Comparative Advantage in Innovation and Production
Online Appendix

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[*]All the views expressed in this paper are mine and do not necessarily represent those of the Federal Reserve Board.

A Theoretical Appendix

A.1 Section I

A.1.1 Derivation of equation (1)

Proof. The arrival of ideas regarding new production techniques for any final good (z,ω) in country i has the following characteristics: (i) follows an inhomogeneous Poisson process $\mathcal{P}(z,\omega,i)$ with arrival rate $\iota_i^{\omega}\left(L_{it}^{R,\omega}\right)^{v}$, where $L_{it}^{R,\omega}$ denotes the total number of researchers targeting industry ω at time t; (ii) for any pair $(z,\omega,i),(z',\omega',i')$ such that $(z,\omega,i)\neq(z',\omega',i')$, $\mathcal{P}(z,\omega,i)$ and $\mathcal{P}(z',\omega',i')$ are independent. The total number of techniques for good (z,ω) discovered up to time t in country i is then a random variable distributed Poisson with parameter T_{it}^{ω} given by (2) in the text. As I discuss in more detail below, the assumptions about the distributions of Z and X imply that the efficiency of the best and second best techniques up to time t for a good in industry ω are random variables $\{X_{it}^{\omega,(1)},X_{it}^{\omega,(2)}\}$ with joint cdf F_{it}^{ω} given by (1). Assuming a law of large numbers across the continuum of goods within each industry, F_{it}^{ω} also represents the cdf of the joint distribution of the best and second best techniques across goods within industry ω in country i.

I will now show that $\{X_{it}^{\omega,(1)},X_{it}^{\omega,(2)}\}$ have a joint cdf given (1) in the text. The analysis that follows applies to any country i, good (z,ω) and time t, so all references to country, good and time are dropped. Let n be the number of techniques available up to time t for good (z,ω) in country i. As discussed above, the efficiency of these techniques is obtained as independent draws from the Pareto distribution H. Let $X_1,...,X_n$ be the random variables corresponding to each of these n draws, and let Y_j represent the j-th best draw among the X_is. We are interested, in the joint distribution of the best and second best draws conditional on n, i.e., the joint distribution of Y_1,Y_2 conditional on n. Following Hogg and Craig (1995) section 4.6, the joint pdf of $Y_1,..,Y_n$ is

$$\begin{aligned} g\left(y_1,...,y_n|n\right) &= n!h\left(y_1\right)h\left(y_2\right)...h\left(y_n\right) \text{ for } \infty > y_1 > y_2 > \cdots > y_n > 1 \\ &= 0 \text{ elsewhere} \end{aligned}$$

where h is the pdf corresponding to H. Integrating over $y_3,...,y_n$, the marginal joint density of Y_1,Y_2 is

$$\begin{aligned} f\left(y_1,y_2|n\right) &= \int_1^{y_2}...\int_1^{y_{n-2}}\int_1^{y_{n-1}} h\left(y_n\right)...h\left(y_1\right)dy_n...dy_1 \\ &= \frac{n!}{(n-2)!}H\left(y_2\right)^{n-2}h\left(y_2\right)h\left(y_1\right) \end{aligned}$$

Once we know $f(y_1, y_2)$, the joint cdf of Y_1, Y_2 conditional on n can be obtained as

$$\begin{aligned} F(x_1, x_2|n) &= \Pr(Y_1 \leq x_1, Y_2 \leq x_2|n) \\ &= \int_1^{x_1} \int_1^{\min\{x_2, y_1\}} g(y_1, y_2)\, dy_2 dy_1 \\ &= \int_1^{x_1} \int_1^{x_2} g(y_1, y_2)\, dy_2 dy_1 - \int_1^{x_2} \int_1^{y_2} g(y_1, y_2)\, dy_1 dy_2 \\ &= nH(x_2)^{n-1} H(x_1) - H(x_2)^n (n-1) \end{aligned}$$

Finally, the unconditional joint distribution of Y_1, Y_2 can be obtained by taking expectation over n. Recalling that n is distributed Poisson with parameter T, this yields

$$\begin{aligned} F(x_1, x_2) &= \sum_{n=0}^{\infty} \frac{F(x_1, x_2|n)\, T^n e^{-T}}{n!} \\ &= \sum_{n=0}^{\infty} \frac{nH(x_2)^{n-1} H(x_1)\, T^n e^{-T}}{n!} - \sum_{n=0}^{\infty} \frac{nH(x_1)^n\, T^n e^{-T}}{n!} + \sum_{n=0}^{\infty} \frac{H(x_1)^n\, T^n e^{-T}}{n!} \\ &= H(x_1)\, T e^{-T[1-H(x_2)]} - H(x_2)\, T e^{-T[1-H(x_2)]} + e^{-T[1-H(x_2)]} \\ &= \left[1 + T\left(x_2^{-\theta} - x_1^{-\theta}\right)\right] e^{-Tx_2^{-\theta}}, \end{aligned}$$

which is the desired result.

Recalling that the joint distribution used in BEJK (2003) for the best and second best technique is $K(x_1, x_2) = \left[1 + T\left(x_2^{-\theta} - x_1^{-\theta}\right)\right] e^{-Tx_2^{-\theta}}$ for $x_1 \geq x_2 \geq 0$, the only difference between F and K is that F is valid only for $x_2 \geq 1$. This discrepancy arises from the fact hat the minimum efficiency level in the present context is 1, implying that $F(x_1, x_2) = 0$ for $x_2 < 1$. According to K, $\Pr(x_2 \leq 1) = K(\infty, 1) = [1 + T]\, e^{-T} = \Pr(n \leq 1)$, i.e., the difference is attributable to the fact that at any time t, there is a set of goods with strictly positive mass such that no more than one technique has been discovered. However, notice that this probability approaches zero as $T \to \infty$.

Formally, F_T converges in distribution to K_T as $T \to \infty$, where the subscript T make explicit the dependence of the distributions on the parameter T. Given that T is growing, we need to first normalize the variables and then analyze the convergence of the normalized variables. Let Y_{iT}^d be the random variable representing the i-th best technique according to distribution d and for parameter T, and consider the variables $Z_{iT}^d = Y_i^d T^{1/\theta}$ for $i = 1, 2$ and $d = F_T, K_T$. In what follows I show that $F_T, K_T \to K$, where $K(z_1, z_2) = \left[1 + \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-z_2^{-\theta}}$ for $z_1 \geq z_2 \geq 0$. Notice that

$$\begin{aligned} F_T^Z(z_1, z_2) &= \Pr\left(Z_{1T}^{F_T} \leq z_1, Z_{2T}^{F_T} \leq z_2\right) \\ &= \Pr\left(Y_{1T}^{F_T} \leq T^{-1/\theta} z_1, Y_{2T}^{F_T} \leq T^{-1/\theta} z_2\right) \\ &= \left[1 + \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-z_2^{-\theta}} \text{ for } z_1 \geq z_2 \geq T^{-1/\theta} \end{aligned}$$

and so it is clear that $F_T^Z(z_1, z_2) \to K(z_1, z_2)$ for all $z_1 \geq z_2 \geq 0$. Similarly,

$$\begin{aligned} K_T^Z(z_1, z_2) &= \Pr\left(Z_{1T}^{K_T} \leq z_1, Z_{2T}^{K_T} \leq z_2\right) \\ &= \Pr\left(Y_{1T}^{K_T} \leq T^{-1/\theta} z_1, Y_{2T}^{K_T} \leq T^{-1/\theta} z_2\right) \\ &= \left[1 + \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-z_2^{-\theta}} \text{ for } z_1 \geq z_2 \geq 0 \end{aligned}$$

and so $K_T^Z(z_1, z_2) = K(z_1, z_2)$ for all $z_1 \geq z_2 \geq 0$. Both results imply that $F_T \to K_T$. ■

A.1.2 Costs, Markups and Prices

Firms in country i produce under constant returns to scale with a unit cost of serving country j given by $w_{it}\tau_{ij}^{\omega}/x(z)$, where $x(z)$ is the efficiency of the firm. The distribution of technologies (1) has the following implications:

I.1. Let $a_{ij}^{\omega(1)}(z)$ be the unit cost of serving market (z, ω) in country j for the most efficient producer of the good in country i. Then the cdf of $a_{ij}^{\omega(1)}(z)$ is

$$G_{ijt}^{\omega}(a) = 1 - e^{-T_{it}^{\omega}\left(w_{it}\tau_{ij}^{\omega}\right)^{-\theta} a^{\theta}}$$

I.2. Let $a_j^{\omega(1)}(z), a_j^{\omega(2)}(z)$ be the costs corresponding to the producers around the world with the lowest and the second lowest unit costs of serving market (z, ω) in country j, respectively. The joint cdf of $a_j^{\omega(1)}$ and $a_j^{\omega(2)}$ is

$$G_{jt}^{\omega}(a_1, a_2) = 1 - e^{-\Phi_{jt}^{\omega} a_1^{\theta}} - \Phi_{it}^{\omega} a_1^{\theta} e^{-\Phi_{it}^{\omega} a_2^{\theta}}$$

where Φ_{jt}^{ω} is given in (12). Moreover, G_{jt}^{ω} is also the joint distribution of $a_{jt}^{\omega(1)}(z), a_{jt}^{\omega(2)}(z)$ conditional on country i being the lowest cost supplier.

I.3. At any given moment in time, there are many alternative techniques in each country to produce a given final good (z, ω) that differ in their respective efficiencies. Price competition implies that the producer with the lowest marginal cost of serving that market becomes the sole supplier of the good to that market, and charges the minimum between the monopoly price and the maximum price that keeps competitors at bay.[1] Recalling that $a_j^{\omega(1)}(z)$ and $a_j^{\omega(2)}(z)$ are the lowest and the second lowest unit costs of serving market (z, ω) in country j, the price charged by the sole supplier of good i in that market is

$$p_{jt}^{\omega}(z) = \min\left\{\bar{m}(\sigma^{\omega})\, a_{jt}^{\omega(1)}(z), a_{jt}^{\omega(2)}(z)\right\}$$

where $\bar{m}(\sigma^{\omega})$ is the optimal monopoly markup corresponding to the iso-elastic demand for good (z, ω) which is given by $\bar{m}(\sigma^{\omega}) = \sigma^{\omega}/(\sigma^{\omega} - 1)$ if $\sigma^{\omega} > 1$ and $\bar{m}(\sigma^{\omega}) = \infty$ if $\sigma^{\omega} \leq 1$. Moreover, an immediate implication of (I.2) is that the distribution of prices in industry ω and country j does not depend on the source country.

[1] The monopoly price is the optimal price charged by a monopoly that faces the residual demand corresponding to good (z, ω).

I.4. Let us define the cost gap in country j, $m_{jt}^{\omega}(z) \equiv a_{jt}^{\omega(2)}(z)/a_{jt}^{\omega(1)}(z)$. Then (I.2) implies that $m_{j}^{\omega}(z)$ is Pareto distributed:

$$M_{jt}^{\omega}(m) = \Pr\left(m_{jt}^{\omega}(z) \leq m\right) = 1 - m^{-\theta}$$

Moreover, the distribution of the cost gap is independent of the source country and of $a_{jt}^{\omega(2)}(z)$.

I.5. With the previous definitions, markups $m_{jt}'^{\omega}(z)$ are given by

$$m_{jt}'^{\omega}(z) = \min\left\{m_{jt}^{\omega}(z), \bar{m}(\sigma^{\omega})\right\}$$

I.6. The exact price index for industry ω in country i is given by expression (13) in the text.

A.1.3 Derivation of equation (13)

Proof. In what follows I drop subscripts since the analysis applies to any time, country and industry. Starting with the definition of P, we have

$$\begin{aligned} P^{1-\sigma} &= \int_0^1 \left[a^{(1)}(z)\, m'(z)\right]^{1-\sigma} dz = \int_0^1 \left[a^{(2)}(z)\,\frac{m'(z)}{m(z)}\right]^{1-\sigma} dz \\ &= \mathbb{E}_t\left[\left(a^{(2)}\right)^{1-\sigma}\right] \mathbb{E}_t\left[\left(m'(z)/m(z)\right)^{1-\sigma}\right] \end{aligned}$$

where in the first line I used the fact that price equals cost times markup along with the definition of $m(z)$; in the third line I used the fact that m is independent of $a^{(2)}$ (see I.4).

Using the marginal distribution of $a^{(2)}$ we get

$$\mathbb{E}_t\left[\left(a^{(2)}\right)^{1-\sigma}\right] = \Phi^{-(1-\sigma)/\theta}\Gamma\left(\frac{1-\sigma+2\theta}{\theta}\right)$$

Finally, (I.4) and (I.5) imply

$$\begin{aligned} \mathbb{E}_t\left[\left(m'(z)/m(z)\right)^{1-\sigma}\right] &= \int_1^{\bar{m}} dM(m) + \int_{\bar{m}}^{\infty} (\bar{m}/m)^{1-\sigma}\, dM(m) \\ &= 1 + \bar{m}^{-\theta}\frac{(\sigma-1)}{[\theta-(\sigma-1)]}. \end{aligned}$$

Using the last two results in the expression above we get the result. ■

A.1.4 Cost Share in Revenues

Proof. I start by showing that the distribution of costs and markups faced by country j imply that production costs represent a fraction $\theta/(1+\theta)$ of its expenditure in industry ω. As the analysis is valid for any industry, country and time, I eliminate the subscripts $\{\omega, i, t\}$ in the proof of this result.. Let

$Cost\left(z\right) \equiv a^{(1)}\left(z\right) q\left(z\right)$ be the total cost of production of country j's demand of good $\left(z, \omega\right)$. Then

$$\begin{aligned} Cost\left(z\right) &= \frac{E\left(z\right)}{m'\left(z\right)} = \frac{E\ P^{\sigma-1}}{m'\left(z\right)} p\left(z\right)^{1-\sigma} = \frac{E\ P^{\sigma-1}}{m'\left(z\right)} \left(a^{(1)}\left(z\right) m'\left(z\right)\right)^{1-\sigma} \\ &= \frac{E\ P^{\sigma-1}}{m'\left(z\right)} \left[a^{(2)}\left(z\right)\left(\tfrac{m'(z)}{m(z)}\right)\right]^{1-\sigma}, \end{aligned}$$

and integrating over z we get

$$Cost = E\ P^{\sigma-1}\mathbb{E}_t\left[a^{(2)}\left(z\right)\right]\mathbb{E}_t\left[\frac{m'\left(z\right)^{-\sigma}}{m\left(z\right)^{1-\sigma}}\right] = \frac{\theta}{1+\theta}E.$$

Given that the distribution of costs and markups is independent of the source country, $\theta/\left(1+\theta\right)$ also represents the share of production costs in country j's expenditure on the goods from any source country. Put another way, for any country i, production costs represent a fraction $\theta/\left(1+\theta\right)$ of that country's sales to country j. Since this is true for any destination country j, we obtain the expression in the text. ■

A.1.5 Probability of Staying in the Market

Proof. What is the probability that a lowest cost producer in country j and industry ω in period t is still the state of the art in period $s > t$? Letting $X^{\omega}_{jt} \equiv 1/a^{\omega(1)}_{jt}$, I.2. implies $X^{\omega}_{jt} \sim Fréchet(\Phi^{\omega}_{jt}, \theta)$. The quality of the best ideas discovered between t and s in each country k is distributed $F\left(x\right) = e^{-T'_k x^{-\theta}}$ for $x \geq 1$ and where $T'^{\omega}_k = T^{\omega}_{ks} - T^{\omega}_{kt}$. Notice that this distribution is independent of the distribution of state of the art ideas at time t. Letting $X'_j\left(\omega\right)$ denote the random variable representing the inverse of lowest costs in country j at time s of the ideas generated between t and s, we know $X'^{\omega}_j \sim Fréchet(\Phi'^{\omega}_j, \theta)$, where

$$\Phi'^{\omega}_j = \sum_{k=1}^{N} T'^{\omega}_k \left(w_{ks}\tau^{\omega}_{ij}\right)^{-\theta} = \Phi^{\omega}_{js} - \Phi^{\omega}_{jt}$$

Since the ideas generated between period t and s are independent from the ideas up to time t, the distribution of X'^{ω}_j is also independent from the distribution of X^{ω}_{jt}. Setting $w_{it} = 1$ for all t, the probability that an idea from country i is still in the market in period s conditional on being in the market in period t is equal to $\Pr\left(X^{\omega}_{jt} \geq X'^{\omega}_j\right)$. Given the Fréchet distribution of inverse costs we obtain

$$\Pr\left(X^{\omega}_{jt} \geq X'^{\omega}_j\right) = \frac{\Phi^{\omega}_{jt}}{\Phi^{\omega}_{jt} + \Phi'^{\omega}_{jt}} = \frac{\Phi^{\omega}_{jt}}{\Phi^{\omega}_{js}}.$$

■

A.1.6 Definition of Equilibrium

Definition A.1 *A market equilibrium is a set of functions* r_{it}, w_{it}, P_{it}, R_{it}, E_{it}, $C_{it} : [0, \infty) \to \mathbb{R}_+$ *and* C^{ω}_{it}, P^{ω}_{it}, R^{ω}_{it}, E^{ω}_{it}, V^{ω}_{it}, $L^{q,\omega}_{it}$, $L^{R,\omega}_{it}$, $T^{\omega}_{it} : \Omega \times [0, \infty) \to \mathbb{R}_+$ *for each country* $i = 1, ..., N$ *such that conditions (2),(7)-(19) hold.*

A.2 Section II

A.2.1 Growth Rates and Research Intensity in the BGP

Lemma A.2 *In any balanced growth path of this economy the following condition holds:*
(i) wages, trade shares and interest rates are constant;
(ii) the interest rate is the same across countries:

$$r_{it} = r = [n + n\upsilon/\theta]\,\eta + \rho - n\upsilon/\theta$$

(iii) growth rates are given by[2]

$$\widetilde{L}_{it}^{q,\omega} = \widetilde{L}_{it}^{R,\omega} = \widetilde{R}_{it}^{\omega} = \widetilde{V}_{ijt}^{\omega} = n, \qquad \widetilde{T}_{it}^{\omega} = \upsilon n, \qquad \widetilde{P}_{it} = -n\upsilon/\theta, \qquad \widetilde{C}_{it} = n + n\upsilon/\theta$$

(iv) the research intensity $L_{it}^{R,\omega}/L_{it}^{\omega}$ is constant and it is the same for all industries and countries:

$$\kappa\left(n, \rho, \eta, \theta, \upsilon\right) \equiv \frac{L_{it}^{R,\omega}}{L_{it}^{\omega}} = \frac{\upsilon^2 n}{\upsilon^2 n + \theta\left[r - (1-\upsilon)\, n\right]}$$

Before getting into the formal proof of the Lemma, I start with an informal discussion of the results. A higher rate of population growth n raises the expected profits from R&D through a higher expected increase in the size of the market for successful ideas, leading to more innovation and growth as we can see in (iii). High values of υ and low values of θ are associated with better R&D possibilities since they represent weaker decreasing returns in R&D and a fatter upper tail of the distribution from which the efficiency of an idea is drawn, respectively. These better R&D possibilities are reflected in higher growth rates for the stock of ideas, and the consumption aggregator as we can see in (iii).

The first two terms in the expression for the interest rate in (ii) represent the real interest rate, while the last term is the change in the price level. Notice that the expression in brackets in the first term of (ii) is just the growth rate of the consumption aggregator, $\widetilde{C}_{it}$. The higher $\widetilde{C}_{it}$ is, the steeper is the expected increase in consumption over time, which leads individuals wanting to smooth their consumption to increase their borrowing at any given rate, pushing up the equilibrium real interest rate.

Lemma A.2 shows that country size, research productivity and openness have no effect on the BGP-growth rates, i.e. all the additional effects on the innovation process brought about by the additional margin of adjustment emphasized in this paper are reflected on the levels of manufacturing technology, a topic to which I turn next.

Proof of parts (i)-(iii). $L_{it}^{q,\omega}, L_{it}^{R,\omega}$ growing at constant rates together with L_{it} growing at the constant rate n, necessarily implies that there the share of labor allocated to each industry ω is constant in the

[2] For any variable X, $\widetilde{X}_t \equiv \dot{X}_t/X_t$ denotes its instantaneous growth rate.

BGP and so $\widetilde{L}_{it}^{q}(\omega) = \widetilde{L}_{it}^{R}(\omega) = n$. Differentiating (2) with respect to time yields

$$\widetilde{T}_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon} / T_{it}^{\omega}$$

Recalling that $\widetilde{T}_{it}^{\omega}$ is constant in a BGP, log-differentiating the last expression with respect to time yields $\upsilon \widetilde{L}_{it}^{R,\omega} = \widetilde{T}_{it}^{\omega}$ which in turn implies $\widetilde{T}_{it}^{\omega} = \upsilon n$.

Combining equations (11)-(15) and (18) yield

$$w_{it} L_{it}^{q} = \sum_{j=1}^{N} \lambda_{ijt} w_{jt} L_{jt}^{q} \tag{A.1}$$

with $\lambda_{ijt} = \int_{\Omega} \alpha_j^{\omega} \lambda_{ijt}^{\omega} d\omega$. Given the technology levels T_{kt}^{ω} and labor allocations $L_{kt}^{q,\omega}$ for all countries k and industries ω, the previous equation determines the equilibrium wages at time t, w_{kt}. It should be evident that if $L_{kt}^{q,\omega}$ and T_{kt}^{ω} grow at the same constant rates across countries, then a vector of wages w_k that solves (A.1) at time t should also solve it at any time $s > t$. Then if we set the w_i as the numeraire, the wages of all countries are constant in the BGP.

Once we know wages are constant, it is easy to see that $\widetilde{R}_{it} = \widetilde{E}_{it} = \widetilde{R}_{it}^{\omega} = n$. From (12) we have $\widetilde{\Phi}_{kt}^{\omega} = \widetilde{T}_{kt}^{\omega} = n\upsilon$ for all k, and (11) imply that trade shares are constant.

From (8) and (13) we get $\widetilde{P}_{it} = -n\upsilon/\theta$, which in turn implies that real wages in country i grow at $n\upsilon/\theta$. The relations in (9) imply $\widetilde{C}_{it} = n + n\upsilon/\theta$ and using (10) the interest rate is constant and is given by $r = n\eta + (\eta - 1)\frac{n\upsilon}{\theta} + \rho$. Using this in (16) we get $\widetilde{V}_{ijt}^{\omega} = n$. ■

Proof of part (iv). Using the results of Lemma A.2 regarding $\widetilde{T}_{it}^{\omega}$ and r_{it} in the BGP, the expression for the value of an idea (16) yields $V_{ijt}^{\omega} = E_{tjt}^{\omega} / \{(1+\theta)[r - (1-\upsilon)n]\}$. Then

$$\begin{aligned} \sum_{j=1}^{N} \lambda_{ijt}^{\omega} V_{ijt}^{\omega} &= \frac{\sum_{j=1}^{N} \lambda_{ijt}^{\omega} E_{jt}^{\omega}}{(1+\theta)[r-(1-\upsilon)n]} = \frac{R_{it}^{\omega}}{(1+\theta)[r-(1-\upsilon)n]} \\ &= \frac{w_{it} L_{it}^{q,\omega}}{\theta[r-(1-\upsilon)n]} \end{aligned}$$

where in the first line I used (15) and in the second line I used (14). Using the last expression in (17) and solving for T_{it}^{ω} yields

$$T_{it}^{\omega} = \frac{\iota_i^{\omega} \upsilon \left(L_{it}^{R,\omega} \right)^{\upsilon - 1} L_{it}^{q,\omega}}{\theta[r-(1-\upsilon)n]}.$$

Differentiating (2) with respect to time yields $\widetilde{T}_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon} / T_{it}^{\omega}$, and using the result of Lemma A.2 regarding $\widetilde{T}_{it}^{\omega}$ we get

$$T_{it}^{\omega} = \frac{\iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon}}{\upsilon n}.$$

The two previous expressions imply that $L_{it}^{q,\omega} / L_{it}^{R,\omega} = \theta[r-(1-\upsilon)n] / \upsilon^2 n$ for all ω and i. ■

A.2.2 Derivation of equation (20)

Given that in the BGP the interest rate is equalized across countries, Lemma A.2 and (16) imply $V^{\omega}_{ijt} = \frac{E^{\omega}_{jt}}{(1+\theta)[r-n(1-\upsilon)]} \equiv V^{\omega}_{jt}$ for any country i. Consequently, V^{ω}_{jt} represents the expected present value of profits generated by country j's stream of expenditure in industry ω.

Differentiating (2) with respect to time yields $\widehat{T}^{\omega}_{it} = \iota^{\omega}_{i}\left(L^{R,\omega}_{it}\right)^{\upsilon}/T^{\omega}_{it}$, and using the result of Lemma A.2 regarding $\widehat{T}^{\omega}_{it}$ we get $(T^{\omega}_{it}\upsilon n/\iota^{\omega}_{i})^{\frac{\upsilon-1}{\upsilon}} = \left(L^{R,\omega}_{it}\right)^{\upsilon-1}$, and using this back in the first order condition (17) (FOC of research firms) we get

$$T^{\omega}_{it} = \iota^{\omega}_{i}\upsilon^{\upsilon}\left(\upsilon n\right)^{\upsilon-1}\left[\frac{\sum_{j=1}^{N}\lambda^{\omega}_{ijt}V^{\omega}_{jt}}{w_i}\right]^{\upsilon}$$

The previous equation relates the level of the stock of ideas at time t, T^{ω}_{it}, with the expected present value of the profits generated by firms in country i and industry ω. Notice that in the BGP, expression (16) for V^{ω}_{ijt} implies $V^{\omega}_{jt} = E^{\omega}_{jt}\zeta$, where $\zeta = \{(1+\theta)\left[r-n\left(1-\upsilon\right)\right]\}^{-1}$. Then we can write

$$\begin{aligned}
\sum_{j=1}^{N}\lambda^{\omega}_{ijt}V^{\omega}_{jt} &= \sum_{j=1}^{N}\lambda^{\omega}_{ijt}\alpha^{\omega}_{j}E_{jt}\zeta \\
&= \left[\sum_{j=1}^{N}\lambda^{\omega}_{ijt}\frac{\alpha^{\omega}_{j}E_{jt}\zeta}{\sum_{k=1}^{N}\alpha^{\omega}_{k}E_{kt}\zeta}\right]V^{\omega}_{t} \\
&= \left[\sum_{j=1}^{N}\lambda^{\omega}_{ijt}\frac{\alpha^{\omega}_{j}\left(E_{jt}/E_{t}\right)}{\sum_{k=1}^{N}\alpha^{\omega}_{k}\left(E_{kt}/E_{t}\right)}\right]V^{\omega}_{t} \\
&= \left[\sum_{j=1}^{N}\lambda^{\omega}_{ijt}\frac{\alpha^{\omega}_{j}\beta^{E}_{j}}{\sum_{k=1}^{N}\alpha^{\omega}_{k}\beta^{E}_{k}}\right]V^{\omega}_{t} \\
&= \left[\sum_{j=1}^{N}\lambda^{\omega}_{ijt}\beta^{E,\omega}_{j}\right]V^{\omega}_{t} \\
&= \beta^{R,\omega}V^{\omega}_{t}
\end{aligned}$$

where in the second line I divided and multiplied by $V^{\omega}_{t} = \zeta\sum_{k=1}^{N}\alpha^{\omega}_{k}E_{kt}$; in the third line I divided numerator and denominator by world expenditure $E_t \equiv \sum_{k=1}^{N}E_{kt}$; in the forth line I used the definition $\beta^{E}_{j} \equiv E_{jt}/E_{t}$; in the fifth line I used the fact that $\beta^{E,\omega}_{j} \equiv E^{\omega}_{jt}/E^{\omega}_{t} = \alpha^{\omega}_{j}\beta^{E}_{j}/\left(\sum_{k=1}^{N}\alpha^{\omega}_{k}\beta^{E}_{k}\right)$; in the fifth line I used the definition of $\beta^{R,\omega}$ together with (15). Using the last result in the previous expression yields the desired result.

A.2.3 Derivation of equation (21)

Proof. Notice that $\widetilde{T}_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon} / T_{it}$ and Lemma A.2 imply that in the BGP we have

$$T_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon} / \upsilon n$$

Dividing and multiplying the RHS of the last expression by $L_{it}^{\upsilon} \left(L_{it}^{\omega} \right)^{\upsilon}$ and using the fact that the research intensity is constant and the same for every industry in the BGP we get

$$T_{it}^{\omega} = B_T' \iota_i^{\omega} \left[\delta_{it}^{\omega} L_{it} \right]^{\upsilon}$$

where $\delta_i^{\omega} \equiv L_{it}^{\omega} / L_{it}$ denotes the share of resources allocated to industry ω and $B_T' \equiv (\upsilon n)^{-1} \kappa^{\upsilon}$ is a constant. ■

A.2.4 Derivation of equation (22)

Proof. Notice that $\widetilde{T}_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon} / T_{it}$ and Lemma A.2 imply that in the BGP, $T_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega} \right)^{\upsilon} / \upsilon n$. Combining this with (21) we get

$$\left(L_{it}^{R,\omega} \right)^{\upsilon} = \upsilon n B_T \left[\beta_i^{R,\omega} V_t^{\omega} / w_i \right]^{\upsilon}$$

Recalling that $\alpha^{\omega} \equiv E_t^{\omega} / E_t$ denotes the share of world expenditure allocated to industry ω, the Cobb-Douglas upper tier utility function implies that we can write $\alpha^{\omega} = \sum_{k=1}^{N} \alpha_k^{\omega} \beta_{kt}^{E}$. Then from the definition of V_t^{ω} we get $V_t^{\omega} = \zeta \alpha^{\omega} E_t$, where ζ is the same constant defined above. With this last relationship in mind, we can take the ratio of the last equation for two industries ω and ω' to get

$$\frac{L_{it}^{R,\omega'}}{L_{it}^{R,\omega}} = \frac{\beta_i^{R,\omega'} \alpha^{\omega'}}{\beta_i^{R,\omega} \alpha^{\omega}}$$

This immediately implies that

$$\frac{L_{it}^{R,\omega}}{L_{it}^{R}} = \frac{\beta_i^{R,\omega} \alpha^{\omega}}{\int_{\Omega} \beta_i^{R,\omega} \alpha^{\omega} d\omega} = \frac{\beta_i^{R,\omega} \alpha^{\omega}}{\beta_i^{R}}$$

Recalling that the research intensity is constant in the BGP we get

$$\delta_t^{\omega} \equiv \frac{L_{it}^{\omega}}{L_{it}} = \frac{\beta_i^{R,\omega} \alpha^{\omega}}{\beta_i^{R}} = \frac{\beta_i^{R,\omega}}{\beta_i^{E,\omega}} \alpha_i^{\omega}$$

where the last equality is obtained using the balanced trade condition, $\beta_i^{E,\omega} = \alpha_i^{\omega} \beta_i^{E} / \sum_{k=1}^{N} \alpha_k^{\omega} \beta_k^{E}$ and $\alpha^{\omega} = \sum_{k=1}^{N} \alpha_k^{\omega} \beta_k^{E}$. ■

A.2.5 Proof of Proposition 1

The description of equations (23.1)-(23.4) can be found in the text. Here I concentrate on the description of the remaining equations and on the proof of the existence of a BGP in which all market shares are strictly positive.

Starting from $R_{it} = \int_{\Omega} R_{it}^{\omega} d\omega$, I obtain (23.5) by dividing both sides by world output R_t and using the facts $E_t = R_t$ and $R_{it}^{\omega} = \beta_i^{R,\omega} \alpha^{\omega} E_t$. Starting from the definition of world expenditure in industry ω, $E_t^{\omega} = \sum_{j=1}^{N} \alpha_j^{\omega} E_{jt}^{\omega}$, I obtain (23.6) by dividing both sides of this expression by total world expenditure E_t. Starting from $E_{it}^{\omega} = \alpha_i^{\omega} E_{it}$, I obtain (23.7) by dividing both sides by world expenditure in industry ω. Finally, I obtain equation (23.8) using the fact that labor income is proportional to total output.

Let us now turn to proof of the existence of a solution to the system (23). In order to obtain this existence result, it is convenient to reduce the system of equations (23) as follows. Using (23.7) and (23.4) in (23.1) and the result in (23.2) we get

$$\beta_i^{R,\omega} = \sum_{j=1}^{N} \frac{\iota_i^{\omega} \left(\tau_{ij}^{\omega}\right)^{-\theta} \left(L_{it}/\beta_i^R\right)^{\theta+\upsilon} \left(\beta_i^{R,\omega}\right)^{\upsilon}}{\sum_{k=1}^{N} \iota_k^{\omega} \left(\tau_{kj}^{\omega}\right)^{-\theta} \left(L_{kt}/\beta_k^R\right)^{\theta+\upsilon} \left(\beta_k^{R,\omega}\right)^{\upsilon}} \beta_j^{E,\omega}. \tag{A.2}$$

Given this reduction of the system, we need to prove that the system of equations given by (A.2), (23.3) and (23.5)-(23.8) has a solution. The goal is to show that a solution to the reduced system can be expressed as a fixed point of a continuous self-map defined over a compact and convex set (the simplex Δ^N). Once we do this we can apply Brouwer's Fixed Point Theorem to obtain the result.

Notice that for a given set of countries' total expenditure shares β_i^E, equations (23.3) pin down total revenue shares $\beta_i^R\left(\beta_1^E, ..., \beta_N^E\right)$ and equations (23.7) pin down countries's expenditure shares in each industry, $\beta_i^{E,\omega}\left(\beta_1^E, ..., \beta_N^E\right)$. This means that for any given set of β_i^E, equations (A.2) represent a system of N equations in the N unknowns (N is the number of countries) for each industry ω that can be used to solve for markets shares $\beta_i^{R,\omega}$ as functions of expenditure shares $\left(\beta_1^E, ..., \beta_N^E\right)$. As I state formally in the following Lemma, (A.2) has a unique interior solution.

Lemma A.3 *For $\upsilon \in (0,1)$, the system (A.2) has a unique solution with $\beta_i^{R,\omega} > 0$ for all $i = 1, ..., N$.*

Proof. The main idea behind the proof is to show that the system of equations (A.2) characterizes the solution of a maximization problem for which there exists a unique solution. Before getting into the details of the proof, it is convenient to simplify the notation. Given that the proof is valid for any industry, in what follows I eliminate industry references (whenever it does not create confusion) and I use $x_i \equiv \beta_i^{R,\omega}$ and $\delta_{ij} \equiv \iota_i^{\omega} \left(\tau_{ij}^{\omega}\right)^{-\theta} \left(L_{it}/\beta_i^R\right)^{\theta+\upsilon}$. With this notation, the system (A.2) can be written as

$$x_i = \sum_{j=1}^{N} \lambda_{ij}\left(x_{1,...,}x_N\right) \beta_j^{E,\omega} = \sum_{j=1}^{N} \frac{\delta_{ij} x_i^{\upsilon}}{\sum_{k=1}^{N} \delta_{kj} x_k^{\upsilon}} \beta_j^{E,\omega}. \tag{A.3}$$

Consider the following maximization problem

$$\max_{x_1,\ldots,x_N} \sum_{j=1}^{N} \beta_j^{E,\omega} \ln\left(\sum_{i=1}^{N} \delta_{ij} x_i^{\upsilon}\right) \text{ subject to } x_k \geq 0 \text{ and} \sum_{k=1}^{N} x_k = 1. \tag{P1}$$

A few remarks about this problem are in order. (i) The problem has at least one solution. The objective function is continuous and the feasible set is compact. (ii) There can be at most one solution. The objective function is strictly concave when $\upsilon \in (0,1)$ and the feasible set is convex. (iii) The solution must be interior. Notice that the objective function satisfies Inada conditions in each variable when $\upsilon \in (0,1)$.

In addition, given that the objective function is differentiable and strictly concave, $(x_1, ..., x_N)$ is a solution to P1 if and only if $(x_1, ..., x_N)$ satisfies the first order conditions (FOC) for an interior solution. This, together with points (i)-(iii) above, implies that there exists a unique solution to the system of equations determined by the FOC of this problem. Consequently, the Lemma is proved if we show that (A.3) corresponds to the FOC of P1, which I do next.

Letting μ be Lagrange multiplier associated with the constraint, the FOC of the previous problem can be written (after rearrangement) as

$$\sum_{j=1}^{N} \beta_j^{E,\omega} \frac{\delta_{ij} x_i^{\upsilon}}{\sum_{k=1}^{N} \delta_{kj} x_k^{\upsilon}} = \frac{\mu}{\upsilon} x_i \text{ for } i = 1, ..., N \tag{FOC}$$

Summing the FOCs side by side and recalling that $\sum_{k=1}^{N} x_k = 1$ we get

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \beta_j^{E,\omega} \frac{\delta_{ij} x_i^{\upsilon}}{\sum_{k=1}^{N} \delta_{kj} x_k^{\upsilon}} = \frac{\mu}{\upsilon} \Rightarrow \frac{\mu}{\upsilon} = 1.$$

Comparing the systems (A.3) and (FOC) with $\mu/\upsilon = 1$, we can see that they are identical, which is the desired result. ■

The last Lemma shows that for a given set of countries' total expenditure shares $\left(\beta_1^E, ..., \beta_N^E\right)$, there exists a unique set of market shares $\beta_i^{R,\omega}\left(\beta_1^E, .., \beta_N^E\right)$ that solves (A.2). In addition it is not hard to see that the function $\left(\beta_1^E, .., \beta_N^E\right) \to \beta_i^{R,\omega}$ defined in this way is continuous. Finally, once we have a set of market shares $\beta_i^{R,\omega}$, we can use equations (23.5) to obtain $\beta_i^R\left(\beta_1^E, .., \beta_N^E\right)$ and then the balanced trade condition (23.3) to obtain a new set of countries' expenditure shares $\beta_i'^E\left(\beta_1^E, .., \beta_N^E\right)$. In other words, we have defined in this way a continuous self-map $\left(\beta_1^E, .., \beta_N^E\right) \to \left(\beta_1'^E, .., \beta_N'^E\right)$ on the N-dimensional simplex. Then, Brouwer's Fixed Point Theorem implies that there exists a $\left(\beta_1^{*E}, .., \beta_N^{*E}\right)$ that is a fixed point of this self-map. Notice that by construction of this self-map, $\left\{\beta_i^{*E}, \beta_i^R\left(\beta_1^{*E}, .., \beta_N^{*E}\right), \beta_i^{R,\omega}\left(\beta_1^{*E}, .., \beta_N^{*E}\right), \beta_i^{E,\omega}\left(\beta_1^{*E}, ..., \beta_N^{*E}\right)\right\}$ is a solution to the reduced system given by equations (A.2), (23.3) and (23.5)-(23.8).

A.2.6 Proof of Lemma 1

Proof. The characterization under autarky is obtained by combining equations (21) and (22) together with the fact that in autarky $\beta_i^{R,\omega}/\beta_i^{E,\omega} = 1$ for all countries and industries.

Let us turn now to the characterization corresponding to a zero gravity world. Using equations (23.4) and (23.8) in (23.1) for the special case of zero gravity yields

$$\lambda_{ij}^{\omega} = \frac{\iota_i^{\omega}\left(\beta_i^{R,\omega}\right)^{\upsilon} w_i^{-(\theta+\upsilon)}}{\sum_k \iota_k^{\omega}\left(\beta_k^{R,\omega}\right)^{\upsilon} w_k^{-(\theta+\upsilon)}}.$$

Note that λ_{ij}^{ω} does not depend on the importer j, so the exports of a country represent the same share of any importer's expenditure, i.e., $\lambda_{ij}^{\omega} = \lambda_{ij'}^{\omega}$ for all i, j, j'. This together with equation (23.2) imply that $\beta_i^{R,\omega} = \lambda_{ii}^{\omega}$ for every country i. Consequently, the ratio of the market shares of two countries i, j is given by

$$\frac{\beta_i^{R,\omega}}{\beta_j^{R,\omega}} = \frac{\lambda_{ii}^{\omega}}{\lambda_{jj}^{\omega}} = \frac{\iota_i^{\omega}\left(\beta_i^{R,\omega}\right)^{\upsilon} w_i^{-(\theta+\upsilon)}}{\iota_j^{\omega}\left(\beta_j^{R,\omega}\right)^{\upsilon} w_j^{-(\theta+\upsilon)}}$$

and solving for $\beta_i^{R,\omega}/\beta_j^{R,\omega}$ yields

$$\frac{\beta_i^{R,\omega}}{\beta_j^{R,\omega}} = \left[\frac{\iota_i^{\omega}}{\iota_j^{\omega}}\right]^{\frac{1}{1-\upsilon}} \left[\frac{w_i}{w_j}\right]^{-\frac{(\theta+\upsilon)}{1-\upsilon}}$$

Finally, taking double ratios in equation (23.4) and using the last expression yields the result in the text

$$\frac{T_{it}^{\omega}/T_{it}^{\omega'}}{T_{jt}^{\omega}/T_{jt}^{\omega'}} = \left[\frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_j^{\omega}/\iota_j^{\omega'}}\right]\left[\frac{\beta_i^{R,\omega}/\beta_i^{R,\omega'}}{\beta_j^{R,\omega}/\beta_j^{R,\omega'}}\right]^{\upsilon} = \left[\frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_j^{\omega}/\iota_j^{\omega'}}\right]^{1+\frac{\upsilon}{1-\upsilon}}.$$

■

A.2.7 Proof of Lemma 2

Proof. Before turning to the proof of Lemma, it is convenient to first find an expression for the market shares in the case of frictionless trade, i.e. $\tau = 1$. In this case, equation (23.1) implies that $\lambda_{ij}^{\omega} = \lambda_{ij'}^{\omega}$ for all i, j, j'. Combining this with equation (23.2) we get $\lambda_{ij}^{\omega} = \lambda_{ij'}^{\omega} = \beta_i^{R,\omega}$ for all i. Using this, together with the expression resulting from using equation (23.4) in (23.1), we get

$$\begin{aligned}
\beta_i^{R,\omega} &= \frac{\iota_i^{\omega}\left(\beta_i^{R,\omega}\right)^{\upsilon} w_i^{-(\theta+\upsilon)}}{\sum_{k=1}^{2} \iota_k^{\omega}\left(\beta_k^{R,\omega}\right)^{\upsilon} w_k^{-(\theta+\upsilon)}} \\
&= \frac{\left(\iota_i^{\omega}\right)^{\frac{1}{1-\upsilon}} w_i^{-\frac{\theta+\upsilon}{1-\upsilon}}}{\left[\sum_{k=1}^{2} \iota_k^{\omega}\left(\beta_k^{R,\omega}\right)^{\upsilon} w_k^{-(\theta+\upsilon)}\right]^{\frac{1}{1-\upsilon}}}
\end{aligned}$$

The last condition implies $\beta_i^{R,\omega}/\beta_j^{R,\omega} = \left(\iota_i^\omega/\iota_j^\omega\right)^{\frac{1}{1-\upsilon}} (w_i/w_j)^{-\frac{\theta+\upsilon}{1-\upsilon}}$, and using this together with the fact that $\sum_k \beta_k^{R,\omega} = 1$ we get

$$\beta_i^{R,\omega} = \frac{(\iota_i^\omega)^{\frac{1}{1-\upsilon}} w_i^{-\frac{\theta+\upsilon}{1-\upsilon}}}{\sum_{k=1}^N (\iota_k^\omega)^{\frac{1}{1-\upsilon}} w_k^{-\frac{\theta+\upsilon}{1-\upsilon}}} \tag{A.4}$$

Let us now turn to the derivation of the results in the Lemma which I prove with the next three results.

Condition (28) implies that country 1 is a net importer in industry ω when $\tau = 1$— Letting NX_i^ω denote country i's net exports in industry ω, we have that $NX_1^\omega < 0$ if and only if $X_{12}^\omega/X_{21}^\omega < 1$, where X_{ij}^ω denote the total exports from country i to country j in industry ω. When $\tau = 1$, we have

$$\frac{X_{12}^\omega}{X_{21}^\omega} = \frac{\lambda_{12}^\omega \alpha_2^\omega E_2}{\lambda_{21}^\omega \alpha_1^\omega E_1} = \left(\frac{\iota_1^\omega}{\iota_2^\omega}\right)^{\frac{1}{1-\upsilon}} \frac{\alpha_2^\omega}{\alpha_1^\omega}$$

where the second equality is obtained using the fact that with frictionless trade $\lambda_{ij}^\omega = \beta_i^{R,\omega}$, $E_1 = E_2$ from the symmetry assumption and equation (A.4). Consequently,

$$NX_1^\omega < 0 \Longleftrightarrow \left(\frac{\iota_1^\omega}{\iota_2^\omega}\right)^{\frac{1}{1-\upsilon}} < \frac{\alpha_1^\omega}{\alpha_2^\omega} \tag{A.5}$$

Due to the fact that the expenditure shares of each country across industries must add up to one, condition (27) implies $\alpha_1^\omega/\alpha_2^\omega > 1$ and $\alpha_1^{\omega'}/\alpha_2^{\omega'} < 1$. In a similar way, the assumption about symmetric countries and condition (27) imply $(\iota_1^\omega/\iota_2^\omega) < 1$ and $\left(\iota_1^{\omega'}/\iota_2^{\omega'}\right) > 1$. The last two conditions together with condition (A.5) yield $NX_1^\omega < 0$, i.e., country 1 is a net importer in industry ω when there are no frictions to trade.

Condition (27) implies that country 1 is a net exporter in industry ω for a sufficiently high value of τ— As before, $NX_1^\omega > 0$ if and only if $X_{12}^\omega/X_{21}^\omega > 1$. For the general case of $\tau > 1$, we have

$$\frac{X_{12}^\omega}{X_{21}^\omega} = \frac{\lambda_{12}^\omega \alpha_2^\omega}{\lambda_{21}^\omega \alpha_1^\omega} = \frac{\alpha_2^\omega}{\alpha_1^\omega} \frac{\iota_1^\omega}{\iota_2^\omega} \left(\frac{\beta_1^{R,\omega}}{\beta_2^{R,\omega}}\right)^\upsilon \frac{\left[\iota_1^\omega \left(\beta_1^{R,\omega}\right)^\upsilon + \iota_2^\omega \left(\beta_2^{R,\omega}\right)^\upsilon \tau^{-\theta}\right]}{\left[\iota_1^\omega \left(\beta_1^{R,\omega}\right)^\upsilon \tau^{-\theta} + \iota_2^\omega \left(\beta_2^{R,\omega}\right)^\upsilon\right]}, \tag{A.6}$$

where the second equality is obtained using the definitions of λ_{ij}^ω given in equation (23.1) together with equation (23.4). In autarky $\beta_i^{R,\omega} = \beta_i^{E,\omega}$, which in turn implies that $\beta_1^{R,\omega}/\beta_2^{R,\omega} = \alpha_1^\omega/\alpha_2^\omega$. Taking the limit as $\tau \to \infty$ in the last expression we get

$$\lim_{\tau\to\infty} \frac{X_{12}^\omega}{X_{21}^\omega} > 1 \Longleftrightarrow \left(\frac{\alpha_1^\omega}{\alpha_2^\omega}\right)^{\upsilon - \frac{1}{2}} > \frac{\iota_2^\omega}{\iota_1^\omega}. \tag{A.7}$$

Notice that the RHS of the last equivalence is just condition (27). This implies that if condition (27) holds, then there is a value $\bar{\tau}$ such that for all $\tau > \bar{\tau}$, country 1 is a net exporter in industry ω.

There is only one level of transport costs $\tau \in (0,1)$ such that trade is balanced in each industry— Notice that if trade is balanced in industry ω, then $\beta_i^{R,\omega} = \beta_i^{E,\omega}$, which in turn implies that $\beta_1^{R,\omega}/\beta_2^{R,\omega} = \alpha_1^{\omega}/\alpha_2^{\omega}$. Using this in (A.6), and letting $z \equiv \tau^{-\theta}$, trade is balanced for some $z \in (0,1)$ if, and only if,

$$f(z) \equiv c\frac{[a+bz]}{[az+b]} = 1,$$

where $c \equiv \left(\frac{\alpha_2^{\omega}}{\alpha_1^{\omega}}\right)^{1-\upsilon} \frac{\iota_1^{\omega}}{\iota_2^{\omega}}$, $a \equiv \iota_1^{\omega} (\alpha_1^{\omega}/\alpha_2^{\omega})^{\upsilon}$, and $b \equiv \iota_2^{\omega}$. Let us now analyze the behavior of the function $f(z)$. First, condition (A.7) implies that $f(0) > 1$. Second, $f(1) = c < 1$ since $\alpha_2^{\omega}/\alpha_1^{\omega} < 1$ and $\iota_1^{\omega}/\iota_2^{\omega} < 1$. Finally, notice that

$$f'(z) = c\frac{\left[b^2 - a^2\right]}{[az+b]^2} < 0$$

for all $z \in (0,1)$, since condition (28) implies $b \equiv \iota_2^{\omega} < \iota_1^{\omega} (\alpha_1^{\omega}/\alpha_2^{\omega})^{\upsilon} \equiv a$. Consequently, there is at most one $z \in (0,1)$ such that trade is balanced in industry ω.

The results proved above imply that as countries move from autarky to trade, they display a unique reversal in their export profile. ■

A.2.8 The BGP in Changes

The next corollary summarizes the extension of the DEK's method to the present model.

Corollary A.4 *Let $\widehat{X} \equiv X'/X$ denote the relative change in variable X from X to X'. Given the constant parameters υ, θ and information about the endogenous trade shares λ_{ij}^{ω}, countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, and world-wide industries' expenditure shares α^{ω} in the initial BGP; the change in those same endogenous variables between the initial and the new BGP associated with exogenous changes in research productivities $\widehat{\iota}_i^{\omega}$, labor endowments $\widehat{L}_{it}$, preference parameters $\widehat{\alpha}_i^{\omega}$ and trade costs $\widehat{\tau}_{ij}^{\omega}$ can be computed as*

$$\begin{aligned}
&\widehat{\lambda}_{ij}^{\omega} = \frac{\widehat{\iota}_i^{\omega}\left(\widehat{L}_{it}/\widehat{\beta}_i^R\right)^{(\upsilon+\theta)}\left(\widehat{\beta}_i^{R,\omega}\right)^{\upsilon}\left(\widehat{\tau}_{ij}^{\omega}\right)^{-\theta}}{\sum_{k=1}^N \widehat{\iota}_k^{\omega}\left(\widehat{L}_{kt}/\widehat{\beta}_k^R\right)^{(\upsilon+\theta)}\left(\widehat{\beta}_k^{R,\omega}\right)^{\upsilon}\left(\widehat{\tau}_{kj}^{\omega}\right)^{-\theta}\lambda_{kj}^{\omega}} \quad \text{(A.8.1)} \qquad && \widehat{\beta}_i^R = \int_{\Omega} \widehat{\alpha}^{\omega}\widehat{\beta}_i^{R,\omega}\frac{\alpha^{\omega}\beta_i^{R,\omega}}{\beta_i^R}d\omega \quad \text{(A.8.4)} \\
&\widehat{\beta}_i^{R,\omega} = \sum_{j=1}^N \widehat{\lambda}_{ij}^{\omega}\widehat{\beta}_j^{E,\omega}\frac{\lambda_{ij}^{\omega}\beta_j^{E,\omega}}{\beta_i^{R,\omega}} \quad \text{(A.8.2)} && \widehat{\alpha}^{\omega} = \sum_{j=1}^N \widehat{\alpha}_j^{\omega}\widehat{\beta}_j^E\beta_j^{E,\omega} \quad \text{(A.8.5)} \\
&\widehat{\beta}_i^E = \widehat{\beta}_i^R \quad \text{(A.8.3)} && \widehat{\beta}_i^{E,\omega} = \frac{\widehat{\alpha}_i^{\omega}\widehat{\beta}_i^E}{\widehat{\alpha}^{\omega}} \quad \text{(A.8.6)}
\end{aligned} \qquad \text{(A.8)}$$

for all i, j and ω.

The previous system is obtained directly from the system (23) with a reduction in the total number of equations that is obtained using equations (23.4) and (23.8) in equation (23.1).

A few comments are in order. First, notice that out of the exogenous components in the previous system, the shocks to the parameters $\left\{\widehat{\iota}_i^{\omega}, \widehat{L}_{it}, \widehat{\alpha}_i^{\omega}, \widehat{\tau}_{ij}^{\omega}\right\}$ are provided by the evaluator and the information regarding the initial BGP $\{\lambda_{ij}^{\omega}, \beta_i^{R,\omega}, \beta_i^R, \beta_i^{E,\omega}, \beta_i^E, \alpha^{\omega}\}$ is readily obtainable from the data. Consequently, the only exogenous parameters that need to be calibrated/estimated are the decreasing returns parameter υ and the shape parameter θ. All the relevant information regarding the initial distribution of research productivities ι_i^ω, labor endowments L_{it}, preference parameters α_i^ω and trade costs τ_{ij}^ω is summarized in the levels of trade shares λ_{ij}^ω; countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, and world-wide industries' expenditure shares α^ω in the initial BGP.

Second, the previous system shows the differences between the counterfactual predictions of the model with directed research and the benchmark model with no innovation, and how the decreasing returns parameter υ controls those differences. To see this more clearly, consider a change in parameters that are exogenous in both models, i.e., changes in labor endowments $\widehat{L}_{it}$, preference parameters $\widehat{\alpha}_i^\omega$ and trade costs $\widehat{\tau}_{ij}^\omega$. To analyze the effect of those changes in the present model we only need to modify equation (A.8.1) setting $\widehat{\iota}_i^\omega = 1$. The only difference between the system of equations in changes corresponding to a model with no innovation and (A.8) is equation (A.8.1), that captures the endogenous change in manufacturing technology. Moreover, setting $\upsilon = 0$ in (A.8.1) yields the exact same system in changes that is obtained from applying DEK approach to the benchmark model with no innovation.

A.2.9 Proof of Proposition 2

Proof of part (i). To simplify the exposition I consider the case of two symmetric countries that have the same preferences and two industries. However, nothing in the proof depends on countries having the same preferences or the number of industries. The only requirement is that the countries are mirror images of each other.

Countries differ in their research productivities across two industries $\omega = 1, 2$. The mirror symmetry assumption for the two countries implies that research productivities satisfy $\iota_i^\omega = \iota_j^{\omega'}$ and for $i \neq j$. Let us now consider the BGP of this world economy. The system of equations (23) to obtain the BGP of the economy reduces to

$$\lambda_{ij}^\omega = \frac{\iota_i^\omega \left(\beta_i^{R,\omega}\right)^\upsilon \left(\tau_{ij}^\omega\right)^{-\theta}}{\sum_{k=1}^2 \iota_k^\omega \left(\beta_k^{R,\omega}\right)^\upsilon \left(\tau_{kj}^\omega\right)^{-\theta}}; \quad \beta_i^{R,\omega} = \sum_{j=1}^N \lambda_{ij}^\omega \frac{1}{2} \tag{A.9}$$

where $\tau_{kj}^\omega = \tau$ for $k \neq j$, and $\beta_j^{E,\omega} = 1/2$ due to symmetry and equal preferences. Given that $\beta_i^{R,\omega} = 1 - \beta_j^{R,\omega}$, with the previous set of equations we can solve for $\beta_i^{R,\omega}$. The symmetry assumption together with $\tau_{kj}^\omega = \tau$ for $k \neq j$ imply that market shares in the other industry satisfy $\beta_i^{R,\omega} = \beta_j^{R,\omega'}$. In addition, symmetry implies that this solution satisfies the balanced trade condition. From the solution to this system, we can obtain the manufacturing technology levels

$$T_i^\omega = B_T' \iota_i^\omega \left[\beta_i^{R,\omega} L\right]^\upsilon \tag{A.10}$$

where L denotes the labor endowment in both countries.

Now consider the following maximization problem (P2),

$$U(\tau) = \max \frac{1}{2} \frac{1}{\theta} \sum\nolimits_{i=1}^2 \sum\nolimits_{\omega=1}^2 \ln \Phi_i^\omega$$

subject to

$$\begin{aligned}
&\Phi_i^\omega = T_i^\omega + T_j^\omega \tau^{-\theta} && \lambda_{ij}^\omega = \frac{T_i^\omega \tau^{-\theta}}{\Phi_j^\omega} \\
&T_i^\omega = B_T' \iota_i^\omega \left(L_i^{R,\omega}\right)^\upsilon && \beta_i^{R,\omega} = \sum_{j=1}^{2} \lambda_{ij}^\omega \tfrac{1}{2} \\
&\textstyle\sum_{\omega=1}^{2} L_i^{R,\omega} = L &&
\end{aligned} \tag{P2}$$

for all ω, i. The objective function in this problem is proportional to the geometric average of the inverse of the price levels in each country. The proof of the Lemma is based on the following claim.

Claim 1 *The solution to the equations in (A.9) and (A.10) are a solution to problem P2.*

Proof. The first order condition with respect to $L_i^{R,\omega}$ yields

$$L_i^{R,\omega} = \frac{\upsilon}{\mu_i \theta} \left[\frac{1}{2}\lambda_{ii}^\omega + \frac{1}{2}\lambda_{ij}^\omega\right] = \frac{\upsilon}{\mu_i \theta} \beta_i^{R,\omega}$$

where μ_i is the Lagrange multiplier associated with the labor feasibility constraint, and the second equality is obtained using the definition of β_i^ω in the constraints. Using this back in the definition of T_i^ω and in the expression for λ_{ij}^ω, and recalling that symmetry implies $\mu_i = \mu_j$, we arrive to system (A.9) above. All the symmetry assumptions made imply that for any variable X, the solution to the previous problem satisfies $X_i^\omega = X_j^{\omega'}$. Consequently, from the solution of the system we can obtain the rest of the variables corresponding to the solution of the previous problem. In particular, the technology levels are given by

$$T_i^\omega = B_T' \iota_i^\omega \left[\frac{\beta_i^{R,\omega}}{\beta_i^{R,\omega} + \beta_i^{R,\omega'}} L\right]^\upsilon = B_T' \iota_i^\omega \left[\beta_i^{R,\omega} L\right]^\upsilon$$

since $\beta_i^{R,\omega} + \beta_i^{R,\omega'} = 1$. ■

Armed with the last claim, we are ready to prove the Lemma. Consider a change in real income associated with a change in trade costs $\widehat{\tau}$. We are interested in comparing the predicted changes in real income between the model with innovation ($\upsilon > 0$) and the model with no innovation ($\upsilon = 0$), as predicted by solving the system in changes (A.8) specialized to the symmetric case under consideration, i.e., we are interested in the predicted changes in real income conditional on observed trade shares and market shares in the original equilibrium. These conditional changes are in line with the analysis in Arkolakis, Costinot and Rodriguez-Clare (2012). However, given that the model with and without innovation share the same cross-section structure, we can always assume that the set of exogenous parameters and manufacturing technologies generating the observed initial equilibrium is the same in both models. In this way, this comparison is also compatible with the comparative static exercises in Melitz and Redding (2014). When the two models are set up in this way, real income is also the same across models in the original equilibrium.

Claim 1 implies that the changes in real income associated with the change in trade costs in the model with directed research, $\widehat{W}_{\upsilon>0}$, corresponds to the change in the objective function in problem P2, i.e.,

$\widehat{W}_{\upsilon>0} = U(\tau')/U(\tau)$. In addition, the change in real income in the model with no innovation, $\widehat{W}_{\upsilon=0}$, corresponds to the change in the objective function in problem P2 when technology levels are kept at their initial levels. A straight forward revealed preference argument implies that $\widehat{W}_{\upsilon>0}(\widehat{\tau}) > \widehat{W}_{\upsilon=0}(\widehat{\tau})$ for all $\widehat{\tau} \neq 1$.

Proof of part (ii). Although the logic used in the proof of point i could also be applied to this case, here I present a simpler proof. Consider the effects of raising trade costs to their autarky levels, $\tau_{ij}^{\omega} \to \infty$ for $i \neq j$. For this particular shock, evaluating (30) is straight forward. In autarky, the home share of expenditure must be equal to one in every industry, while the share of each industry in total output must be equal to the share of consumers' total expenditure allocated to the industry i.e., $\lambda_{ii}^{\omega} = 1$ and $\delta_i^{\omega} = \alpha_i^{\omega}$.[3] Consequently, for any $\upsilon \in [0,1)$, the change in real income associated with moving to autarky, can be computed as

$$\frac{W_{it}}{W_{it}^a} = \exp\left\{\int_{\Omega} \log\left(\frac{\delta_i^{\omega}}{\alpha_i^{\omega}}\right)^{\upsilon\alpha_i^{\omega}/\theta} d\omega\right\} \exp\left\{\int_{\Omega} \log\left(\lambda_{ii}^{\omega}\right)^{-\alpha_i^{\omega}/\theta} d\omega\right\} \tag{A.11}$$

where W_{it}^a denotes the real income per capita in autarky. Noticing that Jensen's inequality implies

$$\int_{\Omega} \alpha_i^{\omega} \log\left(\frac{\delta_i^{\omega}}{\alpha_i^{\omega}}\right) d\omega < \log\left(\int_{\Omega} \alpha_i^{\omega} \frac{\delta_i^{\omega}}{\alpha^{\omega}} d\omega\right) = \log\left(\int_{\Omega} \delta_i^{\omega} d\omega\right) = 0$$

we can write (A.11) as follows

$$W_{it}/W_{it}^a = A_i^{\frac{\upsilon}{\theta}} \exp\left\{\int_{\Omega} \log\left(\lambda_{ii}^{\omega}\right)^{-\alpha_i^{\omega}/\theta} d\omega\right\}$$

where $A_i = \exp\left\{\int_{\Omega} \alpha^{\omega} \log\left(\frac{\delta_i^{\omega}}{\alpha_i^{\omega}}\right) d\omega\right\} < 1$. In other words, the benchmark model with no innovation overestimates the reductions in real income per capita from moving to autarky relative to the model with directed research.

A.2.10 Home Market Effect

In this section, I follow Krugman (1980) and define the home market effect as the situation in which the country with the relatively larger domestic market in an industry becomes the net exporter in that industry.

To analyze the home market effect, it is convenient to consider a world with only two countries and two industries, $\Omega = 2$. Countries are mirror images of each other and they only differ in their preferences. In particular, (i) countries have the same research productivity across industries, eliminating any specialization due to comparative advantage; (ii) countries have the same size as captured by population size, eliminating weak scale effects on technology; (iii) $\tau_{ij}^{\omega} = \tau_{ji}^{\omega} = \tau$. Their preferences satisfy $\alpha_i^{\omega} = \alpha_j^{\omega'}$, and of course $\alpha_i^{\omega'} = (1 - \alpha_i^{\omega})$.

[3]Recall that $\delta_i^{\omega} = \frac{\beta_i^{R,\omega}}{\beta_i^{E.\omega}} \alpha_i^{\omega}$, which can differ from α_i^{ω} only if $\beta_i^{R,\omega}/\beta_i^{E,\omega} \neq 1$, i.e., only if trade is not balanced in the industry.

In what follows I show how the difference in preferences stated above affects the trade patterns for different values of the decreasing returns parameter υ. The previous conditions guarantee that in equilibrium both countries have the same wage -which I normalize to one- which implies that both countries also have the same total expenditure. Moreover, we only need to focus on one industry since the other industry will just be mirror image of it.

Under these conditions we have

$$\begin{aligned} \beta_i^{R,\omega} &= \lambda_{ii}^{\omega}\beta_i^{E,\omega} + \lambda_{ij}^{\omega}\beta_j^{E,\omega} \\ &= \frac{\left(\beta_i^{R,\omega}\right)^{\upsilon}}{\left(\beta_i^{R,\omega}\right)^{\upsilon} + \left(\beta_j^{R,\omega}\right)^{\upsilon}\tau^{-\theta}} \frac{\alpha_i^{\omega}}{\alpha_i^{\omega}+\alpha_j^{\omega}} + \frac{\left(\beta_i^{R,\omega}\right)^{\upsilon}\tau^{-\theta}}{\left(\beta_i^{R,\omega}\right)^{\upsilon}\tau^{-\theta} + \left(\beta_j^{R,\omega}\right)^{\upsilon}} \frac{\alpha_j^{\omega}}{\alpha_i^{\omega}+\alpha_j^{\omega}} \end{aligned}$$

for $i = 1, 2$ and $j \neq i$, and where in the second line I used the definition of $\beta_1^{E,\omega}$ and the fact that $E_1 = E_2$.

Dividing each side of the last equation by $\beta_i^{R,\omega}$, subtracting the equation corresponding to $i = 2$ from the one corresponding to $i = 1$, and solving for $\alpha_2^{\omega}/\alpha_1^{\omega}$ we obtain

$$\frac{\alpha_2^{\omega}}{\alpha_1^{\omega}} = \frac{\left[\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{\upsilon}\tau^{-\theta} + 1\right]}{\left[\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{\upsilon} + \tau^{-\theta}\right]} \frac{\left[1 - \left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{1-\upsilon}\tau^{-\theta}\right]}{\left[\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{1-\upsilon} - \tau^{-\theta}\right]} \tag{A.12}$$

Defining $\beta \equiv \beta_1^{R,\omega}/\beta_2^{R,\omega}$ and $\alpha \equiv \alpha_2^{\omega}/\alpha_1^{\omega}$, the last equation defines β as an implicit function of α, $\beta(\alpha)$. Noticing that in the range $\left(\tau^{-\frac{\theta}{1-\upsilon}}, \tau^{\frac{\theta}{1-\upsilon}}\right)$, the right hand side of (A.12) is strictly decreasing in β and varies from infinity to zero, the function $\beta(\alpha)$ satisfies

(A.12.i) $d\beta/d\alpha < 0$ for any value of $\upsilon \in [0, 1]$ and $\tau > 1$.

(A.12.ii) $\beta = 1$ for $\alpha = 1$ for any value of $\upsilon \in [0, 1]$ and $\tau > 1$

(A.12.iii) $\beta(\alpha) \in \left(\tau^{-\frac{\theta}{1-\upsilon}}, \tau^{\frac{\theta}{1-\upsilon}}\right)$ for any $\alpha \in [0, \infty)$.

Now let us consider the trade balance (net exports) of country 2 in industry ω. We have

$$TB_2^{\omega} = \alpha_1^{\omega}E_1\lambda_{21}^{\omega} - \alpha_2^{\omega}E_2\lambda_{12}^{\omega}$$

where the first term is country 2's exports to country 1 and the second term is country 2's imports from country 1. Using the definitions of λ_{ij}^{ω}, (A.12) and the fact that in equilibrium we must have $E_1 = E_2$, we can write the above equation

$$TB_2^{\omega} = \frac{\alpha_1^{\omega}E_1\tau^{-\theta}}{\left[\beta^{\upsilon} + \tau^{-\theta}\right]} \left[1 - \beta^{\upsilon}\frac{\left[1 - \beta^{1-\upsilon}\tau^{-\theta}\right]}{\left[\beta^{1-\upsilon} - \tau^{-\theta}\right]}\right] \tag{A.13}$$

To analyze the effect of the domestic market size on trade patterns it is instructive to consider first the extreme cases of $\upsilon = 0$ and $\upsilon = 1$. When $\upsilon = 0$ there are no R&D possibilities in the model and the model becomes essentially a two industry version of Eaton and Kortum (2002) (specifically, a two industry version of BEJK). In this case (A.13) becomes

$$TB_2^\omega = \tfrac{\alpha_1^\omega E_1 \tau^{-\theta}}{\left[1+\tau^{-\theta}\right]}\left[1 - \tfrac{\left[1-\beta\tau^{-\theta}\right]}{\left[\beta-\tau^{-\theta}\right]}\right]$$

Now consider the case in which country 2 has a larger domestic market for industry ω, i.e., $\alpha > 1$. Conditions (A.12.i) and (A.12.ii) above imply that $\beta < 1$ if $\alpha > 1$, and using this in the last expression we get $TB_2^\omega < 0$. In other words, when $\upsilon = 0$, the country with the larger home market in a given industry is a net importer in that industry.

When $\upsilon = 1$ there are constant returns to R&D. In this case (A.13) becomes

$$TB_2^\omega = \tfrac{\alpha_1^\omega E_1 \tau^{-\theta}}{\left[\beta+\tau^{-\theta}\right]}\left[1-\beta\right]$$

As before, if country 2 has a larger home country for industry ω, then $\alpha > 1$ and $\beta < 1$, which in turn imply $TB_2^\omega > 0$. As in Krugman (1980), the country with the larger home market in a given industry is a net exporter in that industry.

Let us now turn to the intermediate cases when $\upsilon \in (0,1)$. As before, I consider differences in domestic market size in which country 2 has a relatively larger domestic market in industry ω –which correspond to values of α in the range $[1,\infty)$– and I focus on how this difference affects country 2's net exports in that industry. Equations (A.12) and (A.13) define the balance of trade as a function of α, $TB_2^\omega(\alpha)$, and according to our definition, a home market effect is present if $TB_2^\omega(\alpha) > 0$ for $\alpha \in [1,\infty)$. Consequently, to analyze the home market effect, we need to study the sign of $TB_2^\omega(\alpha)$ for values of α in the range $[1,\infty)$.

It is convenient to start with the analysis of the effects of small deviations from the benchmark case of no differences in home market size, $\alpha = 1$. In this case, (A.12) and (A.13) imply trade is balanced at the industry level, $TB_2^\omega(1) = 0$. A home market effect is present for small differences in market size if $dTB_2^\omega/d\alpha|_{\alpha=1} > 0$, i.e., a small relative increase in the size of country 2's domestic market in industry ω induces a trade surplus in that industry. Recalling that

$$\frac{dTB_2^\omega}{d\alpha} = \frac{\partial TB_2^\omega}{\partial\beta}\frac{\partial\beta}{\partial\alpha},$$

(A.12.i) and (A.12.ii) imply that $dTB_2^\omega/d\alpha|_{\alpha=1} > 0$ if and only if, $\partial TB_2^\omega/\partial\beta|_{\beta=1} < 0$. Deriving (A.13) with respect to β and evaluating at $\beta = 1$ we get that $TB_2^\omega/\partial\beta|_{\beta=1} < 0$ if, and only if, $\upsilon > \left(1+\tau^{-\theta}\right)/2$. In other words, if the decreasing returns to R&D are sufficiently weak (υ sufficiently high), then there is a home market effect for small differences in the relative size of the home market.

However, from (A.12) we can see that as the relative size of the domestic market in country 2 approaches infinity, $\alpha \to \infty$, the relative market share of country 1 approaches its lower bound, $\beta \to \tau^{-\frac{\theta}{1-\upsilon}}$. This implies that the denominator of the second term of the expression in brackets in (A.13) approaches zero, which in turn implies that $TB_2^\omega < 0$ for sufficiently large α. This means that even for those values of υ at which there is a home market effect for small differences in domestic markets' sizes, a sufficiently (relative) large domestic market eventually translates into a trade deficit in the corresponding industry.

The intuition of this result is simple. When a country has a relatively larger domestic market in industry ω and $\tau > 1$, relatively more domestic resources are allocated to that industry for any value of υ. When there are no R&D possibilities ($\upsilon = 0$) those additional resources allocated to production do not compensate the larger domestic demand in the industry, and as a result there is a trade deficit in the industry.

When $\upsilon > 0$, the reallocation of resources also involves the redirection of R&D efforts towards industry ω, which endogenously increases the level of technology in that industry giving the country a comparative advantage in production that industry. Notice that the greater domestic demand and the endogenous increase in technology generated by a large domestic market have opposite effects on the trade balance, and consequently, the net effect depends on the relative strength of these two effects.

When $\upsilon = 1$ there are constant returns to R&D and the effect of a larger domestic market on technology is strongest. In this case, the greater technology effect always dominates the greater demand effect, generating a home market effect for any difference in relative domestic market size.

When $\upsilon > \left(1 + \tau^{-\theta}\right)/2$, the effect on technology is strong enough to generate a home market effect for small differences in relative domestic market size. However, as the differences in domestic market size increase and more resources are allocated to industry ω in country 2, the decreasing returns in R&D kick in and the endogenous changes in technology cannot compensate the greater domestic demand.

Finally, when $\upsilon \leq \left(1 + \tau^{-\theta}\right)/2$ the decreasing returns to R&D are so strong that there is no home market effect for any difference in relative market size. Notice that this means that if $\upsilon \leq 1/2$, then there are no home market effects regardless of the level of trade costs.

With the previous analysis we have proved the following Lemma.

Lemma A.5 *Let $\alpha \equiv \alpha_2^\omega/\alpha_1^\omega$ be the relative market size and let $TB_2^\omega(\alpha)$ the net exports of country 2 in industry ω. In the economy described above the following holds:*
(i) If $\upsilon = 1$, $TB_2^\omega(\alpha) > 0$ for all $\alpha \in (0,1)$.
(ii) If $\left(1 + \tau^{-\theta}\right)/2 < \upsilon < 1$, there is a $\alpha_\upsilon^* \in (0,1)$ such that (a) $TB_2^\omega(\alpha) > 0$ for $\alpha \in (\alpha_\upsilon^*, 1)$; (b) $TB_2^\omega(\alpha_\upsilon^*) = 0$; and (c) $TB_2^\omega(\alpha) < 0$ if $\alpha \in (0, \alpha_\upsilon^*)$.[4]
(iii) If $\upsilon \leq \left(1 + \tau^{-\theta}\right)/2$, $TB_2^\omega(\alpha) < 0$ for all $\alpha \in (0,1)$.

[4] The subscript in α_υ^* emphasizes the dependence of the cutoff value on the parameter υ.

A.3 Section III

A.3.1 Estimation of Comparative Advantage in Production

I will follow Costinot, Donaldson and Komunjer (2012) to estimate comparative advantage across countries. Equation (23.1) can be expressed as follows:

$$x_{ijt}^{\omega} = \frac{T_{it}^{\omega} w_{it}^{-\theta} \left(\tau_{ij}^{\omega}\right)^{-\theta}}{\Phi_{jt}^{\omega}} E_{jt}^{\omega},$$

where the only new variable x_{ijt}^{ω} represents country i's total exports of goods in industry ω to country j in period t. Taking logs in the previous expression yields

$$\log x_{ijt}^{\omega} = \log T_{it}^{\omega} w_{it}^{-\theta} + \log \frac{E_{jt}^{\omega}}{\Phi_{jt}^{\omega}} - \theta \log \tau_{ij}^{\omega}.$$

Using $\ln \tau_{ij}^{\omega} = \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right] + \left(\ln \tau_{ij}^{\omega} - \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]\right)$ in the last equation, the resulting expression can be estimated as

$$\ln x_{ij}^{\omega} = \xi_i^{\omega} + \psi_j^{\omega} + \xi_{ij} + \varepsilon_{ij}^{\omega},$$

where ξ_i^{ω}, ψ_j^{ω}, ξ_{ij} are exporter-industry, importer-industry and importer-exporter fixed effects, and $\varepsilon_{ij}^{\omega}$ is an error term:

$$\xi_i^{\omega} = \ln T_{it}^{\omega} w_{it}^{-\theta}; \qquad \psi_j^{\omega} = \ln E_{jt}^{\omega}/\Phi_{jt}^{\omega}$$
$$\xi_{ij} = \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]; \qquad \varepsilon_{ij}^{\omega} = \ln \tau_{ij}^{\omega} - \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]$$

Given the structure of the fixed effects, the regression can only identify $\left(\xi_i^{\omega'} - \xi_i^{\omega}\right) - \left(\xi_j^{\omega'} - \xi_j^{\omega}\right)$, which can be use to construct measures of revealed comparative advantage

$$CA_{i,i't}^{\omega,\omega'} \equiv \frac{T_{it}^{\omega}/T_{it}^{\omega'}}{T_{i't}^{\omega}/T_{i't}^{\omega'}} = \exp\left\{\left(\xi_i^{\omega} - \xi_i^{\omega'}\right) - \left(\xi_{i'}^{\omega} - \xi_{i'}^{\omega'}\right)\right\}$$

In order to avoid issues related to the particular choice of base year and base industry, I define comparative advantage relative to an "average" industry and country as in the text. Then, starting from a base country i' and a base industry ω'

$$CA_{i,i't}^{\omega,\bar{\omega}} \equiv \frac{CA_{i,i't}^{\omega,\omega'}}{\prod_{\omega=1}^{\Omega}\left[CA_{i,i't}^{\omega,\omega'}\right]^{\frac{1}{\Omega}}}$$

Finally, we can define comparative advantage relative to "average" industry $\bar{\omega}$ and country $\bar{\imath}$ as follows:

$$CA_{it}^{\omega} \equiv \frac{T_{it}^{\omega}/T_{it}^{\bar{\omega}}}{T_{\bar{\imath}t}^{\omega}/T_{\bar{\imath}t}^{\bar{\omega}}} = \frac{CA_{i,i't}^{\omega,\bar{\omega}}}{\prod_{i=1}^{N}\left[CA_{i,i't}^{\omega,\bar{\omega}}\right]^{\frac{1}{N}}}$$

A.3.2 Proof of Lemma 3

Proof. Multiplying both sides of equations (CAE) and (DE) by $\bar{E}_i^\omega$ and $\bar{\iota}_i^\omega$ respectively and taking expectations yields

$$\begin{aligned}\mathbb{E}\left[\bar{T}_i^\omega \bar{E}_i^\omega\right] &= \upsilon \mathbb{E}\left[\bar{\beta}_i^{R,\omega} \bar{E}_i^\omega\right] + \mathbb{E}\left[\bar{\iota}_i^\omega \bar{E}_i^\omega\right], \\ \mathbb{E}\left[\bar{\iota}_i^\omega \bar{E}_i^\omega\right] &= \tfrac{\sigma-1}{\theta} \mathbb{E}\left[\bar{\iota}_i^\omega \bar{\Phi}_i^\omega\right],\end{aligned}$$

where in the second line (equation (DE)) I used $\mathbb{E}\left[\bar{\iota}_i^\omega \bar{\gamma}_i^\omega\right] = 0$. As discussed in the text, the presence of high trade frictions implies $\mathbb{E}\left[\bar{\iota}_i^\omega \bar{\Phi}_i^\omega\right] > 0$ and $\mathbb{E}\left[\bar{\beta}_i^{R,\omega} \bar{E}_i^\omega\right] > 0$. Using these results and $\sigma > 1$ after taking probability limits in (33) yields

$$plim\left(\widehat{\upsilon}_1\right) = \frac{\mathbb{E}\left[\bar{T}_i^\omega \bar{E}_i^\omega\right]}{\mathbb{E}\left[\bar{\beta}_i^{R,\omega} \bar{E}_i^\omega\right]} = \upsilon + \tfrac{\sigma-1}{\theta} \frac{\mathbb{E}\left[\bar{\iota}_i^\omega \bar{\Phi}_i^\omega\right]}{\mathbb{E}\left[\bar{\beta}_i^{R,\omega} \bar{E}_i^\omega\right]} > \upsilon,$$

which is the desired result. ■

A.3.3 Proof of Lemma 4

The estimator $\widehat{\upsilon}_2$ involves the following steps: $\bar{\gamma}_i^\omega, \bar{\Phi}_i^\omega, \bar{T}_i^\omega \bar{\beta}_i^{R,\omega}, \widehat{\bar{\gamma}}_i^\omega$

Proof. As the first step, I show that the OLS estimator of $(\sigma-1)/\theta$ in equation (DE) is biased upwards. Let $c \equiv (\sigma-1)/\theta$ and let $\widehat{c}$ be the OLS estimator of c. Notice that (i) equation (DE) implies that $\bar{\gamma}_i^\omega$ and $\bar{E}_i^\omega$ are positively correlated; (ii) high trade costs imply that $\bar{E}_i^\omega$ and $\bar{\beta}_i^{R,\omega}$ are positively correlated; (iii) equation (CAE) implies that $\bar{\beta}_i^{R,\omega}$ and $\bar{T}_i^\omega$ are positively correlated if $\upsilon > 0$; and (iv) as discussed above $\bar{\Phi}_i^\omega$ and $\bar{T}_i^\omega$ are positively correlated if trade frictions are high. This sequence of correlations imply that $\bar{\gamma}_i^\omega$ and $\bar{\Phi}_i^\omega$ are positively correlated, so the OLS estimator $\widehat{c}$ is biased upwards:

$$\bar{c} \equiv plim\left(\widehat{c}\right) = c + \frac{\mathbb{E}\left[\bar{\Phi}_i^\omega \bar{\gamma}_i^\omega\right]}{\mathbb{E}\left[\left(\bar{\Phi}_i^\omega\right)^2\right]} > c$$

Now I show that the last result implies that $\widehat{\upsilon}_2$ is biased downwards. By construction, $\widehat{\bar{\gamma}}_i^\omega = (c - \widehat{c})\, \bar{\Phi}_i^\omega + \bar{\gamma}_i^\omega$, and taking probability limits yields $plim(\widehat{\bar{\gamma}}_i^\omega) = (c - \bar{c})\, \bar{\Phi}_i^\omega + \bar{\gamma}_i^\omega$. Using this and (CAE), the probability limit of estimator $\widehat{\upsilon}_2$ is given by

$$plim\left(\widehat{\upsilon}_2\right) = \frac{\mathbb{E}\left[\bar{T}_i^\omega \times plim(\widehat{\bar{\gamma}}_i^\omega)\right]}{\mathbb{E}\left[\bar{\beta}_i^{R,\omega} \times plim(\widehat{\bar{\gamma}}_i^\omega)\right]} = \upsilon + (c - \bar{c}) \frac{\mathbb{E}\left[\bar{\iota}_i^\omega \bar{\Phi}_i^\omega\right]}{\mathbb{E}\left[\bar{\beta}_i^{R,\omega} \times plim(\widehat{\bar{\gamma}}_i^\omega)\right]}$$

As discussed above, the structure of the model implies $(c - \bar{c}) < 0$ and $\mathbb{E}\left[\bar{\iota}_i^\omega \bar{\Phi}_i^\omega\right] > 0$. In addition, $\widehat{b} \equiv \frac{1}{\Omega N} \sum_{i,\omega} \bar{\beta}_i^{R,\omega} \widehat{\bar{\gamma}}_i^\omega$ is a consistent estimator of $\mathbb{E}\left[\bar{\beta}_i^{R,\omega} \times plim\left(\widehat{\gamma}_n\right)\right]$ and from the data we have $\widehat{b} > 0$. Then $plim\left(\widehat{\upsilon}_2\right) < \upsilon$, which is the desired result. ■

A.3.4 Non-homothetic preferences and biases in the estimation of υ

I present a brief discussion about the potential biases affecting the estimations above if the true underlying preferences are non-homothetic. Specifically, estimator $\widehat{\upsilon}_1$ is biased upwards if richer countries have comparative advantage in innovation in industries with higher income elasticity of demand. In this case, higher income per-capita is associated with higher relative expenditure and comparative advantage innovation in the same set of industries, $\mathbb{E}\left[\bar{E}_i^{\omega}\bar{\iota}_i^{\omega}\right] > 0$.[5] However, this misspecification bias is unlikely to be important in the present context. First, although non-homothetic preferences are considered to be an important factor affecting relative expenditure between the manufacturing and service sectors, there is less agreement about their importance to explain differences in expenditure patterns between industries within the manufacturing sector, which is the focus of this paper. Second, as most of the countries in the sample used in this paper are OECD's members, income per-capita does not vary much among them, reducing the potential importance of this type of bias. In fact, if this bias is quantitatively important, then reducing the dispersion of income among the countries in the sample should alleviate the bias and lead to lower estimates. However, computing estimator $\widehat{\upsilon}_1$ for the richest and poorest fifteen countries in the sample yields point estimates of 0.813 and 0.764, respectively, falling in both cases within the 95-percent confidence interval corresponding to the point estimate obtained including the whole sample. This suggests that this type of bias is not quantitatively important for the purposes of this paper.

B Extensions of the Model

In this appendix I extend the baseline model to include multiple factors of production, heterogenous trade elasticities across industries and intermediate inputs, and discuss how these extensions may affect the quantitative results above. I argue that many of these results are more general than what the simple structure of baseline model would suggest, as some of these extensions just affect the interpretation of some elements of the model. Accounting for multiple factors of production and heterogenous trade elasticities has little impact on the estimated value of the R&D parameter υ and on our conclusions regarding the *relative* importance of directed research for the determination of comparative advantage in production (CAP) and for welfare evaluations. Interestingly, including intemediate goods reduces the estimated value of υ, but has little impact on many of the other results. This is the case because the presence of intermediate goods tends to amplify the overall effect of directed research on CAP for a given value of υ, and this overall effect is what the estimations in the baseline model are capturing. In all cases, the main messages of the paper go through, i.e. directed research is an important determinant of CAP and trade flows, but it is a somewhat less important factor to understand the effects of trade in manufactured goods on aggregate real income.

In some of these extensions, trade can affect the overall incentives to innovation even if research is undirected.[6] Then, I isolate the impact of directed research by comparing the predictions of a model with

[5] The proof of this result can be obtained making small modifications to the proof of Lemma 3 in the paper.

[6] This is not true in a single-factor model with undirected research. In fact, such a model shares the same predictions with a single-factor model with no innovation along all the dymensions analyzed in this paper.

undirected innovation with those of a model with directed research. Below I present a summary of the results, relegating the detailed description of the extended models and all derivations to the additional appendix C in my personal webpage.[7]

B.1 Multiple Factors of Production

I consider a simple extension of the model with two factors of production, labor and capital. Through the lenses of this extended model, the measures of revealed comparative advantage (RCA) estimated in section III of the paper now reflect the two sources of CAP in the model, differences in relative manufacturing productivities and in relative factor prices. Moreover, said measures of RCA provide consistent estimates of overall CAP in the extended model, $CA_{i,i't}^{\omega,\omega'}$, so the comparative advantage equation (CAE) estimated in section III now has a different structural interpretation.[8] If c_i^{ω} denotes the unit-price of the bundle of factors used by manufacturing firms in country i and industry ω, then, according to the extended model, in section III we estimated the following equation,

$$\overline{CA}_i^{\omega} = \upsilon \bar{\beta}_i^{R,\omega} + \overline{\xi}_i^{\omega}, \text{ with error term } \overline{\xi}_i^{\omega} = \overline{\iota}_i^{\omega} - \theta \overline{c}_i^{\omega}, \tag{B.1}$$

and instrumented relative market shares $\bar{\beta}_i^{R,\omega}$ with relative preference parameters $\overline{\alpha}_i^{\omega}$.

In light of these observations, two comments are in order about estimation of parameter υ. First, in appendix C I show that the general estimation strategy proposed in section III can be also be applied to the multi-factor model, although its implementation now requires additional information on relative prices of industry factor-bundles. Second, any potential upward bias in the estimates of parameter υ obtained in the baseline model are unlikely to be significant.[9] Given assumption A0, equation (B.1) implies that the estimates of υ in section III are biased upward if and only if the covariance between $\overline{\alpha}_i^{\omega}$ and $\overline{c}_i^{\omega}$ is negative. In appendix C I show that, in the two-factor model, this is the case if countries with relatively higher wages, typically richer countries, expend relatively more on more capital-intensive industries. As most of the countries in the sample used in this paper are OECD's members, the relative rewards to labor and capital do not differ much among them, reducing the potential importance of this type of bias. Intuitively, a low dispersion in relative factor prices leads to a low dispersion in relative prices of industry factor-bundles, reducing the absolute value of the covariance between $\overline{\alpha}_i^{\omega}$ and $\overline{c}_i^{\omega}$ and the size of the bias. Moreover, if this type of bias is quantitatively important, focusing on a more homogeneous set of countries should alleviate the bias and lead to lower estimates. However, the estimates do not differ much when I repeat the estimations for the subsamples including only the richest and poorest fifteen countries in the sample.[10]

As the estimates of parameter υ are largely unchanged by the inclusion of multiple factors of production, our conclusions regarding the contribution of directed research to the variance of *overall* CAP are

[7] https://marianosomale.wixsite.com/scientist-site

[8] $CA_{i,i't}^{\omega,\omega'}$ is defined in equation (C.7) of appendix C.

[9] I focus on overestimation risks as this case leads to an overstatement of the importance of directed research.

[10] See the discussion about potential biases induced by the presence of non-homothetic preferences in appendix A.3.4.

also largely unchanged.[11] According to the multi-factor model, the variance decompositions presented in section IV are based on equation (B.1), so what we denoted the "exogenous component" in the baseline model now captures exogenous differences in innovation productivities and endogenous differences in factor prices, $\overline{\xi}_i^\omega = \overline{\iota}_i^\omega - \theta\overline{c}_i^\omega$. I argue that the results in second row of table 2 in the paper are still valid for overall CAP in the extended model, as the dispersion of $\overline{\xi}_i^\omega$ in the observed open equilibrium in 2006 still captures the part of the variance of CAP that is not driven by directed research.[12] First, the variance of $\overline{\xi}_i^\omega$ is largely driven by the exogenous component $\overline{\iota}_i^\omega$, as the relative rewards to labor and capital do not differ much across the countries in the sample. And second, any dispersion in relative factor prices across countries is most likely reflecting small differences in factor endowments.[13] This discussion illustrates the fact that many supply-side determinants of CAP, not explicitly included in the baseline model, are captured in the exogenous component of CAP in that model.

Finally, the welfare implications of directed research in the extended model are also little changed relative to the baseline model. A measure of the *absolute* impact of directed research on the welfare evaluation of shocks is given by the differences in the corresponding log-changes in real income implied by the models with directed and undirected research. In particular, the expressions in (B.2) below can be used to assess the *absolute* impact of directed research on (a) the ex-post evaluation of the change in real income induced by a foreign shock, and (b) ex-ante evaluation of the change in real income associated with moving to autarky,

$$\text{(a)}\ \ln\widehat{W}_{it}^D - \ln\widehat{W}_{it}^U = \frac{\upsilon}{\theta}\int_\Omega \alpha_i^\omega \ln\widehat{\delta}_i^\omega\, d\omega, \qquad \text{(b)}\ \ln\frac{W_{it}^{a,D}}{W_{it}^D} - \ln\frac{W_{it}^{a,U}}{W_{it}^U} = \frac{\upsilon}{\theta}\int_\Omega \alpha_i^\omega \ln\left(\frac{\alpha_i^\omega}{\delta_i^\omega}\right) d\omega. \quad \text{(B.2)}$$

The last expressions are identical to the corresponding expressions in the baseline model, which, together with unchanged estimates of the parameters $\{\alpha_i^\omega, \upsilon, \theta\}$, implies that the single- and multi-factor models provide the same quantitative answers regarding the *absolute* importance of directed research for the welfare evaluations considered above.[14] What is the *relative* importance of directed research in these evaluations? Answering this question amounts to assessing the quantitative importance of the differences in (B.2) *relative* to the log-changes in real income predicted by the model with undirected research, where the latter now includes the effect of changes in factor prices. Although little can be said about the effect of changes in factor prices in the generality of case (a), for case (b) the literature has found that these effects are very small relative to the overall gains from trade. Then, the gains from trade predicted by the multi-factor model with undirected research and the single-factor model with no innovation are unlikely to differ significantly, implying the single- and multi-factor models give similar quantitative answers about

[11] Note that the variance decomposition results in section IV apply to overall CAP, as differences in relative manufacturing technologies are the only source of CAP in the baseline model.

[12] Note that this implies that role of directed research in shaping relative manufacturing technologies is even more important.

[13] Computing the variance decomposition of CAP for the counterfactual cases of autarky and zero gravity is beyond the scope of this paper, as it requires a full-fledged quantitative version of the extended model to compute the impact of changes in factor prices on $\overline{\xi}_i^\omega$. However, relative factor endowments should not differ much among the countries in the sample, suggesting that $\overline{\xi}_i^\omega$ is to unlikely to change much in these counterfactuals scenarios. This implies that the results in rows 1 and 3 of table 2 in the paper are unlikely to be significately different in the extended model.

[14] Assessing the impact of directed research on ex-ante welfare evaluations of trade liberalizations in the extended model is beyond the scope of this paper, as it requires solving a quantitative version of the model.

the *relative* importance of directed research in case (b).

B.2 Heterogeneous Trade Elasticities

I consider a model with industry-specific technology shape parameters, θ^{ω}, implying that the trade elasticity differs across industries. In appendix C, I show that these parameters affect the growth rate of industry price indices in the BGP, so a straightforward extension of the model along this dimension can make the model incompatible with a BGP featuring constant labor allocations across sectors and industries if consumer's preferences differ from the baseline Cobb-Douglas assumption. Given the uncertainty about the elasticity of substitution across industries at this level of aggregation, I consider a model that satisfy the following condition, (i) *industry price indices faced by consumers grow at the same pace in the BGP*. Specifically, I introduce industry-specific retailers in the model and make assumptions about their technology such that condition (i) is satisfied.

In this extension of the model, the interaction between heterogenous trade elasticities across industries and wage differences across countries leads to an additional source of CAP, i.e. everything else equal, countries with higher wages have manufacturing comparative advantage in industries with lower trade elasticity. Moreover, the measures of RCA estimated in section III of the paper provide consistent estimates of *overall* CAP in the extended model, $CA_{i,i't}^{\omega,\omega'}$, affecting the structural interpretation of the estimations in that section.[15] If we define $\overline{w}_{it} \equiv \ln \frac{w_{it}}{w_{\bar{i}t}}$ and $\bar{\theta} \equiv \sum_{\omega=1}^{\Omega} \frac{\theta^{\omega}}{\Omega}$, then, according to this extended model, in section III we estimated the following equation

$$\overline{CA}_i^{\omega} = \upsilon \bar{\beta}_i^{R,\omega} + \bar{\xi}_i^{\omega}, \text{ with error term } \bar{\xi}_i^{\omega} \equiv \bar{\iota}_i^{\omega} - (\theta^{\omega} - \bar{\theta})\overline{w}_{it}, \tag{B.3}$$

and instrumented relative market shares $\bar{\beta}_i^{R,\omega}$ with relative preference parameters $\overline{\alpha}_i^{\omega}$.

The implications of these observations regarding the estimation of parameter υ are similar to those discussed in the case of the multi-factor extension. First, the general estimation strategy proposed in section III can be applied also in this case, although its implementation now requires additional information on wages across countries and trade elasticities across industries. Second, any potential upward bias in the estimates of υ are unlikely to be significant. Given assumption A0, equation (B.3) implies that the estimates of υ in section III are biased upward if and only if the covariance between $\overline{\alpha}_i^{\omega}$ and $(\theta^{\omega} - \bar{\theta})\overline{w}_{it}$ is negative, i.e. if countries with higher income per-capita expend relatively more on industries with lower trade elasticity. As income per-capita does not differ much among the countries in the sample, the relevance of this type of bias should be limited.[16] Moreover, if this type of bias is quantitatively important, then reducing the dispersion of income among the countries in the sample should alleviate the bias and lead to lower estimates. However, the estimates do not differ much when I repeat the estimations for the subsamples including only the richest and poorest fifteen countries in the sample.

With the estimates of parameter υ largely unchanged in this extension of the model, our conclusions

[15] See equation (C.16) for a definition of $CA_{i,i't}^{\omega,\omega'}$ in this extension of the model.

[16] Intuitively, a low dispersion in income per-capita (proxy for $\overline{w}_{it}$) reduces the absolute value of the covariance between $\overline{\alpha}_i^{\omega}$ and $(\theta^{\omega} - \bar{\theta})\overline{w}_{it}$ and the size of the bias.

regarding the contribution of directed research to the variance of *overall* CAP are also largely unchanged. According to the current model, the variance decomposition presented in section III is based on equation (B.3), so what we denoted the "exogenous component" in the baseline model, now captures exogenous differences in innovation productivities and the interaction between wage differences across countries and heterogenous trade elasticities across industries, $\bar{\xi}_i^\omega \equiv \bar{\iota}_i^\omega - (\theta^\omega - \bar{\theta})\overline{w}_{it}$. As before, the dispersion of $\bar{\xi}_i^\omega$ in the observed open equilibrium continues to capture the part of the variance of CAP that is not driven by directed research, so the results in the second row of table 2 are still valid for *overall* CAP in the extended model.[17] This is the case because the variance of $\bar{\xi}_i^\omega$ is largely driven by the dispersion in exogenous relative research productivities $\bar{\iota}_i^\omega$, as income per-capita (proxy for $\overline{w}_{it}$) does not differ much across the countries in the sample.[18]

Condition (i) above affects the welfare implications of directed research in the extended model. The differences in log-changes in real income in (B.4) can be used to assess the *absolute* impact of directed research on (a) the ex-post evaluation of the changes in real income induced by a foreign shock, and (b) the predicted change in real income associated with moving to autarky,

$$\text{(a)}\ \ln \widehat{W}_{it}^D - \ln \widehat{W}_{it}^U = \frac{\upsilon}{\theta^*}\int_\Omega \alpha_i^\omega \ln \widehat{\delta}_i^\omega \, d\omega, \qquad \text{(b)}\ \ln \frac{W_{it}^{a,D}}{W_{it}^D} - \ln \frac{W_{it}^{a,U}}{W_{it}^U} = \frac{\upsilon}{\theta^*}\int_\Omega \alpha_i^\omega \ln\left(\frac{\alpha_i^\omega}{\delta_i^\omega}\right) d\omega, \quad \text{(B.4)}$$

where θ^* is a parameter that affects the growth rate of industry price indices in the BGP. The last expressions are almost identical to the corresponding ones in the baseline model, with the exception that in the former, θ^* takes the place of θ in the latter. In appendix C, I explore alternative ways of calibrating this parameter, obtaining values for θ^* that are similar to the value of θ used in the baseline model. These observations, together with unchanged estimates of $\{\alpha_i^\omega, \upsilon\}$, imply that in the extended model, the *absolute* impact of directed research on the two welfare evaluations considered above is largely unchanged relative to the baseline model. Finally, these results suggest that the *relative* importance of directed research for case (b) may be even smaller in this extension of the model, as the quantitative trade literature typically finds larger gains from trade once heterogeneity in trade elasticities are accounted for.

B.3 Intermediate Goods

In this section I introduce intermediate goods into the model. To keep things simple, I assume that intermediates goods are used only in the manufacturing sector, abstract from interindustry linkages, assume that all countries have the same technology to produce industry-specific bundles of inputs comprising labor and intermediate goods and assume that the share of value added in total output in industry ω, ε^ω, is the same across industries, $\varepsilon^\omega = \varepsilon$.[19] The main difference with the previous two cases is that, in

[17] As before. this implies that role of directed research in shaping relative manufacturing technologies is even more important.

[18] Computing the variance decomposition of CAP for the counterfactual cases of autarky and zero gravity is beyond the scope of this paper, as it requires a full-fledged quantitative version of the extended model to compute the impact of changes in wages across countries on $\bar{\xi}_i^\omega$.

[19] In appendix C, I show that the share of value added in total output, ε^ω, affects the pace at which the price index of an industry grows in the BGP, so variation in ε^ω across industries leads to the same problems discussed in the case of

the presence of intermediate inputs and high trade frictions, the endogenous changes in technology induced by directed research also affect the relative price of inputs across industries, amplifying the overall impact of directed research on *overall* CAP for a given value of the R&D parameter υ. Then, through the lenses of this extended model, our estimate of υ in section III is biased upwards, as it conflates these two effects. However, despite the bias in the estimate of υ, the conclusions regarding the importance of directed research for *overall* CAP and welfare evaluations are little changed in many dimensions.

In this extended model, differences in the prices of industry input-bundles, c_i^ω, are an additional source of comparative advantage. Moreover, the measures of RCA estimated in section III provide consistent estimates of *overall* CAP in the extended model, $CA_{i,i't}^{\omega,\omega'}$, affecting the structural interpretation of the baseline estimations. In appendix C I show that, accroding to the extended model, in section III of the paper we estimated the following equation

$$\overline{CA}_i^\omega = \upsilon \bar{\beta}_i^{R,\omega} + \bar{\xi}_i^\omega \text{ with error term } \bar{\xi}_i^\omega \equiv \bar{\iota}_i^\omega - \theta \bar{c}_i^\omega, \tag{B.5}$$

and instrumented relative market shares $\bar{\beta}_i^{R,\omega}$ with $\bar{\psi}_i^\omega$, where $\psi_i^\omega \equiv \left[\alpha_i^\omega \frac{\theta\varepsilon+1}{1+\theta} + \frac{\theta(1-\varepsilon)}{1+\theta}\delta_{it}^\omega\right]$.

The structure of the extended model implies that this IV strategy yields an upward-biased estimator of υ, as it reveals two channels through which the error term in equation (B.5), $\bar{\xi}_i^\omega$, and the instrument, $\bar{\psi}_i^\omega$, are positively correlated. The first channel is related to the incorrect specification of the instrument in the baseline model, as $\bar{\psi}_i^\omega$ does not reflect relative preference parameters. However, in appendix C I argue that this channel is unlikely to be quantitatively important in the presence of high trade frictions. The second channel, which is active even if the instrument is correctly specified, is related to the incorrect interpretation of CAP in baseline model. The simple structure of the model with intermediate goods implies $\bar{c}_i^\omega = (1-\varepsilon)\bar{P}_{it}^\omega$, so $\bar{\alpha}_i^\omega$ is likely to be negatively correlated correlated with $\bar{P}_{it}^\omega$ (positively with $\bar{\xi}_i^\omega$) when trade costs are high. Intuitively, the relative cost parameter $\bar{\Phi}_{it}^\omega$ is largely driven by $\bar{T}_i^\omega$ in this case, so $\bar{T}_i^\omega$ is negatively correlated with $\bar{P}_{it}^\omega$. In addition, $\bar{T}_i^\omega$ is positively correlated with $\bar{\alpha}_i^\omega$ through the impact of the latter on relative market shares and the direction of innovation. In appendix C I show that in the extreme case of infinite trade costs, the strategy above yields a consistent estimator of $\upsilon/\varepsilon > \upsilon$, providing a clear illustration of the bias induced by this mechanim.

In light of the these observations, in appendix C I re-do some of the estimations and quantitative exercises in sections III and IV of the paper to assess the reboustenss of the results. First, I re-estimate parameter υ following the same general estimation strategy outlined in section III, after computing the appropriate measures of relative manufacturing productivities $\overline{T}_i^\omega$ and preferences parameters $\bar{\alpha}_i^\omega$ in the extended model. Relative to the results obtained with estimator $\widehat{\upsilon}_1$ in column 2, the estimated value of υ goes down, as expected, with the new point estimate given by $\widehat{\upsilon}_I = 0.584$.[20] Second, armed with a new estimate for υ, I recalculate the contribution of directed research to the variance of CAP. The discussion in the previous paragraph implies that, in the presence of intermediate goods and high trade frictions, the term $\bar{\xi}_i^\omega$ in equation (B.5) is partly driven by directed research, as the latter affects the

heterogenous trade elasticities. For the sake simplicity, I deal with these issues by assuming $\varepsilon^\omega = \varepsilon$.

[20] See table 1 in appendix C.

relative price of industry input-bundles through its impact on relative manufacturing technology. Then, unlike the previous two extensions, a decomposition of the variance of CAP according to (B.5) would tend to underestimate the impact of directed research on CAP.[21] Interestingly, after accounting for the impact of directed research on $\overline{\xi}_i^{\omega}$, the contribution of directed research to the variance of *overall* CAP in the observed open equilibrium is 51.5%, which is only slightly below the 52.8% obtained under the benchmark calibration of the baseline model.[22]

The differences in log-changes in real income in (B.6) can be used to assess the *absolute* impact of directed research in this extended model on (a) the ex-post evaluation of the changes in real income induced by a foreign shock, and (b) the predicted change in real income associated with moving to autarky,

$$\text{(a)}\ \ln \widehat{W}_{it}^{D} - \ln \widehat{W}_{it}^{U} = \frac{\upsilon}{\theta\varepsilon} \int_{\Omega} \alpha_i^{\omega} \ln \widehat{\delta}_i^{\omega}\, d\omega, \qquad \text{(b)}\ \ln \frac{W_{it}^{a,D}}{W_{it}^{D}} - \ln \frac{W_{it}^{a,U}}{W_{it}^{U}} = \frac{\upsilon}{\theta\varepsilon} \int_{\Omega} \ln \alpha_i^{\omega} \left(\frac{\delta_i^{\omega}}{\alpha_i} \right) d\omega. \quad \text{(B.6)}$$

There are three differences between the expressions in (B.6) and the corresponding expression in the baseline model given by (B.2). First, the preference parameters are calibrated slightly differently in these models. Second, the estimated value of parameter υ is lower in the the extended model. And third, the share of value added in total output, ε, appears in the denominator of (B.6) in the extended model. In appendix C I argue that the first of these differnces is not quatitatively important, while the net effect of remaining two is to almost double the *absolute* impact of directed research on these welfare evaluations. However, I show that the *relative* importance of directed research for these welfare evaluations is smaller in the extended model, as the presence of intermediate goods also amplifies the changes in real income in the model with undirected research.

[21] Not accounting for the impact of directed research on $\overline{\xi}_i^{\omega}$ in (B.5) reduces its contribution to the variance of CAP to 46.1%.

[22] The simple structure of the model with intermediate goods permits a decomposition of the variance of *overall* CAP in the counterfactual cases of autarky and zero gravity. The contribution of directed research in said decompositions is 40.6% and 82.6% respectively. See appendix C for more details.