AEA Continuing Education Program

DSGE Models and the Role of Finance

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Two-Period Version of Gertler-Karadi, Gertler-Kiyotaki Financial Friction Model

Lawrence J. Christiano

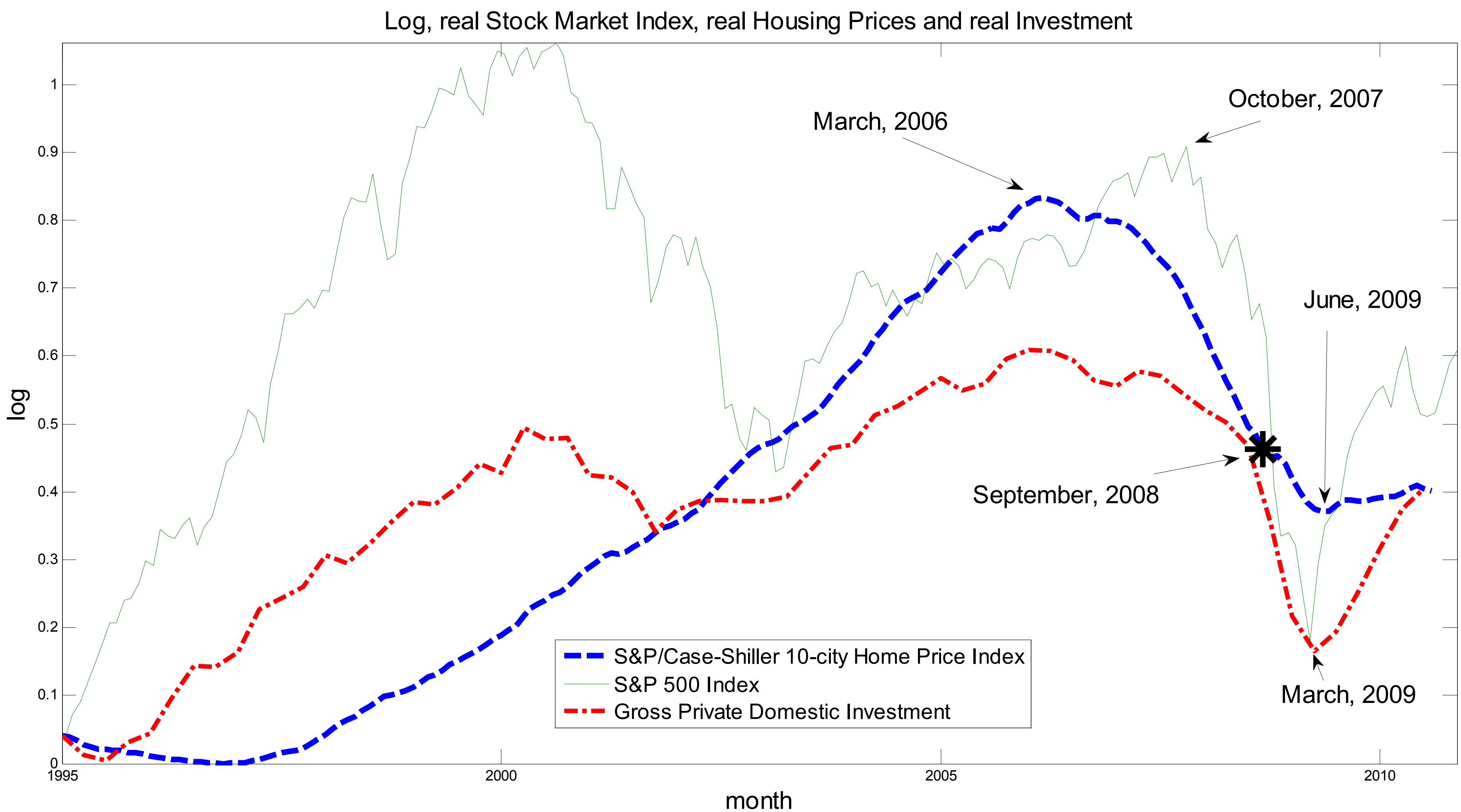
Summary of Christiano-Ikeda, 2012, ‘Government Policy, Credit Markets and Economic Activity,’ in
Federal Reserve Bank of Atlanta conference volume,

A Return to Jekyll Island: the Origins, History, and Future of the Federal Reserve, Cambridge University Press.

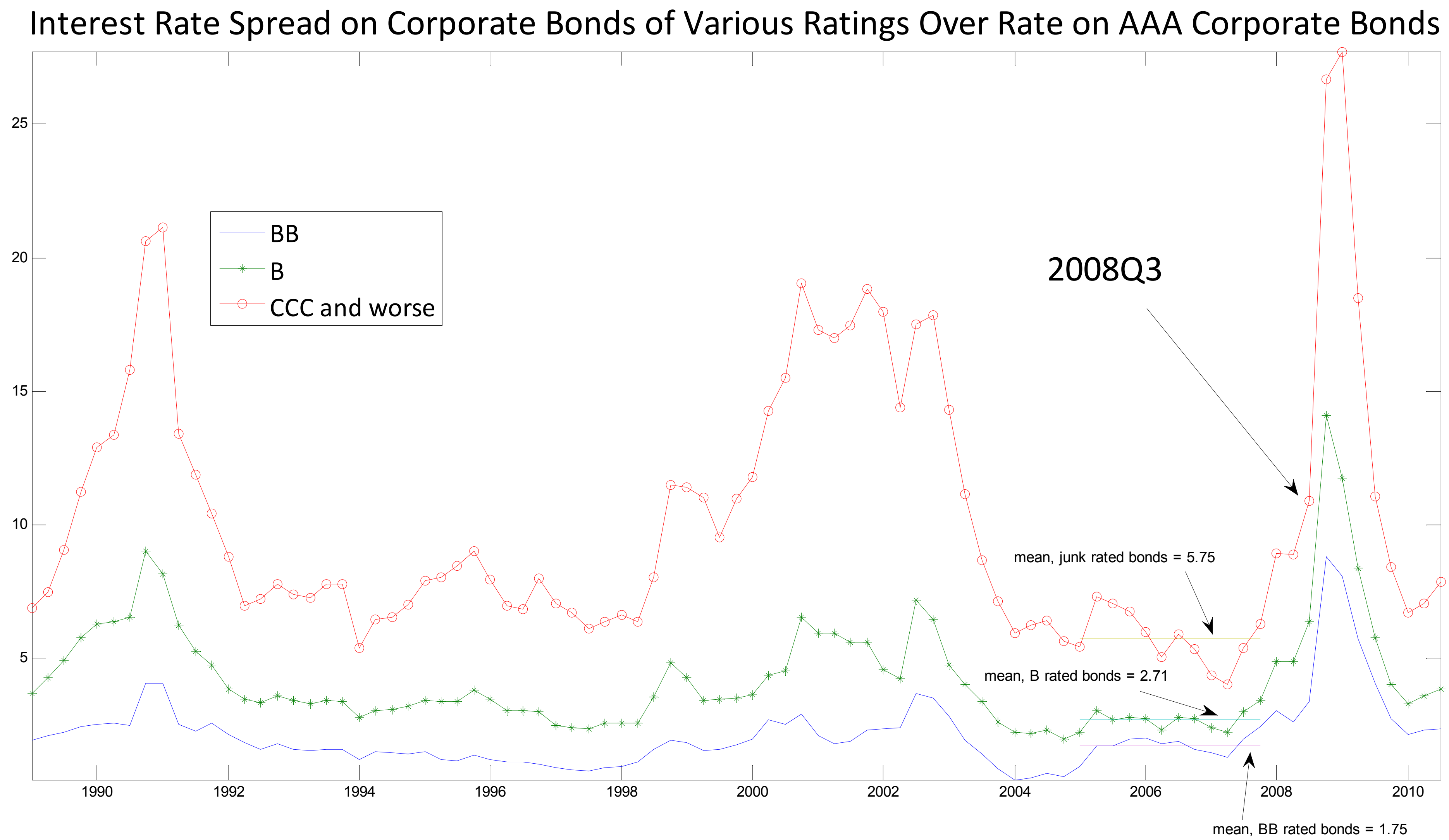
Motivation

- Beginning in 2007 and then accelerating in 2008:
  - Asset values (particularly for banks) collapsed.
  - Intermediation slowed and investment/output fell.
  - Interest rates spreads over what the US Treasury and highly safe private firms had to pay, jumped.
  - US central bank initiated unconventional measures (loans to financial and non-financial firms, very low interest rates for banks, etc.)
- In 2009 – the worst parts of 2007-2008 began to turn around.

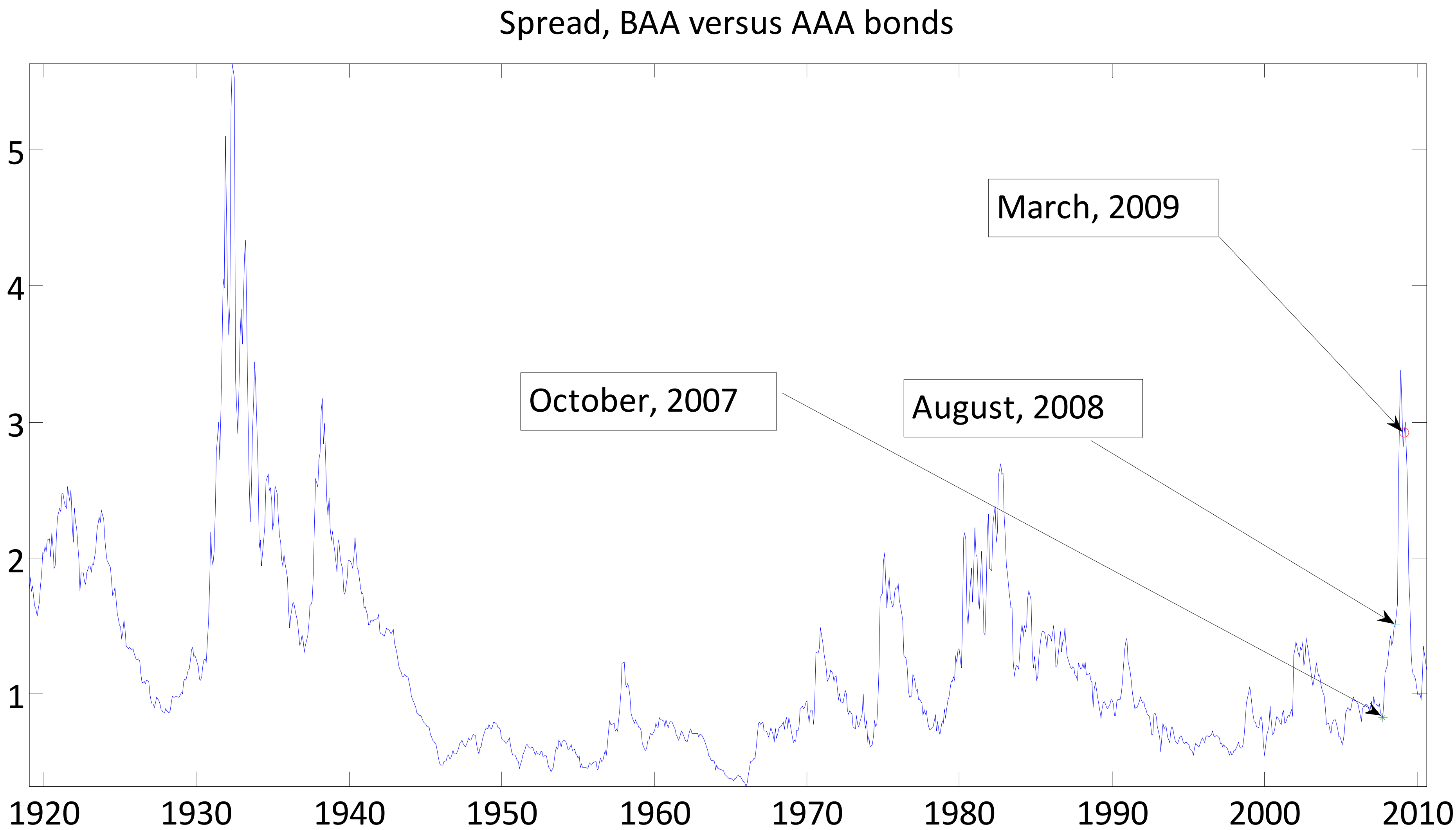
Collapse in Asset Values and Investment



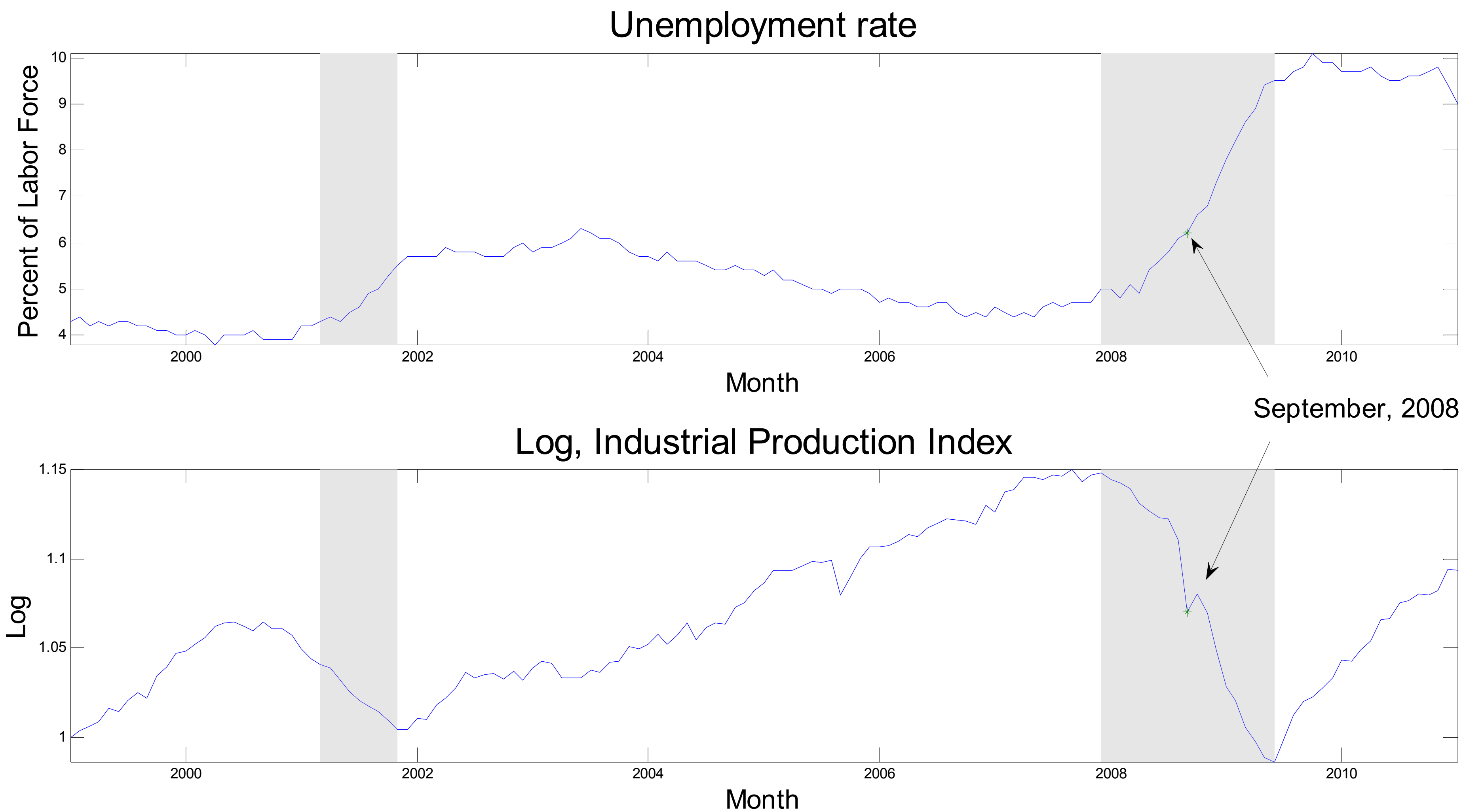
Spreads for 'Risky' Firms Shot Up in Late 2008



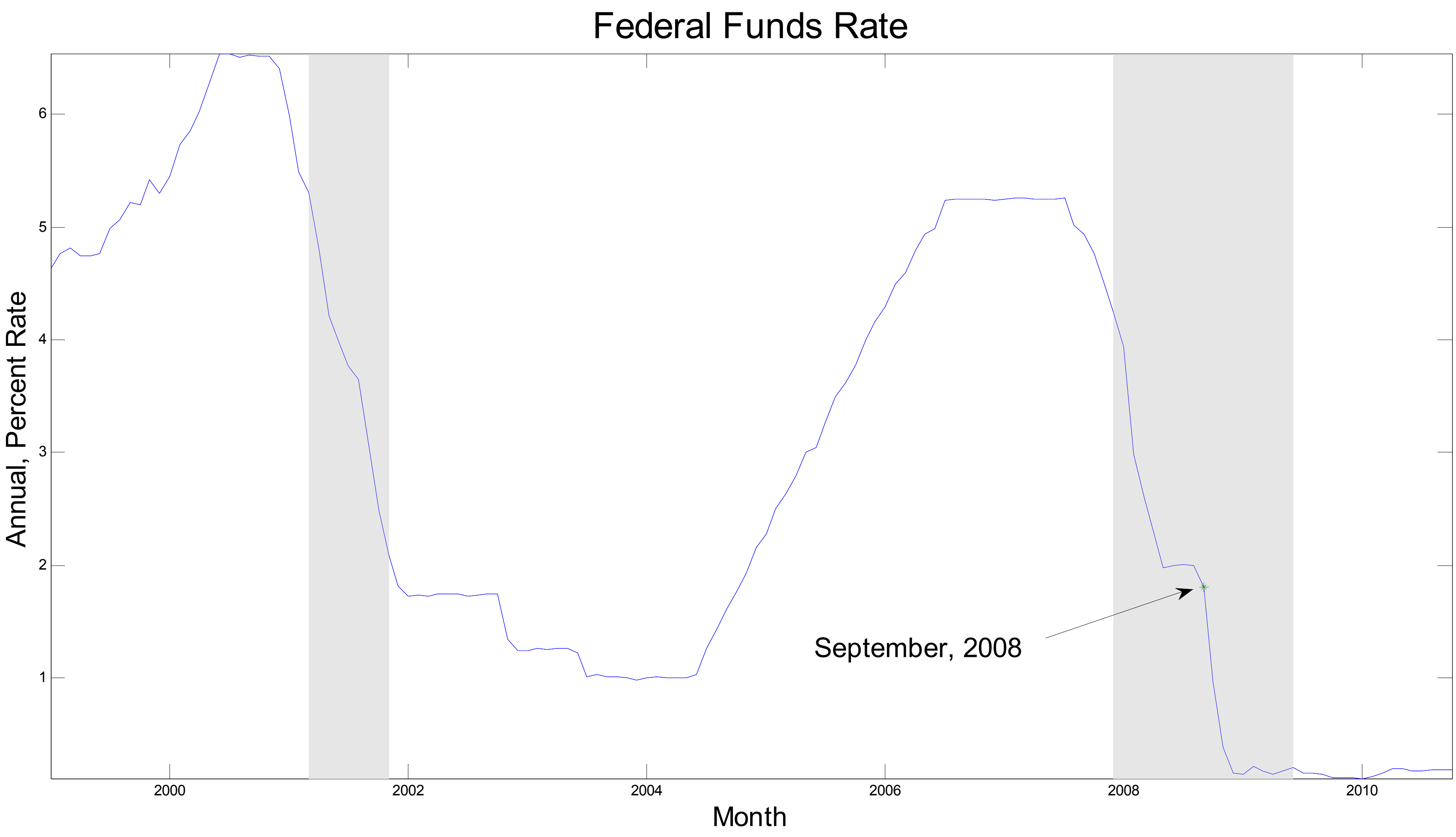
Must Go Back to Great Depression to See Spreads as Large as the Recent Ones



Economic Activity Shows (anemic!) Signs of Recovery June, 2009



Banks' Cost of Funds Low



Characterization of Crisis to be Explored Here

- Bank Asset Values Fell.
- Banking System Became ‘Dysfunctional’
  - Interest rate spreads rose.
  - Intermediation and economy slowed.
- Monetary authority:
  - Transferred funds on various terms to private companies and to banks.
  - Sharply reduced cost of funds to banks.
- Economy in (tentative) recovery.
- Seek to construct models that links these observations together.

Objective

- Keep analysis simple and on point by:
  - Two periods
  - Minimize complications from agent heterogeneity.
  - Leave out endogeneity of employment.
  - Leave out nominal variables: just look ‘behind the veil of monetary economics’

- Models:
  - Gertler-Kiyotaki/Gertler-Karadi
  - In two-period setting easy to study an interesting nonlinearity that is possible:
    - Participation constraint may be binding in a crisis and not binding in normal times.

Two-period Version of GK Model

- Many identical households, each with a unit measure of members:
  - Some members are 'bankers'
  - Some members are 'workers'
  - Perfect insurance inside households…everyone consumes same amount.
- Period 1
  - Workers endowed with y goods, household makes deposits, d, in a bank
  - Bankers endowed with N goods, take deposits and purchase securities, d, from a firm.
  - Firm issues securities, s, to produce sR^k in period 2.
- Period 2
  - Household consumes earnings from deposits plus profits, π, from banker.
  - Goods consumed are produced by the firm.

Problem of the Household		
	period 1	period 2
budget constraint	$c + d \leq y$	$C \leq R^d d + \pi$
problem	$\max_{c,C,d}[u(c) + \beta u(C)]$	

Solution to Household Problem	
$\frac{u'(c)}{\beta u'(C)} = R^d$	$c + \frac{C}{R^d} = y + \frac{\pi}{R^d}$

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Household budget constraint when gov't buys private assets using tax receipts, T, and gov't gets the same rate of return, R^d, as households:

No change!
(Ricardian-Wallace
Irrelevance)

$$c + \frac{C}{R^d} = y - T + \frac{\pi + TR^d}{R^d} = y + \frac{\pi}{R^d}$$

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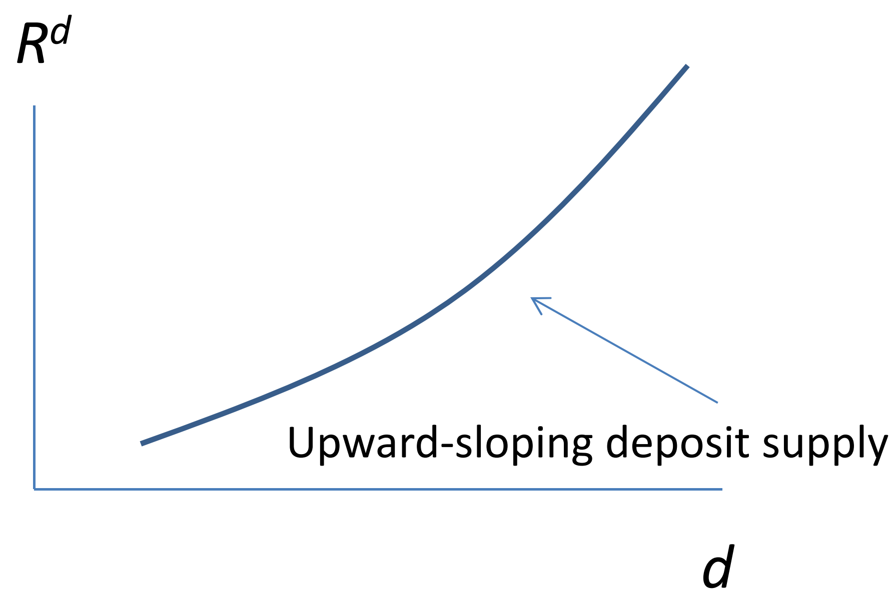
Household Supply of Deposits

- For given π, *d* rises or falls with R^d, depending on parameter values.
- But, in equilibrium $\pi=R^k(N+d)-R^d d$.
- Substituting into the expression for *c* and solving for *d*:

$$d=\frac{(\beta R^d)^{\frac{1}{\gamma}}-\frac{N}{y}R^k}{(\beta R^d)^{\frac{1}{\gamma}}+R^k}y$$



Household Supply of Deposits

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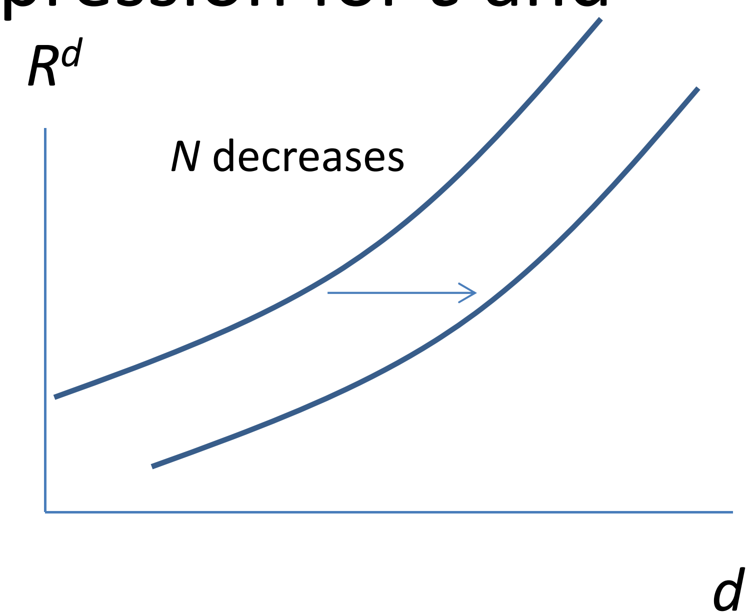
- But, in equilibrium $\pi = R^k(N+d) - R^d d$.
- Substituting into the expression for c and solving for d:

$$d = \frac{(\beta R^d)^{\frac{1}{\gamma}} - \frac{N}{y} R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k} y$$



Properties of Equilibrium Household Supply of Deposits

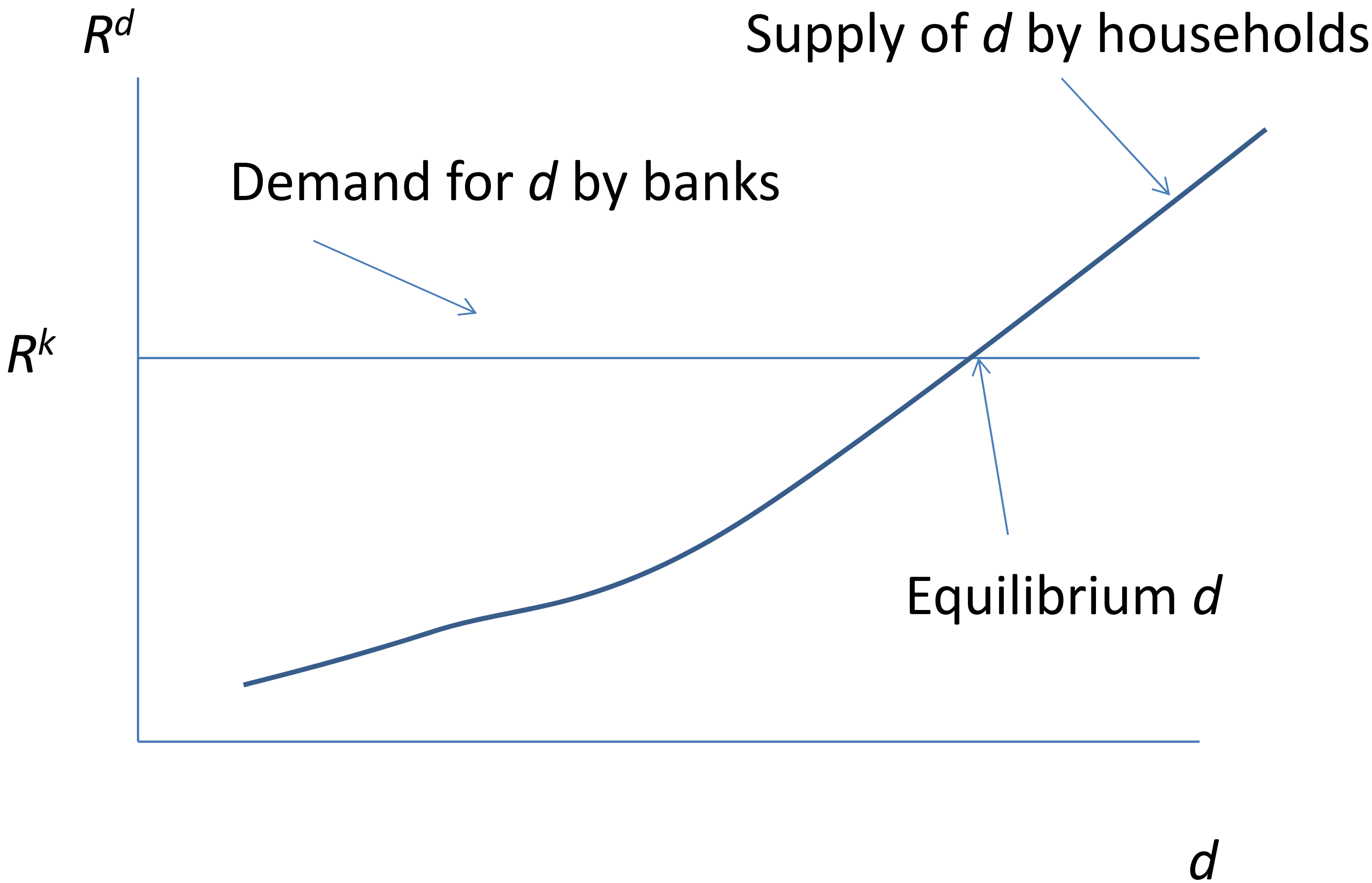
- Deposits increasing in R^d.
- Shifts right with decrease in N because of wealth effect operating via bank profits, π.
  - rise in deposit supply smaller than decrease in N.

$$\frac{\partial d}{\partial N} = -\overbrace{\left[\frac{R^k}{(\beta R^d)^{\frac{1}{\gamma}} + R^k}\right]}^{>0,\ <1}$$

Efficient Benchmark

Problem of the Bank	
period 1	period 2
take deposits, d	pay dR^d to households
buy securities, $s = N + d$	receive sR^k from firms
problem: $\max_d [sR^k - R^d d]$	

Bank demand for *d*



Equilibrium in Absence of Frictions

Interior Equilibrium: R^d, π, d, c, C

(i) $c, d, C > 0$

(ii) household problem is solved

(iii) bank problem is solved

(iv) goods and financial markets clear

- Properties:
  - Household faces true social rate of return on saving:

$$R^k = R^d$$

  - Equilibrium is 'first best', i.e., solves

$$\max_{c,C,k,} u(c) + \beta u(C)$$

$$c + k \leq y + N,\ C \leq kR^k$$

Friction

- bank combines deposits, *d*, with net worth, *N*, to purchase *N+d* securities from firms.

- bank has two options:
  - ('no-default') wait until next period when $(N+d)R^k$ arrives and pay off depositors, $R^d d$, for profit:

$$(N+d)R^k - R^d d$$

  - ('default') take $\theta(N+d)$ securities, refuse to pay depositors and wait until next period when securities pay off:

$$\theta(N+d)R^k$$

– Bank must announce what value of *d* it will choose at the beginning of a period.

Incentive Constraint

- Recall, banks maximize profits
- Choose 'no default' iff

$$\overbrace{(N+d)R^k - R^d d}^{\text{no default}} \quad \geq \quad \overbrace{\theta(N+d)R^k}^{\text{default}}$$

- Next: derive banking system's demand for deposits in presence of financial frictions.

Result for a no-default equilibrium:

- Consider an individual bank that contemplates defaulting.
- It sets a *d* that implies default,

$R^k(N+d) - R^d d < \theta R^k(d+N)$, or

$$\overbrace{R^d}^{\text{what the household gets in the other banks}} > \overbrace{\frac{(1-\theta)R^k(d+N)}{d}}^{\text{what the household gets in the defaulting bank}}$$

- A deviating bank will in fact receive no deposits.
- **An optimizing bank would never default**

Problem of the bank in no-default, interior equilibrium

- Maximize, by choice of *d*,

$$R^k(N+d) - R^d d$$

subject to:

$$R^k(N+d) - R^d d - R^k\theta(N+d) \geq 0,$$

or,

$$(1-\theta)R^k N - [R^d - (1-\theta)R^k]d \geq 0.$$

If interest rate is REALLY low, then bank has no incentive to default because it makes lots of profits not defaulting

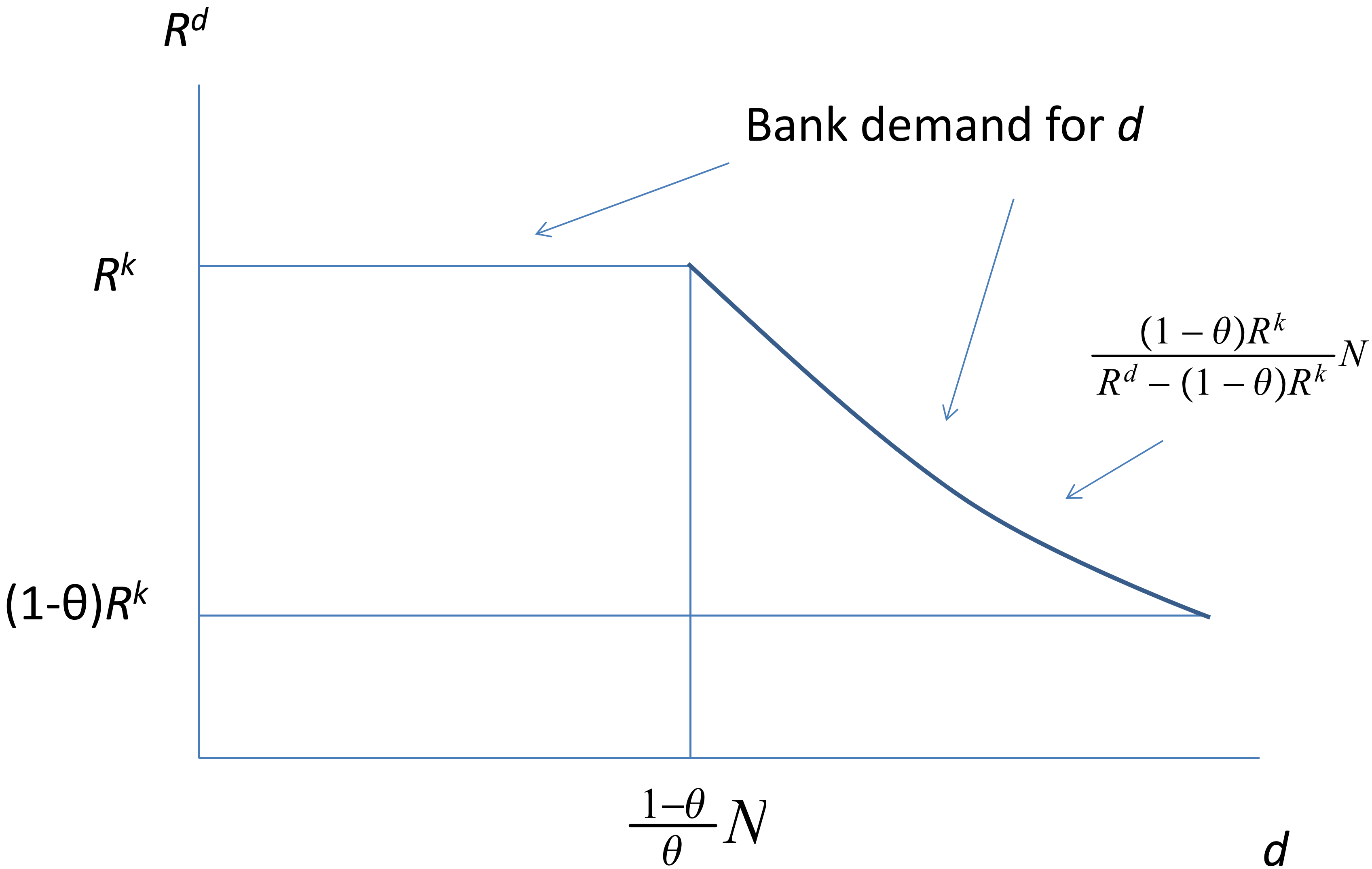
- Note that $0 < d < \infty$ requires

$$\boxed{(1-\theta)R^k \overbrace{<}^{\text{if not, then } d=\infty} R^d \overbrace{\leq}^{\text{if not, then } d=0} R^k.}$$

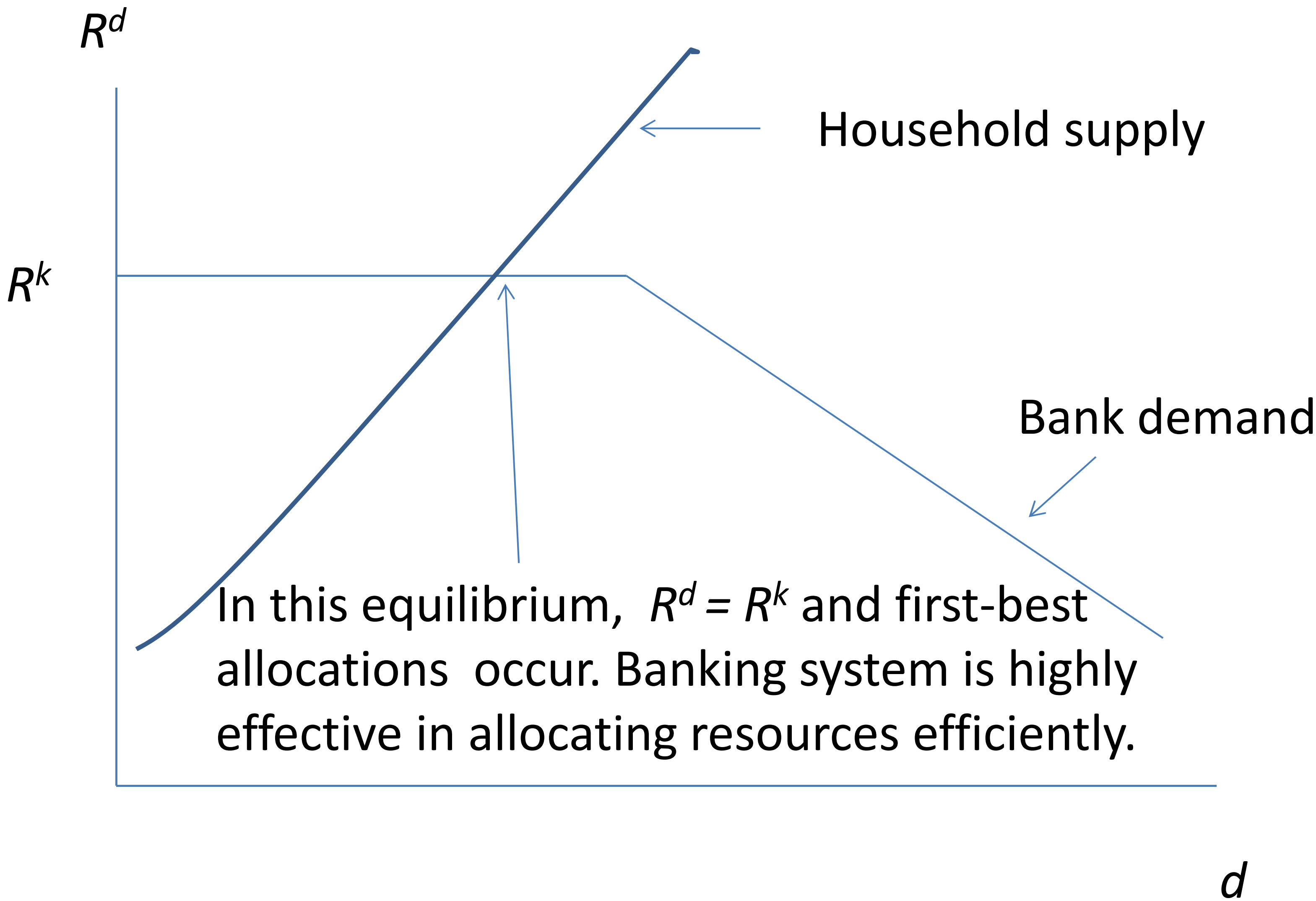
Problem of the bank in no-default, interior equilibrium, cnt'd

- For $R^d = R^k$
  - a bank makes no profits on d so – absent default considerations - it is indifferent over all values of $0 \leq d$
  - Taking into account default, a bank is indifferent over $0 \leq d \leq N(1\text{-}\theta)/\theta$
- For $(1\text{-}\theta)R^k < R^d < R^k$
  - Bank wants d as large as possible, subject to incentive constraint.
  - So, $d = R^k N(1\text{-}\theta)/(R^d\text{-}(1\text{-}\theta)R^k)$

Bank demand for *d*



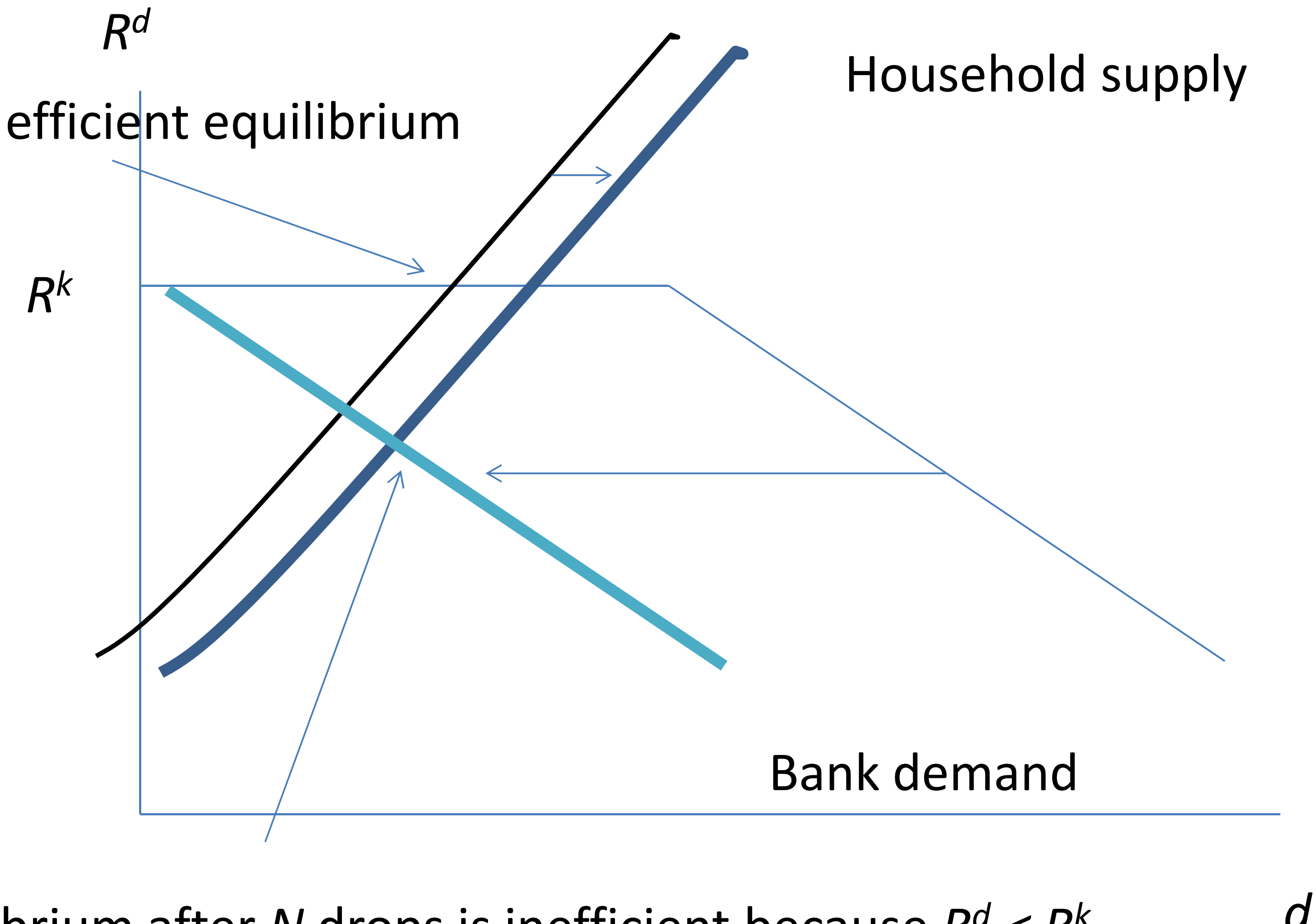
Interior, no default equilibrium



Collapse in Bank Net Worth

- Suppose that the economy is represented by a sequence of repeated versions of the above model.
- In the periods before the 2007-2008 crisis, net worth was high and the equilibrium was like it is on the previous slide: efficient, with zero interest rate spreads.
  - In practice, spreads are always positive, but that reflects various banking costs that are left out of this model.
- With the crisis, *N* dropped a lot, shifting demand to the right and supply to the left.

Effect of Substantial Drop in Bank Net Worth



Equilibrium after N drops is inefficient because $R^d < R^k$.

Government Intervention

- Equity injection.
  - Government raises T in period 1, provides proceeds to banks and demands $R^k T$ in return at start of period 2.
  - Rebates earnings to households in 2.

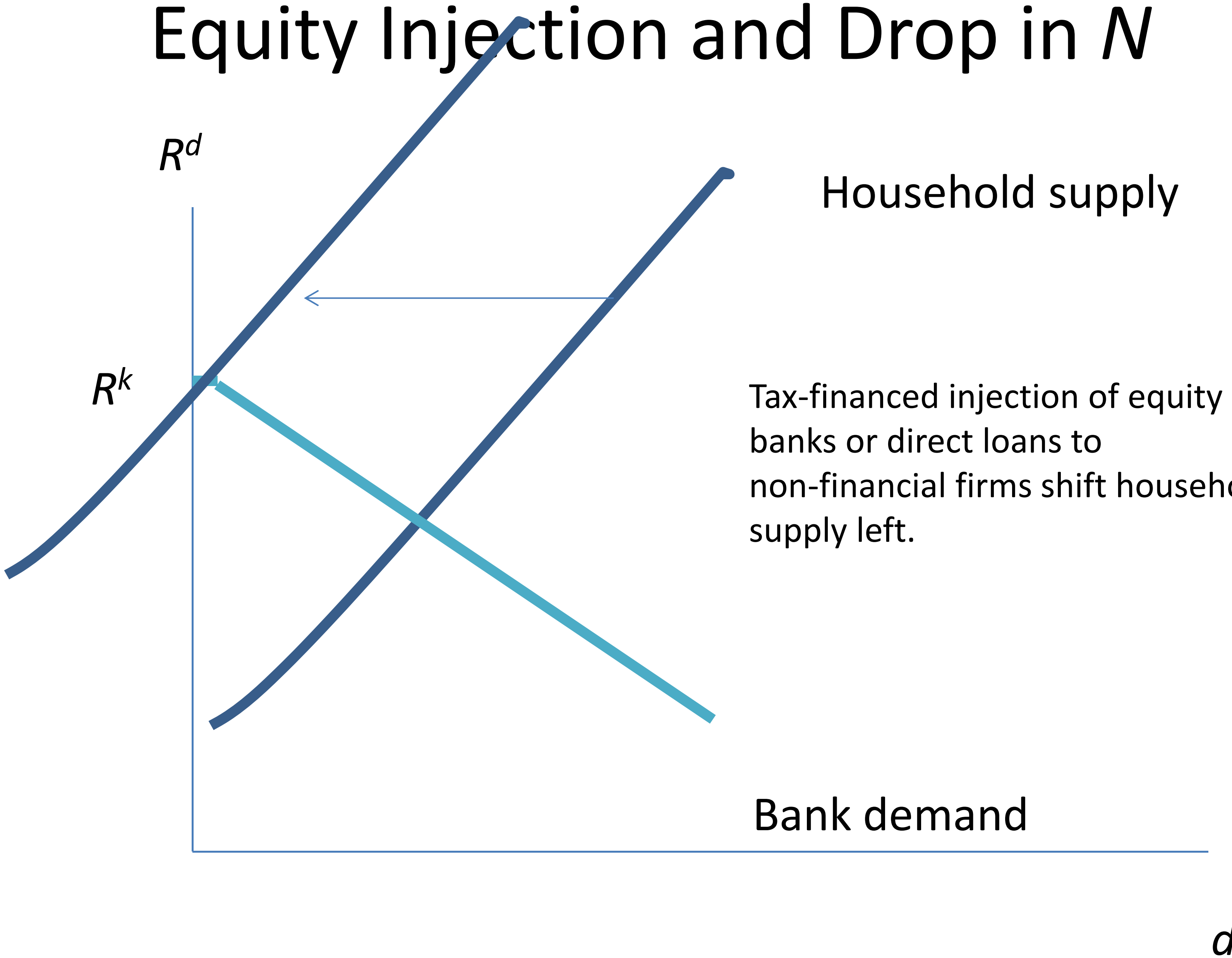
- Has no impact on demand for deposits by banks (no impact on default incentive or profits).

- Reduces supply of deposits by households.
  - $d+T$ rises when T rises (even though d falls) because R^d rises.

- Direct, tax-financed government loans to firms work in the same way.

- An interest rate subsidy to banks will shift their demand for deposits to the right….no impact on supply curve when subsidy financed by period 2 lump sum tax on households.

Equity Injection and Drop in N



Tax-financed injection of equity into banks or direct loans to non-financial firms shift household supply left.

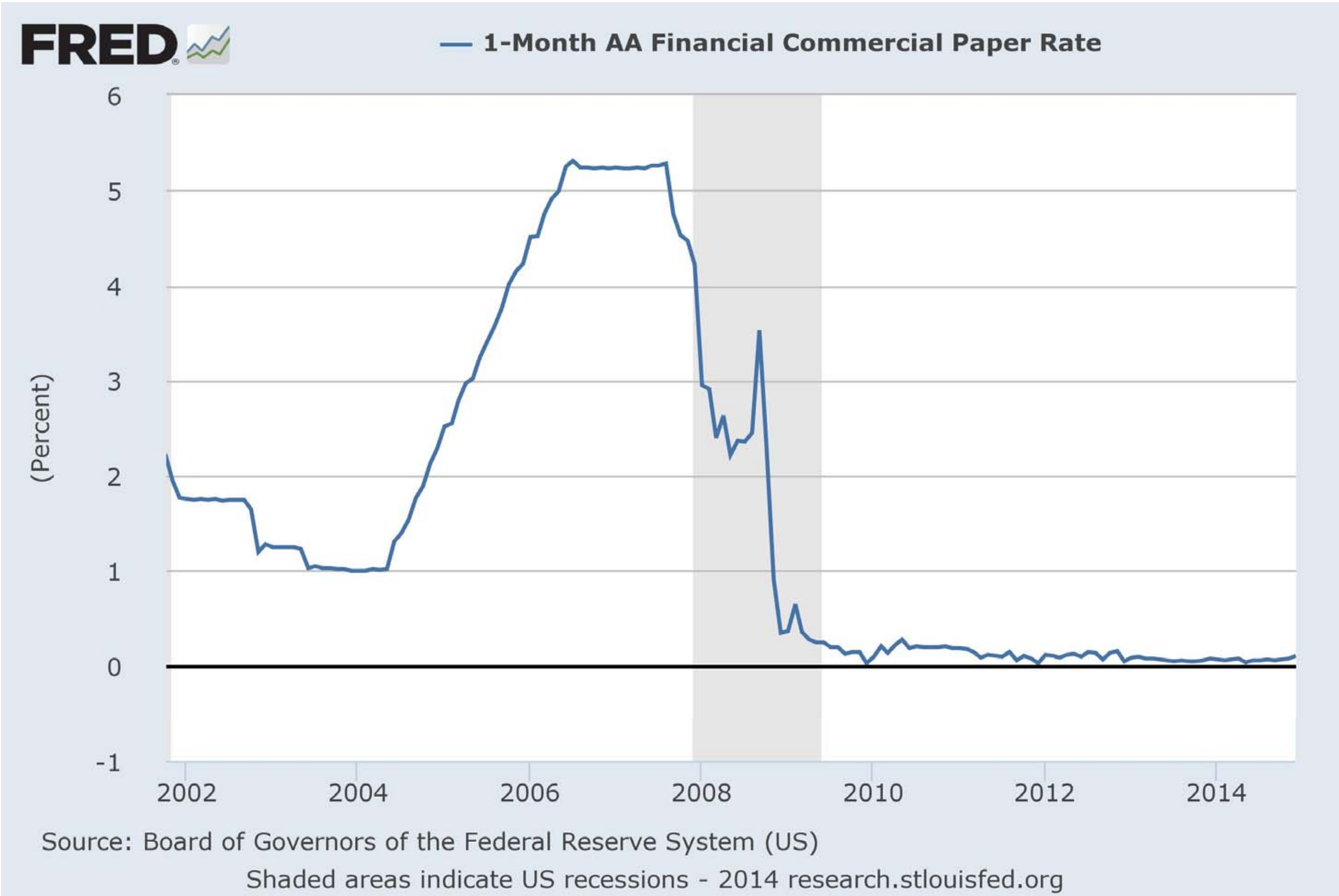
Recap

- Basic idea:
  - Bankers can run away with a fraction of bank assets.
  - If banker net worth is high relative to deposits, friction not a factor and banking system efficient.
  - If banker net worth falls below a certain cutoff, then banker must restrict the deposits.
    - Bankers fear (correctly) that otherwise depositors would lose confidence and take their business to another bank.
  - Reduction in banker demand for deposits:
    - makes deposit interest rates fall and so spreads rise.
    - Reduced intermediation means investment drops, output drops.
  - Equity injections by the government can revive the banking system.

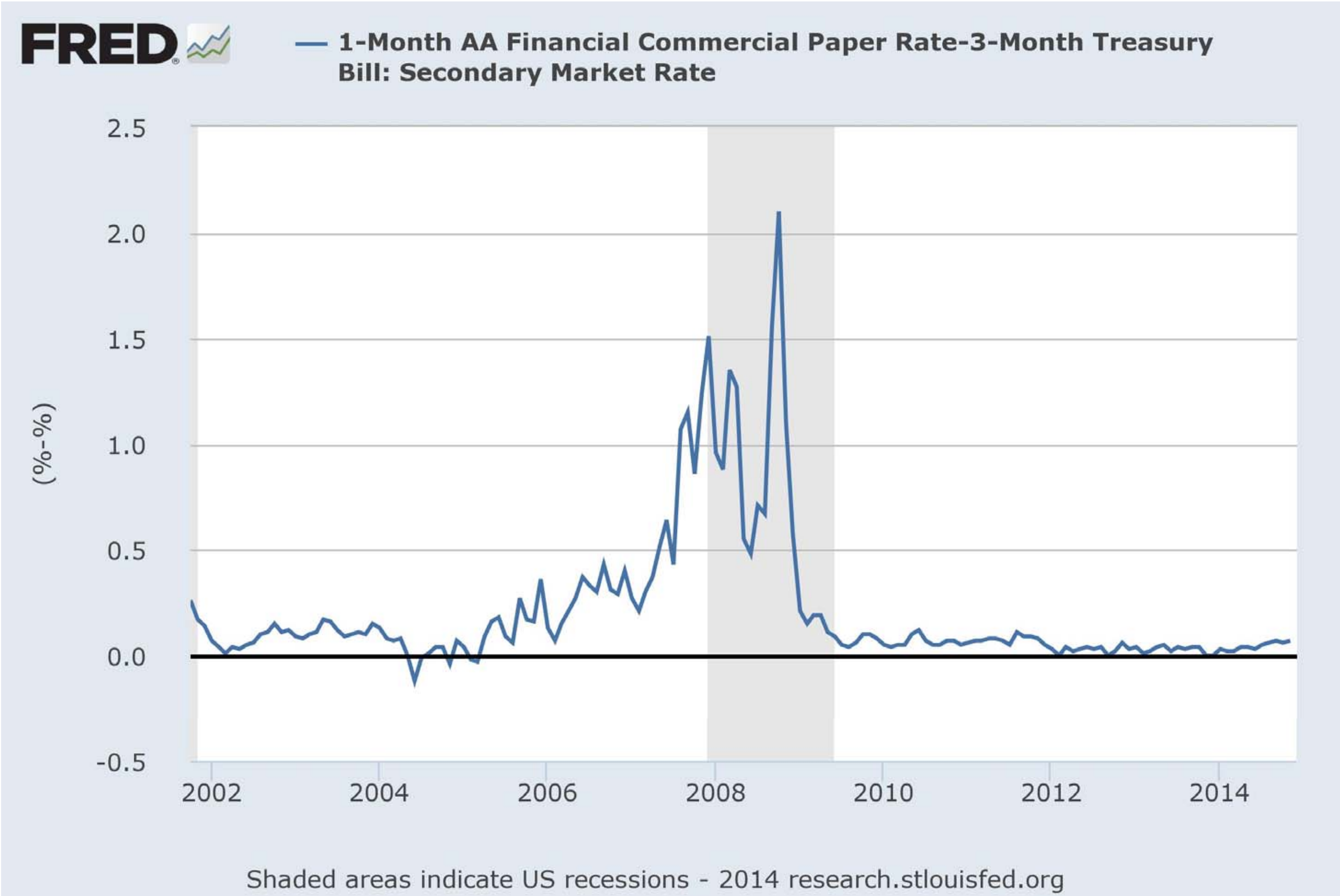
Is the Model Narrative Consistent with the Evidence?

- Model says that reduced intermediation of funds through the financial system reflected reduced demand for credit by financial institutions.

- Prediction: interest rate to financial institutions fall.


FRED
1-Month AA Financial Commercial Paper Rate
6
5
4
3
2
1
0
-1
(Percent)
2002
2004
2006
2008
2010
2012
2014
Source: Board of Governors of the Federal Reserve System (US)
Shaded areas indicate US recessions - 2014 research.stlouisfed.org

- Model prediction for decline in cost of funds to financial institutions seems verified.
- But, other ‘risk free’ interest rates fell even more.
  - Interest rates on US government debt fell more than interest rate on financial firm commercial paper.


FRED
1-Month AA Financial Commercial Paper Rate-3-Month Treasury Bill: Secondary Market Rate
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
(%-%)
2002
2004
2006
2008
2010
2012
2014
Shaded areas indicate US recessions - 2014 research.stlouisfed.org

Assessment

- Fact that interest rates on US government debt went down more than cost of funds to financial institutions suggests that a complete picture of financial crisis may require two additional features:
  - Risky Banks:
    - Banks in the model are risk free. Default only occurs out of equilibrium.
    - Increased actual riskiness of banks is perhaps also an important part of the picture.
  - Liquidity:
    - Low interest rates on US government debt consistent with idea that high demand for liquidity played an important role in the crisis.

Macro Prudential Policy

- In recent years there has been increased concern that banks may have a tendency to take on too much debt.

- Has accelerated thinking about debt restrictions on banks.

- There are several models of financial frictions in banks, but they do not necessarily provide a foundation for thinking about debt restrictions on banks.
  - A CSV model of banks implies they issue too *little* debt. (See Christiano-Ikeda).
  - The 'running away' model of banks does *not* rationalize debt restrictions. (See next).

Optimal Debt Restriction in Two-Period Running Away Banking Model

- Debt restriction on banks:

$$d \leq \bar{d}$$

- What is the socially optimal level of $\bar{d}$?
- To answer this, must take into account structure of private economy
  - The way households choose debt in competitive markets
  - The fact that banks will not choose a debt level that violates incentive constraints.

Social Welfare Function

$$u(c) + \beta u(C)$$

$$= u\left(\overbrace{c}^{=y-d}\right) + \beta u\left(\overbrace{C}^{=\overbrace{R^d d}^{=\text{earnings on deposits}} \overbrace{+R^k(N+d)-R^d d}^{=\text{bank profits}}}\right)$$

$$= u(y-d) + \beta u(R^k(N+d)).$$

Household Saving

- Optimization:

$$u'(y-d) = R^d u'(C)$$

plus budget constraint and definition of profits (see above) implies:

$$d = \frac{\left(\beta R^d\right)^{\frac{1}{\gamma}} - R^k \frac{n}{y}}{\left(\beta R^d\right)^{\frac{1}{\gamma}} + R^k} y$$

or

$$R^d = \frac{1}{\beta}\left(\frac{d+n}{y-d} R^k\right)^{\gamma} \equiv f(d)$$

Implementability Constraint

- Let d^* denote the value of deposits that a benevolent planner wishes the banks would choose.

- Planner must take into account:
  - banks will not choose a level of *d* which implies a violation of the incentive constraint.
  - market arrangement in which households make their deposit supply decision.
  - these considerations restrict *d* as follows:

$$(1-\theta)(N+d)R^k - f(d)d \geq 0$$

Planning Problem

- d^* is solution to the following problem:

$$\max_d u(y-d) + \beta u(R^k(N+d)) + \mu[(1-\theta)(N+d)R^k - f(d)d]$$

- Fonc

$$-u'(y-d) + \overbrace{\beta u'(C)}^{=u'(y-d)/R^d \text{ by households}} \times R^k + \mu[(1-\theta)R^k - f'(d)d - f(d)] = 0$$

$$\mu \geq 0, [(1-\theta)(N+d)R^k - f(d)d] \geq 0, \mu[(1-\theta)(N+d)R^k - f(d)d] = 0 \quad .$$

Planning Problem

- d^* is solution to the following problem:

$$\max_d u(y-d) + \beta u(R^k(N+d)) + \mu[(1-\theta)(N+d)R^k - f(d)d]$$

- Fonc

$$u'(y-d)\left[\frac{R^k}{f(d)} - 1\right] + \mu[(1-\theta)R^k - f'(d)d - f(d)] = 0$$

Complementary Slackness

$$\mu \geq 0, [(1-\theta)(N+d)R^k - f(d)d] \geq 0, \mu[(1-\theta)(N+d)R^k - f(d)d] = 0$$

Planning Problem

- First order conditions:

$$u'(y-d)\left[\frac{R^k}{f(d)}-1\right]+\mu[(1-\theta)R^k - f'(d)d - f(d)] = 0$$

Complementary Slackness

$$\mu \geq 0, [(1-\theta)(N+d)R^k - f(d)d] \geq 0, \mu[(1-\theta)(N+d)R^k - f(d)d] = 0$$

- Solving the problem:
  - Try $\mu = 0$ and solve ('saving supply crosses horizontal line at R^k) $R^k = f(d)$
  - Check incentive constraint. If satisfied, $R^k = f(d^*)$
  - Otherwise, conclude $\mu > 0$ and

$$(1-\theta)(N+d^*)R^k - f(d^*)d^* = 0$$

  - ('Savings supply crosses incentive constraint').

No Borrowing Restrictions Desired

- Deposits selected by government coincide with equilibrium deposits when there is no borrowing restriction.
- So, according to the model, restriction on bank borrowing not necessary.
- Model is not a good laboratory for thinking about leverage restrictions on banks, if you’re firmly convinced that leverage restrictions are required.

Rollover Crisis in DSGE Models

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Why Didn't DSGE Models Forecast the Financial Crisis and Great Recession?

- Bernanke (2009) and Gorton (2008):
  - By 2005 there existed a very large and highly-levered Shadow Banking system.
    - It relied on short-term debt to fund long-term liabilities.
    - So, it was vulnerable to a run.

- The overwhelming majority of academics, regulators and practitioners simply did not recognize this development, or understand its significance.

- The widespread belief (baked into DSGE models) was that if a country had deposit insurance, bank runs were a thing of the past.

Integrating Rollover Crisis into DSGE Models

- Will talk, *at an intuitive level*, about Gertler-Kiyotaki (AER2015).
- More full-blown models by Gertler-Kiyotaki-Prestipino

This is what a bank run looked like in the 19th century: Diamond-Dybvig run.

Bank runs in 2007 and 2008 were different and did not look like this at all (Gorton)!

It was a rollover crisis in a shadow (invisible to normal people) banking system.

Rolling over

- Consider the following bank:

Assets	Liabilities
120	Deposits: 100
	Banker net worth 20

- This bank is ‘solvent’: at current market prices could pay off all liabilities.

- Suppose that the bank’s assets are long term mortgage backed securities and the liabilities are short term (six month) commercial paper.

  - The bank relies on being able to *roll over* its liabilities every period.
  - Normally, this is not a problem.

Rolling over

- Now suppose the bank cannot roll over its liabilities.

- In this case, the bank would have to sell its assets.

    - If only one bank had to do this: no problem, since the bank is solvent.

- But, suppose *all banks face a roll over problem*.

    - Now there may be a *big* problem!
    - In this case, assets must be sold to another part of the financial system, a part that may have no experience with the assets (mortgage backed securities).

Rollover crisis (Nash) equilibrium

- Suppose an individual depositor, Jane, believes all other depositors will refuse to roll over.

- Suppose Jane believes that the fire sales of assets will wipe out bank net worth.

  - Then, Jane can expect to lose money on the deposit she made with the bank in the previous period.

  - But, that loss is *sunk*, and nothing can be done about it.

  - Need some other friction to guarantee that Jane will herself refuse to roll over her deposit.

Rollover crisis (Nash) equilibrium

- Absent other frictions, Jane would just renew her own deposit and the rollover crisis would *not* be a Nash equilibrium.

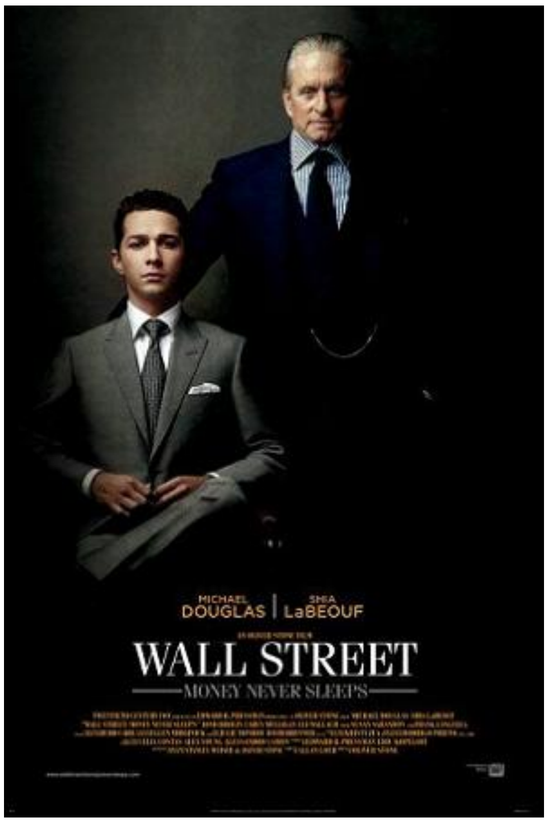
- So, Gertler-Kiyotaki assume that bankers can run away with a fraction of bank assets.

  - With zero net worth, banks would definitely run away.
  - This is why Jane would choose not to roll over her deposit, if she believed everyone else would also choose not to roll over.

- The logic of the rollover crisis equilibrium is a little different from the bank run equilibrium:

  - Suppose Jane thinks everyone else will take their money out of the bank.
  - Then, it makes sense for Jane to run faster than everyone else, to get to the front of the line.

The Drama of a Roll Over Crisis Brought to Life in Some Great Movies!



Why firesales?

- A rollover crisis: when all banks in an industry (e.g., mortgage backed securities industry) are unable to roll over their liabilities.

- The only buyers of the securities have no experience with them, so they won't buy without a price cut (*firesale*).

- Interestingly, the buyers of the securities will all complain at how *complex* they are and how *non-transparent* they are.
  - But, the real problem is that buyers in a fire sale are simply inexperienced.
  - The rollover crisis hypothesis contrasts with the *Big Short hypothesis*: assets were fundamentally *bad* (Mian and Sufi).

Rollover crisis

- When the whole industry has to sell, then bank balance sheets could suddenly look like this:

Fire sale value of assets:

Assets	Liabilities
90	Deposits: 100
	Banker net worth -10

- Multiple equilibrium: balance sheet could be the above, with run, or the following, with no run:

Assets	Liabilities
120	Deposits: 100
	Banker net worth 20

  - A run could happen, or not.

- This is exactly the sort of financial fragility that regulators want to avoid!

  - Under rollover crisis hypothesis, this was the situation in summer 2007.

Rollover Crisis: Role of Housing Market

- What matters is the actual value of assets and their firesale value.

- If bank is solvent under (firesale value), then probability of run is zero.

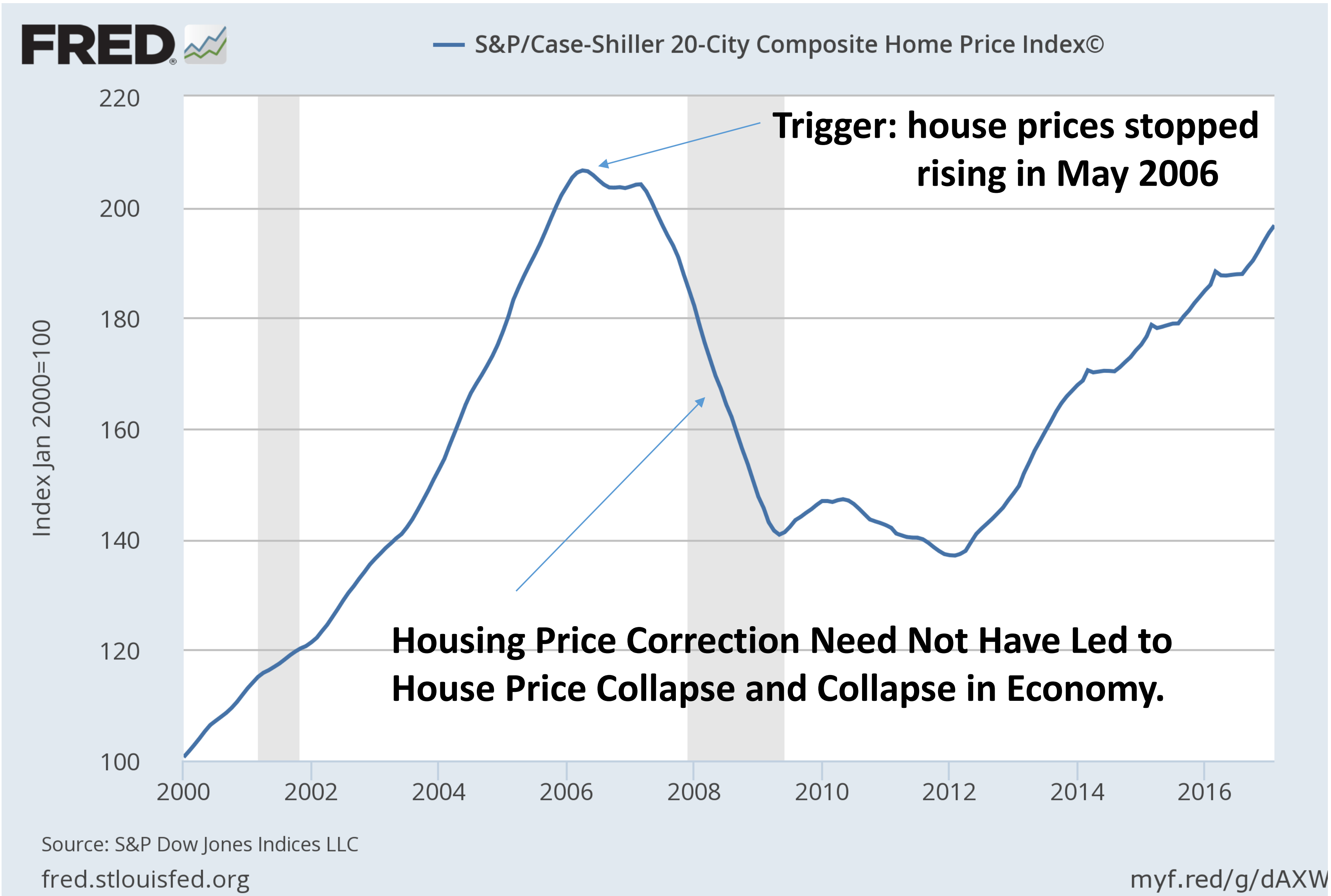
Pre-housing market correction

Assets	Liabilities
120 (105)	Deposits: 100
	Banker net worth 20 (5)

Post-housing market correction

Assets	Liabilities
110 (95)	Deposits: 100
	Banker net worth 10 (-5)

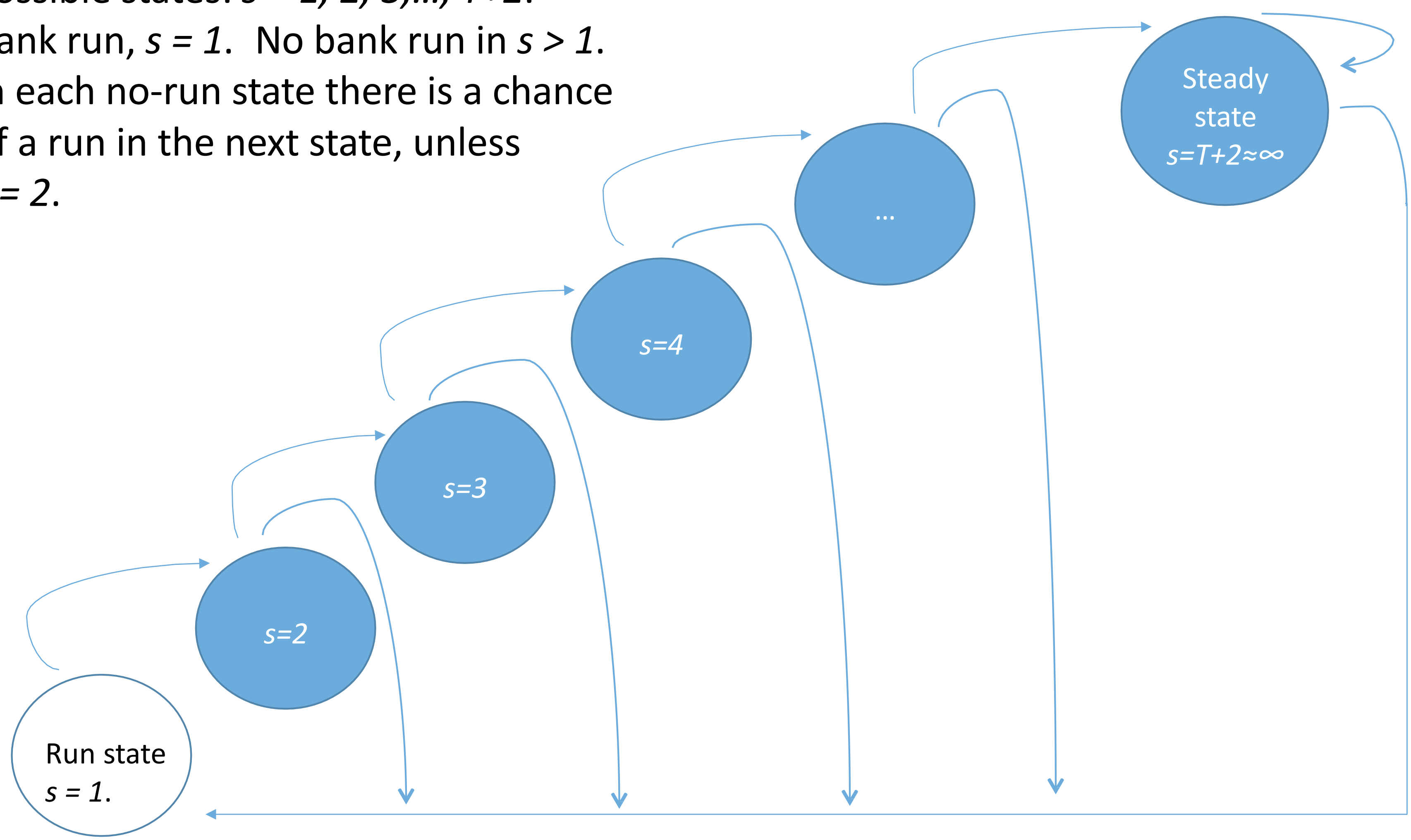
- Rollover Crisis Hypothesis:
  - pre-2005, no crisis possible,
  - post-2005 crisis possible.


FRED
S&P/Case-Shiller 20-City Composite Home Price Index©
Trigger: house prices stopped rising in May 2006
Housing Price Correction Need Not Have Led to House Price Collapse and Collapse in Economy.
Index Jan 2000=100
220
200
180
160
140
120
100
2000
2002
2004
2006
2008
2010
2012
2014
2016
Source: S&P Dow Jones Indices LLC
fred.stlouisfed.org
myf.red/g/dAXW

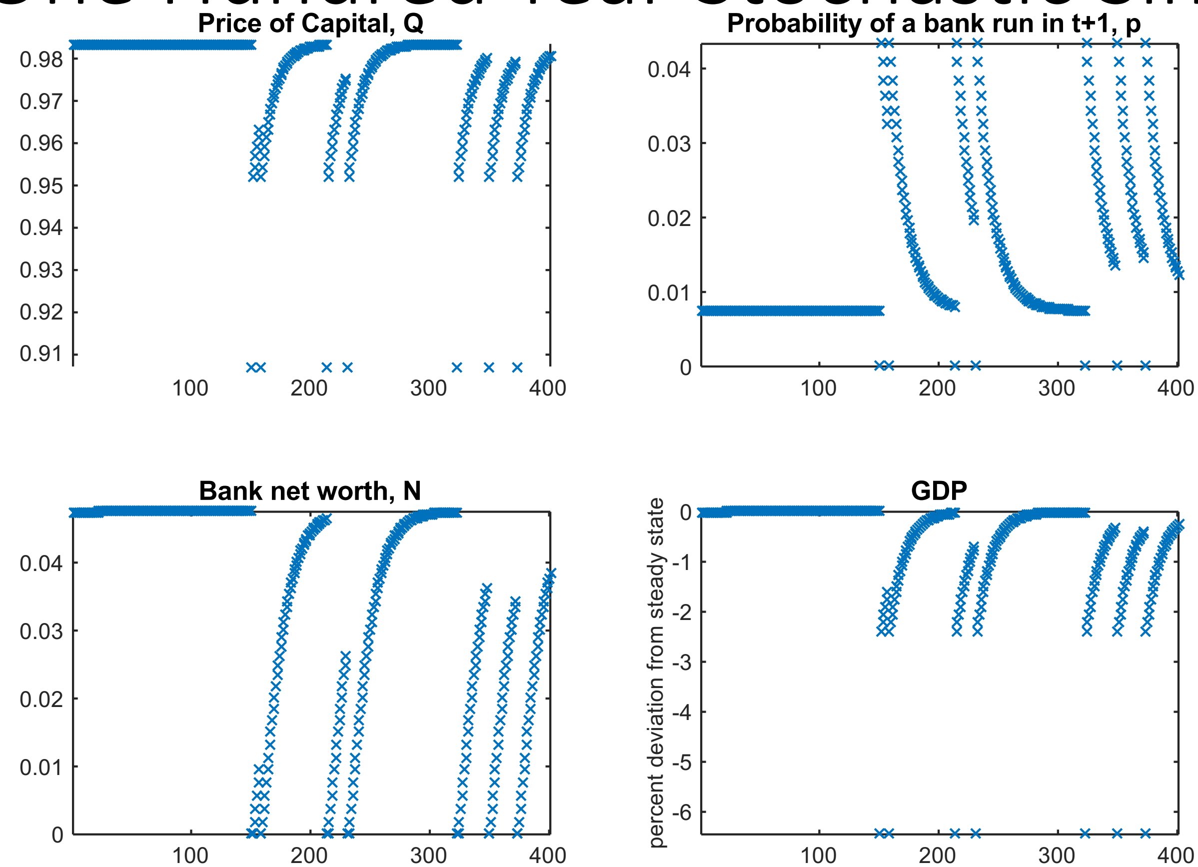
How to think about regulation when the risk is of a rollover crisis.

- One possibility: model the rollover crisis directly.
- Serious model of rollover crisis at this time: Gertler-Kiyotaki (AER2015).
  - They adapt the rollover crisis model of sovereign debt created by Cole-Kehoe (JIE1996).
  - Cole-Kehoe related to Diamond-Dybvig.

Possible states: *s = 1, 2, 3,…, T+2*.
Bank run, *s = 1*. No bank run in *s > 1*.
In each no-run state there is a chance of a run in the next state, unless *s = 2*.



One Hundred Year Stochastic Simulation



Policy Use of Model

- Investigate the impact on financial stability of leverage restrictions.
- But, this analysis is hard! Clearly, it is only in its infancy…
- At the heart of the analysis:
  - Assume that people know what can happen in a crisis, together with the associated probabilities.
  - This seems implausible, given the fact that a full-blown crisis is a two or three times a century rare event.
  - Safe to conjecture that factors such as aversion to ‘Knightian uncertainty’ play an important role driving fire sales in a crisis.
  - Still, research on various types of crises is proceeding at a rapid pace, and we expect to see substantial improvements in DSGE models on the subject.

Conclusion

- Models of rollover risk seem important in light of the crisis.
- These models are in their infancy, a long way from being operational for quantitative policy analysis.
- Possibility: assume that governments will always act as lender of last resort.
  - Use toy models to illustrate the idea of rollover crisis.
  - For quantitative analysis, use models that do not allow rollover crisis, but do capture moral hazard implications of bailouts.
  - Monitor the Shadow Banking system closely.

Notes on Financial Frictions Under Asymmetric Information and Costly State Verification

by

Lawrence Christiano

Incorporating Financial Frictions into a Business Cycle Model

- General idea:
  - Standard model assumes borrowers and lenders are the same people..no conflict of interest
  - Financial friction models suppose borrowers and lenders are different people, with conflicting interests
  - Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
  - Original analysis of Townsend (1978), Bernanke-Gertler.
- Integrating the csv model into a full-blown dsge model.
  - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
  - Empirical analysis of Christiano, Motto and Rostagno (2003,2012).

Simple Model

- There are entrepreneurs with all different levels of wealth, *N*.
  - Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.
- For each value of *N*, there are many entrepreneurs.
- In what follows, we will consider the interaction between entrepreneurs with a specific amount of *N* with competitive banks.
- Later, will consider the whole population of entrepreneurs, with every possible level of *N*.

Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1+R^k)\omega$$

- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

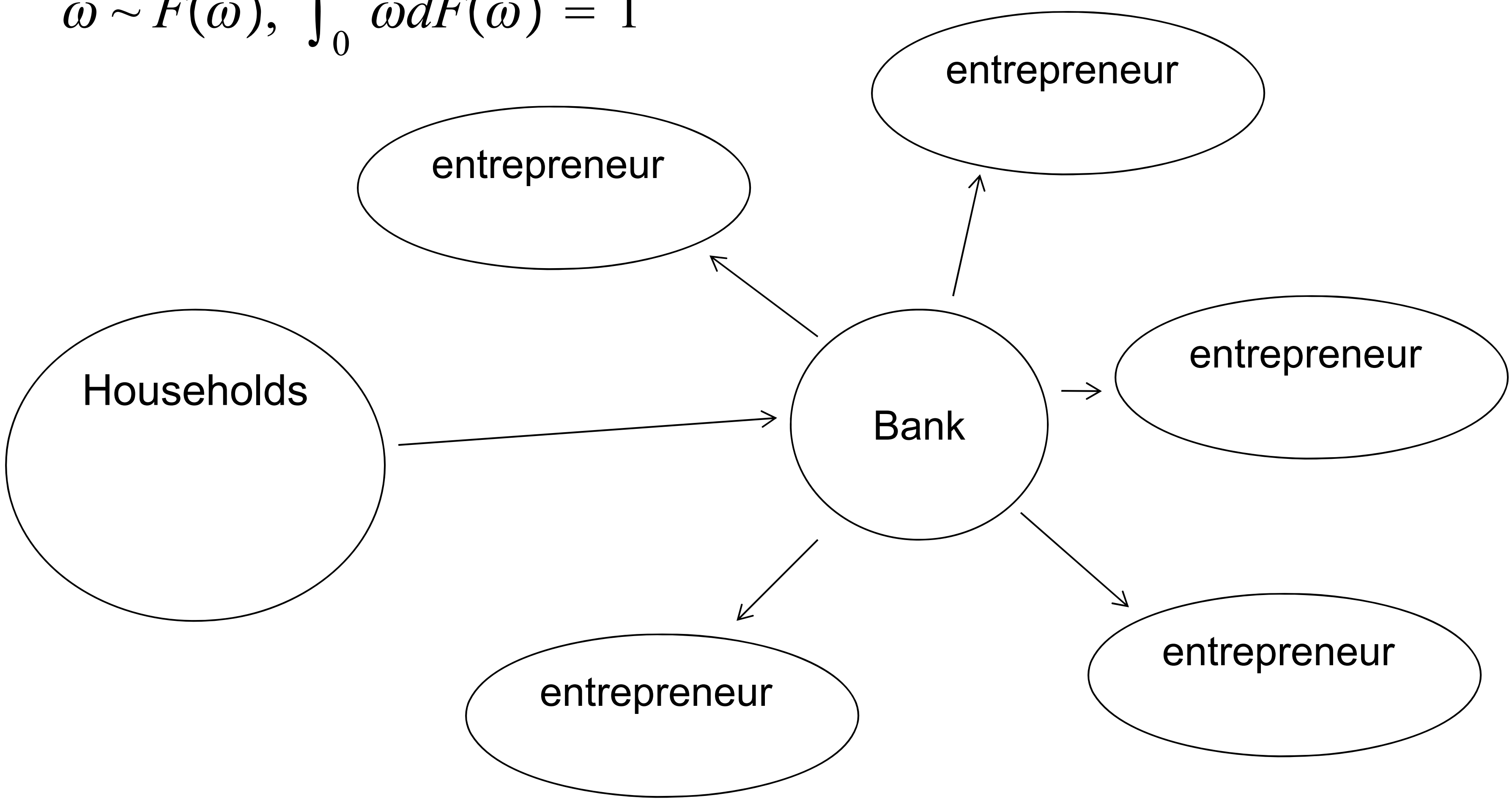
$$\int_0^\infty \omega dF(\omega) = 1$$

- The shock, ω , is privately observed by the entrepreneur.

- *F* is lognormal cumulative distribution function.

Banks, Households, Entrepreneurs

$$\omega \sim F(\omega),\ \int_0^\infty \omega dF(\omega) = 1$$



Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, *Z*, and a loan amount, *B*.
  - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, loses everything.

- Cutoff, $\bar{\omega}$

$$\overbrace{(1+R^k)\bar{\omega}}^{\text{gross rate of return experience by entrepreneur with 'luck', }\bar{\omega}} \times \overbrace{A}^{\text{total assets}}$$

$$= \overbrace{ZB}^{\text{interest and principle owed by the entrepreneur}}$$

$$(1+R^k)\bar{\omega}A = ZB \;\rightarrow$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{\frac{B}{N}}{\frac{A}{N}} = \frac{Z}{(1+R^k)}\frac{\overbrace{\frac{A}{N}}^{\text{leverage}=L}-1}{\frac{A}{N}} = \frac{Z}{(1+R^k)}\frac{L-1}{L}$$

- Cutoff higher with:
  - higher leverage, *L*
  - higher $Z/(1+R^k)$

- Expected return to entrepreneur from operating risky technology, over return from depositing net worth in bank:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^{k})\omega A - ZB]dF(\omega)}{N(1+R)}$$

For lower values of ω , entrepreneur receives nothing 'limited liability'.

gain from depositing funds in bank ('opportunity cost of funds')

- Rewriting entrepreneur's rate of return:

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L}$$

Gets smaller with *L*

Larger with *L*

- Rewriting entrepreneur's rate of return:

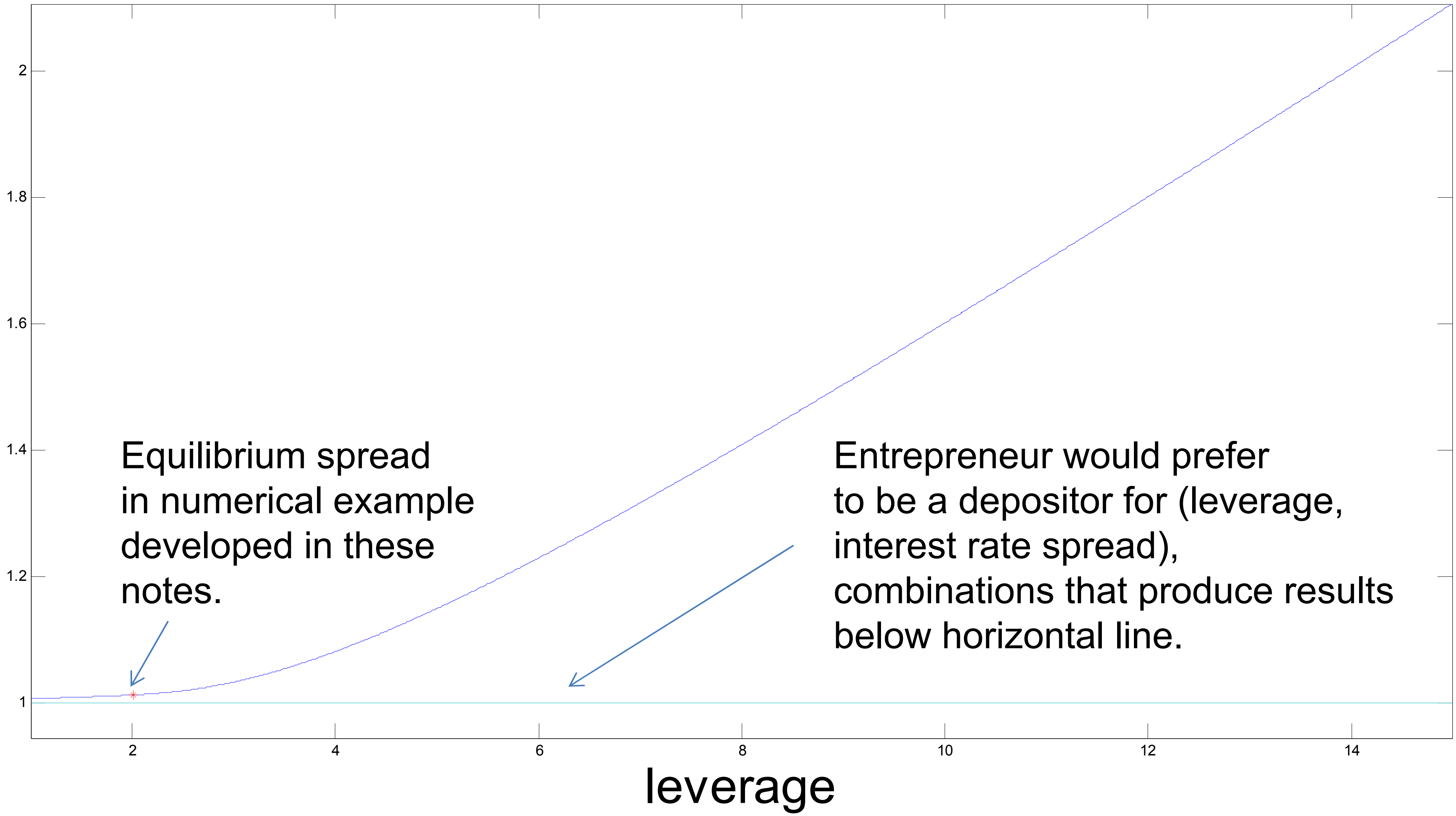
$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L} \rightarrow_{L\rightarrow\infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

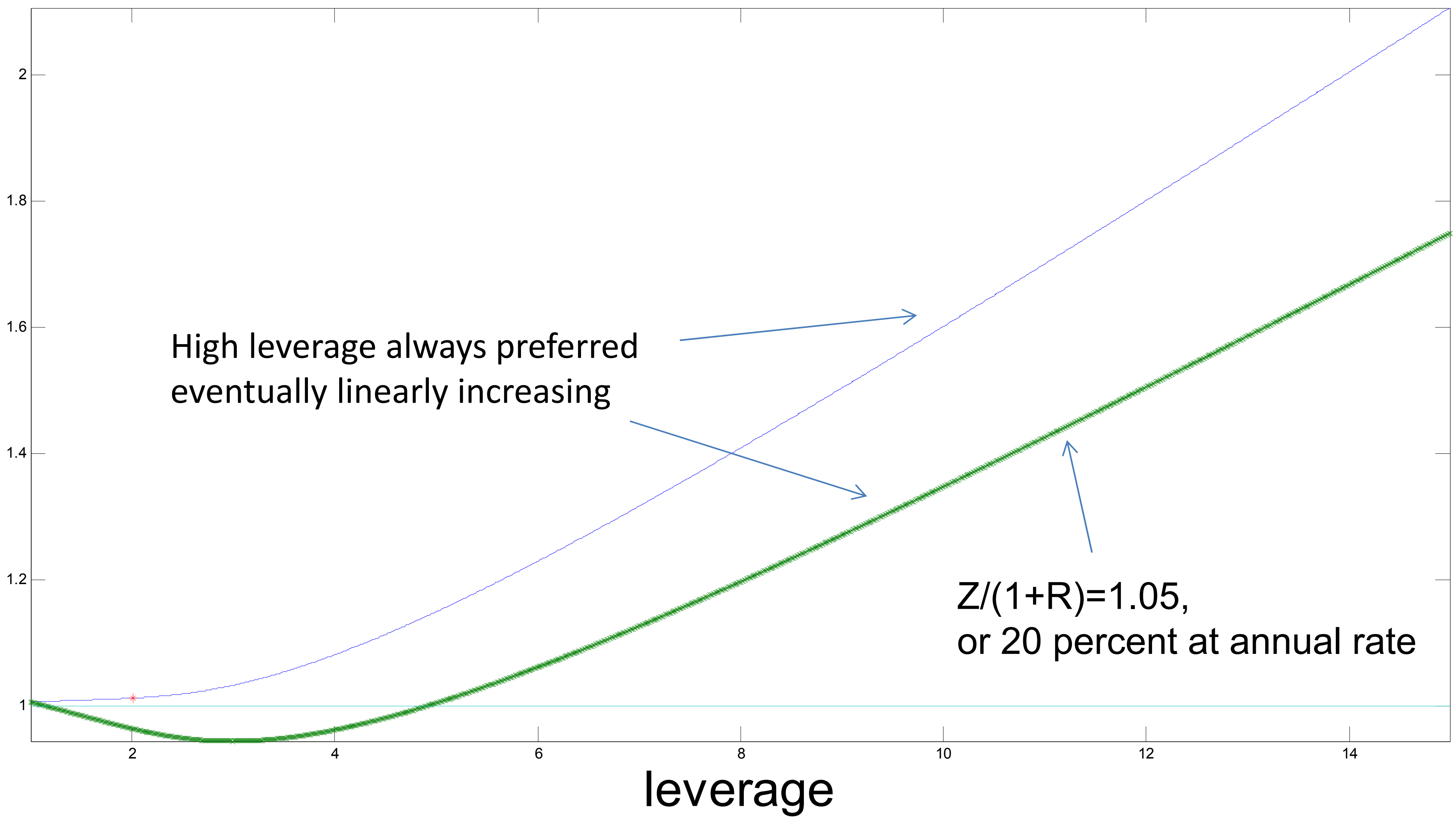
Expected entrepreneurial return, over opportunity cost, N(1+R)



Interest rate spread, Z/(1+R), = 1.0016, or 0.63 percent at annual rate $\sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

Expected entrepreneurial return, over opportunity cost, N(1+R)



Interest rate spread, Z/(1+R), = 1.0016, or 0.63 percent at annual rate $\sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.

- In equilibrium, bank can’t lend an infinite amount.

- This is why a loan contract must specify *both* an interest rate, Z, and a loan amount, B.

Simplified Representation of Entrepreneur Utility

- Utility:

$$\int_{\bar{\omega}}^{\infty}[\omega-\bar{\omega}]dF(\omega)\frac{1+R^k}{1+R}L$$

$$=[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L$$

- Where

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}(1-F(\bar{\omega}))+G(\bar{\omega})$$

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}}\omega dF(\omega)$$

Share of gross entrepreneurial earnings kept by entrepreneur

- Easy to show:

$$0 \leq \Gamma(\bar{\omega}) \leq 1$$

$$\Gamma'(\bar{\omega}) = 1-F(\bar{\omega}) > 0,\ \Gamma''(\bar{\omega}) < 0$$

$$\lim_{\bar{\omega}\to 0}\Gamma(\bar{\omega}) = 0,\ \lim_{\bar{\omega}\to\infty}\Gamma(\bar{\omega}) = 0$$

$$\lim_{\bar{\omega}\to 0}G(\bar{\omega}) = 0,\ \lim_{\bar{\omega}\to\infty}G(\bar{\omega}) = 1$$

Banks

- Source of funds from households, at fixed rate, R
- Bank borrows B units of currency, lends proceeds to entrepreneurs.
- Provides entrepreneurs with standard debt contract, (Z,B)

Banks, cont'd

- Monitoring cost for bankrupt entrepreneur with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1+R^k)\omega A$$

- Bank zero profit condition

$$\overbrace{[1-F(\bar{\omega})]}^{\text{fraction of entrepreneurs with } \omega>\bar{\omega}} \quad \overbrace{ZB}^{\text{quantity paid by each entrepreneur with } \omega>\bar{\omega}}$$

$$+ \quad \overbrace{(1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A}^{\text{quantity recovered by bank from each bankrupt entrepreneur}}$$

$$= \quad \overbrace{(1+R)B}^{\text{amount owed to households by bank}}$$

Banks, cont'd

- Zero profit condition:

$$[1-F(\bar{\omega})]ZB+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)(1+R^k)A = (1+R)B$$

$$\frac{[1-F(\bar{\omega})]ZB+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)(1+R^k)A}{B} = (1+R)$$

The risk free interest rate here is equated to the 'average return on entrepreneurial projects'.
This is a source of inefficiency in the model. A benevolent planner would prefer that the market price savers correspond to the *marginal* return on projects (Christiano-Ikeda).

Banks, cont'd

- Simplifying zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

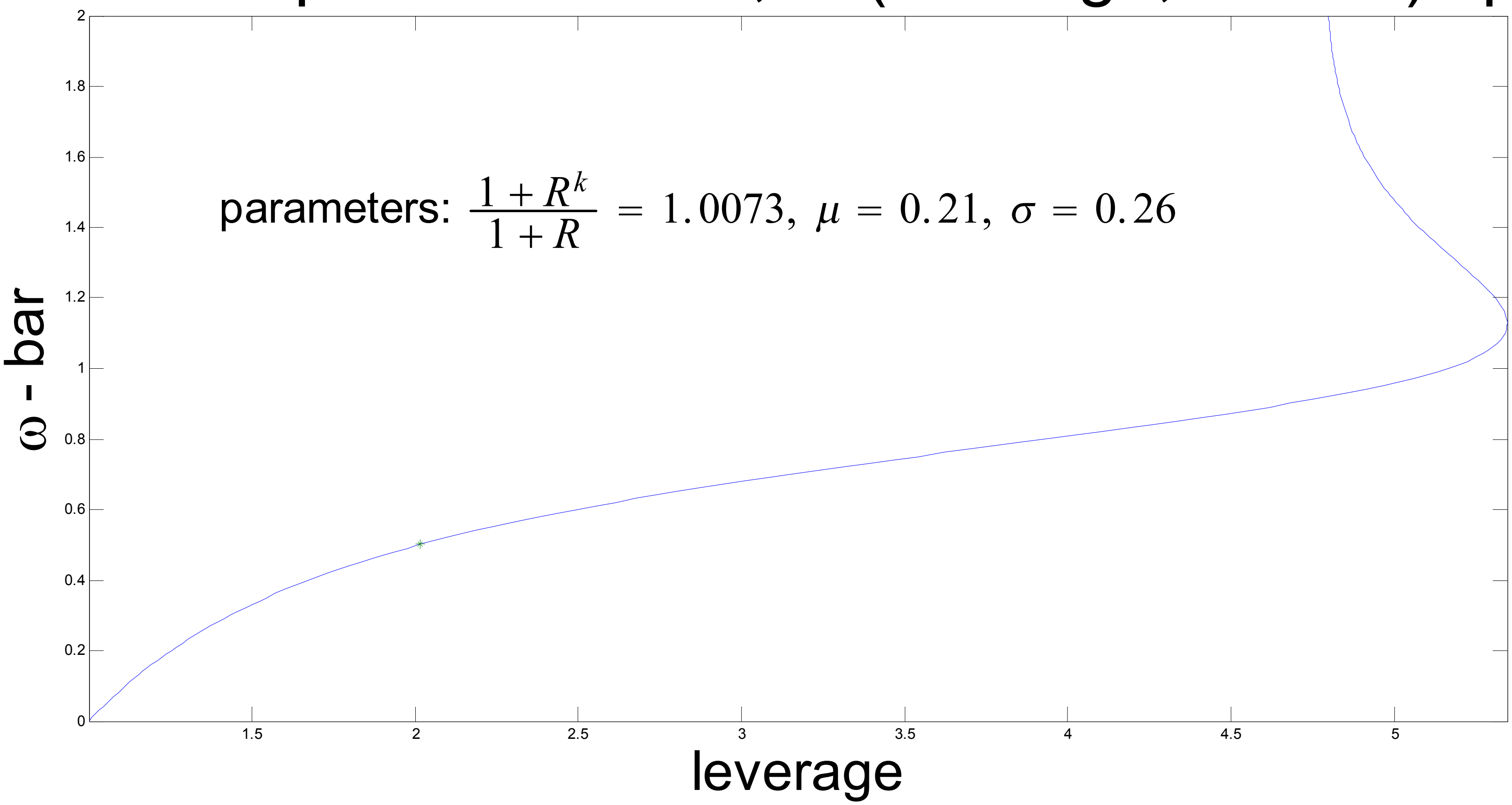
$$[1 - F(\bar{\omega})]\bar{\omega}(1+R^k)A + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$\overbrace{\left([1 - F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)\right)}^{\text{share of gross return, } (1+R^k)A, \text{ (net of monitoring costs) given to bank}} \qquad (1+R^k)A = (1+R)B$$

$$\begin{aligned}[1 - F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1+R}{1+R^k}\frac{B/N}{A/N} \\ &= \frac{1+R}{1+R^k}\frac{L-1}{L}\end{aligned}$$

Expressed naturally in terms of $(\bar{\omega}, L)$

Bank zero profit condition, in (leverage, ω - bar) space



Our value of $\frac{1+R^k}{1+R}$, 290 basis points at an annual rate, is a little higher than the 200 basis point value adopted in BGG (1999, p. 1368); the value of μ is higher than the one adopted by BGG, but within the range, 0.20-0.36 defended by Carlstrom and Fuerst (AER, 1997) as empirically relevant; the value of $Var(\log\omega)$ is nearly the same as the 0.28 value assumed by BGG (1999,p.1368).

Expressing Zero Profit Condition In Terms of New Notation

$$\overbrace{(1-F(\bar{\omega}))\bar{\omega}+(1-\mu)\int_0^{\bar{\omega}}\omega dF(\omega)}^{\text{share of entrepreneurial profits (net of monitoring costs) given to bank}} = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$\Gamma(\bar{\omega})-\mu G(\bar{\omega}) = \frac{1+R}{1+R^k}\frac{L-1}{L}$$

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

Equilibrium Contract

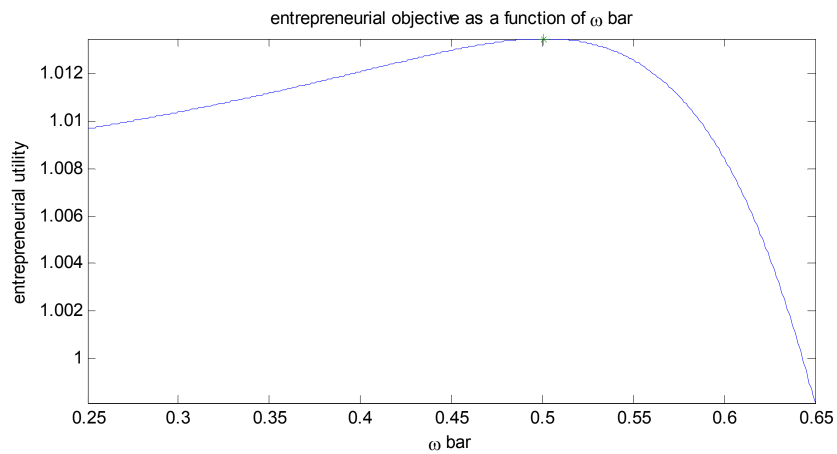
- Entrepreneur selects the contract is optimal, given the available menu of contracts.

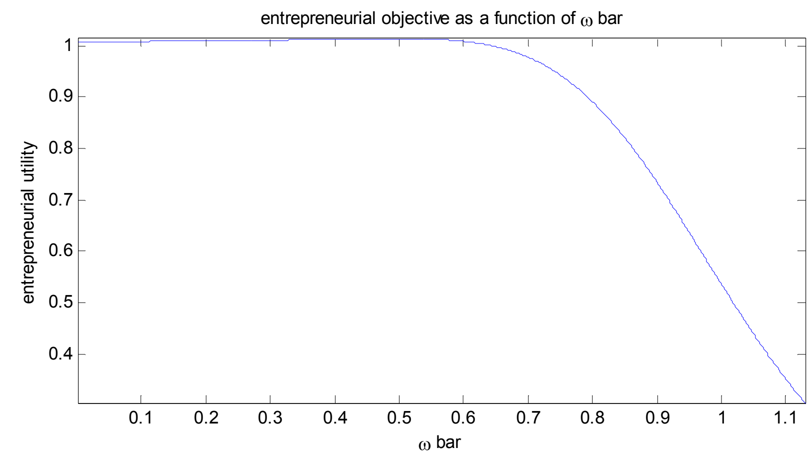
- The solution to the entrepreneur problem is the $\bar{\omega}$ that maximizes, over the relevant domain (i.e., $\bar{\omega} \in [0, 1.13]$ in the example):

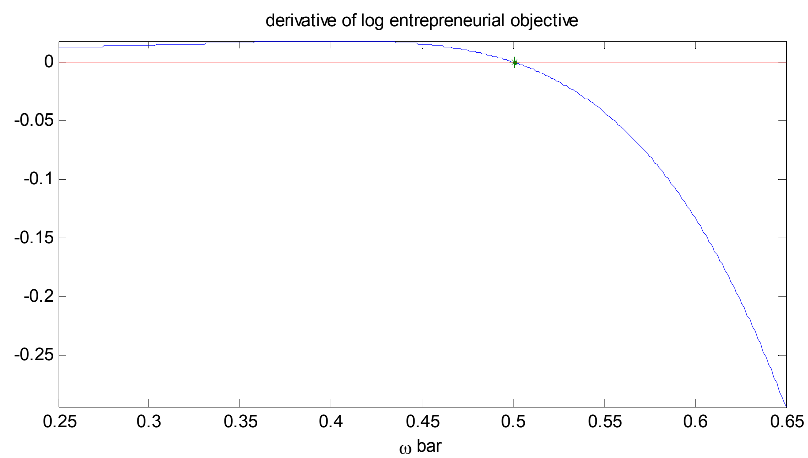
$$\log\left\{ \overbrace{\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1+R^k}{1+R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

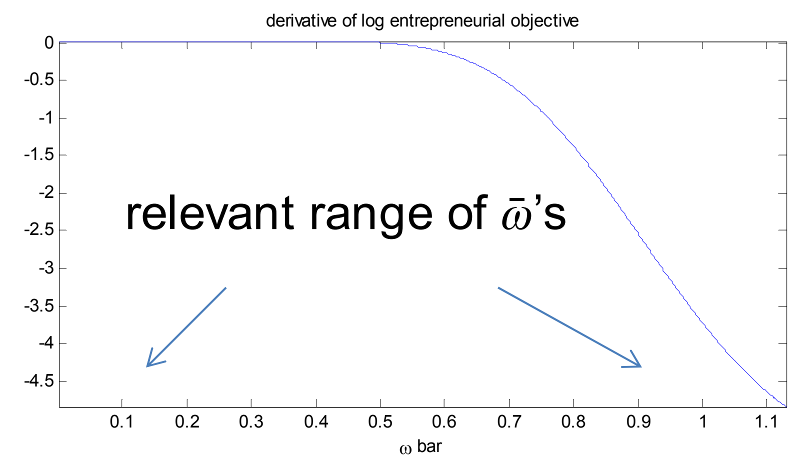
$$= \log \overbrace{[1 - \Gamma(\bar{\omega})]}^{\text{higer } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log \frac{1+R^k}{1+R} \overbrace{-\log\left(1 - \frac{1+R^k}{1+R}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

Entrepreneur Objective









Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, $\bar{\omega}$:

$$\overbrace{\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})}}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}} = \overbrace{\frac{\frac{1+R^k}{1+R}\left[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})\right]}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R}\bar{\omega}\frac{L}{L-1}$$

Result

- Leverage, *L*, and entrepreneurial rate of interest, *Z*, **not a function of net worth**, *N*.

- Quantity of loans proportional to net worth:

$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

$$B = (L-1)N$$

- To compute *L, Z/(1+R)*, must make assumptions about *F* and parameters.

$$\frac{1+R^k}{1+R},\ \mu,\ F$$

Numerical Example

- Parameters:

Percent of average product of entrepreneurial Projects, absorbed by monitoring costs: 0.06%

$$\frac{1+R^k}{1+R} = 1.0073,\ \sigma = 0.26,\ \mu = 0.21$$

- (Micro) equilibrium quantities:

$$\overbrace{\bar{\omega} = 0.50}^{\text{cutoff } \omega},\quad \overbrace{\Gamma(\bar{\omega}) = 0.5008}^{\text{fraction of gross entrepreneurial earnings going to lender}},\quad \overbrace{F(\bar{\omega}) = 0.0056}^{\text{bankruptcy rate: 0.56\%}},\quad \overbrace{G(\bar{\omega}) = 0.0026}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 2.02}^{\text{leverage}},\quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0015}^{0.62 \text{ (APR)}},\quad \overbrace{[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}L = 1.0135}^{\text{avg earnings of entrepreneur, divided by opportunity cost}} > 1$$

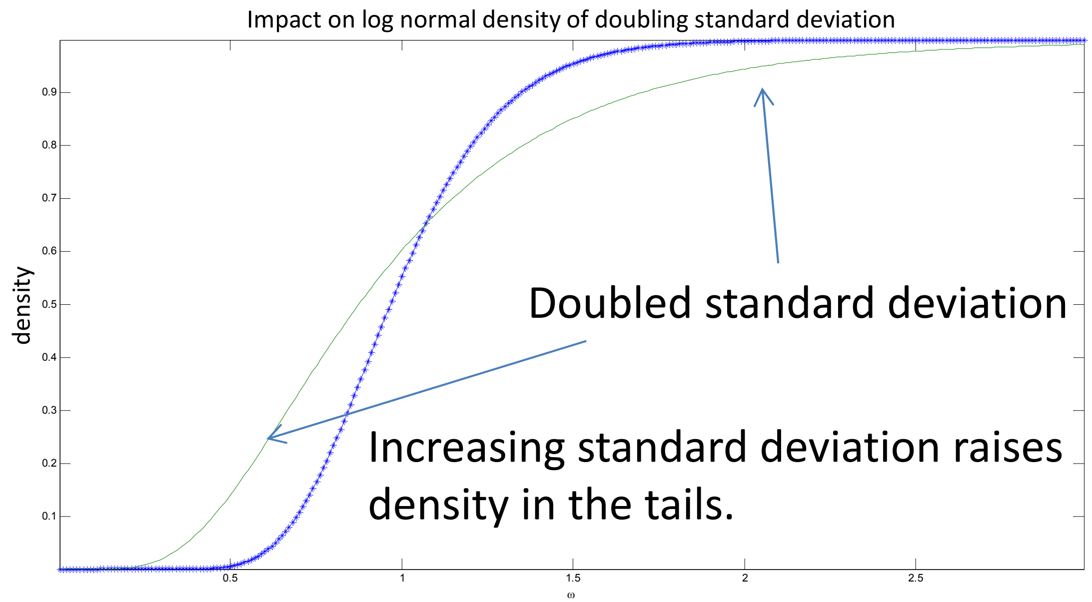
- Note: on average, entrepreneur better off leveraging net worth and investing in project, rather than depositing net worth in bank.

Effect of Increase in Risk, σ

- Keep

$$\int_0^\infty \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying *F*.



Jump in Risk

- σ replaced by $\sigma \times 3$

$$\overbrace{\bar{\omega} = 0.12}^{\text{cutoff } \omega}, \quad \overbrace{\Gamma(\bar{\omega}) = 0.12}^{\text{fraction of gross entrepreneurial earnings going to lender}}, \quad \overbrace{F(\bar{\omega}) = 0.0108}^{\text{bankruptcy rate: 1.08\%}}, \quad \overbrace{G(\bar{\omega}) = 0.0011}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 1.1418}^{\text{leverage}}, \quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0041}^{1.66 \text{ (APR)}}, \quad \overbrace{[1 - \Gamma(\bar{\omega})] \frac{1 + R^k}{1 + R} L = 1.0080}^{\text{avg earnings of entrepreneur, per unit of net worth}} \quad > 1$$

- Comparison with benchmark:

$$\overbrace{\bar{\omega} = 0.50}^{\text{cutoff } \omega}, \quad \overbrace{\Gamma(\bar{\omega}) = 0.5008}^{\text{fraction of gross entrepreneurial earnings going to lender}}, \quad \overbrace{F(\bar{\omega}) = 0.0056}^{\text{bankruptcy rate: 0.56\%}}, \quad \overbrace{G(\bar{\omega}) = 0.0026}^{\text{average } \omega \text{ among bankrupt entrepreneurs}},$$

$$\overbrace{L = 2.02}^{\text{leverage}}, \quad \overbrace{\frac{Z}{R}}^{\text{interest rate spread}} = \overbrace{1.0015}^{0.62 \text{ (APR)}}, \quad \overbrace{[1 - \Gamma(\bar{\omega})] \frac{1 + R^k}{1 + R} L = 1.0135}^{\text{avg earnings of entrepreneur, divided by opportunity cost}} \quad > 1$$

Simple New Keynesian Model without Capital

Lawrence J. Christiano

January 5, 2018

Objective

- Review the foundations of the basic New Keynesian model without capital.
  - Clarify the role of money supply/demand.
- Derive the Equilibrium Conditions.
  - Small number of equations and a small number of variables, which summarize everything about the model (optimization, market clearing, gov't policy, etc.).
- Look at some data through the eyes of the model:
  - Money demand.
  - Cross-sectoral resource allocation cost of inflation.
- Some policy implications of the model will be examined.
  - Many policy implications will be 'discovered' in later computer exercises.

Outline

- The model:
  - Individual agents: their objectives, what they take as given, what they choose.
    - Households, final good firms, intermediate good firms, gov't.
  - Economy-wide restrictions:
    - Market clearing conditions.
    - Relationship between aggregate output and aggregate factors of production, aggregate price level and individual prices.

- Properties of Equilibrium:
  - *Classical Dichotomy* - when prices flexible monetary policy irrelevant for real variables.
  - Monetary policy *essential* to determination of all variables when prices sticky.

Households

- Households' problem.
- Concept of Consumption Smoothing.

Households

- There are many identical households.
- The problem of the typical ('representative') household:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \gamma Z_t log \left(\frac{M_{t+1}}{P_t} \right) \right),$$

$$\text{s.t. } P_t C_t + B_{t+1} + M_{t+1}$$
$$\leq W_t N_t + R_{t-1} B_t + M_t$$
$$+\text{Profits net of government transfers and taxes}_t.$$

- Here, B_t and M_t are the beginning-of-period t stock of bonds and money held by the household.

Household First Order Conditions

- The household first order conditions:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} (5)$$

$$C_t N_t^{\varphi} = \frac{W_t}{P_t}.$$

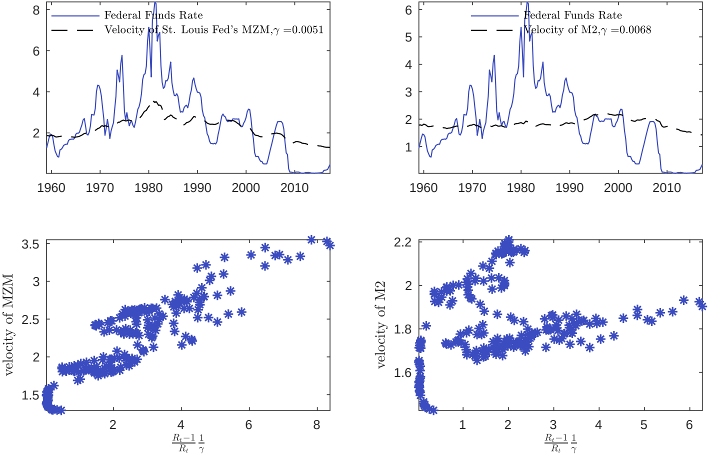
$$m_t = \left(\frac{R_t}{R_t - 1} \right) \gamma C_t \; (7),$$

  where

$$m_t \equiv \frac{M_{t+1}}{P_t}.$$

- All equations are derived by expressing the household problem in Lagrangian form, substituting out the multiplier on budget constraint and rearranging.
- The last first order condition is real money demand, increasing in C_t and decreasing in $R_t \geq 1$.

Figure: Money Demand, Relative to Two Measures of Velocity



Notes: (i) velocity is GDP/M, (ii) With the MZM measure of money, the money demand equation does well qualitatively, but not qualitatively because the theory implies the scatters in the 2,1 and 2,2 graphs should be on the 45^0.

Consumption Smoothing: Example

- Problem:

$$\begin{aligned} max_{c_1,c_2} log\left(c_1\right) + \beta log\left(c_2\right) \\ \text{subject to :} \quad c_1 + B_1 \leq y_1 + rB_0 \\ c_2 \leq rB_1 + y_{2.} \end{aligned}$$

- where y_1 and y_2 are (given) income and, after imposing equality (optimality) and substituting out for B_1,

$$c_1 + \frac{c_2}{r} = y_1 + \frac{y_2}{r} + rB_0,$$
$$\frac{1}{c_1} = \beta r \frac{1}{c_2},$$

second equation is fonc for B_1.

- Suppose $\beta r = 1$ (this happens in 'steady state', see later):

$$c_1 = \frac{y_1 + \frac{y_2}{r}}{1 + \frac{1}{r}} + \frac{r}{1 + \frac{1}{r}} B_0$$

Consumption Smoothing: Example, cnt'd

- Solution to the problem:

$$c_1 = \frac{y_1 + \frac{y_2}{r}}{1 + \frac{1}{r}} + \frac{r}{1 + \frac{1}{r}} B_0.$$

- Consider three polar cases:
  - *temporary change in income*: $\Delta y_1 > 0$ and $\Delta y_2 = 0 \implies \Delta c_1 = \Delta c_2 = \frac{\Delta y_1}{1+\frac{1}{r}}$
  - *permanent change in income*: $\Delta y_1 = \Delta y_2 > 0 \implies \Delta c_1 = \Delta c_2 = \Delta y_1$
  - *future change in income*: $\Delta y_1 = 0$ and $\Delta y_2 > 0 \implies \Delta c_1 = \Delta c_2 = \frac{\frac{\Delta y_2}{r}}{1+\frac{1}{r}}$
- Common feature of each example:
  - When income rises, then - assuming r does not change - c_1 increases by an amount that can be maintained into the second period: **consumption smoothing**.

Goods Production

- We turn now to the technology of production, and the problems of the firms.
- The technology requires allocating resources across sectors.
  - We describe the *efficient* cross-sectoral allocation of resources.
  - With price setting frictions, the market may not achieve efficiency.

Final Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Each intermediate good, $Y_{i,t}$, is produced by a monopolist using the following production function:

$$Y_{i,t} = e^{a_t} N_{i,t}, \qquad a_t \sim \text{exogenous shock to technology.}$$

- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient allocation of resources across i.

Efficient Sectoral Allocation of Resources

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$.
- It is optimal to run them all at the same rate, *i.e.*, $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0, 1]$.
- For given N_t, *allocative efficiency*: $N_{i,t} = N_{j,t} = N_t$, for all $i, j \in [0, 1]$.

  In this case, final output is given by

$$Y_t = \left[\int_0^1 \left(e^{a_t} N_{i,t} \right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}} = e^{a_t} N_t.$$

- One way to understand allocated efficiency result is to suppose that labor is *not* allocated equally to all activities.
- Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0, 1]$.

Suppose Labor *Not* Allocated Equally

- Example:

$$N_{it} = \begin{cases} 2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\ 2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] \end{cases}, \; 0 \leq \alpha \leq 1.$$

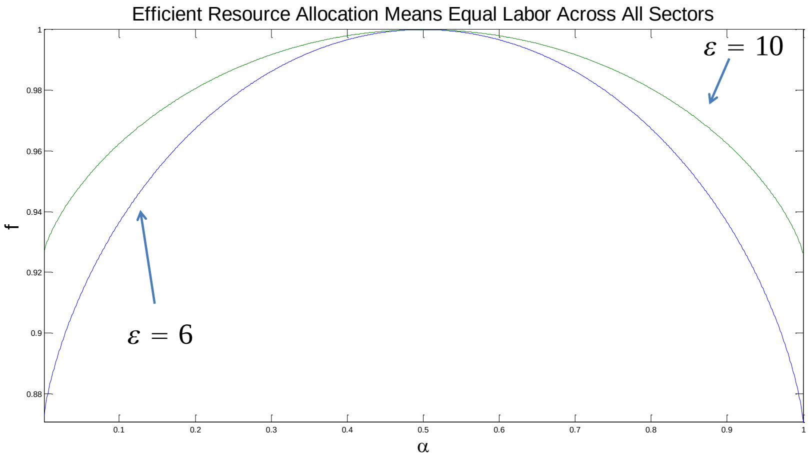
- Note that this is a particular distribution of labor across activities:

$$\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha) N_t = N_t$$

Labor *Not* Allocated Equally, cnt'd

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[\int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{aligned}
$$

$$f(\alpha) = \left[\frac{1}{2}(2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}(2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$



Final Good Producers

- Competitive firms:
  - maximize profits

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to P_t, $P_{i,t}$ given, all $i \in [0,1]$, and the technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Foncs:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon} \rightarrow \overbrace{P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}}^{\text{"cross price restrictions"}}$$

Intermediate Good Producers

- The i^{th} intermediate good is produced by a monopolist.
- Demand curve for i^{th} monopolist:

$$Y_{i,t} = Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\varepsilon}.$$

- Production function:

$$Y_{i,t} = e^{a_t} N_{i,t},\ a_t \text{ ~ exogenous shock to technology.}$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}.$$

Marginal Cost of Production

- An important input into the monopolist's problem is its marginal cost:

$$s_t = \frac{dCost}{dOutput} = \frac{\frac{dCost}{dWorker}}{\frac{dOutput}{dWorker}} = \frac{(1-\nu)\,\frac{W_t}{P_t}}{e^{a_t}}$$
$$= \frac{(1-\nu)\,C_t N_t^{\varphi}}{e^{a_t}}$$

  after substituting out for the real wage from the household intratemporal Euler equation.
- The tax rate, ν, represents a subsidy to hiring labor, financed by a lump-sum government tax on households.
- Firm's job is to set prices whenever it has the opportunity to do so.
  - It must always satisfy whatever demand materializes at its posted price.

Present Discounted Value of Intermediate Good Revenues

- i^{th} intermediate good firm's objective:

$$E_t^i \sum_{j=0}^{\infty} \beta^j \ v_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}}$$

v_{t+j} - Lagrange multiplier on household budget constraint

- Here, E_t^i denotes the firm's expectation over future variables, including the future probability that the firm gets to reset its price.

Decision By Firm that Can Change Its Price

- Let

$$\tilde{p}_t = \frac{\widetilde{P}_t}{P_t}.$$

- The firm's profit-maximizing choice of $\tilde{P}_t$ satisfies:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} \frac{Y_{t+j}}{C_{t+j}} (\beta\theta)^j \left(X_{t,j}\right)^{1-\varepsilon}},$$

the present discounted value of the markup, $\varepsilon / (\varepsilon - 1)$ over real marginal cost.

Decision By Firm that Can Change Its Price

- Recall,

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_t}{F_t}$$

  The numerator has the following simple representation:

$$\begin{aligned} K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j} \\ &= \frac{\varepsilon}{\varepsilon-1} \frac{(1-\nu) Y_t N_t^{\varphi}}{e^{a_t}} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{-\varepsilon} K_{t+1} \ (1), \end{aligned}$$

  after using $s_t = (1-\nu) e^{\tau_t} C_t N_t^{\varphi} / e^{a_t}$.

- Similarly,

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left(\frac{1}{\bar{\pi}_{t+1}}\right)^{1-\varepsilon} F_{t+1} \ (2)$$

Moving On to Aggregate Restrictions

- Link between aggregate price level, P_t, and $P_{i,t}$, $i \in [0,1]$.
  - Potentially complicated because there are MANY prices, $P_{i,t}$, $i \in [0,1]$.

- Link between aggregate output, Y_t, and N_t.
  - Potentially complicated because of earlier example with $f(\alpha)$.
  - Analog of $f(\alpha)$ will be a function of degree to which $P_{i,t} \neq P_{j,t}$.

- Market clearing conditions.
  - Money and bond market clearing.
  - Labor and goods market clearing.

Aggregate Price Index

- Important Calvo result:

$$
\begin{aligned}
P_t &= \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} \\
&= \left((1-\theta)\, \tilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

- Divide by P_t :

$$
1 = \left((1-\theta)\, \tilde{p}_t^{1-\varepsilon} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}
$$

- Rearrange: $\tilde{p}_t = \left[\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$

Aggregate Output vs Aggregate Labor and Tech (Tack Yun, JME1996)

- Define Y_t^*:

$$Y_t^* \equiv \int_0^1 Y_{i,t} di \qquad \left(= \int_0^1 e^{a_t} N_{i,t} di = e^{a_t} N_t \right)$$

$$\overbrace{=}^{\text{demand curve}} \quad Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 \left(P_{i,t}\right)^{-\varepsilon} di$$

$$= Y_t P_t^{\varepsilon} \left(P_t^*\right)^{-\varepsilon}$$

where, using 'Calvo result':

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[(1-\theta) \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Then

$$Y_t = p_t^* Y_t^*, \; p_t^* = \left(\frac{P_t^*}{P_t} \right)^{\varepsilon}.$$

Gross Output vs Aggregate Labor

- Relationship between aggregate inputs and outputs:

$$Y_t = p_t^* Y_t^*$$

or,

$$Y_t = p_t^* e^{a_t} N_t.$$

- Note that p_t^* is a function of the ratio of two averages (with different weights) of $P_{i,t}$, $i \in (0,1)$

- So, when $P_{i,t} = P_{j,t}$ for all $i, j \in (0,1)$, then $p_t^* = 1$.

- But, what is p_t^* when $P_{i,t} \neq P_{j,t}$ for some (measure of) $i, j \in (0,1)$?

Tack Yun Distortion

- Consider the object,

$$p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\varepsilon},$$

where

$$P_t^* = \left(\int_0^1 P_{i,t}^{-\varepsilon} di\right)^{\frac{-1}{\varepsilon}},\ P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di\right)^{\frac{1}{1-\varepsilon}}$$

- Follows easily from (intuition) and Jensen's inequality:

$$p_t^* \leq 1.$$

Law of Motion of Tack Yun Distortion

- We have

$$P_t^* = \left[(1-\theta)\, \tilde{P}_t^{-\varepsilon} + \theta \left(P_{t-1}^* \right)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}$$

- Dividing by P_t:

$$p_t^* \equiv \left(\frac{P_t^*}{P_t} \right)^{\varepsilon} = \left[(1-\theta)\, \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}$$

$$= \left((1-\theta) \left[\frac{1-\theta \left(\bar{\pi}_t \right)^{\varepsilon-1}}{1-\theta} \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \theta \frac{\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right)^{-1} \qquad (4)$$

using the restriction between $\tilde{p}_t$ and aggregate inflation developed earlier.

Evaluating the Distortions

- Tack Yun distortion:

$$p_t^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}.$$

  - Potentially, NK model provides an 'endogenous theory of TFP'.

- Standard practice in NK literature is to set $p_t^* = 1$ for all t.
  - First order expansion of p_t^* around $\bar{\pi}_t = p_t^* = 1$ is:

$$p_t^* = p^* + 0 \times \bar{\pi}_t + \theta \left(p_{t-1}^* - p^* \right), \text{ with } p^* = 1,$$

  so $p_t^* \to 1$ and is invariant to shocks.

Empirical Assessment of Tack Yun Distortion

- Do 'back of the envelope' calculations in a steady state when inflation is constant and p^* is constant.

- Can also use

$$p_t^* = \left[(1-\theta) \left(\frac{1-\theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}.$$

  to compute times series estimate of p_t^*.

  – But, results very similar to what you find with steady state calculations.

Cost of Three Alternative Permanent Levels of Inflation

$$p^* = \frac{1-\theta\bar{\pi}^{\varepsilon}}{1-\theta}\left(\frac{1-\theta}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Table: Percent of GDP Lost Due to Inflation, $100(1-p_t^*)$

steady state inflation	markup, $\frac{\varepsilon}{\varepsilon-1}$		
	1.20	1.15	1.10
1970s: 8%	2.41	3.92	10.85
proposal for dealing with ZLB: 4%	0.46	0.64	1.13
recent average: 2%	0.10	0.13	0.21

Tack Yun Distortion

- The magnitude of the distortion is typically small.
  - Explains why standard literature abstracts from the distortion by linearizing about zero inflation.
  - To first order approximation, $p^* = 1$ at zero inflation (see later).
  - Could have $p^* = 1$ to first order around positive inflation if price indexation is assumed, as in CEE.
    - But, prices don't appear to be indexed.
- Caution: distortion may be small because of simplicity of the model.
  - Distortions at least two times bigger when production occurs in networks of firms. See this and this.
  - Distortions may be bigger when there are intermediate good firm-specific idiosyncratic shocks to demand and supply of intermediate good firms.

Government

- Government budget constraint: expenditures = receipts

$$\overbrace{P_t G_t}^{\text{purchases of final goods}} + \overbrace{\nu W_t N_t}^{\text{subsidy payments}} + \overbrace{B_{t+1}^g}^{\text{gov't bonds (lending, if positive)}}$$

$$+ \overbrace{T_t^{trans}}^{\text{transfer payments to households}}$$

$$= \overbrace{M_t \mu_t}^{\text{money injection, if positive}} + \overbrace{T_t^{tax}}^{\text{tax revenues}} + R_{t-1} B_t^g$$

where μ_t denotes money growth rate.

- Government's choice of μ_t determines evolution of money supply:

$$M_{t+1} = (1 + \mu_t) M_t,\ \mu_t \sim \text{money growth rate.}$$

Government

- The law of motion for money places restrictions on m_t:

$$m_t \equiv \frac{M_{t+1}}{P_t} = \frac{M_{t+1}}{M_t}\frac{M_t}{P_{t-1}}\frac{P_{t-1}}{P_t}$$

$$\rightarrow m_t = \left(\frac{1+\mu_t}{\bar{\pi}_t}\right) m_{t-1} \,(8),$$

for $t = 0, 1, \ldots$.

Market Clearing

- We now summarize the market clearing conditions of the model.
  - Money, labor, bond and goods markets.

Money Market Clearing

- We temporarily use the bold notation, $\mathbf{M}_t$, to denote the per capita supply of money at the start of time t, for $t = 0, 1, 2, \ldots$.
- The supply of money is determined by the actions, μ_t, of the government:

$$\mathbf{M}_{t+1} = \mathbf{M}_t + \mu_t \mathbf{M}_t,$$

  for t=0,1,2,...
- Households being identical means that in period $t = 0$,

$$\mathbf{M}_0 = M_0,$$

  where M_0 denotes beginning of time $t = 0$ money stock of the representative household.
- Money market clearing in each period, $t = 0, 1, \ldots$, requires

$$\mathbf{M}_{t+1} = M_{t+1},$$

  where M_{t+1} denotes the representative household's time t choice of money.
- From here on, we do not distinguish between $\mathbf{M}_t$ and M_t.

Other Market Clearing Conditions

- Bond market clearing:

$$B_{t+1} + B_{t+1}^{g} = 0,\ t = 0, 1, 2, ...$$

- Labor market clearing:

$$\overbrace{N_t}^{\text{supply of labor}} = \overbrace{\int_0^1 N_{i,t} di}^{\text{demand for labor}}$$

- Goods market clearing:

$$\overbrace{C_t + G_t}^{\text{demand for final goods}} = \overbrace{Y_t}^{\text{supply of final goods}},$$

and, using relation between Y_t and N_t:

$$C_t + G_t = p_t^* e^{a_t} N_t \ (6)$$

Next

- Collect the equilibrium conditions associated with private sector behavior.
- Comparison of NK model with RBC model (i.e., $\theta = 0$)
  - *Classical Dichotomy*: In flexible price version of model real variables determined independent of monetary policy.
  - Fiscal policy still matters, because equilibrium depends on how government deals with the monopoly power, i.e., selects value for subsidy, ν.
  - In NK model, markets don't necessarily work well and good monetary policy essential.
- To close model with $\theta > 0$ must take a stand on monetary policy.

Equilibrium Conditions

- 8 equations in 8 unknowns: $m_t, C_t, p_t^*, F_t, K_t, N_t, R_t, \bar{\pi}_t$, and 3 policy variables: ν, μ_t, G_t.

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu)\, Y_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}, \quad \frac{K_t}{F_t} = \left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad \text{(4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \quad C_t + G_t = p_t^* e^{a_t} N_t \text{ (6)}$$

$$m_t = \frac{\gamma C_t}{\left(1 - \frac{1}{R_t}\right)} \,(7), \quad m_t = \left(\frac{1+\mu_t}{\bar{\pi}_t}\right) m_{t-1} \,(8)$$

Classical Dichotomy Under Flexible Prices

- *Classical Dichotomy*: when prices flexible, $\theta = 0$, then real variables determined regardless of the rule for μ_t (i.e., monetary policy).
  - Equations (2),(3) imply:

$$F_t = K_t = \frac{Y_t}{C_t},$$

  which, combined with (1) implies

$$\frac{\varepsilon(1-\nu)}{\varepsilon - 1} \times \overbrace{CN_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

  - Expression (6) with $p_t^* = 1$ (since $\theta = 0$) is

$$C_t + G_t = e^{a_t} N_t.$$

- Thus, we have two equations in two unknowns, N_t and C_t.

Classical Dichotomy: No Uncertainty

- Real interest rate, $R_t^* \equiv R_t / \bar{\pi}_{t+1}$, is determined:

$$R_t^* = \frac{\frac{1}{C_t}}{\beta \frac{1}{C_{t+1}}}.$$

- So, with $\theta = 0$, the following are determined:

$$R_t^*, C_t, N_t,\ t = 0, 1, 2, ...$$

- What about the nominal variables?
  - Suppose the monetary authority wants a given sequence of inflation rates, $\bar{\pi}_t$, $t = 0, 1, ...$.
  - Then,

$$R_t = \bar{\pi}_{t+1} R_t^*,\ t = 0, 1, 2, ...$$

  - What money growth sequence is required?
    - From (7), obtain m_t, $t = 0, 1, 2, ...$. Also, m_{-1} is given by initial M_0 and P_{-1}.
    - From (8)

$$1 + \mu_t = \frac{m_t}{m_{t-1}} \bar{\pi}_t,\ t = 0, 1, 2, ...$$

Classical Dichotomy versus New Keynesian Model

- When $\theta = 0$, then the Classical Dichotomy occurs.
- In this case, monetary policy (i.e., the setting of μ_t, $t = 0, 1, 2, ...$) cannot affect the real interest rate, consumption and employment.
  - Monetary policy simply affects the split in the real interest rate between nominal and real rates:

$$R_t^* = \frac{R_t}{\bar{\pi}_{t+1}}.$$

  - For a careful treatment when there is uncertainty, see.
- When $\theta > 0$ (NK model) then real variables are not determined independent of monetary policy.
  - In this case, monetary policy matters.

Monetary Policy in New Keynesian Model

- Suppose $\theta > 0$, so that we're in the NK model and monetary policy matters.
- The standard assumption is that the monetary authority sets μ_t to achieve an interest rate target, and that that target is a function of inflation:

$$R_t/R = (R_{t-1}/R)^{\alpha} \exp\left[(1-\alpha)\,\phi_{\pi}(\bar{\pi}_t - \bar{\pi}) + \phi_x x_t\right] \quad (7)',$$

  where x_t denotes the log deviation of actual output from target.
- This is a *Taylor rule*, and it satisfies the *Taylor Principle* when $\phi_{\pi} > 1$.
- Smoothing parameter: α.
  - Bigger is α the more persistent are policy-induced changes in the interest rate.

Equilibrium Conditions of NK Model with Taylor Rule

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu) Y_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}, \quad \frac{K_t}{F_t} = \left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad \text{(4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \quad C_t + G_t = p_t^* e^{a_t} N_t \text{ (6)}$$

$$R_t / R = (R_{t-1}/R)^{\alpha} \exp\left[(1-\alpha)\phi_{\pi}(\bar{\pi}_t - \bar{\pi}) + \phi_x x_t\right] \text{ (7)'}.$$

Conditions (7) and (8) have been replaced by (7)'.

Equilibrium Conditions of NK Model

- The model represents 7 equations in 7 unknowns:

$$C, p_t^*, F_t, K_t, N_t, R_t, \bar{\pi}_t$$

- After this system has been solved for the 7 variables, equations (7) and (8) can be used to solve for μ_t and m_t.
  - This is rarely done, because researchers are uncertain of the exact form of money demand and because m_t and μ_t are in practice not of direct interest.

Natural Equilibrium

- When $\theta = 0$, then

$$\frac{\varepsilon(1-\nu)}{\varepsilon - 1} \times \overbrace{C_t N_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

  so that we have a form of efficiency when ν is chosen to that $\varepsilon(1-\nu) / (\varepsilon - 1) = 1$.

- In addition, recall that we have allocative efficiency in the flexible price equilibrium.

- So, the flexible price equilibrium with the efficient setting of ν represents a natural benchmark for the New Keynesian model, the version of the model in which $\theta > 0$.
  - We call this the *Natural Equilibrium*.

- To simplify the analysis, from here on we set $G_t = 0$.

Natural Equilibrium

- With $G_t = 0$, equilibrium conditions for C_t and N_t:

$$\overbrace{C_t N_t^{\varphi}}^{\text{Marginal Cost of work}} = \overbrace{e^{a_t}}^{\text{marginal benefit of work}}$$

aggregate production relation: $C_t = e^{a_t} N_t$.

- Substituting,

$$e^{a_t} N_t^{1+\varphi} = e^{a_t} \rightarrow N_t = 1$$

$$C_t = \exp\left(\mathrm{a}_t\right)$$

$$R_t^* = \frac{\frac{1}{C_t}}{\beta E_t \frac{1}{C_{t+1}}} = \frac{1}{\beta E_t \frac{C_t}{C_{t+1}}} = \frac{1}{\beta E_t \exp\left(-\Delta a_{t+1}\right)}$$

Natural Equilibrium, cnt'd

- Natural rate of interest:

$$R_t^* = \frac{\frac{1}{C_t}}{\beta E_t \frac{1}{C_{t+1}}} = \frac{1}{\beta E_t \exp\left(-\Delta a_{t+1}\right)}$$

- Two models for a_t :

$$DS: \; \Delta a_{t+1} = \rho \Delta a_t + \varepsilon_{t+1}^a$$
$$TS: \; a_{t+1} = \rho a_t + \varepsilon_{t+1}^a$$

Natural Equilibrium, cnt'd

- Suppose the ε_t's are Normal. Then,

$$E_t \exp\left(-\Delta a_{t+1}\right) = \exp\left(-E_t \Delta a_{t+1} + \frac{1}{2}V\right),$$

where

$$V = \sigma_a^2$$

- Then, with $r_t^* \equiv \log R_t^*$

$$r_t^* = -\log\beta + E_t \Delta a_{t+1} - \frac{1}{2}V.$$

- Useful: consider how natural rate responds to ε_t^a shocks under DS and TS models for a_t.
  - To understand how r_t^* responds, consider implications of consumption smoothing in absence of change in r_t^*.
  - Hint: in natural equilibrium, r_t^* steers the economy so that natural equilibrium paths for C_t and N_t are realized.

Conclusion

- Described NK model and derived equilibrium conditions.
  - The usual version of model represents monetary policy by a Taylor rule.
- When $\theta = 0$, so that prices are flexible, then monetary policy is (essentially) neutral.
  - Changes in money growth move prices and wages in such a way that real wages do not change and employment and output don't change.
- When prices are sticky, then a policy-induced reduction in the interest rate encourages more nominal spending.
  - The increased spending raises W_t more than P_t because of the sticky prices, thereby inducing the increased labor supply that firms need to meet the extra demand.
  - Firms are willing to produce more goods because:
    - The model assumes they *must* meet all demand at posted prices.
    - Firms make positive profits, so as long as the expansion is not too big they still make positive profits, even if not optimal.

Simple New Keynesian Model without Capital, II

Lawrence J. Christiano

January 5, 2018

Standard New Keynesian Model

- Taylor rule: designed so that in steady state, inflation is zero $(\bar{\pi} = 1)$

- Employment subsidy extinguishes monopoly power in steady state:

$$(1 - \nu) \frac{\varepsilon}{\varepsilon - 1} = 1$$

Equilibrium Conditions of NK Model with Taylor Rule

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu) Y_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}, \quad \frac{K_t}{F_t} = \left[\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)}$$

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \quad \text{(4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \quad C_t + G_t = p_t^* e^{a_t} N_t \text{ (6)}$$

$$R_t / R = (R_{t-1}/R)^{\alpha} \exp\left[(1-\alpha)\phi_{\pi}(\bar{\pi}_t - \bar{\pi}) + \phi_x x_t\right] \text{ (7)'}.$$

In steady state: $R = \frac{1}{\beta}, p^* = 1, F = K = \frac{1}{1-\beta\theta}, N = 1$

Natural Rate of Interest

- Intertemporal Euler equation in Natural equilibrium:

$$\overbrace{a_t}^{\text{optimal consumption in } t} = -\left[r_t^* - rr\right] + E_t a_{t+1}$$

where a * indicates 'natural' equilibrium and $r_t^* = log\left(R_t^*\right).$

- Back out the natural rate and ignoring constant

$$r_t^* = E_t \Delta a_{t+1}$$

- Law of motion for technology:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t^a.$$

NK IS Curve

- Euler equation in two equilibria (ignoring variance adjustment):

$$\overbrace{c_t = -\left[r_t - E_t\pi_{t+1} - rr\right] + E_tc_{t+1}}^{\text{Taylor rule equilibrium, NK model}}$$

$$\overbrace{c_t^* = -\left[r_t^* - rr\right] + E_tc_{t+1}^*}^{\text{Natural equilibrium } (\theta=0,\nu \text{ kills monopoly power})}$$

where lower case letters mean log, $\pi_t = log\left(\bar{\pi}_t\right)$and * means 'natural equilibrium'.

- Subtract:

$$\overbrace{x_t}^{\text{output gap}} = \left[r_t - E_tx_{t+1} - r_t^*\right]$$

Output in the NK Equilibrium

- Aggregate output relation:

$$y_t = log\left(p_t^*\right) + n_t + a_1,\ log\left(p_t^*\right) = \begin{cases} = 0 & \text{if } P_{i,t} = P_{j,t} \text{ all i,j} \\ \leq 0 & \text{otherwise} \end{cases}.$$

- To first approximation (given that we set inflation to zero in steady state):

$$p_t^* = 1.$$

Phillips Curve

- Equations pertaining to price setting:

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{(1-\nu)\, Y_t N_t^{\varphi}}{A_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}\ , \quad \frac{K_t}{F_t} = \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}}$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1}$$

- Log-linearly expand about zero-inflation steady state:

$$\hat{\bar{\pi}}_t = \frac{(1-\theta)(1-\theta\beta)}{\theta} (1+\varphi)\, x_t + E_t \hat{\bar{\pi}}_{t+1}$$

where $\hat{z}_t$ denotes $z_t - z)/z$

- See this for details.

Collecting the Log-linearized Equations

$$
\begin{aligned}
x_t &= x_{t+1} - [r_t - \pi_{t+1} - r_t^*] \\
r_t &= \phi_\pi \pi_t \\
\pi_t &= \beta \pi_{t+1} + \kappa x_t \\
r_t^* &= E_t (a_{t+1} - a_t),
\end{aligned}
$$

where

$$
\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\varphi).
$$

The Equations, in Matrix Form

- Representation of the shock:

$$s_t = \Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t$$
$$s_t = P s_{t-1} + \epsilon_t$$

- Matrix representation of system

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -1 & 1 \\ \phi_\pi & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r^*_t \end{pmatrix}$$
$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} s_{t+1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} s_t$$

- $E_t \left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0$

Solving the Model

- Linearized equilibrium conditions:

$$E_t \left[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t\right] = 0$$
$$s_t = P s_{t-1} + \epsilon_t$$

- Data generated using this equation,

$$z_t = A z_{t-1} + B s_t$$

with A and B chosen as follows:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0, \quad (\beta_0 + \alpha_0 B) P + [\beta_1 + (\alpha_0 A + \alpha_1) B] = 0$$

- Standard strategy:
  - pick the unique A with all eigenvalues less 1 in absolute value, that solves the first equation,
  - conditional on A, choose B to solve the second equation.
  - Strategy breaks down if there is no such A or there is more than one.

Simulation

- Draw $\epsilon_1, \epsilon_2, ..., T$
- Solve

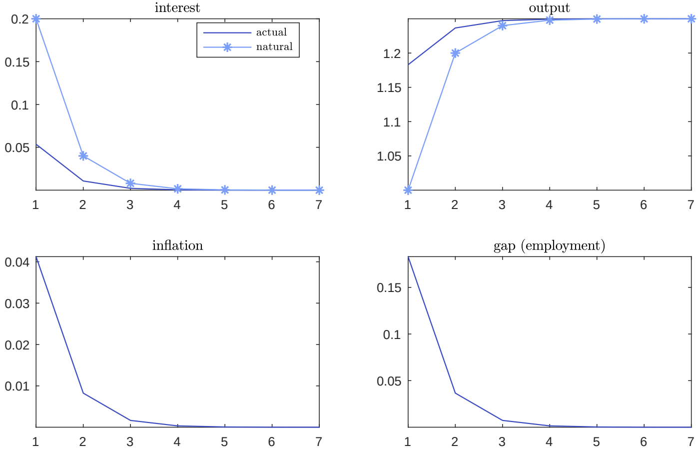
$$
\begin{aligned}
s_t &= Ps_{t-1} + \epsilon_t \\
z_t &= Az_{t-1} + Bs_t
\end{aligned}
$$

,for $t = 1, ..., T$, and given z_0 and s_0.

- If $\epsilon_t = 0$ for $t > 1$ and $\epsilon_1 = 1$, this is an *impulse response function*.
- If ϵ_t, $t = 1, 2, ..., T$ are *iid* and drawn from a random number generator, then it is a *stochastic simulation*.

Impulse Response Function

Figure: Dynamic Response to a Technology Shock



Note: $\beta = 0.97, \phi_\pi = 1.5, \rho = 0.2, \varphi = 1, \theta = 0.75, \kappa = 0.1817.$

Interpretation

- What happened in the figure?
  - The positive technology shock creates a surge in spending because of consumption smoothing. The natural rate has to rise, to prevent excessive consumption (an 'aggregate demand problem'). In the efficient allocations, log, natural consumption is equal to a_t and natural employment is constant.
  - The Taylor rule response is too weak, so the surge in aggregate demand is not pulled back and the economy over reacts to the shock. The excessive aggregate demand causes the economy to expand too much, which raises wages and costs and inflation.
  - The weak response under the Taylor rule is not a function of the value of ρ, whether the time series representation is difference or trend stationary (see).
- How to get a better response?
  - Put the natural rate of interest in the Taylor rule:

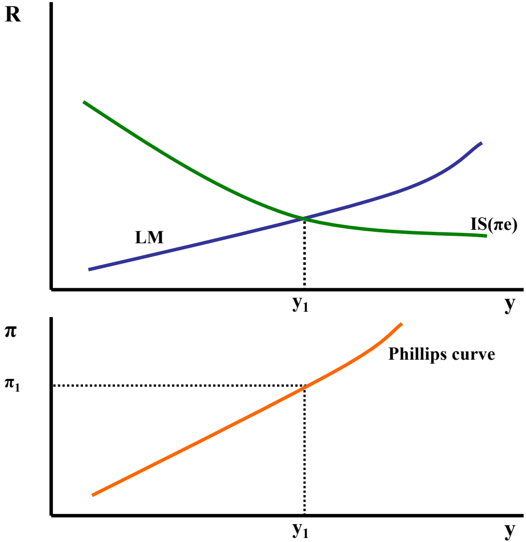
$$r_t = r_t^* + \phi_\pi \pi_t$$

  - Either measure it exactly, or put in a proxy.

How to Proxy the Natural Rate?

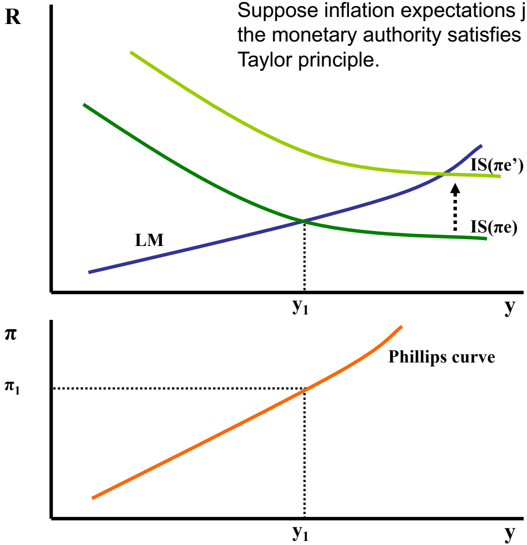
- How to proxy the natural rate?
  - It is consumption growth that appears in the natural rate, so something that signals good future consumption prospects would be a good proxy.
    - Credit growth?
    - Stock market growth?
- DSGE model simulations can be helpful for thinking about a good proxy.
  - For an example, see Christiano-Ilut-Motto-Rostagno (Jackson Hole, 2011).
- Why keep inflation in the Taylor rule?
  - To have a locally unique equilibrium, must have $\phi_\pi > 1$, i.e., *Taylor Principle*.

Uniqueness Under Taylor Principle



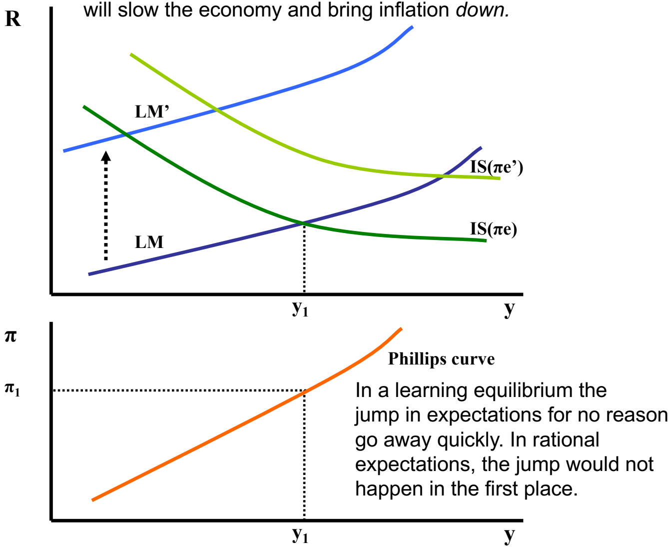
Uniqueness Under Taylor Principle

Suppose inflation expectations jump and the monetary authority satisfies the Taylor principle.



Uniqueness Under Taylor Principle

The sharp rise in interest rate
will slow the economy and bring inflation *down.*



Conclusion

- With sticky prices, the real allocations of the economy are not determined independently of monetary policy.
- So, performance of the economy will depend in part monetary policy design.
  - Put the natural rate (or some good proxy) in the Taylor rule, and policy works well.
  - Violate the Taylor principle and you could have trouble ('sunspots').
  - Other potential dysfunctions associated with poorly designed monetary policy are described in Christiano-Trabandt-Walentin (Handbook of Monetary Economics 2011).
- Without the proper design of monetary policy, aggregate demand can go wrong.
  - In the example described above, the response of aggregate demand to a shock is excessive.
  - Other examples can be found in which the response of aggregate demand is inadequate, and even the wrong sign.

Risk Shocks

Lawrence Christiano (Northwestern University),
Roberto Motto (ECB)
and Massimo Rostagno (ECB)
Based on *AER* publication, January 2014.

Finding

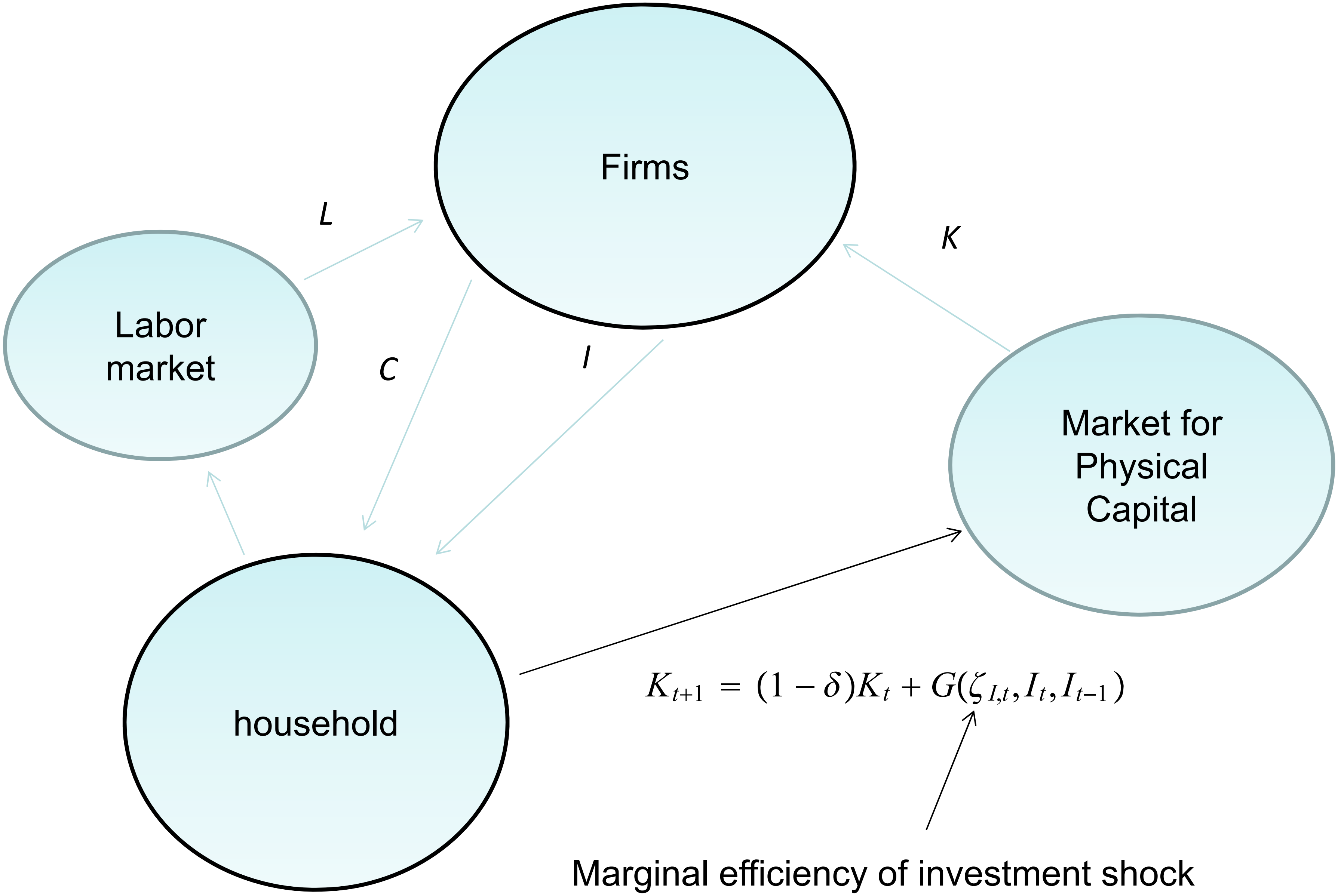
- Countercyclical fluctuations in the cross-sectional variance of a technology shock, when inserted into an otherwise standard macro model, can account for a substantial portion of economic fluctuations.
  – Complements empirical findings of Bloom (2009) and Kehrig (2011) suggesting greater cross-sectional dispersion in recessions.
  – Complements theory findings of Bloom (2009) and Bloom, Floetotto and Jaimovich (2009) which describe another way that increased cross-sectional dispersion can generate business cycles.

- ‘Otherwise standard model’:
  – A DSGE model, as in Christiano-Eichenbaum-Evans or Smets-Wouters
  – Financial frictions along the line suggested by BGG.

Outline

- Rough description of the model.
- Summary of Bayesian estimation of the model.
- Explanation of the basic finding of the analysis.

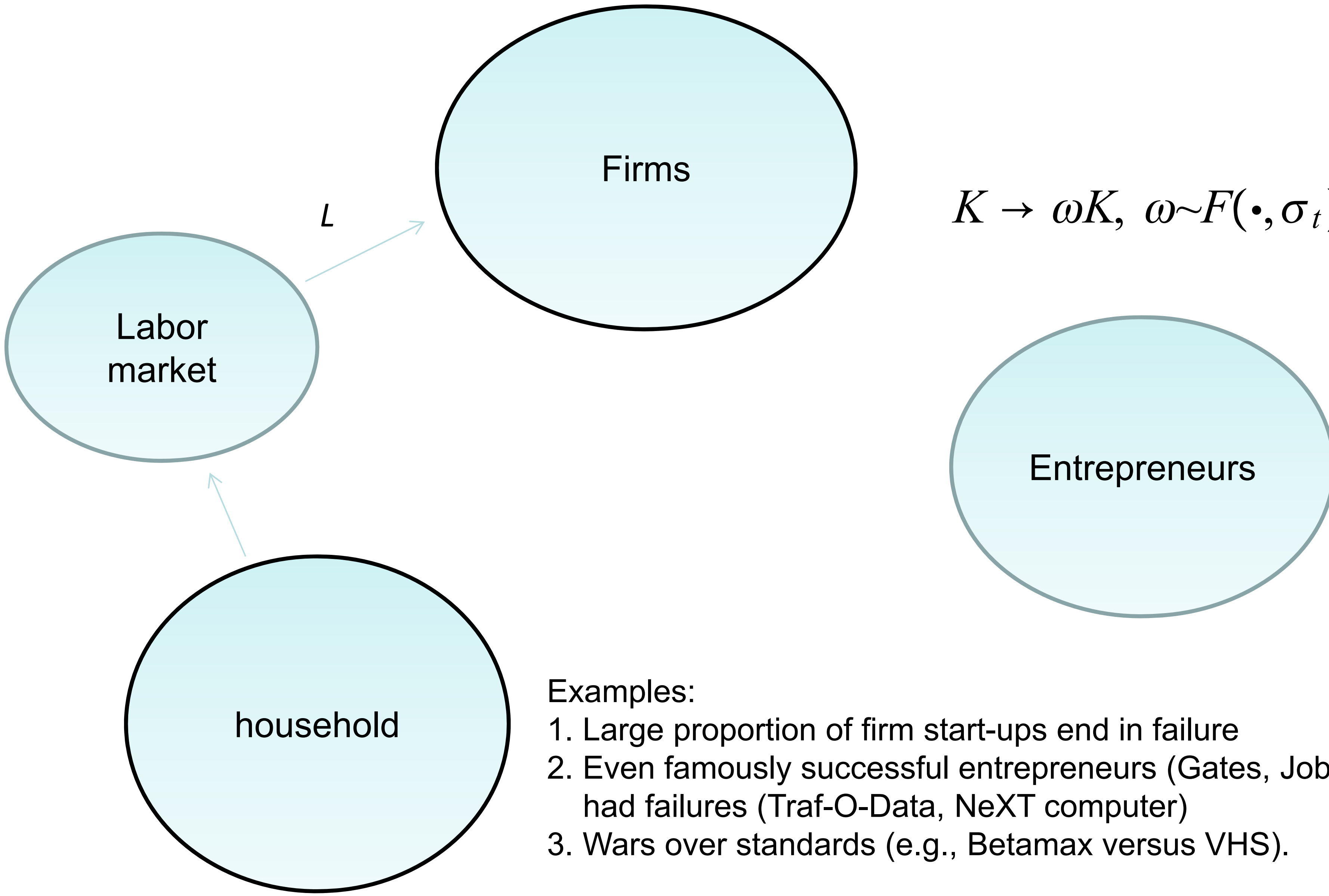
Standard Model



$$K_{t+1} = (1-\delta)K_t + G(\zeta_{I,t}, I_t, I_{t-1})$$

Marginal efficiency of investment shock

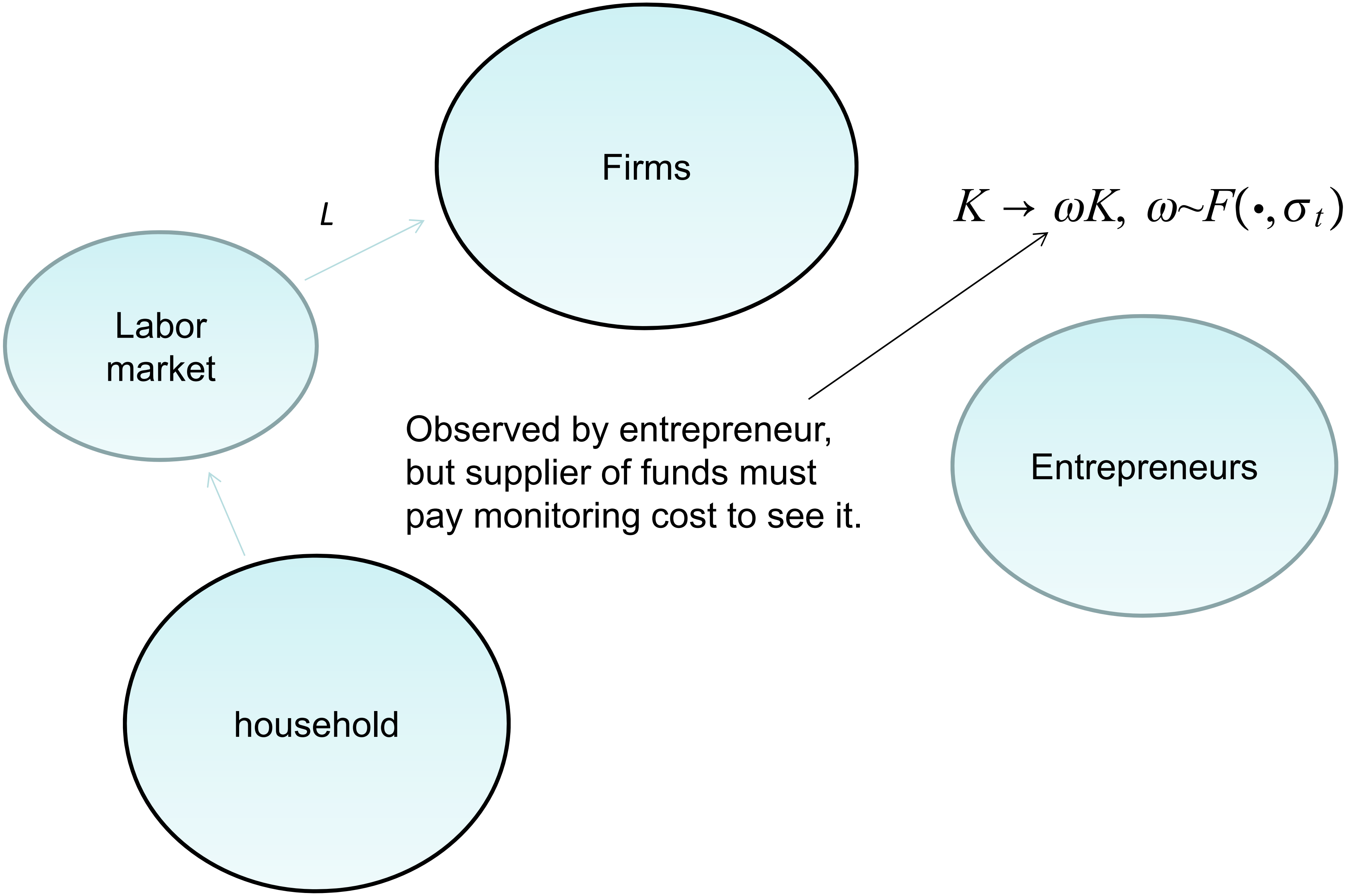
Standard Model with BGG



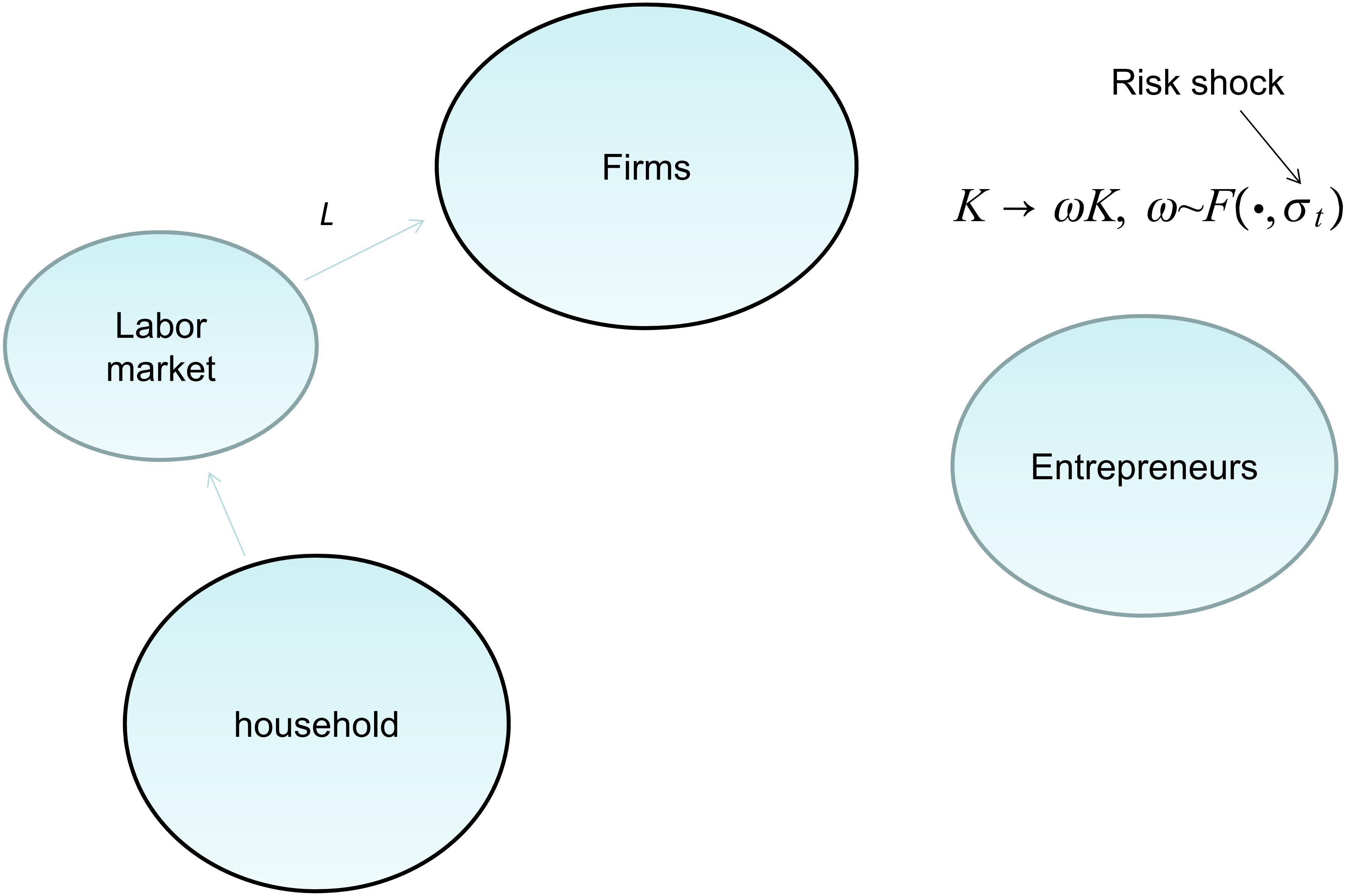
$$K \rightarrow \omega K, \ \omega \sim F(\bullet, \sigma_t)$$

Examples:
1. Large proportion of firm start-ups end in failure
2. Even famously successful entrepreneurs (Gates, Jobs) had failures (Traf-O-Data, NeXT computer)
3. Wars over standards (e.g., Betamax versus VHS).

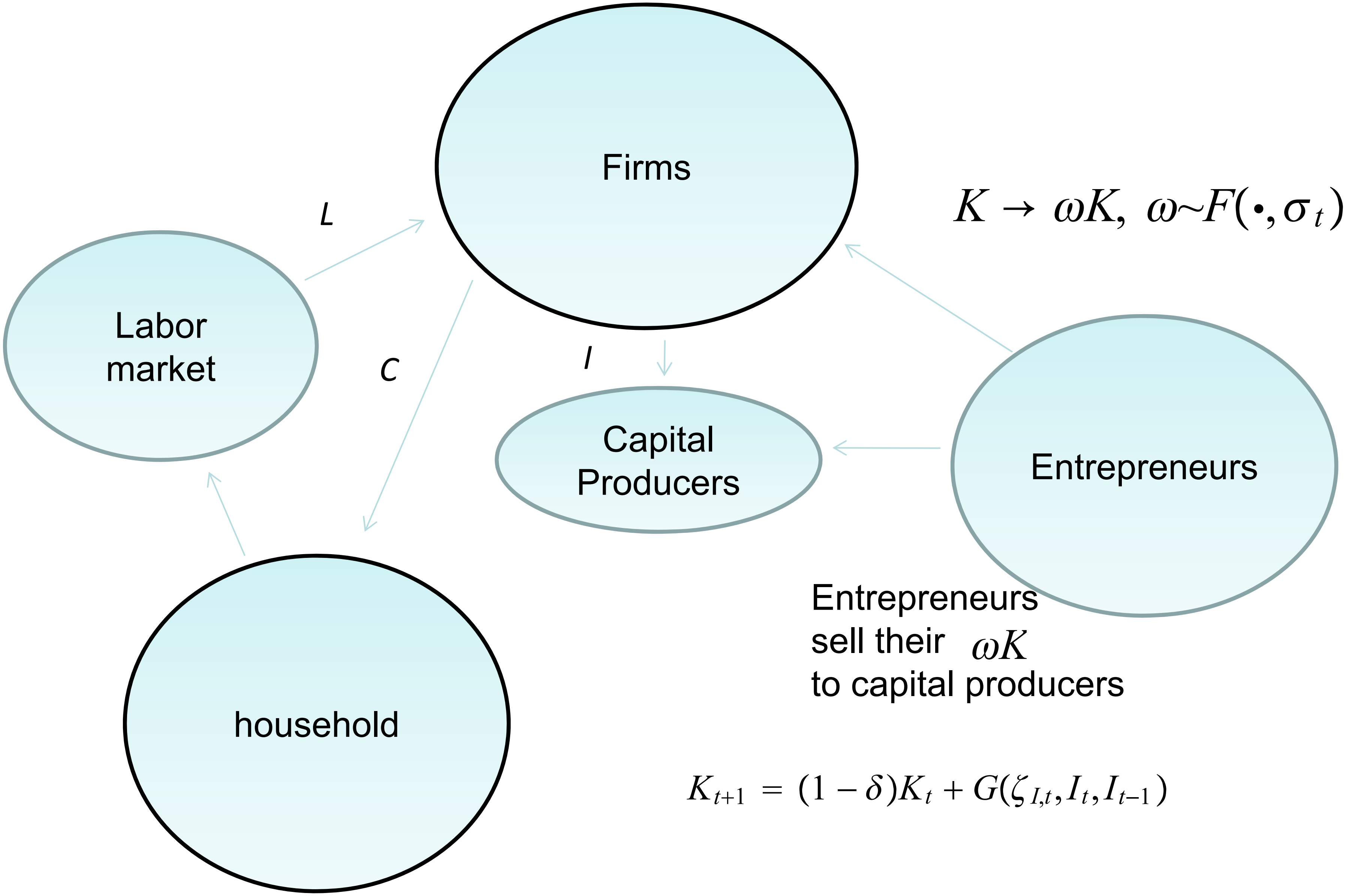
Standard Model with BGG



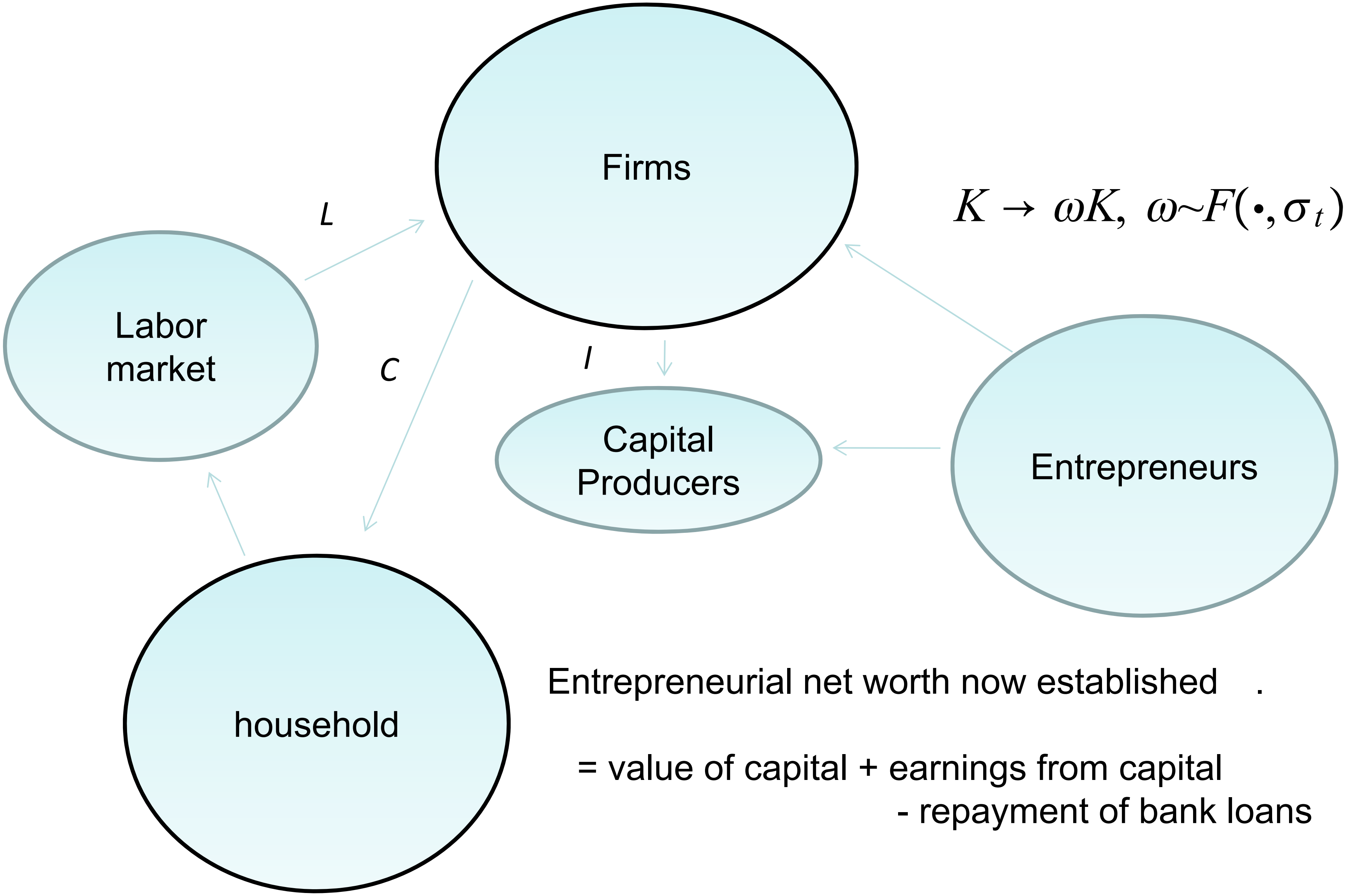
Standard Model with BGG



Standard Model with BGG



Standard Model with BGG

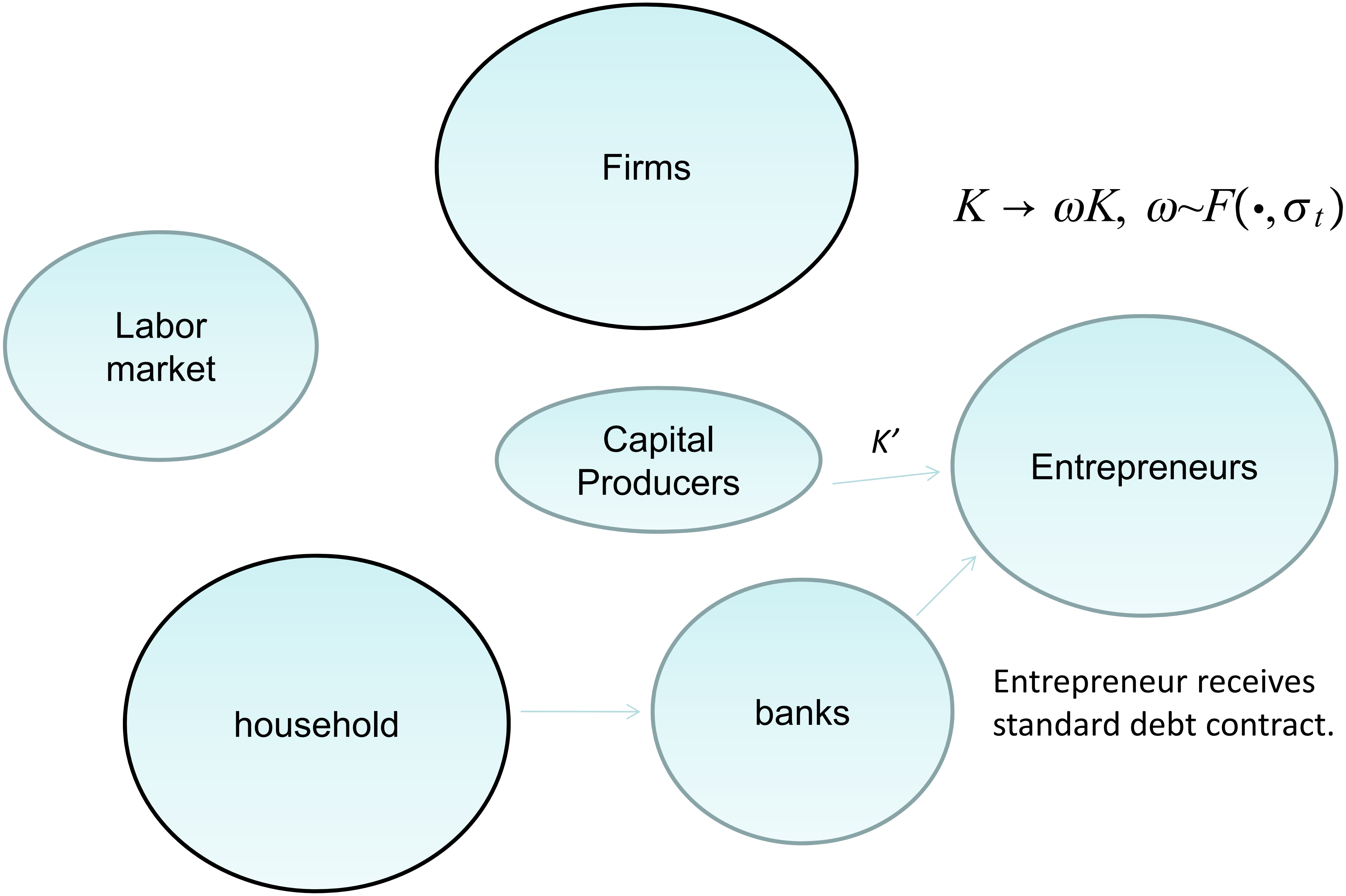


$$K \to \omega K,\ \omega \sim F(\bullet, \sigma_t)$$

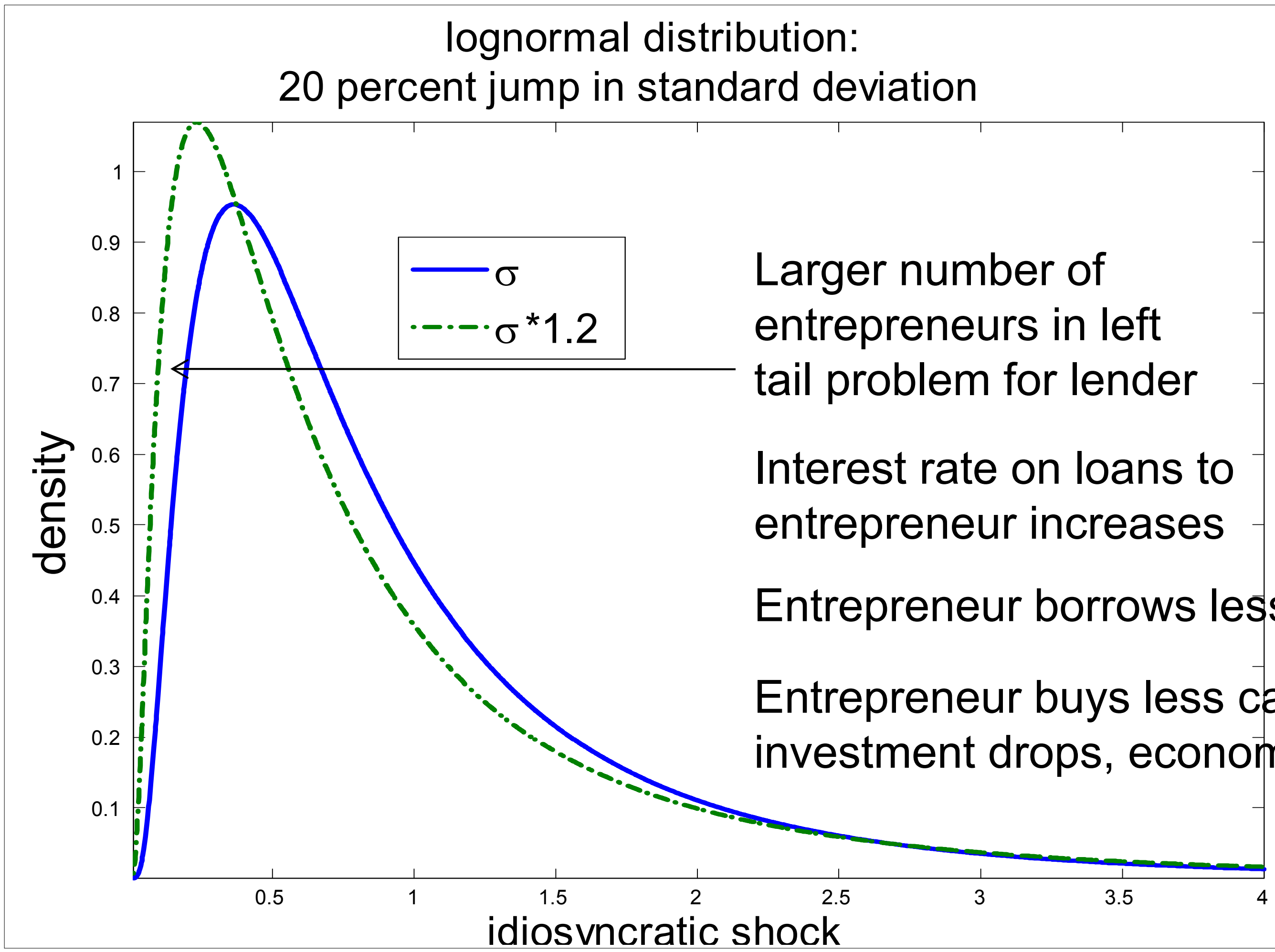
Entrepreneurial net worth now established .

= value of capital + earnings from capital
- repayment of bank loans

Standard Model with BGG



Economic Impact of Risk Shock



Larger number of entrepreneurs in left tail problem for lender

Interest rate on loans to entrepreneur increases

Entrepreneur borrows less

Entrepreneur buys less capital, investment drops, economy tanks

Five Adjustments to Standard DSGE Model for CSV Financial Frictions

- Drop: household intertemporal equation for capital.

- Add: equations that characterize the loan contract –
  - Zero profit condition for suppliers of funds.
  - Efficiency condition associated with entrepreneurial choice of contract.

- Add: Law of motion for entrepreneurial net worth (source of accelerator and Fisher debt-deflation effects).

- Introduce: bankruptcy costs in the resource constraint.

Risk Shocks

- We assume risk has a first order autoregressive representation:

$$\hat{\sigma}_t = \rho_1 \hat{\sigma}_{t-1} + \overbrace{u_t}^{\text{iid, univariate innovation to } \hat{\sigma}_t}$$

- We assume that agents receive early information about movements in the innovation ('news').

Risk Shock and News

- Assume

$$\hat{\sigma}_t = \rho_1 \hat{\sigma}_{t-1} + \overbrace{u_t}^{\text{iid, univariate innovation to } \hat{\sigma}_t}$$

- Agents have advance information about pieces of u_t

'signals' or 'news'

$$u_t = \xi_t^0 + \xi_{t-1}^1 + \ldots + \xi_{t-8}^8$$

$$\xi_{t-i}^i \sim \text{iid}, E(\xi_{t-i}^i)^2 = \sigma_i^2$$

$$\xi_{t-i}^i \sim \text{piece of } u_t \text{ observed at time } t-i$$

News on Risk Shocks Versus News on Other Shocks

	Marginal likelihood
DSGE Baseline	4493.85
DSGE without Signals	4098.43
DSGE with Signals on Equity Shock (γ) and No Signals on Risk Shock (σ)	4422.46
DSGE with Signals on Monetary Policy and No Signals on Risk Shock (σ)	4427.59
DSGE with Signals on Exogenous Spending Shock (g) and No Signals on Risk Shock (σ)	4096.62
DSGE with Signals on Technology Shocks and No Signals on Risk Shock (σ)	4334.47

Monetary Policy

- Nominal rate of interest function of:
  - Anticipated level of inflation.
  - Slowly moving inflation target.
  - Deviation of output growth from ss path.
  - Monetary policy shock.

12 Shocks

- Trend stationary and unit root technology shock.
- Marginal Efficiency of investment shock (perturbs capital accumulation equation)

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + G(\zeta_{i,t}, I_t, I_{t-1})$$

- Monetary policy shock.
- Equity shock.
- Risk shock.
- 6 other shocks.

Estimation

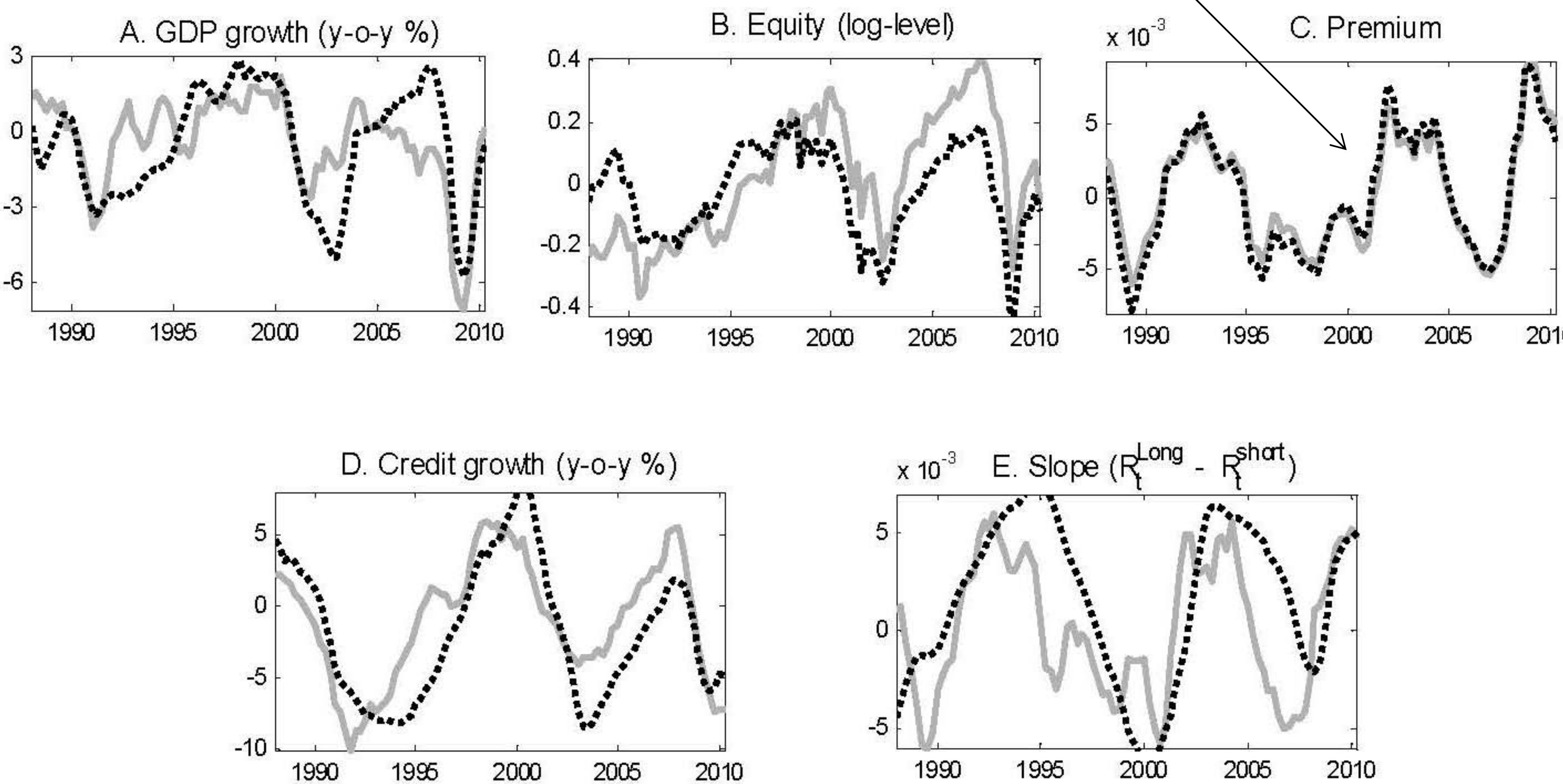
- Use standard macro data: consumption, investment, employment, inflation, GDP, price of investment goods, wages, Federal Funds Rate.
- Also some financial variables: BAA - 10 yr Tbond spreads, value of DOW, credit to nonfinancial business, 10 yr Tbond – Funds rate.
- Data: 1985Q1-2010Q2

Results

- Risk shock most important shock for business cycles.
- Quantitative measures of importance.
- Why are they important?
- What shock do they displace, and why?

Risk shock closely identified with interest rate premium.

Role of the Risk Shock in Macro and Financial Variables



Notes: The grey solid line represents the (two-sided) fitted data. The dotted black line is the model simulations.

Percent Variance in Business Cycle Frequencies Accounted for by Risk Shock

variable	*Risk*, σ_t
GDP	62
Investment	73
Consumption	16
Credit	64
Premium $(\mathbf{Z}-\mathbf{R})$	95
Equity	69
$R^{10\text{ year}}-R^{1\text{ quarter}}$	56

Risk shock closely identified with interest rate premium

Note: 'business cycle frequencies means' Hodrick-Prescott filtered data.

Why Risk Shock is so Important

- A. Our econometric estimator ‘thinks’

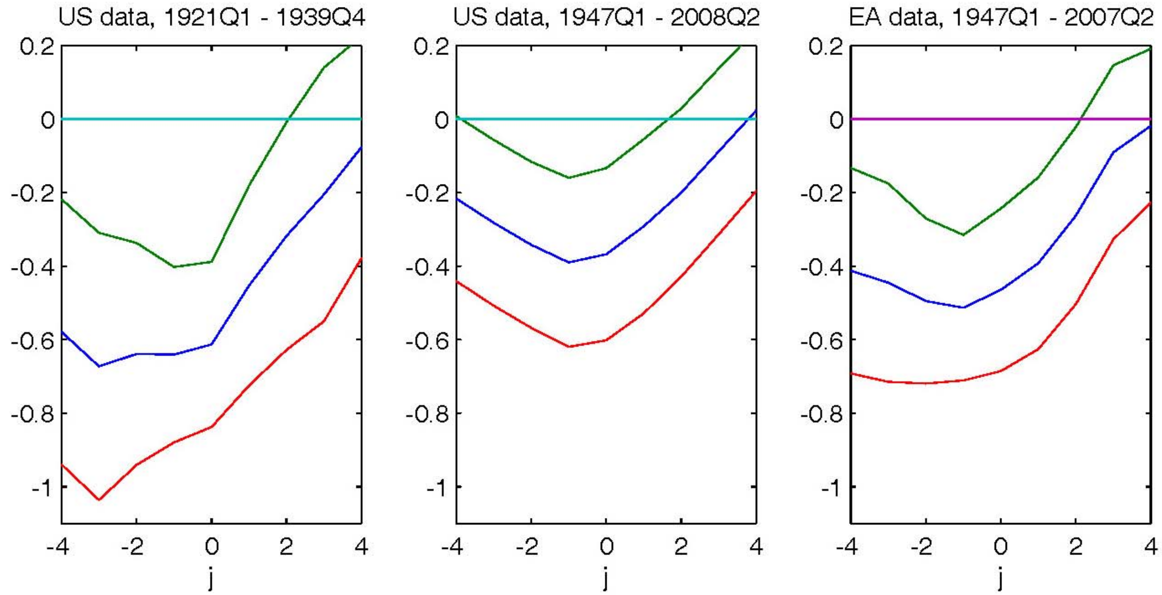
  risk spread ~ risk shock.

- B. In the data: the risk spread is strongly negatively correlated with output.

- C. In the model: bad risk shock generates a response that resembles a recession

- A+B+C suggests risk shock important.

Correlation (risk spread(t),output(t-j)), HP filtered data, 95% Confidence Interval

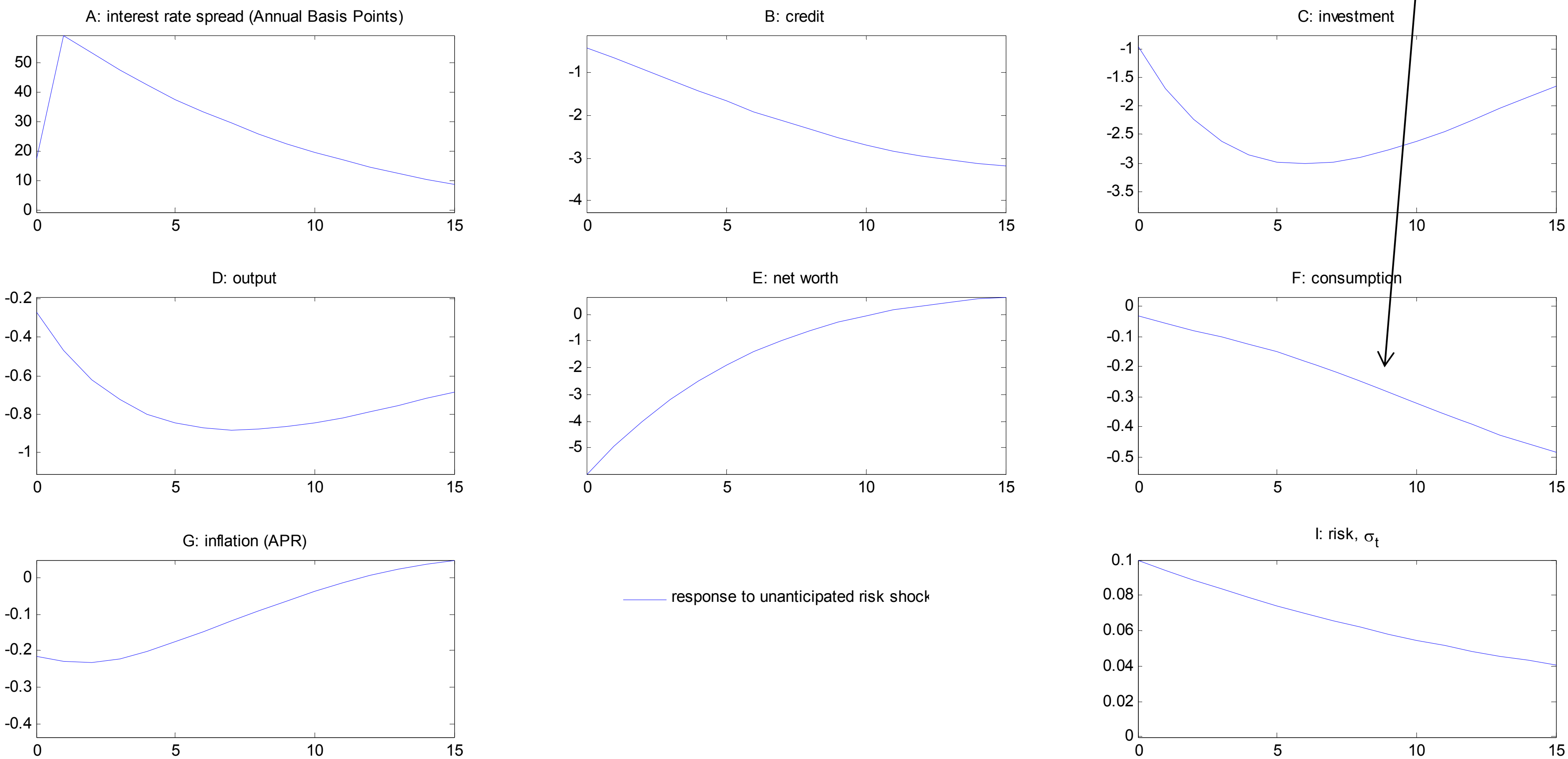


The risk spread is significantly negatively correlated with output and leads a little.

Notes: Risk spread is measured by the difference between the yield on the lowest rated corporate bond (Baa) and the highest rated corporate bond (Aaa). Bond data were obtained from the St. Louis Fed website. GDP data were obtained from Balke and Gordon (1986). Filtered output data were scaled so that their standard deviation coincide with that of the spread data.

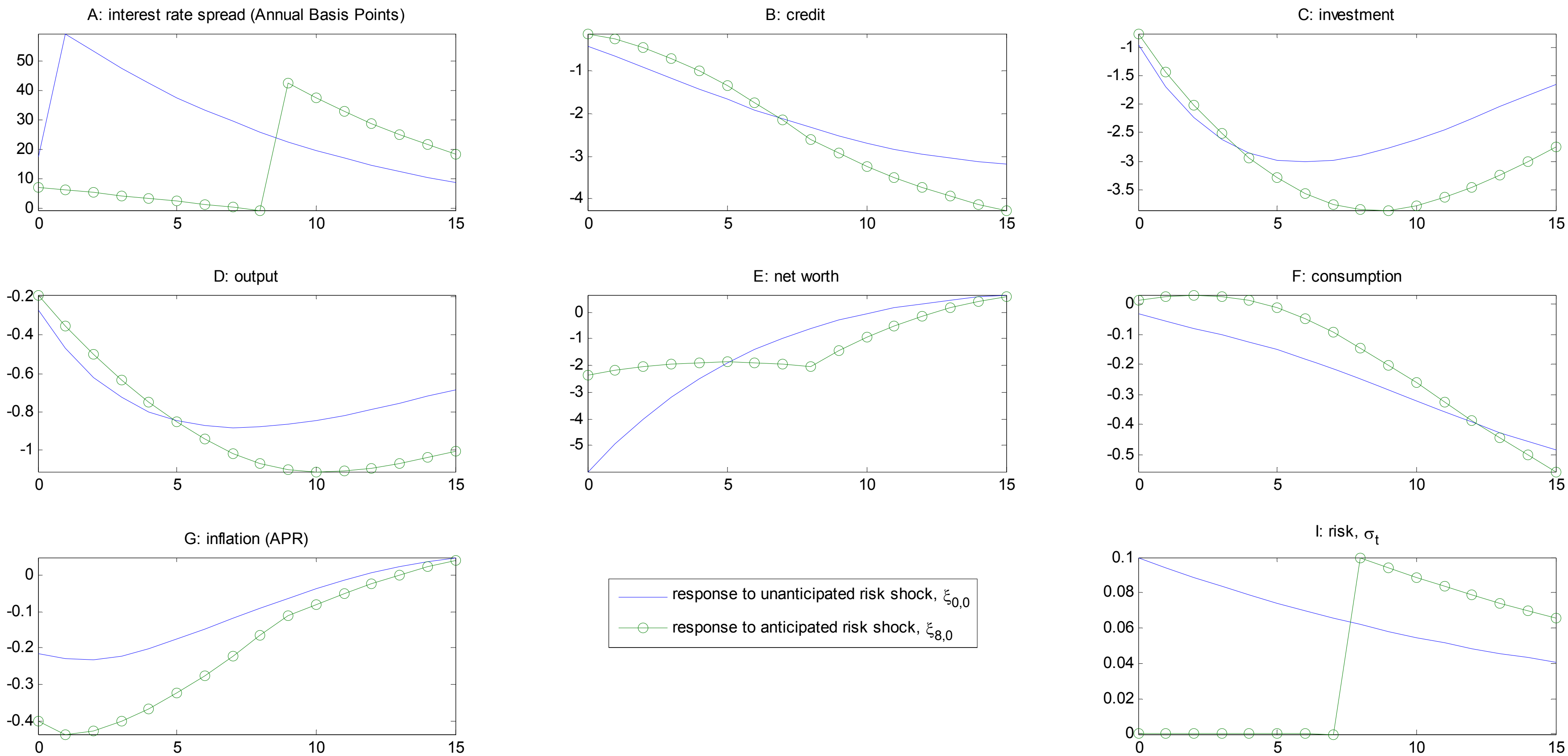
Surprising, from RBC perspective

Figure 3: Dynamic Responses to Unanticipated and Anticipated Components of Risk Shock



Looks like a business cycle

Figure 3: Dynamic Responses to Unanticipated and Anticipated Components of Risk Shock



What Shock Does the Risk Shock Displace, and why?

- The risk shock mainly crowds out the marginal efficiency of investment.
  - But, it also crowds out other shocks.

- Compare estimation results between our model and model with no financial frictions or financial shocks (CEE).

- Baseline model mostly ‘steals’ explanatory power from m.e.i., but also from other shocks:

big drop in marginal efficiency of investment

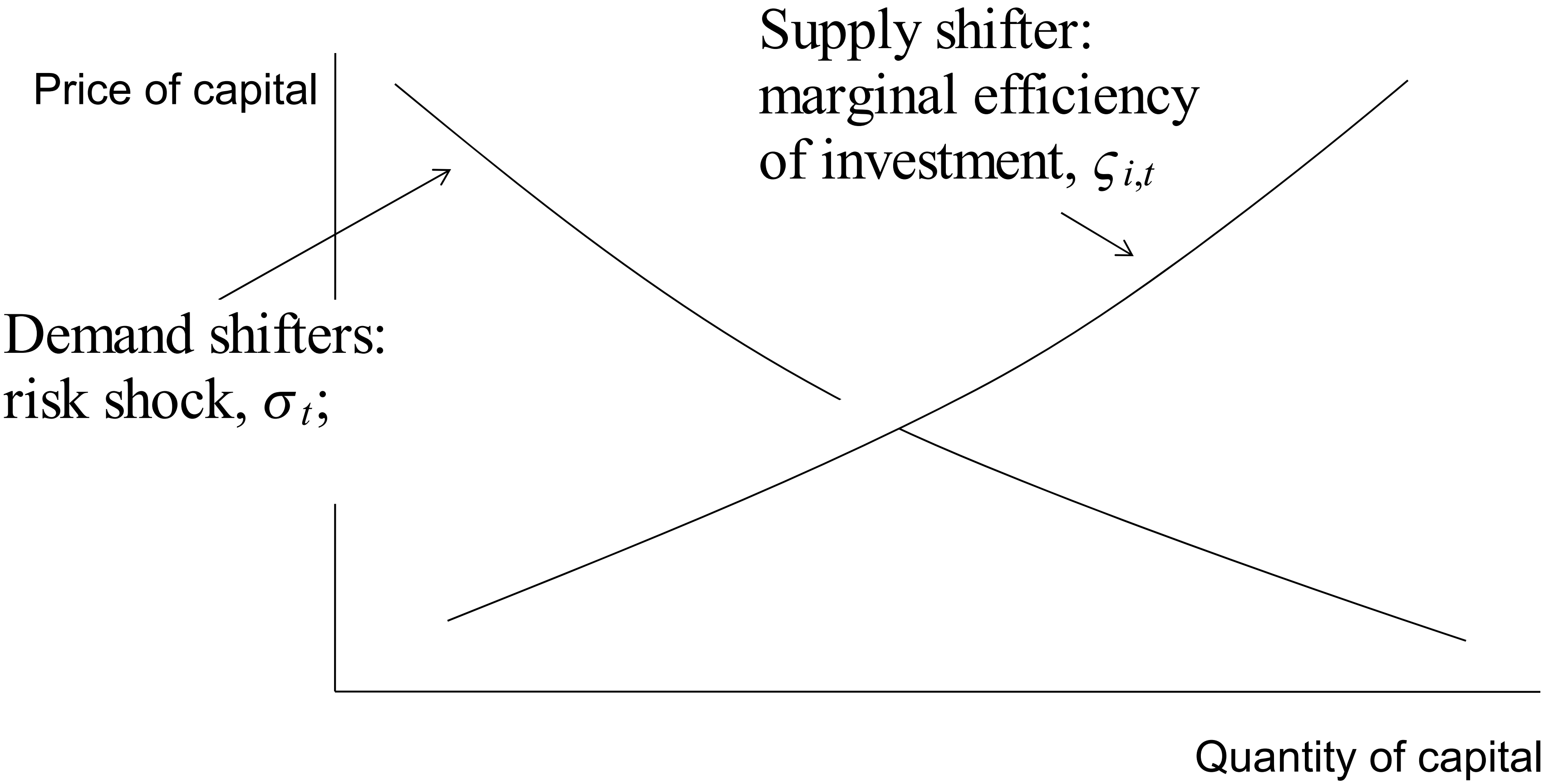
Variance Decomposition of GDP at Business Cycle Frequency (in percent)									
shock	*Risk* σ_t	*Equity* γ_t	*M.E.I.* $\zeta_{I,t}$	*Technol.* ε_t, $\mu_{z,t}$,	*Markup* $\lambda_{f,t}$,	*M.P.* ϵ_t	*Demand* $\zeta_{c,t}$	*Exog.Spend.* g_t	*Term*
Baseline model	62	0	13	2	12	2	4	3	0
CEE	[–]	[–]	[39]	[18]	[31]	[4]	[3]	[5]	[-]

- Baseline model mostly ‘steals’ explanatory power from m.e.i., but also from other shocks:

technology goes from small to tiny

Variance Decomposition of GDP at Business Cycle Frequency (in percent)									
shock	*Risk* σ_t	*Equity* γ_t	*M.E.I.* $\zeta_{I,t}$	*Technol.* $\varepsilon_t,\ \mu_{z,t},$	*Markup* $\lambda_{f,t},$	*M.P.* ϵ_t	*Demand* $\zeta_{c,t}$	*Exog.Spend.* g_t	*Term*
Baseline model	62	0	13	2	12	2	4	3	0
CEE	[–]	[–]	[39]	[18]	[31]	[4]	[3]	[5]	[-]

Why does Risk Crowd out Marginal Efficiency of Investment?



- Marginal efficiency of investment shock can account well for the surge in investment and output in the 1990s, *as long as the stock market is not included in the analysis.*

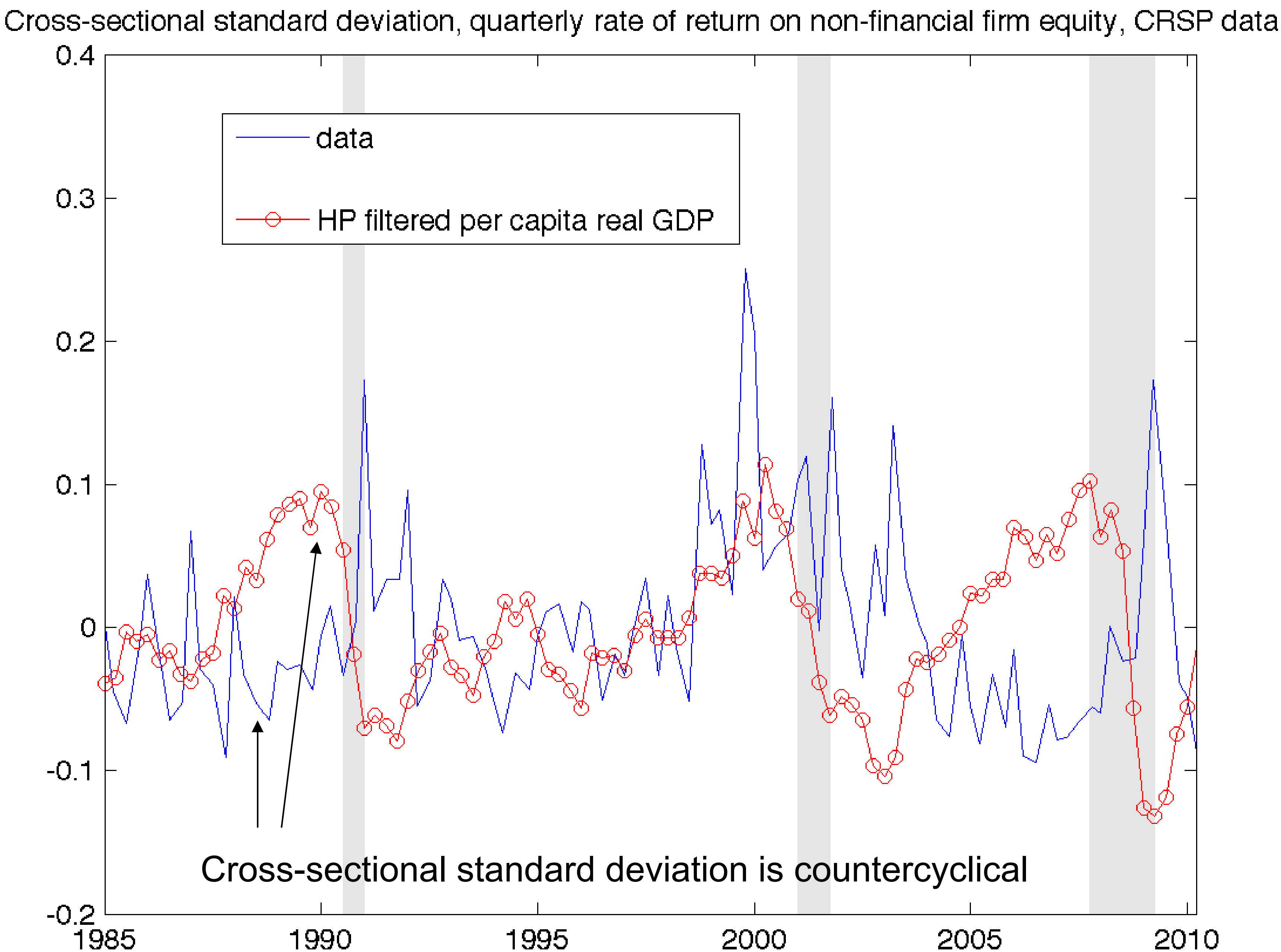
- When the stock market is included, then explanatory power shifts to financial market shocks.

- When we drop ‘financial data’ – slope of term structure, interest rate spread, stock market, credit growth:

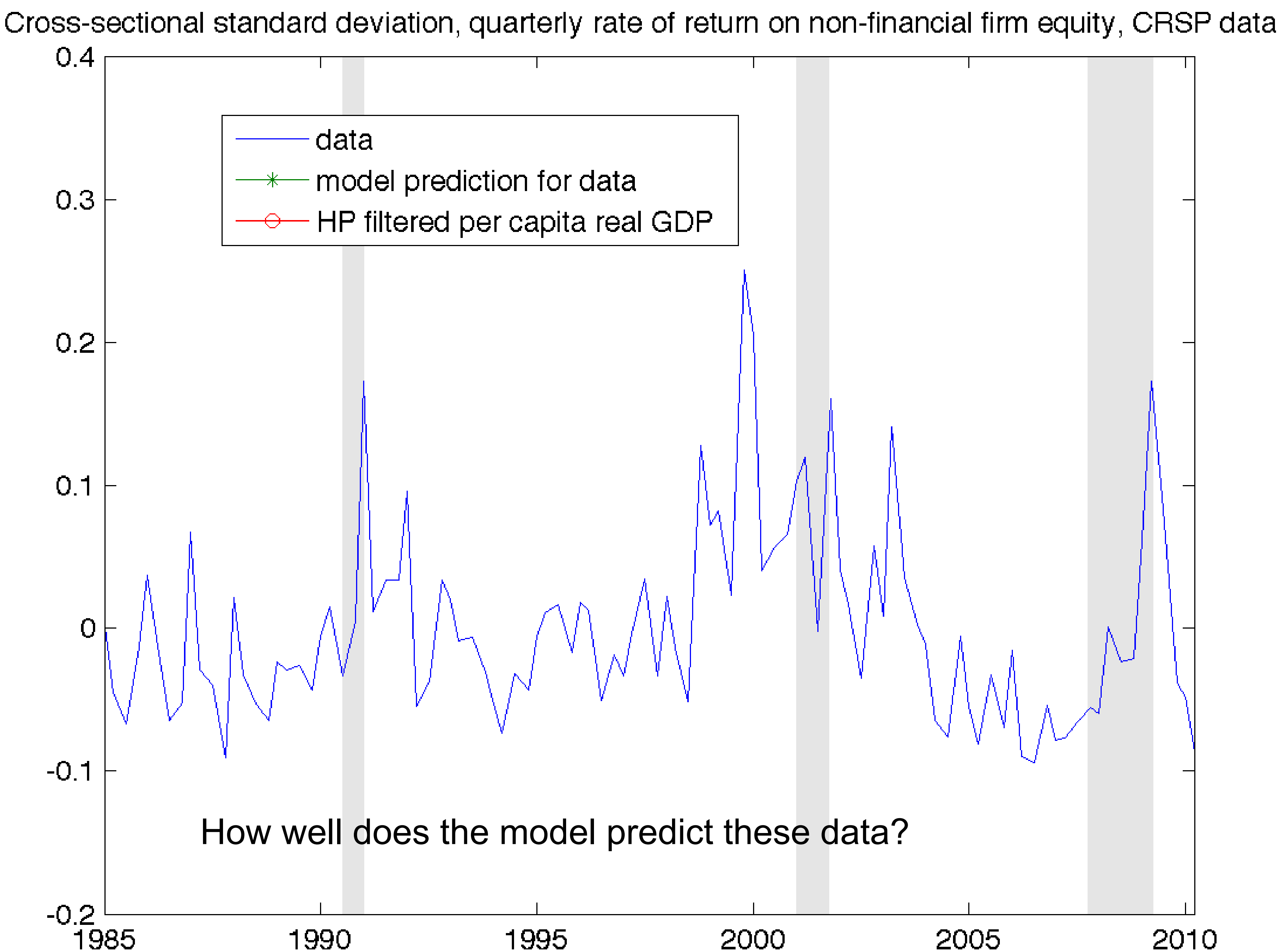
  – Hard to differentiate risk shock view from marginal efficiency of investment view.

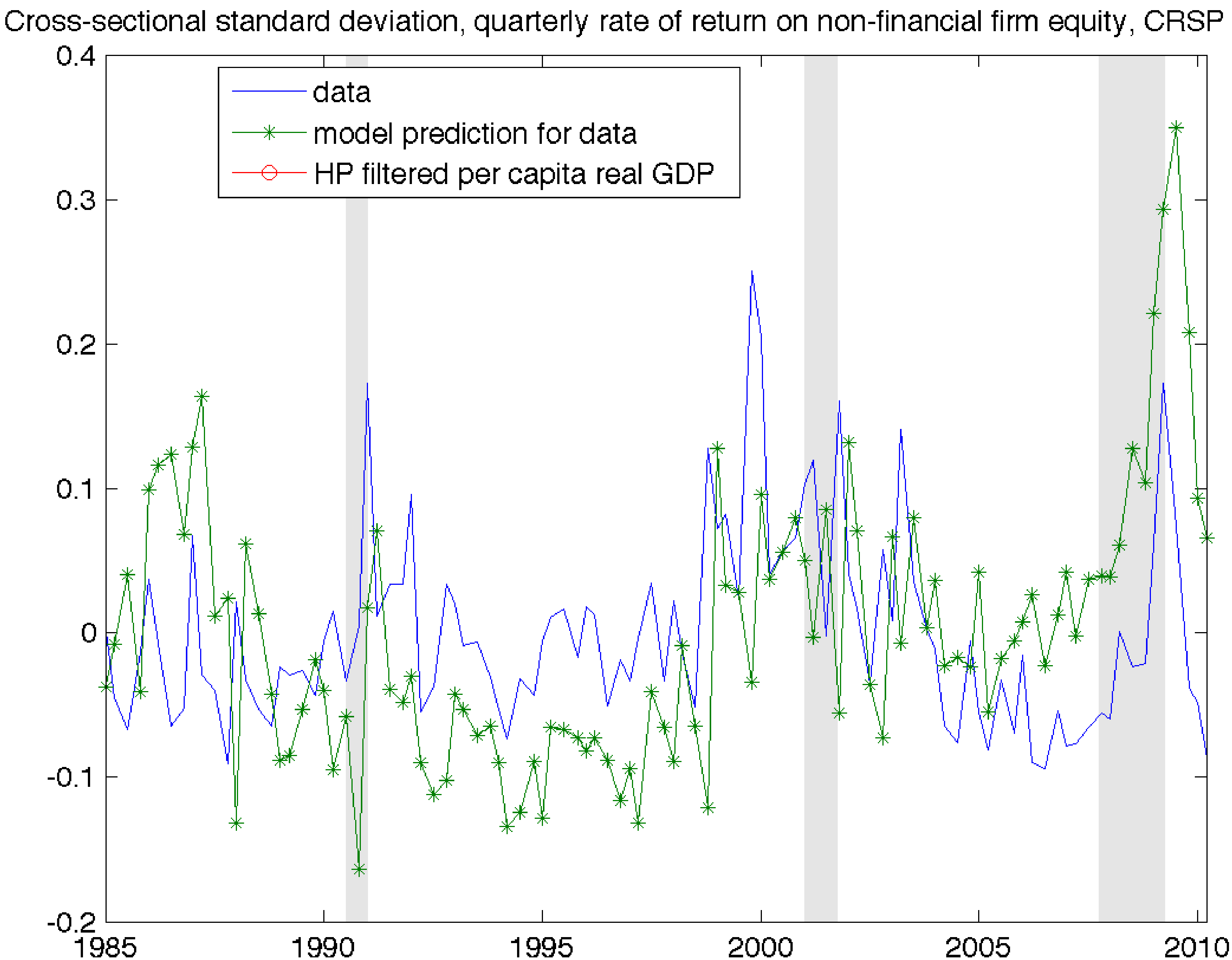
Is There Independent Evidence for Risk Shocks?

- Cross-sectional standard deviation of rate of return on equity in CRSP rises in recessions (Bloom, 2009).
- This observation played no role in the construction or estimation of the model.
- Compute the model’s best guess (Kalman Smoother) about the cross-sectional standard deviation of equity returns, and compare with data.



Cross-sectional standard deviation, quarterly rate of return on non-financial firm equity, CRSP data
data
HP filtered per capita real GDP
Cross-sectional standard deviation is countercyclical
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1985
1990
1995
2000
2005
2010


Cross-sectional standard deviation, quarterly rate of return on non-financial firm equity, CRSP data
data
model prediction for data
HP filtered per capita real GDP
How well does the model predict these data?
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1985
1990
1995
2000
2005
2010


Cross-sectional standard deviation, quarterly rate of return on non-financial firm equity, CRSP data
data
model prediction for data
HP filtered per capita real GDP
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1985
1990
1995
2000
2005
2010

Policy

- How should the monetary authority respond to a jump in interest rate spreads?
  - Depends on why the spread jumped.
  - If the jump is because of an increase in risk (uncertainty), then cut policy rate more than simple Taylor rule would dictate.

Conclusion

- Incorporating financial frictions and financial data changes inference about the sources of shocks:
  - risk shock.
- Interesting to explore mechanisms that make risk shock endogenous.
- Models with financial frictions can be used to ask interesting policy questions:
  - When there is an increase in risk spreads, how should monetary policy respond?
  - How should monetary policy respond to variations in credit growth, stock prices?

The Effects of Balance Sheet Constraints on Non Financial Firms

Lawrence J. Christiano

January 5, 2018

Background

- Several shortcomings of standard New Keynesian model.
  - It assumes that the interest rate satisfies an Euler equation with the consumption of a single, representative household.
  - Evidence against that Euler equation is strong (Hall (JPE1978), Hansen-Singleton (ECMA1982), Canzoneri-Cumby-Diba (JME2007)
- Here, discuss Buera-Moll (AEJ-Macro2015) model of heterogeneous households and firms.
  - Shows how a model with heterogeneous households breaks Euler equation.
  - Shows how deleveraging can lead to many of the things observed in the Financial Crisis and Great Recession.
    - fall in output, investment, consumption, TFP, real interest rate.
- 'Toy' model that can be solved analytically, great for intuition.
- Earlier, similar models: Kahn-Thomas (JPE2013), Liu-Wang-Zha (ECMA2013).

Outline

- Hand-to-mouth workers
- Entrepreneurs (where all the action is)
- Aggregates: Loan Market, GDP, TFP, Consumption, Capital, Consumption
- Equilibrium
  - Computation.
  - Parameter values.
  - The dynamic effects of deleveraging.

Hand-to-mouth Workers

- Hand-to-mouth workers maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^W\right)^{1-\sigma}}{1-\sigma} - \frac{1}{1+\chi} L_t^{1+\chi} \right]$$

  subject to:

$$C_t^W \leq w_t L_t.$$

- Solution:

$$L_t^{\frac{\chi+\sigma}{1-\sigma}} = w_t, \tag{1}$$

  and labor supply is upward-sloping for $0 < \sigma < 1$.

Entrepreneurs

- i^{th} entrepreneur would like to maximize utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}\right),\ u\left(c\right) = \log c.$$

- i^{th} entrepreneur can do one of two things in t:
  - use time t resources plus debt, $d_{i,t} \geq 0$, to invest in capital and run a production technology in period $t+1$.
    - will do this if i's technology is sufficiently productive.
  - use time t resources to make loans, $d_{i,t} < 0$, to financial markets.
    - will do this if i's technology is unproductive.

Rate of Return on Entrepreneurial Investment

- i^{th} entrepreneur can invest $x_{i,t}$ and increase its capital in $t+1$:

$$k_{i,t+1} = (1-\delta)\, k_{i,t} + x_{i,t},\ \delta \in (0,1)$$

- In $t+1$ entrepreneur can use $k_{i,t+1}$ to produce output:

$$y_{i,t+1} = (z_{i,t+1} k_{i,t+1})^{\alpha}\, l_{i,t+1}^{1-\alpha},\ \alpha \in (0,1)\,,$$

  where $l_{i,t+1}$ ~ amount of labor hired in $t+1$ for wage, w_{t+1}.

- Technology shock, $z_{i,t+1}$, observed at time t, and
  - independent and identically distributed:
    - across i for a given t,
    - across t for given i.
  - Density of z, $\psi(z)$; CDF of z, $\Psi(z)$.

Rate of Return on Entrepreneurial Investment

- i^{th} entrepreneur's time $t+1$ profits:

$$\max_{l_{i,t+1}} \left[\left(z_{i,t+1} k_{i,t+1} \right)^{\alpha} l_{i,t+1}^{1-\alpha} - w_{t+1} l_{i,t+1} \right]$$
$$= \pi_{t+1} z_{i,t+1} k_{i,t+1}$$

$$\pi_{t+1} \equiv \alpha \left(\frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}.$$

- Rate of return on one unit of investment in t :

$$\pi_{t+1} z_{i,t+1} + 1 - \delta.$$

The Decision to Invest or Lend

- The i^{th} entrepreneur can make a one period loan at t, and earn $1 + r_{t+1}$ at $t + 1$.
- Let $\bar{z}_{t+1}$ denote value of $z_{i,t+1}$ such that return on investment same as return on making a loan:

$$\pi_{t+1}\bar{z}_{t+1} + 1 - \delta = 1 + r_{t+1}.$$

- If $z_{i,t+1} > \bar{z}_{t+1}$,
  - borrow as much as possible, subject to collateral constraint:

    $$d_{i,t+1} \leq \theta_t k_{i,t+1},\ \theta_t \in [0, 1],$$

    and invest as much as possible in capital.
  - In this case borrow:

    $$d_{i,t+1} = \theta_t k_{i,t+1}.$$

- If $z_{i,t+1} < \bar{z}_{t+1}$, then set $k_{i,t+1} = 0$ and make loans, $d_{i,t} < 0$.

Entrepreneur's Problem

- At t, maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(c_{i,t+j}\right)$$

subject to:

- given $k_{i,t}$ and $d_{i,t}$
- borrowing constraint
- budget constraint:

$$c_{i,t} + \overbrace{k_{i,t+1} - (1-\delta)\, k_{i,t}}^{\text{investment, } x_{it}} \leq \overbrace{\pi_t z_{i,t} k_{i,t}}^{y_{i,t} - w_t l_{i,t}, \text{ if entrepreneur invested in } t-1}$$

$$+ \overbrace{d_{i,t+1} - (1+r_t) d_{i,t}}^{\text{increase in debt, net of financial obligations}}$$

- Alternative representation of budget constraint:

$$c_{i,t} + \overbrace{a_{i,t+1}}^{\equiv k_{i,t+1} - d_{i,t+1}, \text{'net worth'}} \leq \overbrace{\left[\pi_t z_{i,t} + 1 - \delta\right] k_{i,t} - (1+r_t) d_{i,t}}^{\equiv m_{i,t}, \text{'cash on hand'}}$$

Entrepreneur's Problem

- At t, maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(c_{i,t+j}\right),$$

$u(c) = log(c)$, subject to:
  - given $k_{i,t}$ and $d_{i,t}$
  - borrowing constraint
  - budget constraint:

$$c_{i,t} + a_{i,t+1} \leq m_{i,t}$$

where,

$$a_{i,t+1} = k_{i,t+1} - d_{i,t+1}, \quad m_{i,t} = \left[\pi_t z_{i,t} + 1 - \delta\right] k_{i,t} - (1 + r_t) d_{i,t}$$

- Optimal choice of next period's net worth:

$$a_{i,t+1} = \beta m_{i,t}, \quad c_{i,t} = (1 - \beta)\, m_{i,t}.$$

Entrepreneur's Problem

- For $z_{i,t+1} \geq \bar{z}_{t+1}$, max debt and capital:

$$d_{i,t+1} = \theta_t k_{i,t+1} = \theta_t \left(d_{i,t+1} + a_{i,t+1} \right)$$
$$\rightarrow d_{i,t+1} = \frac{\theta_t}{1-\theta_t} a_{i,t+1}, \quad k_{i,t+1} = \frac{1}{1-\theta_t} a_{i,t+1}$$

  - Example:
    - if $\theta_t = \frac{2}{3}$, then leverage $= 1/\left(1-\theta_t\right) = 3$.
    - if net worth, $a_{i,t+1} = 100$, then $k_{i,t+1} = 300$ and $d_{i,t+1} = 200$.

- For $z_{i,t+1} < \bar{z}_{t+1}$, $k_{i,t+1} = 0$ and $d_{i,t} < 0$ (i.e., lend)
  - upper bound on lending: $d_{i,t} = -m_{i,t}$, all cash on hand.
  - won't go to upper bound with log utility.

Aggregates: Loan Market

- Demand for loans = supply in period t:

$$\overbrace{\frac{\theta_t}{1-\theta_t}}^{\text{borrowing, per unit of net worth, by average investing entrepreneur}} \times \overbrace{\left[1-\Psi\left(\bar{z}_{t+1}\right)\right]}^{\text{fraction of investing entrepreneurs}}$$

$$= \overbrace{1}^{\text{lending, per unit of net worth, by average non-investing entrepreneur}} \times \overbrace{\Psi\left(\bar{z}_{t+1}\right)}^{\text{fraction of non-investing entrepreneurs}}$$

- Rearranging:

$$\Psi\left(\bar{z}_{t+1}\right) = \theta_t. \tag{2}$$

The (endogenous) fraction of non-investing entrepreneurs, $\Psi\left(\bar{z}_{t+1}\right)$, equals the (exogenous) collateral constraint.

Aggregates: Gross Domestic Product

- The i^{th} firm's production function is:

$$y_{it} = (z_{i,t}k_{i,t})^{\alpha}\, l_{i,t}^{1-\alpha} = \overbrace{\left(\frac{z_{i,t}k_{i,t}}{l_{i,t}}\right)^{\alpha}}^{\text{same for each } i \text{, because all face same } w_t} l_{i,t}.$$

- Ratios equal ratio of sums:

$$\frac{z_{i,t}k_{i,t}}{l_{i,t}} = \frac{\int_i z_{i,t}k_{i,t}di}{\int_i l_{i,t}di} = \frac{\int_i z_{i,t}k_{i,t}di}{L_t}.$$

- GDP

$$Y_t = \int_i y_{i,t}di = \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^{\alpha} \int_i l_{i,t}di$$

$$= \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^{\alpha} L_t$$

Aggregates: GDP, TFP and wage

- With some algebra, can establish:

$$Y_t = \left(\overbrace{\int_i z_{i,t} k_{i,t} di}^{=E[z|z>\bar{z}_t]\times K_t} \right)^{\alpha} L_t^{1-\alpha} = Z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{3}$$

$$Z_t \equiv \left(E\left[z|z>\bar{z}_t\right]\right)^{\alpha}. \tag{4}$$

- Simple intuition:
  - Aggregate output, Y_t, a function of aggregate capital and labor, and (endogenous) TFP, Z_t.
  - Z_t average TFP of firms in operation.

- Aggregate wage:

$$w_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{5}$$

Aggregates: Consumption

- Integrating over entrepreneurs' budget constraints:

$$\int_i \left[c_{i,t} + k_{i,t+1} - d_{i,t+1}\right] di$$

$$= \int_i \left[y_{i,t} - w_t l_{i,t} + (1-\delta)\, k_{i,t} - (1+r_t) d_{i,t}\right] di$$

- Using loan market clearing, $\int_i d_{i,t} di = 0$:

$$C_t^E + K_{t+1} - (1-\delta)\, K_t = Y_t - \overbrace{(1-\alpha)\, Y_t}^{\overbrace{= w_t \int_i l_{i,t} di}^{=C_t^W}},$$

where

$$C_t^E = \int_i c_{i,t} di$$

- Then,

$$C_t^E + K_{t+1} - (1-\delta)\, K_t = \alpha Y_t. \tag{6}$$

Aggregates: Capital Accumulation

- Entrepreneur decision rule:

$$\begin{aligned} a_{i,t+1} &\equiv k_{i,t+1} - d_{i,t+1} \\ &= \beta \left[y_{i,t} - w_t l_{i,t} + (1-\delta) k_{i,t} - (1+r_t) d_{i,t} \right] \end{aligned}$$

- Integrating over all entrepreneurs (using $\int_i d_{i,t} di = 0$):

$$K_{t+1} = \beta \left[\alpha Y_t + (1-\delta) K_t \right] \tag{7}$$

- Note: K_{t+1} is not a direct function of θ_t.
  - If θ_t falls, then borrowing/lending drops and a lower r_{t+1} encourages unproductive entrepreneurs to buy capital using their own resources, rather than make loans to more productive entrepreneurs so that they can buy capital.
  - That is, leverage falls:

$$\frac{1}{1-\theta_t}.$$

Aggregates: Consumption Euler Equation

- Interestingly, aggregate entrepreneurial consumption satisfies Euler equation:

$$\begin{aligned}
\frac{C_{t+1}^E}{C_t^E} &= \frac{(1-\beta)\left[\alpha Y_{t+1} + (1-\delta) K_{t+1}\right]}{(1-\beta)\left[\alpha Y_t + (1-\delta) K_t\right]} \\
&= \beta \frac{\alpha Y_{t+1} + (1-\delta) K_{t+1}}{K_{t+1}} \\
&= \beta \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right]
\end{aligned}$$

- But,
  - does not hold for aggregate consumption, $C_t = C_t^W + C_t^E$.
  - does not hold relative to the interest rate.

Equilibrium

- Seven variables:

$$L_t, w_t, C_t^E, Y_t, K_{t+1}, \bar{z}_t, Z_t.$$

- Seven equations: (1), (2), (3), (4),(5), (6), (7).
- Exogenous variables:

$$K_1, \theta_0, \theta_1, \theta_2, ..., \theta_T$$

Equilibrium Computation

- Responses to exogenous variables:
  - For $t = 1, 2, ..., T$, $\bar{z}_t = \Psi^{-1}(\theta_{t-1})$ using (2); $Z_t \equiv (E[z|z > \bar{z}_t])^{\alpha}$ using (4),
  - Using (1) and (5) for L_t and w_t; (3) for Y_t; (7) for K_{t+1}; and (6) for C_t^E:

$$
\begin{aligned}
L_t &= \left[(1-\alpha) Z_t K_t^{\alpha}\right]^{\frac{1-\sigma}{\chi+\sigma+(1-\sigma)\alpha}} \\
w_t &= L_t^{\frac{\chi+\sigma}{1-\sigma}} \\
Y_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} \\
K_{t+1} &= \beta\left[\alpha Y_t + (1-\delta) K_t\right] \\
C_t^E &= (1-\beta)\left[\alpha Y_t + (1-\delta) K_t\right]
\end{aligned}
$$

sequentially, for $t = 1, 2, 3, ..., T$.

Equilibrium Computation

- Other variables: interest rate and profits for $t = 1, 2, ..., T$:

$$\pi_t = \alpha \left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}}$$
$$1 + r_t = \pi_t \bar{z}_t + 1 - \delta$$

- Pareto distribution:

$$\psi(z) = \eta z^{-(\eta+1)},\ \eta = 2.1739,\ 1 \leq z$$
$$\Psi(\bar{z}) = 1 - \bar{z}^{-\eta},\ Ez = \frac{\eta}{\eta - 1} = 1.85.$$

Parameter Values and Steady State

- Other parameters:

$$\alpha = 0.36, \delta = 0.10, \beta = 0.97, \chi = 1, \sigma = 0.9.$$

- Steady state, with $\theta = \frac{2}{3}$:

$$\begin{aligned} Y &= 3.45, K = 9.50, L = 1.04, C = 2.50, \\ w &= 2.12, \bar{z} = 1.66, Z = 1.50, \\ Z^{\frac{1}{\alpha}} &= E\left[z|z > \bar{z}\right] = 3.07, 1 + r = 0.97, \\ C^E/C &= 0.12, C^W/C = 0.88, \end{aligned}$$

$()$after rounding.

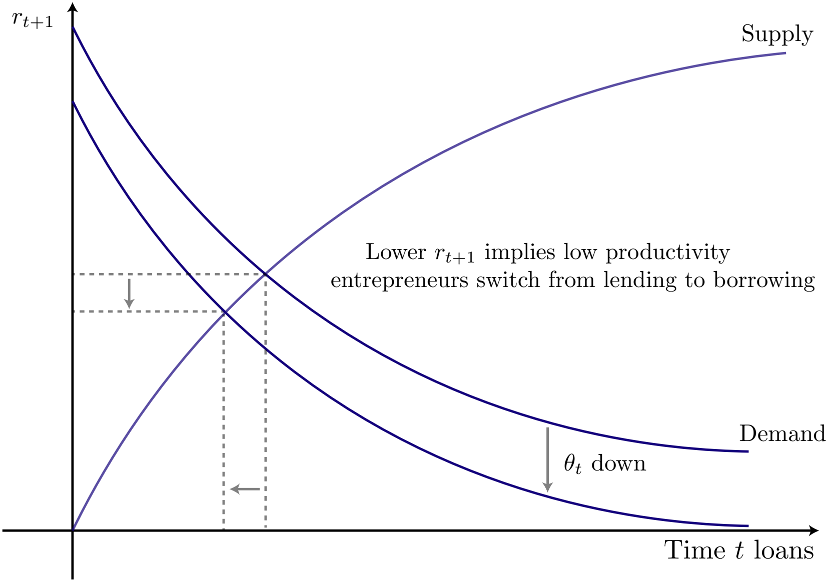
Tighter Lending Standards: θ_t down

- 'MIT shock'
  - economy in a steady state, $t = -\infty, ..., 1, 2$, and expected to remain there.
  - In $t = 3$, θ_3 drops unexpectedly from 0.67 to 0.60, and gradually returns to its steady state level:
    - $\theta_3 = \theta \times 0.9$, $\theta_t = (1 - \rho)\,\theta + \rho\theta_{t-1}$, for $t = 4, 5, ...$
    - $\rho = 0.8.$

Immediate Impact of Negative θ_t Shock

- Period $t = 3$ impact of shock:
  - Deleveraging associated with drop in θ_3 reduces demand for debt by each investing entrepreneur, driving down period $t = 3$ interest rate, r_4.
  - Marginally productive firms which previously were lending, switch to borrowing and making low-return investments with the drop in r_4.
    - No impact on total investment in period $t = 3$, as the cut-back by high productivity entrepreneurs is replaced by expanded investment by lower productivity entrepreneurs.
    - No impact on consumption, wages, etc., in period $t = 3$.

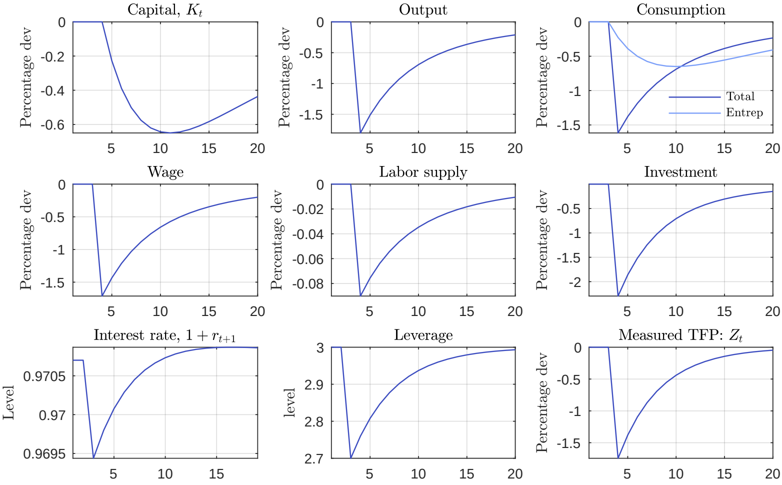
Immediate Impact of Negative θ_t Shock



Dynamic Effects of Drop in θ_t

- The cut in leverage by highly productive, but collateral-poor, firms is the trigger for the over 1.8 percent drop in TFP in period $t = 4$.
  - Until the drop in capital is more substantial, by say period $t = 20$, the drop in TFP is the main factor driving GDP down.
- Total consumption drops substantially, driven by the drop in income of hand-to-mouth workers, who consume $2/3$ of GDP.
  - Entrepreneurial consumption, directly related to GDP, also drops.
- Investment drops by over 2 percent.
- Employment drops by (a modest) 0.1 percent.

Response to Collateral Constraint Shock



Conclusion

- Buera-Moll model gives a flavor of the sort of analysis one can do with heterogeneous agent models with balance sheet constraints.
  - Illustrates the value of simple models for gaining intuition.
- Model provides an ‘endogenous theory of TFP’.
  - Stems from poor allocation of resources due to frictions in financial market.
  - See also Song-Storesletten-Zilibotti (AER2011).
- Deleveraging shock gets a surprising number of things right, but
  - how important was deleveraging per se, for the crisis?
  - what is the ‘deleveraging shock a stand-in for?’