Multiproduct-Firm Oligopoly: An Aggregative Games Approach

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ASSA–ES Meetings, Philadephia, 2018

Introduction

- Even when defined at the NAICS 5-digit level, multiproduct firms (MPFs) account for 41% of the total number of firms and 91% of total output in the U.S. (Bernard, Redding and Schott, 2010).
- In U.S. manufacturing, the average (resp. median) NAICS 5-digit industry has a C4 of 35% (resp. 33%). (Source: Census of U.S. Manufacturing, 2002).
  Suggests that many markets are characterized by oligopolistic competition.
- Ubiquitousness of MPFs and oligopoly is reflected in modern empirical IO literature.

Introduction

What is special about MPFs in oligopoly?

- Must choose not only how aggressive to be in the market place, but also how to vary markup across products.
- Must take self-cannibalization into account when setting markups and deciding which products to offer.

Issues:

- What determines within-firm markup structure, between-firm markup differences, and industry-wide markup level?
- Along which dimensions are markups and product offerings distorted by oligopolistic behavior?

This paper: Develop an aggregative games approach to address these and related issues.

What We Do

- Introduce new class of (integrable) quasi-linear demand systems, derived from discrete/continuous choice.
  - Nests CES and MNL.
- Study a multiproduct-firm pricing game with arbitrary product portfolios and product heterogeneity.
  - Pricing game is aggregative.
  - Prove existence (uniqueness) under weak (stronger) conditions.
  - Approach circumvents technical difficulties (failure of quasi-concavity, (log-)supermodularity, upper semi-continuity).
- Decompose welfare distortions from oligopolistic competition between MPFs.
- Study the determinants of firms scope.
- Rank equilibria and perform comparative statics on set of equilibria.

What We Do

- Extensions:
  - Nested demand systems
  - General equilibrium
  - Non-linear pricing
  - Quantity competition
- Type aggregation under (nested) CES or MNL demands
- Two sets of applications in (nested) CES/MNL demands case:
  1. Merger analysis.
     - Both static and dynamic.
  2. Trade liberalization.
     - Impact on inter- and intra-firm size distributions, average industry-level productivity, and welfare.

Related Literature

- MPF oligopoly pricing with horizontally differentiated products.
  - Equilibrium existence and uniqueness. Spady (1984), Konovalov and Sandor (2010), Gallego and Wang (2014).
  - Applied. Anderson and de Palma (1992, 2006), Shaked and Sutton (1990), Dobson and Waterson (1996).
- MPFs in international trade.
  - Monopolistic competition. Bernard, Redding and Schott (2010, 2011), Dhingra (2013), Nocke and Yeaple (2014), Mayer, Melitz and Ottaviano (2014).
  - Oligopoly. Eckel and Neary (2010).
- Aggregative games.
  - Equilibrium existence. Selten (1970), McManus (1962, 1964), Szidarovsky and Yakowitz (1977), Novshek (1985), Kukushkin (1994).
  - Comparative statics. Corchon (1994), Acemoglu and Jensen (2013).
  - Single-product oligopoly. Anderson, Erkal and Piccinin (2013)
- Multiproduct monopoly.
  - Armstrong and Vickers (2016).

The Baseline Model: Demand

- Set $\mathcal{N}$ of (differentiated) products, and an outside good.
- Consumers' indirect utility: $V(p) = \log(H(p)) + y$, where y is income, and $H(p) = \sum_{j \in \mathcal{N}} h_j(p_j) + H^0$.
- Implied demand system:

$$D_i(p) = \widehat{D}_i(p_i, H(p)) = \frac{-h_i'(p_i)}{H(p)}$$

.

- Two special cases: CES ($h(p) = ap^{1-\sigma}$) and MNL ($h(p) = e^{\frac{a-p}{\lambda}}$).
- Demand system can equivalently be derived from *discrete/continuous choice* with i.i.d. Gumbel taste shocks.

The Baseline Model: Firms

- Set of firms, $\mathcal{F}$, is a partition of $\mathcal{N}$.
- Constant marginal cost of product $i \in \mathcal{N}$, $c_i > 0$.
- Each firm $f \in \mathcal{F}$ sets profile of prices $p^f = (p_k)_{k \in f}$.
- Firm f's profit:

$$\Pi^f(p^f, H(p)) = \sum_{j \in f} (p_j - c_j) \widehat{D}_j(p_j, H(p)).$$

- Allow for infinite prices: If $p_k = \infty$, $k \in f$, firm f does not make any profit on product k.
- Pricing game is *aggregative*: $\Pi^f(p^f, H(p))$ depends on prices set by rival firms only through uni-dimensional aggregator H.

Existence of a Pricing Equilibrium

Standard approaches to equilibrium existence fail because:

(i) Action spaces are not bounded or payoff functions not upper semi-continuous.

(ii) Payoff functions are not (log-)supermodular.

(iii) Profit functions are not quasi-concave.

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Our existence proof relies on an aggregative games approach:

- Fix H and look for $(p_k)_{k \in f}$ such that all of firm f's FOCs hold. Obtain a vector $(p_k(H))_{k \in f}$ for every f.
- Then, look for an H such that

$$\sum_{f \in \mathcal{F}} \sum_{k \in f} h_k \left(p_k(H) \right) = H.$$

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- First-order condition for product $k \in f$:

$$0 = \frac{d\Pi^f}{dp_k} = \widehat{D}_k + (p_k - c_k)\frac{\partial \widehat{D}_k}{\partial p_k} + \frac{\partial H}{\partial p_k}\left(\sum_{j\in f}(p_j - c_j)\frac{\partial \widehat{D}_j}{\partial H}\right),$$

$$= \widehat{D}_k\left(1 - \frac{p_k - c_k}{p_k}\left|\frac{\partial \log \widehat{D}_k}{\partial \log p_k}\right| + \frac{\frac{\partial H}{\partial p_k}}{\widehat{D}_k}\left(\sum_{j\in f}(p_j - c_j)\frac{\partial \widehat{D}_j}{\partial H}\right)\right).$$

- Re-arranging:

$$\frac{p_k - c_k}{p_k}\underbrace{\left|\frac{\partial \log \widehat{D}_k}{\partial \log p_k}\right|}_{=p_k\frac{-h_k''(p_k)}{h_k'(p_k)}=\iota_k(p_k)} = \overbrace{1 + \sum_{j\in f}(p_j - c_j)\underbrace{\frac{\frac{\partial H}{\partial p_k}}{\widehat{D}_k}\frac{\partial \widehat{D}_j}{\partial H}}_{=\widehat{D}_j}}^{\text{independent of } k}.$$

Existence of a Pricing Equilibrium

- The fact that the right-hand side is independent of k follows as the marginal impact on H of an increase in p_k is proportional to the demand of product k. (Follows from IIA property, which implies that demand is multiplicatively separable in the aggregator.)
- IIA also implies: LHS of FOC independent of H.
- Hence, if $(p_k)_{k \in f}$ satisfies the FOCs, then for every $i, j \in f$,

$$\frac{p_i - c_i}{p_i} \iota_i(p_i) = \frac{p_j - c_j}{p_j} \iota_j(p_j)$$

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  We say that $(p_k)_{k \in f}$ satisfies the common ι-markup property.
- Within-firm markup structure: Lerner index is inversely proportional to the "perceived" price elasticity of demand.

Existence of a Pricing Equilibrium

Assume that function $p_k \mapsto \frac{p_k - c_k}{p_k} \iota_k(p_k)$ can be nicely inverted for every $k \in f$.

- Denote the inverse function by $r_k(\mu^f)$.
- Firm f's optimality conditions boil down to a single equation:

$$\mu^f = 1 + \Pi^f((r_k(\mu^f))_{k \in f}, H).$$

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Assume that this equation has a unique solution for every H.

- Denote the solution by $m^f(H)$. $m^f(.)$ is firm f's fitting-in function.

Existence of a Pricing Equilibrium

- Let
$$\Gamma(H) = \sum_{f \in \mathcal{F}} \sum_{k \in f} h_k \left(r_k(m^f(H)) \right).$$
- H is an equilibrium aggregator level if and only if $\Gamma(H) = H$.
- Γ is called the aggregate fitting-in function.
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Assume that such a fixed point exists.

- Then, the pricing game has an equilibrium.
- The nested fixed point structure gives rise to an efficient way of computing the equilibrium.

Existence of a Pricing Equilibrium

Two ways in which dimensionality is reduced:

- Firm f's pricing problem reduces to looking for the right (uni-dimensional) μ^f, i.e., the right ι-markup.
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Of course, we still need to check that:

- FOCs are necessary and sufficient for optimality.
- $p_k \mapsto \frac{p_k - c_k}{p_k} \iota_k(p_k)$ can be nicely inverted.
- Fitting-in functions are well defined.
- The aggregate fitting-in function has a fixed point.
- Also need to deal with infinite prices.

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Need one more assumption to get there.

Existence of a Pricing Equilibrium

Assumption:

(i) For every $k \in \mathcal{N}$, ι_k is non-decreasing.

Note:

- Under monopolistic competition, where firms take H as given, Assumption (i) means that the perceived price elasticity of demand is non-decreasing (Marshall's second law of demand).
- Under MNL demand, $\iota_k(p_k) = \frac{p_k}{\lambda_k}$. Under CES demand, $\iota_k(p_k) = \sigma$.

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Theorem

Under Assumption (i), the pricing game has an equilibrium for every $(c_i)_{i \in \mathcal{N}}$ and $\mathcal{F}$.

We also establish equilibrium uniqueness (under stronger conditions) by showing that $\Gamma'(H) < 1$ whenever $\Gamma(H) = H$.

Other Results

- **Firm scope.** Firm f is "more likely" to offer any given product k in equilibrium, the larger is the equilibrium aggregator H ("fighting brand"). *Intuition:* The more competitive is the market (the larger is H), the less the firm cares about self-cannibalizing its more profitable products (and the more it cares about stealing business from rivals).

- **Welfare analysis.** The equilibrium exhibits only two types of distortions:
  1. The equilibrium aggregator, H^*, is smaller than the welfare-maximizing aggregator, $H^{FB} = \sum_{k \in \mathcal{N}} h_k(c_k)$.
  2. Conditional on H^*, the firm-level aggregators are too small for some firms and too large for others.

Comparing Equilibria

Suppose H^1 and H^2 are equilibrium aggregator levels with $H^1 < H^2$. Then:

- Consumers prefer H^2 to H^1.
- Every firm prefers H^1 to H^2.
- The set of active products at H^1 is contained in the set of active products at H^2.

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Monotone comparative statics: Suppose the aggregate fitting-in function shifts upward (say, because import tariffs are reduced or entry takes place). Then, in the lowest and highest equilibrium:

- Prices go down, consumers are better off, (domestic) firms are worse off.
- The set of active products expands.

Productivity improvements have more ambiguous effects.

- An increase in marginal cost can increase H and thus make consumers better off.

Extensions and Type Aggregation

- **Extensions.**
  - Non-linear pricing.
  - Quantity competition.
  - Generalized IIA demands and nests.
  - General equilibrium.

- **(Nested) CES/MNL demands: Type aggregation.**
  - All information about firm f's behavior/performance (markup, market share, profit) can be summarized by its (uni-dimensional) type T^f, which is independent of H. In CES case: $T^f = \sum_{k \in f} a_k c_k^{1-\sigma}$; in MNL case: $T^f = \sum_{k \in f} \exp(\frac{a_k - c_k}{\lambda})$.
  - Type aggregation useful for:
    - ★ Merger analysis.
    - ★ Defining firm-level productivity.
    - ★ Computational tractability.

Applications to Merger Analysis and International Trade

For the cases of (nested) CES/MNL demands (for which type aggregation obtains), we apply the model to:

1. Static merger analysis, extending Farrell and Shapiro (1990)
   - Consumer/aggregate surplus effects
   - External effects
2. Dynamic merger analysis, extending Nocke and Whinston (2010)
3. Analysis of (Unilateral) Trade Liberalization
   - Effects on inter- and intra-firm size distribution
   - Productivity effects
   - Domestic welfare effects

Conclusion

- Main contribution: Tractable approach to MPF oligopoly.
  - Simple, yet powerful existence, uniqueness, and characterization results.
  - Computationally efficient algorithm.
  - Simple decomposition of welfare distortions.
  - Predictions on how markups and firm scope vary with competitive environment.
- Secondary contribution: Complete characterization of class of demand systems derivable from discrete/continuous choice with i.i.d. Gumbel taste shocks.
  - By going beyond CES and MNL demands, allow for richer patterns of markups.
- Policy contribution: Merger control and trade liberalization with MPFs.
  - Shown how well-known results on static and dynamic merger analysis obtained in homogeneous-goods Cournot settings carry over to price competition with MPFs.
  - Show that a unilateral trade liberalization, despite increasing industry-level productivity, may reduce domestic welfare if the domestic industry is sufficiently concentrated.