Techniques of Empirical Econometrics

Oscar Jordà
UC Davis, FRB San Francisco

Karel Mertens
FRB Dallas, CEPR

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Overview

Dynamic Causal Effects/Structural Impulse Repsponse

Dynamic Causal Effect

The dynamic causal effect of a unit intervention in $\epsilon_{j,t} \in \epsilon_t$ on z_{t+h} is $E[z_{t+h}|\epsilon_{j,t} = 1, \epsilon_{t-1}, ...] - E[z_{t+h}|\epsilon_{j,t} = 0, \epsilon_{t-1}, ...]$

Also known as **'structural' impulse response function (IRF) coefficients** and equal to $\partial z_{t+h}/\partial \epsilon_{j,t}$ for $h = 0, 1, ...$ in linear models.

The structural impulse response coefficients of z_t to shock $\epsilon_{j,t}$ at horizon h are the elements in the j-th column of M_h in the SMA(∞) representation

$$z_t = M(L)\epsilon_t = \sum_{h=0}^{\infty} M_h \epsilon_{t-h}$$

The SMA(∞) representation contains the dynamic causal effects/structural impulse responses to all shocks ϵ_t

Estimating Structural Impulse Responses

Shock realizations $\epsilon_{j,t}$ are observed:

- **Distributed Lag Model**: $z_t = \sum_{h=0}^{H-1} M_h^j \epsilon_{j,t-h} + w_t$
- **Local Projection**: $z_{t+h} = M_h^j \epsilon_{j,t} + w_{h,t}$

(Partial) Fundamentalness and known $M_0^j = \mathcal{D}_j$ (j-th column in impact matrix $\mathcal{D}$)

- **VAR** Model: $B(L)z_t = u_t \Rightarrow M_h^j = G_h \mathcal{D}_j, \quad G(L) = B(L)^{-1}$
- **Local Projections**: $z_{t+h} = G_h z_t + \sum_{i=1}^{\infty} \delta_i z_{t-i} + w_{h,t} \quad , \quad M_h^j = G_h \mathcal{D}_j$

Direct Measurement of Shocks

One option is to obtain direct measures of an economic 'shock' $\epsilon_{j,t}$

Relevance and Exogeneity Assumptions

Suppose we observe a scalar m_t satisfying

$$E[m_t \epsilon_{j,t}] = \phi \neq 0 \qquad \text{(A1 Relevance)}$$

$$E[m_t \epsilon_{-j,t}] = 0 \qquad \text{(A2 Contemporaneous Exogeneity)}$$

$$E[m_t \epsilon_s] = 0 \text{ for all } s < t \qquad \text{(A3 Lag Exogeneity)}$$

$$E[m_t \epsilon_s] = 0 \text{ for all } s > t \qquad \text{(A4 Lead Exogeneity)}$$

Note: there is no assumption of (partial) fundamentalness of $\epsilon_{j,t}$ for z_t

Direct Measurement of Shocks

Distributed Lag Projection on m_t

If $z_t = \sum_{h=0}^{\infty} M_h \epsilon_{t-h}$ and A1-A4 hold, the distributed lag projection:

$$z_t = \sum_{h=0}^{H-1} \beta_h^j m_{t-h} + w_t$$

yields $\beta_h^j = \phi G_h \mathcal{D}_j = \phi M_h^j$

The projection coefficients are the impulse responses to $\epsilon_{j,t}$ up to scale ϕ

The Scale Factor ϕ

Ideally, the shock measure m_t has an interpretable scale

It is common to rescale the impulse response estimates to imply a fixed impact on one of the variables in z_t

However, this amounts to an instrumental variables procedure that scales all estimates by a random variable in finite samples, see later in this Section

This can create problems in small samples (weak instrument bias and size distortions), see later in Section 5.

Examples of Direct Shock Measures

Examples of direct shock measures (including event dummies)

- Narrative tax shocks — Romer and Romer (2010), Cloyne (2013)
- Narrative tax news shocks — Mertens and Ravn (2012)
- Military spending changes — Ramey and Shapiro (1998), Edelberg, Eichenbaum, and Fisher (1999)
- Military spending news — Ramey (2011)
- Narrative monetary policy events — Friedman and Schwartz (1963), Romer and Romer (1989)
- High frequency monetary surprises — Kuttner (2001) . . . Bauer and Swanson (2022)
- Oil shocks — Hamilton (2003), Kilian (2008)
- Financial shocks — Gilchrist and Zakrajšek (2012)
- Housing credit policy shocks — Fieldhouse, Mertens, and Ravn (2018)
- Uncertainty Shocks — Bloom (2009)

Example: Romer and Romer's Narrative Tax Shocks

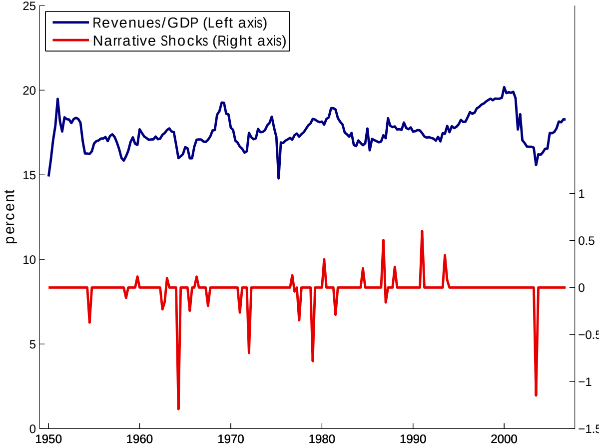
Romer and Romer (2010) classify US postwar tax reforms according to:

1. **size** as measured by the implied tax liability change
2. **motivation** (narrative identification)
   - Endogenous; Countercyclical: "A tax action designed to return output growth to normal"
   - Endogenous; Spending: "Tax change motivated by a change in government spending" both correlated with current economic conditions
   - Exogenous; Long-Run: "A tax change motivated by fairness, efficiency, incentives, belief in smaller government"
   - Exogenous; Deficit: "A tax change designed to reduce an inherited budget deficit"
3. The **dates** at which:
   - the tax act was signed by the President
   - the tax change was implemented

Retain 'unanticipated' shocks , cfr. Mertens and Ravn (2012)

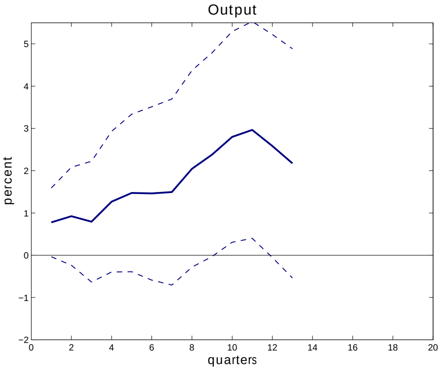
Divide tax liability changes by (lagged) GDP.

Example: Romer and Romer's Narrative Tax Shocks



Example: Romer and Romer's Narrative Tax Shocks

Unit Innovation in τ_t (Change in Tax Liabilities in % of GDP)



$$\Delta Y_t = \beta_0 \tau_t + \beta_1 \tau_{t-1} + \ldots + \beta_h \tau_{t-h} + w_t$$

Download the code here

2. Identification of Dynamic Causal Effects

2.1 Direct Measurement of Shocks

2.2 Covariance Restrictions

2.3 Instrumental Variables

2.4 Higher Order Moments

Covariance Restrictions: The Identification Problem

Under fundamentalness

$$E[u_t u_t'] = \Sigma_u = \mathrm{Var}(\mathcal{D}\epsilon_t) = \mathcal{D}\mathrm{Var}(\epsilon_t)\mathcal{D}' = \mathcal{D}\mathcal{D}'$$

Symmetric positive definiteness of Σ_u provides $N_z \times (N_z + 1)/2$ restrictions on the N_z^2 elements of $\mathcal{D}$. Not sufficient to uncover any of the columns of $\mathcal{D}$.

Identification Problem

Suppose ϵ is orthonormal white noise and $u_t = \mathcal{D}\epsilon_t$, then

$$\Sigma_u = \mathcal{D}\mathcal{D}' = \mathcal{D}QQ'\mathcal{D}' = \mathcal{D}^*\mathcal{D}^*$$

where Q is any orthogonal matrix ($QQ' = \mathcal{I}_{N_z}$).
$\mathcal{D}$ and $\mathcal{D}^* = \mathcal{D}Q$ are observationally equivalent

Conditions for (Local) Identification

- **Order condition**: We need at least $N_z \times (N_z - 1)/2$ covariance restrictions to identify all N_z^2 elements of $\mathcal{D}$
- **Rank condition**: The derivative w.r.t vec($\mathcal{D}$) of the system of identifying equations needs to have full rank

Also need a signing convention for the diagonal elements of $\mathcal{D}$

Covariance Restrictions

Various combinations of covariance restrictions can be imposed on

- the **impact matrix** $\mathcal{D}$,
  i.e. the contemporaneous response to shocks
- the **inverse impact matrix** $\mathcal{D}^{-1}$,
  i.e. the linear contemporaneous relationship between the variables in z_t.
- the **horizon** ***h*****-impulse response coefficients** $M_h\mathcal{D}$,
  i.e. the response after h periods
- the **infinite horizon (cumulative) impulse responses** $M(1)\mathcal{D}$, i.e. the long run (cumulative) response to the shock

Subject to order and rank conditions for (local/global) identification

Rubio-Ramírez, Waggoner, and Zha (2010)

Note, this generally involves solving a system of nonlinear equations.

Common Covariance Restrictions

- Recursivity
- Block Recursivity and Partial Identification
- Nonrecursive Short-Run Restrictions
- Long Run Restrictions
- Sign Restrictions
- Max Share Restrictions

Recursive Identification Scheme

Recursive Identification Sims (1980)

Zero (or timing) restrictions on the impact matrix, lower triangular $\mathcal{D}$:

$$\mathcal{D} = \begin{bmatrix} d_{11} & 0 & \dots & 0 \\ d_{21} & d_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \dots \\ d_{n1} & \dots & \dots & d_{nn} \end{bmatrix}$$

Adds $\frac{N_z \times (N_z - 1)}{2}$ restrictions such that all N_z^2 elements of $\mathcal{D}$ are identified.

Easy computation through the lower triangular factorization (**Cholesky decomposition**) of Σ_u, which factors a positive semi-definite matrix P into the product of a lower triangular matrices and its transpose, $\Sigma_u = \mathcal{D}\mathcal{D}'$.

Block Recursive Schemes and Partial Identification

Partition $z_t = [z_{1,t},\ z_{2,t},\ z_{3,t}]'$ and $\epsilon_t = [\epsilon_{1,t},\ \epsilon_{2,t},\ \epsilon_{3,t}]$ and consider the **lower block triangular** matrix

$$\mathcal{D}_b = \begin{bmatrix} \underset{n_1 \times n_1}{d_{11}} & \underset{n_1 \times 1}{0} & \underset{n_1 \times n_2}{0} \\ \underset{1 \times n_1}{d_{21}} & \underset{1 \times 1}{d_{22}} & \underset{1 \times n_2}{0} \\ \underset{n_2 \times n_1}{d_{31}} & \underset{n_2 \times 1}{d_{32}} & \underset{n_2 \times n_2}{d_{33}} \end{bmatrix} \quad N_z = n_1 + 1 + n_2$$

Block Recursive Partial Identification Christiano, Eichenbaum, and Evans (1999)

All $\mathcal{D}_b$ satisfying $\Sigma_u = \mathcal{D}_b \mathcal{D}_b'$ have the same elements in the $n_1 + 1$-th column

Block recursive structure ($n_1 + n_2 + n_1 n_2$ zero restrictions) suffices to identify the $n_1 + 1$-th column of $\mathcal{D}_b$

Wlg assume d_{11} and d_{33} are lower triangular and take the $n_1 + 1$-th column of the Choleski decomposition of Σ_u

Example: Christiano, Eichenbaum, and Evans (2005) Monetary SVAR

Block Recursive SVAR:

$$B(L)\begin{bmatrix} z_{1,t} \\ ffr_t \\ z_{3,t} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & d_{33} \end{bmatrix}\begin{bmatrix} \epsilon_{1t} \\ \epsilon_t^{ffr} \\ \epsilon_{3t} \end{bmatrix}$$

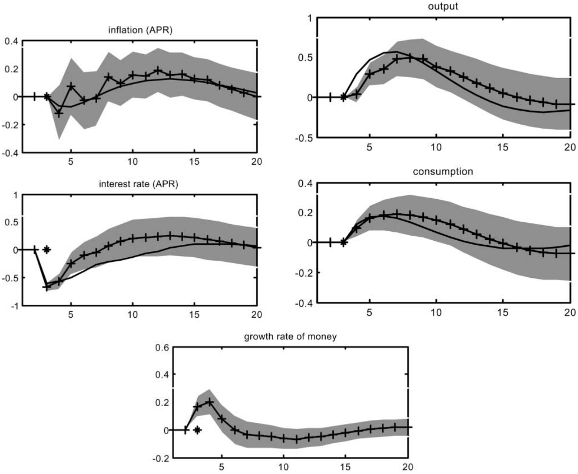
$\mathbf{z_{1,t}}$: gross domestic product, consumption, the GDP deflator, investment, real wage, and labor productivity (log levels)
$\mathbf{ffr_t}$: effective federal funds rate
$\mathbf{z_{3,t}}$: real profits and the growth rate of M2 money supply

Quarterly data 1965Q3-1995Q3

Key assumptions:

- ffr_t equation is an interest rate (Taylor) rule, ϵ_t^{ffr} are suprise deviations
- variables in $z_{1,t}$ do not respond contemporaneously to a monetary policy shock ϵ_t^{ffr}
- Monetary policy does not respond contemporaneously to variables in $z_{3,t}$

Example: Christiano, Eichenbaum, and Evans (2005) Monetary SVAR



Nonrecursive Covariance Restrictions

Recursivity assumptions on the impact matrix $\mathcal{D}$ often lack theoretical justification

For some variables, a recursive causal ordering is implausible even with higher frequency data

For example, where to put indicators of financial conditions in the CEE system? Before or after ffr_t?

Nonrecursive Short Run Restrictions

Non-recursive restrictions are general restrictions on A and B in

$$Au_t = B\epsilon_t \quad , \quad \mathcal{D} = A^{-1}B$$

where none of the elements in $\mathcal{D}$ are necessarily zeros

Example: (Hausman and Taylor (1983))

$$u_{1t} = \beta_{12}u_{2t} + e_{1t} \quad , \quad u_{2t} = \beta_{23}u_{3t} + e_{2t} \quad , \quad u_{3t} = \beta_{31}u_{1t} + e_{3t}$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} = \frac{1}{1-\beta_{12}\beta_{23}\beta_{31}} \begin{bmatrix} 1 & \beta_{12} & \beta_{12}\beta_{23} \\ \beta_{23}\beta_{31} & 1 & \beta_{23} \\ \beta_{31} & \beta_{12}\beta_{31} & 1 \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}, Var(e_t) = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Example: Blanchard and Perotti (2002) Fiscal Policy Shocks

Observables $z_t = [T_t, G_t, Y_t]'$, quarterly sample 1950Q1-2006Q4

T_t : Log Real Federal Tax Revenues per capita
G_t : Log Real Federal Government Spending on Final Goods per capita
Y_t : Log Real GDP per capita

Estimate of Σ_u provides six independent restrictions, need three more.

Blanchard and Perotti (2002) consider

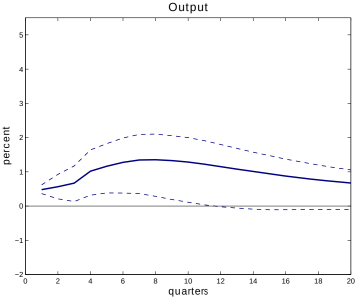
$$\begin{aligned} u_t^T &= \theta_G \sigma_G \epsilon_t^G + \theta_Y u_t^Y + \sigma_T \epsilon_t^T \\ u_t^G &= \gamma_T \sigma_T \epsilon_t^T + \gamma_Y u_t^Y + \sigma_G \epsilon_t^G \\ u_t^Y &= \zeta_T u_t^T + \zeta_G u_t^G + \sigma_Y \epsilon_t^Y \end{aligned}$$

and impose

- $\gamma_Y = \gamma_T = 0$ based on decision and recognition lags
- $\theta_Y = 2.08$ based on outside estimates

Example: Blanchard and Perotti (2002) Tax Shocks

Tax shock of 1% of GDP



Note: much smaller output effects than Romer and Romer (2010)

Download the code here

Long Run Restrictions

Suppose $z_t = \Delta x_t$ are growth rates, then the long-run impact of ϵ_t on levels x_t is

$$\sum_{h=0}^{\infty} M_h = M(1) = M_0 + M_1 + M_2 + M_3 + \ldots$$

$M(1)$ is the cumulative impact of ϵ_t on z_t at $h = \infty$, and therefore the permanent level effect on x_t

Long Run Covariance Restrictions Blanchard and Quah (1989)

Let $B(L)z_t = \mathcal{D}\epsilon_t$, $G(L) = B(L)^{-1}$. Restrictions on $M(1)$ can identify $\mathcal{D}$:

$$M(1) = G(1)\mathcal{D} \Rightarrow \mathcal{D} = G(1)^{-1}M(1) = B(1)M(1)$$

Long run (zero) restrictions are often theoretically more appealing than zero restrictions on impact matrix $\mathcal{D}$

Recursivity of $M(1)$ easily implemented by Cholesky decomposition of $G(1)\Sigma_u G(1)'$

Examples: Blanchard and Quah (1989) (supply shocks), Galí (1999) (technology shocks), Beaudry and Portier (2006) (technology news shocks)

Example: Blanchard and Quah (1989) SVAR

Assumption: supply shocks explain all output movements in the long run

$$\begin{bmatrix} \Delta\text{output}_t \\ \text{unempl}_t \end{bmatrix} = M(L) \begin{bmatrix} \epsilon_t^s \\ \epsilon_t^d \end{bmatrix}, \quad M(1) = \begin{bmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{bmatrix}$$

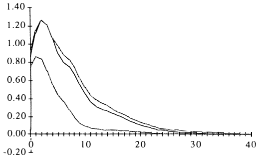


FIGURE 3. OUTPUT RESPONSE TO DEMAND

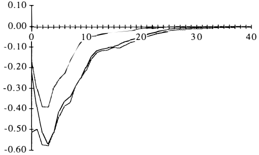


FIGURE 5. UNEMPLOYMENT RESPONSE TO DEMAND

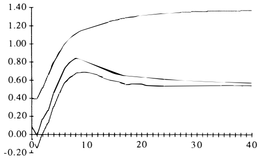


FIGURE 4. OUTPUT RESPONSE TO SUPPLY

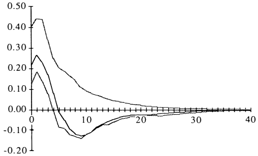


FIGURE 6. UNEMPLOYMENT RESPONSE TO SUPPLY

Max Share Restrictions

Long-run restrictions can be unreliable in realistic samples.

Chari, Kehoe, and McGrattan (2008), Kascha and Mertens (2009)

An alternative approach to identifying $\mathcal{D}_j$ is to require that $\epsilon_{j,t}$ explains the largest possible fraction of the FEV of variable $z_{i,t} \in z_t$ at some finite horizon h

Faust (1998), Uhlig (2004), Barsky and Sims (2011)

Forecast Error Variance (FEV) Decomposition

The share of the FEV for $z_{i,t}$ at horizon h explained by $\epsilon_{j,t}$ is

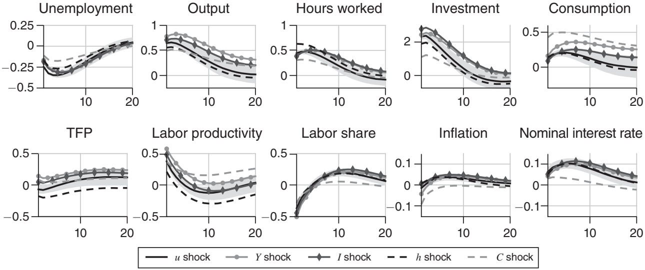
$$\Omega_h = \frac{\sum_{n=0}^{h}(m_n^j(i))^2}{\sum_{l=1}^{N_z}\sum_{n=0}^{h}(m_n^l(i))^2}$$

where $m_h^j(i)$ is the i-th element in M_h^j

Angeletos, Collard, and Dellas (2020) similarly choose $\mathcal{D}_j$ to maximize the contribution of $\epsilon_{j,t}$ to the spectral density $z_{i,t}$ over a frequency band $[\underline{\omega}, \overline{\omega}]$.

Example: Angeletos, Collard, and Dellas (2020) Business Cycle Anatomy

Assumption: shock ϵ_{jt} contains the maximal share of all the information in the data about the volatility of macroeconomic variable j at business cycle frequencies (6 to 32 quarters in the time domain) .



All shocks have very similar impulse responses, suggesting a single 'Main Business Cycle Shock'.

Sign Restrictions

$$\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}$$

Covariance restrictions can be inequalities on the elements of the impact matrix $\mathcal{D}$, e.g. $d_{11}, d_{21}, d_{22} > 0$, $d_{12} < 0$

Inequalities can also be imposed on M_h for any h and across different h

Among all D that satisfy $DD' = \Sigma_u$, only admit those that satisfy the inequality restrictions

The estimates of M_h are no longer points, but sets containing all $\tilde{M}_h$'s generated by admissable D's.

Example: Mountford and Uhlig (2009) Tax Shocks

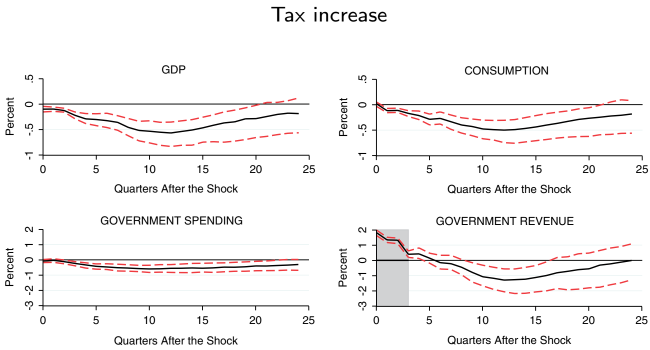
Table I. Identifying sign restrictions

	Gov. revenue	Gov. spending	GDP, cons, non-res.inv.	Interest rate	Adjusted reserves	Prices
Non-fiscal shocks						
Business cycle	+		+			
Monetary policy				+	−	−
Basic fiscal policy shocks						
Government revenue	+					
Government spending		+				

This table shows the sign restrictions on the impulse responses for each identified shock. 'Cons' stands for private consumption and 'Non-res. inv.' stands for non-residential investment. A '+' means that the impulse response of the variable in question is restricted to be positive for four quarters following the shock, including the quarter of impact. Likewise, a '−' indicates a negative response. A blank entry indicates that no restrictions have been imposed.

A tax shock is identified as a shock that is orthogonal to the business cycle and monetary policy shock and where government revenue rises for a year after the shock.

Example: Mountford and Uhlig (2009) Tax Shocks



Implied output effects are very large

Identification with Instrumental Variables

Equivalence Between Covariance Restrictions and IV Hausman and Taylor (1983)

If a linear system of equations is identifiable, covariance restrictions cause residuals to behave as instrumental variables

Covariance restrictions generate **internal instruments**, and the elements of $\mathcal{D}$ can also be obtained by IV methods.

IV estimation:

$$y_t = \beta x_t + w_t \quad , \quad E[x_t w_t] \neq 0$$

Let m_t be a valid instrument for x_t satisfying

$$E[m_t x_t] \neq 0 \qquad \text{(relevance)}$$
$$E[m_t w_t] = 0 \qquad \text{(exogeneity)}$$

Two Stage Least Squares (2SLS):

1. **First Stage:** Regress x_t on m_t and obtain fitted values $\hat{x}_t$
2. **Second Stage:** Regress y_t on $\hat{x}_t$ to obtain consistent estimate of β

Example: (Block) Recursive Identification

u_{2t} is scalar, u_{1t} and u_{3t} are of arbitrary dimension

$$\begin{aligned} u_{1,t} &= a_{11} e_{1,t} \\ u_{2,t} &= a_{21} u_{1,t} + e_{2,t} \\ u_{3,t} &= a_{31} u_{1,t} + a_{32} u_{2,t} + a_{33} e_{3,t} \end{aligned}$$

1. Project z_t on z_{t-1}, z_{t-2} to obtain prediction errors u_t
2. Project $u_{2,t}$ on $u_{1,t}$ and obtain $e_{2,t} = u_{2,t} - a_{21} u_{1,t}$
3. Project u_t on $u_{2,t}$ using $e_{2,t}$ as an instrument to obtain D_2

The impact response to a unit innovation in $e_{2,t}$ is given by D_2

Multiply by std($e_{2,t}$) to get the impact $\mathcal{D}_2$ of a one std shock

Note projecting u_t directly on $e_{2,t}$ gives the same answer

Example: Blanchard and Perotti (2002) Fiscal Policy Shocks

$$
\begin{aligned}
u_t^T &= \theta_G \sigma_G e_t^G + \theta_Y u_t^Y + \sigma_T e_t^T \\
u_t^G &= \gamma_T \sigma_T e_t^T + \gamma_Y u_t^Y + \sigma_G e_t^G \\
u_t^Y &= \zeta_T u_t^T + \zeta_G u_t^G + \sigma_Y e_t^Y
\end{aligned}
$$

Identification restrictions:

- $\gamma_Y = \gamma_T = 0$ based on decision and recognition lags
- $\theta_Y = 2.08$ based on outside estimates

The other 6 unknown parameters can be obtained as follows:

- σ_G is the std of u_t^G
- Project $u_t^T - 2.08 u_t^Y$ on u_t^G to identify θ_G and σ_T
- $u_t^T - 2.08 u_t^Y$ and u_t^G are valid instruments for identifying ζ_T, ζ_G and σ_Y in $u_t^Y = \zeta_T u_t^T + \zeta_G u_t^G + \sigma_Y e_t^Y$

Example: Shapiro and Watson (1988)

Block-recursive long run restrictions:

$$\begin{bmatrix} \Delta \text{hours}_t \\ \Delta \text{output}_t \\ \Delta \pi_t \\ i - \pi_t \end{bmatrix} = M(L) \begin{bmatrix} \epsilon_t^{ls} \\ \epsilon_t^{tech} \\ \epsilon_t^{d1} \\ \epsilon_t^{d2} \end{bmatrix} \quad , \quad M(1) = \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

Demand shocks $\epsilon_t^{d1}, \epsilon_t^{d2}$ have no permanent effect on hours and output
Permanent technology shocks ϵ_t^{tech} have no permanent effect on hours

The recursive IV approach is analogous to before but applied to $\tilde{u}_t = G(1)u_t$.

Shapiro and Watson (1988) show a different IV implementation

Since $\mathcal{D} = B(1)M(1)$, define $B^c(L) = B(1)^{-1}B(L)$, and estimate the VAR

$$B^c(L)z_t = M(1)\epsilon_t = u_t^c$$

imposing the parameter constraints that $B^c(1) = \mathcal{I}$

In each equation, the parameter constraints free up one lag of the three contemporaneous variables as instrumental variables to estimate the off-diagonal coefficients in $B^c(0)$

Identification with Proxy Variables/External Instruments

Shocks identified with internal instruments/covariance restrictions often look unrelated to known historical events Rudebusch (1998)

'Known historical events' are in direct measures of shocks m_t

Can we incorporate m_t to help identify shocks?

Yes, use m_t as **external instruments** to generate covariance restrictions

Think of m_t as 'proxy' measures of unobserved structural shocks

Identification with proxies avoids (often implausible) short run exclusion restrictions

See Stock (2008), Stock and Watson (2012), Mertens and Ravn (2013)

Identification with Proxy Variables/External Instruments

External Instrument Validity

Suppose we have access to a (mean zero) scalar variable m_t satisfying

$$E[m_t \epsilon_{j,t}] = \phi \neq 0 \qquad \text{(A1 Relevance)}$$
$$E[m_t \epsilon_{-j,t}] = 0 \qquad \text{(A2 Contemporaneous Exogeneity)}$$

Identification (up to scale) in Projection with VAR Residuals

Fundamentalness and A1-A2 imply that $E[u_t m_t] = E[\mathcal{D}\epsilon_t m_t] = \phi\mathcal{D}_j$. Therefore the projection

$$u_t = \beta m_t + w_t$$

yields $\beta = \mathcal{D}_j \phi$

The projection coefficient is the impact response up to an (unknown) scale ϕ

Note that now lead/lag exogeneity assumptions are not required.
These are effectively replaced by the fundamentalness assumption

Identification with Proxy Variables/External Instruments

Identification (up to scale) in VARX Projection Paul (2020)

Under fundamentalness and A1-A2, the projection

$$z_t = \sum_{i=1}^{\infty} B_i z_{t-i} + \beta m_t + w_t$$

yields $\beta = \mathcal{D}_j \phi$

The projection coefficient β is the impact response up to an (unknown) scale ϕ

In finite samples, this is not equivalent to regressing VAR residuals $\hat{u}_t$ on m_t

Instead it is equivalent to regressing the VAR residuals $\hat{u}_t$ on $m_t^{\perp}$ where $m_t^{\perp}$ is the residual in the regression of m_t on $z_{t-1}, \ldots, z_{t-p}$ (Frisch-Waugh Theorem)

Identification with Proxy Variables/External Instruments

Without loss of generality, suppose that $j = 1$, i.e. the shock of interest is ordered first, and partition

$$\mathcal{D} = \begin{bmatrix} \underset{1\times 1}{d_{11}} & \underset{1\times(N_z-1)}{d_{12}} \\ \underset{(N_z-1)\times 1}{d_{21}} & \underset{(N_z-1)\times(N_z-1)}{d_{22}} \end{bmatrix}, \quad u_t = \begin{bmatrix} \underset{1\times 1}{u_{1,t}} \\ \underset{(N_z-1)\times 1}{u_{2,t}} \end{bmatrix}$$

Proxy SVAR Identification Mertens and Ravn (2013)

Under fundamentalness, the conditions in A1 and A2 provide $N_z - 1$ covariance restrictions that suffice to identify $\mathcal{D}_1 = [d_{11} \ d_{21}']'$

Since $E[u_{1,t}m_t] = \phi d_{11}$ and $E[u_{2,t}m_t] = \phi d_{21}$,

$$E[u_{2,t}m_t]/E[u_{1,t}m_t] = d_{21}/d_{11}$$

which identifies $\mathcal{D}_1$ up to the scalar d_{11}

The scalar d_{11} is pinned down by the restrictions provided by $\Sigma_u = \mathcal{D}\mathcal{D}'$
See Mertens and Ravn (2013) for the closed form solution.

Identification with Proxy Variables/External Instruments

$$E[u_{2,t} m_t]/E[u_{1,t} m_t] = d_{21}/d_{11}$$

This is the impulse response to a unit innovation in $z_{1,t}$ driven by $\epsilon_{1,t}$

This impulse is now on a specific scale, determined by the choice of $z_{1,t}$

In finite samples, the estimate of d_{21}/d_{11} is simply the 2SLS estimate of δ in $\hat{u}_{2,t} = \delta \hat{u}_{1,t} + v_t$ using m_t as an instrumental variable.

In population, the following is equivalent

$$E[u_{2,t} m_t^{\perp}]/E[u_{1t} m_t^{\perp}] = d_{21}/d_{11}$$

where $m_t^{\perp}$ is the residual in the projection of m_t on $z_{t-1}, z_{t-2}, \ldots$

In finite samples, the estimate of d_{21}/d_{11} is the 2SLS estimate of δ in $\hat{u}_{2,t} = \delta \hat{u}_{1,t} + v_t$ using $m_t^{\perp}$ as an instrumental variable.

Proxy SVARs are also referred to as SVAR-IV (Stock and Watson (2018))

Identification with Proxy Variables/External Instruments

Some Equivalence Results

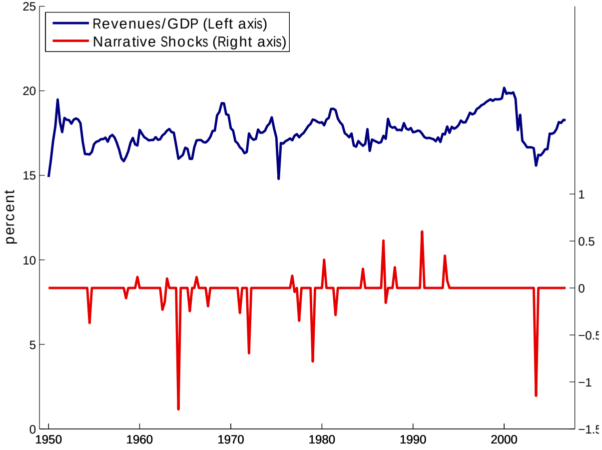
- Proxy SVAR identification of d_{21}/d_{11} with m_t is equivalent to regressing VAR residuals on m_t and rescaling the coefficients to normalize the impact on z_{1t}
- Proxy SVAR identification of d_{21}/d_{11} with $m_t^{\perp}$ is equivalent to OLS estimation of the VARX and rescaling the coefficients on m_t to normalize the impact on z_{1t}

The VAR residuals projection and the VARX projection are just the respective 'reduced form' representations

The rescaling step inevitably turns all impulse response estimators identified by proxies into instrumental variable estimators

Example: Mertens and Ravn (2014) Fiscal Policy Shocks

Recall the (unanticipated) Romer and Romer (2010) narrative tax shocks τ_t



Example: Mertens and Ravn (2014) Tax Shocks

Let's identify the parameters in the Blanchard and Perotti (2002) system using τ_t as a proxy for e_t^T

$$\begin{aligned} u_t^T &= \theta_G \sigma_G e_t^G + \theta_Y u_t^Y + \sigma_T e_t^T \\ u_t^G &= \gamma_T \sigma_T e_t^T + \gamma_Y u_t^Y + \sigma_G e_t^G \\ u_t^Y &= \zeta_T u_t^T + \zeta_G u_t^G + \sigma_Y e_t^Y \end{aligned}$$

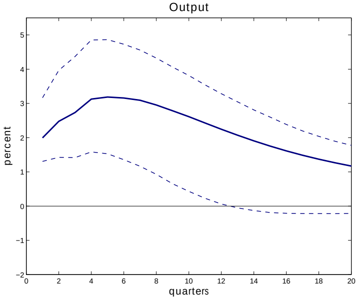
Three identification restrictions:

- $E[\tau_t e_t^G] = E[\tau_t e_t^Y] = 0$
- $\gamma_Y = 0$ based on decision and recognition lags (γ_T remains unrestricted)

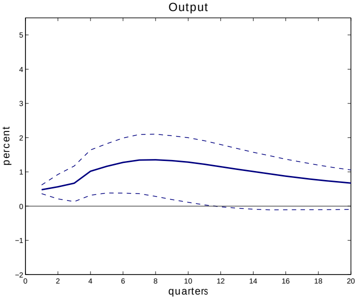
Example: Mertens and Ravn (2014) Tax Shocks

Cut in Tax Revenues of 1% of GDP

Proxy SVAR



BP SVAR



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What is the Difference Across the Identification Schemes?

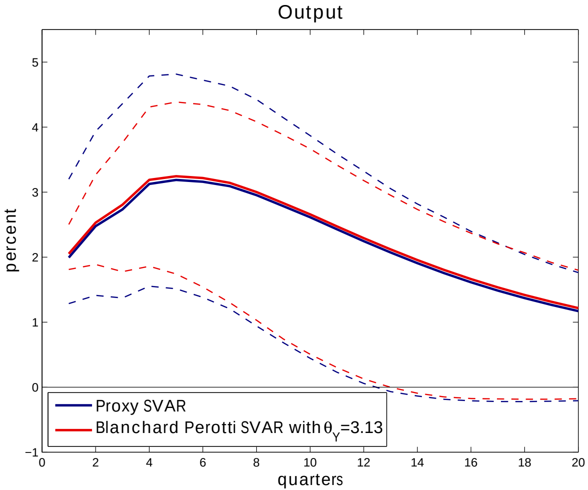
The elasticity of tax revenues wrt GDP

		Proxy SVAR	**Blanchard-Perotti SVAR**	
Equation		Benchmark	$\theta_Y = 2.08$	$\theta_Y = 3.13$
Tax Revenue	θ_G	-0.20	-0.06	-0.13
		$[-0.35, -0.07]$	$[-0.12, -0.03]$	$[-0.19, -0.09]$
	θ_Y	3.13	2.08	3.13
		[2.73, 3.55]	–	–
	$\sigma_T \times 100$	2.54	2.24	2.56
		[2.23, 2.62]	[2.04, 2.19]	[2.34, 2.51]
Spending	γ_T	0.06	0	0
		$[-0.06, 0.17]$	–	–
	γ_Y	0	0	0
		–	–	–
	$\sigma_G \times 100$	2.35	2.36	2.36
		[2.12, 2.30]	[2.13, 2.31]	[2.13, 2.31]
Output	ζ_T	-0.36	-0.08	-0.36
		$[-0.57, -0.24]$	$[-0.11, -0.06]$	$[-0.43, -0.31]$
	ζ_G	0.10	0.07	0.10
		[0.06, 0.13]	[0.06, 0.09]	[0.07, 0.12]
	$\sigma_Y \times 100$	1.54	0.97	1.54
		[1.21, 1.93]	[0.89, 0.98]	[1.37, 1.64]

Values in parenthesis are 95% percentiles computed using 10, 000 bootstrap replications.

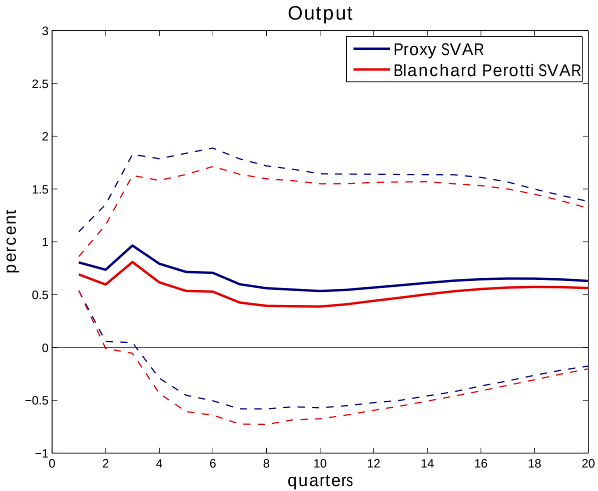
What is the Difference Across the Identification Schemes?

BP SVAR with $\theta_Y = 3.13$



Little Difference for Spending Shocks ϵ_t^G

Spending Increase of 1% if GDP

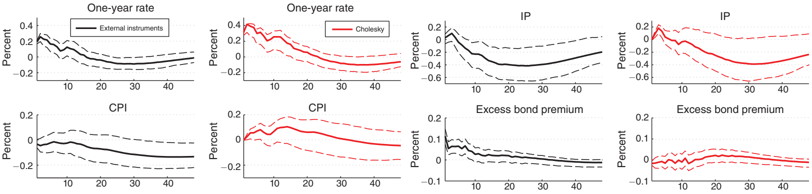


Example: Gertler and Karadi (2015) Monetary Policy Shocks

Block-recursive schemes are not plausible for many variables, e.g. financial indicators

Omitting financial indicators likely leads to a violation of fundamentalness

Gertler and Karadi (2015) use HF ffr futures surprises as proxies for MP shocks



Rate increase tightens financial conditions in the Proxy SVAR, but not in recursive scheme with credit spreads order below ffr_t

Multiple External Instruments

Multiple External Instruments

Let $\epsilon_{j,t}$ be a $K \times 1$ subvector of shocks, and m_t a $K \times 1$ vector of external instruments

$$E[m_t \epsilon'_{j,t}] = \Phi \tag{A1m}$$

$$E[m_t \epsilon'_{-j,t}] = 0 \tag{A2m}$$

where Φ is $K \times K$, unknown and nonsingular, but **not necessarily diagonal**.

Each element in m_t is potentially correlated with multiple shocks

Partition $u_t = \begin{bmatrix} \underset{K\times 1}{u_{1t}} \\ \underset{(N_z-K)\times 1}{u_{2t}} \end{bmatrix}$, $\epsilon_t = \begin{bmatrix} \underset{K\times 1}{\epsilon_{1t}} \\ \underset{(N_z-K)\times 1}{\epsilon_{2t}} \end{bmatrix}$,

ϵ_{1t} are the shocks of interest.

Multiple External Instruments

Proxy SVAR Identification with Multiple Proxies Mertens and Ravn (2013)

Under fundamentalness, A1m and A2m provide $(N_z - K) \times K$ covariance restrictions that identify the first K columns of $\mathcal{D}$ up to a $K \times K$ rotation

Partition $\mathcal{D} = \begin{bmatrix} \underset{k\times k}{\mathcal{D}_{11}} & \underset{k\times n-k}{\mathcal{D}_{12}} \\ \underset{n-k\times k}{\mathcal{D}_{21}} & \underset{n-k\times n-k}{\mathcal{D}_{22}} \end{bmatrix}$, $\mathcal{D}_1 = \begin{bmatrix} \underset{k\times k}{\mathcal{D}_{11}} \\ \underset{n-k\times k}{\mathcal{D}_{21}} \end{bmatrix}$

Assumptions A1m/A2m imply $N_z \times K$ conditions

$$\Phi\mathcal{D}_1' = E[m_t u_t']$$

from which we extract $(N_z - K) \times K$ covariance restrictions

$$\mathcal{D}_{21} = (E[m_t u_{1t}']^{-1} E[m_t u_{2t}'])' \mathcal{D}_{11}$$

that can be used for identifying the first K columns of $\mathcal{D}$

These restrictions identify $\mathcal{D}_{21}\mathcal{D}_{11}{}^{-1}$

An additional $K(K-1)/2$ restrictions are needed to fully identify $\mathcal{D}_1$

Multiple External Instruments

We still need to find the remaining $K(K-1)/2$ restrictions required to identify $\mathcal{D}_1$ and extract ϵ_{1t}

In many applications, however, meaningful impulse responses do not require further restrictions, even if the shocks are not individually identified

Suppose $p_{1,t}$ and $p_{2,t}$ are two scalar policy instruments that are set according to the feedback rules

$$\begin{aligned} p_{1,t} &= \beta_{12} p_{2,t} + \gamma_1' u_t^y + \sigma_1 \epsilon_{1,t}^p \\ p_{2,t} &= \beta_{21} p_{1,t} + \gamma_2' u_t^y + \sigma_2 \epsilon_{2,t}^p \end{aligned}$$

Let $u_t^y = \xi_1 p_{1,t} + \xi_2 p_{2,t} + C_y \epsilon_t^y$ and $u_t = [p_{1,t} \ p_{2,t} \ (u_t^y)']'$.

We have $m_t = [m_{1,t} \ m_{2,t}]$ with

$$\begin{aligned} E[m_{i,t}\epsilon_{j,t}^p] &\neq 0 \text{ for } i,j = 1,2 \\ E[m_t(\epsilon_t^y)'] &= \mathbf{0} \end{aligned}$$

Multiple External Instruments

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} = \underbrace{\frac{1}{1-\beta_{12}\beta_{21}} \begin{bmatrix} \gamma_1' + \beta_{12}\gamma_2' \\ \beta_{21}\gamma_1' + \gamma_2' \end{bmatrix}}_{\alpha} u_t^y + \underbrace{\frac{1}{1-\beta_{12}\beta_{21}} \begin{bmatrix} \sigma_1 & \beta_{12}\sigma_2 \\ \beta_{21}\sigma_1 & \sigma_2 \end{bmatrix}}_{C_p} \begin{bmatrix} \epsilon_{1,t}^p \\ \epsilon_{2,t}^p \end{bmatrix}$$

The proxies m_t identify α, i.e. all the endogenous feedback from u_t^y to the policy instruments, and $\Sigma_p = C_p C_p'$

The proxies m_t are one restriction short of identifying the four unknowns in C_p. The mutual feedback across policy instruments β_{12} and β_{21} is not identified

Consider the upper and lower triangular factorizations of Σ_p

$$\Sigma_p = \eta^U(\eta^U)' \ , \ \eta^U = \begin{bmatrix} \eta_{11}^U & \eta_{12}^U \\ 0 & \eta_{22}^U \end{bmatrix} \ , \ e_t^U = (\eta^U)'(C_p')^{-1} \begin{bmatrix} \epsilon_{1,t}^p \\ \epsilon_{2,t}^p \end{bmatrix}$$

$$\Sigma_p = \eta^L(\eta^L)' \ , \ \eta^L = \begin{bmatrix} \eta_{11}^L & 0 \\ \eta_{21}^L & \eta_{22}^L \end{bmatrix} \ , \ e_t^L = (\eta^L)'(C_p')^{-1} \begin{bmatrix} \epsilon_{1,t}^p \\ \epsilon_{2,t}^p \end{bmatrix}$$

Multiple External Instruments

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} = \alpha u_t^y + \begin{bmatrix} \eta_{11}^U & \eta_{12}^U \\ 0 & \eta_{22}^U \end{bmatrix} \begin{bmatrix} e_{1,t}^U \\ e_{2,t}^U \end{bmatrix}$$

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} = \alpha u_t^y + \begin{bmatrix} \eta_{11}^L & 0 \\ \eta_{21}^L & \eta_{22}^L \end{bmatrix} \begin{bmatrix} e_{1,t}^L \\ e_{2,t}^L \end{bmatrix}$$

$e_{1,t}^U$ is the linear combination of $\epsilon_{1,t}^p$ and $\epsilon_{2,t}^p$ such that there is an exogenous innovation in $p_{1,t}$ but not in $p_{2,t}$

$e_{2,t}^L$ is the linear combination of $\epsilon_{1,t}^p$ and $\epsilon_{2,t}^p$ such that there is an exogenous innovation in $p_{2,t}$ but not in $p_{1,t}$

We can still trace the dynamic causal effects of exogenous changes in p_{1t} and p_{2t}!

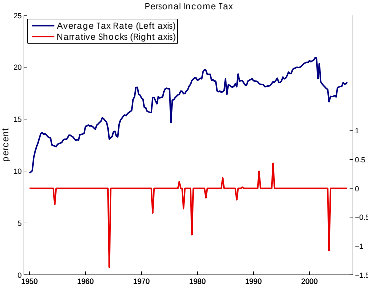
The lower/upper triangularizations are (harmless) rotations that provide what we are typically most interested in, the causal effects of surprise innovations in single policy instrument at a time.

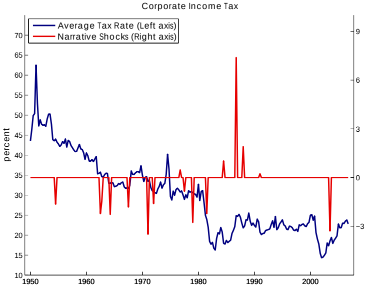
This is poorly understood in some of the literature

Note that p_{1t} (p_{2t}) still responds on impact to $e_{2,t}^L$ ($e_{1,t}^U$) through the impact on u_t^Y

Example: Mertens and Ravn (2013) Personal/Corporate Income Tax Shocks

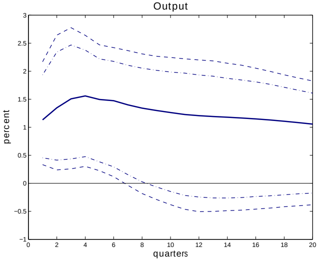
Decomposition of (unanticipated) Romer and Romer (2010) shocks



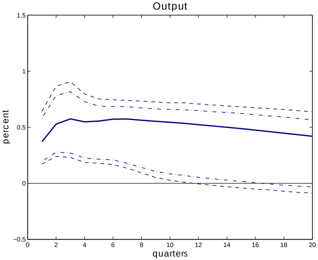


Example: Mertens and Ravn (2013) Personal/Corporate Income Tax Shocks

Personal Income Tax Cut



Corporate Income Tax Cut



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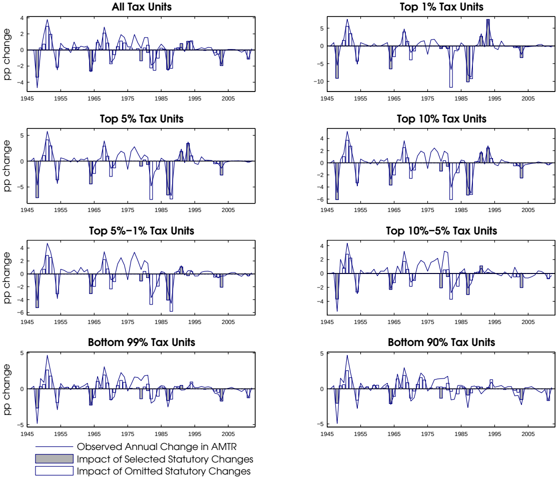
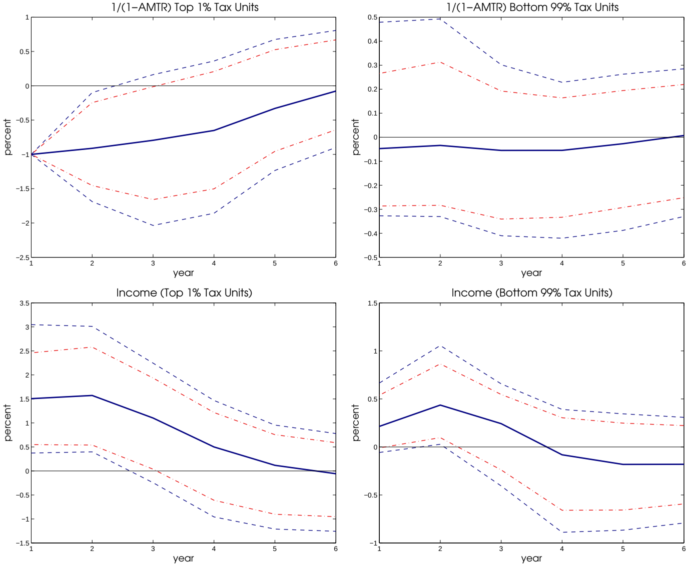
Example: Mertens and Montiel Olea (2018) Marginal Tax Rate Shocks

Proxies for shocks to marginal tax rates across the income distribution



Example: Mertens and Montiel Olea (2018) Top 1% Tax Shocks



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How Fundamental is Fundamentalness?

- DL projection (or Local projections) on m_t:

  Relevance A1+ Contemp. Exo A2+ **Lag Exo A3** (+ Lead Exo A4 for LP)

- Proxy SVAR with m_t:

  Relevance A1+ Contemp. Exo A2 + **Fundamentalness**

Is partial fundamentalness enough for Proxy SVARs?

Partial Invertibility/Fundamentalness

$\epsilon_{j,t}$ is fundamental for z_t if $\epsilon_{j,t}$ is a linear combination of u_t in $z_t = G(L)u_t$

Semi-Structural VAR Representation Stock and Watson (2018), Miranda-Agrippino and Ricco (2019)

Let $\epsilon_{j,t}$ be fundamental for z_t such that $\epsilon_{j,t} = \lambda_j' u_t$. There exists a $\Lambda = [\lambda_j \; \lambda_{-j}]$ where λ_{-j} is $N_z \times (N_z - 1)$ and $\Lambda' \Sigma_u \Lambda = \mathcal{I}$ such that

$$B(L)z_t = \mathcal{D}_j \epsilon_{j,t} + \xi_t \quad , \quad \text{where } \mathcal{D}_j = \Sigma_u \lambda_j, \; \xi_t = \Sigma_u \lambda_{-j} \lambda_{-j}' u_t, \; E[\epsilon_{j,t} \xi_t'] = 0$$

How Fundamental is Fundamentalness?

Limited Lead/Lag Exogeneity Miranda-Agrippino and Ricco (2019)

If $\epsilon_{1:n,t}$ is fundamental for z_t, but $\epsilon_{n+1:N_z,t}$ is nonfundamental for z_t, the exogeneity requirements for Proxy SVARs to identify the causal effect of shock $\epsilon_{j,t} \in \epsilon_{1:n,t}$ are

$$E[m_t \epsilon_{-j,t}] = 0 \qquad \text{(A2 Contemporaneous Exogeneity)}$$

$$E[m_t \epsilon_{n+1:N_z,s}] = 0 \text{ for all } s \neq 0 \qquad \text{(A5 Limited Lag/Lead Exogeneity)}$$

So lead/lag exogeneity is required wrt to the non-invertible shocks

Again, the same for LP-IV but lead exogeneity is required with respect to all the shocks, not just the noninvertible shocks

If the shock of interest $\epsilon_{j,t}$ is nonfundamental for z_t, then Proxy SVAR/SVAR-IV cannot correctly estimate the dynamic causal effects as these are distorted by the Blaschke matrix

Internalizing External Instruments

If $\epsilon_{j,t}$ is nonfundamental for z_t, a solution is to internalize the external instrument

Internal Instrument (II) VAR Projection Plagborg-Møller and Wolf (2021)

Define $\tilde{z}_t = [m_t \ z_t']'$. Define the internal instrument VAR projection

$$\tilde{z}_t = \sum_{i=1}^{\infty} \tilde{B}_i \tilde{z}_{t-1} + \tilde{u}_t$$

Partial Invertibility/Fundamentalness of II-VAR Plagborg-Møller and Wolf (2021)

Under A1-A2 and A4, $\epsilon_{j,t}$ is fundamental for $\tilde{z}_t$

The impulse response function to $\epsilon_{j,t}$ is identified up to scale by the lower triangular factorization (Cholesky decomposition) of $\Sigma_{\tilde{u}} = E[\tilde{u}_t \tilde{u}_t']$ (ordering m_t first)

Note: lead exogeneity needed, unlike in Proxy SVAR/SVAR IV

Some Additional Comments

- Lag/lead exogeneity and fundamentalness are testable assumptions, contemporaneous exogeneity is not testable when $dim(m_t) = dim(\epsilon_{j,t})$

- All examples were just-identified $dim(m_t) = dim(\epsilon_{j,t})$. Extensions to $dim(m_t) \geq dim(\epsilon_{j,t})$ are straighforward

- Relevance requires only non-zero covariance, so m_t can be dummies, signed dummies, measurement-error ridden, censored, ...

- Relevance/exogeneity conditions can also be imposed on prediction errors at other horizons $G_h u_t$, including $h = \infty$

- VAR-LP Equivalence leads to SVAR-IV and LP-IV Equivalence (with additional lead exogeneity requirements for LP-IV)

- II-VAR is asymptotically valid under what are likely the weakest assumptions in practice, small sample performance is another matter

Narrative Sign Restrictions

Narrative Sign Restrictions Antolín-Díaz and Rubio-Ramírez (2018)

$$\epsilon_{j,\tau_+} > 0 \text{ for } \tau_+ \in 1, ..., T \quad , \quad \epsilon_{j,\tau_-} < 0 \text{ for } \tau_- \in 1, ..., T \backslash \tau_+$$

Narrative sign restrictions incorporate information about the sign of shocks at certain dates in the sample

Examples: October 1979 Volcker contractionary monetary policy shock, oil shocks

These and other narrative sign restrictions (e.g. on historical decompositions) eliminate admissible D's in $\Sigma_u = DD'$.

Another approach is to construct a proxy m_t with signed dummies

Plagborg-Møller and Wolf (2021) Giacomini, Kitagawa, and Read (2022)

Identification with Higher Order Moments

So far, all identification schemes have relied on covariance restrictions

Identification can also rely on higher-order moments:

- Heteroskedasticity — Sentana and Fiorentini (2001), Rigobon (2003), Lewis (2021)
- Mutually Independent Non-Gaussian Shocks — Lanne, Meitz, and Saikkonen (2017), Gouriéroux, Monfort, and Renne (2019)

See also Montiel Olea, Plagborg-Møller, and Qian (2022).

Identification with Heteroskedasticity

Example from Lewis (2021)

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & h_{12} \\ h_{21} & 1 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}}_{e_t}, \ E[e_t e_t'] = \underbrace{\begin{bmatrix} \sigma_1 & 0 \\ 0 & \boldsymbol{\sigma_{2t}} \end{bmatrix}}_{\Sigma_t^{1/2}}, \ E[u_t u_t'] = H\Sigma_t H$$

$$\begin{aligned} u_{1t}u_{2t} &= h_{12}e_{2t}^2 + w_{1t} \ , \quad w_{1t} = h_{21}e_{1t}^2 + (1 + h_{12}h_{21})e_{1t}e_{2t} \\ u_{2t}^2 &= e_{2t}^2 + w_{2t} \ , \quad w_{2t} = h_{21}^2 e_{1t}^2 + 2h_{21}e_{1t}e_{2t} \end{aligned}$$

This suggest regressing $u_{1t}u_{2t}$ on u_{2t}^2 and a constant to estimate h_{12}.

Since $Var(w_{2t}) \neq 0$, OLS is generally biased (measurement error bias)

However lagged values of $u_{2,t}^2$ are valid instruments for $u_{2,t}^2$ if σ_{2t} is persistent and e_{1t} is homoskedastic.

Under time-varying volatility, we can identify h_{12} without any other restrictions based on the dynamic covariances of the squared prediction errors

The approach works in general even with time-varying volatility in all the shocks, see Lewis (2021)

Example: Lewis (2021) Fiscal Policy Shocks

Observables $z_t = [T_t, G_t, Y_t]'$, quarterly sample 1950Q1-2006Q4

T_t : Log Real Federal Tax Revenues per capita
G_t : Log Real Federal Government Spending on Final Goods per capita
Y_t : Log Real GDP per capita

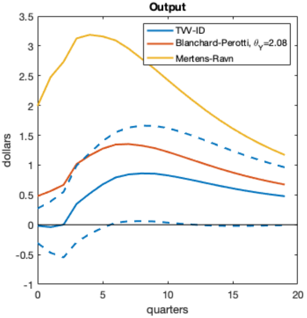
$$
\begin{aligned}
u_t^T &= \theta_G \sigma_G \epsilon_t^G + \theta_Y u_t^Y + \sigma_T \epsilon_t^T \\
u_t^G &= \gamma_T \sigma_T \epsilon_t^T + \gamma_Y u_t^Y + \sigma_G \epsilon_t^G \\
u_t^Y &= \zeta_T u_t^T + \zeta_G u_t^G + \sigma_Y \epsilon_t^Y
\end{aligned}
$$

Identification based on Time-Varying Volatility as in Lewis (2021)

Example: Lewis (2021) Tax Shocks

Table 1: Estimates of structural parameters

	θ_G	θ_Y	γ_T	γ_Y	ξ_T	ξ_G
TVV-ID	-0.13	1.58^{***}	0.11	0.02	-0.00	0.06
	(0.10)	(0.18)	(0.13)	(0.39)	(0.02)	(0.045)
BP	-0.06	2.08	0	0	-0.08	0.07
MR	-0.20	3.13	0.06	0	-0.35	0.10



Non-Gaussianity

Another approach is to assume that ϵ_t are mutually independent and non-Gaussian

Darmois-Skitovich Theorem

If ϵ_t is independently distributed, than linear combinations $\alpha'\epsilon_t$ and $\beta'\epsilon_t$ with $\alpha \neq 0, \beta \neq 0$ are independent only when ϵ_t have normal distributions.

The normal distribution is the only distribution with all cumulants equal to zero except the first two (mean and variance)

If ϵ_t is non-Gaussian i.i.d than the white noise prediction errors $u_t = \mathcal{D}\epsilon_t$ cannot be mutually independent white noise.

Higher order properties of u_t can in that case provides additional identifying information

Key is that ϵ_t are **independently distributed** (not just uncorrelated) and **non-Gaussian**.

Non-Gaussianity

Statistical identification, not based on theoretical or institutional restrictions

Identified shocks in general do not have interpretation without additional economic information.

Example from Montiel Olea, Plagborg-Møller, and Qian (2022):

$$\epsilon_{1t} = \tau_t \zeta_{1t} \quad , \quad \epsilon_{2t} = \tau_t \zeta_{2t}$$

where τ_t, ζ_{1t}, ζ_{2t} are iid, τ_t is a shared stochastic volatility process

Model is no longer linear in the independent shocks.

The impulse response to the (uncorrelated) shocks ϵ_{1t} and ϵ_{2t} are still of interest

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