Fundamental risk and capital structure^{*}

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Abstract

I develop a dynamic capital structure model to examine how the nature of risk affects firm's debt policy. In the model, firm's fundamental risk, captured by its cash flow process, consists of transitory and persistent parts with markedly different dynamics. The model explains the observed dispersion in the risk-leverage relationship. Firms with similar total volatility adopt distinctive debt policies when the composition of their risk differs and issue less debt when their cash flows are more persistent to preserve debt capacity needed to fund investment. The model also provides rationale why the observable dispersion in cash flow persistence is low, which is at odds with the large degree of heterogeneity in other firm characteristics, as well as why persistence and leverage are weakly related in the data.

JEL Classification: G32, G31.

Keywords: Dynamic capital structure, fundamental risk, transitory and persistent shocks, leverage-risk trade-off.

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1 Introduction

The negative relationship between risk and leverage, illustrated in Figure 1, is one of the most well-established phenomena in finance. The association is robust in the data and existing models of capital structure, starting with Merton [1974], Black and Cox [1976] and Leland [1994], provide intuitive theoretical underpinning of how risk affects firm's debt policy. Even practitioners acknowledge that risk plays an important role in shaping firm's capital structure, as according to Graham and Harvey [2001] it constitutes the third most important factor of debt issuance decisions. However, empirical research tends to focus on a single dimension of firm's fundamental risk, usually captured by its cash flow volatility. While there is little doubt that this characteristic plays an important role in determining firm's capital structure, it may not be able to capture more in-depth features of fundamental risk. For example, it could express the degree of total risk in firm's operations while missing out on other important determinants of cash flow dynamics, such as their exposure to aggregate market conditions or the structure of firm's profits. These claims are not unfounded, for instance in a recent study Schwert and Strebulaev [2014] document that asset beta provides additional explanatory power for leverage above and beyond the effect of total volatility, which shows that further dissecting firms' cash flow process could provide additional insights concerning leverage variation in the data.



Figure 1: Scatter plot of average annual book leverage versus average operating profitability volatility for 4-digit SIC industries. Definitions of variables are provided in Appendix B.

In this paper I argue that the nature of risk present in firm's business environment wields influence on its leverage policy through its effect on firm's cash flows and investment. In the model, firm's cash flows can be exposed to both transitory and persistent shocks, which differ in their impact on firm's investment and debt policies. The separation into transitory and persistent shocks represents the fact that some firms may experience frequent but transient cash flow shocks that influence their long-run decisions in a limited way, while others could only face infrequent disturbances, but with permanent impact on cash flows. To capture the distinction, the shocks are modeled using a stationary and a non-stationary process.¹ Firm's fundamental risk can then be directly linked to the composition of cash flows and described by their volatility as well as persistence. Unlike standard dynamic capital structure models in which the firm is exposed to a single transitory shock, this paper can rationalize the mismatch in the risk-leverage relationship in Figure 1 by relating the observed dispersion to differences in risk composition. In particular, the decomposition of fundamental volatility allows to obtain different optimal leverage ratios for firms with the same level of total volatility. Similarly, the model generates firms with high profit persistence even when the composition of their fundamental persistence, which also affects leverage choice, differs.

To see that the way in which firms in different industries generate cash flows is closely related to the business environment they face, we could take an example of two industries: food and apparel. The food industry is expected to have more stable earnings stemming from inelastic demand for food, which results in lower business risk. On the other hand, the apparel industry should have more variable earnings, given that its sales are largely driven by fads and changes in customers' tastes, suggesting that its business risk should also be higher. Consequently, using the risk-leverage trade-off, we could conclude that apparel industry's optimal debt ratio should be lower than food industry's, all else equal. While the data confirms that the 'risk' faced by food is nearly twice as low as the one of apparel, their respective debt ratios are almost the same, contradicting the fundamental intuition of risk-leverage trade-off.² Figure 1 suggests that these two industries are not the only exception. It shows that while the negative correlation between risk and leverage implies that the association holds on average, there also exists a large degree of dispersion in the data. Firms in industries with similar risk adopt markedly different leverage and, as in the example above, firms in industries with similar leverage vary in their riskiness. This paper is able to explain

1. While I do not take a particular stance on what these shocks may represent, the literature typically associates persistent shocks with events that affect long-run prospects of the firm, such as changes to production technology, human capital, tastes. Transitory shocks, on the other hand, subside over time and can result from demand or supply shocks, regulatory changes requiring real adjustments, changes to production cost structure, machine failure or natural disasters.

2. For example, when using operating profitability volatility as a risk proxy one finds $\sigma_{prof}^{agriculture} \approx 0.03$ and $\sigma_{prof}^{apparel} \approx 0.08$ while the average leverage adopted in both industries is approximately 0.25-0.3 depending on the particular measure.

these empirical patterns by considering a more general notion of risk that yields more information about firms' capital structure policies.³

The main prediction of the model is that leverage is a decreasing function of not only total volatility, but also of persistent shock exposure for the same level of total risk. The intuition underpinning this finding results from the fact that leverage choice is closely related to firm's investment decisions which are highly sensitive to persistent shock realizations. Firms want to invest more when being hit by a persistent rather than a transitory shock, as its long-term effects on cash flows are lasting. Therefore, the financial flexibility theory of DeAngelo, DeAngelo, and Whited [2011] holds: firms preserve more debt capacity when investment opportunities are more persistent. In the model, the irreversibility of the persistent shock reinforces this mechanism further. Firms exposed to persistent shocks adopt even more conservative leverage ratios, given that they want to avoid at all cost the prospect of hitting the collateral constraint and having to forgo valuable long-lasting investment opportunities resulting from realizations of the persistent shock. Their behavior is also reflected in the fact that when the persistent shock exposure rises, firms turn more frequently to other sources of financing such as equity issuance or asset sales despite their substantial cost.

Furthermore, the model is able to explain why firms tend to have persistent cash flows despite varying substantially in other dimensions. Taking the empirical evidence by face value, we could think that the persistence of profits has no bearing on investment, leverage or other firm characteristics, which is strongly at odds with model evidence predicting a robust effect of changing shock persistence on these characteristics. In the model, the overall persistence of firm's cash flow process can be very high if it contains a small persistent part. Therefore, two firms with similar observable dynamic properties of cash flows may adopt markedly different leverage and investment policies, depending on the true dynamics of their profits. To this end, the model gives rationale why firms typically have highly persistent profits and why the estimates profit persistence only provide limited explanatory power for explaining variation in capital structure and other firm policies if we do not control for the overall cash flow composition.

Finally, the model shows that firm's risk composition has a significant impact on the dispersion of leverage as well as other characteristics of capital structure such as volatility and persistence. The properties describing the overall composition of cash flow dynamics are therefore bound to provide extra explanatory power above and beyond total volatility in explaining capital structure variation in the data, not only in leverage level but also in its higher-order moments.

3. Appendix B.2 provides more empirical evidence on the relationship between risk and leverage.

Corporate finance literature on persistent and transitory shocks

While the empirical and theoretical literature on capital structure is vast, only a handful of studies deal with the implications of transitory and permanent shocks for corporate policies and even less consider their effects on firm's leverage choice.⁴

Gorbenko and Strebulaev [2010] study financing policy in a model where firms can be exposed to both types of shocks and show that firms with more transitory shock exposure adopt conservative leverage policies, but the shocks interact additively. The shock separation results in an imperfectly correlated firm value and cash flow as well as between earnings and asset volatility. Décamps, Gryglewicz, Morellec, and Villeneuve [2016] extend this analysis by considering the effects of transitory and permanent shocks on investment, financing and liquidity policies. In their model, financing constraints increase the cash-flow sensitivity of cash and firms prefer to hoard liquid assets as their exposure to transitory shocks increases. Even though these papers do address the relationship between shock exposure and capital structure, they do not provide an explicit link with investment policy, which, as this paper shows, is the most important channel affecting firm's leverage through its risk composition.

Other papers investigate the empirical implications of separating the shocks. Chang, Dasgupta, Wong, and Yao [2014] use macroeconometric filters to decompose firm-specific cash flow into trend and cycle components, which can be interpreted as persistent and transitory parts of firm's cash flow. Their analysis implies that a one standard deviation shock to the persistent component of cash flow is associated with a 3.6% increase in investment rate and 2.5% decrease in book leverage, these effects are approximately 50% larger than the ones resulting from a shock to the transitory component. However, most of their analysis focuses on the investment-cash flow sensitivity and financial constraints but not the effect of shock composition. Byun, Polkovnichenko, and Rebello [2016] propose a dynamic investment model with cash in which the firm is subject to a transitory and an idiosyncratic shock and show that each has different implications for the dynamics of savings and investment. Their model contains no debt, however, and all shock processes are stationary, which is different from the setting considered in this paper. Lastly, and perhaps most importantly, this paper is closely related to the work of Gourio [2008], who structurally estimates a dynamic neoclassical model of investment with persistent and transitory shocks and shows that investment policy reacts much stronger to persistent shocks. While the model in this paper can replicate these

^{4.} In general, such decomposition of shocks dates back to Blundell and Preston [1998] who use the permanent income hypothesis to study consumption dynamics. More generally, models with a stationary and a non-stationary shock are popular e.g. in asset pricing, household finance or labor economics. Some (by no means exhaustive) examples include Kaltenbrunner and Lochstoer [2010], Adrian and Rosenberg [2008] Guiso, Pistaferri, and Schivardi [2005] or Zhu [2011].

findings, it also yields further predictions regarding the impact of shocks on capital structure.

Finally, this paper shares many features with the discrete-time neoclassical dynamic investment models of capital structure such as Hennessy and Whited [2005, 2007] or DeAngelo, DeAngelo, and Whited [2011], for example investment is endogenous and debt is risk-free and subject to a collateral constraint.

2 Model

The managers choose firm's investment and financing policy at each date until infinity, thus taking into account the fact that their today's decisions affect potential future choices. In particular, the managers choose real investment policy, debt or equity issuance and payout to debt- or equityholders. Debt is risk-free and debt issuance is costless but subject to a collateral constraint. External equity issuance is costly and subject to linear issuance costs, resulting from the underwriting costs or the adverse selection problem of Myers and Majluf [1984].

2.1 Model setup

Time is discrete and the time horizon is infinite. The risk-neutral firm is governed by managers discounting cash flows at rate r. Their incentives are fully aligned with shareholders. The firm uses capital K to produce output and the per-period profit function $\Pi(K, Z)$ depends on firm's capital K as well as profitability shock Z. The profit function is continuous, concave and satisfies the Inada conditions. The concavity of the profit function reflects the decreasing returns to scale faced by the firm. I further specify that $\Pi(K, Z) = (1 - \tau)ZK^{\theta}$, where τ is the corporate tax rate. The capital evolves according to $K' = I + (1 - \delta)K$ with capital depreciation rate $\delta \in (0, 1)$ and the capital adjustment costs are convex and defined as $A(K, K') = \psi/2 (I/K)^2 K$.⁵ The firm is also able to write off a part of its tax bill due to depreciation tax credit in the magnitude of $\tau \delta K$. The profitability shock Z consists of two components Z_P and Z_T corresponding to permanent and transitory parts, respectively. The law of motion for Z is multiplicative in both shocks and given

^{5.} Normally, as in Cooper and Haltiwanger [2006], a more comprehensive structure for capital adjustment costs which also includes fixed adjustment cost would be preferable, but I choose to focus on a simple formulation to keep the solution of the model tractable. Moreover, other studies such as DeAngelo, DeAngelo, and Whited [2011] or Nikolov and Whited [2014] report insignificant or very small estimated fixed adjustment costs of capital, which shows that they could be difficult to identify.

by:

$$Z = Z_P \times Z_T \iff \log(Z) = \log(Z_P) + \log(Z_T)$$
$$\log(Z'_P) = \log(Z_P) + \sigma_P \varepsilon'_P$$
$$\log(Z'_T) = \rho \log(Z_T) + \sigma_T \varepsilon'_T,$$
(1)

where ε'_i are iid standard normal and $\varepsilon'_T \perp \varepsilon'_P$. The shock Z_P takes values in a compact set $[\underline{Z}_P, \overline{Z}_P]$. The choice of shocks interacting multiplicatively, similar to how they are related in Décamps et al. [2016], is motivated by substantially simplifying the numerical solution. The main difference between an additive and multiplicative way of combining the shocks is connected to how they affect firms of different sizes. Intuitively, small firms are worse off when they face additive shocks, because large firms are less sensitive to shock realizations, whereas shock effects are proportional to firm size when they are modeled multiplicatively, as in this paper.

Firm's financing choices consist of internal funds (cash and current profits), costly external equity and risk-free debt. The stock of net debt P is defined as the difference between the stock of debt (D) and the stock of cash (C). This implies that we can write $D = \max(P, 0)$ and $C = -\min(P, 0)$ and thus P = D - C. Debt takes form of a riskless perpetual bond incurring taxable interest at a rate $r(1-\tau)$. The firm may also choose to hoard liquid assets to save on the costs of external equity issuance or to avoid depleting its debt capacity. However, the interest the firm earns on its cash balance is equal to $r(1-\tau)$, meaning that liquid assets earn a lower rate of return than the risk-free rate. Finally, as in DeAngelo, DeAngelo, and Whited [2011] and Hennessy and Whited [2005], the stock of debt is subject to a collateral constraint proportional to firm's capital: $P' \leq \omega K'$, where $\omega \in [0, 1]$.

This setup implies the following sources and uses of funds constraint defining firm's cash flows, which result in external equity issuance (if negative) or distributions (if positive):

$$E(K, K', P, P', Z_T, Z_P) = (1 - \tau) Z_T Z_P K^{\theta} + \tau \delta K$$

- [K' - (1 - \delta)K] - \psi/2 [(K' - (1 - \delta)K)/K]^2 K (2)
+ P' - [1 + r(1 - \tau)] P.

The firm's problem is to maximize the present value of its future cash flows by choosing the

investment and debt policies subject to equity issuance cost $\Phi(\cdot)$:

$$V(K_0, P_0, Z_{0,T}, Z_{0,P}) = \max_{\{K_{t+1}, P_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left(E(K_t, K_{t+1}, P_t, P_{t+1}, Z_{t+1,T}, Z_{t+1,P}) + \Phi(E(K_t, K_{t+1}, P_t, P_{t+1}, Z_{t+1,T}, Z_{t+1,P})) \right) \right]$$

The Bellman equation for the problem and the laws of motions of the shocks can be written as:

$$V(K, P, Z_T, Z_P) = \max_{K', P'} \left\{ E(K, K', P, P', Z_T, Z_P) + \Phi(E(K, K', P, P', Z_T, Z_P)) + \frac{1}{1+r} \mathbb{E}_{Z'_T, Z'_P} \left[V(K', P', Z'_T, Z'_P) \right] \right\},$$
s.t. $P' \leq \omega K',$

$$K' = I + (1 - \delta)K,$$

$$\log(Z'_P) = \log(Z_P) + \sigma_P \varepsilon'_P, \ \log(Z'_T) = \rho \log(Z_T) + \sigma_T \varepsilon'_T.$$
(3)

where the cost of raising external equity is modeled in reduced form as in Gomes [2001] or Hennessy and Whited [2005, 2007]:

$$\Phi(E(\cdot)) = [\eta E(\cdot)] \mathbb{1}_{E(\cdot) < 0}.$$

The numerical solution of the model is described in detail in Appendix A.

2.2 Optimal financing policy

In this subsection I provide further intuition underpinning the model by analyzing its optimality conditions. These follow closely Hennessy and Whited [2005, 2007] and DeAngelo, DeAngelo, and Whited [2011] given that the model belongs to the same class of discrete time dynamic capital structure models as in these papers. For simplicity, I assume that V is differentiable. I denote the Lagrange multiplier accompanying the collateral constraint as ξ' .

The optimal financing policy, obtained by taking the first-order condition of the Bellman equation with respect to P', satisfies the following equality:

$$1 + \eta \mathbb{1}_{E(\cdot) < 0} = \xi' - \frac{1}{1+r} \mathbb{E}_{Z'_T, Z'_P} \left[V_2(K', P', Z'_T, Z'_P) \right],$$

with $V_2(\cdot)$ being the derivative of the value function with respect to the second argument. The left-hand side contains the marginal benefit of debt financing. If the firm has a financing deficit $(E(\cdot) < 0)$, then an extra dollar of debt financing allows to avoid costly external equity financing today: the benefit of the extra dollar of debt is thus $1+\eta$. If the firm is running a financing surplus, then using an extra dollar of debt means that it can distribute an extra dollar to its shareholders and thus the benefit of debt is 1. To gain more intuition about the expected marginal costs of debt on the right-hand side, we may expand the first-order condition further by using the corresponding envelope condition for P. The first-order condition can thus be expressed as:

$$1 + \eta \mathbb{1}_{E(\cdot) < 0} = \xi' + \frac{[1 + r(1 - \tau)]}{1 + r} \mathbb{E}_{Z'_T, Z'_P} \left[(1 + \eta \mathbb{1}_{E'(\cdot) < 0}) \right].$$

The right-hand side can be seen as the expected principal and interest on debt that must be repaid tomorrow. The term $\eta \mathbb{1}_{E'(\cdot)<0}$ suggests that the marginal cost of debt is higher when the firm is expected to run financing deficit next period: raising an extra dollar of debt today implies debt repayment tomorrow and therefore a higher probability of having to issue costly external equity. The presence of the Lagrange multiplier ξ' implies that the marginal cost of debt is also higher when the firm expects to exhaust its debt capacity next period: choosing a high level of debt today results in less financial flexibility in the future, thus the cost of borrowing today includes the value lost when the firm loses the option to borrow in the future. The equation also shows that firm's financial and real policies are deeply intertwined: if any given firm's characteristic makes it invest more at optimum, it will also imply that the firm will want to preserve its debt capacity right now. This feature is particularly important given the effect of the persistent shock Z_P on firm's investment. This channel will be thoroughly investigated in the following section.

3 Main mechanisms

In this section I provide further explanation of the effects of persistent and transitory shocks, which constitute the main mechanisms underpinning the results of the paper. In particular, I analyze firm's investment and debt policy functions as well as the impulse response functions to the two shocks and demonstrate how shock composition affects various moments resulting from the model. I also discuss whether model-implied moments are informative about the parameters governing firm's risk composition.

Model calibration

I calibrate the model to build intuition about the interactions between the nature of risk faced by the firm and model-implied moments. As I do not want to target any particular moments and since the model belongs to the same class of models as Hennessy and Whited [2005, 2007] or DeAngelo, DeAngelo, and Whited [2011], I set the parameter values close to the estimates resulting from these papers, given that they constitute a plausible starting point. All parameter values are summarized in Table 1. In particular, I set the interest rate r at 0.02, the curvature of the profit function θ at 0.75, the corporate tax rate τ at the statutory rate of 35%, the depreciation rate δ at 0.15, the capital adjustment cost parameter ψ at 0.1 and the external financing cost η at 0.15. The collateral constraint parameter ω is set at 0.6, implying that firms cannot raise more than 60% of their concurrent capital value as debt. This value of the collateral constraint parameter does not appear to be restrictive given than the 95th percentile of firm-level leverage distribution is 0.65 and 0.49 for industry-level data and their net leverage counterparts are 0.59 and 0.4, respectively.

The parameters driving the shock processes used in this exercise were chosen such that the total volatility of the shocks is equal to a value from the interval 0.15–0.35, which is close to the estimates from Nikolov and Whited [2014] and between the relatively high estimates from DeAngelo, DeAngelo, and Whited [2011] and the lower estimates such as those in Hennessy and Whited [2005, 2007] or Riddick and Whited [2009]. For the assumed parametrization, any level of total volatility below the lower bound of the interval produces leverage ratios equal to the collateral constraint. Persistent shock volatility varies between 0.00 and 0.05; the upper end of the interval is close to the value of 0.07 in Gourio [2008].⁶ Finally, the persistence of the transitory shock varies between 0.00 and 0.80.

r	0.02
au	0.35
θ	0.75
δ	0.15
ψ	0.10
η	0.15
ω	0.60
ρ	0.00 - 0.80
σ	0.15 - 0.35
σ_P	0.00 - 0.05
	$ \begin{array}{c} r \\ \tau \\ \theta \\ \delta \\ \psi \\ \eta \\ \omega \\ \rho \\ \sigma \\ \sigma_P \end{array} $

Table 1: Baseline parameters used in the calibration of the model. The persistent shock volatility is implied by the equality $\sigma = \sqrt{\sigma_T^2 + \sigma_P^2}$.

6. As explained in Appendix A, it is not possible to solve the model for an arbitrary value of persistent shock volatility σ_P , as it is closely related to θ , whose higher value limits the plausible range of σ_P . Therefore I only consider 'small' values which are nevertheless consistent with extant literature and intuition concerning persistent shocks.

3.1 Policy functions

I first analyze the simulated policy functions from the model to gain more understanding about the main channels driving the results.⁷ I solve the model using the baseline calibration to investigate how firm's policy functions vary when its risk composition changes. I plot the optimal investment and debt change decisions as a function of the underlying transitory shock.

Figure 2 presents the basic policy functions for investment and net debt change scaled by capital. The graphs reveal that risk composition matter for corporate policies. When the firm becomes more exposed to the persistent shock for the same level of total volatility, its sensitivity to the transitory shock decreases, which is reflected by the fact that its policy functions become flatter. The firm disinvests less when being hit by a negative transitory shock but also invests less when a positive realization of the transitory shock occurs. This result is intuitive given that persistent shock matters more when firm's exposure to this shock rises, but the shapes of policy functions are nevertheless instructive in showing that even small persistent shock exposure may have significant effect on firm's policies.⁸



Figure 2: The figure contains the policy functions for investment and net debt change scaled by capital for different levels of persistent shock exposure. The policy functions are computed at the average capital level and the average debt level. In the graph, transitory shock persistence ρ is set at 0.6 and total volatility σ at 0.15. The computation method for the policy functions is explained in detail in Appendix A.

It is also worth noting that while the shape of the policy function for debt change closely follows that

7. The policy functions take the form of $\{i, \Delta p\} = h(k, p, Z_T)$. The solution method leading to this type of policy function is explained in Appendix A. Note in particular that persistent shocks do not explicitly enter the policy function but rather affect the choice of k and p directly when detrending K and P by the persistent shock.

8. The magnitude of the differences between policy functions for different volatility compositions is unlikely to be explained by changing transitory shock volatility, which only varies by 0.002.

of investment, the 'relative' importance of debt in funding investment differs markedly as volatility composition changes. To see this, note that in the extreme case when the firm is being hit by the highest positive transitory shock, it invests roughly 60% of current capital when $\sigma_P = 0.02$ and 40% otherwise, but while in the first case it funds roughly half of the investment using debt (30% of current capital), in the second case it only funds a quarter. This result is due to the fact that the firm relies more on external equity financing and internal funds so as to preserve its debt capacity to fund higher future investment. This shows that the intuition from DeAngelo, DeAngelo, and Whited [2011], also visible in the optimal financing policy, prevails in the model.

Finally, while not shown in the graphs, when the importance of transitory shocks rises, that is when total volatility increases and persistent shock volatility is kept constant, the policy functions for the two different values of σ_P converge and become qualitatively similar.

3.2 Impulse response functions

I turn to investigating the impulse response functions of the two shocks to gain further intuition about their differential impact on firm's policies. Figure 3 presents the percent deviation from the steady state of capital and debt stock following a one standard deviation transitory or persistent shock.



Figure 3: The figure contain the impulse response functions computed for a firm experiencing a one standard deviation transitory or persistent shock at time t = 6. The left graph examines capital stock while the right graph net debt stock. The values are computed as a % deviation from the 'steady state' value in which the firm is not subject to any shocks. In the graph, transitory shock persistence ρ is set at 0.6, persistent shock volatility σ_P to 0.04 and total volatility σ at 0.15.

There are several implications regarding the distinctive effect of the two shocks suggested by the

figure. First, It is crucial to notice that both shocks matter for firm's policies. However, persistent shock permanently increases the level of capital and net debt, whereas the impact of a transitory shock vanishes over time. Moreover, the effect of a persistent shock is much stronger than that of a transitory shock – at the 'peak' in each graph, the firm hit by a persistent shock departs from its steady state capital (or net debt) more than a firm hit by a transitory shock, even despite the differences in magnitude of the initial shocks. Furthermore, the windows over which different shocks affect the firm also differ: while the effect of a transitory shock to fully translate to a new stable level of capital or net debt. Nevertheless, the 'speed of adjustment' is affected by the capital adjustment cost parameter – higher values imply that it takes longer for the effect of the shock to take a full effect.⁹ Thus, even in absence of convex adjustment costs the adjustment of capital stock is slower, meaning that the observed investment policies exhibiting substantial smoothness, can be caused by either of convex adjustment costs or the presence of persistent shocks, if not both.

All in all, this evidence suggests that each shock has a very different effect on firm policies which translates to markedly different dynamics once the firm is subject to both over time. Even though the long-term impact of each shock is neutral, given that the effect of a transitory shock vanishes over time and that debt and capital stock adjust in the same proportion following a persistent shock, there are nevertheless substantial short run fluctuations and, moreover, different adjustment patterns. Both are bound to affect the dynamic aspects of firm policies in a distinctive way, as the firm is hit by different shocks every period. The analysis of the subsection thus shows that the shocks are truly different and shock dynamics cannot be replicated by a model in which the firm is subject to two transitory shocks of different persistence. Appendix C provides further backing to the claim by formally comparing these two types of models in different dimensions.

3.3 Fundamental risk and model-implied moments

The last important issue related to examining the main mechanisms of the model concerns linking the intuition from the previous two subsections to the actual moments implied by the model. Table 2 contains the values of selected moments resulting from simulating the model using different values of the three parameters describing the nature of risk faced by the firm: the persistent shock volatility σ_P , the transitory shock persistence ρ and the transitory shock volatility σ_T .¹⁰

^{9.} The qualitative differences in IRFs between the results of this paper and those in Gourio [2008] are due to the fact that the latter assumes $\psi \approx 8$ while in this paper $\psi = 0.1$.

^{10.} Appendix D contains the comparative statics of parameters related to real or financing frictions such as the capital adjustment cost ψ , the external equity issuance cost η or the collateral constraint parameter ω .

Results in Table 2 imply that average investment as well as variation in investment increase with persistent shock exposure. This result, consistent with Gourio [2008], is important given that investment constitutes the main reason for debt issuance in the model, thus drives the dynamics of leverage. Persistent shocks appear to have large influence on the dynamics of investment, as shown by the correlations between various moments. In particular, they substantially increase long-run persistence in investment, which is consistent with the evidence from Gourio [2008] and DeBacker et al. [2013] that the effect of persistent shocks can be seen in higher order autocorrelations. Finally, higher persistent shock exposure increases the incidence of disinvestment, in line with the intuition that firm's policies are more sensitive to persistent shock realizations. If the firm experiences a negative persistent shock, then it is more likely to conduct an asset sale, because its cash flows will be forever affected by this shock realization.

The moments related to firm's debt policy, which are extensively discussed in Section 4, reveal the negative relationship between persistent shock exposure and average leverage. Similarly as in case of investment, higher persistent shock volatility increases leverage variation and leverage persistence and these outcomes are closely related to the behavior of investment-related moments.

Not unexpectedly, shock composition also holds significance for the dynamics of profits and profitability as well as for the correlations between profit and growth variables. However, it is important to note the distinction in how persistent shocks affect profitability and log profits: their importance for the former is limited while substantial for the latter, which results directly from detrending profits Π by capital K, both of which are affected by persistent shocks. They also interact with transitory shock persistence in a subtle way, making it difficult to distinguish between the effects of these two channels. Similar rationale explains the decreasing correlation between investment and profitability as well as between (log) growth and profitability. These issues are thoroughly discussed in Section 4.

Finally, it is important to note that persistent shock exposure greatly increases the degree of dispersion in the simulated moments, which is documented by the median absolute deviations of average investment and leverage.

		Total volatility σ	0.15	0.15	0.15	0.15	0.15	0.15	0.25	0.25	0.25	0.25	0.25	0.25
		Transitory shock persistence ρ	0.20	0.20	0.20	0.60	0.60	0.60	0.20	0.20	0.20	0.60	0.60	0.60
		Persistent shock volatility σ_P	0.00	0.02	0.04	0.00	0.02	0.04	0.00	0.02	0.04	0.00	0.02	0.04
1.		Average investment (i/k)	0.151	0.152	0.153	0.154	0.155	0.155	0.151	0.152	0.153	0.156	0.158	0.159
2.		Standard deviation of investment (i/k)	0.031	0.041	0.069	0.093	0.095	0.099	0.041	0.046	0.065	0.106	0.122	0.131
3.	int	Autocorrelation of investment $\phi_1(i/k)$	0.007	0.212	0.315	0.192	0.256	0.285	-0.109	0.132	0.258	0.225	0.214	0.248
4.	me	Autocorrelation of investment $\phi_3(i/k)$	-0.103	-0.028	-0.007	-0.106	-0.106	-0.066	-0.083	-0.029	0.002	-0.095	-0.106	-0.088
5.	est	Frequency of disinvestment $\#(i/k < 0)$	0.000	0.000	0.016	0.042	0.044	0.062	0.000	0.001	0.040	0.053	0.086	0.106
6.	Inv	Corr. inv. and profitability $\operatorname{corr}(i/k, \pi/k)$	0.899	0.693	0.500	0.877	0.883	0.775	0.700	0.603	0.432	0.855	0.846	0.836
7.		Autocorrelation capital stock $\phi_1(K)$	0.667	0.871	0.896	0.775	0.817	0.863	0.564	0.847	0.893	0.794	0.799	0.832
8.		Dispersion of average investment $MAD(\overline{i/k})$	0.002	0.010	0.020	0.010	0.014	0.022	0.003	0.010	0.020	0.012	0.016	0.023
9.		Average leverage (p/k)	0.596	0.590	0.375	0.578	0.548	0.497	0.567	0.512	0.381	0.320	0.201	0.172
10.	ıge	Standard deviation of leverage (p/k)	0.003	0.003	0.030	0.015	0.029	0.030	0.021	0.034	0.041	0.043	0.064	0.074
11.	erê	Persistence of leverage $\rho(p/k)$	0.108	0.267	0.539	0.396	0.455	0.518	0.226	0.491	0.632	0.528	0.693	0.729
12.	Lev	Volatility of leverage $\sigma(p/k)$	0.003	0.003	0.024	0.014	0.020	0.024	0.016	0.028	0.023	0.035	0.043	0.047
13.	_	Dispersion of average leverage $MAD(\overline{p/k})$	0.001	0.001	0.011	0.005	0.013	0.020	0.008	0.012	0.025	0.012	0.025	0.034
14.		Persistence of profitability $\rho(\pi/k)$	0.082	0.089	0.110	0.337	0.335	0.352	0.079	0.088	0.093	0.347	0.337	0.339
15.		Volatility of profitability $\sigma(\pi/k)$	0.066	0.066	0.067	0.069	0.067	0.067	0.123	0.121	0.121	0.144	0.141	0.139
16. 17. 18.	fits	Persistence of log profits $\rho(\log(\Pi))$	0.225	0.406	0.613	0.707	0.739	0.781	0.191	0.259	0.411	0.620	0.661	0.702
	2 ro	Volatility of log profits $\sigma(\log(\Pi))$	0.149	0.160	0.175	0.156	0.157	0.161	0.257	0.268	0.278	0.263	0.266	0.267
	1	Corr. prof. and growth $\operatorname{corr}(\pi/k, k'/k)$	0.903	0.696	0.499	0.880	0.886	0.780	0.704	0.434	0.607	0.858	0.849	0.839
19.		Corr. prof. and log gth. $\operatorname{corr}(\pi/k, \log(k'/k))$	0.903	0.695	0.498	0.872	0.882	0.776	0.699	0.431	0.598	0.843	0.831	0.819

Table 2: Summary statistics of model-implied moments for different values of σ , ρ and σ_P . Remaining parameters are taken as specified in Table 1. An AR(1) model $x_{i,t+1} = a_i + \rho_i x_{i,t} + \varepsilon_{i,t+1}$ was fit for each simulated firm to compute persistence ρ_x or volatility σ_x of variable x. ϕ_k is the k^{th} order autocorrelation. The measure of dispersion MAD is median absolute deviation. The numerical solution and simulation of the model is described in detail in Appendix A.

3.4 Identifying fundamental risk

Seeing that the effects of different parameters describing firm's fundamental risk may wield similar influence on its policies, it is important to ask whether we can infer their magnitude by considering the model-implied moments. Thus, the identification of risk characteristics should result from analyzing all policy choices of the firm. Ideally, we would want to identify each parameter by a single moment, in which case changing the parameter would cause only that moment to vary. However, many directional effects of parameters are similar (cf. Table 2). Therefore, we have to consider the overall relationship between risk characteristics and resulting corporate policies.

The main concerns that we have to address are related to examining the relationship between persistent shock volatility σ_P and capital adjustment cost ψ as well as distinguishing between the effect of persistent shock Z_P and the transitory shock Z_T .

First, the shape of the impulse response functions implies that firm's capital and debt policies are more smooth following persistent shock realization. As such, we could expect that changing persistent shock exposure could have similar implications as increasing the convex capital adjustment cost ψ . However, it turns out that the effect of changing persistent shock volatility σ_P is significantly different than the effect of changing ψ , which increase average leverage and decrease investment variation, thus resulting in lower leverage volatility.¹¹ These effects are qualitatively different than those of varying firm's exposure to persistent shock.

Second, changing volatility and persistence has a similar effect on model-implied moments, no matter whether the source of change comes from the transitory shock or the persistent shock. Considering these characteristics jointly is crucial given the two-faceted nature of persistent shocks, which affect both at the same time. Therefore it is important to ask if we can infer the relative importance of persistent shock volatility σ_P in total volatility σ . There are several moments that could provide insight about the parameters describing firm's fundamental risk. For example, distinguishing between profit persistence and profitability persistence is informative: the former is greatly affected by persistent shocks, while the latter being insensitive, as detrending of profits II by capital K removes a part of their effect. However, there are also other channels which may help tell the two shocks apart. Most importantly, volatility composition as well as total volatility level, while holding transitory shock persistence ρ constant, greatly affect investment, leverage and profit autocorrelation and the direction of change is different for the two shocks. Therefore, these moments are informative about the magnitude of volatility parameters. Considering the distinction between ρ and σ_P , the differential effect of these parameters can be seen in higher-order autocorrelations (which increase with persistent shock exposure but decrease when ρ rises), corre-

^{11.} See also comparative statics of ψ in Table 9 in Appendix D.

lations between investment and profitability or correlations between profitability and log growth (which decrease with σ_P but rise when transitory shock volatility is increased). Furthermore, in spite of the same qualitative effect of both of these parameters on model-implied moments, their sensitivity, that is the quantitative effect, may be different.

Identification of the remaining parameters is fairly standard, e.g. as in DeAngelo, DeAngelo, and Whited [2011] or Nikolov and Whited [2014]. For example, the external equity financing cost η can be identified off its effect on investment and leverage while the collateral constraint parameter ω from the dynamics of leverage, which it significantly affects.¹²

4 Risk composition and capital structure

In this section I analyze the main implications of the model for the relationship between risk and capital structure. I focus on two characteristics of the fundamental risk: volatility and persistence. In particular, I highlight how the composition of firm's fundamental risk affects its capital structure characteristics and provide a thorough analysis of all the parameters describing firm's fundamental risk resulting from its cash flows. I also consider how analyzing risk composition helps explain the observed heterogeneity in corporate policies.

4.1 The fundamental volatility channel

In the model, fundamental risk affects leverage primarily through its effect on investment policy. One dimension of firm's fundamental risk is its fundamental volatility. High total volatility implies that there is a higher chance that large investment is optimal, so the firm preserves its debt capacity as it places a higher value on its option to borrow to fund higher investment. On the other hand, low volatility firms have more predictable cash flows, thus they do not value preserving their debt capacity as much to address their funding needs and adopt higher leverage. The first main channel through which the nature of firm's risk affects its capital structure is related to the composition of its fundamental volatility, which goes beyond the effect of total volatility. This is because persistent shocks reinforce the risk-leverage trade-off by increasing the size of investment outlays and making the profitability of investment more persistent. These effects result in firms placing even higher value on their ability to borrow, which further reduces their optimal leverage ratios. Importantly, firms more exposed to persistent shocks not only use less debt financing, but

^{12.} See also Appendix D for comparative statics of the parameters governing the cost of equity financing η and the tightness of collateral constraint ω .

also more internal funds, which is highlighted by the analysis of policy functions. Furthermore, as suggested by the impulse response functions, firms spread out their investment outlays over time following a positive realization of a persistent shock, which reduces their need to use external finance even further.

Indeed, higher persistent shock exposure results in a lower optimal leverage for the same level of total volatility. Figure 4 illustrates the negative association between persistent shock exposure and leverage for different levels of total volatility σ . It documents that the relationship between volatility composition and leverage crucially depends on total volatility. For high values of σ , firm's exposure to persistent shock is relatively small and increasing it further has muted effect on firm's debt policy. However, when firm's cash flow process contains a relatively larger persistent part, then its leverage is sensitive to changing the volatility composition. Even firms with very low total volatility of $\sigma = 0.15$, which otherwise would lever up to their collateral constraint, prefer to substantially decrease their leverage ratio when increasing the importance of the persistent component in their cash flow process. This observation provides an alternative explanation for the long-standing debt conservatism puzzle, as shock decomposition is not an additional financing friction, but merely allows for a more flexible definition of firms' cash flow process.

Another implication of Figure 4 is that the one-to-one link between total volatility and leverage, present in standard capital structure models, is broken. In other words, while extant models are able to explain the values in the graph when $\sigma_P = 0$, they are unable to generate firms with the same optimal leverage but different total volatility or firms with distinctive debt ratios but the same total volatility. Both of these cases can be obtained in the model, which reinforces the claim that volatility composition may be able to explain a portion of the dispersion in the risk-leverage relationship illustrated in Figure 1.

Fundamental volatility and leverage dynamics

Volatility composition also has important implications for moments describing capital structure dynamics such as leverage variation (represented by its standard deviation) or leverage persistence (captured by its first-order autocorrelation).¹³ As argued by Baranchuk and Xu [2007, 2011], these moments appear to vary at least as much as average debt ratios themselves. Standard leverage factors fail to explain their variation in the data and have even lower explanatory power than for average debt ratios. Figure 5 shows how these characteristics differ depending on firm's exposure to persistent shock and the level of total volatility.

13. Note that these moments are closely related to leverage volatility and persistence, as described in Table 2.



Figure 4: The figure contains the model-implied average leverage as a function of total volatility σ (number above each line) and persistent risk volatility σ_P . The transitory shock volatility σ_T was computed such that the total volatility was constant on each line. In the graph, the transitory shock persistence ρ is set at 0.6.

Leverage variation increases not only when total volatility rises, but also as persistent part of firm's cash flows becomes more important. Furthermore, its sensitivity to σ_P is the greater, the higher firm's total volatility, which highlights the fact that when persistent shock volatility constitutes a lower share of total volatility, their effect could still be visible in certain moments. The channel through which persistent shocks affect leverage volatility is related to variation in investment. As shown by the comparative statics in section 3.3, firm's investment policy becomes more volatile when the share of σ_P in total volatility rises, as it tends to disinvest substantially more. In other words, firm's investment policy is very sensitive to the realizations of the persistent shock, which translates to highly variable debt policy.

Leverage persistence increases with both total volatility and persistent shock exposure, but is much more sensitive to the latter. The differences between various levels of total volatility remain relatively constant when varying firm's shock exposure even when changes made to persistent shock volatility are small in comparison to the magnitude of varying total volatility. For very high values of total volatility, however, leverage persistence may also decrease with σ_P due to the fact that



Figure 5: The figure contains the model-implied average standard deviation of leverage (left graph) or average first-order autocorrelation of leverage (right graph) as a function of total volatility σ (ranging from 0.15 to 0.25) and persistent shock volatility σ_P . The transitory shock volatility σ_T was computed such that the total volatility was constant on each line. In the graph, transitory shock persistence ρ is set at 0.6.

the firm is then increasingly more likely to hold low level of debt or even cash, which mutes the state-dependence of leverage.

4.2 The fundamental persistence channel

The analysis in previous subsection suggests that the composition of total volatility plays an important role in shaping firm's debt policy. However, it focuses on only one particular dimension of firm's fundamental risk related to fundamental volatility, which captures the magnitude of shocks. Another important characteristic of fundamental risk, and one that has not attracted much attention in the literature, concerns how long the effects of shocks are expected to affect cash flows. A firm is likely to behave differently if its cash flows are subject to shocks of large magnitude but which reverse quickly or shocks that may have lower magnitude but whose effects last for many periods. Thus, different persistence of cash flows is bound to result in different firm policies. For example, if hit by a positive shock, the firm may be incentivized to invest more if the effect on cash flows is more lasting to take advantage of the investment opportunity that persists. As such, shock persistence directly affects investment policy and thus firm's financing choices, given that the firm has to raise internal or external funds to cover increased capital expenditure.

Even if these theoretical arguments appear sound, the data suggests that the between-industry variation in profit persistence is smaller than the variation in other firm characteristics. Figure 6

shows that the average estimated coefficients of profit persistence for different industries, computed assuming that firm's log real profits follow an AR(1) process as usually done in practice, are strongly positively skewed and cluster around a high value or 0.8. For approximately 15% of industries they also assume values greater or equal than 1, which further highlights the need to consider a more flexible setting able to cover the potential non-stationarity of profits. Further examination of the data reveals that these estimated coefficients are not significantly related to leverage or other firm characteristics. This is evident when considering correlations between average firm size, leverage, investment, asset tangibility or market-to-book ratio, as well as other variables, and the estimated measures of profit persistence: all resulting values are negligibly small.¹⁴ Moreover, even when extracting the value of ρ using structural estimates for different industries, as in DeAngelo, DeAngelo, and Whited [2011], its explanatory power for the cross-sectional variation in average leverage is still weak or modest at best. These findings are strikingly at odds with the evidence resulting from this model, in which the comparative statics of the transitory shock persistence ρ in Table 2 suggest that it has a strong and robust effect on model-implied moments.



Figure 6: Histogram of the average estimated persistence parameter ρ of log real operating profits $\log(\Pi)$ of firms in 4-digit SIC industries. The estimate of the persistence parameter ρ was computed using an AR(1) fit of log real profits for each firm and then averaged over all firms in an industry. Definitions of variables are provided in Appendix B.

Based on these results and on the intuitive notion of the nature of uncertainty discussed earlier,

14. See appendix B.3 for detailed empirical evidence on the relationship between persistence and leverage as well as other firm characteristics.

we could suspect that changing persistence should play an important role in firms' investment and leverage decisions. However, the channel through which it takes effect must be different than the one implied by standard models with a single transitory shock in which all persistence comes from ρ . I argue that shock composition discussed in this paper offers a convincing alternative explanation for these phenomena and that it is also able to justify the disparity between model-based evidence and the data.

Decomposing fundamental persistence

There are two sources of cash flow persistence in the model. First, any realization of the persistent shock impacts all future values of cash flow, greatly increasing the persistence in firms policies as illustrated by the impulse response functions in Figure 3. However, the share of persistent shock volatility in total volatility may be small, thus it is not clear whether their overall contribution to total persistence is always large. Second, transitory shocks can also affect the overall persistence, as they are path-dependent, but plausibly to a much lesser extent than persistent shocks given their transient nature. As such, both sources could be vital for determining firm's debt policy.

Figure 7 illustrates the differential effect of the two persistence channels by plotting the modelimplied average profit persistence as a function of transitory shock persistence ρ for different levels of persistent shock volatility σ_P . Both ρ and σ_P strongly affect the level of profit persistence. However, it is also important to notice that observable profits can be substantially path-dependent even when $\rho = 0$ if the cash flow process contains a small persistent part. When ρ takes small to moderate values, the effect of risk composition on profit persistence is the stronger, the higher is the share of persistent shock volatility in total volatility. However, when the transitory shock persistence is very high, then the additional persistence stemming from persistent shock is fairly small.

The effect of transitory shock persistence ρ and persistent shock volatility σ_P on model-implied profit persistence may vary depending on the level of total volatility, which determines the relative importance of the two channels. To this end, I analyze the elasticities of model-implied profit persistence to changing these parameters for different values of total volatility σ . Figure 8 contains the results which suggest that the effect of changing σ_P on profit persistence is only important when ρ assumes low or modest values. Its significance also decreases as the total volatility is increased, as then transitory shocks become relatively more important. On the other hand, ρ appears to wield substantial influence on profit persistence for different levels of σ_P , but is less vital when firm's persistent shock exposure is high. The fact that the elasticity of profit persistence to ρ decreases when the firm is not exposed to the persistent shock represents another important result



Figure 7: The figure contains the model-implied average persistence of log profits $\rho(\log(\Pi))$ as a function of transitory shock persistence ρ for different levels of persistent shock volatility σ_P . The total volatility is set to $\sigma = 0.15$.

not directly represented by the graphs, that is the negative relationship between total volatility and persistence. In general, higher values of σ result in lower average profit persistence, all else equal, implying that volatility composition plays a vital role in determining fundamental persistence as well.

Finally, the evidence presented in this subsection suggests that the model provides more flexibility in terms of being able to generate firms with the same level of profit persistence but different values of transitory shock persistence ρ . In particular, this implies that the *true*, unobservable ρ could vary widely between firms or industries despite observing very similar, potentially high, values of profit persistence. As such, it is not surprising that the empirical association between estimated profit persistence and firm characteristics is weak, because it is the variation in unobservable parameters describing the overall fundamental risk of the firms that wields more influence on their characteristics.

Fundamental persistence and leverage

Having examined the extent to which the two channels generate model-implied profit persistence and documented that different parameters affect profit persistence to different extent, it is also



Figure 8: The figure contains the elasticity computed at average moments $(\partial m/\partial x) \times (\overline{m}/\overline{x})$ of log profit persistence $\rho(\log(\Pi))$ to changing persistent shock volatility σ_P for different levels of ρ (left graph) or to changing transitory shock persistence ρ for different levels of σ_P (right graph). The elasticities are computed as functions of total volatility σ .

important to ask how the two channels of persistence affect firm's leverage policy. Figure 9 contains the elasticity of the model-implied average leverage to changing transitory shock persistence ρ for different level of total volatility σ and persistent shock volatility σ_P . While the elasticities are always negative, highlighting that persistence and leverage are negatively related, the graph reaffirms the claim that each source of persistence may have a distinctive quantitative effect on firm's debt policy, depending on its overall fundamental risk. In particular, it shows that ρ can have a different effect on leverage depending on σ_P .

First, changing ρ appears to affect leverage for any given risk composition. Second, firms not exposed to persistent shocks are always more sensitive to changing ρ than firms whose cash flow also contains a small persistent component. Finally, when σ_P constitutes a large share of total volatility (for example when $\sigma = 0.15$ and $\sigma_P = 0.04$), changing ρ may have negligible effect on firm's average leverage. In other cases, however, the effect is expected to be sizeable.

4.3 Implications for capital structure heterogeneity

A vast amount of corporate finance research focusing on understanding the variation in corporate leverage ratios recognizes that capital structure heterogeneity remains largely unexplained by existing factors based on firm characteristics such as size, profitability, asset tangibility or the degree



Figure 9: The figure contains the elasticity of leverage to changing transitory shock persistence computed at average moments $(\partial lev/\partial \rho) \times (\overline{lev}/\overline{\rho})$ as a function of total volatility σ and persistent shock volatility σ_P .

of financial constraints.¹⁵ Lemmon, Roberts, and Zender [2007] and Graham, Leary, and Roberts [2015] examine the underlying reasons as to why these factors fare poorly and point out that firm fixed effects provide substantial incremental explanatory power. It is therefore important to understand what these fixed effects contain. Examining deep structural parameters of the model, for example these governing firm's cash flow process, could therefore help rationalize the importance of the time-invariant effects seen in the data. The evidence presented in this section suggests that studying the nature of risk affecting firms is likely to provide further insight regarding the variation in corporate policies.

There are two main reasons why studying risk exposure could provide further explanatory power above and beyond standard leverage factors. First, the two-industry example from the introduction as well as the evidence in section 4.1 suggest that while two firms may have similar observable risk, their leverage could be markedly different, because the composition of their fundamental volatility is distinctive. As such, separating total volatility into transitory and persistent components would enhance the ability of the standard cross-sectional regression in explaining the data.

^{15.} The factors used in empirical studies are based on various theories of capital structure, summarized e.g. in Harris and Raviv [1991]. Titman and Wessels [1988] and Rajan and Zingales [1995] examine these factors extensively while Strebulaev and Yang [2013] consider whether they can explain the zero-leverage puzzle.

Second, composition of fundamental persistence is also informative of firm's leverage policy. The model has the appealing feature of allowing to change the transitory shock persistence ρ without changing profit persistence $\rho(\log(\Pi))$ much. To illustrate how this helps in explaining variation in firms' policies, let us consider the following example. Suppose that we fit an AR(1) model to firms' log real profits, as frequently assumed in empirical studies. Suppose further that we observe $\hat{\rho} \approx 0.6$ for two firms which also adopt different leverage ratios of 0.59 and 0.36. The ability of $\hat{\rho}$ to explain capital structure variation would be low in this case. However, the model suggests that it could very well be the case that the composition of firms' risk is such that the first one has $\rho = 0.4$ and $\sigma_P = 0$ while the other $\rho = 0$ and $\sigma_P = 0.04$. In this case, both persistent shock volatility σ_P as well as transitory shock persistence ρ would be able to provide additional insight concerning leverage heterogeneity.

Overall, I argue that the three parameters governing the processes $(\rho, \sigma_T \text{ and } \sigma_P)$ may not only provide explanatory power over σ_{total} or $\hat{\rho}$ in explaining the capital structure variation in the data, but that their incremental contribution to explaining the heterogeneity will also vary. The marginal effect of each parameter characterizing firm's fundamental risk will in general depend on the overall risk composition, as seen in the analysis of the elasticities of model-implied moments to these parameters. As an example, when the firm is much more exposed to the transitory shock, then changing its persistence has a much greater effect on firms policies than when the firm is more exposed to the persistent shock.

5 Conclusions

In this paper I argue that the nature of firm's fundamental risk, captured by the composition of its cash flow process, has important implications for firm's capital structure characteristics. Crucially, the two channels through which firm's fundamental risk affects capital structure are fundamental volatility and fundamental persistence. The model is able to explain several empirical patterns, in particular the dispersion in the risk-leverage relationship and the low observed variation in profit persistence. It also highlights that risk composition may provide additional explanatory power for capital structure heterogeneity, above and beyond standard leverage factors.

However, the paper is silent on the actual extent to which fundamental risk affects firms and it has to be taken to the data. Measuring firm's fundamental risk could prove challenging, given that it is inherently unobservable. Empirical studies often resort to stock returns to measure firm's risk, but any stock price-based measure is unlikely to provide much insight about profitability shock persistence, as stock returns are approximately iid. Statistical filtering of cash flow into different components, while well-suited to decompose aggregate processes, may be inappropriate for firm-level analysis given the low number of firm-level observations available and the ambiguity about its true dynamics. It may be unable to identify the correct magnitude of fundamental risk characteristics, because it does not take into account firm's policies, which are informative about risk composition. To alleviate some of these concerns, one can employ structural estimation to extract a measure of risk by quantifying how firms perceive their own risk exposure. In this estimation method, the theoretical structure of the model is used to interpret the observed data by ascertaining that it resembles the model-generated behaviour. In short, the observed corporate policy choices are used to infer the magnitudes of risk exposures of an average firm.

Appendices

A Model solution

A.1 Transforming the problem

Given the non-stationarity of permanent shocks, the state space is unbounded. However, as in Gourio [2008, 2012], one can define 'detrended' variables to make the state space bounded and reduce the dimensionality of the problem. Given the homogeneity of the value function, we can make (and verify) the following guess:

$$V(K, P, Z_T, Z_P) = Z_P^{\frac{1}{1-\theta}} v(k, p, Z_T),$$

which implies the following laws of motion for:

1. Profit function:

$$\Pi = (1 - \tau)ZK^{\theta} = (1 - \tau)Z_P Z_T K^{\theta}$$
$$\iff \pi = \Pi/Z_P^{\frac{1}{1-\theta}} = (1 - \tau)Z_T \left(K/Z_P^{\frac{1}{1-\theta}}\right)^{\theta} = (1 - \tau)Z_T k^{\theta},$$

where $k = K/Z_P^{\frac{1}{1-\theta}}$.

2. Capital:

$$k' = \frac{K'}{Z_P'^{\frac{1}{1-\theta}}} = \frac{K'}{Z_P^{\frac{1}{1-\theta}}} \frac{Z_P^{\frac{1}{1-\theta}}}{Z_P'^{\frac{1}{1-\theta}}} = (k(1-\delta)+i) \exp\left(-(1-\theta)^{-1}\sigma_P \varepsilon_P'\right),$$

where $i = I/Z_P^{\frac{1}{1-\theta}}$.

3. Net debt. Let $\Delta P \equiv P' - P$:

$$p' = \frac{P'}{Z_P'^{\frac{1}{1-\theta}}} = \frac{P'}{Z_P^{\frac{1}{1-\theta}}} \frac{Z_P^{\frac{1}{1-\theta}}}{Z_P'^{\frac{1}{1-\theta}}} = (p + \Delta p) \exp\left(-(1-\theta)^{-1}\sigma_P \varepsilon_P'\right),$$

where $\Delta p = \Delta P/Z_P^{\frac{1}{1-\theta}}$ and $p = P/Z_P^{\frac{1}{1-\theta}}$.

The resulting problem is:

$$v(k, p, Z_T) = \max_{k', p'} \left\{ e(k, k', p, p', Z_T) + \phi(e(k, k', p, p', Z_T)) + \frac{1}{1+r} \mathbb{E}_{Z'_T, \varepsilon'_P} \left[\exp\left((1-\theta)^{-1} \sigma_P \varepsilon'_P\right) v(k', p', Z'_T) \right] \right\}.$$

To remove the problem of the dependence of k' and p' on ε'_P , I transform the problem into an equivalent one by maximizing over i and Δp rather than over k' and p'. This transformation of the problem is without loss of generality given the laws of motion for capital and net debt. The ultimate formulation of the problem is thus:

$$\begin{aligned} v(k, p, Z_T) &= \max_{i, \Delta p} \left\{ e(k, i, p, \Delta p, Z_T) + \phi(e(k, i, p, \Delta p, Z_T)) \\ &+ \frac{1}{1+r} \mathbb{E}_{Z'_T, \varepsilon'_P} \left[e^{(1-\theta)^{-1}\sigma_P \varepsilon'_P} v\left((k(1-\delta)+i)e^{-(1-\theta)^{-1}\sigma_P \varepsilon'_P}, (p+\Delta p)e^{-(1-\theta)^{-1}\sigma_P \varepsilon'_P}, Z'_T \right) \right] \right\} \\ \text{s.t. } e(k, i, p, \Delta p, Z_T) &= (1-\tau)Z_T k^{\theta} + \tau \delta k - i - \frac{\psi}{2k}i^2 + \Delta p - r(1-\tau)p, \\ \Delta p &\leq \omega \left[k(1-\delta) + i \right] - p, \\ \log(Z'_T) &= \rho \log(Z_T) + \varepsilon'_T, \\ \varepsilon'_T \sim i.i.d. \ \mathcal{N}(0, \sigma_T^2), \ \varepsilon'_P \sim i.i.d. \ \mathcal{N}(0, \sigma_P^2), \ \varepsilon'_T \perp \varepsilon'_P. \end{aligned}$$

The equivalent representation of the problem admits a standard numerical solution (described in the following subsection), as the state-space is bounded and the remaining shocks are stationary.

A.2 Numerical solution

The firm's problem is solved by value function iteration on a discrete state-space of k, p, i, Δp , Z_T . As in Gomes [2001], the equivalent specification of the problem implies that k lies in a compact set, with upper bound defined by $(1 - \tau)\pi(\bar{k}, \bar{Z}_T) - \delta \bar{k} = 0$ and where \bar{Z}_T is the highest level of the transitory shock. Therefore capital is discretized into the following grid (containing 81 points):

$$[\bar{k}(1-\delta)^{40},\ldots,\bar{k}(1-\delta),\bar{k}(1-\delta)^{1/2},\bar{k}].$$

Net debt is discretized into an equally-spaced grid of 61 points over the interval $[-\bar{k}/2, \bar{k}]$. A less coarse grid was used for the control variables: investment was discretized over $[-\bar{k}/10, \bar{k}/10]$ and

debt changes over $[-\bar{k}/15, \bar{k}/15]$, using 31 points for each. All grids were chosen so that the optimal choice of investment or debt change never hits the lower/ upper thresholds. The transitory shock was discretized into a Markov chain with 9 grid points using the method of Tauchen [1986]. The process for the persistent shock was approximated with a truncated standard normal distribution using 5 grid points. The numerical procedure is implemented as follows:

- 1. Initial value for the value function in set.
- 2. Linear interpolation of the value function is used to compute the continuation value

$$\mathbb{E}_{Z'_T,\varepsilon'_P}\left[e^{(1-\theta)^{-1}\sigma_P\varepsilon'_P}v\left((k(1-\delta)+i)e^{-(1-\theta)^{-1}\sigma_P\varepsilon'_P},(p+\Delta p)e^{-(1-\theta)^{-1}\sigma_P\varepsilon'_P},Z'_T\right)\right]$$

- 3. For every k, p and Z_T the values of i and Δp are chosen such that the value function in (4) is maximized.
- 4. A new starting value is chosen and the procedure is repeated until convergence. Policy function iteration is also used as a part of the algorithm to speed it up.

As in Gourio [2012] the Blackwell's sufficient conditions for the contraction mapping may not be satisfied when the volatility of the persistent shock is too large, however in practive the convergence is achieved for most reasonable values.

The solution produces a value function $v(k, p, Z_P)$ and policy function $\{i, \Delta p\} = h(k, p, Z_T)$, which is used to compute k' and p' according to the law of motions derived in section 3 (and while making sure that the values stay within the specified grids). When simulating the model, the state space for Z_P is further extended to 120 points and interpolation is used to find the corresponding values of the value function and policy functions. I generate a simulated panel with N = 10000 firms over T = 200 periods and keep the 20 last observations for each firm to make sure that the realized values do not depend on the initial condition of k set at the steady state capital level, p = 0, $Z_P = 1$ and Z_T simulated from its stationary distribution.

B Data

B.1 Data processing and variable definitions

I use the annual Compustat data file for the sample period of 1965–2014. I remove all observations of firms that are not based in the US. As usual in the literature on leverage, I exclude firms in the financial and utility industries. I also exclude all variables with less than \$10M of total book assets, negative book equity or market-to-book ratio above 15. I further remove all observations with missing data on the key variables: total book assets at, debt in current liabilities dlc, long-

term debt dltt, total liabilities lt or operating income before depreciation oibdp. This leaves a dataset of roughly 192141 observations of 16490 firms.

I use the following definitions of variables, with defl being the CPI deflator:

- 1. quasi-market leverage (QML): (lt+pstkl-txditc)/(lt+pstkl-txditc+csho*prcc_f),
- 2. book leverage: (dltt+dlc)/at,
- 3. book net leverage: (dltt+dlc-che)/(at-che),
- 4. real profits: log(oibdp/defl),
- 5. profitability: oibdp/at,
- 6. cash flow growth: $(\texttt{oibdp}_{t}\texttt{-oibdp}_{t-1})/(0.5(\texttt{sale}_{t}\texttt{+sale}_{t-1}))$,
- 7. investment: capx/at
- 8. collateral: (invt+ppent)/at,
- 9. asset tangibility: ppent/at,
- 10. size: $\log(sale/defl)$,
- 11. market-to-book: (csho*prcc_f+lt+pstkl-txditc)/at,
- 12. dividend dummy: 1 if dvt/defl>0.1.

I require that any firm has at least 10 observations when computing persistence (ρ) and volatility (σ) of variables of interest by means of fitting the AR(1) models of the form

$$x_{i,t+1} = a_i + \rho_i x_{i,t} + \sigma_i \varepsilon_{x;i,t+1}.$$

I use the following risk proxies (computed for each firm):

- 1. standard deviation of profitability,
- 2. volatility of profitability,
- 3. volatility of log real profits,
- 4. log volatility of cash flow growth (log of 10-year rolling st. dev. of cash flow growth).

These proxies are winsorized at 1% and 99% and, in most of the analysis, aggregated on industry level using their average values within an industry.

B.2 Further empirical evidence on the risk-leverage relationship

In this section I provide more evidence on the risk-leverage trade-off by considering different risk proxies. Table 3 contains the correlation coefficients between four different risk proxies and four 'measures' of leverage, while Figure 10 contains the corresponding scatter plots for four selected pairs. The evidence suggests what while the correlations are consistently negative, their mag-

nitudes tend to vary. One implication of the data is that the correlations are always lower for the 'residual' leverage, which captures the fact that risk is likely to be jointly determined with other firm characteristics, therefore if one removes a part of leverage heterogeneity due to these observable factors, which results in lower correlation (in absolute terms). Another important issue

Average	\overline{lev}_i	$\overline{\widehat{lev}}_i$	\overline{nlev}_i	$\overline{\widehat{nlev}_i}$
St. dev. of profitability π/k	-0.505	-0.278	-0.590	-0.282
Volatility of profitability π/k	-0.406	-0.220	-0.489	-0.269
Volatility of log real profits $\log(\Pi)$	-0.106	-0.067	-0.079	-0.052
Log cash flow growth volatility $\log(\Delta \Pi)$	-0.193	-0.049	-0.411	-0.206

Table 3: Correlations between averages of risk proxies and average annual book (net) leverage for 4-digit SIC industries. $\overline{lev_i}$ ($\overline{nlev_i}$) is the residual from a fixed-effect model of book (net) leverage regressed on standard leverage factors as in e.g. Lemmon, Roberts, and Zender [2007] (without risk proxy).

concerns the fact that different proxies may capture different *notions* of risk. As an example, cash flow growth volatility and log real profit volatility appear to behave differently from profitability volatility, which scales or de-trends profits using total assets. Table 4 reaffirms this claim by considering the pairwise correlations of risk measures. This gives hope that these measures could in fact be more informative about different parts of riskiness, which motivates this study. In particular, the volatility of log real profits appears to be less related to the other three measures, which on the one hand is a natural consequence of the way in which it was computed (using level variables rather than ratios), but on the other hand suggests that the stark difference between levels and ratios may be suggestive of the presence of some phenomenon that drives both the profits as well as the total assets in the same way. This paper argues that this phenomenon reveals itself by the means of persistent shocks.

Average	$\operatorname{std}(\pi/k)$	$\sigma_{\pi/k}$	$\sigma_{\log(\Pi)}$	$\sigma_{\log(\Delta\Pi)}$
St. dev. of profitability $(\operatorname{std}(\pi/k))$	1.000			
Volatility of profitability $(\sigma_{\pi/k})$	0.774	1.000		
Volatility of log real profits $(\sigma_{\log(\Pi)})$	0.408	0.474	1.000	
Log cash flow growth volatility $(\sigma_{\log(\Delta \Pi)})$	0.635	0.631	0.329	1.000

Table 4: Correlation matrix of four risk proxies computed for 4-digit SIC industries.



Figure 10: Scatter plots of average annual book leverage versus risk proxy for 4-digit SIC industries. The 'residual book leverage' was computed as the difference between observed leverage and a fitted value from a fixed-effect model of book leverage regressed on standard leverage factors (size, profitability, asset tangibility, market-to-book) as in e.g. Lemmon, Roberts, and Zender [2007] (without controlling for a risk proxy).

B.3 Profit persistence as a leverage factor

I estimate profit persistence using an AR(1) model. To alleviate the concern that estimation bias may be at play, I consider different ways to aggregate the data. For example, I use profitability or log real profits computed for individual firms or industries using entity-level or aggregate data. The results, presented in Table 5, are similar in all cases considered and suggest a weak association between persistence and firm characteristics, especially leverage. A similar conclusion can be drawn from running cross-sectional regressions of average book leverage on leverage factors and profit persistence. The results presented in Table 6 indicate that the regression coefficients of $\hat{\rho}$ are nearly always statistically insignificant. They also provide very small incremental explanatory power compared to other variables (not reported).

Average		Firms		Industries					
	$ ho(\pi/k)$	$\rho(\log(\Pi))$	$\overline{ ho}(\pi/k)$	$\overline{\rho}(\log(\Pi))$	$\rho_{agg}(\pi/k)$	$\rho_{agg}(\log(\Pi))$			
Book leverage	-0.018	-0.002	0.009	-0.108	-0.032	-0.139			
Investment	-0.007	-0.033	0.047	-0.009	0.082	0.058			
Market-to-book	0.016	0.037	-0.002	0.040	0.032	0.076			
Size	0.013	0.029	0.070	0.085	0.011	-0.156			
Asset tangibility	-0.006	-0.025	0.020	-0.045	0.031	-0.039			
Collateral	-0.002	-0.002	-0.037	-0.058	-0.038	-0.127			
Volatility of log real profits	-0.022	-0.028	-0.095	-0.191	-0.159	-0.128			
Vol. of agg. log real profits			-0.059	-0.167	-0.312	-0.155			

Table 5: Correlations between firm characteristics and estimated profit persistence. ρ is estimated as the persistence parameter from an AR(1) fit of log real profits log(II) or profitability π/k for each firm and then averaged over all firms in an industry. Industry-specific persistence parameters ρ_{agg} are estimated using the aggregate industry-level data. Industries are defined using the 4-digit SIC code.

	I	Firms		Industries							
	$\overline{ ho}(\pi/k)$	$\overline{\rho}(\log(\Pi))$	$\overline{ ho}(\pi/k)$	$\overline{\rho}(\log(\Pi))$	$\rho_{agg}(\pi/k)$	$\rho_{agg}(\log(\Pi))$					
$\widehat{ ho}$	-0.001	0.001	-0.004	-0.007	-0.009	-0.011					
<i>t</i> -stat	-1.89	1.30	-0.74	-0.69	-0.75	-0.86					
Incr. \overline{R}^2 of $\widehat{\rho}$	0.001	0.000	0.002	0.001	0.001	0.000					
\overline{R}^2	0.262	0.262	0.313	0.332	0.332	0.333					
Industry FE	Yes, 4D-SIC	Yes, 4D-SIC	Yes, $2D$ -SIC	Yes, 2D-SIC	Yes, 2D-SIC	Yes, 2D-SIC					
N	6387	6387	353	353	353	353					

Table 6: Coefficients from cross-sectional regressions of average book leverage on average leverage factors (size, profitability, asset tangibility, market-to-book, volatility of log real profits) and estimated profit persistence $\hat{\rho}$. Standard errors are robust and clustered at 4-digit or 2-digit industry level. ρ is estimated as the persistence parameter from an AR(1) fit of log real profits log(Π) or profitability π/k for each firm and then averaging over all firms in an industry. Industry-specific persistence parameters ρ_{agg} are estimated using the aggregate industry-level data. The estimated profit persistence parameters are normalized by their full-sample standard deviation. Industries are defined using the 4-digit SIC code. All variables were winsorized at 1% and 99%.

C Is the model equivalent to one with two transitory shocks of different persistence?

One important question to ask when analyzing the model is related to whether its main predictions also prevail when we consider a related model in which the firm is exposed to two transitory shocks with different persistence parameter ρ . While certain features of both models are bound to be similar, given that each includes varying the 'overall persistence' of cash flow, there is a number of reasons as to why the persistent and transitory shock model is superior to one with two transitory shocks. I focus on three dimensions when comparing the two classes of models: the dynamics of the shocks and their implications for firm's policies, the effect of changing risk exposure on model-implied moments and the identification of risk characteristics.

Shock dynamics

The analysis in section 3 shows that persistent and transitory shocks result in markedly different capital and debt policies. Their effects differ both quantitatively and qualitatively. Furthermore, if we considered the impulse response functions for two transitory shocks with different persistence, their shapes would be similar, but shifted. However, unlike persistent shocks, transitory shocks are unable to permanently affect firm's policies. From this point of view, the two classes of models are completely distinctive.

Effect on model-implied moments

For the purpose of this subsection, I solve a slightly altered version of the model in which I set the profit function to $\Pi(K, Z) = (1 - \tau)Z_1Z_2^{1-\theta}K^{\theta}$ as in Belo, Lin, and Yang [2016], which allows to use a different solution method and consequently a wider range for σ_P 's (there is still a one-to-one link between the extended model and the one considered in this paper). The Z_2 shock will be represented by either the persistent shock Z_P or a transitory shock Z_{T+} where the '+' represents a higher value for the persistence parameter ρ . The parametrization of the models is summarized in Table 7.

The illustration in this subsection is qualitative in nature and demonstrates the nature of differences between the two classes of models. I solve the models for different exposures to the two types of shocks and compare the elasticity of model-implied moments to changing volatility composition. The results of this exercise are presented in Table 8.

The resulting elasticities suggest that while there are several moments that are affected in the same

	Mod	el 1 $(T+P)$	Model 2 $(T+T_+)$					
	$ ho_i$	σ_i	$ ho_i$	σ_i				
Z_1	0.2	0.24 - 0.15	0.2	0.24 - 0.15				
Z_2	1.0	0.05 - 0.20	0.7	0.05 - 0.20				

Table 7: Parametrization of the extended models. The assumed total volatility is constant and set to $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.25$. All remaining parameter values are taken as in Table 1.

	Moment	Elasticity				
		Model 1 (T+P)	Model 2 $(T+T_+)$			
	Average investment (i/k)	0.073	0.065			
	Standard deviation of investment (i/k)	0.383	0.618			
	Average leverage (p/k)	-0.128	-0.105			
ct	Standard deviation of leverage (p/k)	0.331	0.648			
effe	MAD of average investment (i/k)	1.004	0.571			
ar e	MAD of average leverage (p/k)	1.205	0.362			
nilå	Persistence of profitability $\rho(\pi/k)$	0.103	0.379			
Sir	Volatility of profitability $\sigma(\pi/k)$	0.047	0.857			
	Persistence of log profits $\rho(\log(\Pi))$	0.500	0.149			
	Volatility of log profits $\sigma(\log(\Pi))$	0.536	0.081			
	Corr. inv. and profitability $\operatorname{corr}(i/k, \pi/k)$	-0.633	-0.043			
et	Autocorrelation of investment $\phi_1(i/k)$	1.323	-0.481			
effe	Autocorrelation of investment $\phi_3(i/k)$	3.585	-0.907			
ы	Autocorrelation of capital stock $\phi_1(K)$	0.243	-0.164			
Dif	Autocorrelation of investment $\phi_3(K)$	0.549	-0.361			

Table 8: Elasticities of model-implied moments to changing risk exposure, computed at average moments $(\partial m/\partial \beta) \times (\bar{m}/\bar{\beta})$. The assumed total volatility is constant and set to $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} =$ 0.25. Elasticities are computed by changing risk composition according to Table 7. *MAD* denotes the median absolute deviation, ϕ_k is the k^{th} order autocorrelation. volatility and persistence are the ε_i and rho_i parameters from an AR(1) model $x_{i,t+1} = a_i + \rho_i x_{i,t} + \varepsilon_{i,t+1}$ estimated using corresponding moments.

way in both models (the top panel of Table 8), others behave differently. Moreover, the sensitivity of parameters to changing risk exposure is almost always higher for the model with a persistent shock, except for the variation investment and leverage and moments concerning profitability. This finding suggests that even despite their similarity, the way in which each set of shocks affect firm's policies is distinctive.

Furthermore, the model including a persistent shock appears to positively affect correlations of various moments, unlike the model with two transitory shocks. The intuition underlying this result

is straightforward. While increasing exposure to the persistent shock or to the transitory shock with high ρ raises the overall persistence in the model, at least in terms of profits or profitability, the *type* of persistence is different. There are two forces at play affecting this outcome. On the one hand, higher persistent shock exposure increases the overall persistence of model-implied moments, especially when total volatility is not too high. On the other hand, higher persistence results in the firm being more sensitive to underlying shock realizations. A high shock is likely to be followed by another high realization, therefore the firm is likely to change its investment policy and fund its capital expenditure by issuing debt. Thus, the firm will alter its investment and leverage policies more frequently and with higher magnitude, which lowers the extent to which these policies are path-dependent. Here, the second effect dominates. However, in the other case when the firm is more exposed to a low-persistence transitory shock, then its policy response is in general muted, given that this shock is more 'iid-like', unless a really large and positive realization occurs.

The effect in which persistent shocks affect the persistence of firm's policies is also markedly different. Given that the impact of the shock is spread out over multiple periods, as shown by the analysis of IRFs in section 3.2, the autocorrelation of these policies is bound to increase. This is not to say that the logic of the previous paragraph does not apply here. To the contrary, the firm is also much more sensitive to persistent shock realizations when its exposure to this shock increases, which results in more variable policies. However, this effect does not overtake the impact of 'spreading out' the effect of a persistent shock.

Several remaining remarks concerning the elasticity of other moments:

• Dispersion in firms' policies It is important to notice that the sensitivity of dispersion (as measured by the median absolute deviation MAD) in firm characteristics to changing risk exposure is two or three times as large for the model with a persistent shock. This observation gives further backing to the claim of the paper that the differences in dispersion of within-industry firm characteristics for different industries can be explained to some extent by different exposure to persistent shock, rather than to changing ρ .

• Persistence of log profits/ profitability Transitory shocks affect profitability to a higher extent than log profits, while the opposite is true for persistent shocks. This distinction, already highlighted in Table 2 in the original model, results from detrending profits Π by capital K. However, the quantitative differences remain large.

• Correlation betwen investment and profitability The value of the moment in a model with two transitory shocks is insensitive to changing firm's exposure to more persistent shocks, while the opposite is true in a model with a persistent shock. As already argued by Gourio [2008],

we should be able to identify the extent to which the firm is exposed to persistent shocks by looking at how it responds to being hit by a profitability shock. Here, the negative elasticity of the correlation between investment and profitability to changing risk composition results from the fact that profitability is relatively insensitive to persistent shock exposure, given the detrending.

• Volatility of log profits/ profitability Interestingly enough, the way in which each model affect these two moments is different. However, this result is intuitively related to how log profits and profitability are defined and has been discussed earlier when considering the comparative statics of the model in section 3.3. On the one hand, when we divide profits by capital, this implies that we de-trend profits, which removes most if not all of the variation due to persistent shocks (note that both profits and capital move with this variable). Therefore, this moment is more affected by transitory shocks. On the other hand, the volatility of log profits is much more sensitive to changing risk exposure in the model which includes a persistent shock. Again, this result in intuitive, given that persistent shock affect the growth rate of profits, unlike transitory shocks, which means that even small change in cash flow composition may result in big changes in the volatility of log profits.

Identification?

The last difference between the two classes of models is more subtle and relates to identifying the parameters governing the shock processes. In some sense, as implied by the results in Table 8 and the analysis in previous subsections, changing risk exposure in a model with two transitory shocks of different persistence is comparable to changing ρ in a standard dynamic capital structure model such as DeAngelo, DeAngelo, and Whited [2011]. Therefore, it may be impossible to infer the exact composition of firm's cash flows in such a two-transitory-shock model. On the other hand, a model with persistent shock has a distinctive impact on several key moments, giving hope that the identification is possible.

Finally, apart from the purely technical concerns, there are also a few economically motivated reasons. Despite the fact that the exact nature of persistent and transitory shocks may be unknown, that is we do not know what this shock decomposition *exactly* represents, it is easier still to imagine a firm being exposed to these two sources of risk rather than two transitory shocks with differing persistence. It is not easy to imagine how to attribute the 'less persistent' and 'more persistent' features. It is also relatively easier to think of risk exposure in terms of a very persistent (permanent) shock and a transitory shock with small persistence, especially given the vast macroeconomics literature using persistent shocks to describe the evolution of technology shocks in the economy.

As a final remark it is important to mention that rather than investigating the differences between the models having $\rho = 1$ and $\rho \approx 0.99$, in this paper I am more interested in examining whether a model in which the firm is exposed to a small persistent shock and a transitory shock with lower persistence than usually assumed in the literature is able to provide additional insight regarding variation in observable corporate policies.

D Comparative statics of other parameters

To conclude the analysis of the sensitivity of model-implied moments to model parameters, I analyze the effect of changing capital adjustment cost (ψ), external equity issuance cost (η) and the parameter governing tightness of the collateral constraint (ω). Table 9 presents the resulting moments computed for a 'low' and 'high' value of each of the three specified parameters for different values of persistent shock volatility σ_P .

• Capital adjustment cost ψ

As convex capital adjustment costs increase, firms become less sensitive to shock arrival and, as a result, investment becomes less variable. Therefore, firms also increase their leverage, given that investment opportunities become more predictable and so they can manage their debt capacity less conservatively. One could be concerned that the effect of changing persistent shock volatility is equivalent to that of changing convex capital adjustment costs given the increased smoothness in capital that both induce. However, it turns out that it is possible to disentangle these two effects. For example, the average standard deviation of investment appears to distinguish the two sufficiently well: while it increases as firm's persistent shock exposure grows, it decreases as the magnitude of convex adjustment costs rises. The intuition for this result is related to the fact that while both parameters affect the 'smoothness' of investment, σ_P also impacts the overall time-series variation of firm policies (i.e. high convex costs affect mostly the overall smoothness of firm policies while firms act on shock realizations). Other moments which are affected differently by these parameters concern e.g. the average leverage or the dispersion in average investment or leverage: the dispersion generally increases with persistent shock exposure and decreases as adjustment costs become larger.

• Equity issuance cost η

Higher equity issuance costs result in lower leverage level, as in e.g. Hennessy and Whited [2005, 2007]. They also result in more persistent and volatile leverage but less persistent investment. When equity issuance becomes more costly, firms' policies also become less dispersed. The effect of varying this parameter appears to be stronger when firm's persistent shock exposure is lower.

\bullet Tightness of the collateral constraint ω

As expected, this parameter largely affects average leverage and to some extent also other moments related to firm's debt policy such as leverage variation or dispersion in average leverage, but its effect on other model-implied moments is relatively limited.

		Total volatility σ Persistent shock volatility σ_P	$0.25 \\ 0.02$	0.25 0.02	$0.25 \\ 0.04$	$0.25 \\ 0.04$	$0.25 \\ 0.02$	$0.25 \\ 0.02$	$0.25 \\ 0.04$	$0.25 \\ 0.04$	$0.25 \\ 0.02$	$0.25 \\ 0.02$	$0.25 \\ 0.04$	$0.25 \\ 0.04$
			Capital adj. cost ψ			low	Equity iss. cost η			(low	Collateral constraint ω			
1. 2. 3. 4. 5. 6. 7.	Investment	Average investment (i/k) Standard deviation of investment (i/k) Autocorrelation of investment $\phi_1(i/k)$ Autocorrelation of investment $\phi_3(i/k)$ Frequency of disinvestment $\#(i/k < 0)$ Corr. inv. and profitability corr $(i/k, \pi/k)$ Autocorrelation capital stock $\phi_1(K)$	0.158 0.126 0.194 -0.107 0.078 0.834 0.791	$\begin{array}{c} 0.161\\ 0.069\\ 0.211\\ -0.043\\ 0.035\\ 0.696\\ 0.833\\ 0.014 \end{array}$	0.159 0.132 0.218 -0.090 0.105 0.812 0.827	$\begin{array}{c} 0.163\\ 0.076\\ 0.324\\ -0.011\\ 0.052\\ 0.666\\ 0.893\\ 0.021 \end{array}$	0.165 0.180 0.230 -0.109 0.156 0.879 0.781	$\begin{array}{c} 0.157\\ 0.115\\ 0.163\\ -0.103\\ 0.068\\ 0.784\\ 0.788\\ 0.015\\ \end{array}$	$\begin{array}{c} 0.167\\ 0.183\\ 0.245\\ -0.105\\ 0.170\\ 0.864\\ 0.805\\ 0.027\end{array}$	0.158 0.124 0.179 -0.087 0.105 0.787 0.818	0.158 0.124 0.192 -0.105 0.098 0.825 0.793	0.161 0.145 0.211 -0.109 0.087 0.866 0.789	0.159 0.134 0.195 -0.085 0.121 0.808 0.820	0.161 0.144 0.217 -0.098 0.103 0.840 0.818
8. 9. 10. 11. 12. 13.	Leverage	Dispersion of average investment $MAD(i/k)$ Average leverage (p/k) Standard deviation of leverage (p/k) Persistence of leverage $\sigma(p/k)$ Volatility of leverage $\sigma(p/k)$ Dispersion of average leverage $MAD(\overline{p/k})$	$\begin{array}{r} 0.016\\ 0.172\\ 0.062\\ 0.513\\ 0.045\\ 0.022 \end{array}$	$\begin{array}{c} 0.014\\ 0.227\\ 0.061\\ 0.616\\ 0.041\\ 0.018\end{array}$	$\begin{array}{c} 0.023 \\ 0.154 \\ 0.077 \\ 0.609 \\ 0.059 \\ 0.036 \end{array}$	$\begin{array}{c} 0.021\\ 0.213\\ 0.079\\ 0.655\\ 0.054\\ 0.024 \end{array}$	0.021 0.549 0.024 0.228 0.023 0.006	$\begin{array}{c} 0.015\\ 0.174\\ 0.071\\ 0.683\\ 0.049\\ 0.036\end{array}$	$\begin{array}{c} 0.027\\ \hline 0.5511\\ 0.025\\ 0.152\\ 0.024\\ 0.006\end{array}$	$\begin{array}{c} 0.021 \\ 0.142 \\ 0.080 \\ 0.673 \\ 0.055 \\ 0.037 \end{array}$	$\begin{array}{r} 0.016\\ \hline 0.154\\ 0.070\\ 0.643\\ 0.051\\ 0.024 \end{array}$	$\begin{array}{c} 0.018\\ 0.296\\ 0.082\\ 0.688\\ 0.054\\ 0.031 \end{array}$	$\begin{array}{c} 0.023\\ \hline 0.118\\ 0.073\\ 0.622\\ 0.055\\ 0.024 \end{array}$	0.024 0.227 0.089 0.711 0.058 0.041
14. 15. 16. 17. 18. 19.	Profits	Persistence of profitability $\rho(\pi/k)$ Volatility of profitability $\sigma(\pi/k)$ Persistence of log profits $\rho(\log(\Pi))$ Volatility of log profits $\sigma(\log(\Pi))$ Corr. prof. and growth $\operatorname{corr}(\pi/k, k'/k)$ Corr. prof. and log gth. $\operatorname{corr}(\pi/k, \log(k'/k))$	$\begin{array}{c} 0.333 \\ 0.141 \\ 0.665 \\ 0.267 \\ 0.838 \\ 0.823 \end{array}$	$\begin{array}{c} 0.432 \\ 0.160 \\ 0.757 \\ 0.288 \\ 0.707 \\ 0.723 \end{array}$	$\begin{array}{c} 0.336 \\ 0.138 \\ 0.700 \\ 0.270 \\ 0.816 \\ 0.799 \end{array}$	$\begin{array}{c} 0.377 \\ 0.144 \\ 0.626 \\ 0.267 \\ 0.667 \\ 0.661 \end{array}$	$\begin{array}{c} 0.300 \\ 0.135 \\ 0.732 \\ 0.274 \\ 0.882 \\ 0.866 \end{array}$	$\begin{array}{c} 0.342 \\ 0.141 \\ 0.646 \\ 0.267 \\ 0.790 \\ 0.780 \end{array}$	$\begin{array}{c} 0.304 \\ 0.134 \\ 0.754 \\ 0.276 \\ 0.868 \\ 0.849 \end{array}$	$\begin{array}{c} 0.341 \\ 0.137 \\ 0.686 \\ 0.270 \\ 0.791 \\ 0.776 \end{array}$	$\begin{array}{c} 0.332 \\ 0.138 \\ 0.666 \\ 0.268 \\ 0.830 \\ 0.818 \end{array}$	$\begin{array}{c} 0.324 \\ 0.141 \\ 0.686 \\ 0.268 \\ 0.869 \\ 0.849 \end{array}$	0.332 0.137 0.704 0.271 0.813 0.797	0.329 0.139 0.711 0.270 0.844 0.826

Table 9: Summary statistics of model-implied moments for different values of σ_P and parameters describing real or financing frictions: convex capital adjustment cost ψ , linear equity financing costs η and collateral constraint ω . Persistence of transitory shock ρ is set at 0.6. Remaining parameters are taken as specified in Table 1. An AR(1) model $x_{i,t+1} = a_i + \rho_i x_{i,t} + \varepsilon_{i,t+1}$ was fit for each simulated firm to compute persistence ρ_x or volatility σ_x of variable x. ϕ_k is the k^{th} order autocorrelation. The measure of dispersion MAD is median absolute deviation. The numerical solution and simulation of the model is described in detail in Appendix A.

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