Online Appendix to "Credit Risk and Disaster Risk"

Not for Publication

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1 Organization of the Appendix

This appendix contains (1) some additional comparative statics and comparative dynamics; (2) a list of the equations characterizing the benchmark model equilibrium, (3) the full set of equations for the variant of the model with state-contingent debt, (4) a description of the computational method used to solve the model.

2 Additional Comparative Statics

Tables 1, 2, 3 and 4 of this appendix provide some additional comparative statics for the model, reporting respectively the parameter values; the business cycle and asset return statistics; the spread, default, and leverage statistics; the AAA asset spread, default and leverage.

For each of the parameter studied, we provide two results: one where the parameter is above the benchmark value, and one where it is below the benchmark. More precisely, version 6 is the benchmark model; versions 9 and 10 vary idiosyncratic risk σ_ε only; versions 11 and 12 vary the tax shield χ only; versions 13 and 14 vary the recovery rate θ only; versions 15 and 16 vary the persistence of disaster probability ρ_p only; versions 17 and 18 vary risk aversion γ only; versions 19 and 20 vary the standard deviation of disaster (log) size σ_b only; versions 21 and 22 vary the mean size of disasters $\bar{b}$ only; versions 23 and 24 vary the mean disaster probability; versions 25 and 26 vary the recovery rate during disaster.

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Version	$\overline{b_k}$	$\overline{b_z}$	σ_{b_k}	σ_p	ρ_p	γ	ψ	χ	σ_ε	θ_{nd}	θ_d
6:benchmark	.15	.15	.30	.70	.75	10	.50	4.20	.19	.70	.70
9:higher σ_ε	.15	.15	.30	.70	.75	10	.50	4.20	.21	.70	.70
10:lower σ_ε	.15	.15	.30	.70	.75	10	.50	4.20	.17	.70	.70
11:lower χ	.15	.15	.30	.70	.75	10	.50	3.0	.19	.70	.70
12:higher χ	.15	.15	.30	.70	.75	10	.50	5.5	.19	.70	.70
13:lower θ	.15	.15	.30	.70	.75	10	.50	4.2	.19	.50	.50
14:higher θ	.15	.15	.30	.70	.75	10	.50	4.2	.19	.95	.95
15:higher ρ_p	.15	.15	.30	.70	.90	10	.50	4.2	.19	.70	.70
16:lower ρ_p	.15	.15	.30	.70	.20	10	.50	4.2	.19	.70	.70
17:higher γ	.15	.15	.30	.70	.75	12	.50	4.2	.19	.70	.70
18:lower γ	.15	.15	.30	.70	.75	8	.50	4.2	.19	.70	.70
19:lower σ_b	.15	.15	.25	.70	.75	10	.50	4.2	.19	.70	.70
20:higher σ_b	.15	.15	.35	.70	.75	10	.50	4.2	.19	.70	.70
21:lower $\overline{b}$	.10	.10	.30	.70	.75	10	.50	4.2	.19	.70	.70
22:higher $\overline{b}$	.20	.20	.30	.70	.75	10	.50	4.2	.19	.70	.70
23:low Ep	.15	.15	.30	.70	.75	10	.50	4.2	.19	.70	.70
24:high Ep	.15	.15	.30	.70	.75	10	.50	4.2	.19	.70	.70
25:lower θ_d	.15	.15	.30	.70	.75	10	.50	4.2	.19	.70	.50
26:higher θ_d	.15	.15	.30	.70	.75	10	.50	4.2	.19	.70	1.00

Table 1: **Parameter values.**

3 Equilibrium characterization: list of equations

The equilibrium is characterized by the following set of sixteen equations, in the sixteen unknowns $C_t, I_t, B_{t+1}, K^w_{t+1}, L_t, W_t, z_t, K_{t+1}, R^K_{t+1}, N_t, Y_t, M_{t+1}, U_t, x_{t+1}, V_t, \varepsilon^*_t$:

$$L_t = \frac{B_{t+1}}{K^w_{t+1}}, \tag{1}$$

$$\varepsilon^*_{t+1} = \frac{B_{t+1}}{R^K_{t+1} K^w_{t+1}} = \frac{L_{t+1}}{R^K_{t+1}}, \tag{2}$$

$$R^K_{t+1} = e^{x_{t+1} b_{k,t+1}} \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}}\right), \tag{3}$$

$$Y_t = K^\alpha_t (z_t N_t)^{1-\alpha}, \tag{4}$$

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{(1-\upsilon)C_t}{\upsilon\,(1-N_t)}, \tag{5}$$

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\psi)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\psi)} \left(\frac{U_{t+1}}{E_t\left(U^{1-\gamma}_{t+1}\right)^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma}, \tag{6}$$

$$U_t = \left((1-\beta)u(C_t, N_t)^{1-\psi} + \beta E_t \left(U^{1-\gamma}_{t+1}\right)^{\frac{1-\psi}{1-\gamma}}\right)^{\frac{1}{1-\psi}}, \tag{7}$$

	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$E(R^{aaa})$	$E(R^{baa})$	$E(R^{e})$
6	1.53	1.12	5.28	1.17	.36	1.14	5.13
9	1.51	1.08	4.99	1.08	.42	1.13	4.98
10	1.56	1.19	5.57	1.25	.35	1.22	5.30
11	1.46	1.00	4.76	.95	.38	.85	4.89
12	1.61	1.28	5.68	1.39	.29	1.36	5.35
13	1.52	1.11	5.26	1.13	.24	.96	4.71
14	1.39	.90	3.47	.75	.98	1.49	6.14
15	1.52	1.09	5.08	1.10	0.38	1.15	5.12
16	1.47	1.02	4.58	.99	.34	1.13	5.13
17	1.56	1.19	5.72	1.25	-1.06	-.03	5.75
18	1.48	1.04	4.68	1.02	1.32	1.87	4.67
19	1.46	1.01	4.42	.96	.94	1.43	4.71
20	1.61	1.26	6.19	1.37	-.54	.69	5.61
21	1.49	1.04	4.67	1.02	.95	1.54	4.62
22	1.57	1.22	5.88	1.30	-.48	.57	5.72
23	1.43	.96	4.07	.87	1.57	2.00	3.92
24	1.56	1.21	5.85	1.28	-.87	.07	6.09
25	1.54	1.14	5.43	1.18	.27	1.05	4.74
26	1.36	.79	2.85	.50	.45	.66	5.64

Table 2: **Business cycle and asset return statistics.** Annual volatility of the growth rates of investment, consumption, hours and output; and mean return on the AAA and BAA bonds and equity. Model statistics are computed in a sample without disasters.

	Credit Spread			Default rate	Loss given Default	Leverage B/K	
	Mean	SD	Corr with I	Mean	Mean	Mean	SD
6	.90	.40	-.44	39.03	33.76	56.97	6.33
9	.85	.34	-.48	45.58	34.23	55.14	5.29
10	0.98	.47	-.39	33.47	33.31	58.98	7.44
11	.53	.24	-.26	18.60	33.45	53.30	7.37
12	1.29	.60	-.57	64.10	34.01	59.58	4.98
13	.79	.25	-.09	11.71	52.33	5.47	7.74
14	1.18	.54	-.61	687.54	12.49	74.00	0.59
15	.90	.39	-.40	39.24	33.77	57.04	5.99
16	.91	.40	-.42	38.60	33.76	56.93	6.45
17	1.12	.46	-.12	26.49	33.50	53.47	8.41
18	.70	.36	-.60	49.74	33.95	59.23	4.17
19	.62	.23	-.52	42.61	33.88	58.42	4.20
20	1.34	.72	-.28	37.37	33.65	55.25	9.13
21	.73	.34	-.55	46.08	33.90	58.69	4.51
22	1.16	.54	-.21	31.84	33.61	54.86	8.21
23	.60	.37	-.64	61.52	34.07	6.63	2.70
24	1.02	.37	-.10	24.45	33.48	53.33	8.01
25	.85	.28	-.02	2.61	33.36	51.62	8.57
26	.43	.14	-.27	97.75	34.30	62.97	.16

Table 3: **Credit spread, leverage and default.** Mean, volatility and correlation with investment of the credit spread; mean default rate; mean loss given default; mean and volatility of leverage. Annualized statistics. Model statistics are computed in a sample without disasters.

	E(B/K)AAA	E(Def AAA)	E(LGD AAA)
6	45.38	3.97	33.76
9	44.39	5.16	34.23
10	46.19	2.93	33.31
11	45.51	3.81	33.45
12	44.91	3.79	34.01
13	39.38	.69	52.33
14	65.64	186.16	12.49
15	45.43	3.94	33.77
16	45.25	3.81	33.76
17	4.38	1.77	33.50
18	49.38	7.03	33.95
19	49.37	5.70	33.88
20	40.13	3.01	33.65
21	48.61	6.12	33.90
22	41.24	2.42	33.61
23	51.90	10.87	34.07
24	41.03	1.62	33.48
25	39.66	1.14	33.36
26	58.73	34.99	34.30

Table 4: **Leverage, default and loss given default for the AAA asset.**

$$E_t\left(M_{t+1}R_{t+1}^K\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon_{t+1}^*\right)+(\chi-1)\,\varepsilon_{t+1}^*\left(1-H\left(\varepsilon_{t+1}^*\right)\right)\right)\right)=1, \tag{8}$$

$$E_t\left(M_{t+1}\left(\chi\left(\theta-1\right)\varepsilon_{t+1}^*h\left(\varepsilon_{t+1}^*\right)+(\chi-1)\left(1-H\left(\varepsilon_{t+1}^*\right)\right)\right)\right)=0, \tag{9}$$

$$K_{t+1}=K_{t+1}^w e^{x_{t+1}b_{k,t+1}}, \tag{10}$$

$$C_t+I_t=Y_t, \tag{11}$$

$$I_t=K_{t+1}^w-(1-\delta)K_t, \tag{12}$$

$$V_{t+1}=R_{t+1}^K K_t^w, \tag{13}$$

$$\log z_{t+1} = \log z_t+\mu+\sigma_e e_{t+1}+x_{t+1}b_{z,t+1}, \tag{14}$$

$$p_{t+1}|p_t \text{ is Markov with transition } Q, \tag{15}$$

$$\begin{aligned}\Pr(x_{t+1} &= 1)=p_t\\ \Pr(x_{t+1} &= 0)=1-p_t\end{aligned}$$

Bond price;

$$q_t=E_t\left(M_{t+1}\left(1-H\left(\varepsilon_{t+1}^*\right)+\frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}}\Omega\left(\varepsilon_{t+1}^*\right)\right)\right),$$

Equity price:

$$\begin{aligned}S_t &= K_{t+1}^w-\chi q_t B_{t+1}\\ &= E_t\left(M_{t+1}\max(V_{t+1}-B_{t+1},0)\right)\\ &= E_t\left(M_{t+1}\int_{\varepsilon_{t+1}^*}^{\infty}\left(\varepsilon K_{t+1}^w R_{t+1}^K-B_{t+1}\right)dH(\varepsilon)\right)\\ &= E_t\left(M_{t+1}\left(\left(1-\Omega(\varepsilon_{t+1}^*)\right)K_{t+1}^w R_{t+1}^K-(1-H(\varepsilon_{t+1}^*))B_{t+1}\right)\right).\end{aligned}$$

Portfolio return on equity:

$$R_{t+1}^e=\frac{\left(1-\Omega(\varepsilon_{t+1}^*)\right)K_{t+1}^w R_{t+1}^K-(1-H(\varepsilon_{t+1}^*))B_{t+1}}{S_t},$$

Portfolio return on corporate bonds:

$$R_{t+1}^c=\frac{1-H\left(\varepsilon_{t+1}^*\right)+\frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}}\Omega\left(\varepsilon_{t+1}^*\right)}{q_t},$$

Fictitious risk-free rate:

$$R_{t+1}^f=\frac{1}{E_t\left(M_{t+1}\right)}.$$

Recall that $\Omega(x)=\int_0^x s dH(s)$. In the model with constant leverage, the equations are identical, except that $L_{t+1}=B_{t+1}/K_{t+1}^w=\overline{L}$. As a result, the first order conditions are identical, except that we discard equation (9) which determines optimal debt.

4 Variant of the model with state-contingent debt

This section summarizes the model with state-contingent debt that is sketched in section 3. The budget constraint of the firm is

$$K_{t+1}^{w} = S_t + \chi \left(q_t^{nd} B_{t+1}^{nd} + \sum_{b'} q_t^{d}(b') B_{t+1}^{d}(b') \right),$$

where B_{t+1}^{nd} (resp. $B_{t+1}^{d}(b')$) is the face value of the debt to be repaid in non-disaster (resp. disaster of size b') states, and q_t^{nd} (resp. $q_t^{d}(b')$) the associated price:

$$q_t^{nd} = E_t \left((1 - x_{t+1}) M_{t+1} \left(\int_{\varepsilon_{t+1}^*}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}^{nd}} \int_0^{\varepsilon_{t+1}^*} \varepsilon R_{t+1}^K K_{t+1}^w dH(\varepsilon) \right) \right),$$

$$q_t^{d}(b') = E_t \left(\widetilde{x}_{t+1}(b') M_{t+1} \left(\int_{\varepsilon_{t+1}^*}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}^{d}(b')} \int_0^{\varepsilon_{t+1}^*} \varepsilon R_{t+1}^K K_{t+1}^w dH(\varepsilon) \right) \right),$$

where $(1 - x_{t+1})$ is a dummy equal to 1 if no disaster happens.

Note that in these two equations, ε_{t+1}^* (like other variables) is state-contingent, and in particular depend on x_{t+1} (disaster realization):

$$\varepsilon_{t+1}^* = \frac{\sum_{b'} \widetilde{x}_{t+1}(b') B_{t+1}(b') + B_{t+1}^{nd}(1 - x_{t+1})}{R_{t+1}^K K_{t+1}^w}$$

The firm problem is

$$\max_{B_t^{nd}, B_t^d, S_t, K_{t+1}^w} E_t \left(M_{t+1} \left(\left(1 - \Omega(\varepsilon_{t+1}^*)\right) K_{t+1}^w R_{t+1}^K - \left(1 - H\left(\varepsilon_{t+1}^*\right)\right) \sum_{b'} \widetilde{x}_{t+1}(b') B_{t+1}(b') + B_{t+1}^{nd}(1 - x_{t+1}) \right) \right) - S_t$$

$$s.t. : K_{t+1}^w = S_t + \chi \left(q_t^{nd} B_{t+1}^{nd} + \sum_{b'} q_t^{d}(b') B_{t+1}^{d}(b') \right),$$

and the bond pricing equations above. The FOCs yield:

$$E_t \left(M_{t+1} R_{t+1}^K \left(\begin{array}{c} 1 + (\chi - 1) L_{t+1}^{nd}(1 - x_{t+1}) \left(1 - H\left(\varepsilon_{t+1}^{*nd}\right)\right) \\ + (\chi - 1) \sum_{b'} L_{t+1}^{d}(b') \widetilde{x}_{t+1}(b') \left(1 - H\left(\varepsilon_{t+1}^{*d}(b')\right)\right) \\ + (\theta\chi - 1)\, \Omega(\varepsilon_{t+1}^*) \end{array} \right) \right) = 1,$$

where $L_{t+1}^{d}(b') = B_{t+1}^{d}(b') / K_{t+1}^{w}$; and

$$\begin{aligned} \frac{\chi - 1}{\chi} E_t \left((1 - x_{t+1}) M_{t+1} \left(1 - H\left(\varepsilon_{t+1}^*\right)\right) \right) &= (1 - \theta)\, E_t \left(M_{t+1} \varepsilon_{t+1}^* h\left(\varepsilon_{t+1}^*\right) (1 - x_{t+1}) \right), \\ \frac{\chi - 1}{\chi} E_t \left(\widetilde{x}_{t+1}(b') M_{t+1} \left(1 - H\left(\varepsilon_{t+1}^*\right)\right) \right) &= (1 - \theta)\, E_t \left(M_{t+1} \varepsilon_{t+1}^* h\left(\varepsilon_{t+1}^*\right) \widetilde{x}_{t+1}(b') \right). \end{aligned}$$

In terms of the recursive representation, the only change is that we need to solve for policy functions $L^{nd}(k, p; b')$.

5 Computational Method

The model is solved using projection methods (see Aruoba et al. (2009) and Caldara et al. (2010) for an introduction and discussion of implementation with Epstein-Zin preferences). I use the characterization of the equilibrium using the recursive formulation of the three policy functions, consumption $c(k, p)$,

employment $N(k,p)$, leverage $L(k,p)$, and the value function $g(k,p)$. We have the three first-order conditions:

$$(1-\alpha)\left(\frac{k}{N(k,p)}\right)^{\alpha} = \frac{1-\upsilon}{\upsilon}\frac{c(k,p)}{1-N(k,p)}, \tag{16}$$

$$E_{\omega'|p}\left(M\left(k,p,\omega'\right)R^{K}(k,p,\omega')\left(\begin{array}{c}1+\left(\chi\theta-1\right)\Omega\left(\varepsilon^{*}(k,p,\omega')\right)\\+\left(\chi-1\right)\varepsilon^{*}(k,p,\omega')\left(1-H\left(\varepsilon^{*}(k,p,\omega')\right)\right)\end{array}\right)\right)=1, \tag{17}$$

$$E_{\omega'|p}\left(M\left(k,p,\omega'\right)\left(\begin{array}{c}\chi\left(\theta-1\right)\varepsilon^{*}(k,p,\omega')h\left(\varepsilon^{*}(k,p,\omega')\right)\\+\left(\chi-1\right)\left(1-H\left(\varepsilon^{*}(k,p,\omega')\right)\right)\end{array}\right)\right)=0, \tag{18}$$

where $\varepsilon^{*}(k,p,\omega')=\frac{L(k,p)}{R^{K}(k,p,\omega')}$, $R^{K}(k,p,\omega')=e^{x'b'_k}\left(1-\delta+\alpha\frac{y(k',p')}{k'}\right)$, and the value u is defined as

$$u\left(k,p\right)=(1-\beta)c(k,p)^{\upsilon(1-\psi)}(1-N(k,p))^{(1-\upsilon)(1-\psi)}+\beta e^{\mu\upsilon(1-\psi)}\left(E_{\omega'|p}e^{\upsilon(1-\gamma)\left(\sigma_e e'+x'b'_z\right)}u\left(k',p'\right)^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{1-\psi}{1-\gamma}}. \tag{19}$$

In the formulas above, the stochastic discount factor is

$$\begin{aligned}M(k,p,\omega') \quad = \quad & \beta e^{(\upsilon(1-\gamma)-1)\left(\mu+\sigma_e e'+x'b'_z\right)}\\ & \times\left(\frac{c(k',p')}{c(k,p)}\right)^{\upsilon(1-\psi)-1}\left(\frac{1-N(k',p')}{1-N(k,p)}\right)^{(1-\upsilon)(1-\psi)}\\ & \times\frac{u(k',p')^{\frac{\psi-\gamma}{1-\psi}}}{E_{\omega'|p}\left(e^{\upsilon(1-\gamma)(\mu+\sigma_e e'+x'b'_z)}u(k',p')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{\psi-\gamma}{1-\gamma}}},\end{aligned}$$

where k' is given by the law of motion for the single endogenous state variable k,

$$k'=k'(k,p,\omega')=\frac{e^{x'b'_k}\left(\left(1-\delta\right)k+i(k,p)\right)}{e^{x'b'_z+\mu+\sigma_e e'}},$$

as well as the law of motions for z and p. Recall that p follows a Markov chain and hence is discrete, with N_p values.

I approximate each of the four functions $c(k,p), N(k,p), L(k,p), g(k,p)$ through an expression $\sum_{j=1}^{N_c}\alpha_{ip}T_i(k)$, where T_i is the Chebychev polynomial of degree i, adjusted to have a domain $\left[\underline{k},\overline{k}\right]$. That is, for each discrete value of p, we approximate the function $c(k,p)$ by a one-dimensional Chebychev polynomial. We then stack all the first order conditions 16-19, evaluated at the nodes of the Chebychev polynomial, and each value of p. This gives a system of $4N_cN_p$ equations with $4N_cN_p$ unknowns, which is solved using a quasi-Newton algorithm. To help with convergence, we start with a small N_c, small N_p, and small risk aversion parameter θ and small disaster size $\overline{b_k}=\overline{b_z}$, and progressively increase these parameters, using as initial guess the previous solution. This allows to solve the model for $N_c=10$ or more, and $N_p=7$ or more. Expanding the number of points beyond this does not alter the solution significantly, so we use $N_c=5$ and $N_p=8$. Once the policy functions are calculated, asset prices are easily found using the definitions in the text. We obtain model moments by running 200 simulations of the model for 500 periods, and cutting off the first 300 periods.

The model with state-contingent debt requires specifying many more policy functions, since there is a leverage for each disaster size, but the method applies almost identically. I also solved a version where debt is a state variable due to default costs being real rather than a transfer. The same method applies, except now we need to approximate policy functions $c(k,b,p)$ as $\sum_{j=1}^{N_c}\sum_{l=1}^{N_b}\alpha_{i,l,p}T_i(k)T_l(b)$.

References

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