} = \prod_{s=0}^t e^{g_s}$. z and g are Normal AR(1) processes. The growth rate of A can be written as $\ln(g_t^A) \equiv \ln\left(\frac{A_t}{A_{t-1}}\right) = \alpha g_t + z_t - z_{t-1}$. Under imperfect information, the agent optimally decomposes the signals, $\ln(g_t^A)$, into *trend growth*, g_t , and *change in the cycle*, $z_t - z_{t-1}$. This, in turn, implies that when updating the beliefs about the changes in the cycle, she updates her beliefs not only about the contemporaneous cycle shock but also its first lag. This backward

¹Apart from the imperfect information and associated learning, our model is a canonical small open economy RBC model featuring production with endogenous capital and labor, with capital adjustment costs. The representative agent has access to international capital markets through a one-period non-contingent bond.

 $^{^{2}\}alpha$ is labor share of output and appears in the definition of total TFP because of the labor augmenting trend shock assumption. See Section 3 for a more detailed description.

revision has no implications for the already executed decisions in the previous period. However, it implies that in response to a positive signal, the agent may improve her beliefs about the change in the cycle, i.e., $z_t - z_{t-1}$, by not only improving her beliefs about the contemporaneous cycle shock, z_t , but also by lowering her beliefs about its first lag, z_{t-1} . Therefore, a given upward updating of $z_t - z_{t-1}$ can be attained by improving the beliefs about contemporaneous cycle shock, z_t , by less than she would in a setting without the backward revision of z_{t-1} (e.g., trend plus pure noise).

These two key mechanisms-due to imperfect information coupled with stronger reactions of the policy functions to the trend growth shocks compared with the cycle-are sufficient for the model to generate "permanent-like" responses even when trend growth shocks are not predominant. Calibrated to Mexico, the imperfect information model can generate a higher variability of consumption relative to output and a strong negative correlation between the trade balance and output for a wide range of relative variance of trend shocks. A standard deviation of trend shocks relative to cycle shocks in the interval [0.5, 5] allows the model to match the key features of emerging market moments reasonably well.³

To reconcile the key differences between emerging and developed economy business cycles, we study an extension of our baseline model in which we introduce an additional noisy signal that reveals information about the permanent component of the TFP. This allows us to vary the degree of information imperfection while keeping all other structural parameters unchanged including for the TFP.⁴ Starting from the baseline imperfect information model for Mexico and reducing the noisiness (variance) of the signal, the model moments move closer to those of developed economies regarding variability of consumption and cyclical behavior of the trade balance. This structural analysis shows that the degree of uncertainty that agents face while formulating expectations can help explain key differences of EME business cycles compared to developed countries.

Why is imperfect information relevant for accounting for the EME business cycles? Because EMEs are likely to be surrounded by an additional layer of uncertainty due to lesser transparency, poorer institutional quality, and greater political uncertainty. These features would make it harder to predict how policy makers would react to a given shock, which, in turn, would make it more difficult for agents to form expectations about whether a given shock will have a permanent or transitory impact.

To provide suggestive evidence about the severity of informational frictions, we analyze the

 $^{^{3}}$ We generate our baseline results with Cobb-Douglas preferences. However, we also show in Section 4.4 that our results are robust to using Greenwood-Hercowitz-Huffmann (GHH) preferences (Greenwood et al., 1988).

⁴Notice particularly that in this case, the relative variance of trend shocks would remain unchanged.

behavior of the GDP growth forecasting errors for EMEs and developed countries. Four facts emerge from this analysis. First, we find that the root mean squared error (RMSE) of these errors for EMEs is twice that of developed economies. This unpredictability declines considerably with the level of development also in relative terms (i.e., considering the standard deviation relative to the variation of the underlying series). Second, these errors are more likely to have non-zero means in EMEs, a symptom of systematic errors. Third, the data reveal significant first order autocorrelation for some EMEs, while none of the developed countries show this pattern. Finally, the dispersion of analysts' forecasts for GDP growth is twice as much for emerging market economies (EMEs) than that for developed countries. We realize that informational frictions are difficult to measure since they are abstract in nature and therefore our findings using forecast errors are suggestive rather than conclusive, which is one of our motivations for building a structural model.

Our paper mainly contributes to the emerging market business cycles literature including Aguiar and Gopinath (2007) (AG) and Garcia-Cicco, Pancrazzi and Uribe (2010) (GPU), Mendoza (1995, 2010), Neumeyer and Perri (2005), Oviedo (2005), and Uribe and Yue (2006), among others.⁵ AG show that introducing trend shocks to an, otherwise, standard small open economy real business cycle model can account for the salient features of economic fluctuations in EMEs.⁶ In order for the perfect information model to account for the two key features of EME business cycles, a high variability of innovation to trend shocks as well as a low autocorrelation of the trend growth shocks are necessary. Our imperfect information model relaxes these assumptions considerably. More specifically, our model can generate the salient features of emerging market business cycles for a wider range of standard deviation and persistence of trend growth shocks. In a related paper, GPU argue that RBC model could imply spurious dynamics such as a need for highly dominant trend shocks as well as a near-unit root behavior of the trade balance-GDP ratio.⁷ Our imperfect information model performs well on these two dimensions since our model does not need to resort highly dominant trend growth shocks, and implies a stationary, downward slopping behavior for the trade balance-GDP ratio.

 $^{^{5}}$ An early contribution in this literature includes Mendoza (1991), who provides a workhorse real business cycle model for small open economies. Mendoza's model calibrated to Canada proves successful in explaining the observed persistence and variability of output fluctuations as well as counter-cyclicality of trade balance.

⁶The intuition for this result relies on the permanent-income theory of consumption. If faced with a positive trend growth shock to output, the agent increases her consumption by more than the increase in current output since she expects an even higher output in the following period. This mechanism generates a consumption profile that is more volatile than output and also a trade balance deficit in response to a positive trend growth shock for the agent to finance a consumption level above output.

 $^{^{7}}$ In a related paper, Chang and Fernandez (2008) also study the driving forces of the emerging market business cycles using a Bayesian approach. Their estimation assigns relatively less weight for the importance of trend growth shocks.

Our paper does not undermine the importance of trend shocks, financial frictions or terms of trade shocks. Our paper complements the earlier studies by explicitly modeling a friction that has largely been overlooked. Also our paper makes an important methodological contribution to a large literature on macro models with learning. To our knowledge, ours is the first paper to incorporate a learning problem with permanent shocks as well as persistent AR(1) transitory shocks using Kalman filtering techniques into a dynamic stochastic general equilibrium growth model.⁸

Last but not the least, our model contributes to the "news shocks" literature (e.g., Jaimovich and Rebelo (2009) and Lorenzoni (2006), Schmitt-Grohe and Uribe (2008), among others). As shown by these studies, the standard RBC model with Cobb-Douglas preferences implies counterfactual dynamics on labor supply in response to positive news shocks; labor supply drops on impact due to positive wealth effect—similar to the dynamics of labor supply in response to highly persistent trend growth shocks. Many of the recent studies have focused on building frameworks that deliver empirically-plausible dynamics of labor. Jaimovich and Rebelo (2009), for example, propose "quasi-GHH preferences" to contain the large wealth effect. Our analysis shows that an alternative modeling approach could be the introduction of learning in an environment with trend growth shocks. Highly persistent trend growth shocks have similar economic interpretation as news shocks and the gradual learning in our framework leads to realistic dynamics of labor supply.

The rest of the paper is structured as follows. The next section presents our empirical findings. Section 3 introduces the model as well as the information structure and the consequent learning process. Section 4 presents our baseline quantitative analysis and sensitivity. Section 5 concludes and discusses extensions for further research.

2 Empirical Evidence: Comparison of Forecast Errors

To provide a suggestive evidence as to whether there are any differences in the uncertainty faced in EMEs compared with developed economies, we calculate the standard deviations of forecast errors, check the efficiency of these forecasts, and also examine forecast errors' autocorrelation structure.

⁸In this literature, Nieuwerburgh and Veldkamp (2004) study U.S. business cycle asymmetries in an RBC framework with asymmetric learning regarding transitory TFP shocks. Boz (2009) investigates the business cycle implications of learning about persistent productivity shocks. This model does not allow simultaneously for both, permanent and transitory shocks. Edge, Laubach and Williams (2004) show that uncertainty with respect to the nature of productivity shocks (permanent shifts versus transitory shocks) helps explain some of the U.S. business cycle characteristics. They model signals as trend plus iid shocks, whereas we model signals as trend plus AR(1) cycle shocks. Similarly, Guvenen (2007) studies learning about earnings utilizing a signal extraction problem with AR(1) plus noise shocks. Blanchard, L'Huillier and Lorenzoni (2008) use a similar learning framework with trend growth and transitory shocks to explore the contribution of news and noise shocks to macroeconomic volatility.

Let the forecast for period t + 1 based on information available at time t be defined by $\hat{y}_{t+1,t}$ and actual GDP growth be y_{t+1} . Then, the one-step-ahead forecast error can be defined as:

$$e_{t+1,t} = y_{t+1} - \hat{y}_{t+1,t} \tag{1}$$

First, we investigate the RMSE of forecast errors based on Consensus Forecasts, IMF's World *Economic Outlook* forecasts, and finally by estimating an ARMA model using TFP data. Table 1 summarizes the RMSE of Consensus Forecasts' forecast errors $(e_{t+1,t})$ for quarterly GDP growth (at annualized rates) for a set of developed and emerging market countries until the third quarter of 2007 since - at most - the last quarter of 1998.⁹ This table suggests that the RMSE of forecast errors for EMEs are systematically higher than those of developed economies. On average, the RMSE of these errors are 0.95 percentage points for EMEs and 0.38 percentage points for developed countries, less than half that of EMEs. The same result holds if we consider the median RMSE for both groups. In this case, EMEs median value is 0.81 versus 0.39 for developed countries. Thus, forecasts are subject to more uncertainty in EMEs than in developed countries. Similar evidence is reported by Timmermann (2006) regarding the IMF's World Economic Outlook forecast errors. For example, for Western Hemisphere the standard deviation of forecast errors is 2.41%, Asia (2.22%), Middle East (6.38%), Africa (3.19%), and Central and Eastern Europe (3.49%), while for advanced economies it is 1.36%. Finally, estimating a simple country-specific ARMA model including or not a time trend for our TFP yields standard deviations of forecast errors for the emerging market sample that are on average 78% larger than for developed countries.¹⁰

It could be argued that the comparison of RMSE of forecast errors in levels does not take into account the fact that GDP growth shocks in emerging market economies have a larger standard deviation. Thus, next we present a measure of relative predictability frequently used to compare the accuracy of forecasts across series with different variability. The statistic used is the Theil (1961) U_i indicator for country i, defined by:

$$U_{i} = \sqrt{\frac{\frac{1}{N} \sum_{t=1}^{N} e_{i,t}^{2}}{\frac{1}{N} \sum_{t=1}^{N} y_{i,t}^{2}}},$$

where the he nominator is the RMSE of forecast errors and the denominator the standard deviation

⁹The GDP growth data are from Bloomberg and refer to quarterly year-on-year growth rates. We report only those countries for which we have at least 10 quarters of forecasts available.

¹⁰More detailed results on the ARMA estimations are available upon request.

of real GDP growth.

Clearly, when this statistic is equal to 0, it means that the forecast is perfect, whereas larger values imply less forecasting accuracy. We compute this statistic for all countries in our sample and plot its relationship with GDP per capita in Figure 1. As seen in the graph, there is a significantly negative correlation between Theil's U statistic and GDP per capita. The simple correlation coefficient between both variables is -0.46, significant at conventional levels of confidence. Thus, the figure provides further evidence on the fact that forecasting real GDP growth in less developed countries is less accurate, even in relative terms.

Further, in emerging market economies forecasts are more likely to be inefficient, in the sense that the sample mean of forecasting errors differs significantly from zero which would imply that forecasters make systematic errors when projecting GDP growth. While in the case of developed countries there are just two cases out of nine where the forecast errors are biased, for EMEs in almost 50% of the cases (8 out of 18) the sample mean of forecast errors differs significantly from zero at a 10% level of significance. This result suggests again that there are serious difficulties in forecasting the relevant economic variables for EMEs.

In the fifth column of Table 1, we examine the first order autocorrelations of forecast errors. These autocorrelations are positive and significant for the cases of Argentina, Malaysia and Mexico; however, there is no developed country with a significant autocorrelation. This positive autocorrelation implies that if e.g., the current GDP growth forecast is below the actual realization, next period, it will probably underestimate growth again. This type of errors are likely to occur if a trend shock hits and the agent is uncertain about it. In the case of a positive (negative) trend shock, she would underestimate (overestimate) until she learns that a structural break took place.¹¹

Finally in the last two columns of Table 1, we examine the dispersion of analysts' forecasts reported in the consensus forecasts. To do this, for a given quarter, we calculate the standard deviation of the forecasts across analysts and calculate the median over the sample period to arrive at the dispersion reported for each country in the second to last column.¹² The last column normalizes the dispersion with the standard deviation of the GDP growth for each country over the sample period. As illustrated in these columns, the dispersion of analysts' forecasts is systematically

¹¹For both Argentina and Mexico, quarters of extreme collapses in output are not included due to lack of Consensus Forecast data. We conjecture the results would be much stronger in the case of Argentina, if the two quarters of 2002 where output collapsed at year-on-year rates greater than -10% were included in our sample. Consensus Forecasts are unavailable for these particular quarters, which per se is an indicative of the degree of uncertainty surrounding this kind of episodes.

¹²Calculating the average of dispersion leads to similar results to those with median. Due to space limitations, we report only the median dispersions.

higher for EMEs than developed countries. Even after normalizing with the variability of the GDP growth in EMEs, the dispersion still appears higher for EMEs than developed countries.

The evidence that EMEs are surrounded by greater uncertainty we documented can be due to a lack of transparency, lower institutional quality, and greater political uncertainty faced by these economies.¹³ These differences could also contribute to the uncertainty as to how policy makers would react to any given shock. This uncertainty could, in turn, contribute to the difficulty in forming expectations as to whether a given shock will have a permanent or transitory impact on the economy.

3 Model

Motivated by the observations outlined in the last section, we consider a standard small open economy real business cycle model with trend shocks similar to that utilized by AG and GPU. Unlike these two studies, in our emerging market economy model, the representative agent is imperfectly informed about the trend-cycle decomposition of the TFP shocks and, thereby, solves a signal extraction problem as explained in detail below.

The model features production with endogenous capital and labor. There are costs associated with adjusting capital which are typically introduced in the literature to match the variability and the persistence in investment. The agent can borrow and lend in international capital markets. We assume incomplete asset markets, such that the only financial instrument available is a one-period non-contingent bond that pays an interest rate that increases with the debt level to account for possible risk premia charged due to a higher default risk when debt increases. At the beginning of every period, the agent observes the realization of the TFP shock, updates expectations regarding the components of TFP, makes investment, labor, level of debt, and consumption decisions.

The production function takes a standard Cobb-Douglas form,

$$Y_t = e^{z_t} K_t^{1-\alpha} (\Gamma_t L_t)^{\alpha},$$

where $\alpha \in (0,1)$ is the labor's share of output. z_t is the transitory shock that follows an AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

¹³Acemoglu et al. (2003) and references therein document that EMEs are characterized by poorer institutional quality and greater political uncertainty compared with developed countries.

with $|\rho_z| < 1$, and ε_t^z is independently and identically and normally distributed, $\varepsilon_t^z \sim N(0, \sigma_z^2)$. Γ_t represents the cumulative product of growth shocks and is defined by

$$\Gamma_t = e^{g_t} \Gamma_{t-1} = \prod_{s=0}^t e^{g_s},$$

and

$$g_t = (1 - \rho_g)\mu_g + \rho_g g_{t-1} + \varepsilon_t^g,$$

where $|\rho_g| < 1$, and ε_t^g is independently and identically and normally distributed with $\varepsilon_t^g \sim N(0, \sigma_q^2)$. The term μ_g represents the long run mean growth rate. Combining trend growth and transitory shocks, we define a single productivity shock A:¹⁴

$$\ln(A_t) \equiv z_t + \alpha \ln(\Gamma_t).$$

and growth rate of A as g^A :

$$\ln(g_t^A) \equiv \ln\left(\frac{A_t}{A_{t-1}}\right) = z_t - z_{t-1} + \alpha g_t.$$
⁽²⁾

The representative agent's utility function is in Cobb-Douglas form:

$$u_t = \frac{(C_t^{\gamma}(1-L_t)^{1-\gamma})^{1-\sigma}}{1-\sigma}.$$
¹⁵

The agent maximizes expected present discounted value of utility subject to the following resource constraint:

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \mu_g\right)^2 K_t - B_t + q_t B_{t+1}.$$

 C_t, K_t, q_t , and B_t denote consumption, the capital stock, price of debt and the level of debt, respectively. We assume that capital depreciates at the rate δ , and adjustments to capital stock requires quadratic adjustment cost where ϕ is adjustment cost parameter. μ_g denotes the unconditional mean of the growth rate of A.

We assume that the small open economy faces a debt-elastic interest-rate premium, such that

¹⁴This follows directly from the fact that the production function could be written alternatively as Y_t = $A_t K_t^{1-\alpha}(L_t)^{\alpha}$, where $A_t = e^{z_t} \Gamma_t^{\alpha}$. ¹⁵We explore the case with GHH preferences in Section 4.4.

the interest rate paid is given by:

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + \psi \left[e^{\frac{B_{t+1}}{A_t} - b} - 1 \right],$$

where b is the aggregate level of debt that the representative agent takes as given.¹⁶

Since realizations of shock g_t permanently affect Γ_t , output is nonstationary. To induce stationarity, we normalize all the variables by A_{t-1} .¹⁷ We use the notation that a variable with a hat denotes its detrended counterpart. After detrending, the resource constraint becomes:

$$\widehat{C}_t + \widehat{K}_{t+1}g_t^A = \widehat{Y}_t + (1-\delta)\widehat{K}_t - \frac{\phi}{2}\left(\frac{\widehat{K}_{t+1}}{\widehat{K}_t}g_t^A - \mu_g\right)^2\widehat{K}_t - \widehat{B}_t + g_t^A q_t\widehat{B}_{t+1}$$

The recursive representation of the representative agent's problem can be formulated as follows:

$$V(\widehat{K}_t, \widehat{B}_t, \widetilde{z}_t, \ln(\widetilde{g}_t), g_t^A) = \max\left\{ u(\widehat{C}_t, L_t) + \beta(g_t^A)^{\gamma(1-\sigma)} E_t V(\widehat{K}_t, \widehat{B}_{t+1}, \widetilde{z}_{t+1}, \ln(\widetilde{g}_{t+1}), g_{t+1}^A) \right\},$$

where \tilde{z}_t and $\ln(\tilde{g}_t)$ are the beliefs regarding the transitory and permanent shock, respectively.

subject to the budget constraint:

$$\widehat{C}_t + \widehat{K}_{t+1}g_t^A = \widehat{Y}_t + (1-\delta)\widehat{K}_t - \frac{\phi}{2}\left(\frac{\widehat{K}_{t+1}}{\widehat{K}_t}g_t^A - \mu_g\right)^2\widehat{K}_t - \widehat{B}_t + g_t^A q_t\widehat{B}_{t+1}$$

Defining investment as X_t , we can summarize the evolution of the capital stock as follows:

$$g_t^A \widehat{K}_{t+1} = (1-\delta)\widehat{K}_t + \widehat{X}_t - \frac{\phi}{2} \left(\frac{\widehat{K}_{t+1}}{\widehat{K}_t} g_t^A - \mu_g\right)^2 \widehat{K}_t.$$

The first order conditions for the competitive equilibrium are:

$$\gamma \widehat{C}^{\gamma(1-\sigma)-1} (1-L_t)^{(1-\gamma)(1-\sigma)} \left(g_t^A \phi \left(g_t^A \frac{\widehat{K}_{t+1}}{\widehat{K}_t} - \mu_g \right) + g_t^A \right) = -\beta g_t^{A\gamma(1-\sigma)} E_t \frac{\partial V}{\partial \widehat{K}_{t+1}}, \quad (3)$$

¹⁶The debt elastic interest rate premium is introduced so as to induce stationarity to the asset holdings in the stochastic steady state. Other formulations used in the literature for this purpose include Mendoza (1991)'s endogenous discounting, and Aiyagari (1994)'s preferences with the rate of time preference higher than the interest rate. Schmitt-Grohé and Uribe (2003) survey some of the alternative methods used for this purpose and concludes that quantitative differences among the approaches applied to linearized systems are negligible.

¹⁷Note that perfect information model can be normalized by Γ_{t-1} . In our imperfect information setting, however, Γ_{t-1} is not in the information set of the agent. Y_{t-1} and A_{t-1} are other plausible candidates for normalization as they grow at the same rate as A and are in emerging market representative agent's information set. We choose to normalize by A_{t-1} , but normalizing by Y_{t-1} would yield identical results.

$$\gamma \widehat{C}^{\gamma(1-\sigma)-1} (1-L_t)^{(1-\gamma)(1-\sigma)} g_t^A q_t = \beta(g_t^A)^{\gamma(1-\sigma)} E_t \frac{\partial V}{\partial \widehat{B}_{t+1}},\tag{4}$$

$$\frac{\widehat{K}_t}{1-L_t} = \frac{\gamma}{1-\gamma} \frac{\partial \widehat{Y}_t}{\partial L_t}.$$
(5)

Equation (3) is the Euler Equation that relates the marginal benefit of investing an additional unit of resource in capital to marginal cost of not consuming that unit. Equation (4) is the Euler Equation related to the level of debt and equation (5) is the first order condition concerning the labor-leisure choice.

3.1 Filtering Problem

We assume that the representative agent is imperfectly informed about the true decomposition of the TFP shocks into its trend growth and cycle components and forms expectations about this decomposition using the Kalman filter. Her information set as of time t includes the entire history of TFP shocks; $I_t \equiv \{A_t, A_{t-1}, ...\}$. We also assume that underlying probabilistic distributions of Γ and z are known to the agent. Thus, we abstract from any consideration regarding model uncertainty to concentrate exclusively on the implications of learning under imperfect information about the realizations of the shocks.

In order to use the Kalman filter, we express the filtering problem in state space form as described in Harvey (1989). This form is composed of a measurement equation and a transition equation. The measurement equation is just a vector reformulation of Equation (2). It describes the relationship between the observed variable g^A , and the unobserved variables z and g, and is given by:

$$\ln(g_t^A) = \underbrace{\left[\begin{array}{ccc} 1 & -1 & \alpha \end{array}\right]}_{\mathbf{Z}} \underbrace{\left[\begin{array}{c} z_t \\ z_{t-1} \\ g_t \end{array}\right]}_{\alpha_t}.$$

The measurement equation includes the lagged value of transitory shock, z_{t-1} . Because, to make the learning problem stationary, the relationship between the observed and unobserved variables needs to be formulated in growth rates. The transition equation summarizes the evolution of unobserved variables and is given by:

$$\underbrace{ \begin{bmatrix} z_t \\ z_{t-1} \\ g_t \end{bmatrix}}_{\boldsymbol{\alpha}_t} = \underbrace{ \begin{bmatrix} \rho_z & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_g \end{bmatrix}}_{\mathbf{T}} \underbrace{ \begin{bmatrix} z_{t-1} \\ z_{t-2} \\ g_{t-1} \end{bmatrix}}_{\boldsymbol{\alpha}_{t-1}} + \underbrace{ \begin{bmatrix} 0 \\ 0 \\ (1-\rho_g)\mu_g \end{bmatrix}}_{\mathbf{c}} + \underbrace{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{R}} \underbrace{ \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^g \end{bmatrix}}_{\boldsymbol{\eta}_t}$$
(6)

where $\eta_t \sim N(0, \mathbf{Q})$ and $\mathbf{Q} \equiv \begin{bmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_g^2 \end{bmatrix}$. Equation (6) simply summarizes the autoregressive processes of trend growth and transitory components of TFP in matrix notation. Given the normality of the disturbances, the optimal estimator that minimizes the mean squared error is linear. The matrices \mathbf{Z} , \mathbf{d} , \mathbf{T} , \mathbf{c} , \mathbf{R} and \mathbf{Q} are the system matrices. Following the notation of Harvey (1989), we denote the optimal estimator of α_t based on information set, I_t by \mathbf{a}_t :

$$\mathbf{a}_t \equiv E[\boldsymbol{\alpha}_t | I_t].$$

The covariance matrix of the estimation error is given by \mathbf{P}_t :

$$\mathbf{P}_t \equiv E[(\boldsymbol{\alpha}_t - \mathbf{a}_t)(\boldsymbol{\alpha}_t - \mathbf{a}_t)'].$$

In this setting, the updating rule converges monotonically to a time-invariant solution for the error covariance matrix.¹⁸ In addition, the steady state error covariance matrix can be calculated as a solution to the following *algebraic Riccati equation*:

$$\mathbf{P} = \mathbf{T}\mathbf{P}\mathbf{T}' - \mathbf{T}\mathbf{P}\mathbf{Z}'(\mathbf{Z}\mathbf{P}\mathbf{Z}')^{-1}\mathbf{Z}\mathbf{P}\mathbf{T}' + \mathbf{R}\mathbf{Q}\mathbf{R}'.$$
(7)

Finally, using I_{t-1} and the transition equation (6), we have:

$$\mathbf{a}_{t|t-1} = \mathbf{T}\mathbf{a}_{t-1} + \mathbf{c}.$$

The updating rule sets the posteriors \mathbf{a}_t to be a convex combination of prior beliefs $\mathbf{a}_{t|t-1}$ and the new signal $\ln(g_t^A)$:

¹⁸See Harvey (1989) pp. 123 for a proof of this statement.

$$\mathbf{a}_{t} = \underbrace{\left[\mathbf{I} - \mathbf{P}\mathbf{Z}'(\mathbf{Z}\mathbf{P}\mathbf{Z}')^{-1}\mathbf{Z}\right]}_{k^{1}} \mathbf{a}_{t|t-1} + \underbrace{\left[\mathbf{P}\mathbf{Z}'(\mathbf{Z}\mathbf{P}\mathbf{Z}')^{-1}\right]}_{k^{2}} \ln(g_{t}^{A})$$
(8)

where I is an identity matrix of size 3×1 . Equations (3.1) and (8) fully characterize learning.

Equation (8) deserves a closer look. This equation consists of two parts. The first part is priors, $\mathbf{a}_{t|t-1}$ or $E[\alpha|I_{t-1}] = E[z_t, z_{t-1}, g_t|I_{t-1}]$, multiplied by their corresponding weights summarized in the matrix $k_{3\times 3}^1$. The second part is the new signal, g_t^A , multiplied by the Kalman gain $k_{3\times 1}^2$. Weights assigned to the priors and the new signals $(k^1 \text{ and } k^2)$ depend mainly on the relative variance of trend to cycle shocks, σ_g/σ_z . As we will illustrate and explain in detail in the next section, the higher the relative variability of trend shocks, the larger the share of TFP shocks attributed to the permanent component.

4 Quantitative Analysis

This section explains the calibration and estimation procedure of the parameters, documents the estimated parameters, and business cycle moments for Mexico. In addition, it plots impulse response functions and explains in detail the implications of introducing imperfect information.

4.1 Emerging Market Business Cycles: Application to Mexico

We calibrate our model to quarterly Mexican data. We use a combination of calibrated and estimated parameters. For β , γ , b, ψ , α , σ , and δ , we use values that are standard in the literature (see e.g., Mendoza (1991); AG; Schmitt-Grohé and Uribe (2003); Neumeyer and Perri (2005)). The parameter γ is set to 0.36 which implies that around one-third of agent's time is devoted to labor in the steady-state. Note that the coefficient on the interest rate premium is set to a small value, 0.001. The full set of calibrated parameters is summarized in Table 2.

We set μ_g to the average growth rate of output from the data and estimate the remaining structural parameters, σ_g , σ_z , ρ_g , ρ_z , and ϕ using a GMM estimation applied to the imperfect information model.¹⁹ Our estimation, reported in Table 3, yields a standard deviation of transitory component higher than the standard deviation of the trend growth component. The autocorrelation coefficients for both the trend growth and the transitory components are around 0.6.

 $^{^{19}}$ See the appendix for more details, as well as Burnside (1999) for the description and application of the GMM methodology.

4.1.1 Business Cycle Moments and Impulse Response Functions

We solve our model using a first order approximation around the deterministic steady state following the "brute-force iterative procedure" proposed by Binder and Pesaran (1997).²⁰ Table 4 compares the business cycle moments of the imperfect information model with Mexican data as well as with those of the benchmark perfect information model.²¹ For comparison, we also calculate the moments of the perfect information model using the imperfect information model's parameters. We calculate all moments using simulated data series. Simulated data is HP-filtered with a smoothing parameter of 1600, the standard value for quarterly data.

Before examining the model with imperfect information, it is worth revisiting the dynamics of the benchmark model with perfect information. In the perfect information model, when there is a positive transitory shock to output, the representative agent increases her consumption but this increase is lower than the increase in output. Because the agent knows that the output will gradually decline back to its previous level, she saves a portion of the increase in output. This is the standard consumption-smoothing effect in the presence of transitory shocks. When the shock is permanent, however, i.e., there is a positive shock to trend growth rate, the agent observes an increase in output today but she also realizes that future output will be even higher. The agent's optimal response to such positive permanent shocks is to increase her consumption more than the increase in current output. When both shocks are present in such an environment with perfect information, whether the effects of trend growth shocks dominate the transitory shocks depends on the relative importance of each shock.

To measure the relative importance of trend shocks, we examine two related metrics calculated based on our estimated parameters. The first metric is the variability of innovations to trend growth shocks relative to innovations to transitory shocks, σ_g/σ_z . This metric, however, does not incorporate any potential differences in the persistence of these two types of shocks. Therefore, we also examine a second metric that we calculate as follows. We decompose the first log-difference of TFP according to:

$$\Delta \ln A_t = \Delta z_t + \alpha g_t.$$

Computing the variance of this expression, and taking into account that trend and transitory shocks

²⁰The log-linearized system is provided in an Appendix available upon request.

²¹For the perfect information, we use the parameters estimated by AG for Mexico.

are uncorrelated in our model, we obtain

$$\sigma_{\Delta \ln A}^2 = \underbrace{\frac{2\sigma_z^2}{1+\rho_z}}_{\sigma_{\Delta z}^2} + \underbrace{\frac{\alpha^2 \sigma_g^2}{1-\rho_g^2}}_{\sigma_{\alpha g_t}^2}$$

and we define our measure of the relative importance of trend shocks simply as follows:

$$V = \frac{\frac{\alpha^2 \sigma_g^2}{1-\rho_g^2}}{\frac{2\sigma_z^2}{1+\rho_z} + \frac{\alpha^2 \sigma_g^2}{1-\rho_q^2}},$$

which is reported in the last row of Table 3. Note that this metric is bounded in the interval [0,1] where higher values imply higher importance of trend growth shocks.

This metric has several advantages that makes it an accurate measure of relative variability of trend shocks. First, it does not restrict the permanent shocks to follow a random walk. As pointed out by Campbell and Mankiw (1987), if trend growth shocks are persistent (remember that we estimate ρ_g to be 0.61 for Mexico), only a type of metric we use would preserve that trend and cycle shocks remain uncorrelated. This is an important aspect that is also highlighted by Campbell and Mankiw (1987).²² Second, Quah (1990, 1992) shows that when the trend follows a more complicated process, as it does in our model, an unobserved component estimation of trend growth shocks should be pursued, as we do in our analysis.²³ Guided by these studies, we use the metric reported above to measure the relative importance of trend shocks.

Using the two metrics described, we find that the perfect information model requires strong predominance of permanent shocks as well as a low persistence for trend growth shocks. The perfect information model implies a variability for trend growth shocks of 2.55 percent and a variability for transitory shocks of 0.54 percent, which implies $\sigma_g/\sigma_z = 4.02$. In addition, GMM estimation for the perfect information model yields a $\rho_g = 0$. This, together with the estimates for standard deviations, implies V = 0.88. To illustrate the resulting implications of the perfect information model when permanent shocks are not predominant, the last column of Table 4 reports the moments of the perfect information model using the parameters that we estimated with the imperfect information model. In this case, the perfect information model implies a consumption variability less than that

²²Campell and Mankiw indicate that: "...But of course, one usually thinks of trend and cycle as having a low or zero correlation..."

 $^{^{23}}$ Similarly, Oh et al. (2008) also makes a case in favor of a use of type of metric we present when the trend is not a pure random walk."

of output and procyclical trade balance, which is at odds with the observed patterns in the data. Also, the correlation of output with consumption and investment is significantly smaller than in the data.

The imperfect information model matches the key moments of the Mexican data very closely as reported in the third column of Table 4. The ratio of consumption variability to income variability is 1.17, compared with 1.26 in the data. The correlation of net-export with output is -0.69, which compares quite well with the value of -0.75 in the data. The model also matches the other moments closely as illustrated in Table 4. The GMM estimation reveals $\sigma_g/\sigma_z = 0.78$ and V = 0.26, suggesting that the imperfect information model matches the data without a predominance of trend growth shocks.

The imperfect information model performs well with perfect information model parameters, too. When those parameters are fed into the imperfect information model, the model can match key moments reasonably well as illustrated in the fourth column of Table 4. Therefore, the results of the imperfect information model do not hinge on a specific value for relative variability of trend shocks as we explain further below.

To understand the quantitative implications of the imperfect information model we first explain why the perfect information model is subject to some limitations. In the perfect information model, the response of hours to a *persistent trend growth shock*, $\rho_g > 0.2$, is quite strong. In response to a persistent, positive trend growth shock, hours decline significantly due to the wealth effect. The magnitude of this decline increases with the persistence of the trend growth shock. With higher values of ρ_g , the decline in hours becomes so large that it leads output to fall in response to a positive trend growth shock. Investment increases gradually due to the capital adjustment cost. Hence, the increase in capital in response to positive trend growth shock is insufficient to offset the impact of the fall in hours on output.

The strong response of hours to persistent trend growth shocks makes it difficult for the perfect information model to match the correlations of aggregate variables with output observed in the data. This is evident in Figure 2, which plots the impulse response functions to 1-percent shocks to transitory as well as permanent components of TFP in the perfect information model using the imperfect information parameters (remember that estimated ρ_g in this case is 0.61).²⁴ The graph shows that in response to a positive trend growth shock, hours and net exports fall while

²⁴In general, the perfect information model with $\rho_g > 0.2$ cannot generate $\rho(c, y)$ or $\rho(I, y)$ that is greater than 0.9 regardless of σ_g/σ_z .

consumption and investment increase. Note that output also falls in response to this positive shock to trend growth. As a result, the model generates $\rho(c, y)$ and $\rho(I, y)$ that are lower compared with the case when $\rho_g = 0$. In addition, $\rho(nx, y)$ becomes positive because net exports move in the same direction with output both in the case of a cyclical shock and a trend growth shock.

These insights also explain the results reported in the last column of Table 4. The imperfect information parameters fed into the perfect information model lead to a low output-consumption correlation (0.44), a low investment-output correlation (0.31) and a positive trade balance-output correlation (0.38).

The imperfect information model can deliver high $\rho(c, y)$ and $\rho(I, y)$ for relatively higher values of ρ_g . This is because learning leads the agent to realize only gradually that a trend shock hit. Since learning induces gradual realization, even for higher values of ρ_g , the decline in hours is not sufficient to lead to a decline in output. Therefore, the imperfect information model matches the correlations in the data quite well even with persistent trend growth shocks.

This gradual behavior is evident in Figures 3 and 4. Figure 3 plots the response of the imperfect information model to transitory and permanent shocks. In response to a 1-percent transitory shock (top panel), the model displays "permanent-like" responses: consumption increases more than output; net export declines significantly. In response to a 1-percent permanent shock (bottom panel), the model again displays permanent-like responses: consumption responds more than output; net-export declines significantly. Even though imperfect information dampens the response of all variables, for the case of transitory shocks, there is an amplification effect, driven by the fact that the agent assigns a positive probability to the event that the shock might be permanent and, therefore, increases investment and consumption by more than in the perfect information case. In addition, comparing the perfect information model impulse responses depicted in Figure 2 to those of imperfect information model, learning introduces persistence.

To illustrate the learning dynamics implied by the model, in Figure 4, we plot beliefs for permanent and transitory components along with TFP that the agent directly observes. The crossed solid line depicts TFP, the diamond-dashed line plots the evolution of the belief about the permanent component, while the star-dashed line represents the evolution of the belief for the transitory component. In the top panel, the source of fluctuation in TFP is a 1-percent transitory component shock, whereas in the bottom panel, it is a trend shock of the same magnitude.²⁵ A

²⁵In the first panel, interestingly, TFP shock turns negative after the initial positive shock. This is in fact intuitive. Rewriting Equation 2, we have: $\ln(g_t^A) = z_t - z_{t-1} + \alpha g_t$. Thus, g_t is zero as only the transitory component is shocked in the first panel, while z_t increases by 1-percent on impact and $z_{t-1} = 0$ because we start from the steady state.

close investigation of the bottom panel of Figure 4 - the case of a trend growth shock- suggests that, on impact, beliefs regarding the trend growth shock, \tilde{q} , goes up by only half of the increase in the true value of g. The initial period in which a high TFP growth is observed is particularly confusing for the agent. Only after observing another signal, \tilde{g} becomes significantly close to the true value of q in that period. This is because of the nature of learning about cycle and trend. A high TFP growth today can be either a positive trend growth shock or a positive cyclical shock. Therefore, the observation of a high TFP growth by itself is not very informative. However, note that a cyclical shock dies out very differently from a trend shock.²⁶ A positive cyclical shock in period 2 leads to a negative TFP growth starting from period 3. This is because given that the trend does not change, an above trend growth in period 2 has to be followed by below trend growth so that the economy converges back to the same trend as the shock dies out. On the contrary, a positive trend growth shock in period 2 dies out by leading to an even higher trend over time. Given these differences, after observing the initial high TFP growth in period 2, the TFP growth in period 3 becomes crucial for the agent to be able to decompose trend and cycle. Therefore, it is this initial uncertainty and its gradual disappearance that contains the decline in hours and prevents a potential decline in output in response to a persistent trend growth shock.

This establishes why the perfect information model requires $\rho_g = 0$ while the imperfect information model does not. In fact, as mentioned before, imperfect information model can match the key moments with different combinations of ρ_g and σ_g/σ_z . For a given value of ρ_g , imperfect information model in general implies that $\sigma(c)/\sigma(y)$ increases with σ_g/σ_z . Hence, lower values of ρ_g combined with higher values of σ_g/σ_z deliver the desired key moments. In addition, in the imperfect information model, note that $\rho(nx, y)$ rarely turns positive as opposed to the perfect information model where for most calibrations, this correlation is positive. (Compare the lower panels of Figures 6 and 7.) We observe that because of the dynamics through hours and output that the perfect information model most of the time generates a positive trade balance output correlation. Hence, it is not the trade balance that increases in response to a positive trend shock, but it is the output that declines.²⁷

As the shock dies out after the first period, $z_t = \rho_z z_{t-1}$ becomes smaller than z_{t-1} implying a negative value for $z_t - z_{t-1}$. With $z_t - z_{t-1} < 0$ and $g_t = 0$, we have $\ln(g_t^A)$ turning negative after the initial period as depicted in the top panel of Figure 4.

²⁶The comparison of "simulated TFP growth," solid blue line, in the lower and higher panels of Figure 4 reveals this.

²⁷Similar dynamics with hours declining sufficient to lead to a decline in output takes place in the imperfect information model only in the case of unrealistically high values for both ρ_g and σ_g/σ_z . Those values imply output variabilities that are larger than 3 percent - higher than that in the data.

We explore further the imperfect information model to see if it relies on certain values of ρ_g and σ_g/σ_z . Figure 5 shows how key moments change as we change the relative variability of the trend shocks, σ_g/σ_z , while keeping the other parameters constant. As the first panel illustrates, as long as the relative variability of the permanent component relative to the transitory component is higher than approximately 0.7, the model can generate a higher consumption variability relative to output variability. In order for the model to match counter-cyclicality of the trade balance, the relative variability of trend shocks needs to be less than 2. Hence, the imperfect information model can match these two key moments with σ_g/σ_z in 0.7 to 2 range, assuming $\rho_g = 0.61$. However, this does not imply that the imperfect information model requires a σ_g/σ_z in the range of [0.7, 2]. Our analysis suggests that once we allow the other estimated parameters (ρ_z , ρ_g , ϕ) to change, the imperfect information model is able to match the data fairly closely for a wide range of values for σ_g/σ_z .

The ability of the imperfect information model to match the key moments $(\sigma(c)/\sigma(y))$ and $\rho(nx, y)$ for a wide range of relative variability of trend shocks is evident in Figure 6. The top panel of this figure plots $\sigma(c)/\sigma(y)$ for different values of relative variability of trend shocks (y-axis) and ρ_g (x-axis).²⁸ We keep the remaining parameters (ρ_z, μ, ϕ) at their original values from the baseline parametrization of imperfect information model. Similarly, the bottom panel shows $\rho(nx, y)$ for the same sets of parameters. The top panel suggests that, in general, $\sigma(c)/\sigma(y)$ increases with the relative variability of trend shocks and ρ_g . $\sigma(c)/\sigma(y)$ of 1.26 observed in the data can be matched with $(\sigma_g/\sigma_z, \rho_g) \in \{(5,0), (3,0.2), (2,0.4), (1,0.61), (0.5,0.8)\}$. That is, the model can match this moment with higher relative variability of trend shocks if one allows for lower ρ_g . Similarly, the correlation between output and net exports, $\rho(nx, y)$ of -0.75, in the data is implied by the imperfect information model for $(\sigma_g/\sigma_z, \rho_g) \in \{(4.5, 0), (2.2, 0.2), (1.1, 0.4), (0.7, 0.61), (0.5, 0.8)\}$. Likewise, the model can match this moment with several values for relative variability of trend shocks and ρ_g combinations if lower ρ_g 's are combined with higher relative variability of trend shocks.

Finally, we investigate the higher order correlations of the trade balance since GPU finds that one shortcoming of the perfect information model is that it implies a near-unit root behavior of the trade balance-GDP ratio. The imperfect information performs well on this dimension, too. As

²⁸We conducted similar analysis by allowing ρ_z and ϕ to vary along with the relative variability of trend shocks and found that variation in those parameters do not change the relationship between $\sigma(c)/\sigma(y)$, $\rho(nx, y)$, and the relative variability of trend shocks. In other words, regardless of ρ_z and ϕ , $\sigma(c)/\sigma(y)$ and $\rho(nx, y)$ increase with relative variability of trend shocks. Simulations are available upon request.

shown in Figure 8, the imperfect information model implies a stationary behavior for the trade balance-GDP ratio, which converges quickly to zero.

As a separate point, our results make a methodological contribution to the "news shocks" literature (e.g., Jaimovich and Rebelo (2009) and Lorenzoni (2006), Schmitt-Grohe and Uribe (2008), among others) as well. As shown by these studies, the standard RBC model with Cobb-Douglas preferences implies counterfactual dynamics on labor supply in response to positive news shocks; labor supply drops on impact due to positive wealth effect–similar to the dynamics of labor supply in response to highly persistent trend growth shocks. Many of the recent studies in this literature have focused on building frameworks that deliver empirically-plausible dynamics of labor. Jaimovich and Rebelo (2009), for example, propose "quasi-GHH preferences" to contain the large wealth effect. Our analysis shows that an alternative modeling approach could be the introduction of learning in an environment with trend growth shocks. Highly persistent trend growth shocks and the gradual learning in our framework implies realistic dynamics of the labor supply.

4.2 Further Insights on Learning

The Kalman filter assigns slightly higher probability to trend component. This appears counterintuitive considering that the cycle component is more volatile than the trend according to our GMM estimations of the imperfect information model. However, the experiment explained next clarifies the intuition for this finding.

We simulate a case where both 1% permanent shock and 1% transitory shock are given at the same time in the perfect and the imperfect information models. Table 5 documents the true values of these shocks in perfect information case and the beliefs calculated by the agent in imperfect information case under baseline parameterization. As expected, under perfect information, the shocks are 1 % each for g_t and z_t leading to 1.68 % growth in TFP, given that $\alpha = 0.68$. Under imperfect information, however, while decomposing TFP between g_t and Δz_t , the agent assigns 0.65% to \tilde{g}_t , 0.60% to \tilde{z}_t , and -0.63% to \tilde{z}_{t-1} . In other words, the agent, using the Kalman filter, increases \tilde{z}_t while decreasing \tilde{z}_{t-1} , part of the increase in $\Delta \tilde{z}_t$ coming from an update of \tilde{z}_{t-1} . This leads to the increase in \tilde{g}_t to be larger than \tilde{z}_t inducing a dampening of the contemporaneous cyclical component in the imperfect information model. Considering that the policy decisions of time t-1 are already executed at the time when the signal $\ln(g_t^A)$ arrives, the reduction in \tilde{z}_{t-1} does not impact the imperfect information model's long run moments directly. However, as mentioned

earlier, the reduction in \tilde{z}_{t-1} allows the agent to increase $\Delta \tilde{z}_t$ by increasing \tilde{z}_t by a smaller amount than she would otherwise under perfect information scenario. This has a significant impact on the long run moments because it induces the agent to give more weight to permanent shocks relative to the contemporaneous cycle shocks in the imperfect information model.

Note that both \tilde{g}_t and \tilde{z}_t under imperfect information are lower than g_t and z_t under perfect information as reported Table 5. This leads to a dampening in the overall volatilities in imperfect information setting. This dampening manifests itself as a reduction in overall volatilities in the imperfect information model relative to the perfect information model (compare $\sigma(y)$ of 3.21 in the perfect information model vs 2.18 in the imperfect information model in Table 4).

In order to further analyze the implications of learning using the Kalman filter, we conduct further experiments. We report implied beliefs attached to the components of TFP for various values of σ_g/σ_z (Table 6). These experiments reveal that the probability assigned to a given TFP shock being permanent (\tilde{g}_t) monotonically increases with σ_g/σ_z , while that assigned to it being transitory (\tilde{z}_t) decreases. Note that the relative variability of trend shocks that equates \tilde{g}_t to \tilde{z}_t is 0.76, which is slightly lower than 0.78 under baseline parametrization.

This mechanism hinges on the revision of \tilde{z}_{t-1} . This revision of \tilde{z}_{t-1} in case of a positive shock at time t is downwards. This is because the agent assigns positive probability to a scenario with a negative transitory shock in period t-1. A close investigation of the top panel of Figure 4 reveals that for example in the case of a positive transitory shock in period 1, $g_t^A = \alpha g_t + z_t - z_{t-1}$ increases in period 1 with unchanged z_{t-1} and g_t . However, starting with the second period, g_t^A turns negative with $z_t < z_{t-1}$ as the shock dies out gradually. The mirror image of these dynamics occur in the case of a negative shock. Going back to Table 5, observing a positive signal in period t, the agent realizes that a positive transitory or permanent shock might have hit at time t, or a negative transitory shock might have hit in period t-1 and g^A went up in period t as this negative shock dies out. Assigning some probability to each of these scenarios, the agent increases her belief about g_t , z_t , and reduces the one about z_{t-1} .

4.3 From Emerging Market to Developed Economy Business Cycles: Varying Degrees of Information Imperfection

In this subsection, we explore further the hypothesis that the higher degree of information imperfection in EMEs is the main driver of the higher consumption volatility relative to income and countercyclical net exports. In order to do so, we generalize our model to allow for lower levels of information imperfection than the baseline scenario we considered. In our baseline imperfect information model, TFP growth (g_t^A) is the only source of information about the true values of g_t and z_t and therefore the noisiness of signals are inherently determined by the TFP process. In order to separate the TFP process uncertainty from the degree of information imperfection, we introduce an additional publicly observable signal that reveals information about the trend shocks. Note that the baseline imperfect information model is a particular case of this model when the noisiness of this additional signal goes to infinity and therefore it reveals no information. And when it goes to zero and reveals entirely the true trend shock, the model converges to a full information setting.

To make such a modification, we need to alter the filtering problem. Let us define the new additional signal as $s_t = g_t + \epsilon_t^s$ where $\epsilon^s \sim N(0, \sigma_s)$. Note that we could also model this signal as one that reveals information about the cycle (z). This would yield similar results because a more accurate knowledge of g would transform into a more accurate knowledge of z and vice versa. This latter observation is due to the fact that the sum of g and Δz is actually observed (through the TFP growth). Accordingly, the information set is modified to include the realization of these new signals, $I_t \equiv \{A_t, s_t, A_{t-1}, s_{t-1}, ...\}$. The measurement equation now becomes:

$$\begin{bmatrix} \ln(g_t^A) \\ s_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1 & \alpha & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}_{\mathbf{Z}} \underbrace{\begin{bmatrix} z_t \\ z_{t-1} \\ g_t \\ \epsilon_t^s \end{bmatrix}}_{\boldsymbol{\alpha}_t}.$$
(9)

The transition equation is modified as:

$$\underbrace{\begin{bmatrix} z_t \\ z_{t-1} \\ g_t \\ \epsilon_t^s \end{bmatrix}}_{\boldsymbol{\alpha}_t} = \underbrace{\begin{bmatrix} \rho_z & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \rho_g & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} z_{t-1} \\ z_{t-2} \\ g_{t-1} \\ \epsilon_{t-1}^s \end{bmatrix}}_{\boldsymbol{\alpha}_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ (1-\rho_g)\mu_g \\ 0 \end{bmatrix}}_{\mathbf{c}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \underbrace{\begin{bmatrix} \epsilon_t^z \\ \epsilon_t^g \\ \epsilon_t^s \end{bmatrix}}_{\boldsymbol{\eta}_t}$$
(10)

where $\boldsymbol{\eta}_t \sim N(0, \mathbf{Q})$ and $\mathbf{Q} \equiv \begin{bmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_g^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{bmatrix}$. The remaining parts of the model regarding

production, consumption, etc. remain the same as baseline. In this setting with two signals, we

can control the degree of information imperfection by varying σ_s without changing the TFP process.

Table 7 reports the business cycle moments for different degrees of information imperfection, using the previously estimated parametrization for Mexico. The first column with $\sigma_s \to \infty$ is identical to our baseline imperfect information model. We report in the following five columns the results with lower values of σ_s . All the structural parameters in this exercise are kept constant, which, in turn, implies merely the same σ_g/σ_z , and V across columns. Notice that as σ_s falls, the moments get closer to those of developed economies, i.e., consumption variability becomes lower than output variability and trade balance becomes procyclical. (See AG for business cycle statistics of developed economies.) Hence, this exercise illustrates that the differences in the degree of uncertainty faced by agents while formulating their expectations can contribute greatly to the key differences between emerging and developed economy business cycles.

In order to link our evidence on the severity of informational frictions in EMEs with the quantitative implications of our baseline model, we report the standard deviation of forecast errors in the last two rows of Table 7. The forecast errors implied by the model are defined in the same fashion with those in the empirical section (Equation 1). Since we look at one-step ahead forecast errors, these errors are non-zero even in the full information case.

The standard deviation of forecast errors decline as we move towards full information. In the baseline imperfect information case, the standard deviation of these errors is 1.76 while that in the perfect information is 1.59. This result survives even after normalizing the standard deviation of forecast errors with output's standard deviation. We also evaluated the autocorrelation and means of these errors, but did not find them to be significantly different from zero in any of the scenarios reported in Table 7.

4.4 Sensitivity Analysis: The Role of Preferences

In this section, we explore the sensitivity of our baseline results to the preference specification. For this, we revisit the implications of our baseline results with the widely-used GHH preferences (Greenwood et al. 1988), which takes the following form:

$$u_t = \frac{(C_t - \tau L_t^\nu)^{1-\sigma}}{1-\sigma},$$

where $\tau > 0$ and $\nu > 1.^{29}$ The elasticity of the labor supply is given by $\left(\frac{1}{\nu-1}\right)$. The budget constraint and the information structure remains the same as in our baseline model.

Table 8 lists the additional parameters required for the calibration as well as the associated new set of estimated parameters for exogenous shock processes.³⁰ Table 9 summarizes the implied business cycle moments. With those estimated parameters, our imperfect information model does a remarkable job in matching the business cycle moments shown in the first column. Consumption is more variable than output, trade balance is strongly countercyclical, and the model matches the variability of output, investment and trade balance quite well. Further, similar to Cobb-Douglas preferences, the model delivers these results when the relative variability of trend shock is lower than that of transitory shock. When we feed in the same estimated parameters to the perfect information model with GHH preferences, it generates weakly countercyclical trade balance and consumption profile that is less variable than output; and undershoots the variability of investment and trade balance.

This section establishes that our baseline results are robust to the preference specification. Since perfect information model features zero wealth effect on labor supply with GHH, the counterfactual dynamics that we observed in the perfect information model with Cobb-Douglass preferences in response to highly persistent trend growth shocks do not arise. Therefore, GHH preferences can generate a negative correlation between trade balance and output. However, without the predominance of trend growth shocks, in the perfect information model with GHH preferences, the countercyclicality of the trade balance is weak and consumption variability is lower than output variability.

5 Conclusion

In this paper, we provided a framework to explain the key business cycle characteristics of emerging market economies. We showed that when the agents are imperfectly informed about the trend-cycle decomposition of productivity shocks, and they solve a learning problem using the Kalman filter to estimate the components of the TFP, the model performance in matching the salient features of emerging market business cycles improves greatly. The key ingredients for these results are: the existence of trend shocks, the existence of transitory but persistent cycle shocks, and uncertainty

²⁹We assume that $\beta \mu_g^{1-\sigma} < 1$ to ensure that utility is well defined. Further, to have a well-behaved steady state consumption, we impose $\beta (1+r^*)^{1/\sigma} = \mu_g$.

 $^{^{30}\}text{We}$ take the calibration of τ and ν from the existing literature.

regarding the decomposition of TFP into its components.

Our analysis contributes to the emerging market business cycles literature, which has largely emphasized the role of financial frictions, terms of trade shocks, and trend shocks but has overlooked the role of uncertainty and informational frictions. We fill this gap by highlighting the role of uncertainty that the agents face while formulating their expectations about long-run implications of the shocks they face in explaining the emerging market business cycles. Further, by introducing imperfect information and learning about the underlying fundamentals of the economy in a tractable manner, we also open up a new line of research. For example, studying optimal policy (fiscal or monetary) in the framework we provide can deliver interesting insights. Another interesting application could be to build the signal extraction problem developed in this paper into a two country environment allowing different levels of informational frictions across the two economies to explore cross country portfolio allocations, consumption correlations, etc. In addition, the gradual learning inherent in our setup makes an important contribution to the news shocks literature by generating a realistic response of labor in the face of highly persistent trend growth shocks.

A Appendix

A.1 GMM Estimation

This subsection presents the GMM moment conditions and procedures used in our estimations. The estimated structural parameters are $b \equiv (\sigma_g, \sigma_z, \rho_g, \rho_z, \phi)$. In terms of notation, all lower-case variables are in logs and \tilde{x} refers to the Hodrick-Prescott filtered series of x. Net exports, nx, is expressed as a fraction of output. Further, σ refers to the theoretical variance-covariance terms, while S refers to the moments in the data. The moments conditions are given by:

$$u_t = \begin{pmatrix} \sigma_{\tilde{y}}^2 - S_{\tilde{y}}^2 \\ \sigma_{\Delta y}^2 - (\Delta y - \bar{y})^2 \\ \sigma_{\tilde{c}}^2 - S_{\tilde{c}}^2 \\ \sigma_{\tilde{i}}^2 - S_{\tilde{i}}^2 \\ \sigma_{nx}^2 - (nx - n\bar{x})^2 \\ \sigma_{\tilde{y},\tilde{c}} - S_{\tilde{y},\tilde{c}} \\ \sigma_{\tilde{y},\tilde{i}} - S_{\tilde{y},\tilde{i}} \\ \sigma_{\tilde{y},nx} - S_{\tilde{y},nx} \\ \sigma_{\tilde{y}t,\tilde{y}_{t-1}} - S_{\tilde{y}t,\tilde{y}_{t-1}} \\ \sigma_{\Delta yt,\Delta y_{t-1}} - S_{\Delta yt,\Delta y_{t-1}} \end{pmatrix}$$

Let \bar{u} be the sample mean of u_t and $J(b, W) = \bar{u}'W\bar{u}$, with W being a symmetric positive definite weighting matrix. The GMM estimate of b is given by the vector that minimizes J(b, W). The matrix W is estimated using the two-step procedure outlined by Burnside (1999).

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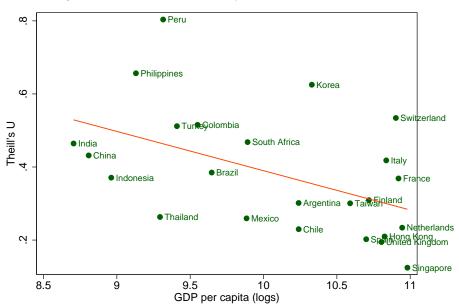


Figure 1: Relative Predictability of Real GDP Growth

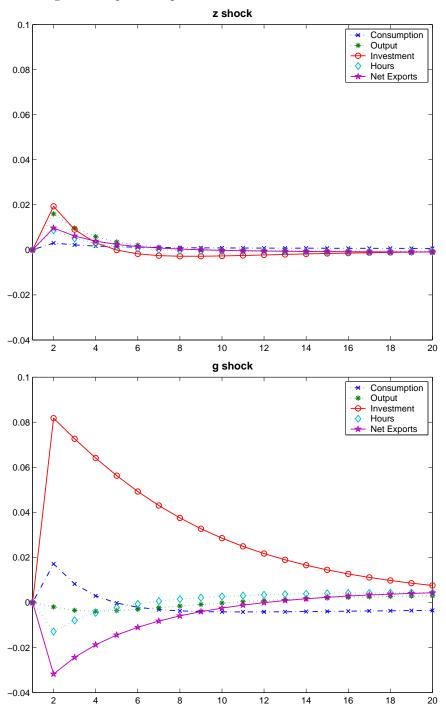


Figure 2: Impulse Responses in the Perfect Information Model

Note: This figure illustrates the response of the endogenous variables to a 1-percent shock to the transitory (top panel) vs. trend growth component (bottom panel) of the TFP.

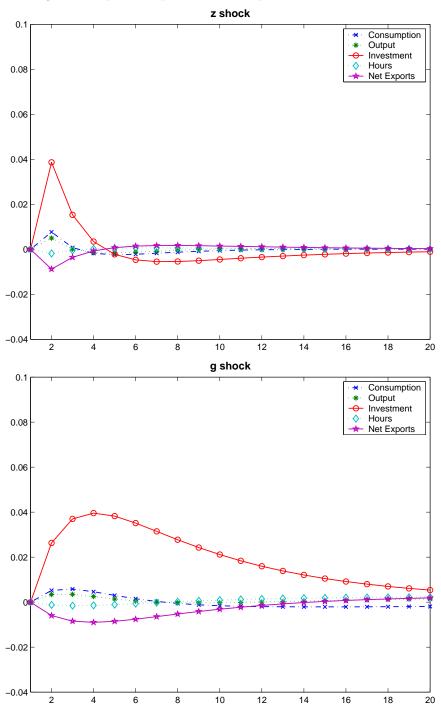


Figure 3: Impulse Responses in the Imperfect Information Model

Note: This figure illustrates the response of the endogenous variables to a 1-percent shock to the transitory (top panel) vs. trend growth component (bottom panel) of the TFP.

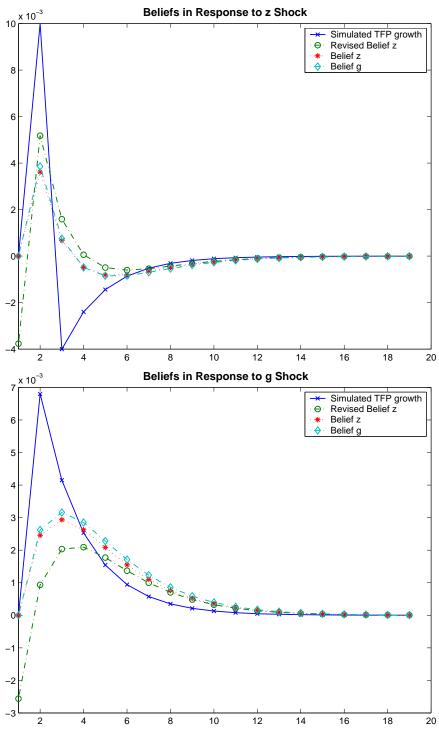


Figure 4: Beliefs Attached to TFP Components

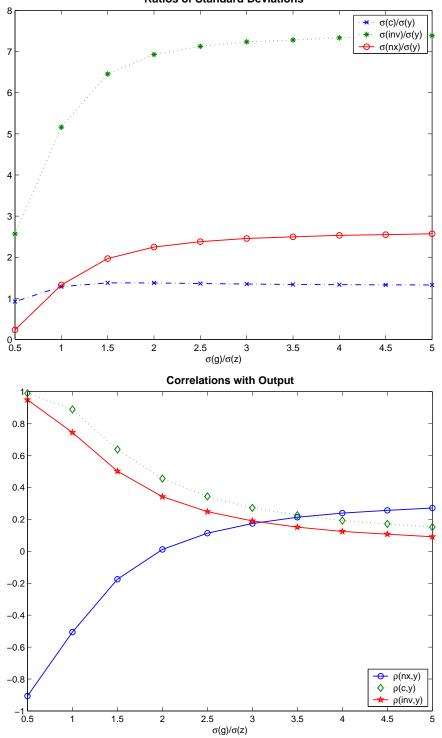


Figure 5: Sensitivity of Moments to the Relative Variability of Trend Shocks Ratios Ratios of Standard Deviations

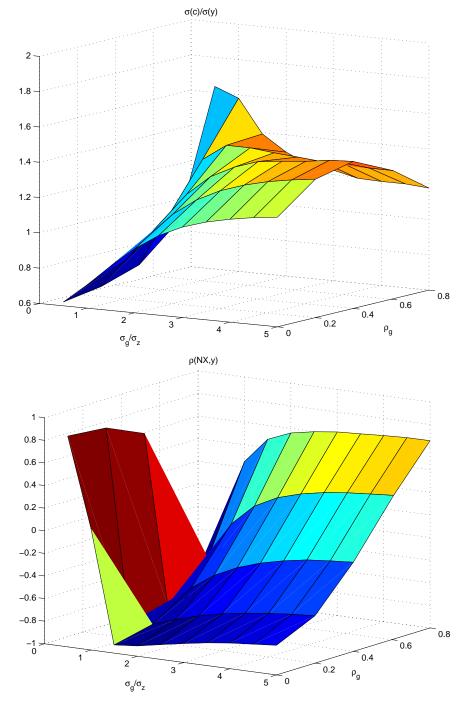


Figure 6: Imperfect Information Model Moments with Different σ_g/σ_z and ρ_g 's

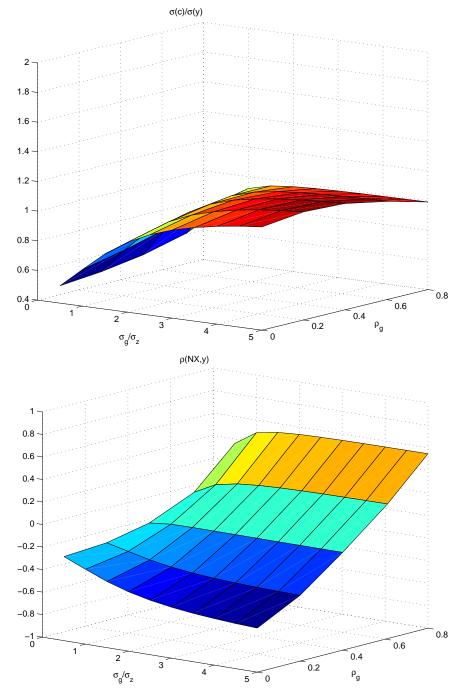


Figure 7: Perfect Information Model Moments with Different σ_g/σ_z and ρ_g 's

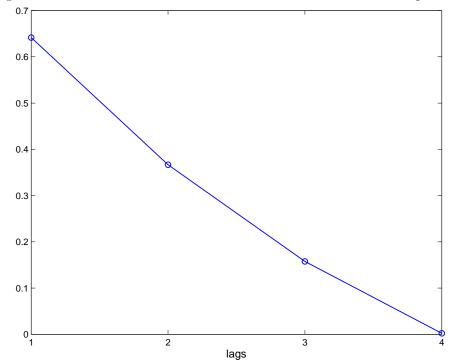


Figure 8: The Predicted Autocorrelation Function of Trade Balance-Output Ratio

Note: This figure shows the estimated autocorrelation function of trade balance-output ratio in the baseline estimation of the imperfect information model.

| Country | # of obs. | Mean | RMSE | $\operatorname{corr}(e_{t+1,t}, e_{t,t-1})$ | Dispersion | Norm. Dispersion |
|--------------|-----------|------------|------|---|------------|------------------|
| DCs | | | | | | |
| Australia | 33 | -0.01 | 0.50 | 0.16 | 0.20 | 0.22 |
| Denmark | 23 | 0.11 | 0.39 | -0.02 | 0.33 | 0.25 |
| Finland | 11 | 0.35^{*} | 0.70 | -0.41 | 0.76 | 0.39 |
| France | 25 | -0.02 | 0.30 | -0.35 | 0.12 | 0.15 |
| Italy | 18 | -0.11 | 0.39 | -0.02 | 0.15 | 0.17 |
| Netherlands | 16 | -0.02 | 0.36 | 0.32 | 0.15 | 0.10 |
| Spain | 20 | 0.04 | 0.15 | -0.13 | 0.10 | 0.13 |
| Switzerland | 14 | 0.14 | 0.46 | 0.08 | 0.16 | 0.19 |
| UK | 36 | 0.05^{*} | 0.14 | 0.01 | 0.09 | 0.13 |
| Average | 21.78 | 0.06 | 0.38 | -0.01 | 0.23 | 0.19 |
| Median | 20.00 | 0.04 | 0.39 | 0.01 | 0.15 | 0.17 |
| EMEs | | | | | | |
| Argentina | 26 | -0.57 | 2.23 | 0.57^{*} | 0.42 | 0.06 |
| Brazil | 28 | -0.28* | 0.83 | 0.06 | 0.43 | 0.20 |
| Chile | 14 | 0.10 | 0.28 | 0.21 | 0.17 | 0.14 |
| China | 21 | 0.30^{*} | 0.55 | -0.33 | 0.39 | 0.31 |
| Colombia | 17 | 0.23 | 0.87 | 0.03 | 0.41 | 0.24 |
| India | 21 | 0.30 | 0.85 | 0.06 | 0.48 | 0.26 |
| Indonesia | 20 | 0.18* | 0.43 | 0.18 | 0.41 | 0.35 |
| Hong Kong | 26 | 0.70^{*} | 0.80 | -0.16 | 0.68 | 0.18 |
| Korea | 23 | 0.23 | 0.86 | -0.10 | 0.31 | 0.23 |
| Malaysia | 28 | 0.01 | 0.99 | 0.62* | 0.45 | 0.19 |
| Mexico | 33 | 0.05 | 0.59 | 0.31* | 0.36 | 0.16 |
| Peru | 61 | 0.43* | 1.45 | -0.13 | 0.54 | 0.30 |
| Philippines | 17 | -0.35* | 0.65 | -0.13 | 0.48 | 0.48 |
| Singapore | 18 | -0.37* | 0.46 | -0.21 | 0.31 | 0.08 |
| South Africa | 23 | -0.01 | 0.80 | 0.28 | 0.28 | 0.16 |
| Taiwan | 22 | -0.16 | 0.86 | 0.21 | 0.61 | 0.21 |
| Thailand | 18 | -0.19* | 0.42 | 0.16 | 0.41 | 0.25 |
| Turkey | 28 | -0.13 | 3.12 | 0.10 | 1.07 | 0.18 |
| Average | 24.67 | 0.03 | 0.95 | 0.10 | 0.46 | 0.22 |
| Median | 22.50 | 0.03 | 0.81 | 0.08 | 0.41 | 0.21 |

Table 1: Moments of Forecast Errors in EMEs vs. Developed Economies

Source: Bloomberg. * Significantly different from 0 at 10% level.

 Table 2: Calibrated Parameters

| β | Discount factor | 0.98 |
|----------|--------------------------------------|-------|
| γ | Consumption exponent of utility | 0.36 |
| b | Steady state normalized debt | 10 |
| ψ | Coefficient on interest rate premium | 0.001 |
| α | Labor exponent | 0.68 |
| σ | Risk aversion | 2 |
| δ | Depreciation rate | 0.05 |

Table 3: Estimated Parameters of the Imperfect Information Model for Mexico

| σ_g | Stdev of permanent component noise | 1.06 (0.00) |
|---------------------|-------------------------------------|------------------|
| σ_z | Stdev of transitory component noise | $1.35 \\ (0.00)$ |
| $ ho_g$ | Persistence of permanent component | 0.61 (0.02) |
| ρ_z | Persistence of transitory component | $0.60 \\ (0.03)$ |
| ϕ | Capital adjustment cost | 1.27 (0.03) |
| μ_g | Growth rate | 0.66 |
| Relativ | ve variance of trend shocks | |
| σ_g/σ_z | | 0.78 |
| V | | 0.26 |

Note: This table summarizes the parameter estimates calculated using generalized method of moments. The moment conditions are provided in the Appendix. The numbers in parentheses are standard errors in percent.

| | Data | PI | GMM with II | II with PI | PI with II param |
|--------------------------------|-------|-------|-------------|------------|------------------|
| $\sigma(y)$ | 2.40 | 2.13 | 2.18 | 1.46 | 3.21 |
| $\sigma(\Delta y)$ | 1.52 | 1.42 | 1.55 | 1.33 | 2.68 |
| $\frac{\sigma(c)}{\sigma(y)}$ | 1.26 | 1.10 | 1.17 | 1.17 | 0.75 |
| $\frac{\sigma(I)}{\sigma(y)}$ | 4.15 | 3.83 | 4.17 | 6.74 | 3.71 |
| $\frac{\sigma(NX)}{\sigma(y)}$ | 0.90 | 0.95 | 0.89 | 1.44 | 1.31 |
| $\rho(y)$ | 0.83 | 0.82 | 0.77 | 0.66 | 0.68 |
| $\rho(\Delta y)$ | 0.27 | 0.18 | 0.27 | 0.04 | 0.10 |
| $\rho(y, NX)$ | -0.75 | -0.50 | -0.69 | -0.69 | 0.38 |
| $\rho(y,c)$ | 0.92 | 0.91 | 0.97 | 0.95 | 0.44 |
| $\rho(y, I)$ | 0.91 | 0.80 | 0.85 | 0.83 | 0.31 |

Table 4: Business Cycle Moments for Mexico

Notes: Moments are calculated using the simulated and HP-filtered data generated by the corresponding model. PI refers to the perfect information model using the parameter values from Aguiar and Gopinath (2007), II refers to the imperfect information model. The column "II with PI param" refers to the imperfect information model using the parameters estimated in column PI, while the column 'PI with II param' reports the moments of the perfect information setup generated using the estimated parameters of the imperfect information setup.

Table 5: Perfect vs Imperfect Information $ln(g_t^A) = \alpha g_t + \Delta z_t$ \widetilde{z}_t \widetilde{g}_t \widetilde{z}_{t-1} 1 % 1 % 0 % ΡI 1.68~%Π 1.68~%0.65~%0.60~%-0.63 %

Note: \tilde{g}_t , \tilde{z}_t , and \tilde{z}_{t-1} are equal to their true values in the perfect information case.

| Model | σ_g/σ_z | $ln(g_t^A) = \alpha g_t + \Delta z_t$ | \widetilde{g}_t | \widetilde{z}_t | \widetilde{z}_{t-1} |
|---------------|---------------------|---------------------------------------|-------------------|-------------------|-----------------------|
| \mathbf{PI} | 0.78 | 1.68~% | $1 \ \%$ | $1 \ \%$ | 0 % |
| II | 0.5 | 1.68~% | 0.36~% | 0.83~% | -0.61 % |
| II | 0.76 | 1.68~% | 0.62~% | 0.62~% | -0.63 % |
| II | 0.78 | 1.68~% | 0.65~% | 0.60 % | -0.63 % |
| II | 1 | 1.68~% | 0.85~% | 0.49~% | -0.61 % |
| II | 1.5 | 1.68~% | 1.25~% | 0.32~% | -0.51 % |
| II | 2 | 1.68~% | 1.54~% | 0.22~% | -0.41 % |
| II | 2.5 | 1.68~% | 1.75~% | 0.16~% | -0.33 % |
| II | 3 | 1.68~% | 1.91~% | 0.12~% | -0.26 % |
| II | 4 | 1.68~% | 2.10~% | 0.08~% | -0.17 % |
| II | 5 | 1.68~% | 2.22~% | 0.05~% | -0.12 % |

Table 6: Further Experiment on Kalman Learning

Notes: This table illustrates the weights or beliefs attached to the components of TFP for various values of relative variability of permanent to transitory shock.

| | $\sigma_s \to \infty$ | $\sigma_s = 0.5$ | $\sigma_s = 0.1$ | $\sigma_s = 0.05$ | $\sigma_s = 0.02$ | $\sigma_s=0.005$ |
|--------------------------------|-----------------------|------------------|------------------|-------------------|-------------------|------------------|
| $\sigma(y)$ | 2.18 | 2.20 | 2.26 | 2.46 | 2.81 | 3.11 |
| $\sigma(\Delta y)$ | 1.55 | 1.56 | 1.61 | 1.77 | 2.04 | 2.30 |
| $rac{\sigma(c)}{\sigma(y)}$ | 1.17 | 1.16 | 1.13 | 1.03 | 0.91 | 0.83 |
| $rac{\sigma(I)}{\sigma(y)}$ | 4.17 | 4.16 | 4.11 | 3.94 | 3.75 | 3.62 |
| $\frac{\sigma(NX)}{\sigma(y)}$ | 0.89 | 0.90 | 0.93 | 1.06 | 1.19 | 1.25 |
| $\rho(y)$ | 0.77 | 0.77 | 0.82 | 0.67 | 0.76 | 0.74 |
| $\rho(\Delta y)$ | 0.27 | 0.25 | 0.43 | 0.32 | 0.30 | 0.30 |
| $\rho(y, NX)$ | -0.69 | -0.64 | -0.53 | -0.15 | 0.17 | 0.32 |
| $\rho(y,c)$ | 0.97 | 0.96 | 0.94 | 0.82 | 0.67 | 0.57 |
| $\rho(y, I)$ | 0.85 | 0.83 | 0.79 | 0.62 | 0.41 | 0.30 |
| Forecast e | rrors | | | | | |
| $\sigma(e)$ | 1.76 | 1.75 | 1.74 | 1.74 | 1.71 | 1.59 |
| $rac{\sigma(e)}{\sigma(y)}$ | 0.81 | 0.81 | 0.80 | 0.78 | 0.69 | 0.50 |

Table 7: Comparison of EMEs vs. Developed Economies: Varying Degrees of Information Imperfection

| Additio | nal Calibrated Parameters | | | | |
|-----------------------------------|-------------------------------------|--|--|--|--|
| ν | Elasticity of labor supply | 1.60 | | | |
| τ | Labor Coefficient | 1.40 | | | |
| Estima | ted Parameters | | | | |
| σ_g | Stdev of permanent component noise | 0.95 (0.00) | | | |
| σ_z | Stdev of transitory component noise | $1.00 \\ (0.00)$ | | | |
| $ ho_g$ | Persistence of permanent component | $\begin{array}{c} 0.75 \ (0.03) \end{array}$ | | | |
| ρ_z | Persistence of transitory component | $\begin{array}{c} 0.75 \ (0.03) \end{array}$ | | | |
| ϕ | Capital adjustment cost | 1.27 (0.03) | | | |
| μ_g | Growth rate | 0.66 | | | |
| Relative variance of trend shocks | | | | | |
| σ_g/σ_z | | 0.95 | | | |
| V | | 0.45 | | | |

Table 8: Parameters with GHH Preferences

Note: This table summarizes the parameter used for the estimation of the imperfect information model with GHH preference. The numbers in parentheses are standard errors in percent.

| | Data | II | PI |
|--------------------------------|-------|-------|-------|
| $\sigma(y)$ | 2.40 | 2.39 | 2.74 |
| $\sigma(\Delta y)$ | 1.52 | 1.59 | 2.13 |
| $\frac{\sigma(c)}{\sigma(y)}$ | 1.26 | 1.06 | 0.87 |
| $rac{\sigma(I)}{\sigma(y)}$ | 4.15 | 5.06 | 2.04 |
| $\frac{\sigma(NX)}{\sigma(y)}$ | 0.90 | 0.86 | 0.19 |
| $\rho(y)$ | 0.83 | 0.84 | 0.72 |
| $\rho(\Delta y)$ | 0.27 | 0.46 | 0.12 |
| $\rho(y, NX)$ | -0.75 | -0.78 | -0.09 |
| $\rho(y,c)$ | 0.92 | 0.99 | 0.98 |
| $\rho(y, I)$ | 0.91 | 0.89 | 0.91 |

Table 9: Business Cycle Moments with GHH Preferences

Notes: Moments are calculated using the simulated and HP-filtered data generated by the corresponding model. II refers to the imperfect information model. PI refers to the perfect information model using the same parameter values for the imperfect information model.