AEA Continuing Education Program

International Trade

Marc Melitz, Harvard University

January 6-8, 2019

AEA Continuing Education 2019
International Trade
Dave Donaldson (MIT) and Marc Melitz (Harvard)

Part 2 (Marc)

Lectures

https://tinyurl.com/AEA2019-Trade2

Lecture 5: Some Empirical Facts on Firms and Trade (Jan 7, 3-4.30 pm)

Lecture 6-7: Monopolistic Competition Models of Producer Heterogeneity: Theory (Jan 7, 4.45-5.45 pm & Jan 8, 8-9.45 am)

- Modeling framework (Closed economy)
- Open Economy
- Competition and Endogenous Markups
- Extensions (Comparative Advantage, Innovation; Not covered: Dynamics)
- Gains from Trade and Policy

- Supplemental Notes:
  - Pareto Distributions
  - Goods Aggregation and Homothetic Preferences
  - Consumer Demand and Monopolistic Competition Pricing with a Continuum of Differentiated Goods

Lecture 8: Boundaries of the Multinational Firm and the Offshoring/Outsourcing Decision (Jan 8, 10-11.15 am)

Lecture 9: Gravity and the Firm-Level Margin of Trade (Jan 9, 11.30 am-12.00 pm)

Aside: US BLS Import and Export Price Index micro data access (soon via US Census RDC)

Background Reading

Lecture 5: Some Empirical Facts on Firms and Trade (Jan 7, 3-4.30 pm)

- Bernard, A.B., Jensen, J.B., Redding, S.J. & Schott, P.K. (2018) Global Firms. *Journal of Economic Literature*, 56 (2): 565–619.
- Bernard, A.B, Jensen, J.B., Redding, S.J. and Schott, P.K. (2007), Firms in International Trade. *Journal of Economic Perspectives* 21(3).
- Bernard, A.B., Jensen, J.B., Redding, S.J. and Schott, P.K (2012), The Empirics of Firm Heterogeneity and International Trade. *Annual Review of Economics*, 4, 283-313.

Lecture 6-7: Monopolistic Competition Models of Producer Heterogeneity: Theory (Jan 7, 4.45-5.45 pm & Jan 8, 8-9.45 am)

- Melitz, M.J. & Redding, S.J. (2015) Heterogeneous Firms and Trade, *In*: Helpman, E., Rogoff, K., & Gopinath, G. eds. *Handbook of International Economics*, Elsevier. pp.1–54. web appendix.
- Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695-1725.
- Demidova, S. & Rodríguez-Clare, A. (2013) The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics*, 90 (2): 266–272.

Endogenous Markups

- Melitz, M. J., & Ottaviano, G. I. P. (2008). Market size, trade, and productivity. *Review of Economic Studies*, 75(1), 295-316
- Zhelobodko, E., Kokovin, S., Parenti, M. & Thisse, J.-F. (2012) Monopolistic Competition: Beyond the Constant Elasticity of Substitution. *Econometrica*, 80 (6): 2765–2784.

Comparative advantage

- Bernard, A. B., Redding, S. J., & Schott, P. K. (2007). Comparative advantage and heterogeneous firms. *Review of Economic Studies*, 74(1).

Innovation

- Burstein A & Melitz M (2013) Trade Liberalization and Firm Dynamics. Advances in Economics and Econometrics: Theory and Applications, Econometric Society Monographs
- Atkeson A & Burstein AT (2010) Innovation, Firm Dynamics, and International Trade. *Journal of Political Economy* 118: 433-84
- Bustos, P. (2011) Trade Liberalization, Exports, and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms. *American Economic Review*, 101 (1): 304–340.
- Lileeva, A. & Trefler, D. (2010) Improved Access to Foreign Markets Raises Plant-Level Productivity…For Some Plants. *The Quarterly Journal of Economics*, 125 (3): 1051 –1099.
- Sampson, T. (2016) Dynamic Selection: An Idea Flows Theory of Entry, Trade and Growth. *Quarterly Journal of Economics*, 131(1), 315-380.

Gains from Trade and Policy

- Arkolakis, C., Costinot, A. & Rodríguez-Clare, A. (2012) New Trade Models, Same Old Gains? *American Economic Review*, 102 (1): 94–130.
- Melitz, M.J. & Redding, S.J. (2015) New Trade Models, New Welfare Implications. *American Economic Review*, 105 (3): 1105–46.
- Dhingra, S. & Morrow, J. (2016) Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity. Mimeo, LSE.

- Epifani, P. & Gancia, G. (2011) Trade, Markup Heterogeneity and Misallocations. Journal of International Economics, 83 (1): 1–13.

Lecture 8: Boundaries of the Multinational Firm and the Offshoring/Outsourcing Decision (Jan 8, 10-11.15 am)

- Antràs, Pol (2003), Firms, Contracts, and Trade Structure, *Quarterly Journal of Economics*, 118:4, pp. 1375--1418.
- Antràs, Pol and Elhanan Helpman (2004), Global Sourcing, *Journal of Political Economy*, 112, pp.552--580.
- Antràs, Pol and Stephen R. Yeaple (2013), Multinational Firms and the Structure of International Trade, Chapter 2, *Handbook of International Economics vol. 4*.
- Helpman, Elhanan, Marc J. Melitz, and Stephen R.Yeaple (2004), Exports versus FDI with Heterogeneous Firms, *American Economic Review*, 94(1): 300-316.

Lecture 9: Gravity and the Firm-Level Margin of Trade (Jan 9, 11.30 am-12.00 pm)

- Chaney, T. (2008). Distorted gravity: The intensive and extensive margins of international trade. *American Economic Review*, 98(4), 1707-1721.
- Eaton J., Kortum S. & Kramarz F. (2011) An Anatomy of International Trade: Evidence From French Firms. *Econometrica* 79: 1453-1498.
- Head, K. & Mayer, T. (2015) Gravity Equations: Workhorse, Toolkit, and Cookbook, *In*: Helpman, E., Rogoff, K., & Gopinath, G. eds. *Handbook of International Economics*, Elsevier. Vol. 4. pp.131–195

Micro-Structure of Firms and Trade: Empirical Evidence

Lecture Notes

Heterogeneity in Micro-Level Data

- Standard deviation of log sales

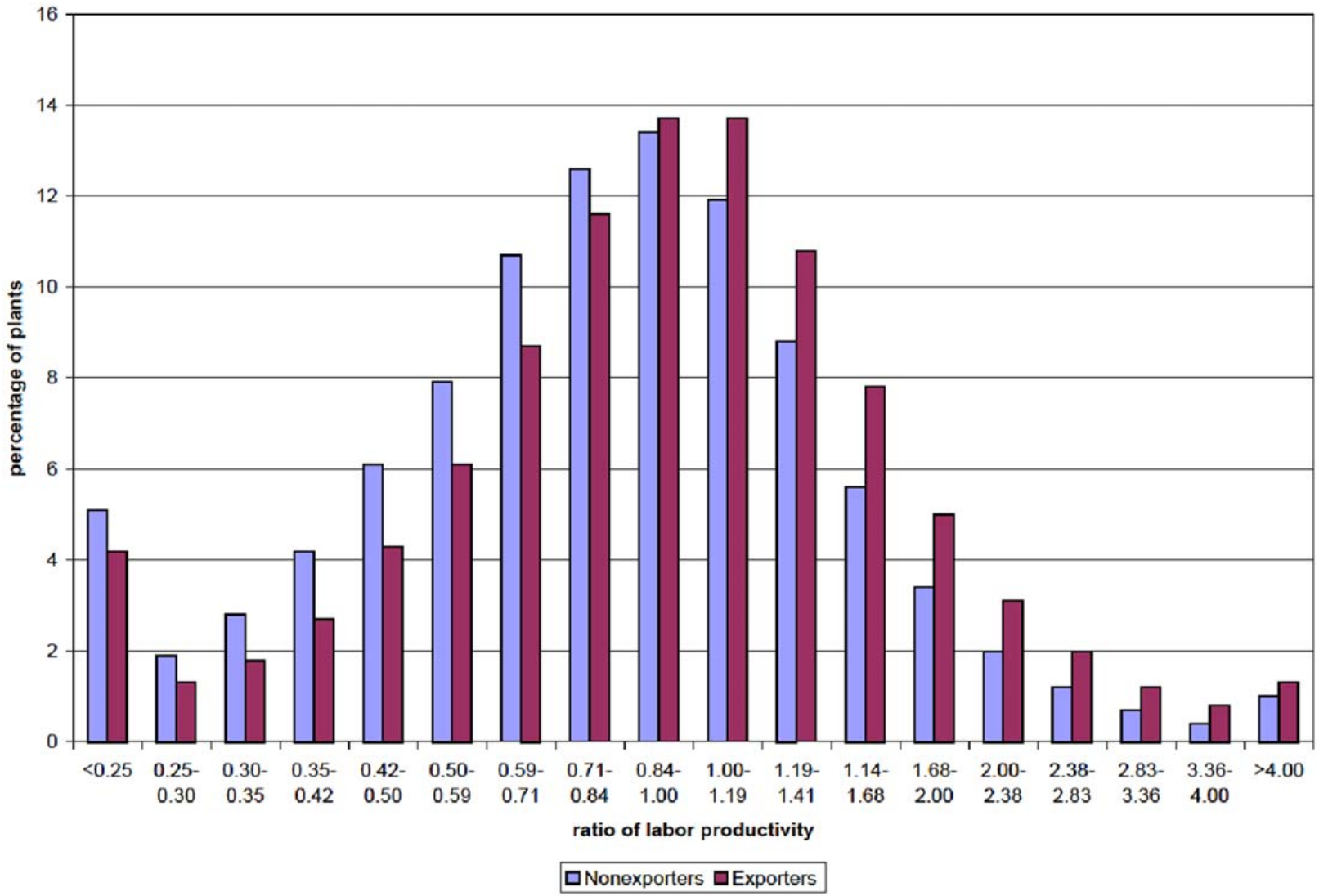
Country	# of producers	Overall	Within Sector (52 Manufacturing Sectors)
France	76,456	1.82	1.70
Italy	39,704	1.33	1.29
Spain	31,446	1.26	1.18
U.S. (plants)	224,009	1.67	

TABLE 2—PLANT-LEVEL PRODUCTIVITY FACTS

Productivity measure (value added per worker)	Variability (standard deviation of log productivity)	Advantage of exporters (exporter less nonexporter average log productivity, percent)
Unconditional	0.75	33
Within 4-digit industries	0.66	15
Within capital-intensity bins	0.67	20
Within production labor-share bins	0.73	25
Within industries (capital bins)	0.60	9
Within industries (production labor bins)	0.64	11

Notes: The statistics are calculated from all plants in the 1992 Census of Manufactures. The "within" measures subtract the mean value of log productivity for each category. There are 450 4-digit industries, 500 capital-intensity bins (based on total assets per worker), 500 production labor-share bins (based on payments to production workers as a share of total labor cost). When appearing within industries there are 10 capital-intensity bins or 10 production labor-share bins.

Heterogeneity in the Data Even Among Exporters



Exporters are a Minority

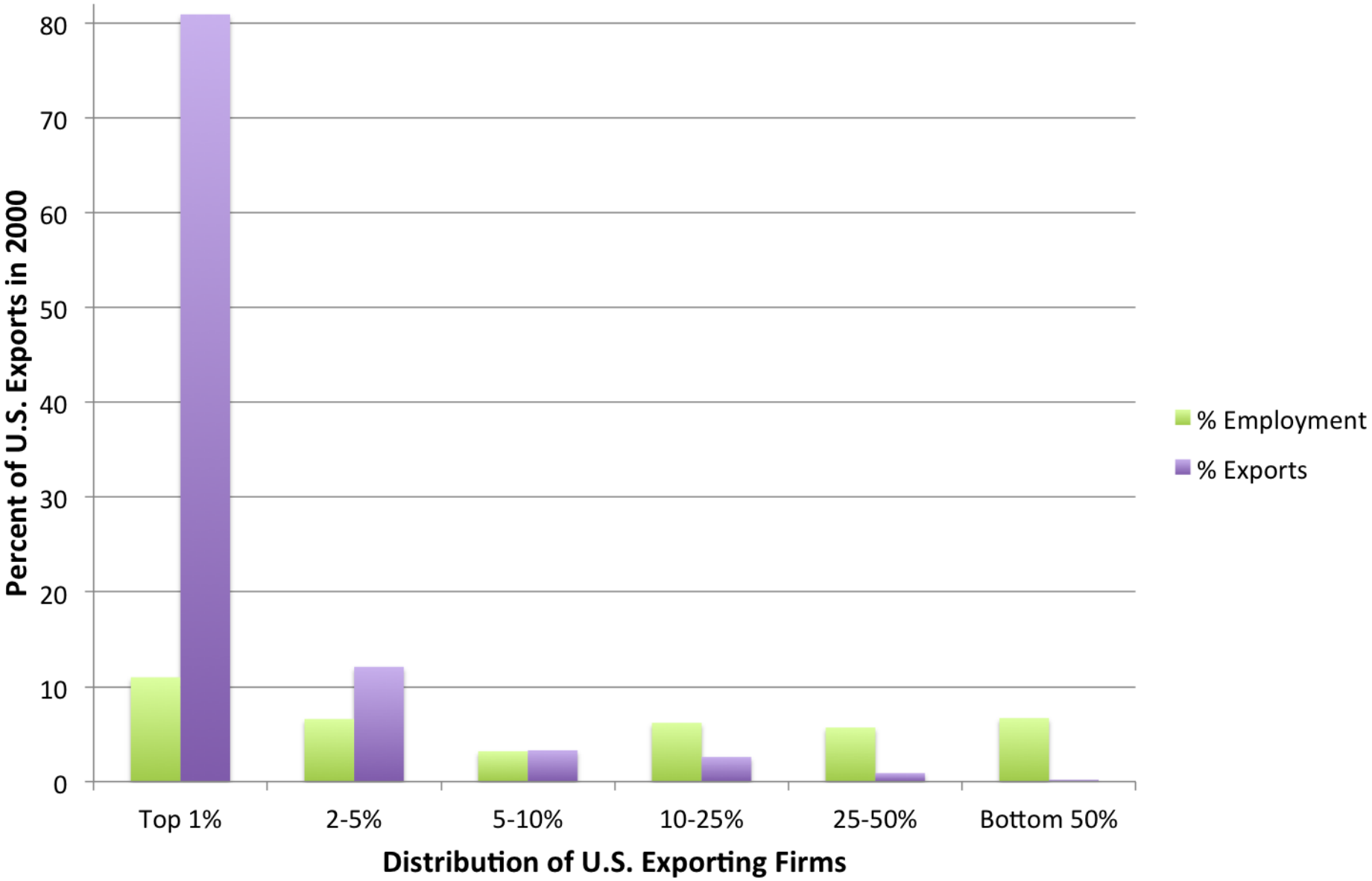
Table 2

Exporting By U.S. Manufacturing Firms, 2002

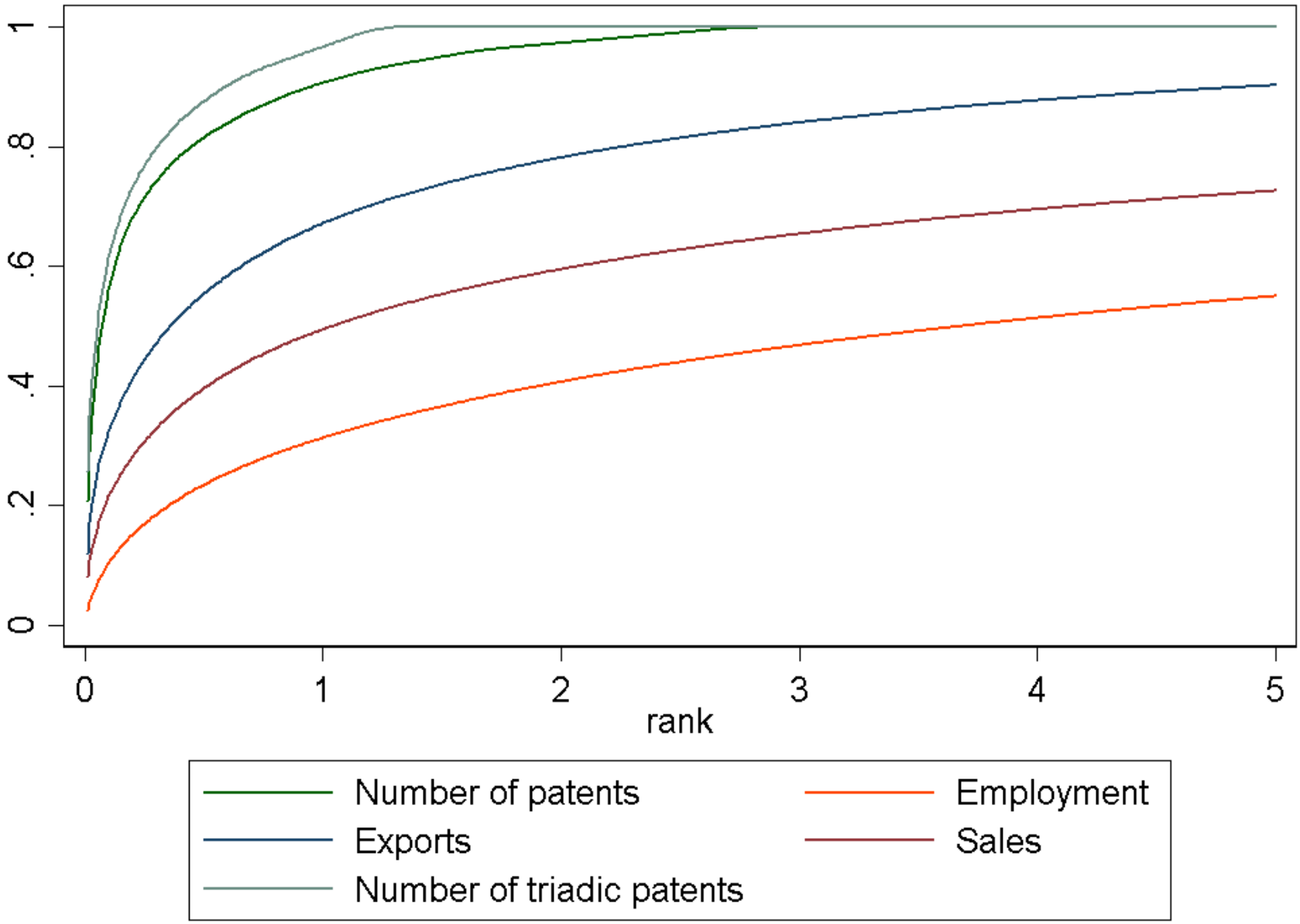
NAICS industry	*Percent of firms*	*Percent of firms that export*	*Mean exports as a percent of total shipments*
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	2	15
Aggregate manufacturing	**100**	**18**	**14**

Sources: Data are from the 2002 U.S. Census of Manufactures.
Notes: The first column of numbers summarizes the distribution of manufacturing firms across three-digit NAICS manufacturing industries. The second reports the share of firms in each industry that export. The final column reports mean exports as a percent of total shipments across all firms that export in the noted industry.

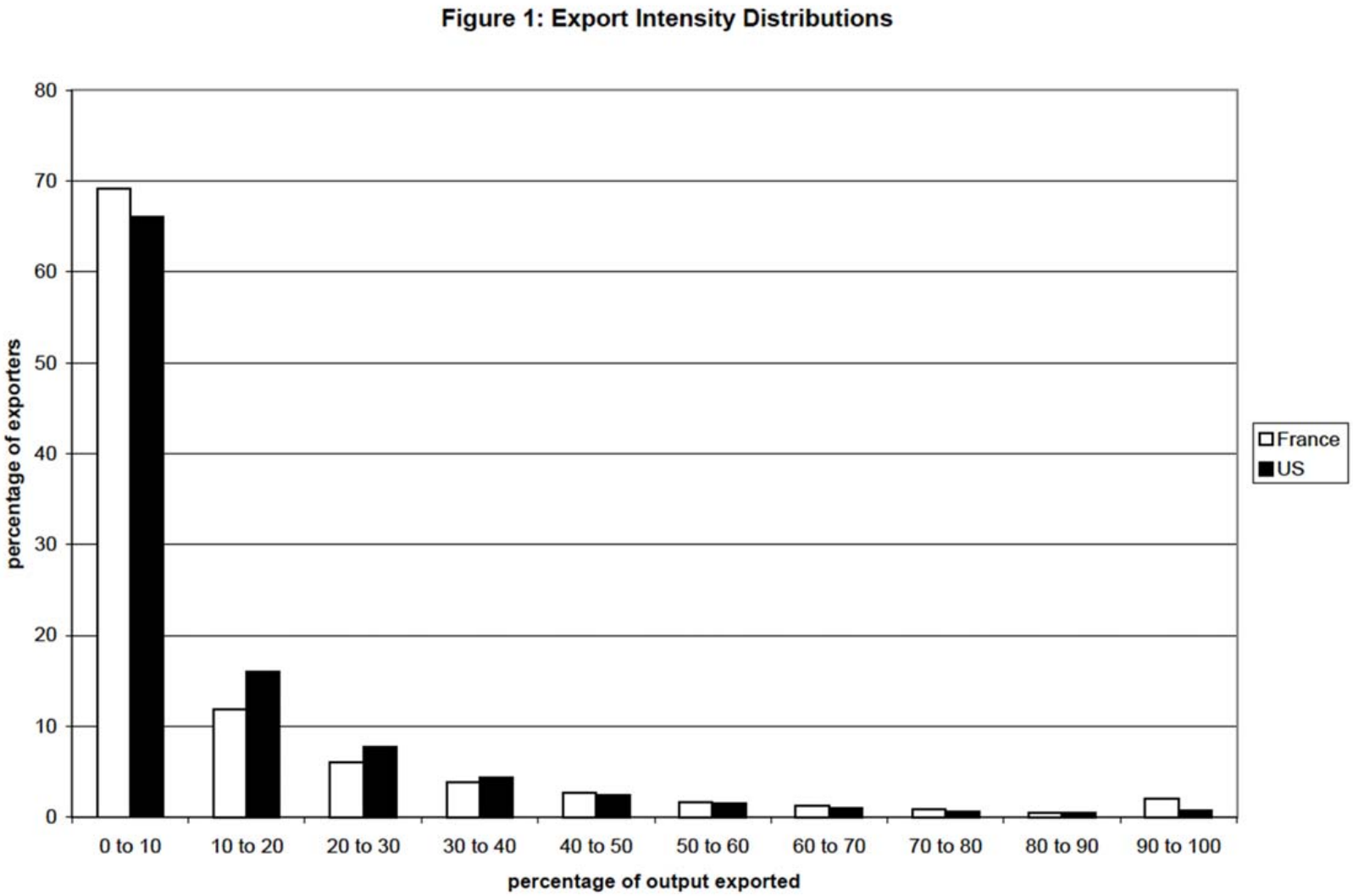
Exporting Activity is Very Skewed



Relative Skewness of Exporting for French Firms



Exporters Export Relatively Little



Exporters Sell to Very Few Markets

Exporting is not just a binary decision: firms decide where to export

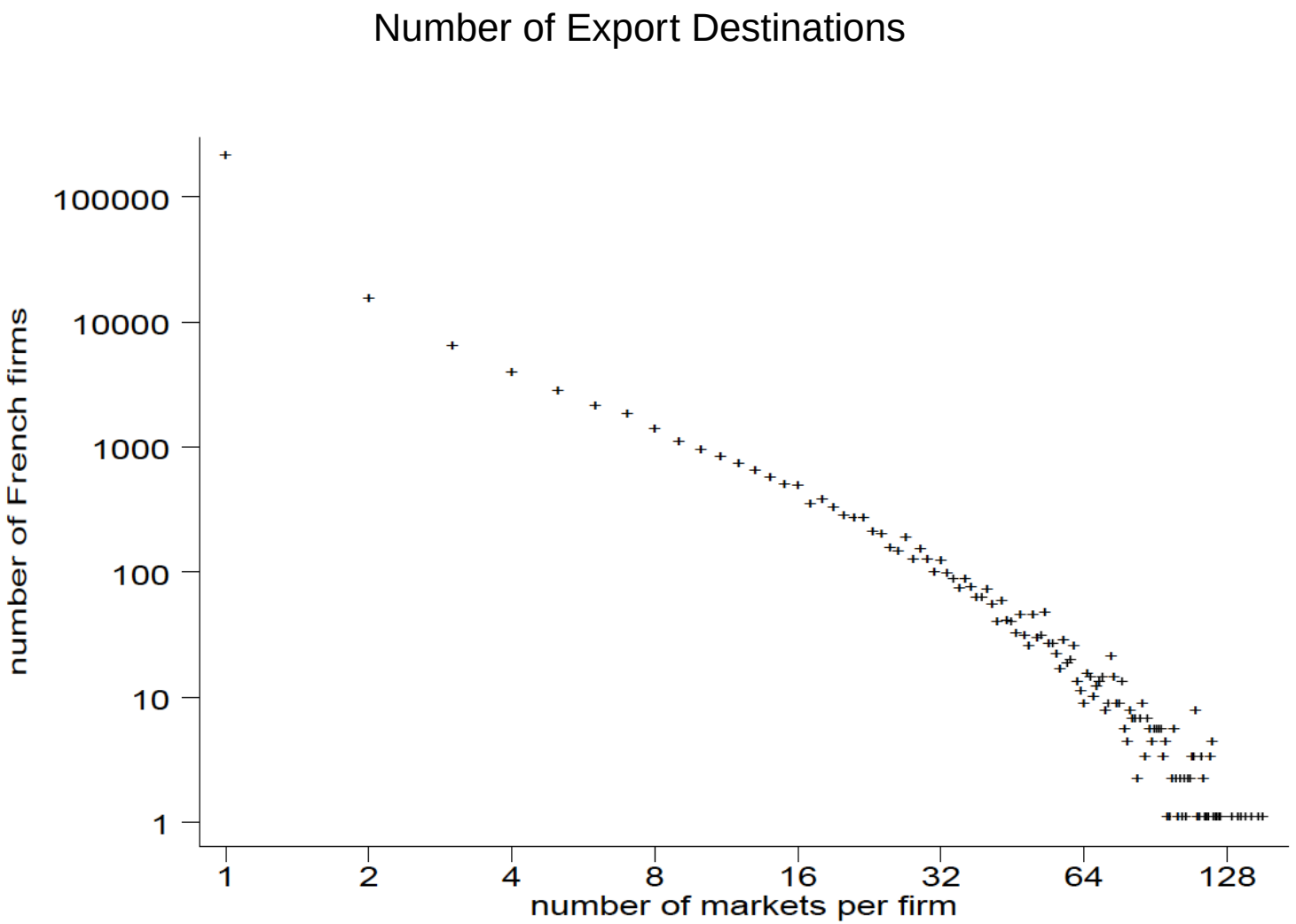
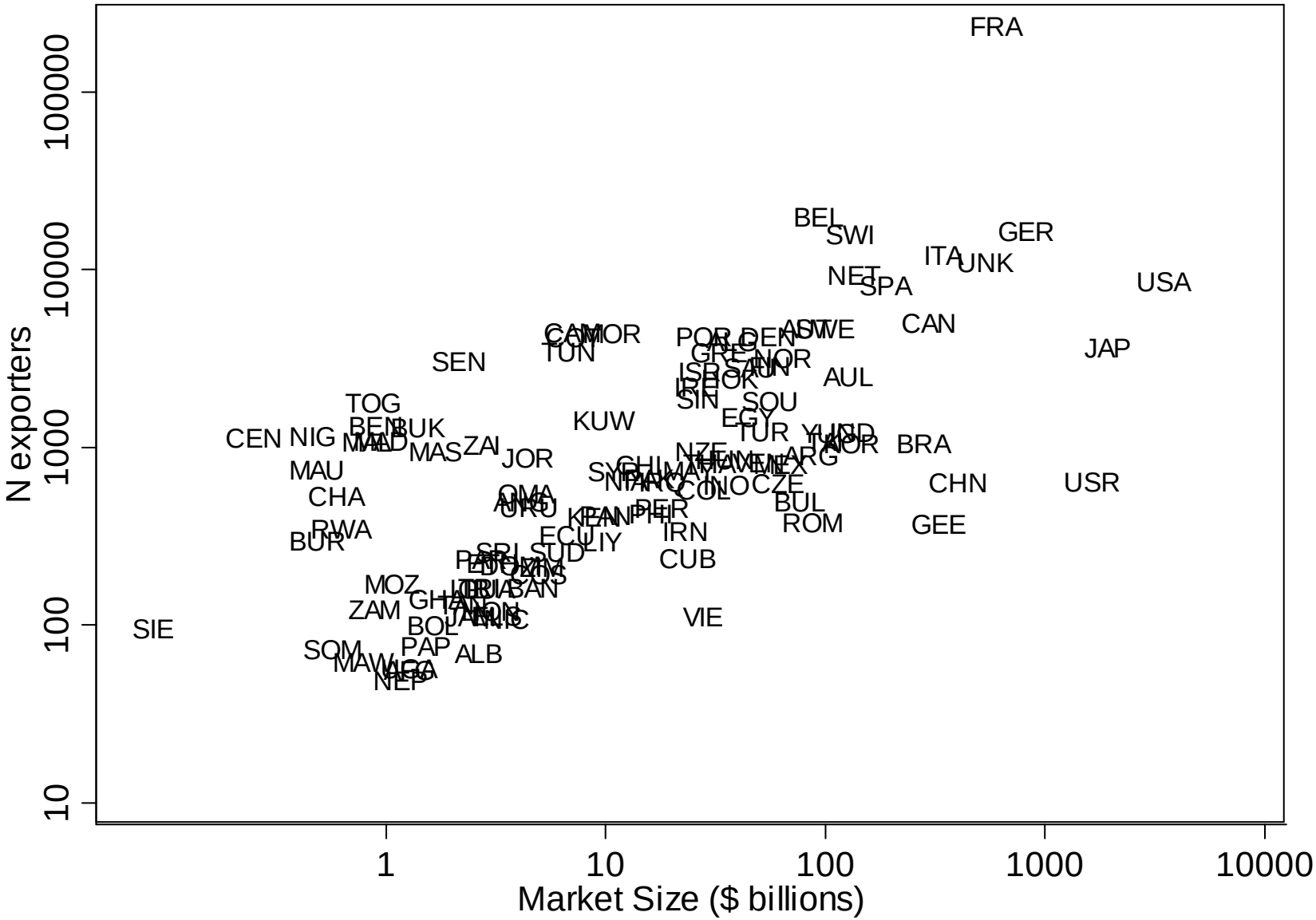
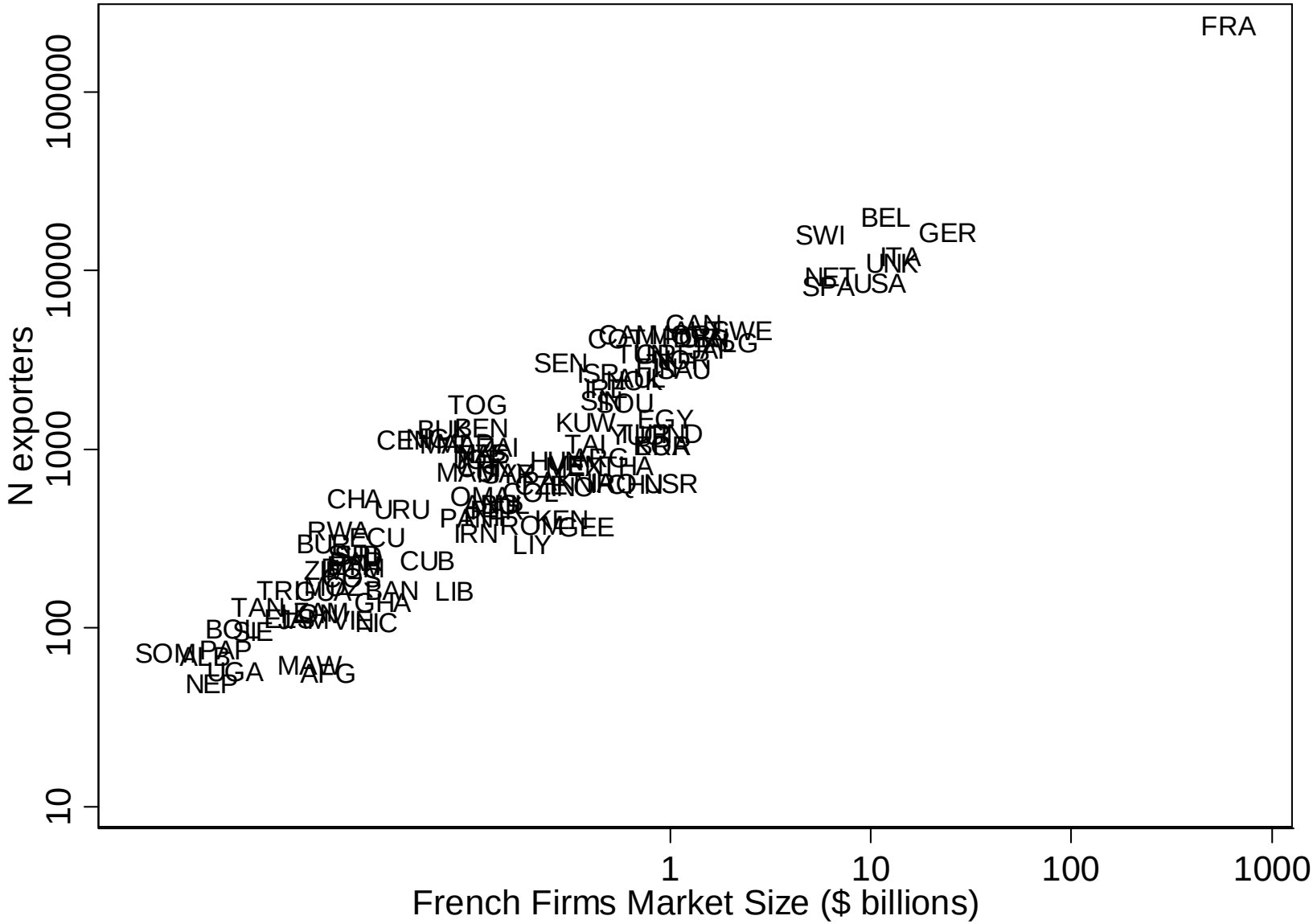


Figure 1A: Entry of French Firms

‘Pecking Order’ for Export Market Destinations



Strong Correlation Between Aggregate Trade and Export Market Participation



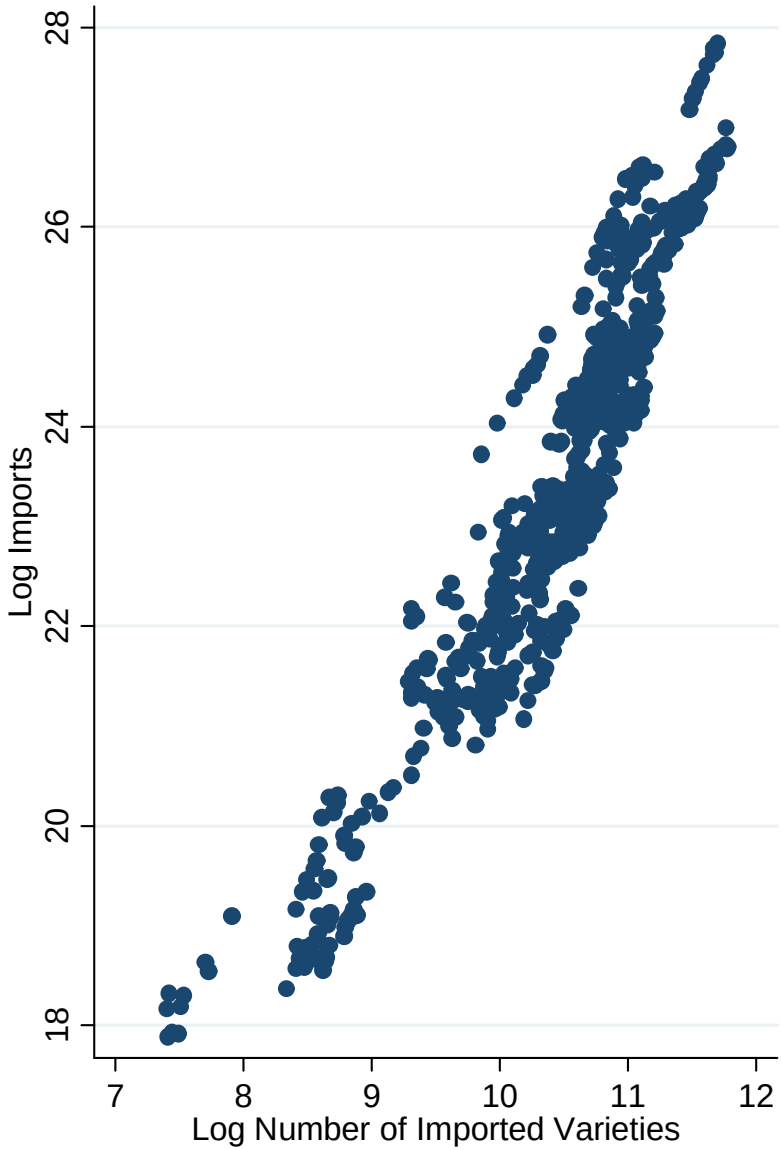
Extensive Margin is Important

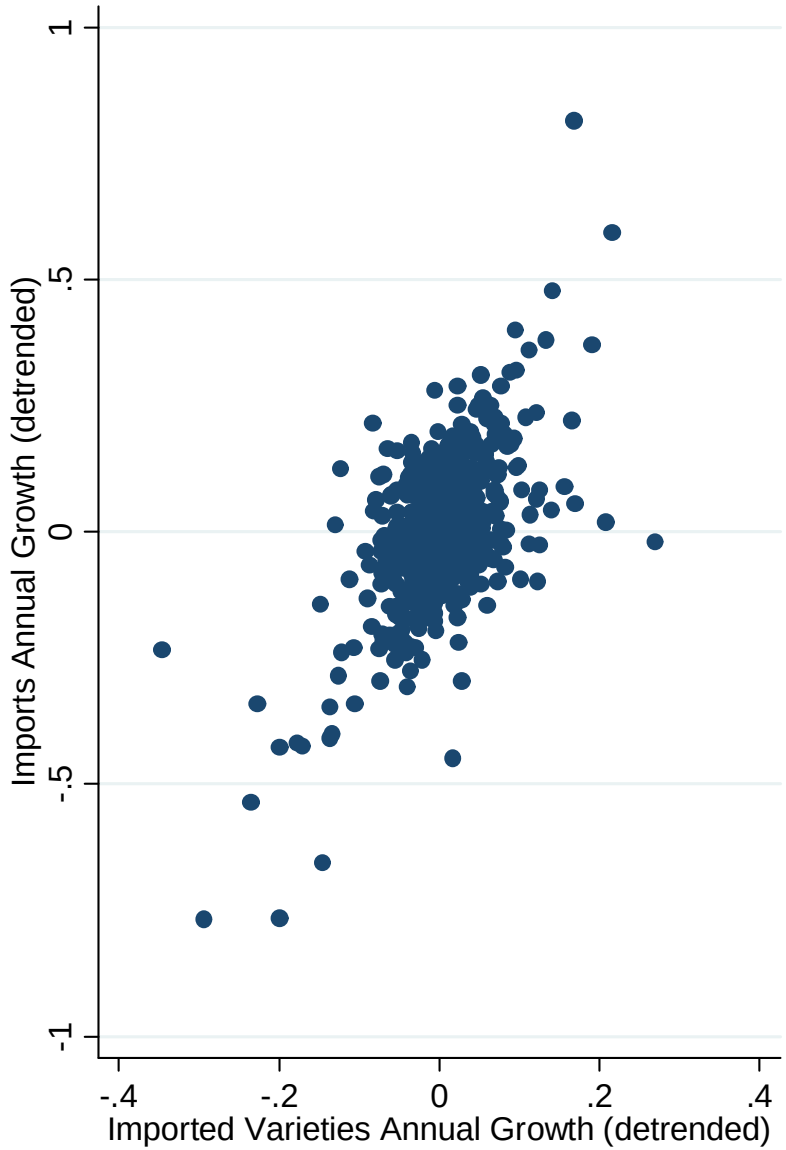
Table 3: Gravity and the Margins of U.S. Exports

	$\ln(\text{Value}_c)$	$\ln(\text{Firms}_c)$	$\ln(\text{Products}_c)$	$\ln(\text{Density}_c)$	$\ln(\text{Intensive}_c)$
$\ln(\text{Distance}_c)$	-1.37	-1.17	-1.10	0.84	0.05
	0.17	0.15	0.15	0.13	0.10
$\ln(\text{GDP}_c)$	1.01	0.71	0.55	-0.48	0.23
	0.04	0.03	0.03	0.03	0.02
Constant	7.82	0.52	3.48	-2.20	6.03
	1.83	1.59	1.55	1.37	1.07
Observations	175	175	175	175	175
R^2	0.82	0.76	0.68	0.66	0.37

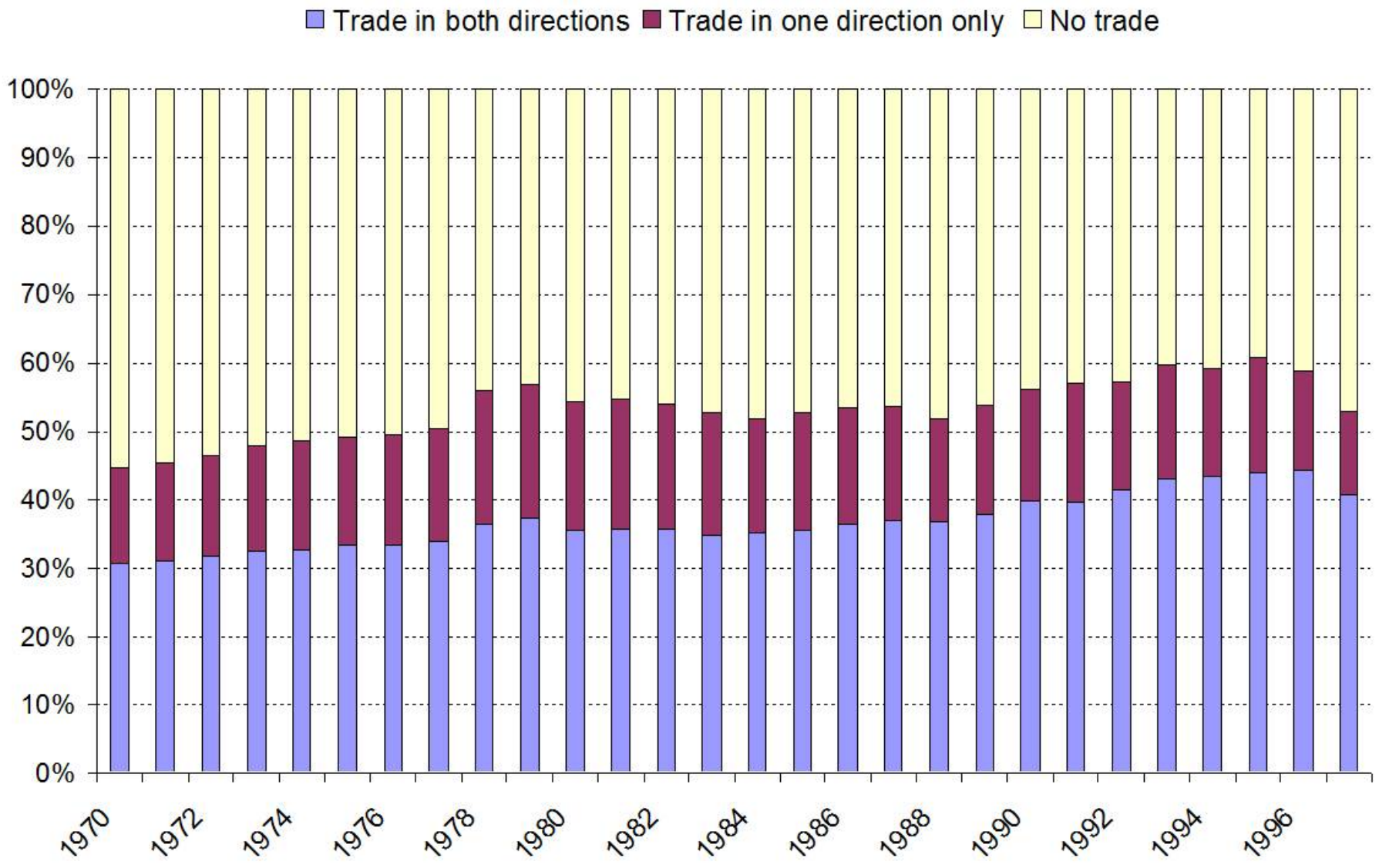
Notes: Table reports results of country-level OLS regressions of U.S. exports or their components on trading-partners' GDP and great-circle distance (in kilometers) from the United States. Standard errors are noted below each coeficient. Data are for 2002.

Extensive Margin is Also Important Over Time





A Lot of Zero Bilateral Trade Flows in the Data



Interaction Between Export Status and Firm Performance

Table 3

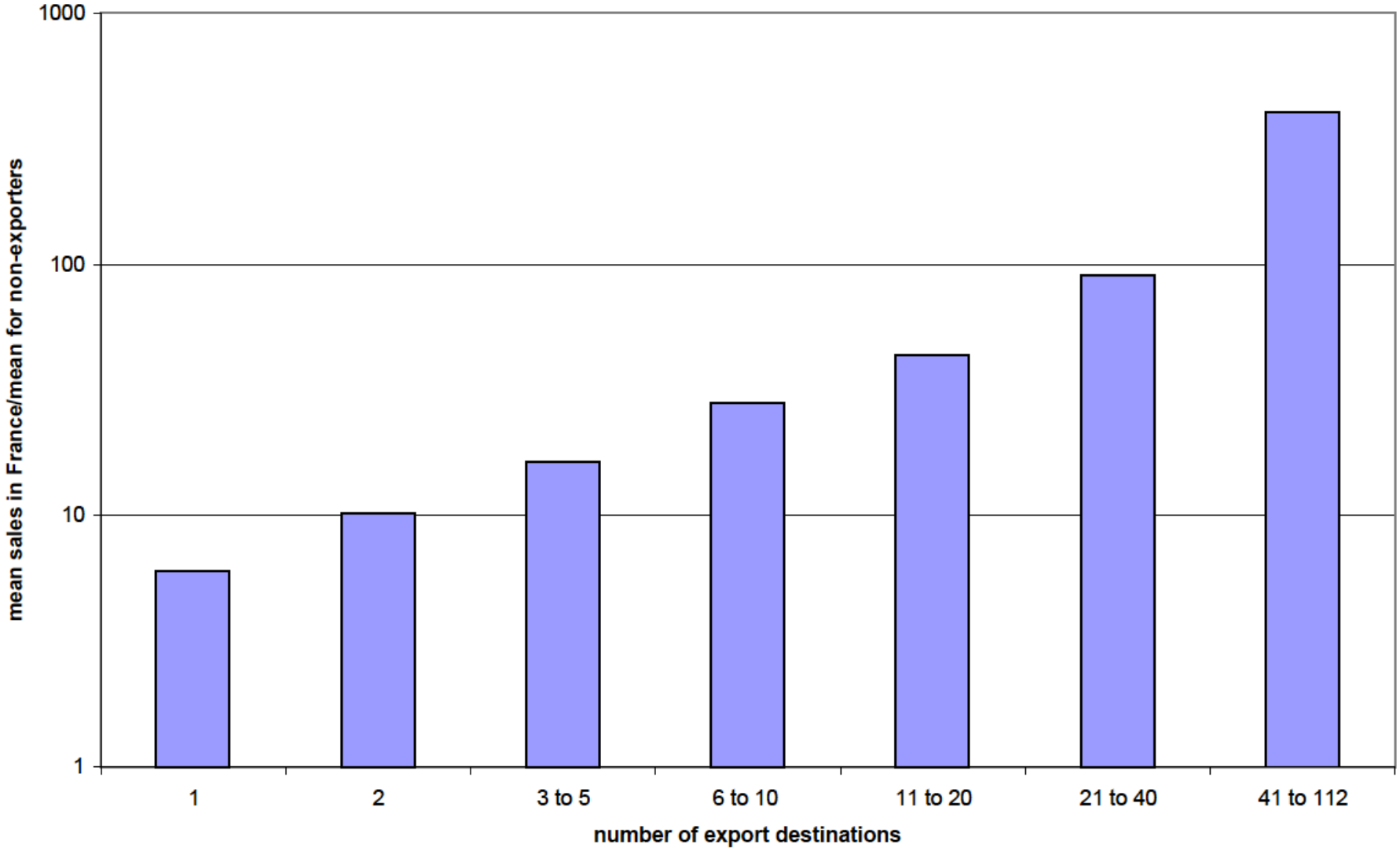
Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	*(2)*	*(3)*
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Sources: Data are for 2002 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm's export status. Regressions in column 2 include industry fixed effects. Regressions in column 3 include industry fixed effects and log firm employment as controls. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982). "Capital per worker" refers to capital stock per worker. "Skill per worker" is nonproduction workers per total employment. All results are significant at the 1 percent level.

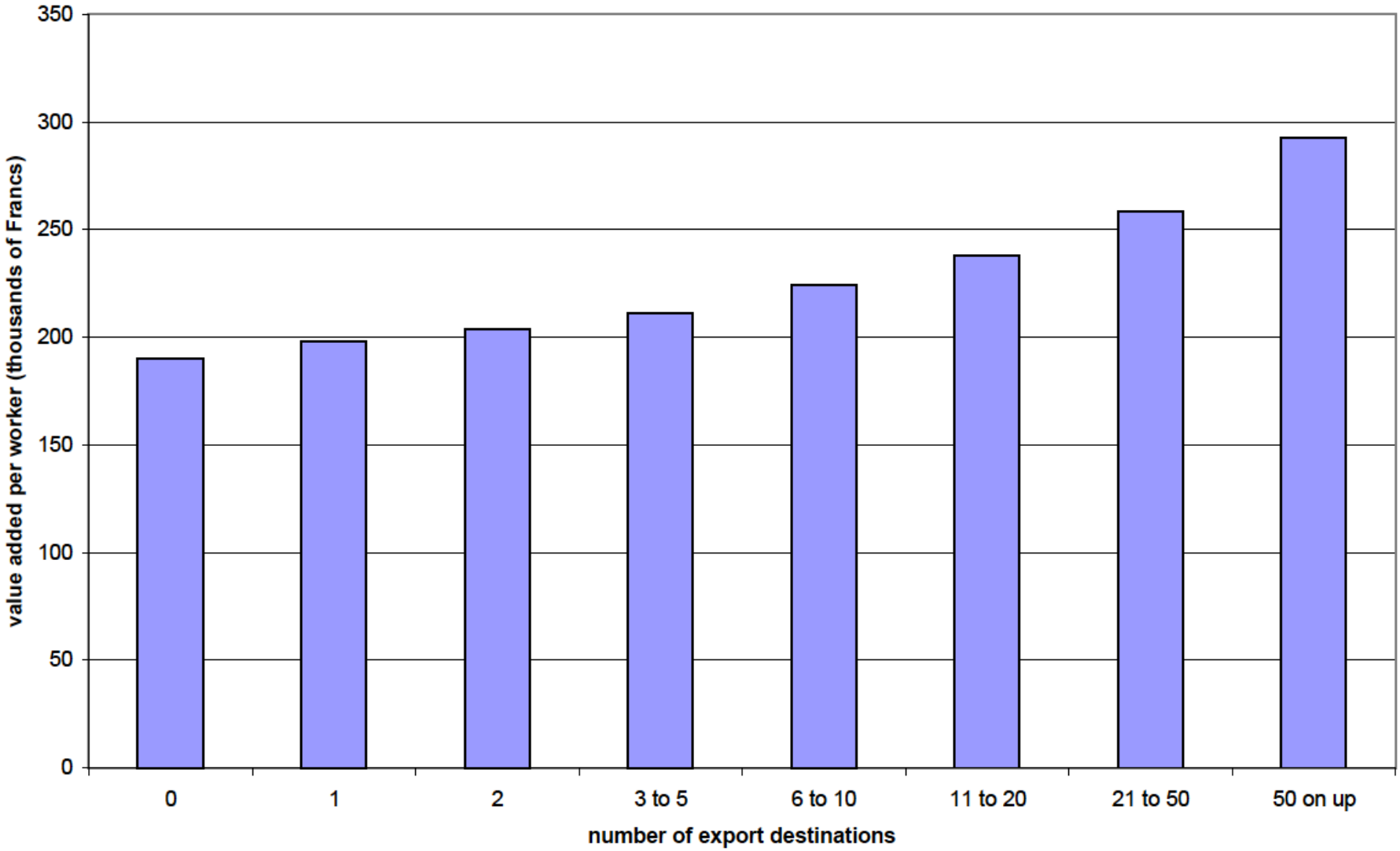
Interaction Between # Destinations and Firm Performance

Figure 7: The Size Advantage of Exporters



Interaction Between # Destinations and Firm Performance

Figure 6: Exporting and Productivity



Similar Export-Performance Structure *Within* Multi-Product Firms

Across Firms

- Stable performance ranking for firms based on performance in any given market (including domestic market) or worldwide sales
- Better performing firms export to more destinations
- Worse performing firms are most likely to exit (overall, or from any given export market)

Across Products within Firms

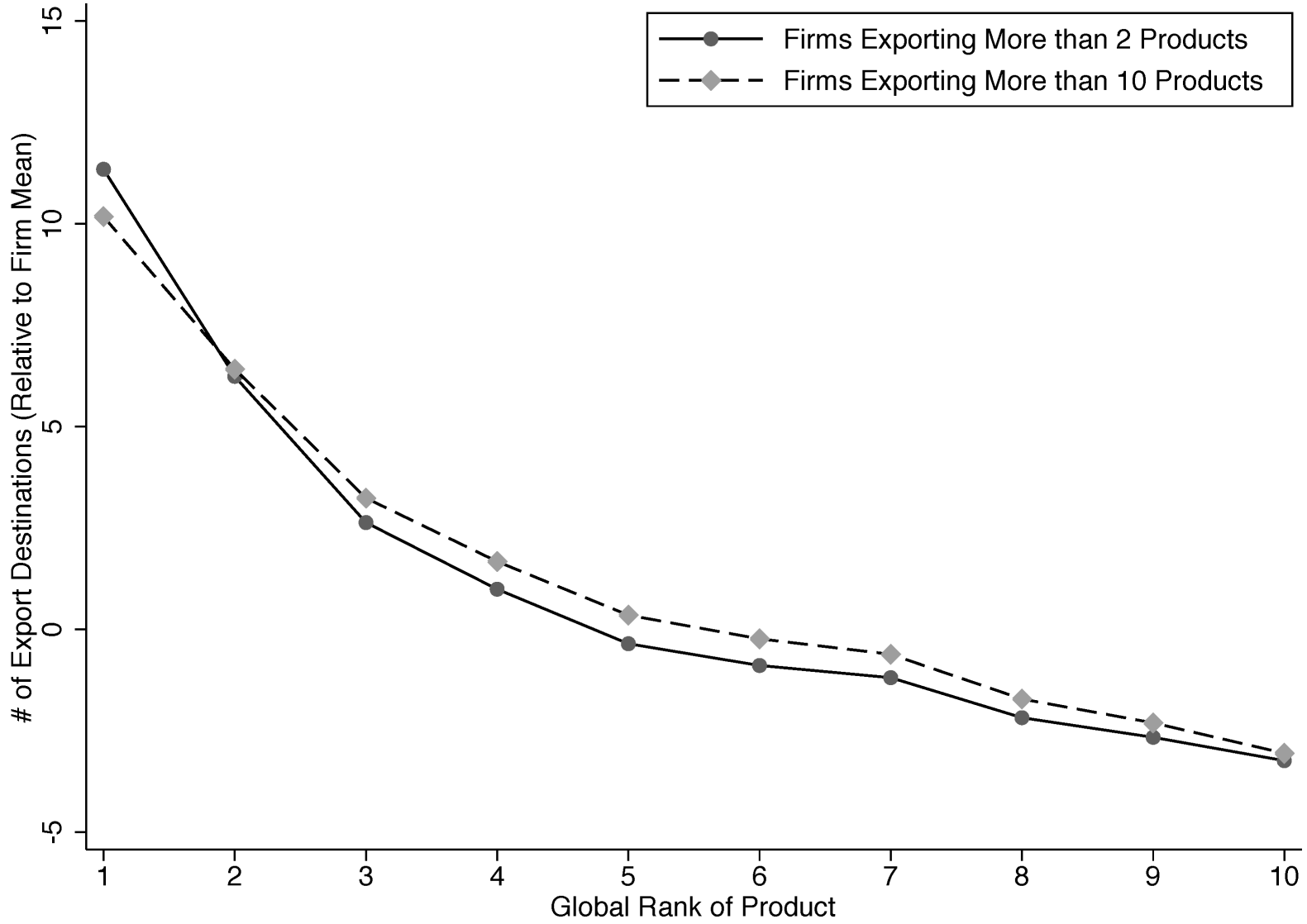
- Stable performance ranking across destinations (and for worldwide sales)
- Better performing products are sold in more destinations
- Worse performing products are most likely to be dropped from any given market

French Multi-Product Firms: Correlations Between Local and Global Product Rankings

Table 1: Spearman Correlations Between Global and Local Rankings

Firms exporting at least:	# products				
to # countries	1	2	5	10	50
1	67.93%	67.78%	67.27%	66.26%	59.39%
2	67.82%	67.74%	67.28%	66.28%	59.39%
5	67.55%	67.51%	67.2%	66.3%	59.43%
10	67.02%	67%	66.82%	66.12%	59.46%
50	61.66%	61.66%	61.64%	61.53%	58.05%

French Multi-Product Firms: Global Ranking and Export Destinations



Reallocation Effects

- In the U.S., on average each year, 1 in every 10 jobs are created and destroyed by entering, exiting, expanding, contracting firms
- Less than 10% of these job "reallocations" reflect shifts across 4-digit sectors (true for other countries)
- For other variables too, (4-digit) industry fixed effects explain little variation in firm dynamics

Table 1

Fraction of Variance of Establishment-Level Growth Rates: Four-Digit Industry Effects

Dependent Variable	R^2
Employment growth	0.057
Capital equipment growth	0.062
Capital structures growth	0.052
Output (gross) growth	0.089
Labor productivity growth (gross output per hour)	0.086
Total factor productivity growth	0.095

Reallocation Effects in International Trade

- Evidence for U.S. (Bernard & Jensen, 2004 OREP):
  - 40% of U.S. manufacturing TFP growth is related to exporters growing faster than non-exporters (in terms of both shipments and employment)
- Evidence for other countries
  - Mexico: Tybout & Westbrook (1995 JIE)
  - Taiwan: Aw, Chen, & Roberts (2000 WBER)
  - Chile: Pavcnik (2002 REStud)
  - Between 1979-86, productivity grew by 19.3% (trade liberalization)
    - 6.6% accounted for by increased productivity within plants
    - 12.7% accounted for by reallocation towards more efficient producers
- Trefler (AER, 2004) for Canada following Canada-U.S. FTA

Interpreting the Evidence

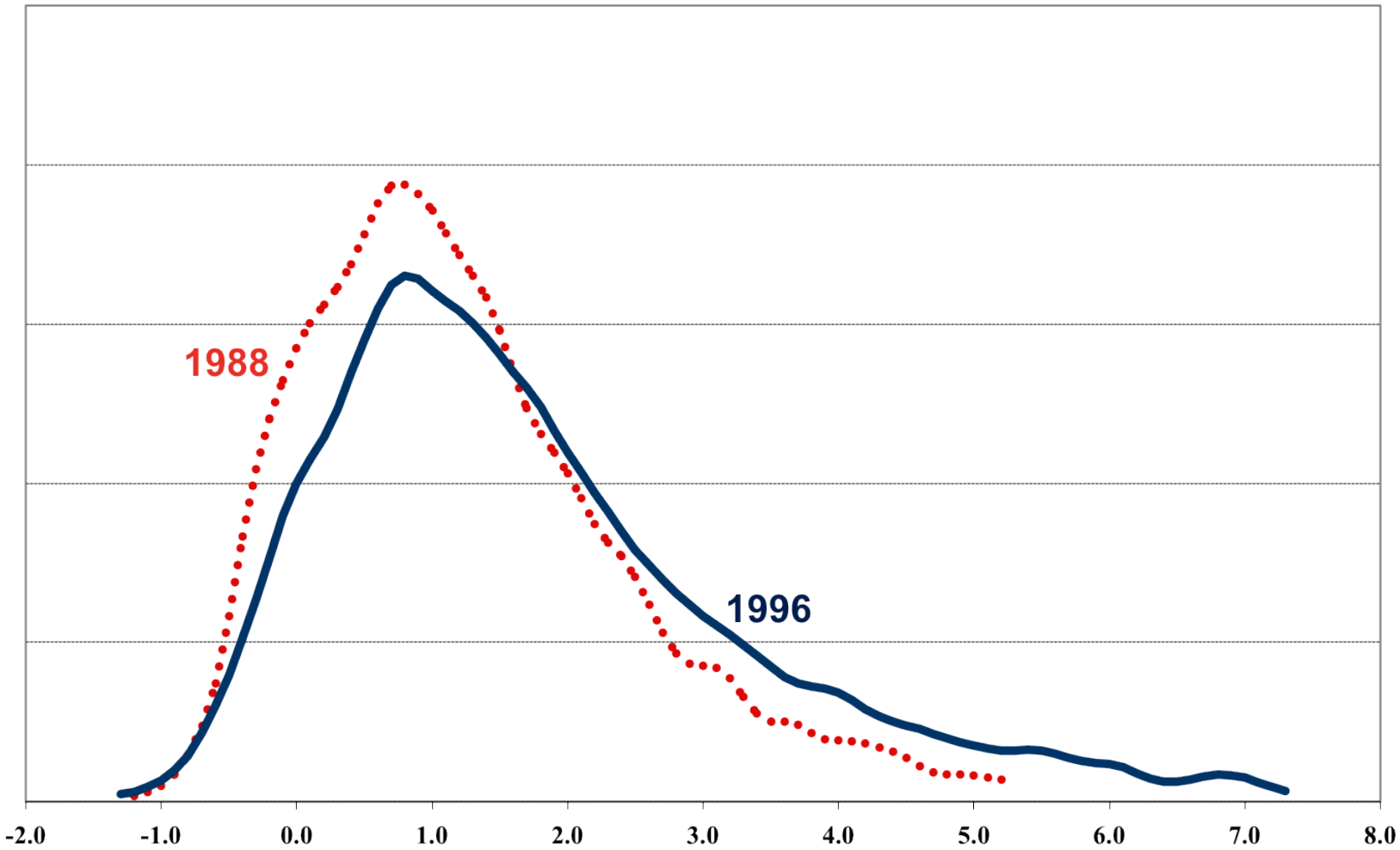
- An obvious question at this point is: Do differences in performance generate selection into exporting, or does exporting generate differences in performance?
- Not straightforward to tease out empirically because firms make joint decisions concerning both export status and technology choice:
  - Verhoogen (2009, QJE): quality upgrade and exports in Mexico
  - Bustos (2010, AER): new exporters in Argentina spend more on technological upgrades
  - Lileeva and Trefler (2010, QJE): similar for Canada
  - see Burstein and Melitz (2011) for overview of theoretical approaches

Reallocation Effects from CUSFTA: Summary

- Trefler (AER 2004), and subsequent work
- Isolates one specific trade liberalization episode
  - Unanticipated, sudden change in trade policy
  - Relatively large changes in trade costs for some sectors
  - Can independently analyze effects of Canadian and US tariff reductions on Canadian firms

CUSFTA: Overall Effect on Productivity

Labor Productivity Distribution of *All* Canadian Manufacturing Plants
1988 and 1996 (employment weighted)



CUSFTA: Effect On Reallocations

- Lileeva (2008) quantifies effect of FTA on exit: Exit rates increased by as much as 16% and is concentrated on less-productive non-exporters
- Effects on export market entry:

CUSFTA: Import Competition vs Export Opportunities

- Effect of lower Canadian tariffs on most impacted import competing sectors:
  - 12% decrease in employment
  - 15% increase in labor productivity
    - Half of gain comes from exit/contraction of low productivity plants
- Effect of lower US tariffs on most impacted export sectors:
  - no significant change in employment
  - 14% increase in labor productivity

CUSFTA: Quantifying the Reallocations

The Effects of the FTA on overall Canadian Manufacturing Productivity
Within- and Between-Plant decomposition

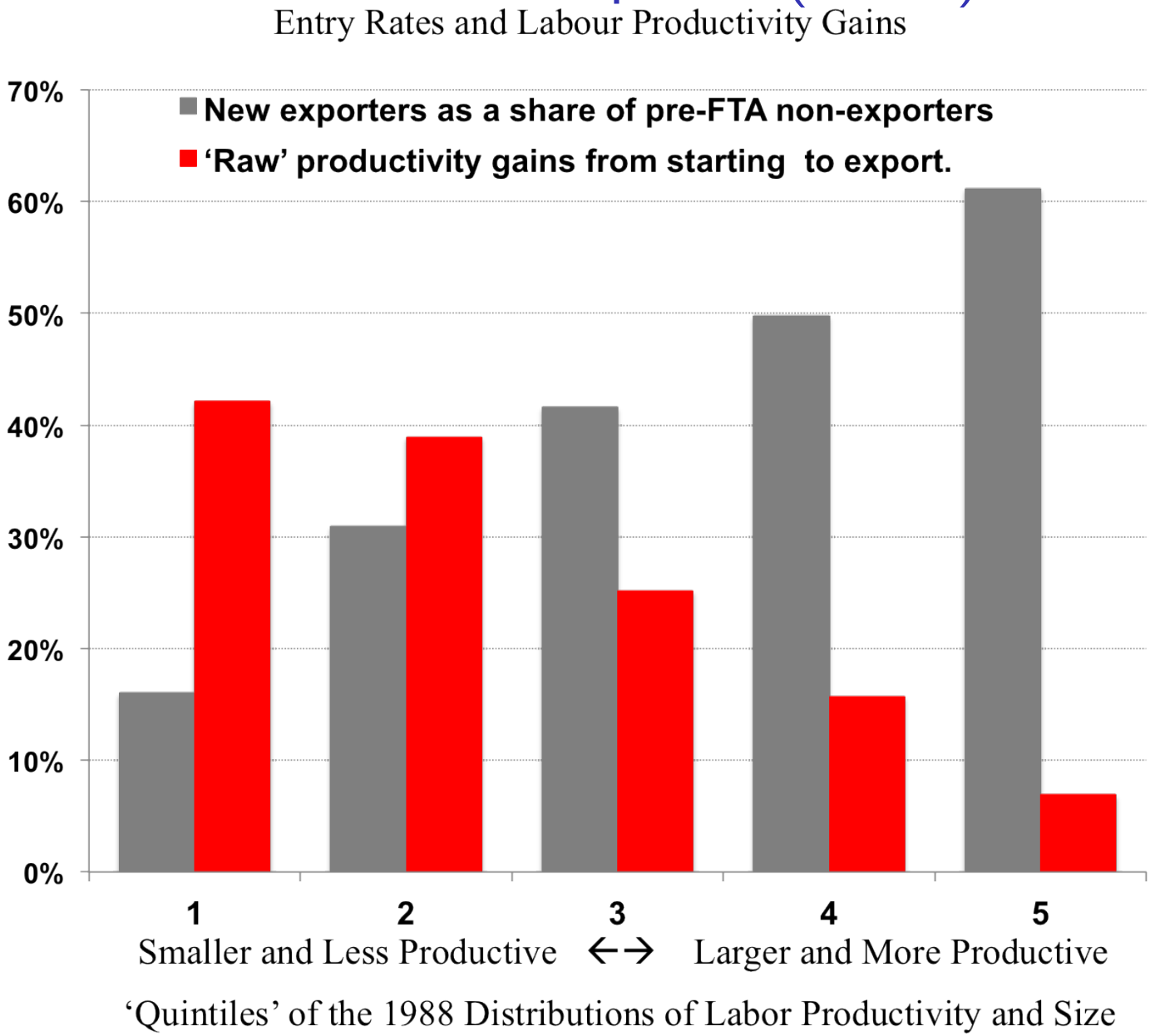
Selection/Reallocation (Between Plants)	
Growth of exporters (most-productive plants)	4.1%
Exit of least-productive plants	4.3%
Within-Plant Growth	
New exporters invest in raising productivity	3.5%
Existing exporters invest in raising productivity	1.4%
Improved access to U.S. intermediate inputs	0.5%
Total	
Total	13.8%

CUSFTA: Firm Innovation Response

Investments in Productivity

	Raw Adoption and Innovation Rates			LATE
	New Exporters	Non-exporters	Difference	Difference
	(1)	(2)	(3)	(4)
Manufacturing Information Systems	16%	6%	10%	7%
Inspection and Communications	18%	10%	8%	8%
Any Product or Process Innovation	30%	20%	10%	8%
Any Product Innovation	26%	14%	12%	7%

CUSFTA: Firm Innovation Response (Cont.)



Exporters and Inovators

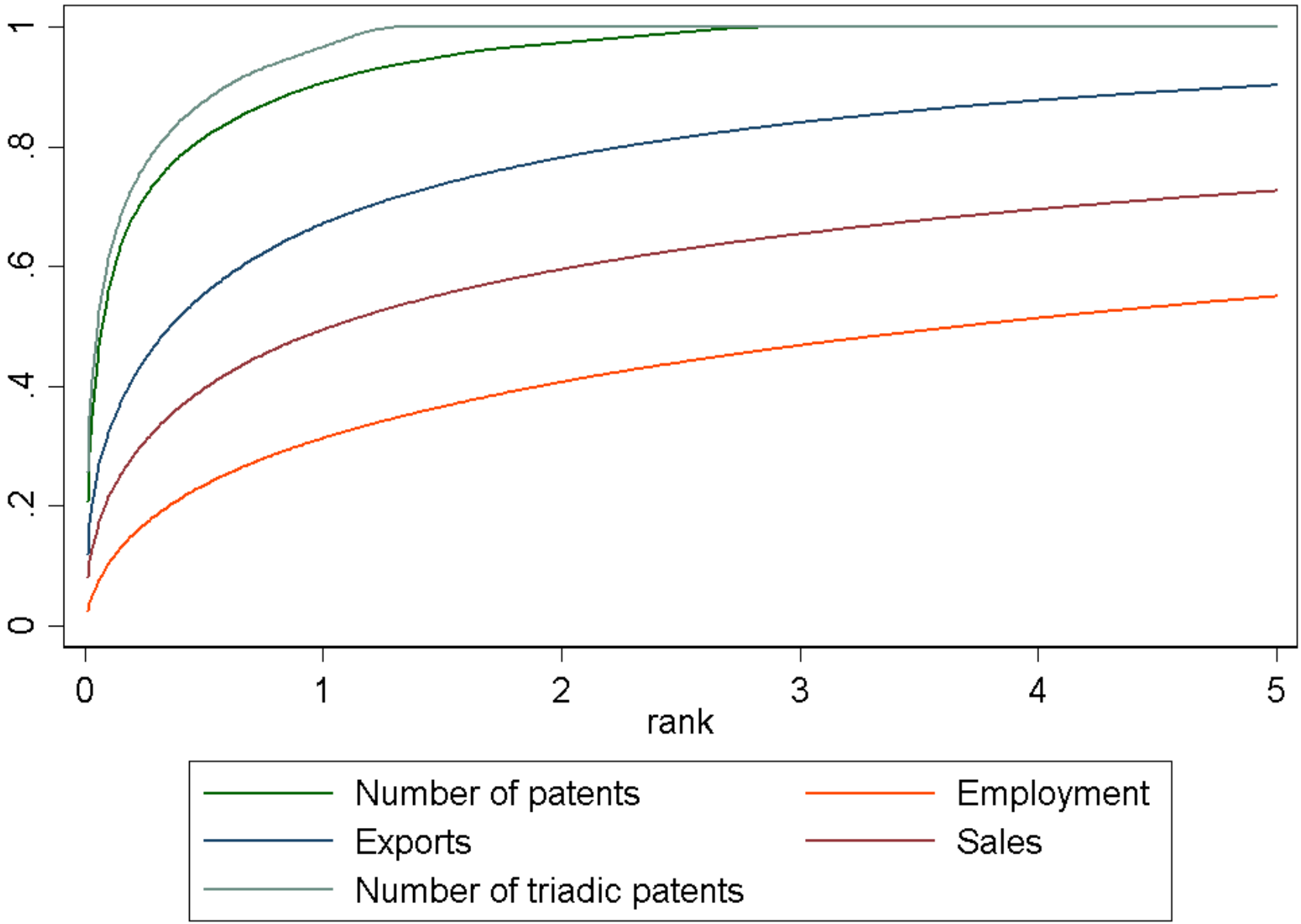
Number of Firms

Number of patents	Number of Export countries						
	1	2	3-5	6-10	11-20	21+	All +
1	4.5	3.5	7.2	6.2	8.9	7.7	38.0
2	1.4	1.0	2.7	2.5	3.1	3.6	14.3
3-5	1.5	0.7	2.9	3.6	3.3	7.6	19.6
6+	0.6	0.9	1.9	2.8	6.7	15.1	28.1
All +	8.1	6.1	14.7	15.1	22.0	34.0	2.7

Value of Exports

Number of patents	Number of Export countries						
	1	2	3-5	6-10	11-20	21+	All +
1	0.0	0.0	0.4	0.3	1.1	6.5	8.4
2	0.0	0.0	0.0	0.2	0.8	3.4	4.5
3-5	0.0	0.0	0.2	0.4	0.6	11.1	12.2
6+	0.0	0.0	0.1	0.8	6.3	67.8	75.0
All +	0.0	0.1	0.7	1.6	8.8	88.8	48.4

Exporters and Innovators: Skewness



Exporters and Innovators: Premia

Panel 1: Premium for being an exporter (among all manufacturing firms)

	(1)	(2)	(3)	Obs.	Firms
log Employment	0.851	0.762	-	931,309	90,688
log Sales	1.613	1.474	0.417	972,956	103,404
log Wage	0.132	0.097	0.110	929,756	90,653
log Value Added Per Worker	0.217	0.171	0.176	918,062	90,055

Panel 2: Premium for being an innovator (among all exporting manufacturing firms)

	(1)	(2)	(3)	Obs.	Firms
log Employment	1.038	0.993	-	639,938	57,267
log Sales	1.277	1.233	0.197	650,013	57,901
log Wage	0.15	0.095	0.110	638,955	57,253
log Value Added Per Worker	0.203	0.173	0.180	629,819	56,920
log Export Sales (Current period exporters)	2.043	1.970	0.859	433,456	56,509
Number of destination countries	13	12	7	656,609	57,991

The Firm FDI Decision

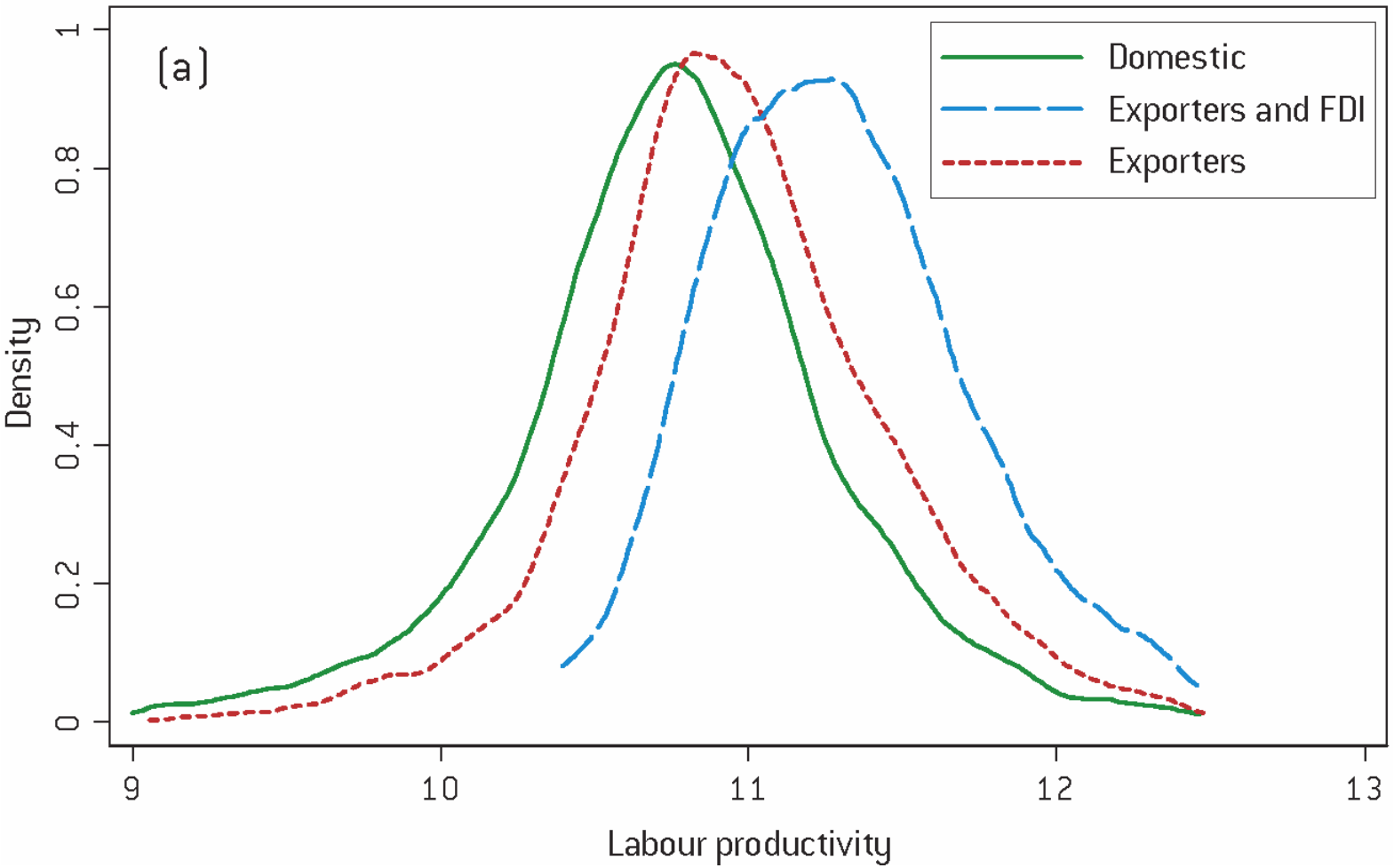
- Firms can also choose to reach foreign customers via horizontal FDI
- Becoming a multinational is associated with an additional "productivity premium" relative to exporting (by non-multinationals)
  - For U.S. publicly held firms (from COMPUSTAT):

Multinational	0.537
	(14.432)
Non-Multinational Exporter	0.388
	(9.535)
Coefficient Difference	0.150
	(3.694)
Number of Firms	3202

Robust T-statistics in parentheses. Coefficients for capital intensity controls and industry effects are suppressed.

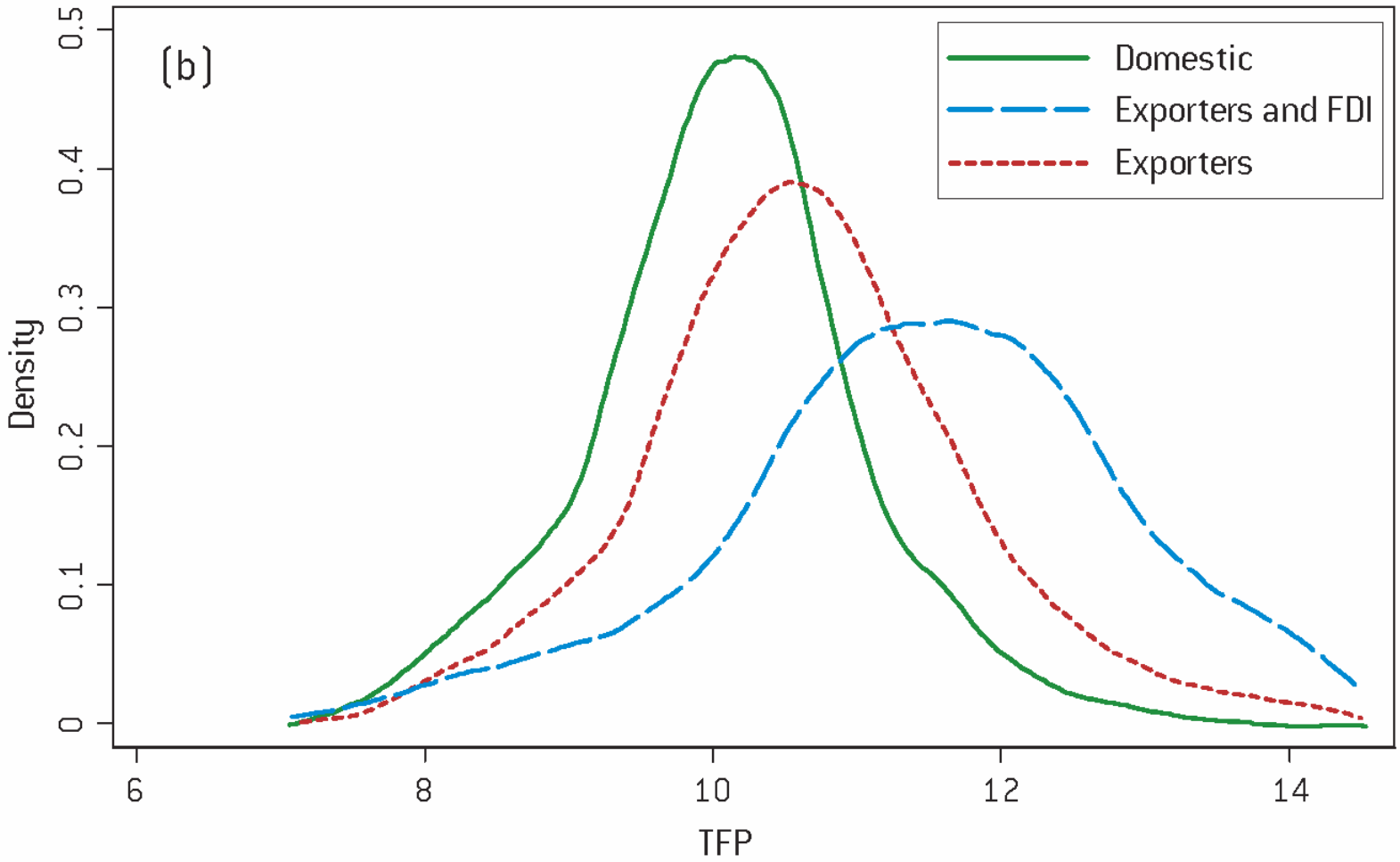
The Firm FDI Decision (Cont.)

- Similar evidence for Belgian firms:



The Firm FDI Decision (Cont.)

- Similar evidence for Belgian firms:



Source: EFIM. Note: Data for Belgium 2004.

Monopolistic Competition, Firm Heterogeneity, and Trade

Lecture Notes

Background: Monopolistic Competition and Firm Heterogeneity

- Start with production/exit decision in closed economy and add export decision in open economy version
  - ... but can also add many other firm-level decisions: innovation, FDI, insource/outsource, finance source, ...
  - All of these decisions can also be modeled over time, but will start with static version
- Key benefit of monopolistic competition: makes modeling firm heterogeneity much more tractable as can use law of large numbers to characterize equilibrium distribution of firms

Preferences and Demand

Assumptions

- Cobb-Douglas preferences over sectors, and C.E.S product differentiation within sectors:

$$U = \sum_j \beta_j \log Q_j \quad , \quad Q_j = \left[\int_{\omega \in \Omega_j} q_j(\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)}$$

  with $\sum_j \beta_j = 1$ and $\sigma > 1$
- Sector $j = 0$ is a homogeneous sector used as numeraire good

Implications

- Let Y be aggregate income, so consumers spend $X_j = \beta_j Y$ on sector j goods
- Within sector j, demand is $q_j(\omega) = A_j p_j(\omega)^{-\sigma}$ where

$$A_j = X_j P_j^{\sigma-1}, \quad P_j = \left[\int_{\omega \in \Omega'_j} p(\omega)^{1-\sigma} d\omega \right]^{1/1-\sigma}$$

Production

- Composite factor L_j with unit cost w_j
- For example, can have $L_j = \bar{\eta}_j S^{\eta_j} U^{1-\eta_j}$ where $\bar{\eta}_j$ such that unit cost $w_j = w_S^{\eta_j} w_U^{1-\eta_j}$
- This factor is used (with same aggregation) in all productive uses $\longrightarrow$ including all fixed costs (overhead, entry, export)
- There is a continuum of firms, each choosing to produce a different variety ω
- Input usage in production is linear in output:

$$l_j = f_j + \frac{q_j}{\varphi}$$

  All firms share the same fixed cost $f_j > 0$ but have different productivity levels indexed by $\varphi > 0$
- Each firm's constant marginal cost is given by

$$MC_j(\varphi) = \frac{w_j}{\varphi}$$

- Assume that numeraire sector is competitive and CRS, so $w_0 = 1$

Firm Behavior

Now focus on equilibrium in sector j and drop all j subscripts

- Each firm faces a residual demand curve with constant elasticity σ
- A firm with productivity φ will set a price

$$p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}$$

leading to revenue

$$r(\varphi) = Ap(\varphi)^{1-\sigma} = A \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} w^{1-\sigma} \varphi^{\sigma - 1}$$

and profit

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf = B\varphi^{\sigma - 1} - wf, \qquad B = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma^{\sigma}} w^{1-\sigma} A$$

- Note that 'variable' or 'gross' profits are proportional to sales (Constant markups and constant AVC/MC across firms sufficient to deliver this)

Firm Performance Measures and Productivity

- Elasticity of substitution amplifies size and profitability differences across firms (given productivity differential):

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma} \quad , \quad \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1} \qquad \forall\ \varphi_1, \varphi_2 > 0$$

- With constant markups (C.E.S.), any productivity gain is passed entirely on to consumers in the form of a lower price
- How does φ relate to empirical measures of firm/plant productivity (based on deflated sales or value-added)?

$$\frac{r(\varphi)}{l(\varphi)} = \frac{w\sigma}{\sigma - 1}\left[1 - \frac{f}{l(\varphi)}\right] \qquad \text{is increasing in } \varphi$$

Firm Entry and Exit: Assumptions

- Firms are identical prior to entry and must pay a fixed investment cost f_E to enter (using same composite factor)
- Upon entry, firms draw their initial productivity level φ from a common distribution $G(\varphi)$
- A firm drawing a low φ may decide to immediately exit and not produce

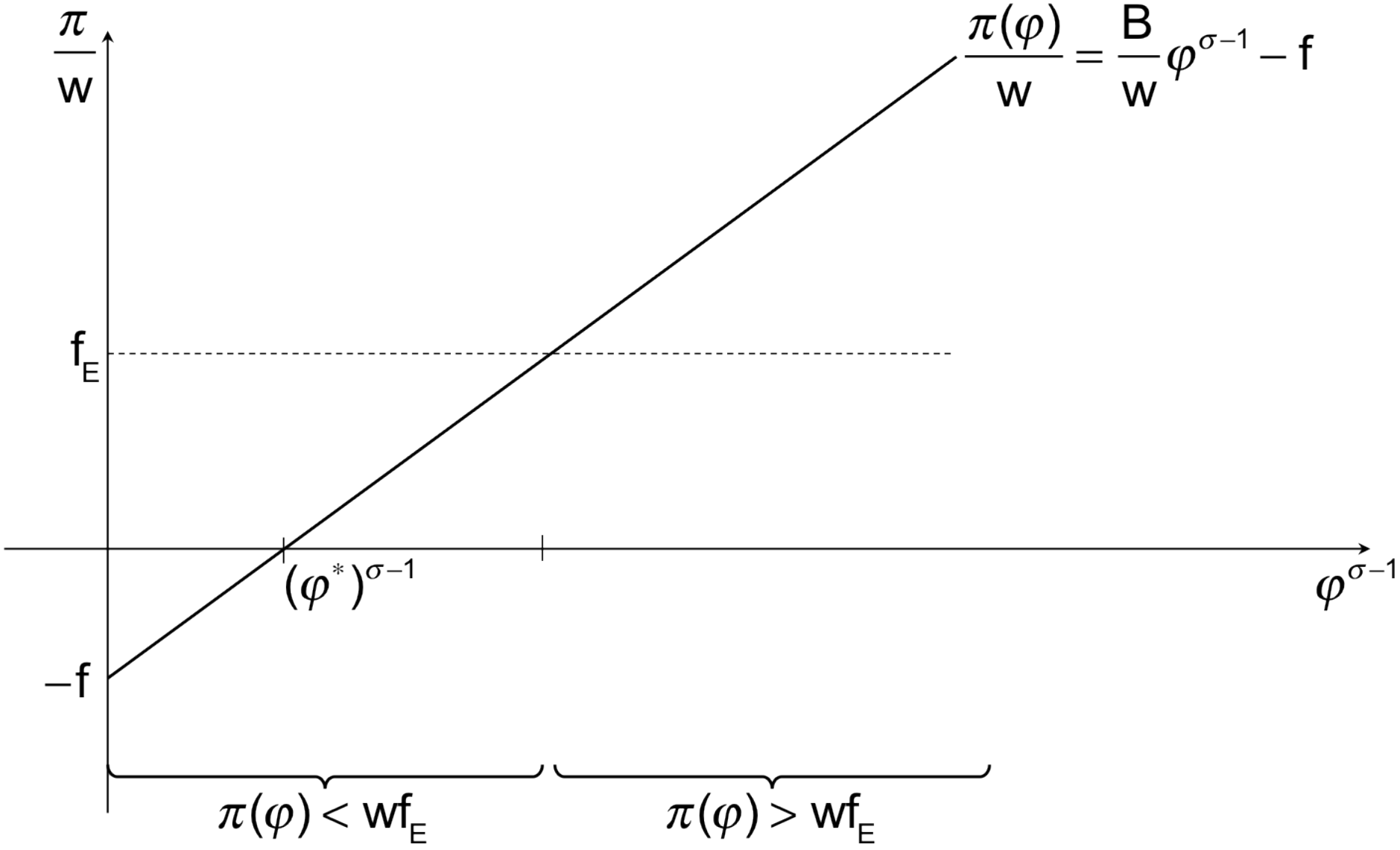
Firm Entry and Exit: Implications

- Survival cutoff φ^* such that $\pi(\varphi^*) = 0$
  - A firms with $\varphi < \varphi^*$ exit and earn $\pi(\varphi) = 0$
- Free entry drives ex-ante (expected) profits (including entry cost) to 0

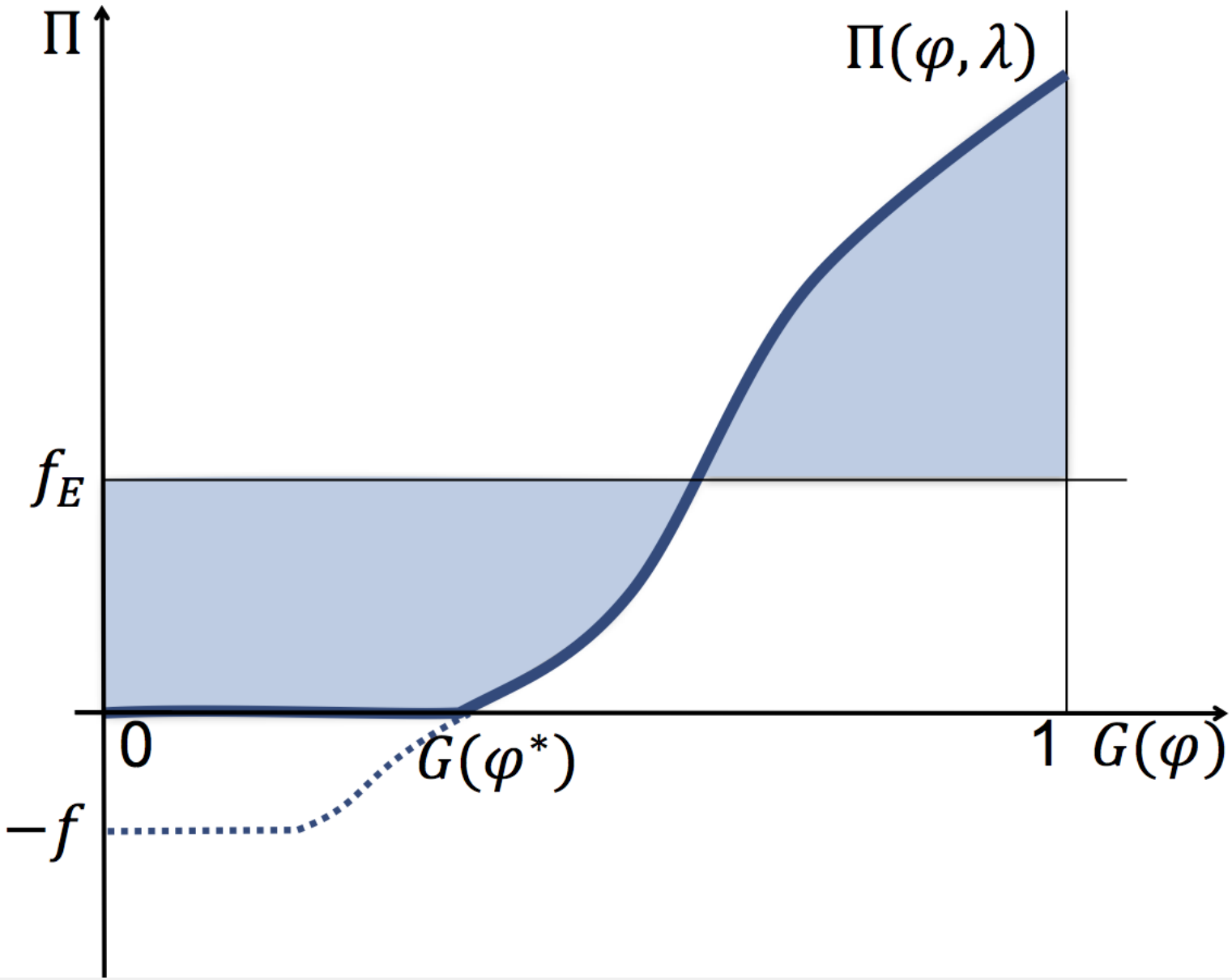
$$\int_0^\infty \pi(\varphi) dG(\varphi) = [1 - G(\varphi^*)] \bar{\pi} = wf_E$$

  where $\bar{\pi}$ is average profit of producing firms
- Recall that $\pi(\varphi) = B\varphi^{\sigma-1} - wf$ so get 2 equilibrium conditions (ZCP & FE) in 2 unknowns: cutoff φ^* and market demand B/w
  $\longrightarrow$ Note that aggregate demand $X = \beta Y$ and wages w do not affect determination of cutoff

Zero Cutoff Profit and Free Entry



Zero Cutoff Profit and Free Entry (Cont.)

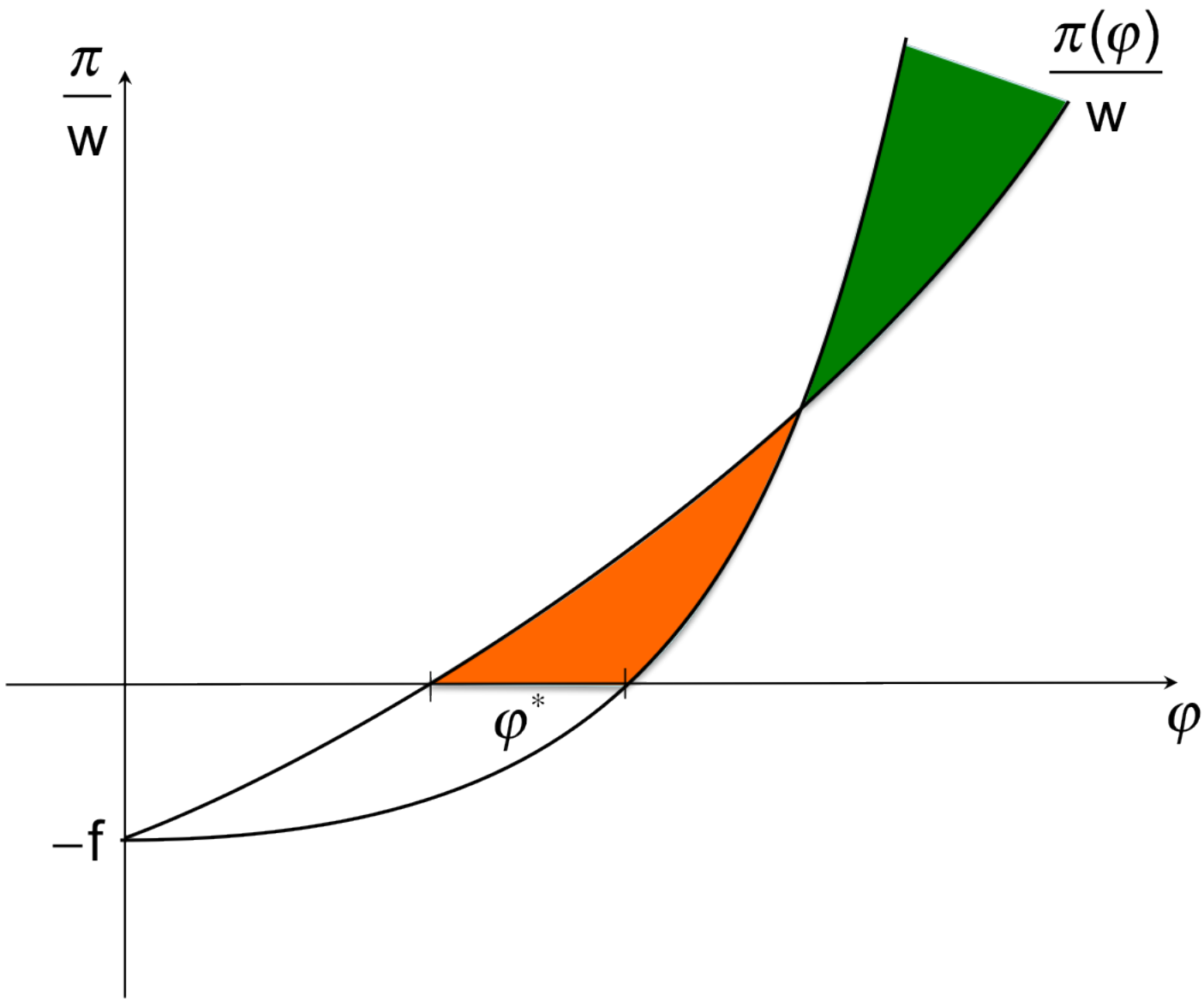


Aggregate Demand and Firm Selection

- Why does aggregate demand $X = \beta Y$ not have any effect on firm selection?
- Partial answer: Aggregate demand does not affect market demand B/w (firm profitability as a function of φ)
- Why does aggregate demand not affect firm profitability?

- Note that CES preferences are key for this result

Zero Cutoff Profit and Free Entry: Non CES Preferences



Aggregate Demand and Entry

What is response of entry M_E to change in aggregate demand X/w?

- Recall that free entry condition pins down average firm profits – and hence average firm revenues:

$$\frac{\bar{\pi}}{w} = \frac{f_E}{1 - G(\varphi^*)}, \qquad \frac{\bar{r}}{w} = \sigma\left(\frac{\bar{\pi}}{w} + f\right)$$

- So aggregate demand X/w does not affect average firm revenue $\bar{r}/w$
  - Recall that cutoff φ^* is determined independently of X/w
- But $X = R \equiv M\bar{r}$ (aggregate sector revenue) in a closed economy
- What does this imply about response of entry M_E and mass of producing firms M to changes in aggregate demand X/w?
- Note: can also use the fact that $A = XP^{\sigma-1} = XM^{-1}\tilde{p}^{\sigma-1}$ remains constant

General Equilibrium

- Simplest way to close GE part of model: assume a single mobile factor $\bar{L}$ $\longrightarrow$ index of country size
- Same wage w in all sectors j
- If numeraire good is produced, then $w = 1$ (otherwise, choose L as numeraire)
- Aggregate income (and expenditure in all sectors) is then exogenously fixed:

$$Y = w\bar{L} \text{ and } R_j = X_j = \beta_j Y$$

Opening to trade as changes in size of integrated world economy

- What is effect of trade on firm selection and welfare?
- What would be the effect of import competition if there were no export opportunities?

Aside: Free Entry and Net Aggregate Profits

Back to sector equilibrium – drop sector j

- Composite sector input is used for both production and entry:

$$\begin{aligned} L &= L_P + L_E \\ &= \frac{R - \Pi}{w} + M_E f_E \end{aligned}$$

- Free entry ensures that aggregate profits $\Pi = M\bar{\pi}$ exactly covers aggregate entry cost $wM_E f_E$
  - $\longrightarrow$ No aggregate profits net of entry cost
- Therefore aggregate sector revenue R is determined by the sector input supply: $R = wL$
  - Note that this is not an accounting identity!

Aside: Average Productivity

- Define $\tilde{\varphi}$ as the productivity level of a firm earning the average profit level $\bar{\pi}$:

$$\pi(\tilde{\varphi}) \equiv \bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{dG(\varphi)}{1 - G(\varphi^*)} \iff \tilde{\varphi}^{\sigma-1} = \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G(\varphi^*)}$$

  (Recall that $\pi(\varphi) \propto \varphi^{\sigma-1}$)
- $\tilde{\varphi}$ is a harmonic average of firm productivity φ, weighted by relative output shares $q(\varphi)/q(\tilde{\varphi})$
- A hypothetical equilibrium with M representative firms with productivity $\tilde{\varphi}$ would feature:
  - Same aggregate statistics, including:

$$Q = M^{\sigma/(\sigma-1)} q(\tilde{\varphi}) \text{ and } P = M^{1/(1-\sigma)} p(\tilde{\varphi})$$

    Note that $p(\tilde{\varphi})$ is the CES weighted average price $\tilde{p}$
- Given the same input supply L and expenditures X, the hypothetical equilibrium would also feature the same M

Aside: Combining ZCP & FE to Solve for Cutoff

$$\begin{aligned}
\pi(\varphi) &= \frac{r(\varphi)}{\sigma} - wf \\
&= \frac{r(\varphi)}{r(\varphi^*)}\frac{r(\varphi^*)}{\sigma} - wf \\
&= \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} wf - wf \qquad \text{using ZCP} \\
&= \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] wf \qquad \forall \varphi \geq \varphi^*
\end{aligned}$$

So FE can be written to solve directly for cutoff:

$$\int_0^\infty \pi(\varphi) dG(\varphi) d\varphi = wf_E \iff J(\varphi^*) wf = wf_E \iff J(\varphi^*) = \frac{f_E}{f}$$

where

$$J(\varphi^*) = \int_{\varphi^*}^\infty \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] dG(\varphi)$$

is an exogenous function that depends only on $G(.)$ and σ

Equilibrium when $G(\varphi)$ is Pareto

If $G(\varphi)$ is Pareto(k) with lower bound $\varphi_{\min}$ then:

- Price $p(\varphi)$ is distributed inverse Pareto(k)
- ... and firm size and gross profit are distributed Pareto$(k/[\sigma-1])$
- $\longrightarrow$ Need $k > \sigma - 1$ for finite average firm size
- Note: Pareto shapes are preserved for truncation on set of producing firms with $\varphi \geq \varphi^*$
- $J(\varphi^*)$ is then:

$$J(\varphi^*) = \frac{\sigma - 1}{k - (\sigma - 1)} \left(\frac{\varphi_{\min}}{\varphi^*} \right)^k$$

so the cutoff is given by

$$(\varphi^*)^k = \frac{\sigma - 1}{k - (\sigma - 1)} \varphi_{\min}^k \frac{f}{f_E}$$

Monopolistic Competition, Firm Heterogeneity, and Trade (Part 2)

Lecture Notes

Open Economy Setup

- Countries $i = 1..N$
- Same consumers preferences in every country: C-D across sectors j and CES σ_j within sectors
- Assume a single homogeneous labor factor with inelastic supply $\bar{L}_i$ across countries
- Assume that homogeneous numeraire good is produced in every country: $\longrightarrow$ wages $w = 1$ in all sectors and countries
- Expenditures by consumers in country i in sector j is $X_{i,j} = \beta_j \bar{L}_i$ (exogenous)
  - $\longrightarrow$ but not longer have balanced trade at the sector level: $X_{ij} \neq R_{ij}$ in general (unless working with a single sector model)

Open Economy Setup (Cont.)

Drop sector j subscript

- Similar process for firm heterogeneity in every country:
  - Firms in country i draw a productivity draw φ from distribution $G_i(\varphi)$ after paying sunk cost f_{Ei}
- Fixed and per-unit trade costs: f_{ni} and τ_{ni} for firms from i to sell to consumers in n
  - Why fixed "market access" cost?
    1. Accounts for distribution, marketing, regulation that do not vary with scale
    2. With CES demand: need fixed cost to induce selection into export markets
- Let f_{ii} be the fixed cost for firms from i to sell in their domestic market
  - Fixed cost combines overhead production cost and "market access": Need not have $f_{ii} < f_{ni}$
- Assume $\tau_{ii} \leq \tau_{ni}$ and set $\tau_{ii} = 1$ and $\tau_{ni} \geq 1$ without loss of generality

Firm Performance Measures

- If firm φ in i sells to consumers in n then it sets a price

$$p_{ni}(\varphi) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ni}}{\varphi}$$

... which generates sales and profits

$$r_{ni}(\varphi) = A_n p_{ni}(\varphi)^{\sigma-1}$$

$$\pi_{ni}(\varphi) = B_n \tau_{ni}^{i-\sigma} \varphi^{\sigma-1} - f_{ni}, \qquad B_n = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} A_n$$

- Account for overhead production cost in "domestic" profit $\pi_{ii}(\varphi)$
  - $\longrightarrow$ Need to assume that firms always sell in their domestic market (This need not be satisfied if there are country asymmetries in market demand A_n or factor prices w_n)

Zero Cutoff Profit and Free Entry Conditions

- ZCP for all i, n:
$$\pi_{ni}(\varphi^*_{ni}) = 0 \iff B_n \left(\tau_{ni}\right)^{1-\sigma} \left(\varphi^*_{ni}\right)^{\sigma-1} = f_{ni}$$
  - If $\varphi < \varphi^*_{ni}$ then firm from i will not sell in n
    ... and $\pi_{ni}(\varphi) = 0$ for those firms
  - φ^*_{ii} is survival cutoff in country i
  - Assume that $\varphi^*_{ii} \leq \varphi^*_{ni}$: Surviving firms always sell in their domestic market
- Total firm profit is $\pi_i(\varphi) = \sum_n \pi_{ni}(\varphi)$
- Same FE condition as in closed economy: $\int_0^\infty \pi_i(\varphi) dG_i(\varphi) = f_{Ei}$
- ZCP & FE jointly determine cutoffs φ^*_{ni} $\forall i, n$ and market demand B_n $\forall n$
  - ... independent of country sizes L_n!
- Still have $B_n \propto A_n = X_n P_n^{\sigma-1}$ with $P_n^{\sigma-1} = M_n^{-1} \tilde{p}_n^{\sigma-1} \implies M_n \propto X_n$
  - If trade is balanced (single sector), then $R_n = X_n = \beta \bar{L}_n$ and $M_n \propto \bar{L}_n$
  - Even if trade is balanced, no longer have $M_{En} \propto \bar{L}_n$: HME for entry
  - Can write M_n and $\tilde{p}_n$ as functions of M_{En} $\forall n$ and φ^*_{ni} $\forall n, i$
    $\longrightarrow$ solve for M_{En} $\forall n$ which yields welfare measure P_n^{-1} $\forall n$

Aside: Solving for Entry, Product Variety, and Prices

- $$M_n = \sum_i M_{ni} = \sum_i \left[1 - G\left(\varphi^*_{ni}\right)\right] M_{Ei}$$
- $$\tilde{p}_n^{1-\sigma} = \frac{1}{\sum_i M_{ni}} \sum_i M_{ni} \tilde{p}_{ni}^{1-\sigma}$$

  where

  $$\tilde{p}_{ni} = \frac{\sigma}{\sigma - 1} \frac{\tau_{ni}}{\tilde{\varphi}_{ni}} \quad \text{and} \quad \tilde{\varphi}_{ni} = \tilde{\varphi}(\varphi^*_{ni})$$

- Can thus use $A_n = X_n P_n^{\sigma-1} = (\beta \bar{L}_n)\left(M_n^{-1} \tilde{p}_n^{\sigma-1}\right)\ \forall n$ to solve for $M_{En}\ \forall n$
  - This also yields $P_n\ \forall n$

Aside: Combining ZCP and FE

- Just as in closed economy, can write profits for all firms as a function of variable profit of cutoff firm:

$$\pi_{ni}(\varphi) = \left[\left(\frac{\varphi}{\varphi^*_{ni}}\right)^{\sigma-1} - 1\right] f_{ni} \qquad \forall \varphi \geq \varphi^*_{ni}, \forall n, i$$

- Define

$$J_i(\varphi^*) = \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] dG_i(\varphi)$$

- Then FE can be written:

$$\int_0^{\infty} \pi_i(\varphi) dG_i(\varphi) = \sum_n \int_0^{\infty} \pi_{ni}(\varphi) dG_i(\varphi) = \sum_n J_i(\varphi^*_{ni}) f_{ni} = f_{Ei}$$

  and note that ZCP $B_n (\tau_{ni})^{1-\sigma} (\varphi^*_{ni})^{\sigma-1} = f_{ni}$ yields φ^*_{ni} as a function of φ^*_{nn}
- So obtain N equations in N cutoffs φ^*_{nn}

Symmetric Trade and Production Costs Across Countries

- Symmetric trade costs: $\tau_{ni} = \tau$ and $f_{ni} = f_X \ \forall n \neq i$
- Symmetric production costs: $f_{ii} = f, f_{Ei} = f_E$ and $G_i(.) = G(.) \ \forall i$

- ZCP for domestic and export markets:

$$B_i \left(\varphi_{ii}^*\right)^{\sigma-1} = f$$
$$B_n \tau^{1-\sigma} \left(\varphi_{ni}^*\right)^{\sigma-1} = f_X$$

- and FE:

$$J(\varphi_{ii}^*) f + \sum_n J(\varphi_{ni}^*) f_X = f_E$$

- Unique solution must be symmetric across countries:

$$B_i = B, \varphi_{ii}^* = \varphi^*, \varphi_{ni}^* = \varphi_X^* \ \forall i, n \text{ and } n \neq i$$

Symmetric Trade and Production Costs Across Countries

- Equilibrium conditions:

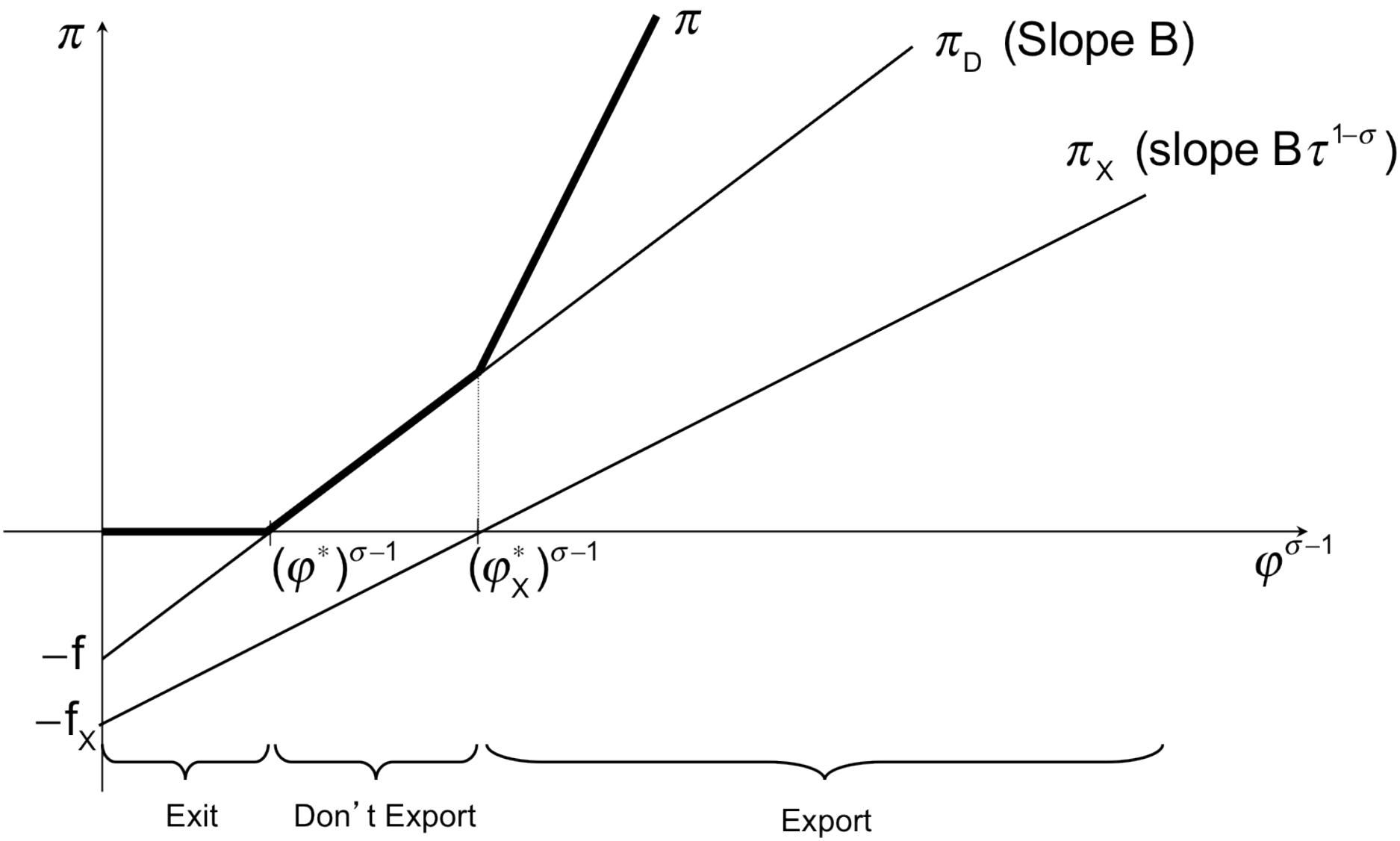
$$B\left(\varphi^*\right)^{\sigma-1} = f$$
$$B\tau^{1-\sigma}\left(\varphi_X^*\right)^{\sigma-1} = f_X$$
$$J(\varphi^*)f + (N-1)\,J(\varphi_X^*)f_X = f_E$$

  jointly determine $\varphi^*, \varphi_X^*, B$
- Must have $\tau^{\sigma-1} f_X > f$ to induce $\varphi_X^* > \varphi^*$ as assumed (necessary condition is $f_X > 0$)
  - Otherwise, get single ZCP condition:

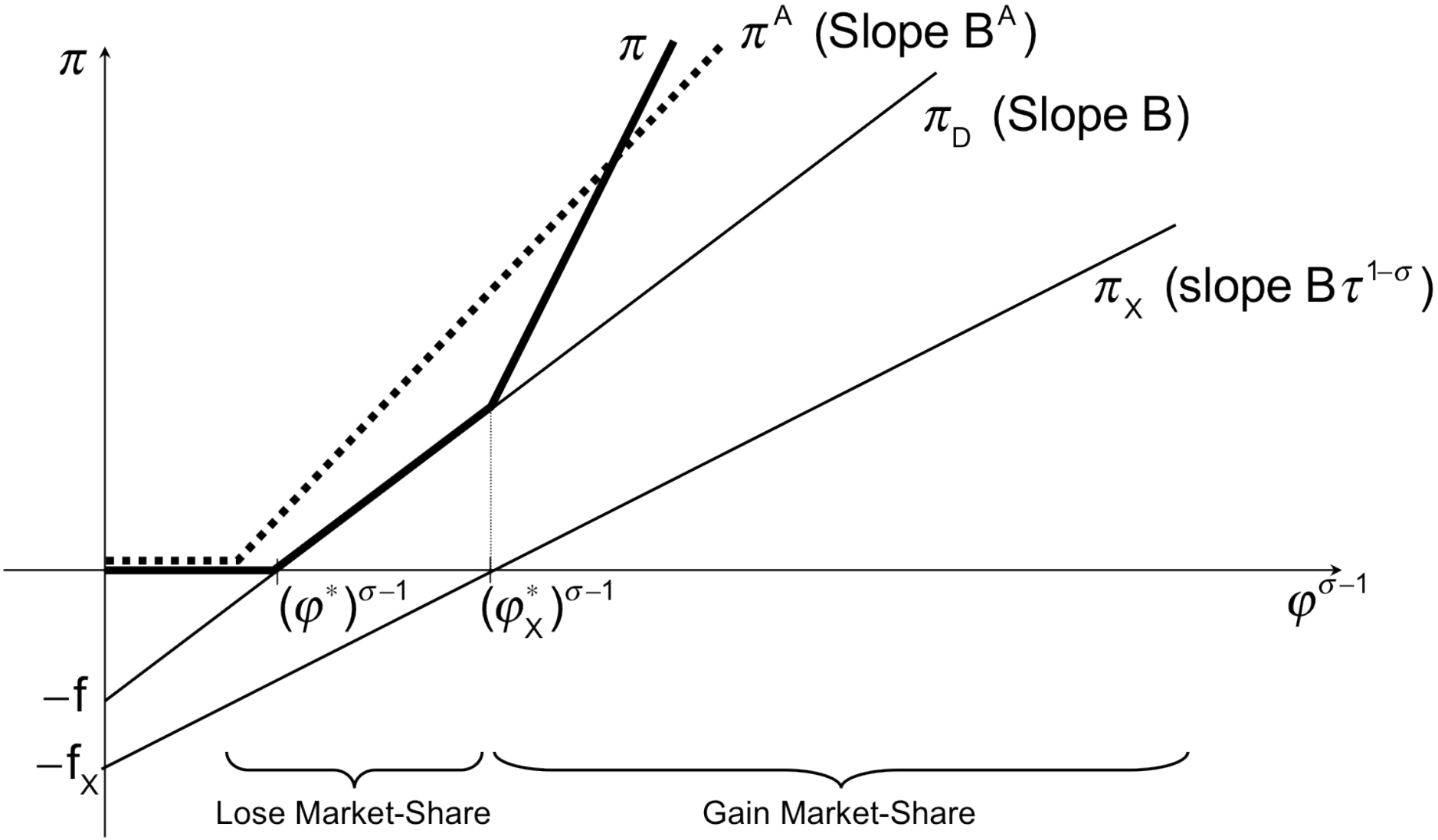
$$\pi(\varphi^*) = B\left(\varphi^*\right)^{\sigma-1}\left[1 + (N-1)\,\tau^{1-\sigma}\right] - f - (N-1)\,f_X = 0$$

- If $G(\varphi)$ is Pareto then $J(\varphi^*) \propto \left(\varphi^*\right)^{-k}$ as in the closed economy example

Symmetric Case



Symmetric Case (Cont.)



Changes in the Exposure to Trade

- Scenarios
  - Increase in the number of trading partners N
  - Decrease in variable trade costs τ
  - Decrease in fixed export costs f_x
- In all cases, an increase in the exposure to trade is associated with:
  - Tougher selection: exit of the least productive firms from the industry ($\varphi^* \nearrow$)
  - Market share reallocations from less productive firms to more productive ones
  - Welfare gain ($P_n \searrow \forall n$)

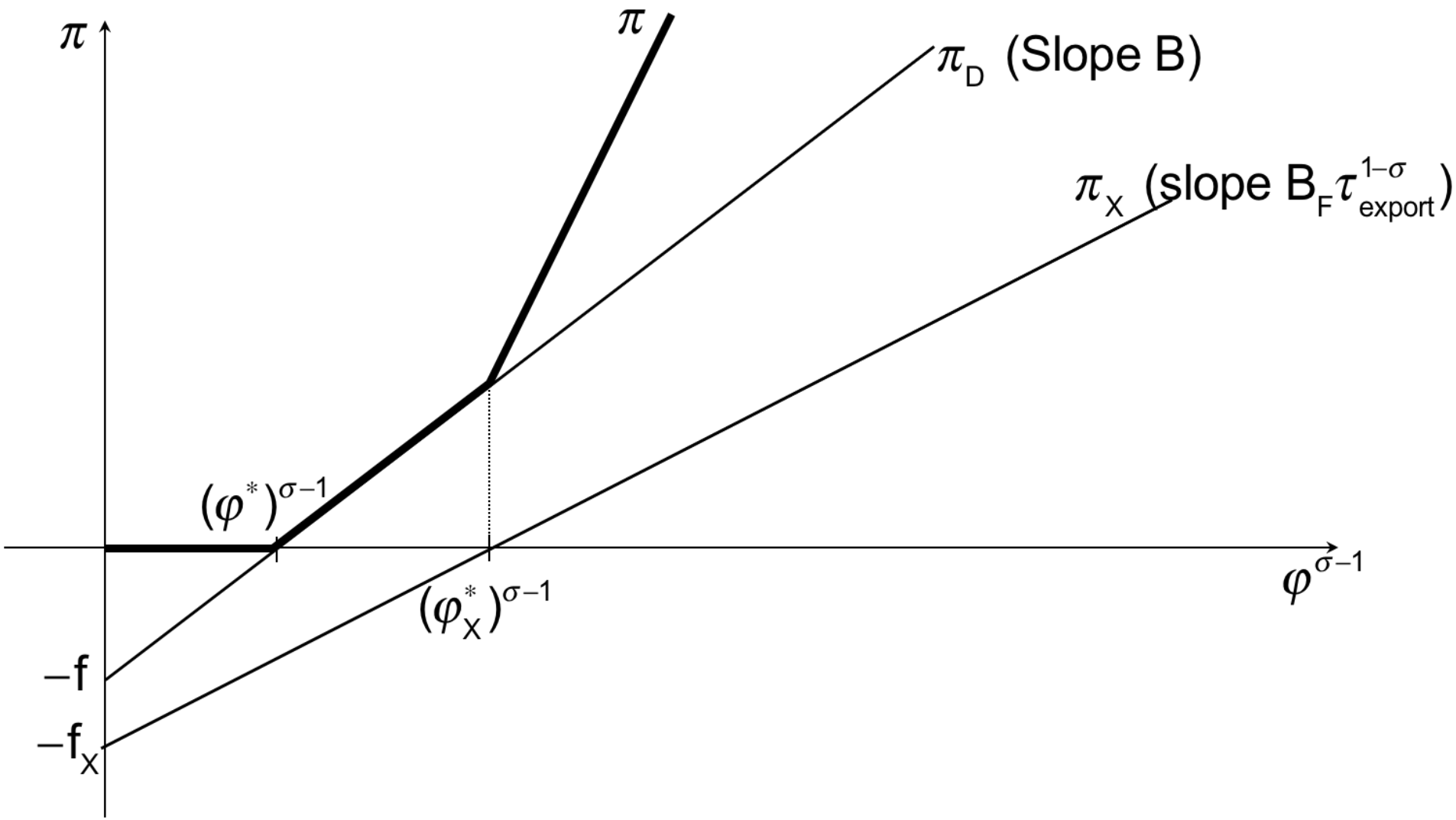
Effects of Trade on Selection: What are the Channels?

- There are two potential channels for the effects of trade on selection:
  - Increased competition from imports
  - Increased competition from entry in domestic market (driven by higher export profit opportunities)
  - With CES product differentiation:
    The effects of increased competition from imports is exactly offset by lower entry
- To see this let's separately consider lower import and export barriers (asymmetric trade costs)
  - To keep things tractable, consider a "small open economy"
    (See Demidova & Rodrigues-Clare, NBER 17521)

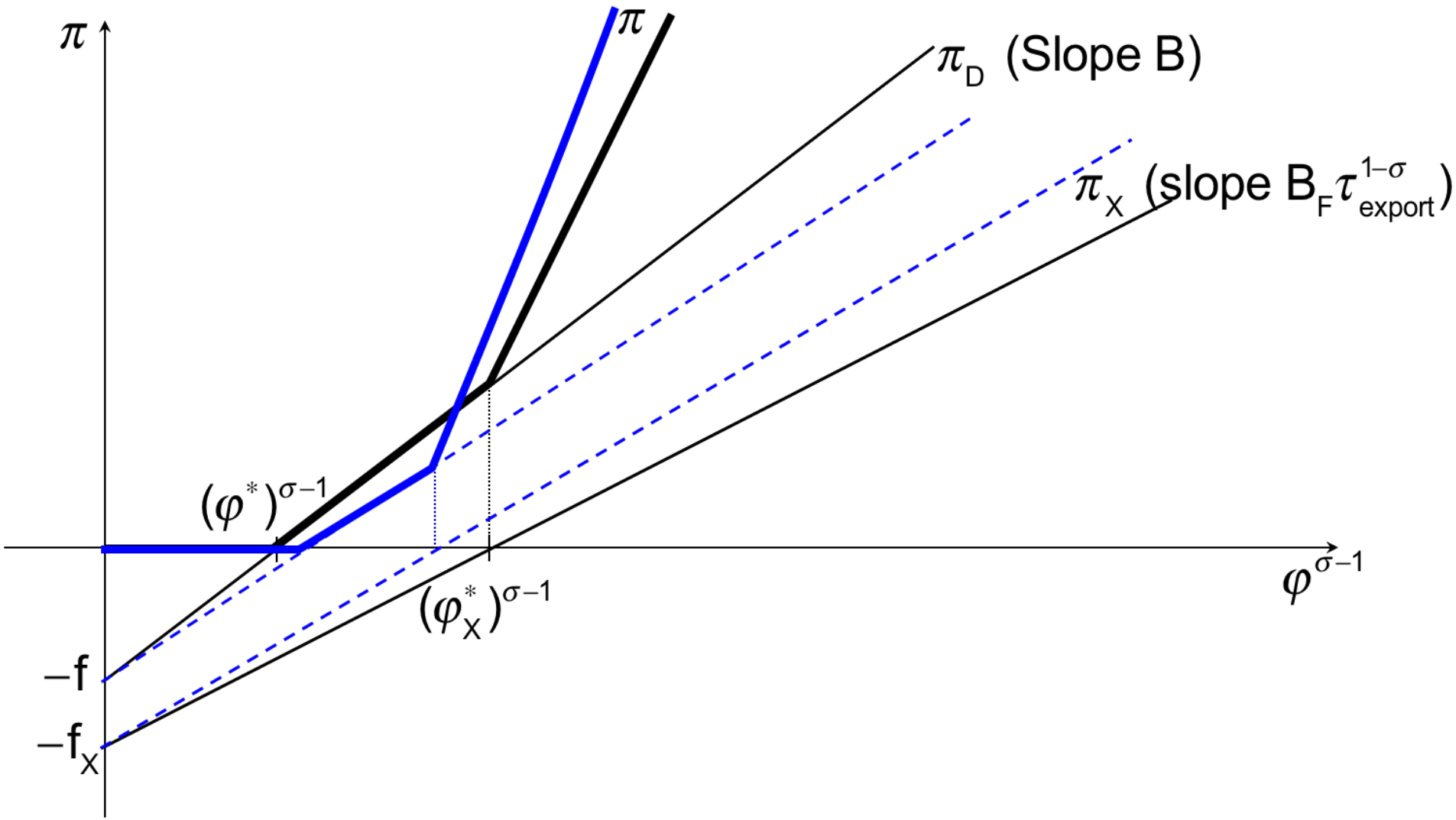
Effects of Trade on Selection: What are the Channels?

- Define a "small open economy":
  - Foreign country is large relative to Home, so country-level variables in Foreign do not respond to changes in Home (or trade costs between Home and Foreign). Specifically:
    - The price index P_F (and B_F), wage w_F, and the distrib. of producing firms (M_{EF} and φ_F^*) are exogenous to changes in Home
  - Note that the Foreign export cutoff to Home φ_{FX}^* is still endogenous
- Consider first case with outside sector and $w = w_F$
  - Effects of τ_{export} $\searrow$: φ_X^* $\searrow$, M_E $\nearrow$, φ^* $\nearrow$
  - Effects of τ_{import} $\searrow$: φ_X^* $\longrightarrow$, M_E $\searrow$, φ^* $\longrightarrow$ (lower M_E exactly compensates for lost sales from increased imports)
- Now consider case with single sector j (no outside sector): w adjusts so labor demand matches exogenous labor supply L and trade is balanced
  - Effects of τ_{export} $\searrow$: w $\nearrow$ (lower exports to re-establish trade balance) and φ_X^* $\searrow$, M_E $\nearrow$, φ^* $\nearrow$
  - Effects of τ_{import} $\searrow$: w $\searrow$ (increase exports to re-establish trade balance with higher imports) and φ_X^* $\searrow$, M_E $\searrow$, φ^* $\nearrow$

Comparative Statics for Small Open Economy



Effects of Lower Export Cost for Small Open Economy



Model Extensions

- Structure of fixed export costs
  - Different export strategies involving different levels of costs
  - Regional and country level externalities
- Dynamics
  - Investment and technological choice linked to export status
  - Effects of anticipated future liberalization
- Labor Markets
  - Long-run unemployment
  - Short-run unemployment due to reallocations
    - Model emphasizes importance of factor market flexibility
    - ... but ignores potential displacements costs
    - ... and potential link between trade liberalization and worker uncertainty over job tenure

VES Preferences: "Competitive" Effects of Trade

Lecture Notes

VES Monopolistic Competition and Trade

- Open economy version of Zhelodbodko et al (2012)
- ... along with long – and growing – literature on trade models with endogenous markups

VES Preferences and Demand

- Demand for differentiated varieties q_i is generated by L^c consumers who solve:

$$\max_{q_i \geq 0} \int u(q_i)di \text{ s.t. } \int p_i q_i di = 1$$

(consumer expenditures on differentiated varieties normalized to 1)

- So long as

(A1) $u(q_i) \geq 0$; $u(0) = 0$; $u'(q_i) > 0$; and $u''(q_i) < 0$ for $q_i \geq 0$

this leads to a downward sloping inverse demand function (per consumer)

$$p(q_i; \lambda) = \frac{u'(q_i)}{\lambda}, \text{ where } \lambda = \int_0^M u'(q_i)q_i di > 0$$

is the marginal utility of income (spent on differentiated varieties)

- $\phi(q_i; \lambda) \equiv [u'(q_i) + u''(q_i)q_i]/\lambda$ is the associated marginal revenue
- Let $\varepsilon_p(q_i) \equiv -u''(q_i)q_i/u'(q_i)$ and $\varepsilon_\phi(q_i) \equiv -\phi'(q_i)q_i/\phi(q_i)$ denote the elasticities of inverse demand and marginal revenue (independent of λ)

Firms and Production

Single factor of production: labor (wage normalized to 1)

- Firm productivity φ (output per worker); same overhead labor cost f
- Firms maximize operating profits per-consumer

$$\pi(\varphi;\lambda) = \max_{q_i \geq 0} [p(q_i)q_i - q_i/\varphi]$$

$\longrightarrow$ Maximized quantity $q(\varphi;\lambda)$ (per-consumer) solves $\phi(q) = \varphi^{-1}$ (MR=MC) so long as **(A2)** $\varepsilon_p(q) < 1$ and **(A3)** $\varepsilon_\phi(q) > 0$ (MR positive and decreasing)

- This leads to standard markup pricing

$$p(\varphi;\lambda) = \frac{1}{1 - \varepsilon_p(q(\varphi;\lambda))}\varphi^{-1}$$

and revenue (per-consumer) $r(\varphi;\lambda) = q(\varphi;\lambda)p(\varphi;\lambda)$

- Total firm profit (across consumers) is

$$\Pi(\varphi;\lambda) = L^c\pi(\varphi;\lambda) - f$$

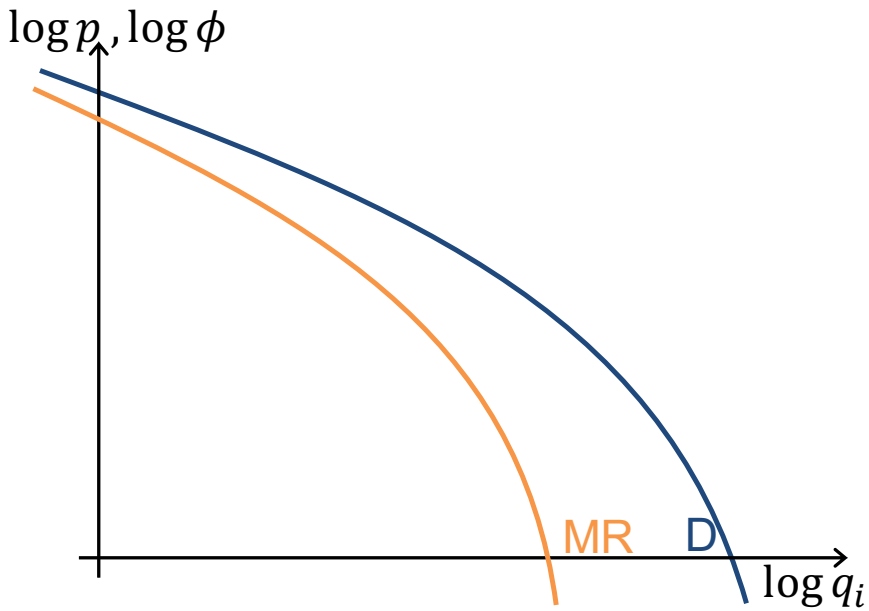
- Assume monopolistic competition: Firms take λ as outside their control

Firms and Production (Cont.)

- λ is a key endogenous market-level variable capturing the extent of competition for a given level of market demand
- Analogous to the CES price index
- Increases in λ shift down/in the firms' demand curves, and lowers the firms' optimal choice of price and quantity, and resulting profits
  $\longrightarrow$ Increased competition

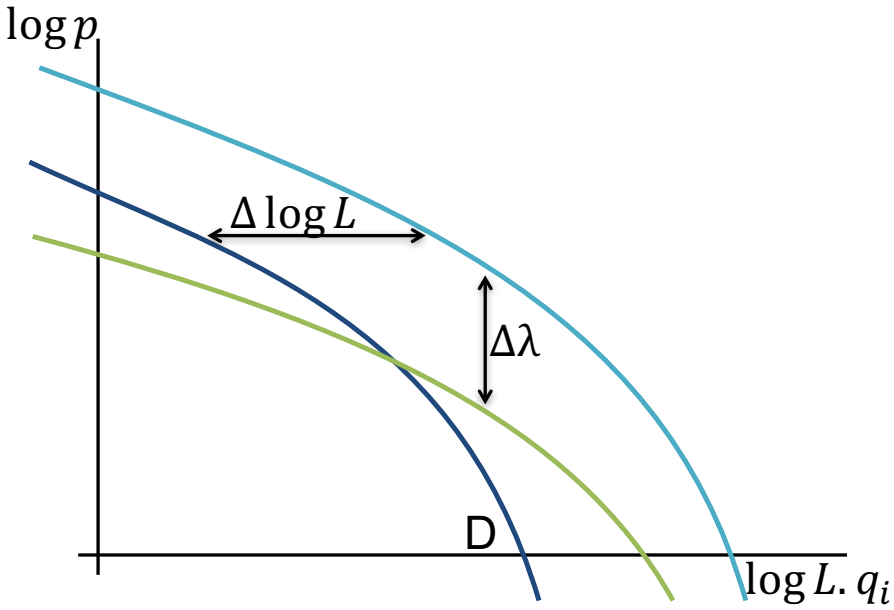
Shape of Demand

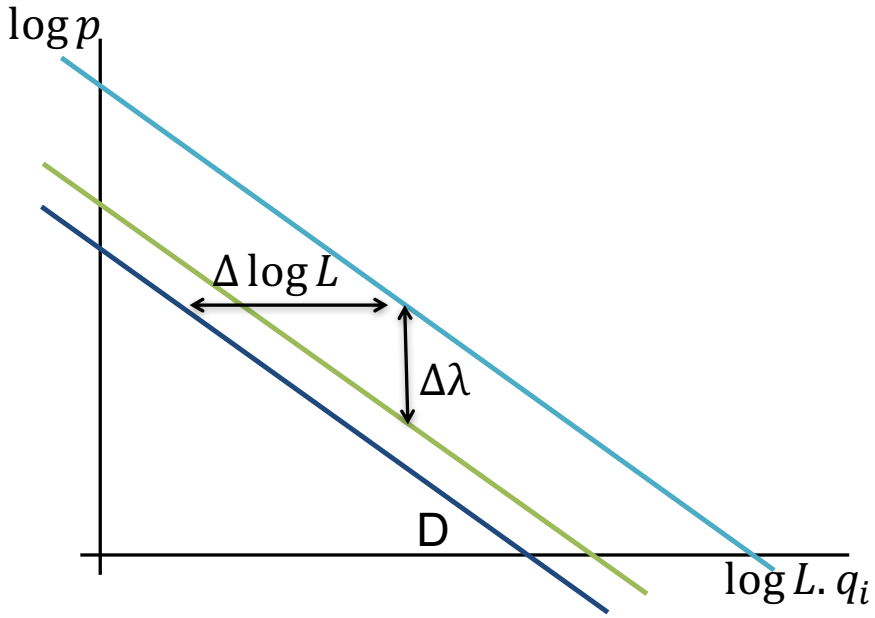
- Assume that VES preferences fall in "price-decreasing" competition case (Zhelobodko et al, 2012) $\longrightarrow$ Demand becomes more elastic as move up the demand curve



- Consistent with vast majority of empirical evidence on firm markups and pass-through:
- Larger, better performing firms set higher markups
  - Incomplete pass-through of cost shocks to prices
    - Additional evidence on 'more' incomplete pass-through for better-performing firms: Berman et al (2012) and Li et al (2015)

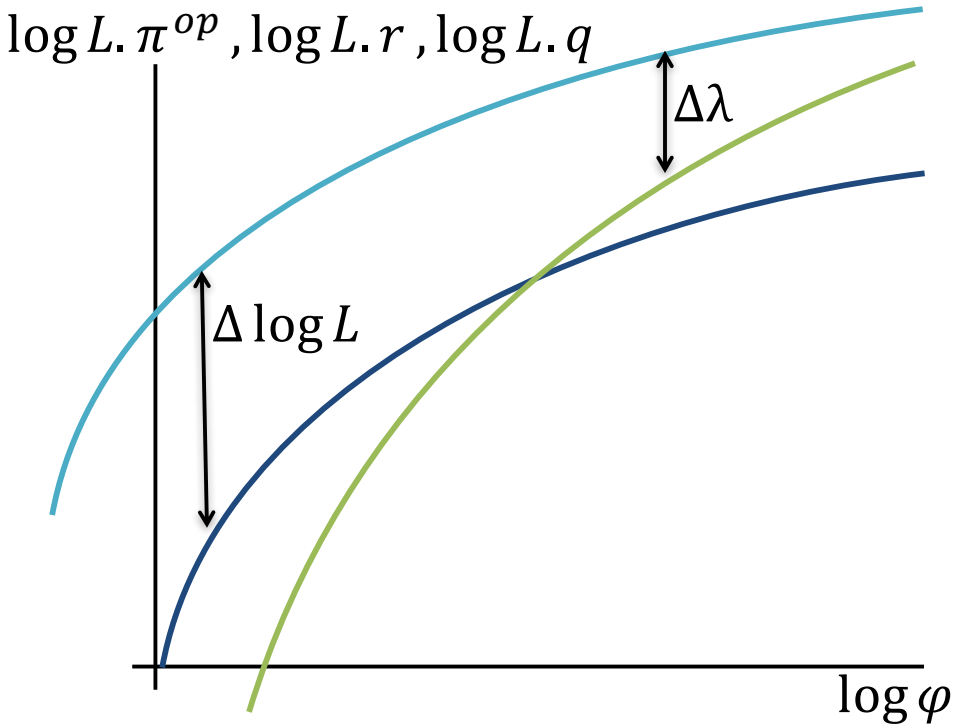
Shape of Demand and Competition

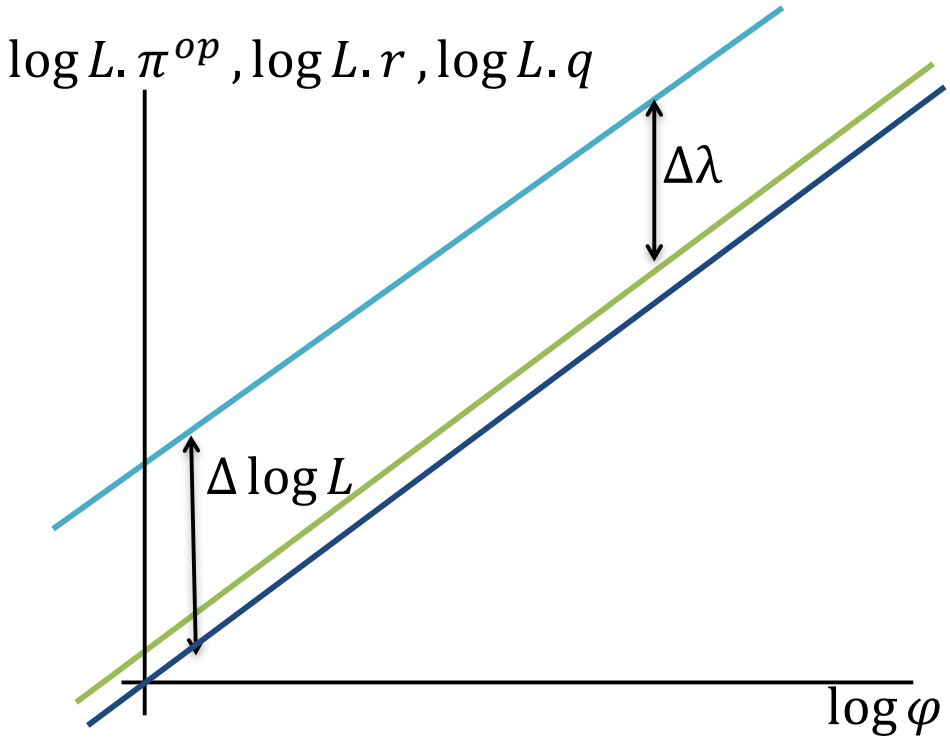




- Increases in competition ($\lambda \nearrow$) shift down $\varepsilon_p(q_i) \longrightarrow$ more elastic demand for all firms

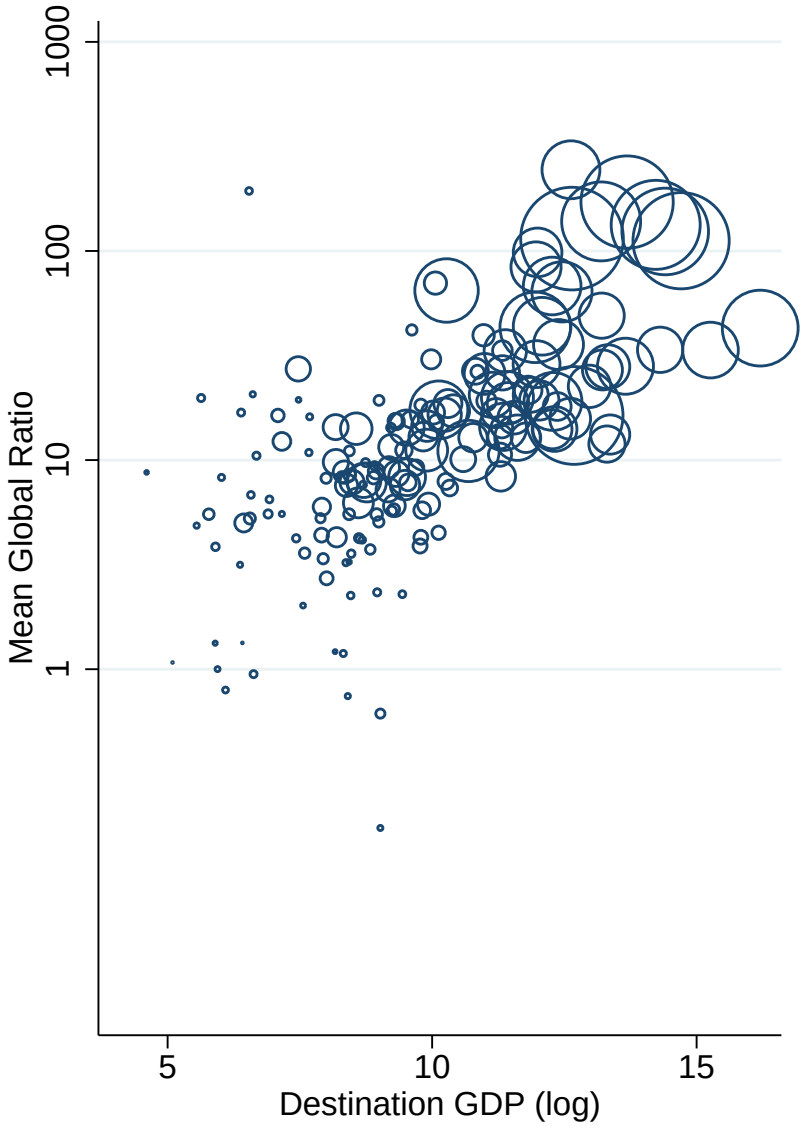
Implications for Competition and Firm Performance



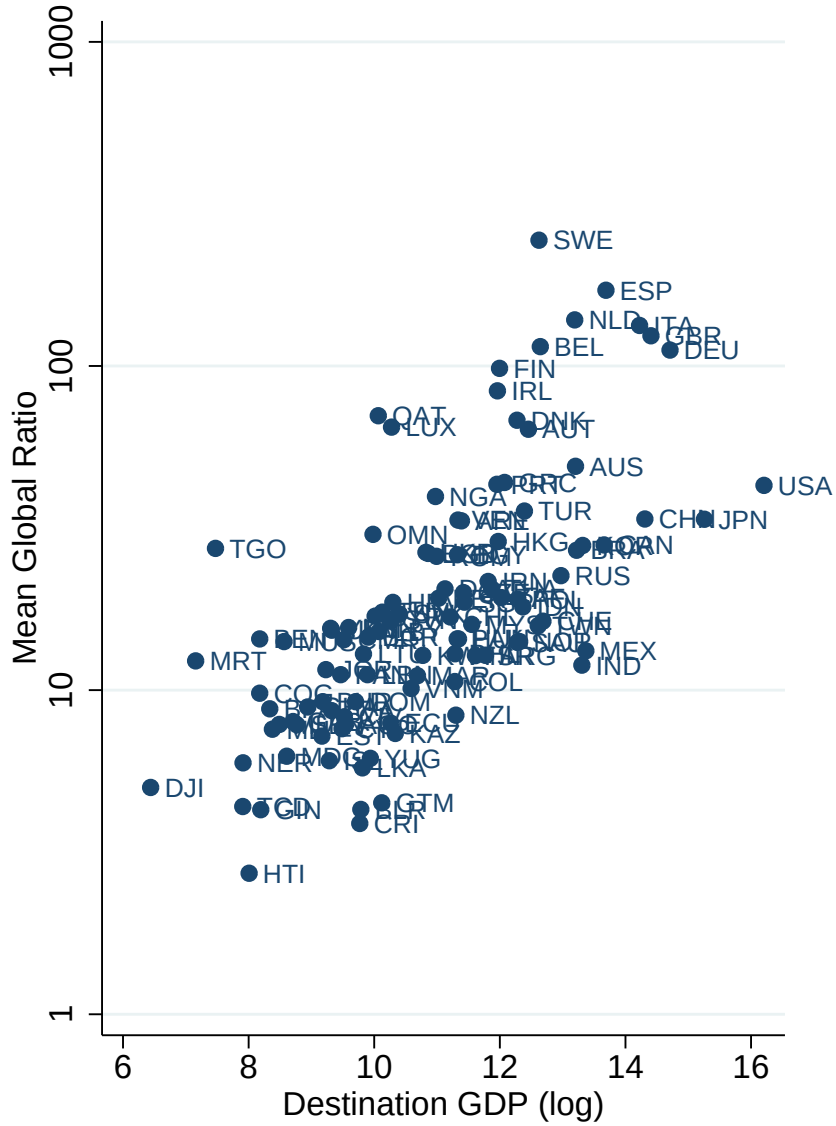


- Increases in competition ($\lambda \nearrow$) increase the elasticity of operating profits, revenues, and output $\longrightarrow$ intensive margin reallocations

Competition Effects: Evidence for French Exporters



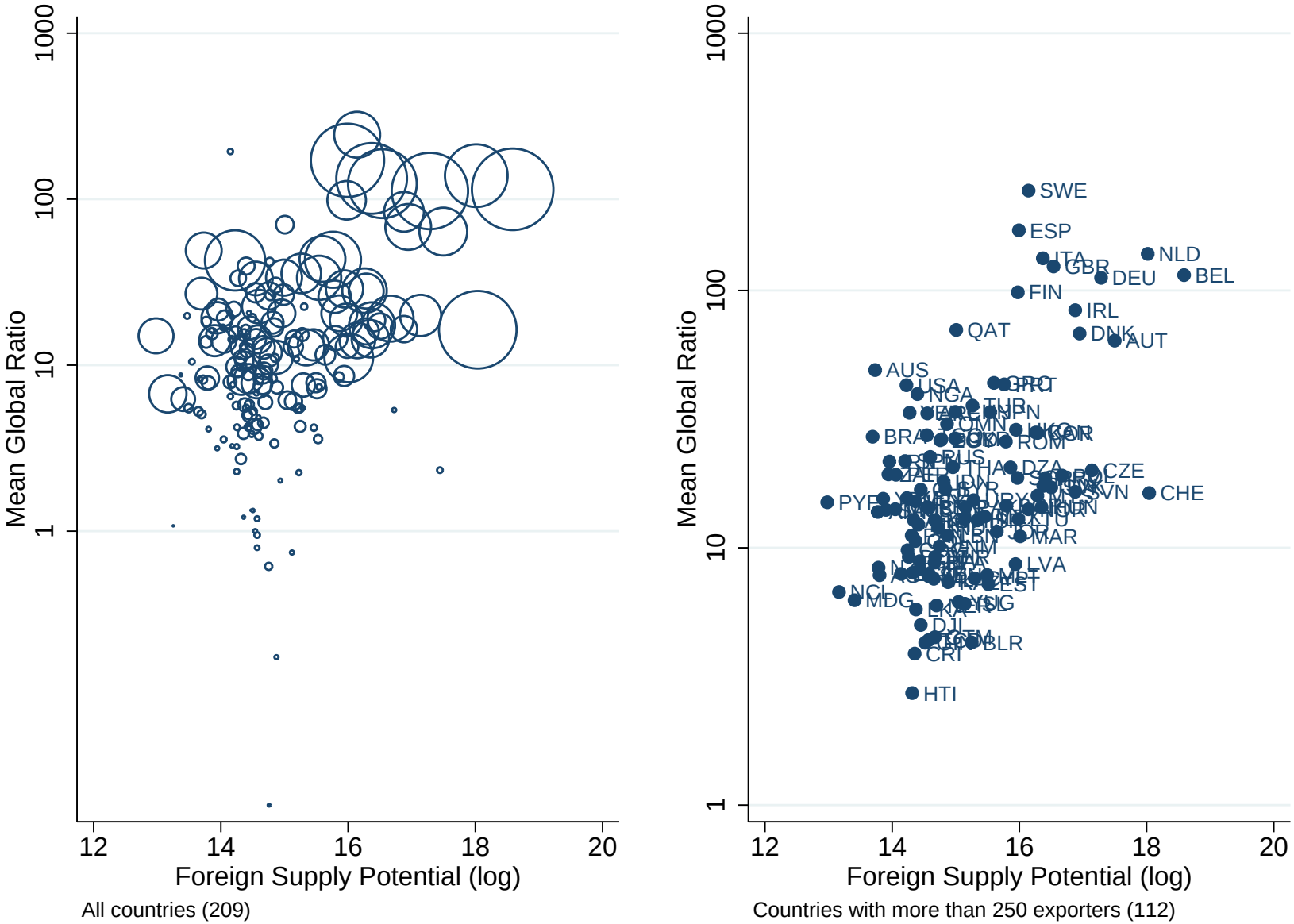
All countries (209)



Countries with more than 250 exporters (112)

Mean Global Sales Ratio and Destination Market Size

Competition Effects: Evidence for French Exporters



Mean Global Sales Ratio and Foreign Supply Potential

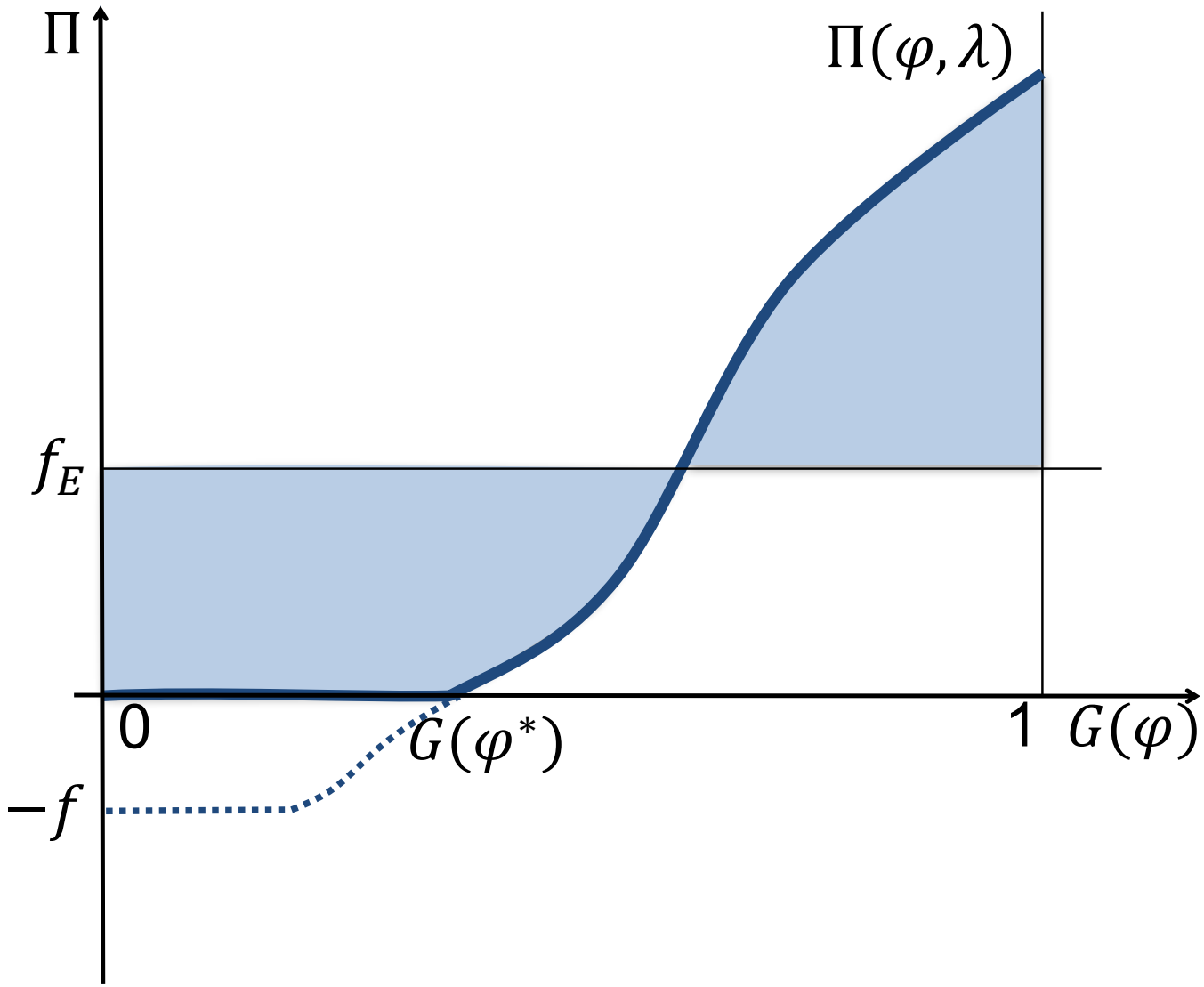
Closed Economy Equilibrium

- Irreversible investment f_E (in labor units) for firms to enter
  - Uncertain return: draw from a productivity distribution $G(\varphi)$
- 2 key equilibrium conditions:
  - Endogenous exit: Zero profit cutoff productivity such that $\Pi(\varphi^*; \lambda) = 0$. Firms with $\varphi < \varphi^*$ exit
  - Free entry:

$$\int_{\varphi^*}^{\infty} \Pi(\varphi; \lambda) dG(\varphi) = f_E$$

- These 2 equilibrium conditions jointly determine cutoff productivity φ^* and competition level λ
  - Response of entry and wages depend on how model is closed:
    - Single sector (GE): Exogenous labor supply of workers $L^w = L^c$. Wages adjust to ensure labor market equilibrium.
    - Multiple sectors (PE): Exogenous expenditures on given sector (L^c consumers). Endogenous labor supply of workers at exogenous economy-wide wage.

Closed Economy Equilibrium



Adjustment Path to Long Run Equilibrium

- Re-establishing free-entry condition after a trade-shock may take time
  - Especially for downward response of entry!
- In the short-run, the endogenous exit (zero-cutoff profit) condition would still hold; but not the free-entry condition
- Instead, the set of (potential) producers is fixed in the short-run
  $\longrightarrow$ Mass $\bar{M}_E$ of firms with productivity distribution $G(\varphi)$
  $\longrightarrow$ So $\bar{M}_E G(\varphi^*)$ firms produce – remaining firms "shut-down" in the short-run
- The cutoff φ^* and competition level λ are given by ZCP and consumer's budget constraint:

$$\bar{M}_E \int_{\varphi^*}^{\infty} r(\varphi;\lambda) dG(\varphi) = 1$$

  - In the long-run, this budget constraint determines the endogenous number of entrants M_E

Labor Market Equilibrium

- Aggregate employment across firms yields aggregate labor demand:

$$L^w = M_E \left\{ f_E + \int_{\varphi^*}^{\infty} \left[f + L^c q(\varphi; \lambda)/\varphi \right] dG(\varphi) \right\}$$

- In long-run, free-entry ensures that $L^w = L^c$ in GE version
- PE version features same equilibrium, so endogenous labor supply L^w adjusts to equalize number of consumers L^c

Industry (Aggregate) Productivity

- Consider the set of producing firms with productivity $\varphi \geq \varphi^*$ with distribution $\mu(\varphi) = G(\varphi)/\left[1 - G(\varphi^*)\right]$
- Symmetry in preferences assumes that quantity units are adjusted for quality (utility-based units) rather than physical quantity units
- A theoretical aggregation of productivity can therefore sum quantity produced per worker:

$$\Phi = \frac{\int_{\varphi^*}^{\infty} q(\varphi;\lambda) d\mu(\varphi)}{L_w}$$

- Can also define an empirical measure of industry productivity as deflated expenditures per worker:

$$\hat{\Phi} = \frac{\int_{\varphi^*}^{\infty} r(\varphi;\lambda) d\mu(\varphi)/P}{L^w}, \qquad \text{where } P = \frac{\int_{\varphi^*}^{\infty} r(\varphi;\lambda) p(\varphi;\lambda) d\mu(\varphi)}{\int_{\varphi^*}^{\infty} r(\varphi;\lambda) d\mu(\varphi)}$$

  is the market-share weighted average of firm prices
- Any changes to L^c, L^w, or λ always move both productivity measures Φ and $\hat{\Phi}$ in the same direction

Aside: Quadratic Functional Form

- Consider $u(q_i) = \alpha q_i - \frac{1}{2}\gamma q_i^2$
- Then demand is linear: $p(q_i) = (\alpha - \gamma q_i)/\lambda$
- Then performance measures (in terms of marginal cost $c = 1/\varphi$) are:

$$\pi(c, \lambda) = \frac{(\alpha - c\lambda)^2}{4\gamma\lambda}$$

$$r(c, \lambda) = \frac{\alpha^2 - (c\lambda)^2}{4\gamma\lambda}$$

$$q(c, \lambda) = \frac{\alpha - c\lambda}{2\gamma}$$

also price as a function of marginal cost c:

$$p(c, \lambda) = \frac{\alpha + c\lambda}{2\lambda}$$

Aside: Quadratic Functional Form (Cont.)

- Do not need $f > 0$ to generate selection (choke prices)
- If $f = 0$ then ZCP does not depend on L^c: $\pi(c^*, \lambda) = 0 \iff \lambda = \alpha/c^*$ $\longrightarrow$ can use cutoff φ^* as measure of competition level (instead of λ)
- If use marginal utility of income as numeraire, obtain:

$$\frac{\pi(c, c^*)}{\lambda} = \frac{(c^* - c)^2}{4\gamma}$$

$$\frac{r(c, c^*)}{\lambda} = \frac{c^{*2} - c^2}{4\gamma}$$

$$q(c, c^*) = \frac{c^* - c}{2\gamma}$$

$$\frac{p(c, c^*)}{\lambda} = \frac{c^* + c}{2}$$

Aside 2: Quadratic with Interaction

- Consider the quasi-linear variant with an interaction term (so not additively separable):

$$U = q_0^c + \alpha \int_{\omega \in \Omega} q_\omega^c d\omega - \frac{1}{2}\gamma \int_{\omega \in \Omega} (q_\omega^c)^2 \, d\omega - \frac{1}{2}\eta \left(\int_{\omega \in \Omega} q_\omega^c d\omega \right)^2$$

- Demand parameters:
  - γ: index of product differentiation
    - As $\gamma \nearrow$ increasing penalty for skewed consumption
  - α and η: substitution with numeraire good
- Linear inverse demand for all varieties:

$$p_\omega = \alpha - \gamma q_\omega^c - \eta Q^c, \qquad Q^c \equiv \int_{\omega \in \Omega} q_\omega^c d\omega$$

- Marginal utility of income λ now constant $(= 1)$

Aside 2: Quadratic with Interaction (Cont.)

- Do not need $f > 0$ to generate selection (choke prices)
- If $f = 0$ then ZCP does not depend on L^c: cutoff c^* is inverse index of endogenous competition
- Obtain same performance measure functions (even though interaction mechanism has changed):

$$\pi(c, c^*) = \frac{(c^* - c)^2}{4\gamma}$$

$$r(c, c^*) = \frac{c^{*2} - c^2}{4\gamma}$$

$$q(c, c^*) = \frac{c^* - c}{2\gamma}$$

$$p(c, c^*) = \frac{c^* + c}{2}$$

$$\mu(c, c^*) = p(c, c^*) - c = \frac{c^* + c}{2}$$

Aside 2: Quadratic with Interaction (Cont.)

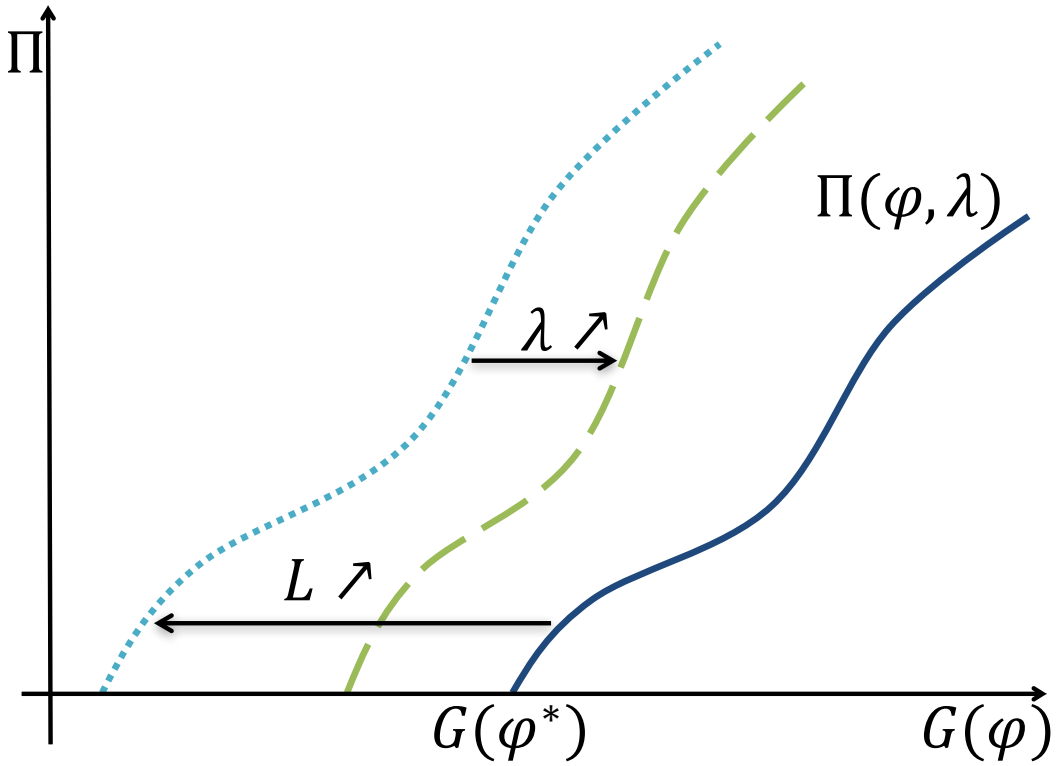
- Assume Pareto(k) distribution for productivity $1/c$ with lower bound $1/c_M$
- Cutoff is then given by:

$$c^* = \left(\frac{\gamma\phi}{L}\right)^{\frac{1}{k+2}}$$

- $\phi = 2(k+1)(k+2)c_M^k f_E$ is an (inverse) index of technology:
- $\phi \nearrow$ with $k \nearrow, c_M \nearrow, f_E \nearrow$
- Comparative Statics:
- Cutoff $c^* \searrow$ (hence higher average productivity) when:
  - $\gamma \searrow$ (varieties are closer substitutes)
  - $c_M \searrow$ (better distribution of cost draws)
  - $f_E \searrow$ (lower sunk costs)
  - $L \nearrow$ (in bigger markets)
  - In all cases: changes induce an increase in the "toughness of competition"
- Welfare varies monotonically (inversely) with cutoff c^*

Market Size Effects: Short Run

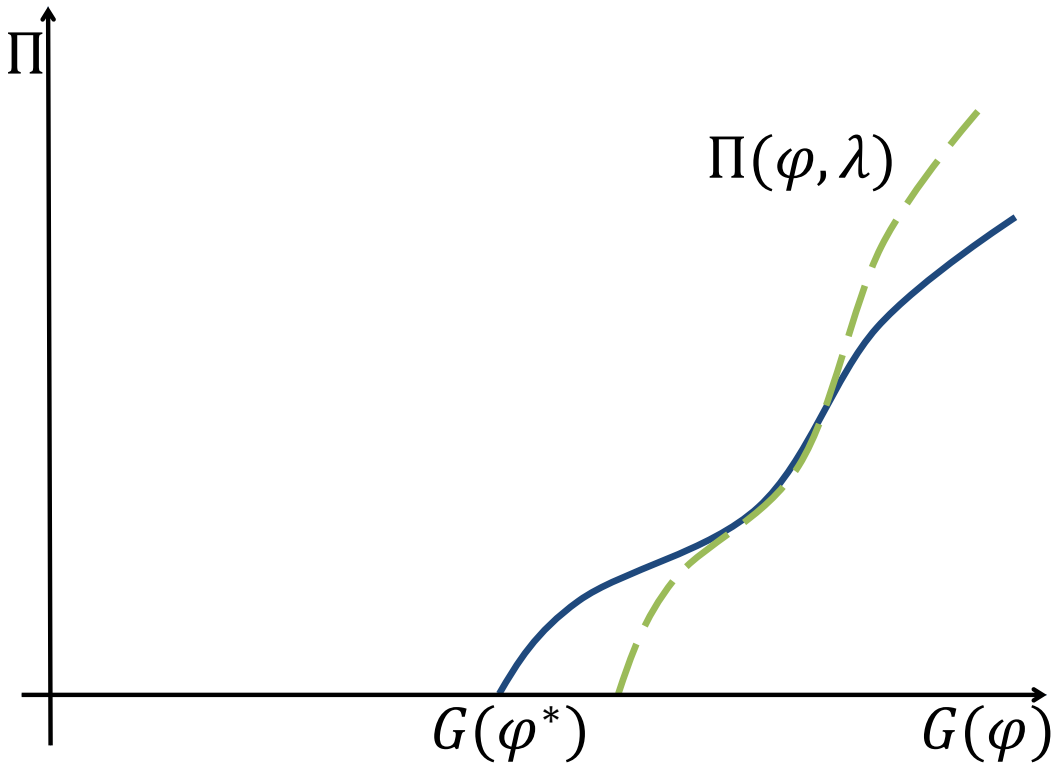
- Consider an increase in the size of "closed" economy – or integrated "world" economy: $L^c \nearrow$



- Increased competition ($\lambda \nearrow$) $\rightarrow$ intensive margin reallocations $\rightarrow \Phi \nearrow$
  (assume that intensive margin reallocations dominate negative extensive margin effects from selection)

Market Size Effects: Long Run

- Consider an increase in the size of "closed" economy – or integrated "world" economy: $L^c \nearrow$



- Further increases in competition ($\lambda \nearrow\nearrow$) due to entry $\longrightarrow$ cutoff $\varphi^* \nearrow$ $\longrightarrow$ extensive and intensive margin reallocations $\longrightarrow \Phi \nearrow$

Opening Economy to Trade

Consider trade with rest of the world F:

- Exports to F:
  - Firms incur a per-unit trade cost τ and fixed export cost f_X to reach F
  - Market size L_F^c and competition level λ_F determine export profits $\Pi_X(\varphi; \lambda_F)$
  - Only firms with $\varphi \geq \varphi_X^*$, such that $\Pi_X(\varphi_X^*; \lambda) = 0$ export (Zero cutoff profit condition for export market)
- Free entry condition: same as in closed economy except that firms anticipate profits

$$\Pi(\varphi; \lambda, \lambda_F) = \mathbf{1}_{[\varphi \geq \varphi^*]} \Pi_D(\varphi; \lambda) + \mathbf{1}_{[\varphi \geq \varphi_X^*]} \Pi_X(\varphi; \lambda_F)$$

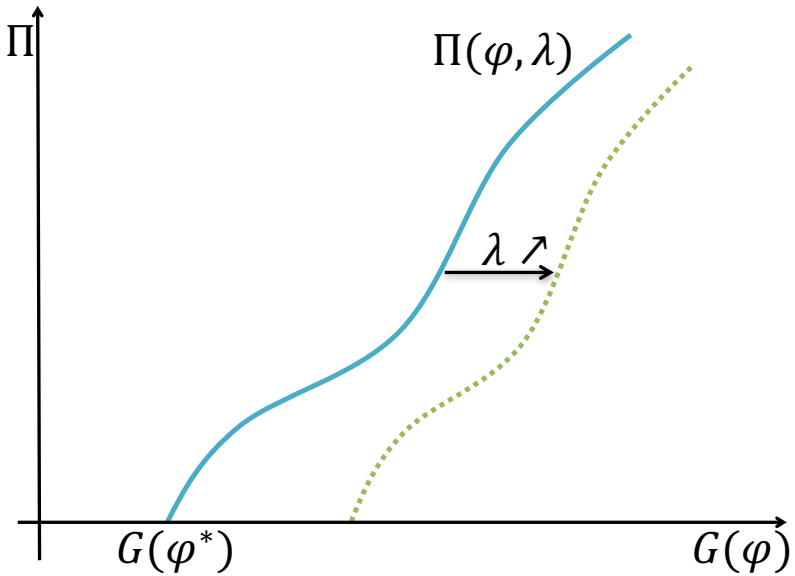
- Same modeling setup in F (generating imports into domestic economy):
  - Mass M_F of firms with productivity distribution $G_F(\varphi)$
  - Firms incur trade costs $\tau_F, f_{F,X}$ and earn profits $\Pi_{F,X}(\varphi; \lambda)$
  - Only firms with $\varphi \geq \varphi_{F,X}^*$ export

Small Open Economy Assumption

- Introduced by Demidova and Rodrigues-Clare (2009,2013) to analyze trade models with heterogeneous firms
- Assume that domestic economy is "small" relative to the rest of the world F:
  - Changes for its economy (market size, trade costs in/out) do not affect aggregate outcomes for F:
    - Set of producing firms in F and competition level λ_F
      $\longrightarrow$ But foreign export cutoff $\varphi^*_{F,X}$ still responds to changes in the economy!

Opening Economy to Trade: Import Competition

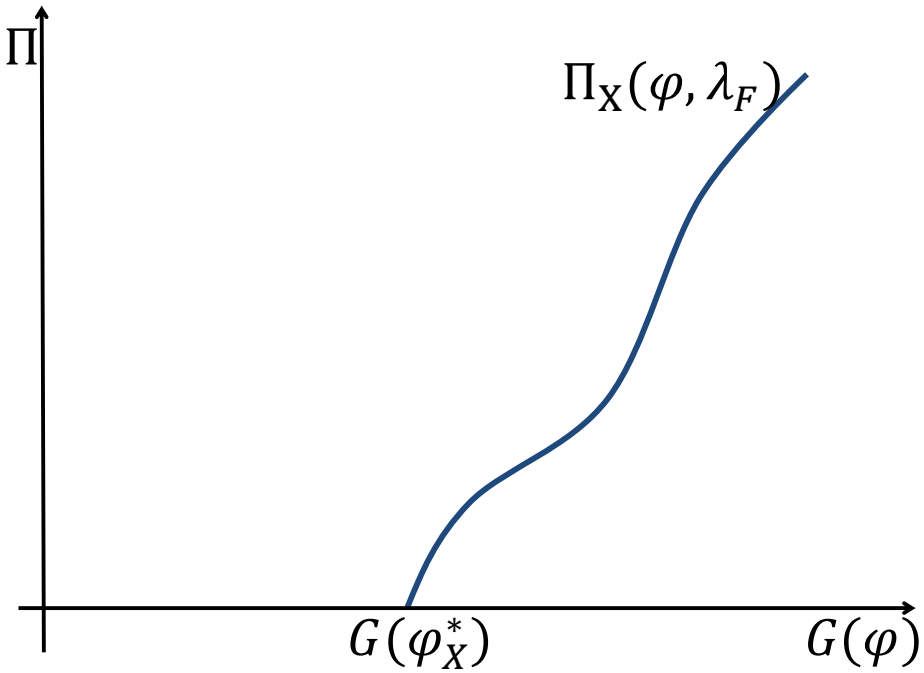
- First only consider imports (from rest of world F) into domestic economy $\longrightarrow$ So PE scenario (no balanced trade)

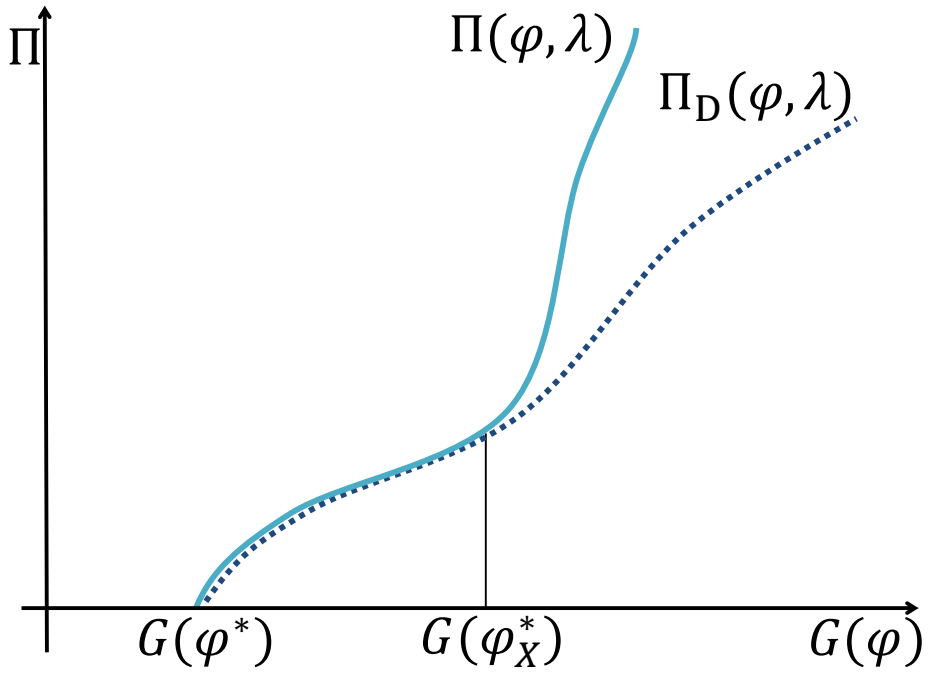


- Short run: Increased competition ($\lambda \nearrow$) $\longrightarrow$ cutoff $\varphi^* \nearrow$ $\longrightarrow$ extensive and intensive margin reallocations $\longrightarrow$ $\Phi \nearrow$
- Long run: reduced entry, return to old equilibrium (very long run)
- Same effect for any asymmetric liberalization of imports: $\tau_F \searrow$ or $f_{F,X} \searrow$

Opening Economy to Export Opportunities

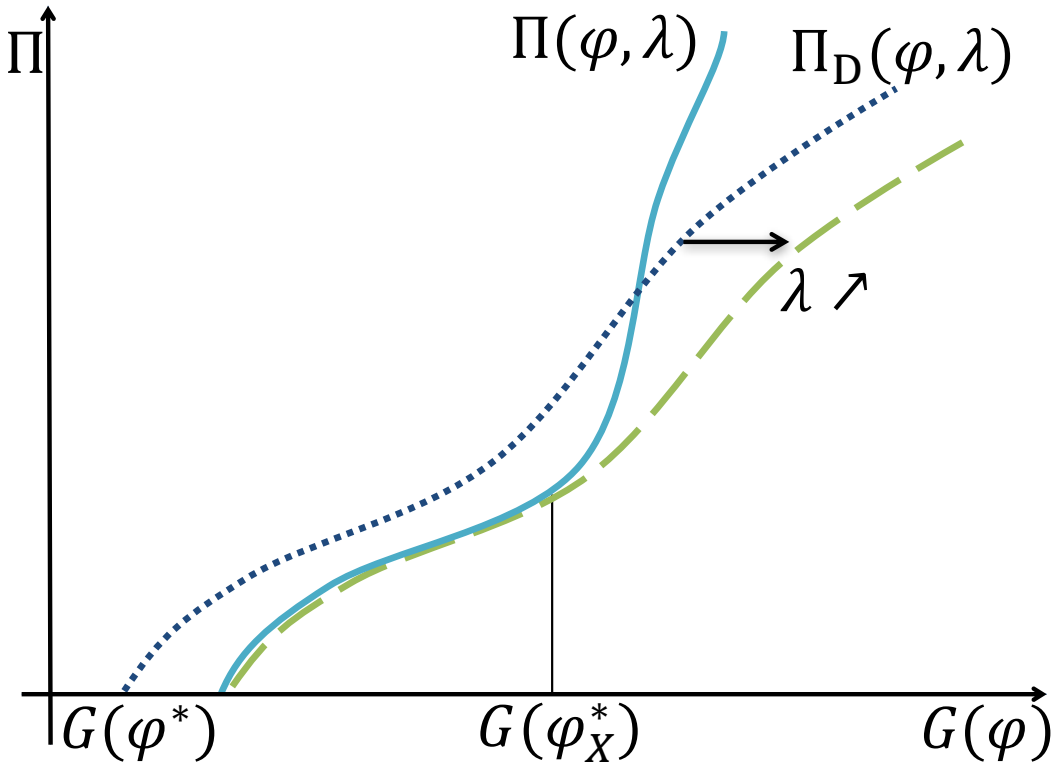
- Now only consider exports from domestic economy to rest of world F $\longrightarrow$ So PE scenario (no balanced trade)
- Short run:





- Extensive margin reallocations from domestic producers to exporters $\longrightarrow$ $\Phi \nearrow$

Opening Economy to Export Opportunities (Long Run)



- Increased entry $\rightarrow$ increased competition ($\lambda \nearrow$) $\rightarrow$ cutoff $\varphi^* \nearrow \rightarrow$ extensive (domestic/export) and intensive margin reallocations $\rightarrow \Phi \nearrow$
- Additional export market selection effects when exports are further liberalized: $\tau \searrow, f_X \searrow, L_F^c \nearrow$
- ... or an increase in labor-cost advantage: $w/w_F \searrow$

2-Way Trade: Import Competition and Export Opportunities

- Partial equilibrium: Just compounding of effects for import competition and export opportunities
- General equilibrium: Trade liberalization now induces adjustments in the relative wage w/w_F (consider only long run adjustment)
  - Export liberalization ($\tau \searrow$): $w/w_F \nearrow$ to restore trade balance
    - But does not overturn direct effect of $\tau \searrow$
  - Import liberalization ($\tau_F \searrow$): $w/w_F \searrow$ to restore trade balance
    - Recall, that before change in relative wage, $\tau_F \searrow$ only generates reduced entry (and no selection effects)
    - Increase in labor-cost advantage now generates export market entry ($\varphi_X^* \searrow$) and increase in domestic competition ($\lambda \nearrow$), which also leads to exit ($\varphi^* \nearrow$)
      $\longrightarrow$All these changes lead to further productivity gains $\Phi \nearrow$

2-Way Trade Between Large Countries

- Symmetric trade liberalization: similar effects as in small open economy
  - In addition, competition increases in both markets ($\lambda \nearrow, \lambda_F \nearrow$) so intensive margin reallocations of exports sales, along with similar reallocations of domestic sales
- Asymmetric trade liberalization: Main difference is new firm de-location in long run
  - Important for welfare and trade policy
  - Also, asymmetric increases in import competition can potentially lead to lower competition in the domestic market in the long run
    - Entry in F reduces export profits for domestic firms
      $\longrightarrow$ Lower export opportunities feeds back to lower competition in domestic market
    - This result only holds in very long run
      ... And reduction in export profits from trade liberalization must represent a non-negligible share of total profits
    - Empirical relevance in a multi-country world?

Monopolistic Competition, Firm Heterogeneity, and Trade – Model Extensions

Lecture Notes

Comparative Advantage and Heterogeneous Firms
Bernard, Redding, and Schott

Main Idea and Results

- 2x2x2 H-O setup with firm heterogeneity:
  - 2 sectors with different skilled/unskilled labor factor intensities: η_S and η_U (No homogeneous good sector)
  - 2 countries with different S/U endowment

2 important set of new results:

- Show how sector level responses (endogenous productivity, reallocations and gross flows) vary with the sector's comparative advantage
  - Comparative advantage sector:
    - Bigger response of cutoff
    - Larger productivity gain (endogenous feedback with Ricardian comparative advantage)
    - Higher level of gross labor flows (job creation & destruction)
- Jointly characterize gross and net factor reallocations driven by trade
  - Response of factor prices: dampening and possible reversing of standard Stolper-Samuelson effect

Autarky Equilibrium

- Country with higher S/U has lower w_S/w_U and hence lower w_j in skill-intensive j sector (relative to other country)
- Recall that differences in factor prices do not induce differences in cutoffs and firm selection: same for sector j across countries
  - Profits, market demand, and firm size shift proportionally to differences in w_j

Free Trade Equilibrium

- Same as autarky equilibrium but now use S/U for the "world"
- All firms export (destination of sales is irrelevant)
- Same cutoffs in both countries (for given sector), hence same average productivity (and same as in autarky)
- Furthermore:
  - (Specialization) Relative to autarky, countries devote a larger share of resources to their comparative advantage industry
  - (Stolper-Samuelson) The move from autarky to free trade increases the relative reward of a country's abundant factor
  - A move from autarky to free trade increases relative average firm size in the comparative advantage industry (driven by change in w_S/w_U)
  - Countries have relatively more firms in their comparative advantage industry
- All of these results would also be obtained with a modified Helpman-Krugman model with representative firms that all have the same productivity level $\tilde{\varphi}_j$ in each sector

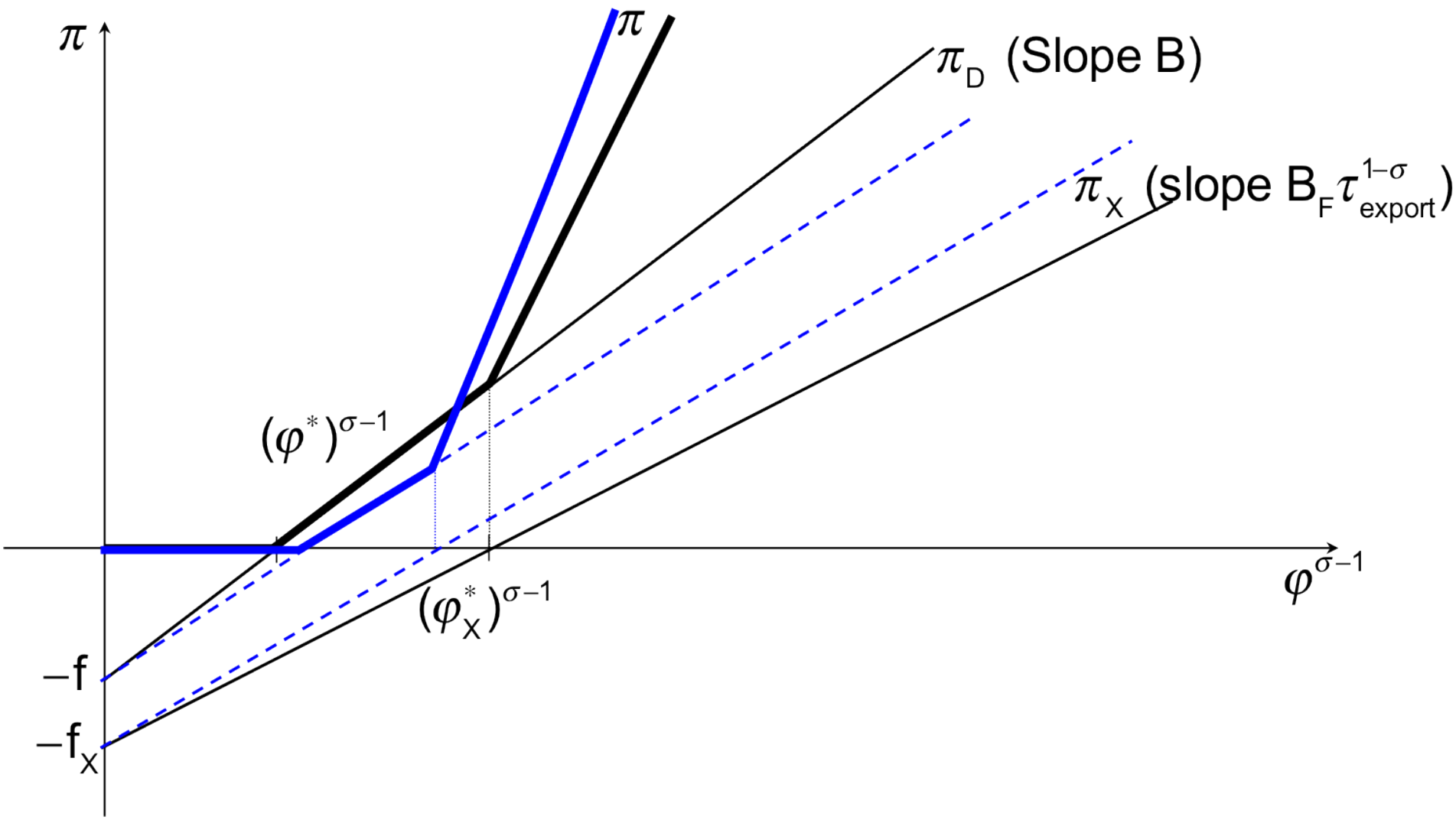
Costly Trade Equilibrium

- Similar trade frictions: τ_j and f_{Xj} (in units of composite labor factor)

Effects of opening to costly trade:

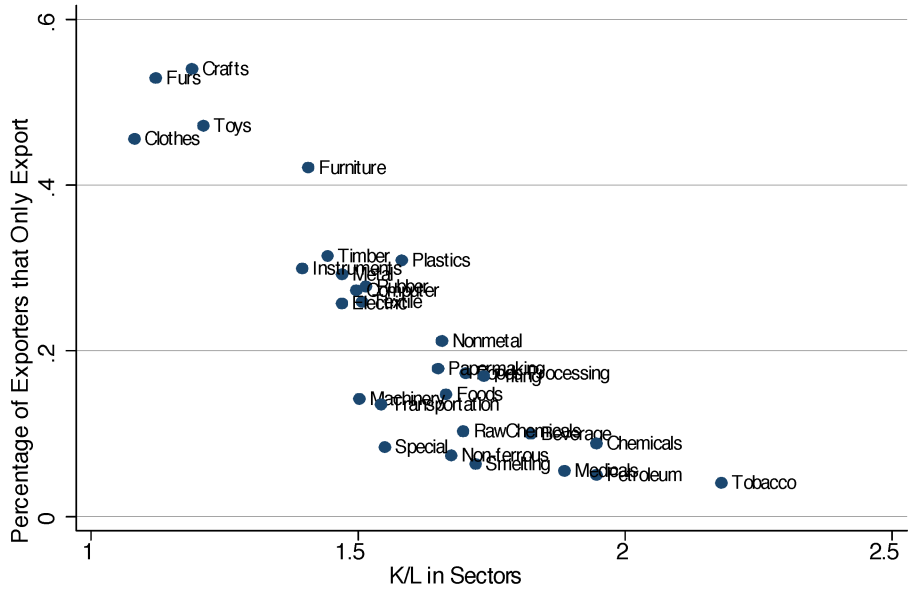
- Cutoffs φ_j^* (and hence $\tilde{\varphi}_j$) increase in all sectors (for both countries)
- But φ_j^* and $\tilde{\varphi}_j$ increase relatively more in the comparative advantage sectors
  - Intuition: Exporters in comparative advantage sector have cost advantage w_j relative to foreign competitors
- This induces an important amplification mechanism:
  - Endogenous Ricardian comparative advantage amplifies H-O comparative advantage based on factor abundances
- Simulations of model can jointly capture effects of trade liberalization for gross and net job flows
  - Gross job flows are larger in comparative advantage sector
  - Trade liberalization induces net job flows towards comparative advantage sector

Comparative Advantage and Selection

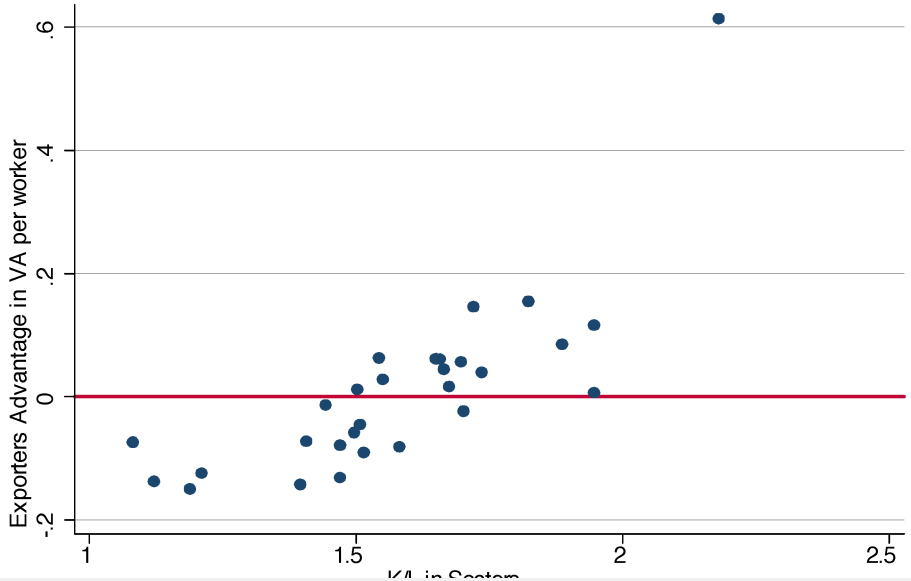


Direction of Export-Selection Effect for China

Figure 10: Percentage of Exporters that Only Export Across Sectors



Productivity Differences Between Exporters and Non-Exporters Across Sectors



Innovation

Joint Innovation and Export Decision

- A binary innovation choice: Adoption of a new technology (Bustos AER 2011)
  - New technology increases productivity φ by a factor $\iota > 1$ to $\iota\varphi$
  - Firm pays fixed cost f_I to upgrade to the new technology
- Implications: There is a treshold φ_I^* for technology adoption
  - Depending on parameters, can have either $\varphi_I^* <> \varphi_X^*$
  - $\longrightarrow$ Strong correlation between export status and technology adoption

Innovation Intensity and Export Decision

Consider the following model for a continuous innovation decision:

- Rescale productivity measure $\phi = \varphi^{\sigma-1}$
  - Changes in ϕ are proportional to firm size and variable profits
- Successful innovation increases productivity ϕ by a factor $\iota > 1$ to $\iota\phi$
- Probability of successful innovation is α
- Innovation intensity: firm choose α given a (convex) innovation cost function $c_I(\alpha)$
  - Innovation cost scales up proportionally to firm size on domestic market
    $\longrightarrow$ Total innovation cost is $\phi c_I(\alpha)$
    $\longrightarrow$ Big firms choose same α
    $\longrightarrow$ Delivers Gibrat's in a dynamic version

Innovation Intensity and Export Decision (Cont.)

Closed Economy

- Consider a firm ϕ that would produce even if innovation were not successful:

$$E[\pi(\phi)] = [(1-\alpha) + \alpha\iota]\, B\phi - \phi c_I(\alpha) - f$$

FOC for α:

$$c_I'(\alpha) = (\iota - 1)\, B$$

so all firms (above a given cutoff) choose same innovation intensity

Open Economy

- Do exporters choose a different innovation intensity?

Trade Policy, Efficiency, and Gains from Trade

Lecture Notes

Introduction

- Consider the incentives of a welfare maximizing planner to use trade policy
  - Will mostly consider 2 types of import barriers: revenue generating tariffs (t) requiring no real resources, and barriers that raise "real costs" to importers (τ)
- If the market equilibrium is not efficient, then planner may have an incentive to use trade policy to move equilibrium towards first-best efficient outcome (if possible)
- Is the market equilibrium with monopolistic competition efficient?
  - If not, what are the source of the inefficiencies?
  - Can they be remediated using trade policy

Planner Problem and Efficiency

- A welfare-maximizing planner would choose:
  1. Labor supply across sectors (if applicable)
  2. Average firm size (or alternatively mass of entrants or producers)

  With firm heterogeneity:
  3. Productivity cutoff (or alternatively average productivity of producers)
  4. Distribution of production across firms

  In open economy:
  5. Share of consumption on imports

- In each case, market outcome may be different than planner's choice leading to inefficiency
- Note: with same CES preferences across sectors, monopolistic competition equilibrium and planner outcomes coincide

Planner Problem and Efficiency (Cont.)

Symmetric Firms

- Labor supply across sectors
  - Under-allocation to sectors with high markups
- Average firm size (or alternatively mass of entrants or producers)
  - 2 offsetting externalities (product variety and business stealing/profit)
  - With "pro-competitive" preferences: Excess entry in market equilibrium (symmetric firms key)

Heterogeneous Firms

- Productivity cutoff (or alternatively average productivity of producers)
  - No immediate intuition...
- Distribution of production across firms
  - Same as across sectors (move labor to high markup firms)

Open economy

- Share of consumption on imports
  - Reduce share of imports as foreign profits not part of domestic welfare (not true for "World Planner") and due to terms of trade

Dynamic Efficiency

- CES preferences and monopolistic competition
  - Dixit-Stiglitz result also extends to dynamic version with sunk entry costs and consumption/saving choice (determining entry)
- Translog preferences
  - Counter-cyclical markups over the business cycle
  - Efficiency requires countercyclical policies for employment in differentiated product sector
  - Even in steady-state, profit incentives for entry are no longer exactly aligned with welfare effect of product varietys

Inter-Sectoral Efficiency and Gains From Trade
Epifani and Gancia (JIE, 2011)

Modeling Setup

- CES preferences across sectors:

$$U = \int_0^1 C_i^{(\sigma-1)/\sigma} di$$

- Within sector (no firm heterogeneity):

$$C_i = (N_i)^{\nu_i+1} c_i \qquad P_i = N_i^{-\nu_i} p_i$$

  - Consistent with CES if $\nu_i = 1/(\sigma_i - 1)$ and Benassy otherwise
  - also consistent with non CES homothetic preferences (e.g. translog)

- Firm markups

$$p_i = \frac{\mu_i w}{\varphi_i}$$

  - $\mu_i = \sigma_i/(\sigma_i - 1)$ if CES or Benassy with monopolistic competition
  - Can also have oligopoly within sector

- Define industry productivity (variety adjusted) as deflated industry sales per worker:

$$\Phi_i \equiv \frac{(P_i C_i)/p_i}{L_i} = \varphi_i N_i^{\nu_i}$$

Restricted Entry

- Market allocation

$$\frac{L_i}{L_j} = \left(\frac{\mu_j}{\mu_i}\right)^{\sigma} \left(\frac{\Phi_i}{\Phi_j}\right)^{\sigma-1}$$

  - Allocation of labor depends on relative productivity and gross-substitutatibility across sectors
  - Conditional on productivity, labor is allocated inversely to relative markups
- Planner allocation

$$\frac{L_i}{L_j} = \left(\frac{\Phi_i}{\Phi_j}\right)^{\sigma-1}$$

- Intuition: planner wants to reverse allocation of labor based on markup differences (#1)
  - High σ magnifies misallocation as it amplifies labor allocation in response to markup differences
- Lerner (1934): Only relative markups matter for welfare (not levels)
- GFT: Welfare losses are possible when trade increases markup dispersion (e.g. lower markups in relatively low markup sectors)

Free Entry

- Introduce fixed (overhead) cost f_i per firm
- Market allocation

$$\frac{L_i}{L_j} = \left(\frac{\mu_j}{\mu_i}\frac{\Phi_i}{\Phi_j}\right)^{\sigma-1}$$

- Planner allocation
  - Same as market allocation in Dixit-Stiglitz case
  - Otherwise, planner solution for firm size/number of firms (#2) does not coincide with market: if markups μ_i are higher than love of variety $\nu_i - 1$, then excess entry in market allocation
- GFT: Assume that markups fall with number of products
  - Welfare losses no longer possible so long as market equilibrium number of firms is below the efficient level
  - This holds even when trade amplifies markup dispersion

Empirical Evidence on Markup Dispersion: US over time

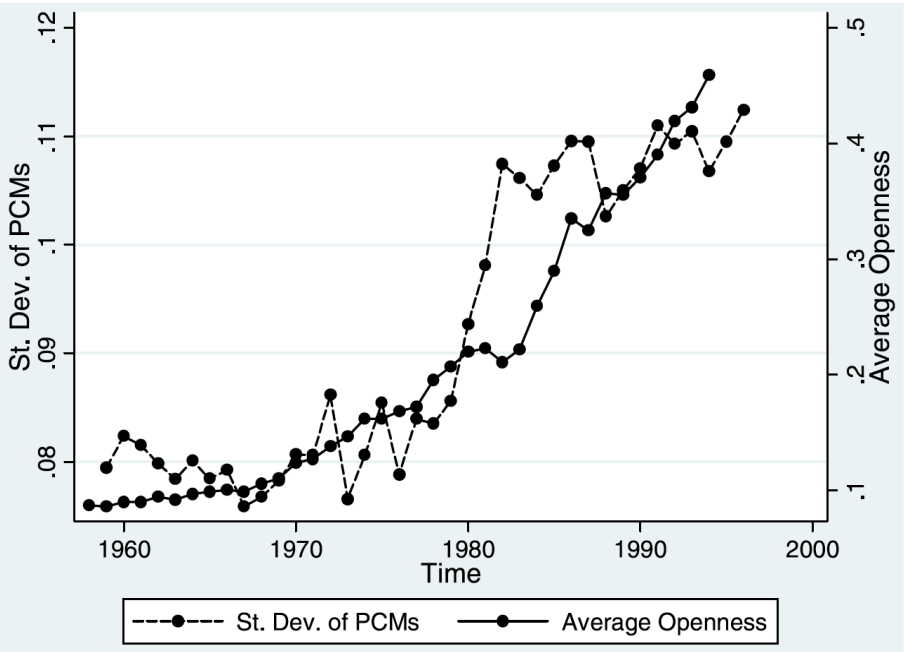


Fig. 2. Openness and cross-industry markup heterogeneity.

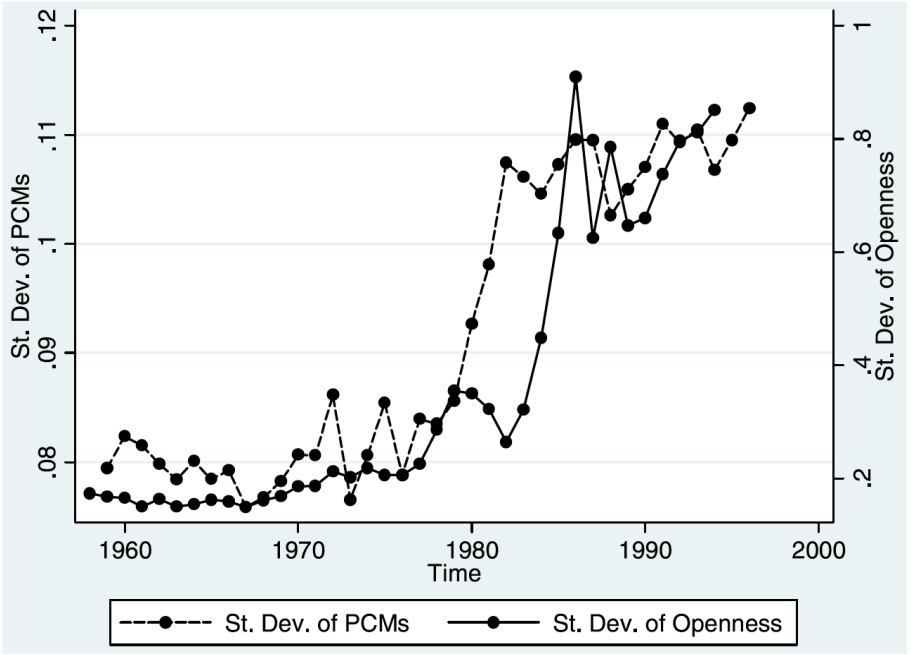


Fig. 3. Asymmetries in trade exposure and markup heterogeneity.

Empirical Evidence on Markup Dispersion: Cross-Country

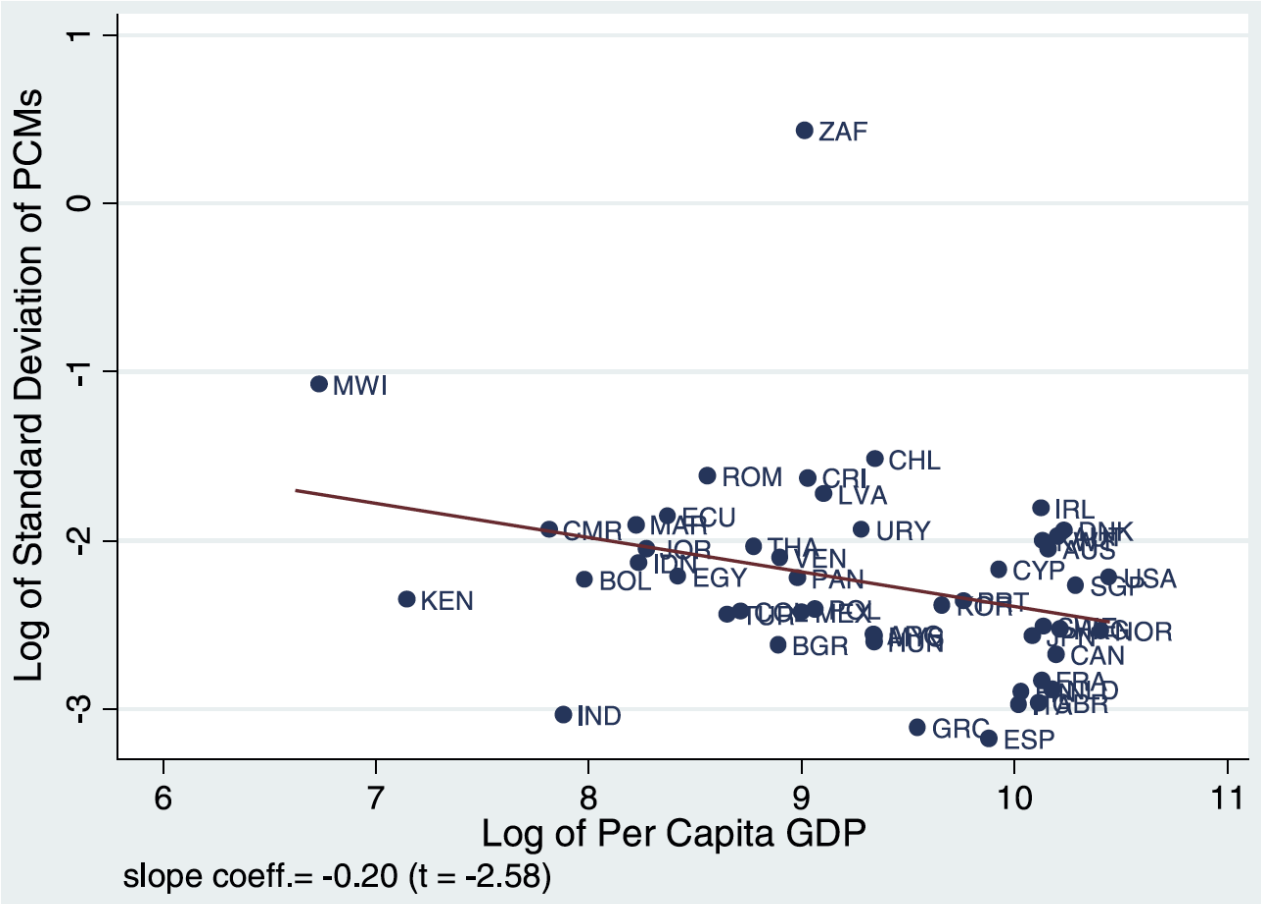


Fig. 4. Economic development and cross-industry markup heterogeneity.

Unilateral Trade Policy and Efficiency under Monopolistic Competition

Implications for Unilateral Trade Policy

What are the incentives to use trade barriers (tariff or "real costs")?

- Small open economy (CES preferences)
  - Single sector: $\tau = 1, t = 1 + 1/(\sigma - 1) = \sigma/(\sigma - 1)$
  - With outside sector?
  - Now add firm heterogeneity (Pareto distribution of productivity for domestic firms and importers)
    - Single sector: $\tau = 1, t = \frac{k - \frac{\sigma - 1}{\sigma}}{k} < \frac{\sigma}{\sigma - 1}$ (but > 1)
    - With outside sector?
- What if economy is now large with respect to trading partner?
- What about non CES preferences?

Efficiency and Gains from Trade with Heterogeneous Firms

Efficiency with Heterogeneous Firms

Dhingra & Morrow (2012): "Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity"
Melitz & Redding (2013): "Firm Heterogeneity and the Welfare Gains from Trade"

- Model ingredients
  - 2 symmetric countries, single factor (inelastically supplied), single C.E.S. sector
  - Heterogeneous firm productivity at entry, single period production
  - Monopolistic competition with free entry (subject to sunk cost)
  - Trade costs: both per-unit (iceberg and fixed)
- Results
  - Market equilibrium is efficient (planner uses same "entry" technology for firms)
  - $\longrightarrow$ Endogenous selection of firms into domestic and export markets is efficient

Efficiency with Heterogeneous Firms: Closed Economy Ex.

- Welfare maximizing planner chooses:
  - Average production (also mass of entrants/producers given labor supply)
  - Dispersion of production
  - Range of producing firms (cutoff)
- First order conditions:
  - Dispersion of production: equate MRS and MRT
  - Average production and cutoff: Conditions are equivalent to:
    - Only firms above cutoff earn non-negative profits
    - On average firm profits equal to sunk entry cost
  - $\longrightarrow$ Planner solution is identical to market equilibrium

Closed Economy Example: Relationship to Dixit-Stiglitz

- Consider heterogeneous firm model with productivity distribution at entry $G(\varphi)$, overhead production cost f_d, and sunk entry cost f_e
- Let φ_d be the equilibrium productivity cutoff for this model
- The heterogeneous firm equilibrium is identical to one generated by representative firms with a productivity average:

$$\tilde{\varphi}^{-1} = \int_{\varphi_d}^{\infty} \varphi^{-1} \frac{q(\varphi)}{q(\tilde{\varphi})} \frac{dG(\varphi)}{1 - G(\varphi)} \qquad \text{(weights } \propto \text{ output shares)}$$

  Along with a single fixed cost

$$F_d = f_d + \frac{f_e}{1 - G(\varphi_d)}$$

- Dixit-Stiglitz result: conditional on choice of cutoff φ_d, replicating the equilibrium with a representative technology $\tilde{\varphi}$ and F_d is welfare maximizing
- Need additional step: how to choose cutoff φ_d?

$$\varphi_d \nearrow \quad \Longrightarrow \quad \tilde{\varphi} \nearrow \text{ and } F_d \nearrow$$

Open Economy and Efficiency

- Now consider planner's problem for open economy version with both per-unit and fixed trade costs:
  - Planner (maximizing world welfare) also chooses subset of firms that produce for the export market (and their relative production levels)

Result:

- Efficiency result extends to open economy: planner chooses same export cutoff (range of exporting firms) as market equilibrium

Implication:

- Effect of trade costs on selection (for both domestic and export market) are second order for welfare
  - ... but second order effects are quantitatively very big!

Aggregating Goods into Sectors and Homothetic Functional Forms

Aggregation Across Goods: Commodity Groups

- Apply consumer and demand theory to groups of goods (instead of individual goods)

Setup:

- Divide vector of goods into two groups: $(\boldsymbol{x}, \boldsymbol{z})$
- ... along with their associated price vectors: $(\boldsymbol{p}, \boldsymbol{q})$
- Initial consumer problem is

$$\max u(\boldsymbol{x}, \boldsymbol{z}) \quad \text{subject to} \quad \boldsymbol{p}.\boldsymbol{x} + \boldsymbol{q}.\boldsymbol{z} = m$$

- Would like to assign a price index $P = f(\boldsymbol{p})$ and quantity index $X = g(\boldsymbol{x})$ such that can analyze consumer's demand over this bundle of goods as

$$\max U(X, \boldsymbol{z}) \quad \text{subject to} \quad PX + \boldsymbol{q}.\boldsymbol{z} = m$$

  where utility U is now defined over the quantity index X
- This can be done only if answers obtained for X and $\boldsymbol{z}$ are identical to the ones obtained for the initial consumer's problem applying the quantity index to the derived $\boldsymbol{x}$

Functional Separability of the Utility Function

- Whenever goods prices within a group do not vary proportionally, then must rely on some functional form assumptions on the utility function (and hence on the preferences)
- First, must assume that utility function is 'separable' in $\boldsymbol{x}$ and $\boldsymbol{z}$ in the following way: $u(\boldsymbol{x}, \boldsymbol{z}) = U(g(\boldsymbol{x}), \boldsymbol{z})$
- Then, if one knows the optimal expenditures on $\boldsymbol{x}$ goods, say $m_x = \boldsymbol{p}.\boldsymbol{x}$
  - Can solve for the demand of the individual $\boldsymbol{x}$ goods as

$$\max_{\boldsymbol{x}} g(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}.\boldsymbol{x} = m_x$$

  - Obtain the same answer for $\boldsymbol{x}$ as when solving the original problem

$$\max_{\boldsymbol{x},\boldsymbol{z}} U(g(\boldsymbol{x}), \boldsymbol{z}) \quad \text{subject to} \quad \boldsymbol{p}.\boldsymbol{x} + \boldsymbol{q}.\boldsymbol{z} = m$$

  - But... how does one solve for m_x???
  - In general, can not without first solving the original consumer's problem

Additional Functional Form Assumption

- If $g(\boldsymbol{x})$ is homogeneous of degree one
  (the preferences over $\boldsymbol{x}$ are homothetic – given spending on $\boldsymbol{x}$ goods m_x)
  - Note that must use H.O.D. 1 representation of those preferences (and not any monotonic transformation)
- Then can think of $X = g(\boldsymbol{x})$ as a quantity index associated with the price index $P = \omega(\boldsymbol{p})$ derived from expenditure minimization using the utility function $g(\boldsymbol{x})$:
$$e(\boldsymbol{p}, X) = \omega(\boldsymbol{p})X$$
- Can then solve
$$\max_{X,\boldsymbol{z}} U(X, \boldsymbol{z}) \quad \text{subject to} \quad PX + \boldsymbol{q}.\boldsymbol{z} = m$$
- We know that $PX = \omega(\boldsymbol{p})X$ will be optimal spending on $\boldsymbol{x}$ goods given any set of prices $\boldsymbol{p}$

Homothetic Sub-Utility and Ideal Price Indices

Start with $u(\boldsymbol{x}, \boldsymbol{z}) = U(g(\boldsymbol{x}), \boldsymbol{z})$ where $g(.)$ is H.O.D. 1

1. Solve utility maximization for $g(\boldsymbol{x})$ at prices $\boldsymbol{p}$ to obtain demand $\boldsymbol{x}(\boldsymbol{p}, m_x)$ and the price index function $P = \omega(\boldsymbol{p})$
2. Can then solve utility maximization for $U(X, \boldsymbol{z})$ using prices $(P, \boldsymbol{q})$
3. Demand over commodities in bundle $\boldsymbol{x}$ is then given by utility maximization in 1 subject to income PX
   - Get exactly same results for demand $\boldsymbol{x}$ and $\boldsymbol{z}$ and welfare as when solve utility maximization $u(\boldsymbol{x}, \boldsymbol{z})$ over all goods (which represents a much bigger problem to solve)

- P is an ideal price index for bundle $\boldsymbol{x}$
  - P summarizes all the information in vector of prices $\boldsymbol{p}$ relevant for demand and welfare
- If there are no other goods $\boldsymbol{z}$ then overall welfare is just given by m/P

Homothetic Sub-Utility: Sector Decompositions

Start with $U = U(g_1(\boldsymbol{x}_1), g_2(\boldsymbol{x}_2), ..., g_J(\boldsymbol{x}_J))$ where $g_j(.)$ is H.O.D. 1 (preferences for goods in sector j)

1. Given prices $\boldsymbol{p}_j$ in sector j, calculate price index P_j
2. Solve for sector level consumption $X_j \equiv g_j(\boldsymbol{x}_j)$ and spending $X_j P_j$:

$$\max_{X_1,...,X_J} U(X_1, ..., X_J) \quad \text{subject to} \quad \sum_{j=1}^{J} P_j X_j = m$$

3. Then solve for demand over goods in sector j, $\boldsymbol{x}_j$, given sector spending $X_j P_j$

- If $U = \prod_{j=1}^{J} X_j^{a_j}$ $\left(\sum_{j=1}^{J} a_j = 1\right)$ is Cobb-Douglas, then sector spending $X_j P_j$ is exogenous: share a_j of income m
  $\longrightarrow$ Can go right to step 3

Cobb-Douglas Price Index with Many Goods

With the Cobb-Douglas preferences, the price index has the same functional form as utility:

$$U = \prod_{k=1}^{M} x_k^{a_k} \text{ with } \sum_{k=1}^{M} a_k = 1$$

then

$$P = \omega(\boldsymbol{p}) = K(\boldsymbol{a}) \prod_{k=1}^{M} p_k^{a_k}$$

where $K(\boldsymbol{a})$ is an exogenous constant

- So P is a geometric average of the individual prices p_k
  ... and changes in the price index P can be calculated as

$$\Delta \ln P = \sum_{k=1}^{M} a_k \Delta \ln p_k$$

C.E.S Price Index With k Symmetric Goods

- Consider C.E.S. preferences where $U = \left(\sum_{k=1}^{M} x_k^{\rho} \right)^{1/\rho}$ with $\rho \leq 1$
- Elasticity of substitution between any two goods $k = i, j$ is defined as:

$$\sigma_{ij} \equiv \frac{\partial \ln \left(\frac{x_i^*}{x_j^*} \right)}{\partial \ln \left(\frac{p_i}{pj} \right)}$$

- In this case, $\sigma_{ij} = 1/(1-\rho) \geq 0$ is constant between any two goods
  - Cobb-Douglas is a special case with $\sigma = 1$

C.E.S. Price Index (Cont.)

- Recall $U = \left(\sum_{k=1}^{M} x_k^{\rho} \right)^{1/\rho}$ with $\rho \leq 1$
- The ideal price index for these preferences can be written

$$P = \left(\sum_{k=1}^{M} p_k^{1-\sigma} \right)^{1/(1-\sigma)}$$

- A very good approximation to this ideal price index is given by

$$\Delta \ln P \simeq \sum_{k=1}^{M} s_k \Delta \ln p_k$$

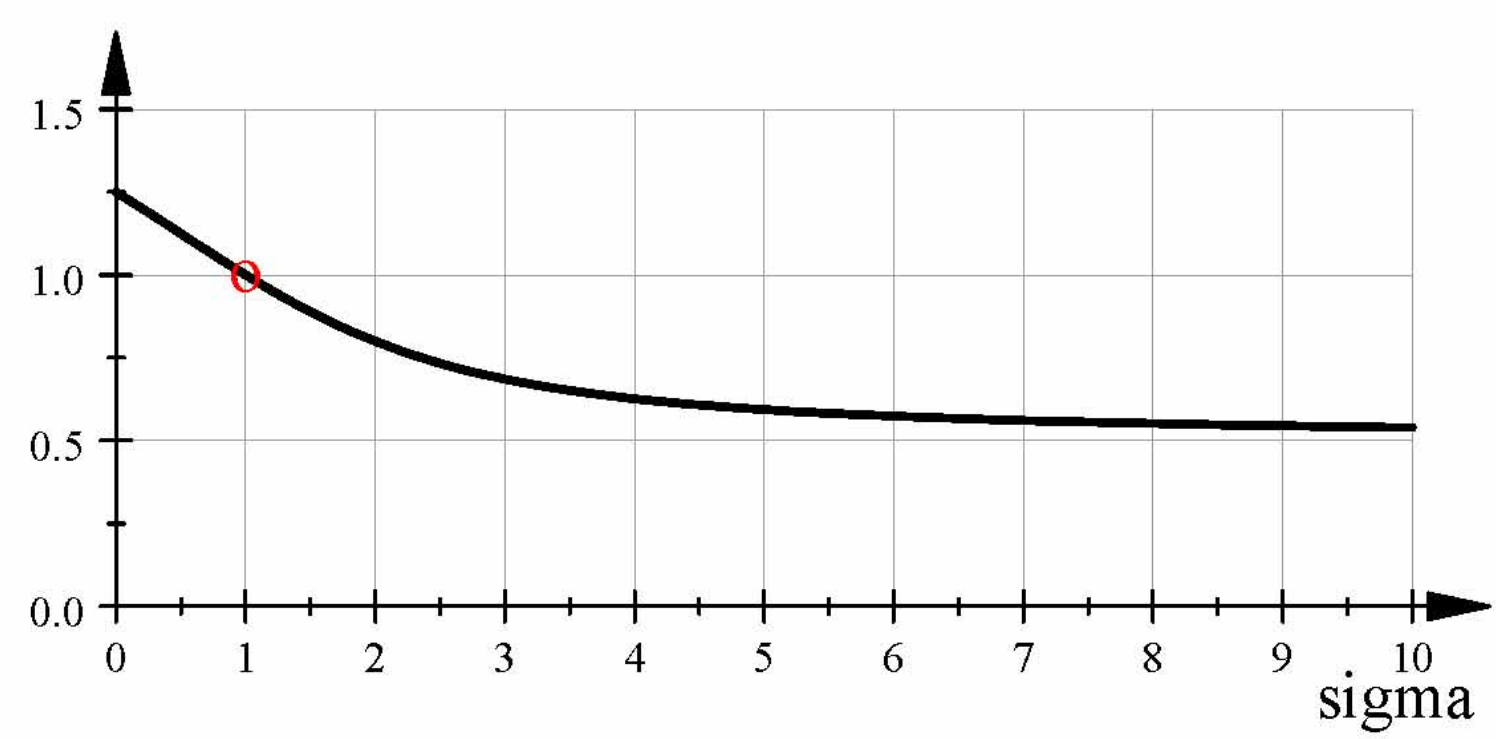
  where s_k is an average expenditure share for good i across the two time periods
- In general the C.E.S. price index is a weighted average of the prices, where the weights are proportional to the goods market shares
  - Higher σ implies a higher weight on the lower priced goods

C.E.S. Price Index with 2 goods

- With 2 goods:

$$P\left(p_1, p_2\right) = \left(p_1^{1-\sigma} + p_2^{1-\sigma}\right)^{1/(1-\sigma)}$$

- P is a weighted average of the prices p_1 and p_2 where the weights on the lower price increases with the elasticity of substitution σ
- Plot of $P(1/2, 2)/P\left(1, 1\right)$ as a function of σ:



C.E.S. Price Index: Love of Variety

- Recall $U = \left(\sum_{k=1}^{M} x_k^{\rho}\right)^{1/\rho}$ with $\rho \leq 1$ and price index

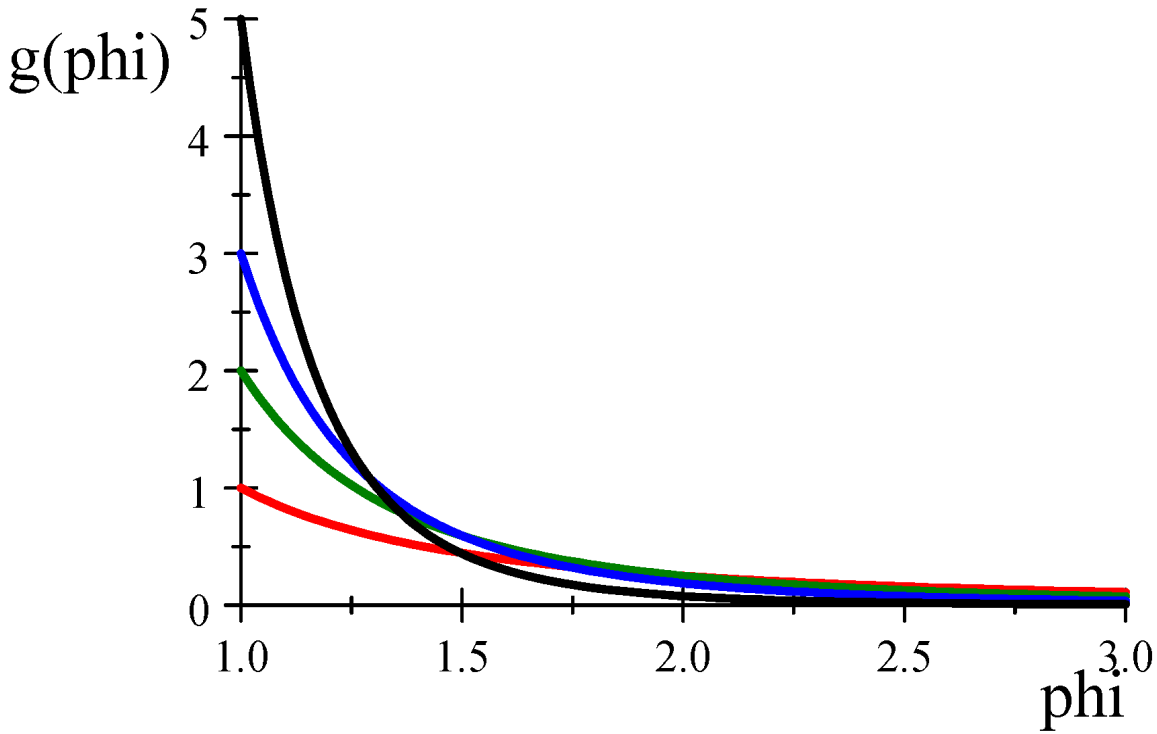
$$P = \left(\sum_{k=1}^{M} p_k^{1-\sigma}\right)^{1/(1-\sigma)}$$

  where $\sigma \equiv 1/(1-\rho)$
- If all M goods have the same price p, then $P = M^{1/(1-\sigma)}p$
  $\longrightarrow$ Holding price p fixed, a 1% increase in the # of products M decreases P by $1/(\sigma-1)\%$ (also proportional to welfare increase)
- This also holds for any given distribution of prices
  - i.e. change # of products M holding the distribution of prices fixed

Notes on Pareto Distribution

Functional Form

- CDF: $\Phi \sim \text{Pareto}(k)$ then $\Pr(\Phi > \varphi) = G(\varphi) = 1 - (\varphi_0/\varphi)^k$ for any $\varphi \geq \varphi_0$ and $k > 1$
- The PDF is $g(\varphi) = k\varphi_0^k/\varphi^{k+1}$
- k is the main 'shape' parameter and φ_0 is a 'location' parameter:



$g(\varphi)$ for $k = 1, 2, 3, 5$ and $\varphi_0 = 1$

Shape Parameter

- Moments:

$$E(\Phi) = \frac{k}{k-1}\varphi_0 \qquad \text{for } k > 1 \text{ (mean is infinite for } k \leq 1)$$

$$\text{var}(\Phi) = \frac{k}{(k-1)^2(k-2)}\varphi_0^2 \qquad \text{for } k > 2 \text{ (variance is infinite for } k \leq 2)$$

- Measures of dispersion:

$$\text{Coeff. of Variation} \equiv \frac{\sqrt{\text{var}(\Phi)}}{E(\Phi)} = \frac{1}{\sqrt{k}\sqrt{k-2}} \text{ and } \text{var}\left(\log\left(\Phi\right)\right) = \frac{1}{k^2}$$

  only depend (monotonically) on shape parameter k (and not location parameter φ_0)
  $\longrightarrow$ lower k indexes more dispersion

- If $k \leq 2$ then variance is infinite, and if $k \leq 1$ mean is also infinite
  - $k \longrightarrow 1$ is limiting case where inverse $1/\varphi$ is distributed uniformly on $[0, 1/\varphi_0]$
  - $k \longrightarrow \infty$ is limiting case where distribution is degenerate at φ_0

Special Properties

- A Pareto distribution preserves its shape with any truncation from below (particularly useful for modeling productivity with lower bound cutoffs). More formally

$$\frac{\Phi}{\bar{\varphi}} \mid \Phi \geq \bar{\varphi} \text{ has identical distribution for any } \bar{\varphi} \geq \varphi_0$$

- Furthermore, for any $\varphi_2 \geq \varphi_1 \geq \varphi_0 \geq 1$:

$$\frac{g(\varphi_2)}{g(\varphi_1)} = g\left(\frac{\varphi_2}{\varphi_1}\right) = \left(\frac{\varphi_2}{\varphi_1}\right)^{-k-1}, \frac{1-G(\varphi_2)}{1-G(\varphi_1)} = 1-G\left(\frac{\varphi_2}{\varphi_1}\right) = \left(\frac{\varphi_2}{\varphi_1}\right)^{-k}$$

  and are independent of location parameter φ_0 (and truncation from below)
- Also, truncation from below does not affect measures of dispersion (only dependent on k)

Transformations

- $\log(\Phi/\varphi_0)$ is distributed Exponential with coefficient $1/k$
- $A\Phi \sim \text{Pareto}(k)$ with location $A\varphi_0$ for any constant $A > 0$
- $\Phi^\alpha \sim \text{Pareto}(k/\alpha)$ with location $(\varphi_0)^\alpha$ for any constant $\alpha > 0$

Application to C.E.S Price Index

- C.E.S preferences with elasticity $\sigma > 0$

$$U = \left(\int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

- A subset $\Omega' \subset \Omega$ of measure M of those goods are produced and sold at price $p(\omega)$
- Assume that $p(\omega)^{-1} \sim \text{Pareto}(k)$ with upper bound price p_0
- How can the C.E.S. price index

$$P = \left(\int_{\omega \in \Omega'} [p(\omega)]^{1-\sigma} d\omega \right)^{1/(1-\sigma)}$$

be written in terms of parameters k, σ, p_0, and M?

Application to C.E.S Price Index (Cont.)

Recall

$$P = \left(\int_{\omega \in \Omega'} \left[p(\omega) \right]^{1-\sigma} d\omega \right)^{1/(1-\sigma)}$$

1. Can solve directly:

$$P = \left(\int_0^{p_0} p^{1-\sigma} M dG(p) \right)^{1/(1-\sigma)}, G(p) = \left(\frac{p}{p_0} \right)^k$$

2. ... or write

$$P = M^{\frac{1}{1-\sigma}} \left[\frac{1}{M} \int_{\omega \in \Omega'} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = M^{\frac{1}{1-\sigma}} \left(E \left[p(\omega)^{1-\sigma} \right] \right)^{\frac{1}{1-\sigma}}$$

   where $p(\omega)^{1-\sigma} \sim \text{Pareto}(k/(\sigma-1))$ with lower bound $p_0^{1-\sigma}$

3. To obtain:

$$P = M^{1/(1-\sigma)} \left[\frac{k}{k-(\sigma-1)} \right]^{1/(1-\sigma)} p_0$$

Pareto Productivity, CES Demand, and Monopolistic Competition

- With CES preferences with elasticity of substitution σ:
  - Firm markups are exogenously fixed at $\sigma/(\sigma-1)$
  - Firm sales are proportional to $\varphi^{\sigma-1}$ across firms
- Hence, irrespective of cutoff productivity level for surviving firms:
  - Inverse of firm price $1/(p(\varphi)$ is distributed Pareto(k)
  - Firm sales/size are distributed Pareto $\left(k/(\sigma-1)\right)$
  - Mean firm size is finite only if $k/(\sigma-1)>1 \iff k>\sigma-1$
  - Standard deviation of log firm size is $(\sigma-1)/k$
- For any $\varphi_2 \geq \varphi_1 \geq \varphi_0 \geq 1$:

$$\frac{\#\text{ of firms with } \varphi \geq \varphi_2}{\#\text{ of firms with } \varphi \geq \varphi_1} \quad \text{and} \quad \frac{\text{Aggregate sales of firms with } \varphi \geq \varphi_2}{\text{Aggregate sales of firms with } \varphi \geq \varphi_1}$$

  only depend on φ_2/φ_1 and exogenous parameters k and σ (for sales). In particular those ratios are independent of any truncation from below (so long as truncation point is below φ_1)

Consumer Demand with a Continuum of Differentiated Goods

Motivation: Formalization of Monopolistic Competition Model

- In monopolistic competition, each firm produces a unique differentiated good or 'variety', which gives every firm some market power
- Continuum of differentiated goods provide formalization of the notion – critical for monopolistic competition – that firms are nevertheless small relative to the market
  - So that the action of any single firm has no impact on market aggregates and on other firms
- Using the approximation with a discrete, but very large number of goods can be problematic:
  - Cannot formalize comparative statics for the total number of produced goods
  - The concept of a 'large enough' number of firms is substantially affected by changes in the size distribution of firms

Continuum of Differentiated Varieties

- Continuum of (symmetric) differentiated varieties $\omega \in \Omega$
- Define preferences over this continuum with a utility function:

$$U = F(C_1, C_2, ...), \qquad C_i = \int_{\omega \in \Omega} f_i\left(c\left(\omega\right)\right) d\omega$$

- Given income Y and a distribution of prices $p(\omega)$ over a subset $\Omega' \subset \Omega$ of produced varieties, utility maximization yields the FOC

$$\sum_i \frac{\partial F}{\partial C_i} f_i'\left(c\left(\omega\right)\right) = \lambda p(\omega)$$

$$\int_{\omega \in \Omega'} p(\omega) c(\omega) d\omega = Y$$

where λ is the marginal utility of income (the Lagrange multiplier associated with the budget constraint)

  - With a slight abuse of notation, the FOC can also be written: $\partial U / \partial c(\omega) = \lambda p(\omega) d\omega$

Continuum of Differentiated Varieties (Cont.)

- Solving the utility maximization problem yields the residual demand curve for each good: $c(\omega)$ as a function of $p(\omega)$ and market aggregates
  - where market aggregates depend on statistics of the distribution of prices $p(\omega)$ and the mass of available varieties in Ω', $M = \int_{\omega \in \Omega'} d\omega$
- The price elasticity of residual demand is given by:

$$\varepsilon(\omega) = -\frac{\partial c(\omega)}{\partial p(\omega)} \frac{p(\omega)}{c(\omega)}$$

- Marginal revenue at any point on the residual demand curve is then $MR = p(\omega)\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)} < p(\omega)$
- If prices are symmetric, then common $p = p(\omega)$ summarizes degenerate distribution of prices and the common price elasticity of demand $\varepsilon = \varepsilon(\omega)$ is uniquely determined by the mass of varieties M and the common price p

Special Example of Preferences: C.E.S.

- $$U = C = \left(\int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

  where $\sigma > 0$ is the symmetric elasticity of substitution between any two varieties ω_1 and ω_2
  - In order to study cases where the range of varieties produced M is endogenous, must further assume $\sigma > 1$
- Given prices $p(\omega)$ and income Y, the FOC for utility maximization are:

$$\left[\frac{c(\omega)}{C} \right]^{-1/\sigma} = \lambda p(\omega)$$

- Since these preferences are homothetic, welfare C also represents a consumption index for a single composite good, along with an associated price index P such that $C = Y/P$ (this is also the indirect utility function)
  - ... and $\lambda = \partial C / \partial Y = 1/P$

Special Example of Preferences: C.E.S. (Cont.)

- Residual demand under C.E.S. preferences is thus given by

$$c(\omega) = C\left[\frac{p(\omega)}{P}\right]^{-\sigma} = Ap(\omega)^{-\sigma}$$

  where $A = YP^{\sigma-1}$ is an aggregate demand shifter
- The price index is given by

$$P = \left(\int_{\omega\in\Omega'} [p(\omega)]^{1-\sigma}\, d\omega\right)^{1/(1-\sigma)}$$

- In this specification of C.E.S. preferences with a continuum of goods, the price elasticity of residual demand is exogenously determined by the elasticity of substitution σ:

$$\varepsilon(\omega) = -\frac{\partial c(\omega)}{\partial p(\omega)}\frac{p(\omega)}{c(\omega)} = \sigma$$

- Note that this is only an approximation in the discrete goods case where the firms internalize the effects of their actions on market aggregates:

$$\partial P/\partial p(\omega) > 0 \qquad \text{and} \qquad \varepsilon(\omega) = \sigma\left[1 - s(\omega)\right]$$

C.E.S. Preferences and 'Love of Variety'

- The C.E.S. price index can be written as a combination of a statistic of the distribution of prices and the mass of varieties M:

$$P = M^{1/(1-\sigma)} \left[\frac{1}{M} \int_{\varepsilon \in \Omega'} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} = M^{1/(1-\sigma)} \tilde{p}$$

  where $\tilde{p}$ is a weighted average of the distribution of prices (where the weights are proportional to the induced relative consumption levels)
- Holding income Y and the distribution of prices $p(\omega)$ fixed (hence $\tilde{p}$ fixed), welfare C will vary with the mass of available varieties M:

$$C = \frac{Y}{P} = M^{1/(\sigma-1)} \frac{Y}{\tilde{p}}$$

- Thus, holding income Y and the distribution of prices $p(\omega)$ fixed, welfare will rise by $1/(\sigma - 1)\,\%$ for every per-cent rise in product variety M
- This is the independent welfare effect of product variety

Another Example: Quasilinear Quadratic Continuum

$$U = c_0 + \alpha \int_{\omega \in \Omega} c(\omega) d\omega - \frac{1}{2}\gamma \int_{\omega \in \Omega} c(\omega)^2 d\omega - \frac{1}{2}\eta \left(\int_{\omega \in \Omega} c(\omega) d\omega \right)^2$$

- Then the FOC yields:
$$p(\omega) = \alpha - \gamma c(\omega) - \eta M \bar{c}$$
where $\bar{c} = \left(\int_{\omega \in \Omega} c(\omega) d\omega \right) / M$ is the average quantity consumed and $M = \int_{\omega \in \Omega'} \omega d\omega$ is the mass of varieties in Ω', the set of consumed goods (with $c(\omega) > 0$)
- The FOC can be re-written as a residual demand curve:
$$c(\omega) = \frac{\alpha}{\eta M + \gamma} - \frac{1}{\gamma} p(\omega) + \frac{\eta M}{\eta M + \gamma} \frac{1}{\gamma} \bar{p}$$
where $\bar{p} = \left(\int_{\omega \in \Omega'} p(\omega) d\omega \right) / M$ is the mean of the price distribution (for consumed goods)
- Note that, once again, the residual demand curve can be written as a function of the mass of varieties M and a statistic of the price distribution

Quasilinear Quadratic Continuum:Threshold Price and Endogenous Price Elasticity

- Note that this residual demand curve exhibits a threshold price

$$p(\omega) \leq \frac{1}{\eta + \gamma/M}\left(\frac{\gamma}{M}\alpha + \eta\bar{p}\right) \equiv p_{\max}$$

  such that $c(\omega) = 0$ for any $p(\omega) > p_{\max}$
- The price elasticity of demand can also be written as a function of the mass of varieties and the mean price:

$$\varepsilon(\omega) \equiv -\frac{\partial c(\omega)}{\partial p(\omega)}\frac{p(\omega)}{c(\omega)} = \left(\frac{p_{\max}}{p(\omega)} - 1\right)^{-1}$$

- Note that this elasticity shifts up (increases at any given price) when M rises or $\bar{p}$ falls
  $\longrightarrow$ Captures an increase in competition (or 'toughness' of competition)
- Also note that bigger firms (those who set lower prices and obtain bigger market shares) face lower price elasticities

Quasilinear Quadratic Continuum: Welfare

- The indirect utility function is:

$$V = Y + \frac{1}{2}\left(\eta + \frac{\gamma}{M}\right)^{-1}(\alpha - \bar{p})^2 + \frac{1}{2}\frac{M}{\gamma}\sigma_p^2$$

  where $\bar{p}$ and σ_p^2 are the mean and variance of the price distribution
- Welfare $\nearrow$ when:
  - $\bar{p} \searrow$
  - $\sigma_p^2 \nearrow$
  - $M \nearrow$ (love of variety)

Another Example: Trans-Log Preferences

- Trans-log preferences are homothetic preferences ($U \equiv C$) that do not have a closed-form for the direct-utility function (See Feenstra, Econ Letters for more details)
- The preferences are defined via a translog expenditure function (which is enough to completely characterize preferences, given homotheticity):

$$\log Y = \log C + \alpha_0 + \frac{1}{2}\frac{\tilde{M} - M}{\gamma M \tilde{M}} + \frac{1}{M}\int_{\omega \in \Omega'} \log p(\omega) d\omega$$
$$+ \frac{1}{2}\frac{\gamma}{M}\int\int_{\omega_i, \omega_j \in \Omega'} \log p(\omega_i) \log p(\omega_j) d\omega_i d\omega_j - \frac{1}{2}\gamma \int_{\omega \in \Omega'} \log p(\omega)^2 d\omega$$

  where M is the mass of consumed goods (measure of Ω') and $\tilde{M}$ is the mass of all potential goods (measure of Ω)
- With C.E.S. preferences, the set Ω can be unbounded, but this is not the case with trans-log preferences
- Note that all the terms following $\log C$ must represent $\log P$, where P is the ideal homothetic price index (i.e. $Y = PC$)

Trans-Log Preferences: Price Index

- The price index P can also be written as a function of the mass of available varieties M and statistics of the distribution of prices:

$$\log P = \alpha_0 + \frac{1}{2}\frac{1 - M/\tilde{M}}{\gamma M} + \overline{\log p} - \frac{1}{2}\gamma M \operatorname{var}(\log p)$$

  where $\overline{\log p} = \left(\int_{\omega\in\Omega'} \log p(\omega) d\omega\right) / M$ and
  $\operatorname{var}(\log p) = \left(\int_{\omega\in\Omega'} \left[\log p(\omega) - \overline{\log p}\right]^2 d\omega\right) / M$

- Note that welfare $\nearrow$ ($P \searrow$) as:
  - $\overline{\log p} \searrow$
  - $\operatorname{var}(\log p) \nearrow$
  - $M \nearrow$ (love of variety, but at a decreasing rate as $M \longrightarrow \tilde{M}$)
    - $\partial \log P / \partial \log M = -1/(\gamma M)$

Trans-Log Preferences: Residual Demand

- Recall that:

$$\log P = \alpha_0 + \frac{1}{2}\frac{\tilde{M} - M}{\gamma M \tilde{M}} + \overline{\log p} - \frac{1}{2}\gamma M \operatorname{var}(\log p)$$

where $\overline{\log p} = \left(\int_{\omega \in \Omega'} \log p(\omega) d\omega\right) / M$ and
$\operatorname{var}(\log p) = \left(\int_{\omega \in \Omega'} \left[\log p(\omega) - \overline{\log p}\right]^2 d\omega\right) / M$

- Since the preferences are homothetic, residual demand for any good is

$$c(\omega)d\omega = C\frac{\partial P}{\partial p(\omega)} \implies c(\omega) = \frac{E}{p(\omega)}\left[\frac{1}{M} + \gamma\left(\overline{\log p} - \log p(\omega)\right)\right]$$

$$\text{(market share)} \implies s(\omega) = \frac{1}{M} + \gamma\left(\overline{\log p} - \log p(\omega)\right)$$

  - Note that, once again, residual demand is a function of the mass of firms and a statistic of the price distribution
  - ... and exhibits a threshold price: $p(\omega) \leq \overline{\log p} + 1/(\gamma M)$

Trans-Log Preferences: Endogenous Price Elasticity

- The price elasticity of residual demand is

$$\varepsilon(\omega) \equiv -\frac{\partial c(\omega)}{\partial p(\omega)}\frac{p(\omega)}{c(\omega)} = 1 + \frac{\gamma}{\frac{1}{M} + \gamma\left(\overline{\log p} - \log p(\omega)\right)}$$
$$= 1 + \frac{\gamma}{s(\omega)} \quad \text{(in terms of market share)}$$

- Note that this elasticity shifts up when M rises or $\overline{\log p}$ falls
  $\longrightarrow$ Captures an increase in competition (or 'toughness' of competition)
- Also note that bigger firms (those who set lower prices and obtain bigger market shares) face lower price elasticities

Additively Separable Preferences

- Consider the general additively separable case (See Krugman, JIE 1979 and Zhelobodko et al, Ecma 2012):

$$U = C = \int_{\omega \in \Omega} f\left(c\left(\omega\right)\right) d\omega$$

where

$$\varepsilon(\omega) \equiv -\frac{\partial c(\omega)}{\partial p(\omega)} \frac{p(\omega)}{c(\omega)} = -\frac{f'(c(\omega))}{c(\omega) f''(c(\omega))}$$

- Competition is price decreasing (increasing) if $\varepsilon(\omega) \searrow$ ($\varepsilon(\omega) \nearrow$) when $q(\omega), s(\omega) \nearrow$ (strongly supported by empirical evidence)
  - CES is limiting case where $\varepsilon(\omega) \longrightarrow$
  - Cross section: Bigger firms set higher markups
  - Across markets: Bigger markets (more competition) have lower prices (bigger firms) and tougher selection (with firm heterogeneity)
- See Dhingra & Morrow (WP) for welfare properties with firm heterogeneity

Additively Separable Preferences – Some Examples

- Recall
$$U = C = \int_{\omega \in \Omega} f\left(c\left(\omega\right)\right) d\omega \quad \text{and} \quad \varepsilon(\omega) = -\frac{f'(c(\omega))}{c(\omega) f''(c(\omega))}$$
- CARA utility (Behrens & Murata, JET 2007):
$$f(\omega) = k - ke^{-c(\omega)/\gamma} \qquad \text{and} \qquad \varepsilon(\omega) = \gamma / c(\omega)$$
- Stone-Geary (Simonovska, WP):
$$f(\omega) = \log\left[c(\omega) + \bar{q}\right]$$
- Bulow-Pfleiderer (JPE 1983) – Constant Pass-Through:
$$p(x) = \alpha + \beta x^{-\theta} \quad \text{with } \alpha > 0 \ \& \ \beta, \theta < 0 \text{ or } \alpha \geq 0 \ \& \ \beta, \theta > 0$$
  - Note: Defined via demand function!
  - This nests CES (as limiting case) and linear demand
- Price decreasing competition in all cases: $\varepsilon(\omega) \searrow$ when $c(\omega), s(\omega) \nearrow$
- Also, preferences are non-homothetic: $\varepsilon(\omega) \searrow$ for richer consumers (more inelastic demand)

Boundaries of the Firm

Lecture Notes

A Simple Transaction-Cost Model of the Firm

- Consider a situation in which the manager of a firm F has a access to a technology for converting an intermediate input into a final good.
  - If the input is of high quality, final-good production generates sales revenues equal to $R(x)$, where x is quantity of high quality input;
  - If the input is of low quality, $R(x) = 0$.
- Assume $R'(x) > 0$, $R''(x) < 0$, $\lim_{x \to 0} R'(x) = +\infty$, and $\lim_{x \to \infty} R'(x) = 0$.
- The manager F has two options for obtaining the intermediate input:
  - she can either manufacture it herself at a constant marginal cost of $\lambda > 1$;
  - she can buy it from an independent supplier (at a lower cost).
- Assuming no frictions inside the firm, this yields the standard MR=MC condition:

$$R'\left(x^V\right) = \lambda.$$

and

$$\Pi^V = R\left(x^V\right) - \lambda x^V.$$

- Both output and profits are decreasing in $MC = \lambda$

Arm's-Length Transacting

- The independent supplier S has access to a technology for producing high-quality intermediate inputs at a marginal cost of $1 < \lambda$.
  - but it can also produce low-quality intermediate inputs at a negligible cost.
- The intermediate input is **specialized** – tailored specifically to the final-good producer.
  - for simplicity, assume it is useless to any other producer.
- Contracts are **incomplete**: The managers F and S are are unable to write an ex-ante enforceable contract specifying the purchase of a specialized intermediate input of a particular quality for a certain price.
  - Third party is unable to verify quality
  - This is the only reason for modelling the low-tech input (never produced in equilibrium).
  - In addition, the parties cannot sign contracts contingent on the volume of sales revenues obtained when the final good is sold.
- Assume Nash bargaining outcome over the generated sales

Generalized Nash Bargaining

- If parties 1 and 2 'cooperate', they generate a net-gain Π (to be split among them)
- If they do not, they are left with their outside options O_1 and O_2
- Generalized Nash Bargaining outcome with $\beta \in (0, 1)$
  - Party 1 obtains $\beta(\Pi - O_1 - O_2) + O_1$
  - Party 2 obtains $(1 - \beta)(\Pi - O_1 - O_2) + O_2$
- Note that:
  - Payoffs sum to Π
  - Must have $\Pi > O_1 + O_2$ for cooperation: neither player can be made worse off by cooperating

The Hold-Up Problem

- Incomplete contract leads to classical **hold-up problem:**
  - The price of the intermediate input will only be determined ex-post
    - Investment incurred by supplier is then sunk and has no value outside the relationship (or lower value...)
  - F will try to push the price of the input as low as possible (but not "too much" if separation is costly to her).
- Nash bargaining determines ex-post price.
- Assume that the outside option for the final-good producer is zero (Can't produce anything if don't have input. This can be relaxed...).
- Let π_i denote the Nash bargaining payoff of agent i, then $\pi_F = \beta R(x)$ and $\pi_S = (1-\beta) R(x)$ (outside options for both F and S are zero)
- At investment stage, S will choose x^O such that (MR=MC condition):

$$(1-\beta) R'\left(x^O\right) = 1$$

1. Is chosen investment level x^O efficient?
2. How does β (bargaining power of F) affect efficiency of chosen investment?

Choice of Organization Form: Integration vs. Outsourcing

- Assume that F chooses the "efficient" organizational form that maximizes joint profits (including S):
  - So F chooses integration if $\Pi^V = R\left(x^V\right) - \lambda x^V \geq \Pi^O = R\left(x^O\right) - x^O$ and outsourcing otherwise.
- Trade off between the governance costs associated with integration ($\lambda > 1$) and the investment inefficiencies associated with outsourcing ($\beta \in (0, 1)$).
  - A high λ depresses x^V and Π^V
  - While a low β depresses x^O and Π^O
- In general, can show that:
  - $\Pi^V > \Pi^O$ if either λ or β are close enough to 1;
  - $\Pi^V < \Pi^O$ if λ is sufficiently high or β is close enough to 0.
- Grossman and Helpman (2002) present an industry equilibrium model of the integration vs. outsourcing decision.

A Simple Property-Rights Model

- The firm faces a demand function

$$y = Ap^{-1/(1-\alpha)}, \quad 0 < \alpha < 1 \quad \text{(so revenue is } R = y^{\alpha}\text{)}$$

- Production of good y requires the development of **two** specialized intermediate inputs h and m. Output is

$$y = \left(\frac{h}{\eta}\right)^{\eta} \left(\frac{m}{1-\eta}\right)^{1-\eta}, \quad 0 < \eta < 1$$

  where a higher η is associated with a more intensive use of h (headquarters) in production.

- So a choice of h and m leads to revenues:

$$R(h, m) = \left(\frac{h}{\eta}\right)^{\alpha\eta} \left(\frac{m}{1-\eta}\right)^{\alpha(1-\eta)}.$$

A Simple Property-Rights Model (Cont.)

- There are two agents engaged in production:
  - Final-good producer (F) who supplies h and produces the final good y.
  - Operator of a manufacturing plant (S) who supplies the input m.
- F can produce h at a constant marginal cost c_h; S can produce m at $MC = c_m$. In addition, production requires a fixed cost $f \cdot g\left(c_h, c_m\right)$.
- Both inputs are tailored specifically to y and are useless to anybody else.

Contractual Structure

- Before the investments h and m are made, the only contractibles are the allocation of residual rights of control (i.e., the ownership structure) and a lump-sum transfer between the two parties.
- Ex-post determination of the price of m results from generalized Nash bargaining.
- Ex-ante, F faces a perfectly elastic supply of potential S agents, so that in equilibrium the initial transfer secures the participation of S in the relationship at minimum cost to F.
- Key features:
  1. Ex-post bargaining takes place both under outsourcing and under integration;
  2. The distribution of surplus is sensitive to the mode of organization, because the outside option of F is higher under integration.
- Outside options are as follows:
  - Under outsourcing, a contractual breach gives 0 to both agents;
  - Under integration, F can fire S and seize the input m, but this reduces output to a fraction $\delta < 1$.

Nash Bargaining

- Assume that y units of output are produced, which generates net gains of $R = y^{\alpha}$
- Outsourcing
  - Outside options for both F and S are zero
  - F gets share β of R (call this β_O)
- Integration
  - Outside option for S is zero, but outside option for F is $(\delta y)^{\alpha}$
  - F gets share β of $\left[y^{\alpha} - (\delta y)^{\alpha}\right]$ plus outside option $(\delta y)^{\alpha}$
    - This represents a share $\delta^{\alpha} + \beta\left(1 - \delta^{\alpha}\right) > \beta$ of R (call this β_V)
- Why is $\beta_V > \beta_O$?

Choice of Organizational Structure

- Again, assume that the optimal ownership structure is chosen (maximizes joint profits)
- The optimal ownership structure k^* is the solution to the following program:

$$\begin{aligned} \max_{k \in \{V,O\}} \quad & \pi_k = R\left(h_k, m_k\right) - c_h \cdot h_k - c_m \cdot m_k - f \cdot g\left(c_h, c_m\right) - \overline{U} \\ s.t. \quad & h_k = \arg\max_h \left\{\beta_k R\left(h, m_k\right) - c_h \cdot h\right\} \\ & m_k = \arg\max_m \left\{\left(1 - \beta_k\right) R\left(h_k, m\right) - c_m \cdot m\right\} \end{aligned}$$

  where $\overline{U}$ is the outside option of the operator S.
- Transfers between F and S ensure that F picks the optimal k^*

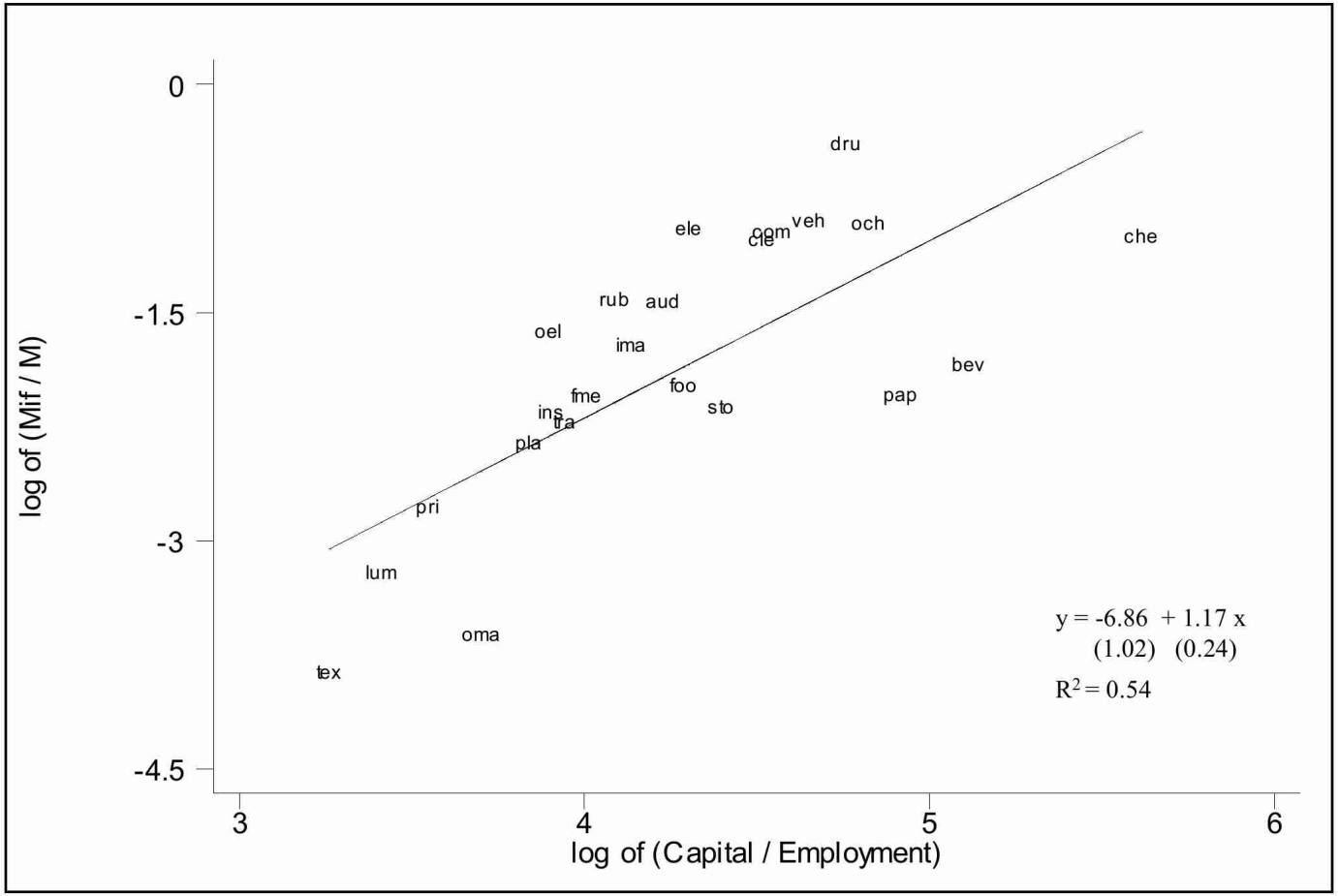
A Useful Result

- Antras (2003) shows that this choice is determined by η:
  - There exists a unique threshold $\widehat{\eta} \in (0,1)$ such that for all $\eta > \widehat{\eta}$, integration dominates outsourcing ($k^* = V$), while for all $\eta < \widehat{\eta}$, outsourcing dominates integration ($k^* = O$).
- As in Grossman and Hart (1986), in a world of incomplete contracts, ex-ante efficiency dictates that residual rights should be controlled by the party undertaking the relatively more important investment:
  - When production is intensive in m, **outsourcing** alleviates best the underinvestment problem;
  - When production is intensive in h, **vertical integration** does a better job.
- Convenient feature: threshold $\widehat{\eta}$ is independent of factor prices (Cobb-Douglas assumption important).

An Open-Economy Formulation: Antras (2003)

- Consider a two-country version of the model in which firms are allowed to locate different parts of the production process in different countries.
- Final good producers choose ownership structure and location of intermediate good production

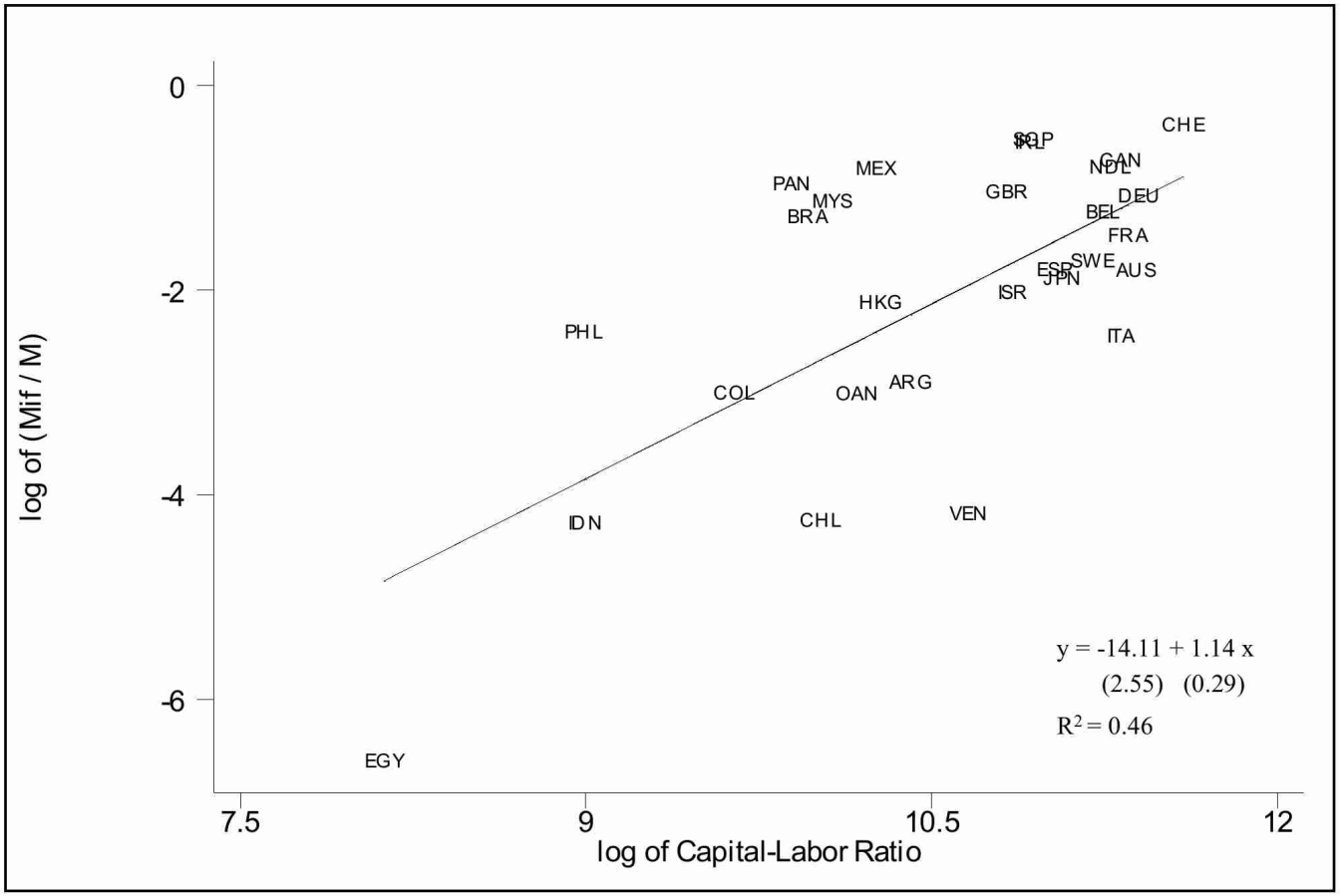
Antras (2003)



Notes: The Y-axis corresponds to the logarithm of the share of intrafirm imports in total U.S. imports for 23 manufacturing industries averaged over 4 years: 1987, 1989, 1992, 1994. The X-axis measures the average log of that industry's ratio of capital stock to total employment, using U.S. data. See Table A.1. for industry codes and Appendix A.4. for data sources.

Share of Intrafirm U.S. Imports and Relative Factor Intensities

Antras (2003)



Notes: The Y-axis corresponds to the logarithm of the share of intrafirm imports in total U.S. imports for 28 exporting countries in 1992. The X-axis measures the log of the exporting country's physical capital stock divided by its total number of workers. See Table A.2. for country codes and Appendix A.4. for details on data sources.

Share of Intrafirm Imports and Relative Factor Endowments

Global Sourcing: Antràs and Helpman (2004)

- **Environment and Preferences:** Two countries, North and South, and a unique factor of production, labor. There is a representative consumer in each country with quasi-linear preferences:

$$U = x_0 + \frac{1}{\mu}\sum_{j=1}^{J} X_j^{\mu},\ 0 < \mu < 1,$$

where x_0 is consumption of a homogeneous good, X_j is an index of aggregate consumption in sector j, and μ is a parameter.

- Aggregate consumption in sector j is a CES function

$$X_j = \left[\int x_j(i)^{\alpha} di\right]^{1/\alpha},\ 0 < \mu < \alpha < 1.$$

and the residual demand function:

$$x_j(i) = X_j^{\frac{\alpha-\mu}{1-\alpha}} p_j(i)^{-1/(1-\alpha)}$$

so revenue is $x_j(i)^{\alpha}$

The Model

- **Technology:** Producers of varieties face a perfectly elastic supply of labor. Let the wage in North be strictly higher than that in South ($w^N > w^S$).
  - Producers needs to incur sunk entry costs $w^N f_E$, after which they learn their productivity $\theta \sim G(\theta)$.
  - As in Antràs (2003), final-good production combines two specialized inputs according to:

$$x_j(i) = \theta \left(\frac{h_j(i)}{\eta_j} \right)^{\eta_j} \left(\frac{m_j(i)}{1-\eta_j} \right)^{1-\eta_j}, \quad 0 < \eta_j < 1.$$

  - h is controlled by a final-good producer (agent F), m is controlled by an operator of the production facility (agent S).
  - Sectors vary in their intensity of headquarter services η_j. Furthermore, within sectors, firms differ in productivity θ.
  - Intermediates are produced using labor with a fixed coefficient (same productivity in both countries):
    - Producing intermediate $m_j(i)$ in South is always cheaper because $w^S < w^N$
  - $h_j(i)$ is produced only in the North, which implies that the headquarters H are always located in North

The Model (continued)

- After observing θ, F decides whether to exit the market or start producing.
- In the latter case additional fixed costs of organizing production are incurred.
  - These additional fixed costs are a function of the structure of ownership and the location of production.
  - In particular, if an organizational form is $k \in \{V, O\}$ and $\ell \in \{N, S\}$, these fixed costs are $w^N f_k^\ell$ and satisfy

$$f_V^S > f_O^S > f_V^N > f_O^N.$$

- Contracting is the same as before, but $\delta^N \geq \delta^S$.
- The ex-post division of surplus is:

	North	South
Outsource	$\beta_O^N = \beta$	$\beta_O^S = \beta$
Integration	$\beta_V^N = (\delta^N)^\alpha + \beta\left[1 - (\delta^N)^\alpha\right]$	$\beta_V^S = (\delta^S)^\alpha + \beta\left[1 - (\delta^S)^\alpha\right]$

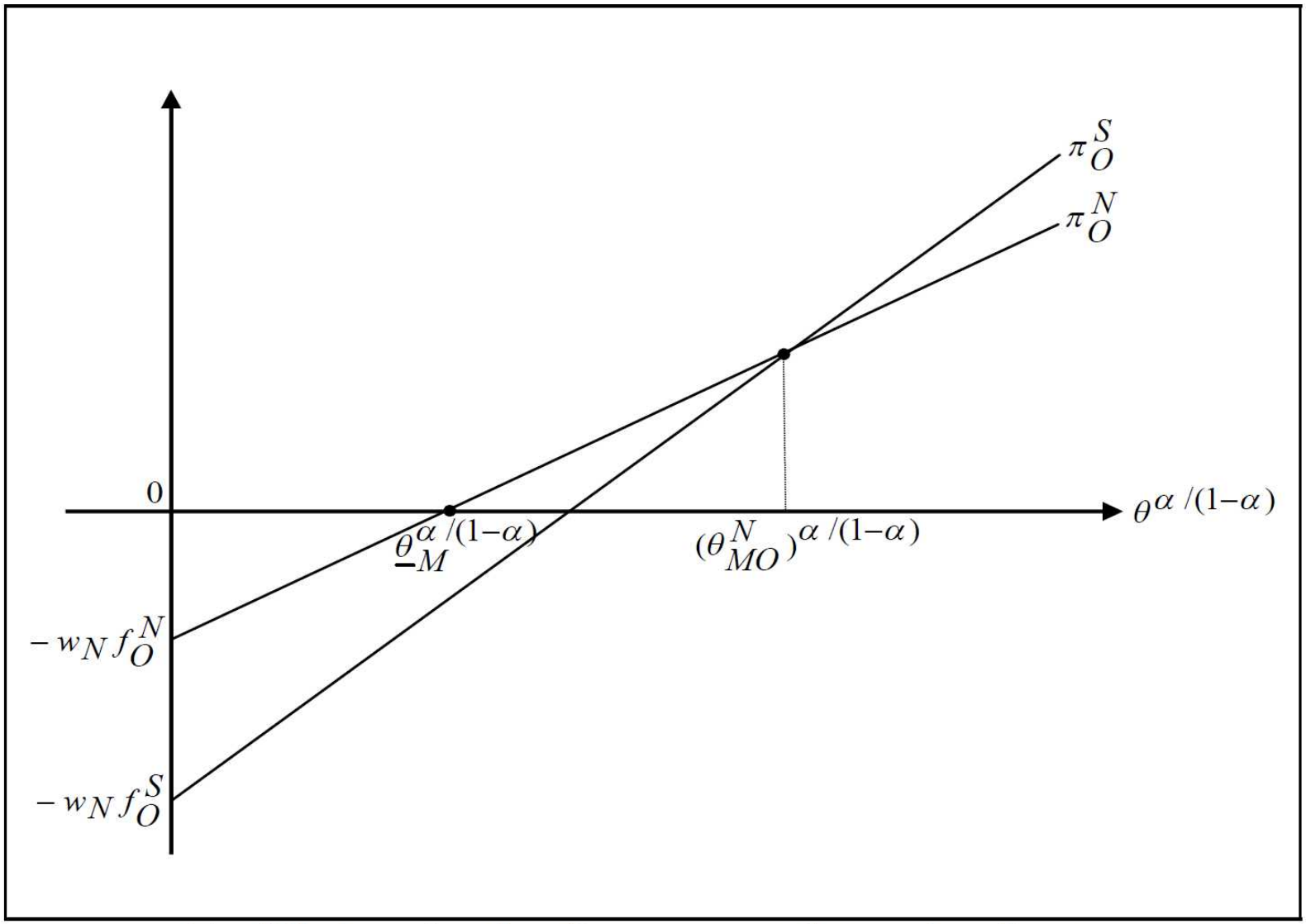
- Notice that

$$\beta_V^N \geq \beta_V^S > \beta_O^N = \beta_O^S = \beta.$$

Tradeoffs

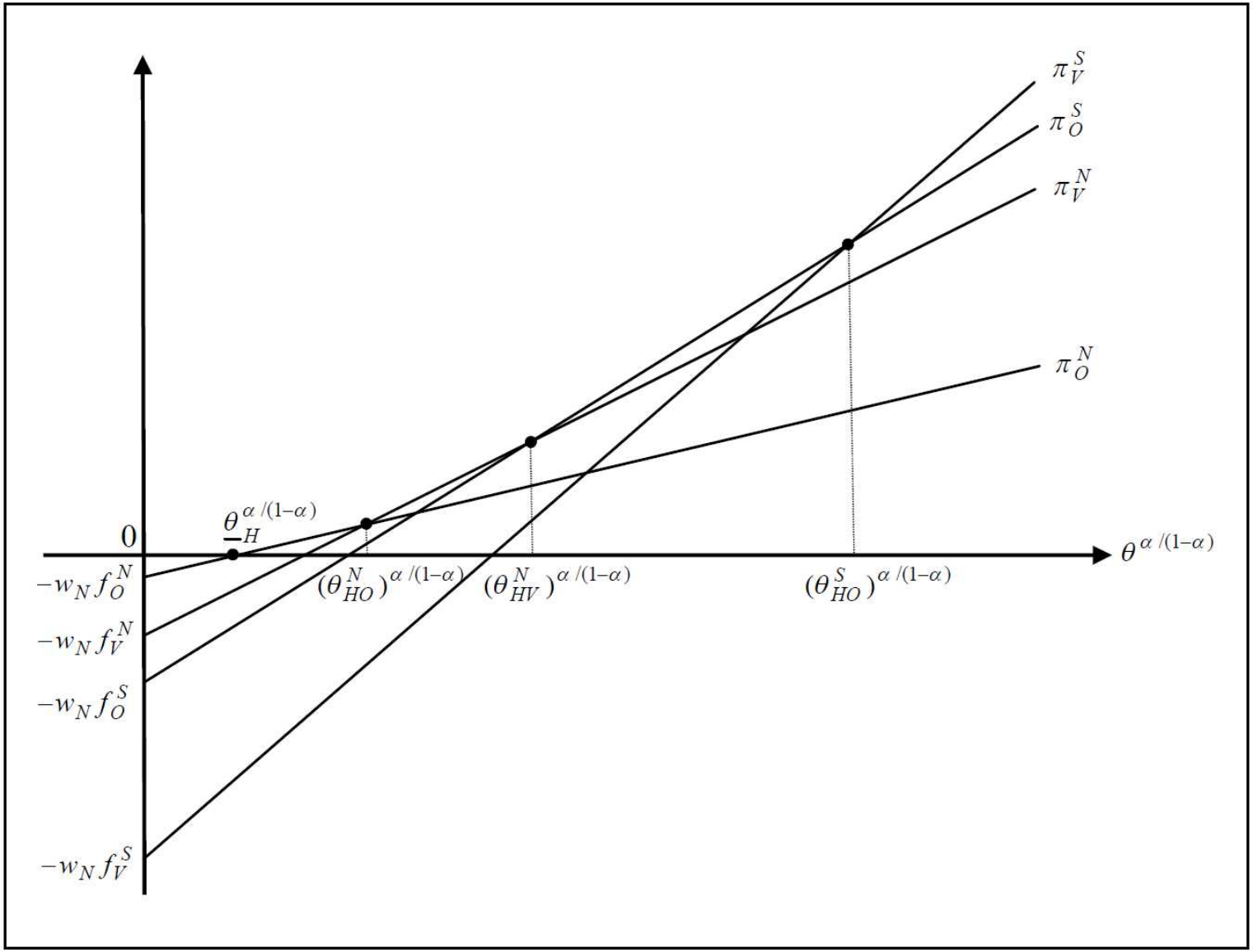
- Two key decisions: integrate – and where to produce (4 outcomes)
- The choice of an organizational form faces two types of tensions:
  - Location decision: variable costs are lower in South, but fixed costs are higher there, and productivity θ affects the profitability of offshoring.
  - Integration decision: integration improves efficiency of variable production when η is high, but involves higher fixed costs. This decision therefore depends on η, and also on θ.
- Focus on two types of sectors:
- A **Component-intensive sector** (low η):
  - Integration in both N and S is dominated in equilibrium
  - So outsource – but need to decide whether produce in N or S
- A **Heaquarter-intensive sector** (high η)
  - Integration leads to lowest unit cost of production – but there is trade-off with higher fixed cost
  - At lower production scale, outsourcing could still be preferred due to lower fixed costs.
  - Rich pattern of integration and outsourcing in both countries

Firm Decisions



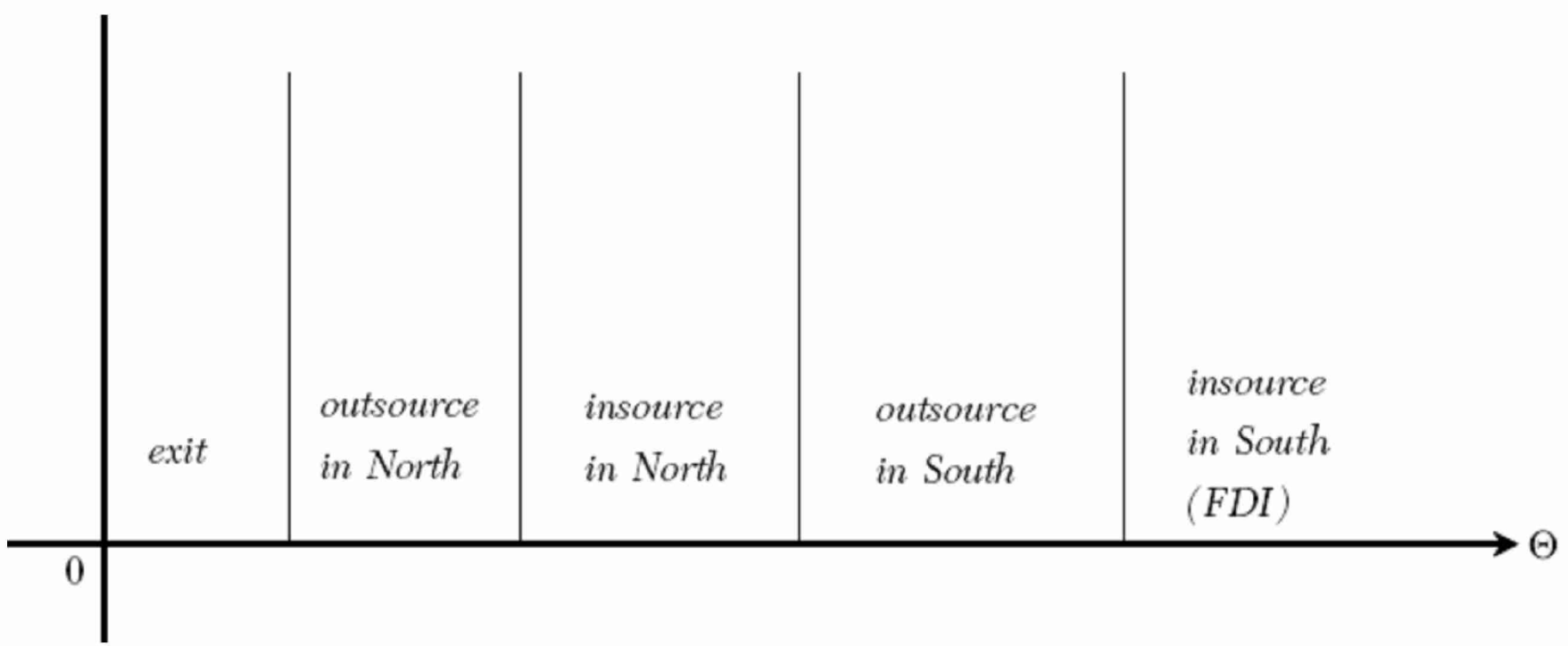
Equilibrium in the Component-Intensive Sector

Firm Decisions (Cont.)



Equilibrium in the Headquarter-Intensive Sector

Firm Sorting: Headquarter Intensive Case



Gravity with Firm Heterogeneity

Lecture Notes

Gravity: Review

- Consider a world with many countries
- Same CES preferences across countries with elasticity of substitution with $\sigma > 1$
- Iceberg trade cost τ_{ni} for trade from i to n
- Consider 2 different modeling assumptions for production side: Armington and monopolistic competition with representative firms

Modeling Assumptions

- Armington:
  - Country produces a single differentiated good
  - Production level is exogenously fixed by country endowment and technology
  - Competitive markets: price differences across importers reflect trade costs: $p_{ni} = \tau_{ni} p_{ii}$
  - Domestic price p_{ii} endogenously adjusts to clear markets
- Monopolistic competition:
  - Firms produce differentiated varieties with unit input requirement a and overhead fixed production f
  - Cross-country differences in technology subsumed in factor endowments L_i
  - Input factor price w_i varies across countries
  - Firms optimize: set constant markup $\sigma / (\sigma - 1)$ over marginal cost $w_i a$ (for country i production)
  - Number of firms in country i, N_i, endogenously adjusts (no need to further specify for gravity results)

Gravity Result

- Consider an exporter i and importer n
- Let Y_i denote value of production for country i and X_n total expenditures for country n
- Let P_n is the CES price index associated with the consumption bundle in country n
- Both models lead to same gravity prediction for aggregate bilateral trade from i to n:

$$X_{ni} = \frac{Y_i}{\Xi_i} \frac{X_n}{P_n^{1-\sigma}} \tau_{ni}^{1-\sigma} \qquad \text{where} \qquad \Xi_i \equiv \sum_n \frac{X_n}{P_n^{1-\sigma}} \tau_{ni}^{1-\sigma}$$

- Importer fixed effect $X_n / P_n^{1-\sigma}$ is CES market demand parameter for any producer selling in n: $q = \left(X_n / P_n^{1-\sigma}\right) p^{1-\sigma}$
  - It combines total import market size X_n as well as competition in market n via the price index there
- Exporter fixed effect Y_i / Ξ_i combines exporter's GDP with its market potential: average of all importers' market demand – weighted by trade costs

Gravity Result: Implications

Recall

$$X_{ni} = \frac{Y_i}{\Xi_i} \frac{X_n}{P_n^{1-\sigma}} \tau_{ni}^{1-\sigma} \qquad \text{where} \qquad \Xi_i \equiv \sum_n \frac{X_n}{P_n^{1-\sigma}} \tau_{ni}^{1-\sigma}$$

- As emphasized by Anderson & van Wincoop (2003), an empirical gravity estimation must control for a country's geography, as well as its market size
  - They also show that when trade costs are symmetric $(\tau_{ni} = \tau_{in})$ and trade is balanced $(Y_i = X_i)$, then the exporter's market potential is $\Xi_i = P_i^{1-\sigma}$, so there is a single country geography index: 'remoteness'
- The elasticity of substitution amplifies the effects of bilateral trade costs

Gravity Result: Implications for Extensive Margin

- What do the Armington and monopolistic competition models predict for the extensive margin of trade w.r.t to:
  - Exporter size?
    - Hummels & Klenow (2005) report an elasticity of .6
  - Importer size?
  - Bilateral trade costs?
- How does this concord with further empirical work on the extensive margin of trade?

Distorted Gravity: Heterogeneous Firms, Market Structure and the Geography of International Trade
by Chaney

Motivation

- It is accepted wisdom (in both the trade and international macro literature) that high degrees of substitutability between goods amplifies the impact of trade costs on trade volumes (the overall "trade elasticity")
- This paper provides strong arguments and evidence overturning this accepted wisdom
- Main insight: trade costs not only affect the intensive margin of trade (how much of an individual good is traded) but also the extensive margin of trade (how many goods are traded)
  - Most standard models neglect this effect on the extensive margin
  - Product substitutability affects the impact of trade costs on both margins in opposite ways
  - ... and the impact on the extensive margin dominates

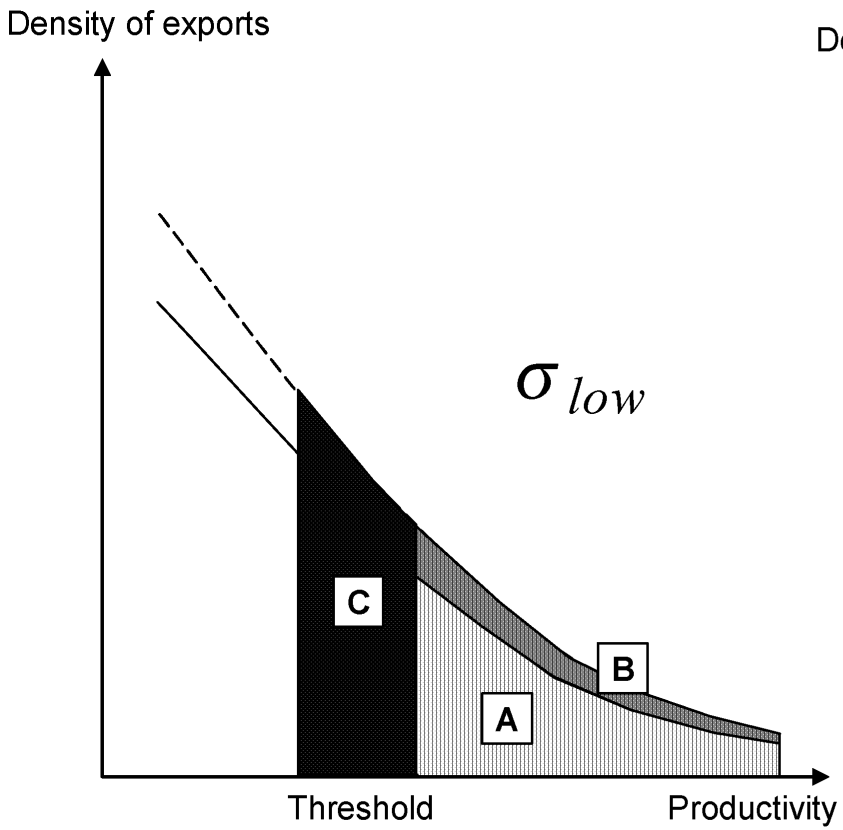
Main Results

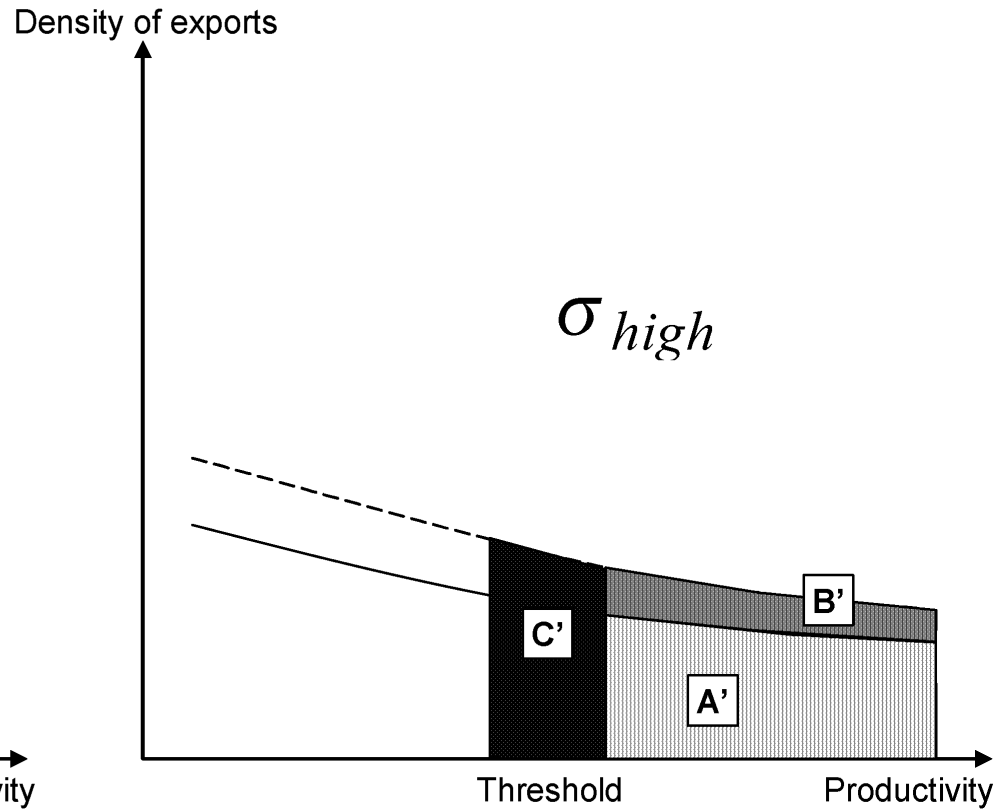
- Start with a gravity model of trade with heterogeneous firms
  - Features both an intensive and extensive margin response to changes in trade costs
- The elasticity of trade with respect to trade costs is higher in a model with firm heterogeneity than in a standard gravity model with representative firms
  - The added effect of the extensive margin

Main Results (Cont.)

- Consider the response of trade volumes to a given change in variable trade costs in 2 sectors (high and low degrees of product substitutability)
  In the sector with high product substitutability (relative to the other sector):
  - The response elasticity from the intensive margin is higher (standard result)
  - However, the response elasticity from the extensive margin is lower
  - Why? Due to differences in the size distribution of firms across sectors
  - For a Pareto size distribution, the two effects cancel out
  - For any distribution with thinner tails, the extensive margin effect dominates
- Now consider the response of trade volumes to a given change in fixed trade costs in 2 sectors (high and low degrees of product substitutability)
  - The response elasticity from the intensive margin is zero
  - The extensive margin response is lower for the sector with high product substitutability
- Putting all this together, when trade costs fall (some combination of fixed and variable trade costs), then the extensive margin response will dominate
- ... and this response is dampened in sectors with high product substitutability (relative to the other sector)

Extensive and Intensive Margin Responses





A General Derivation of Gravity with Heterogeneous Firms

A General Derivation of Gravity with Heterogeneous Firms

- Consider a world with many countries
- Same CES preferences across countries with elasticity of substitution with $\sigma > 1$
- Country i has a measure N_i of potential producers (can think of entrants)
- Potential producers draw their unit input requirement a from a distribution $G(a) = (a/\bar{a})^k$, where $k > \sigma - 1$ and $0 \leq a \leq \bar{a}$
  - Same $G(.)$ across countries but cost of input bundle w_i varies
- Trade costs for trade between i and n:
  - $\tau_{ni} \geq 1$ is variable (per-unit) trade costs (iceberg)
  - $f_{ni} > 0$ is fixed trade cost
  - These costs include costs of serving domestic market where $i = n$ (can assume $\tau_{ii} = 1$ and that f_{ii} includes overhead fixed cost)

Gravity with Heterogeneous Firms (Cont.)

- If a producer in country i with unit cost a exports to n it will set a price $p_{ni}(a)$ and generate export sales $x_{ni}(a)$ and export profits $\pi_{ni}(a)$

$$p_{ni}(a) = \frac{\sigma}{\sigma-1} w_i \tau_{ni} a, \quad x_{ni}(a) = \frac{X_n}{P_n^{1-\sigma}} p_{ni}(a)^{1-\sigma}, \quad \pi_{ni}(a) = \frac{1}{\sigma} x_{ni}(a) - w_i f_{ni}$$

  where X_n is total expenditures in country n and P_n the associated price index with that aggregate consumption bundle
- The cutoff for profitable exports from i to n, a_{ni}, is determined by $\pi_{ni}(a_{ni}) = 0$ (We assume that $\bar{a}$ is high enough such that $a_{ni} \leq \bar{a}\ \forall i, n$)
- Aggregate bi-lateral trade from i to n is then

$$X_{ni} = N_i \int_0^{a_{ni}} x_{ni}(a) dG(a)$$

- Define $Y_i \equiv \sum_n X_{ni}$ as the value of country i's aggregate output. Then (skipping over some tedious algebra) the aggregate bi-lateral trade from i to n can be written:

$$X_{ni} = \frac{Y_i}{\Xi_i} \left(\frac{X_n}{P_n^{1-\sigma}} \right)^{\frac{k}{\sigma-1}} \tau_{ni}^{-k} f_{ni}^{-\frac{k-\sigma+1}{\sigma-1}} \quad , \quad \Xi_i \equiv \sum_n \left(\frac{X_n}{P_n^{1-\sigma}} \right)^{\frac{k}{\sigma-1}} \tau_{ni}^{-k} f_{ni}^{-\frac{k-\sigma+1}{\sigma-1}}$$

Contrast with Previous Gravity Derivations

$$X_{ni} = \frac{Y_i}{\Xi_i}\left(\frac{X_n}{P_n^{1-\sigma}}\right)^{\frac{k}{\sigma-1}} \tau_{ni}^{-k} f_{ni}^{-\frac{k-\sigma+1}{\sigma-1}} \quad , \quad \Xi_i \equiv \sum_n \left(\frac{X_n}{P_n^{1-\sigma}}\right)^{\frac{k}{\sigma-1}} \tau_{ni}^{-k} f_{ni}^{-\frac{k-\sigma+1}{\sigma-1}}$$

Contrast with:

$$X_{ni} = \frac{Y_i}{\Xi_i}\frac{X_n}{P_n^{1-\sigma}}\tau_{ni}^{1-\sigma} \quad , \quad \Xi_i \equiv \sum_n \frac{X_n}{P_n^{1-\sigma}}\tau_{ni}^{1-\sigma}$$

- Per-unit trade costs now affect both the intensive and extensive margin of trade
  - Elasticity of substitution does not magnify effect of per-unit trade costs
- Fixed trade costs only affect the extensive margin of trade
- Importer CES market demand $X_n / P_n^{1-\sigma}$ is amplified by $k/(\sigma-1) > 1$
  - $\longrightarrow$ Effect of market demand on the extensive margin of imports
- Exporter's market potential is computed in identical way (given differences in trade costs and importer fixed effect)

Contrast with Eaton-Kortum Gravity Derivation

- Recall that Eaton-Kortum predict

$$X_{ni} = \frac{Y_i}{\Xi_i} \frac{X_n}{\Phi_n} \tau_{ni}^{-\theta} \quad , \quad \Xi_i \equiv \sum_n \frac{X_n}{\Phi_n} \tau_{ni}^{-\theta}$$

- Elasticity of substitution does not magnify trade costs
- Importer competition effect is now captured by country's technology parameter Φ_n (geography weighted with trading partners)
  - Recall

$$\Phi_n \equiv \sum_{i=1}^{N} T_i \left(w_i d_{ni} \right)^{-\theta}$$

- Exporter's market potential is computed in identical way

Aside: Effect of Per-Unit Trade Costs on Prices

- Just like in Eaton-Kortum model, the elasticity of substitution does not amplify per-unit trade costs because they do not affect the distribution of delivered prices
- Note that export cutoff condition is determined by:

$$\frac{1}{\sigma}\frac{X_n}{P_n^{1-\sigma}}\left(\tfrac{\sigma}{\sigma-1}w_i\tau_{ni}a_{ni}\right)^{1-\sigma} = w_i f_{ni}$$

Excluding GE effects on country level variables, an increase in τ_{ni} is matched by a proportional decrease in the cutoff a_{ij} leaving the distribution of delivered prices $p_{ni}(a) = \frac{\sigma}{\sigma-1}w_i\tau_{ni}a, a \leq a_{ni}$ unchanged.

Predictions for Extensive Margin of Trade

Contrast predictions for the extensive margin of trade between E-K and monopolistic competition with heterogeneous firms:

- Trade costs: both predict an extensive margin response
- Exporter size: both predict that # of exported varieties increase with size (via higher T_i for E-K)
- Importer size: opposite predictions
  - E-K: bigger countries (via higher T_n) import fewer varieties $\longrightarrow$ Competition effect of domestic varieties along with fixed extensive margin of consumption
  - Monop. Compt: bigger countries import more varieties, and also higher extensive margin of consumption
    - However, no independent channel of GDP per capita on extensive margin (independent of overall country GDP)

Extending the E-K Model to Match Gravity Predictions

- Fieler (2011) and Hepenstrick (2010) extend E-K model to non-homothetic preferences
  - In both cases, they derive very similar aggregate bi-lateral trade gravity derivation, but also incorporate an effect of GDP per capita on imports:
    - Fieler: Higher GDP per capita (controlling for GDP) leads to higher imports
    - Hepenstrick also adds effect of GDP per capita on the # of products imported
    - Strong support for both empirically