# Short-Run Equilibrium of International Trade under Heterogeneous Discrete Firms with Multiple Continuous Varieties


Gyu Hyun Kim

Iowa State University


## Background Motivation

- Market integration (or Liberalization)
  - Introduction of foreign entrants and their products
  - Intensive competition
    - Lower markup of a product
    - Product range adjustment

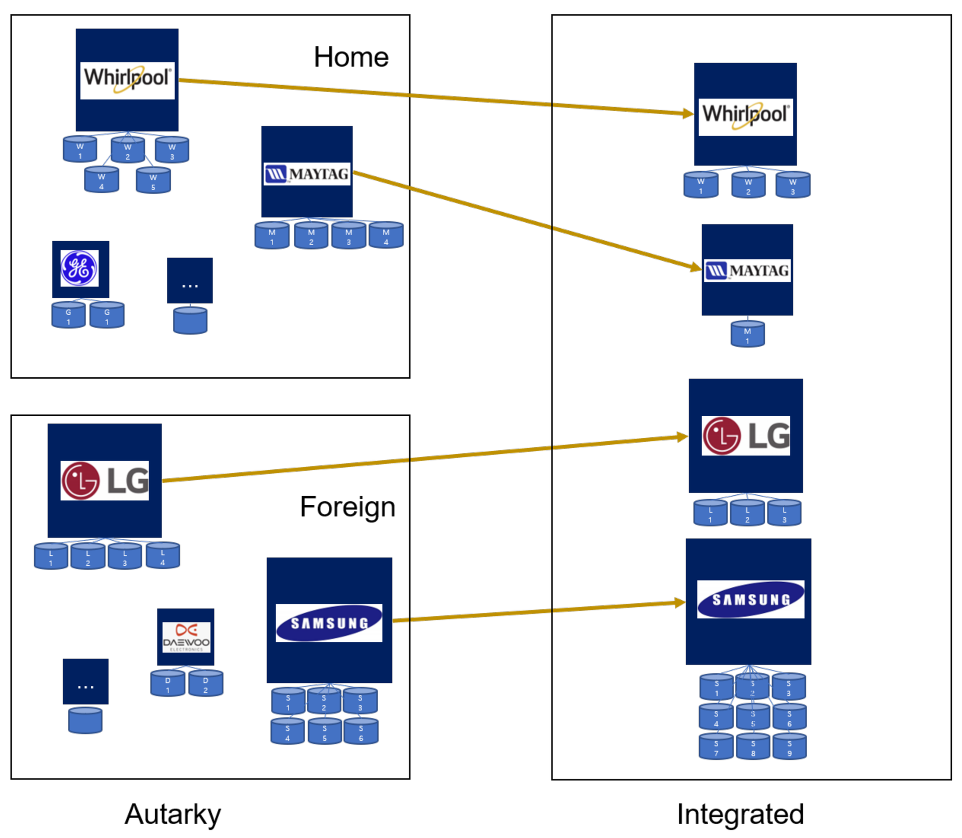



Figure 1. Illustration - Washer Market Integration

## Research Question

How do *discrete* firms *differentiated in productivity* (=heterogeneous) response to *the market integration between symmetric economies* (= international trade) *when they maintain own productivity assigned in autarky* (=short-run)?

## This Paper

- Objective
  - Numerical exploration of market integration impact on an individual firm's decision on (1) markup of a product and (2) product range in the short-run
  - Capturing "Head-to-head competition" making the integrated market pro-competitive
    - Competition among the highest productive firms from each economy
- Overview
  - 1st Part
    - Base Framework: Nested CES demand + Monopolistic Compeittion
    - Defining the concept of market integration in the short-run
      - Fixed productivity (No uncertainty)
      - No entrants
  - 2nd Part: Quantitative analysis with the newly defined short-run environment of market integration
    - Implementing pro-competitive integrated market

## Contribution

This work gives an idea about tractable firm-level optimalities for heterogeneous discrete firms producing multiple products

- **Discrete (Granular) Firms** vs. Continuous firms (Zero-measured)
- **Focusing on transition** vs. Focusing on equilibrium
- **Fixed productivity at the moment of market integration** vs. New random assignment

→ We can figure out how the superstars adjust their markups and product ranges at the moment of market liberalization.

## Model Structure

- Employed the general framework in Feenstra and Ma (2008)
- Integration between symmetric markets
  - Set of firm-level productivity: Identical across regions
  - Zero iceberg trade (transportation) cost, and zero fixed cost for exporting goods
  - Fixed wage $w = 1$: Consistent with symmetricity

## Demand & Supply

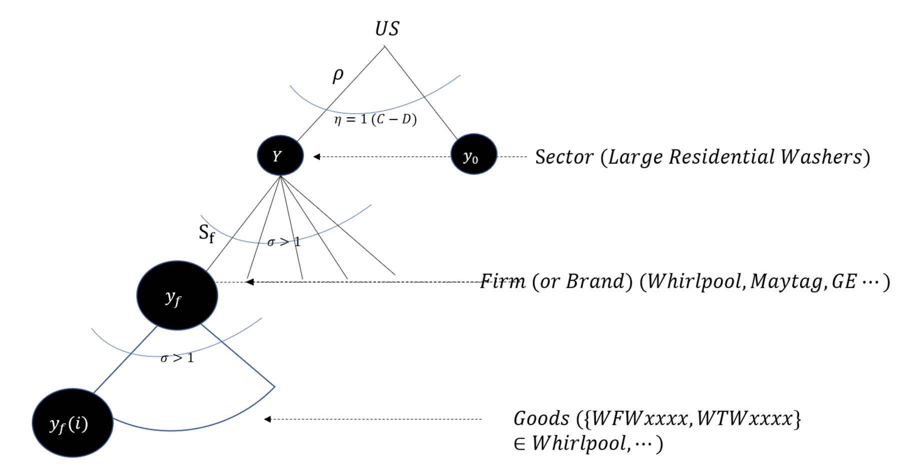



Figure 2. Nested CES Demand

- Marshallian demands and Dixit-Stiglitz price indexes
  - Heterogeneous sector:
    - $Y = \rho I \frac{1}{P} = R \frac{1}{P}$ and $P = \left[\sum_{f \in \Omega} (P_f)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$
  - Het firm:
    - $y_f = \rho I \underbrace{\frac{P_f^{1-\sigma}}{P^{1-\sigma}}}_{S_f} \frac{1}{P_f}$ and $P_f = \left[\int_{i \in \Omega_f} (P_f(i))^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$
- Supply side
  - Homogeneous sector: CRS technology with a unit of labor
    -$P_0 = 1$: A Numeraire and $P_0 y_0 = 1$ as $w = 1$
  - Heterogeneous sector: Heterogeneity in productivity ($\varphi_f$)
  - Assumptions
    -Symmetric tech. across products: No product index $(i)$
    -Fixed cost for expanding a unit range of product $(K_1)$
  - A het-firm's profit function
    $\Pi_f = N_f \left(P_{fi} - \frac{1}{\varphi_f}\right) y_{fi} - w K_1 N_f$
  - A Het-firm's optimal choices
    -Price: $P_{fi} = \frac{1}{\varphi_f}\mu_f = \frac{1}{\varphi_f}\frac{\epsilon_f}{\epsilon_f - 1}$ where $\epsilon_f = \sigma + (1-\sigma) S_f$
    -Product Range: $N_f = \frac{\rho I}{K_1}\frac{1}{\sigma - 1}\frac{\epsilon_f - 1}{\epsilon_f} S_f$

## Zero-Cutoff Profit (ZCP) Condition

- Fixed cost $(K_1)$: Among $M_e$ entrants in an economy, only productive $M_o (< M_e)$ firms survive and produce
- Sorting productivities in a descending order, $\varphi_1 > \varphi_2 > \cdots > \varphi_{M_e}$, the market can be summarized like

$$\begin{cases} \Pi_f \geq 0 \ and \ S_f > 0 & for \ f = 1, \cdots, M_o \\ \Pi_f = 0 \ and \ S_f = 0 & for \ f = M_o + 1, \cdots, M_e \end{cases}$$

- ZCP condition gives the elements of the threshold (marginal) firm
  - Threshold productivity ($\varphi^{ZCP} = \varphi_{M_o}$)
  - Market share of a marginal firm ($S^{ZCP} = S_{M_o}$)

## The Moment of Market Integration (Short-run)

- Bilateral Trade between symmetric economies
  - Liberalized market size: $I^W = 2I$
- No random productivity assignment
  - All entrants in the liberalized market maintain their own productivity assigned in autarky
- No new entrant
  - Survivors in autarky become the only entrants of the integrated market

## Short-run Equilibrium

- At moment of the market integration from trade liberalization
  - $M_e^W = 2M_o$ firms surviving in autarky of symmetric economies $(I)$
  - Fixed firm-level productivity
  - Market size of the integrated market: $I^W$ $(= 2I)$

The Bertrand-Nash equilibrium of trade liberalization in the short-run consists of

- A set of information about the ZCP condition in the heterogeneous sector: $\{S^{W,ZCP}, \varphi^{W,ZCP}\}$
- A vector of the optimality set by the $M_o^W$ surviving firms in the integrated market, including the price of a variety, the range of varieties, and the firm-level market share: $\{P_{fi}, N_f, S_f\}_{f=1}^{M_o^W}$, and
- A sectoral price index within the integrated economy: $P$,

which solves both utility and profit maximization simultaneously.

## Quantitative Analysis - Introduction

- Discrete and heterogeneous firms: Unavailability to employ the Law of Large Numbers (LLN)
  - No analytical closed-form solution
  - Non-stationary short-run equilibrium

  → Even in the simple scenario using symmetric economies, it is not able to get a closed-form of firm-level optimalities.
- Quantitative analysis: Numerically exploring how an individual firm adjust their optimalities in the short-run of market liberalization
- How?
  1. Benchmark replication of the market integration in the previous literature - New productivity assignment at the beginning of market integration
  2. Counterfactual quantitative analysis with the novel concepts of the market integration

## Quantitative Analysis Process

- Followed the mechanism and relevant parameters in Feenstra and Ma (2008)
  - The conventional general procedure to find equilibrium in the granular firms with multiple products framework
- Mechanism: Finding the marginal firm satisfying the ZCP condition
  - The lowest (and unique) number of $\varphi^{ZCP}$ in the productivity set to satisfy

$$\sum_f S(\tau_f) = 1$$

where

$$\tau_f = \frac{\varphi^{ZCP}}{\varphi_f} = \left[\frac{\frac{\epsilon_f}{\epsilon_f - 1}}{\frac{\epsilon^{ZCP}}{\epsilon^{ZCP} - 1}}\right]^{-\frac{\sigma}{\sigma - 1}}$$

$$If \ \tau_f > 1, \ then \ S_f = 0$$

- Productivity set and relevant parameters
  - $\sigma = 6$ and $K_1 = 5$
  - Fixed sectoral share due to the C-D $\succsim$ at the top-level demand
    - $R = \rho I = 1000$ in autarky and $R^W = \rho I^W = 2000$ in the integrated market

## Survivors in Equilibrium (w/ the new concept)

Unlike the previous literature, we can find that the ZCP condition is updated, resulting in firm-level changes in markup and product range.

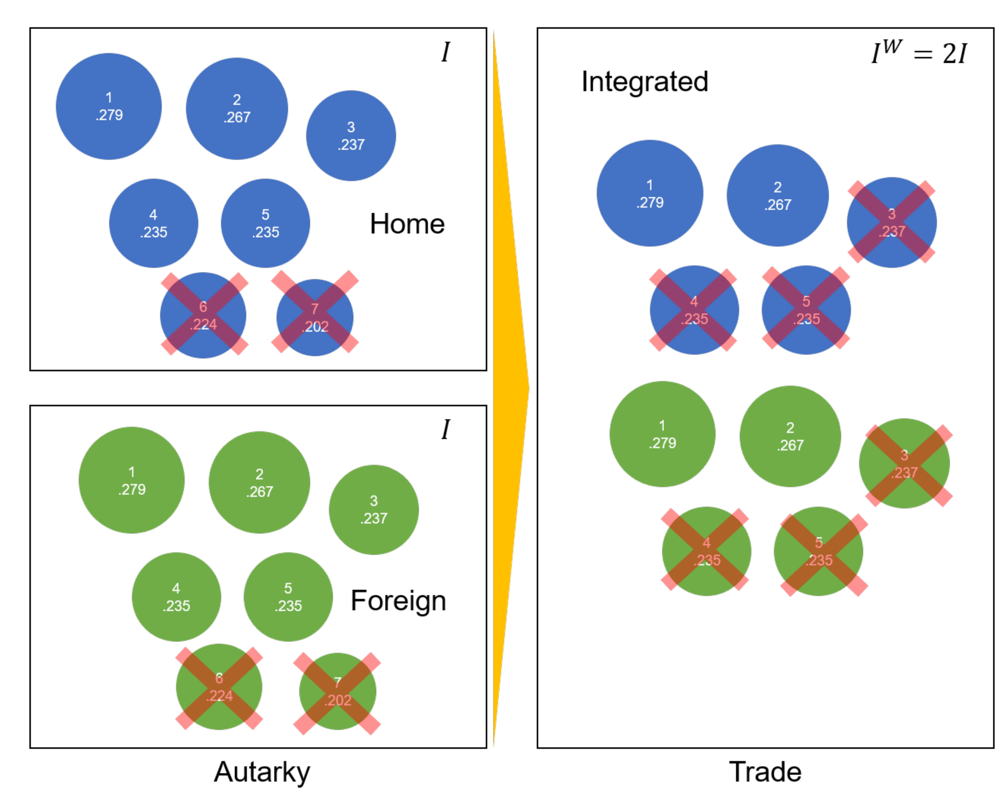


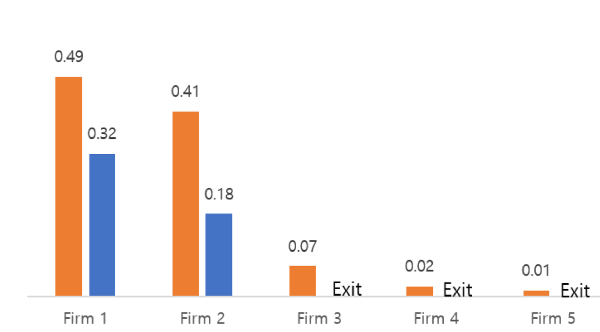

Figure 3. Equilibrium in the Short-run of Market Liberalization

## Numerical Results

- Market shares

$$S_f = S(\tau_f) = 1 - \frac{1}{\left\{\left(\sigma - 1 + \frac{1}{1 - S^{ZCP}}\right)(\tau_f)^{-\frac{\sigma - 1}{\sigma}} - (\sigma - 1)\right\}}$$

  - Higher productive firms get larger market shares.
  - Head-to-head competition with symmetric productive foreign firms -Total of four survivors (two survivors in each economy) with lower market shares
    -Decreasing rate of market share: Firm 2s > Firm 1s

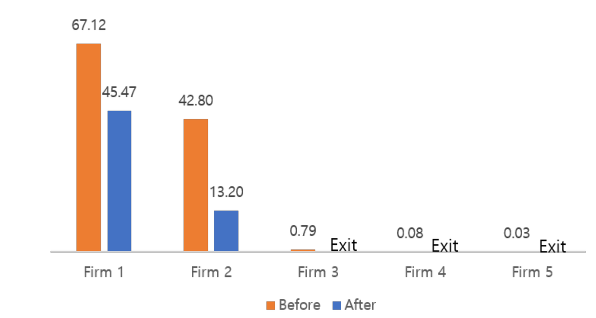


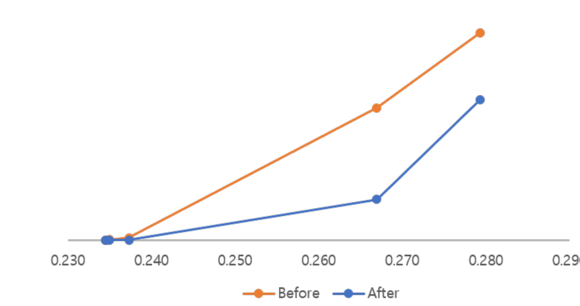

## Numerical Results (Cont'd)

- Profits

$$\Pi_f = \Pi(\tau_f) = \frac{\{S(\tau_f)\}^2}{1 + (\sigma - 1)\{1 - S(\tau_f)\}} R$$

  - Higher productive firms earns more profits.
  - Two opposite effects on profit → No proportional to the market size -Larger market size $(R) \Rightarrow \Pi_f \uparrow$
    -Smaller market share $(S_f) \Rightarrow \Pi_f \downarrow$
    →In this example of productivity set: (1) < (2) for all survivors

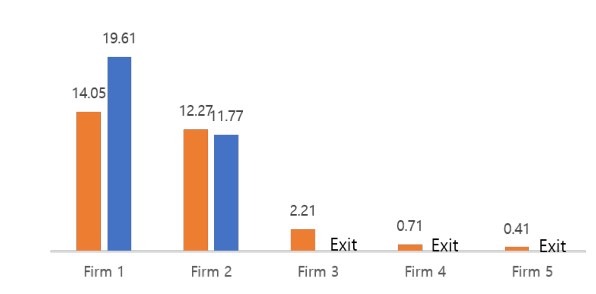


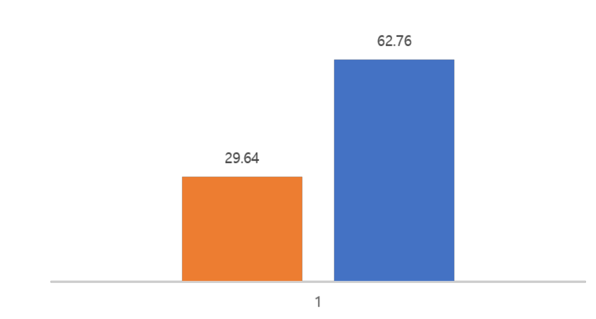




- Range of Products

$$N_f = \frac{\overbrace{\rho I}^{=R}}{K_1}\frac{1}{\sigma - 1}\frac{\epsilon_f - 1}{\epsilon_f} S_f$$

  - Higher productive firms produce a broader range of products.
  - Two opposite effects on profit → No proportional to the market size -Larger market size $(R) \Rightarrow N_f \uparrow$
    -Smaller market share $(S_f) \Rightarrow$
    $N_f \downarrow$ directly and $N_f \uparrow$ indirectly via $\epsilon_f$
    -Two opposite effects on profit → No proportional to the market size
  - Resource are concentrated on the most productive firms (Firm 1s)

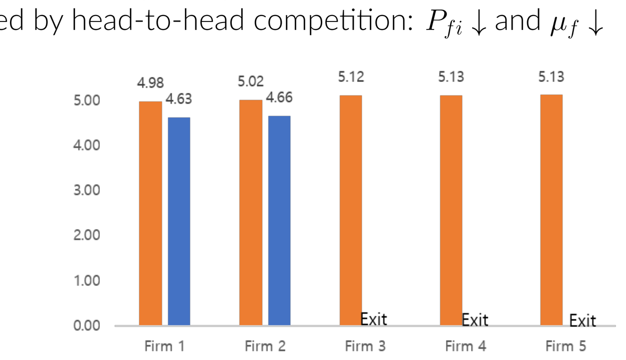


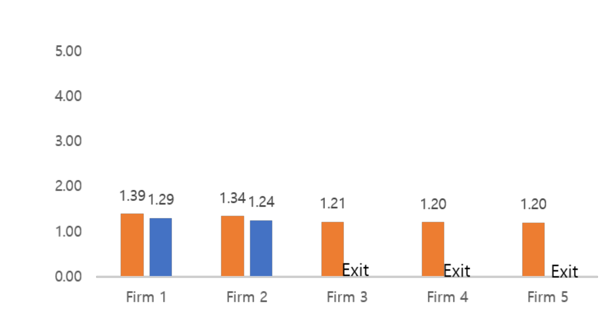




- Price and Markup

$$P_{fi} = \frac{1}{\varphi_f}\mu_f = \frac{1}{\varphi_f}\frac{\epsilon_f}{\epsilon_f - 1} \ where \ \epsilon_f = \sigma + (1 - \sigma) S_f$$

  - Higher productive firms set a higher markup and a lower price.
  - Only $S_f$ affects markup of a product.
  - Lower $S_f$ led by head-to-head competition: $P_{fi} \downarrow$ and $\mu_f \downarrow$




## Thank You

Comments are always welcome!