Government intervention and information aggregation by prices[1]

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Abstract

Governments intervene in firms' lives in a variety of ways. However, efficient intervention depends on various economic variables, about which governments often have only limited information. Consequently, many researchers and policymakers call for governments to at least partially "follow the market" and make intervention decisions based on the information revealed by stock market prices. We analyze the implications of governments' reliance on market information for market prices and government decisions, and show that the use of market information might not come for free. A key point is that price informativeness is endogenous to government policy. In some cases, it is optimal for a government to marginally reduce its reliance on market prices in order to avoid harming traders' incentives to trade and the concomitant aggregation of information into market prices. For similar reasons, it is optimal for a government to limit transparency in some dimensions.

Our paper is motivated by two key observations. First, governments play an important role in the lives of firms and financial institutions, and take actions that have significant implications for their cash flows and stock prices. Second, governments' actions often follow financial market movements; and, closely related, many government officials view market prices as a useful source of information, and a number of policy proposals advocate making more explicit use of this information.

In this paper we analyze the implications of a government's use of market information in light of a key economic force: market prices reflect not only the fundamentals about which a government may wish to learn, but also expected government actions. Consequently, when governments make decisions based on information they glean from market prices, this affects the amount of information the government can ultimately extract from the market. We first analyze the equilibrium effect of these forces, and derive cross-sectional implications. Second, we analyze whether a government should increase or decrease its reliance on the market. Third, we develop implications for other issues—in particular, whether a government should reveal its own information to the market (i.e., transparency).

Before detailing our findings, we expand upon our opening two observations. The first observation is well-illustrated by the course of the recent financial crisis, during which government bailouts of leading financial institutions (e.g., AIG and Citigroup) and other firms (such as in the auto industry) constituted very important events for these firms and institutions. Government actions remain important following the crisis, as exemplified by recent penalties and regulations for financial institutions.

These government actions—and especially transfers made during the crisis—have attracted much controversy, both in policy and academic circles. Critics of government transfers argue that they waste taxpayer funds; unfairly reward both bankers and their shareholders; and engender future moral hazard. On the other side, proponents of government transfers argue that they help to soften the negative externalities that would flow from weak bank balance sheets, notably reduced lending and financial contagion.[1]

Regardless of the balance between the costs and benefits of government intervention, however, there is little debate that it is desirable that a government be in a position to make an informed decision. The concern is that the government conducts major interventions without having very precise information about the fundamentals, costs, and benefits.[2] For example, prior to the collapse of Lehman Brothers, the US government had to quickly decide whether or not to bail out Lehman. Ideally, this decision requires information about the state of Lehman, the implications of its failure for the financial system, and the potential moral hazard that a bailout might create for future episodes. Obtaining and analyzing all this information in a short amount of time is impossible.

This concern leads to the second key observation mentioned above. The challenge of making intervention decisions under limited information is well-understood by policymakers themselves, and one oft-proposed solution is to learn from and base intervention decisions on market prices. Indeed, a basic tenet of financial economics is that market prices aggregate information from many different market participants (Hayek 1945; Grossman 1976; Roll 1984). As such, market prices can provide valuable guidance. As an illustration, consider the following excerpt from a 2004 speech of Ben Bernanke:

> Central bankers naturally pay close attention to interest rates and asset prices, in large part because these variables are the principal conduits through which monetary policy affects real activity and inflation. But policymakers watch financial markets carefully for another reason, which is that asset prices and yields are potentially valuable sources of timely information about economic and financial

[1] For example, government programs to stimulate bank lending (e.g., TARP and TALF) were motivated by concerns that a decrease in lending would hurt firms and deepen the recession. The bailouts of large financial institutions such as AIG and Bear Stearns were driven by fears that the failure of these institutions would bring down the financial system due to the connections across different institutions. The intervention in the auto industry was motivated by fears that bankruptcies of large automakers such as General Motors would have devastating implications for their employees, suppliers, and customers.

[2] In a striking example, in order to determine the size of the bailout needed to save Anglo Irish Bank, the Irish government asked the bank's executives for an estimate of losses. Recently exposed internal tape recordings reveal that the bank's top executives lied to the government about the true extent of losses.

> conditions. Because the future returns on most financial assets depend sensitively on economic conditions, asset prices–if determined in sufficiently liquid markets–should embody a great deal of investors' collective information and beliefs about the future course of the economy.

Other senior Federal Reserve officials—for example, Minneapolis Federal Reserve Bank presidents Gary Stern and Narayana Kocherlakota—have voiced similar opinions.

Inspection of government actions in the recent crisis indeed suggests that policymakers watch prices closely, and often use price movements to justify their actions. For example, the 2011 report of the Special Inspector General for the Troubled Asset Relief Program states that "short sellers were attacking [Citigroup] ... Citigroup's share price fell from around \$13.99 at the markets close on November 3, 2008, to \$3.05 per share on November 21, 2008, before closing that day at \$3.77. In the week leading up to the decision to extend Citigroup extraordinary assistance, Citigroup's stock decreased far more than that of its peers." Beyond anecdotal evidence, empirical studies from before the crisis establish that government actions are significantly affected by market prices.[3]

In addition to existing government responses to financial markets, a range of policy proposals call for governments to make (more) use of market prices, particularly in the realm of bank supervision (e.g., Evanoff and Wall (2004) and Herring (2004)). Such policy proposals are increasingly prominent in the wake of the recent crisis and the perceived failure of financial regulation prior to it (e.g., Hart and Zingales (2011).)

In light of these observations, in this paper we study a model of information aggregation in financial markets, where information aggregated in the price is used by the government to make an intervention decision. A central implication is that relying on market information is not as simple as the public discussion mentioned above seems to suggest and may not so easily solve the problem of the government being uninformed. The problem stems from the fact that prices of financial securities—from which the government attempts to learn—are

[3]See Feldman and Schmidt (2003), Krainer and Lopez (2004), Piazzesi (2005), and Furlong and Williams (2006).

not pure projections of the state variables that the government wishes to learn about. Rather, prices are projections of future cash flows, which are typically affected by government actions. The information in security prices is thus endogenous and is affected by government policies, and to the extent that governments rely on prices. When governments rely on market prices, it is thus important to consider the consequences this has for price informativeness.

For illustration, consider the case of a government bailout or a guarantee for a financial institution. Empirical evidence clearly shows such actions benefit the institution's shareholders, and are reflected in increased share prices. For example, O'Hara and Shaw (1990) show that an increase in expectations that a government will provide a guarantee to large financial institutions is immediately reflected in share price increases for such institutions. Gandhi and Lustig (2013) show that shares of large financial institutions are priced at a premium, reflecting the benefit that their shareholders expect from government intervention. Hence, government actions affect prices, and consequently also affect the ability of the government to learn from prices. This affects the desirability of market-based intervention.

To understand these forces, it is important to consider the process by which prices aggregate information. We analyze how market-based government policy affects speculators' trading incentives, and hence the extent to which financial markets aggregate dispersed information. We build on the canonical model of information aggregation of Grossman (1976), Hellwig (1980), and Admati (1985). Speculators possess heterogenous information about the payoffs of an asset and trade in a market that is subject to noise/liquidity shocks. The equilibrium price of the asset then reflects the aggregated information of speculators with noise. In the existing literature, the asset's cash flows are exogenous. However, if the government (or some other decision maker) uses information in prices when intervening in the firm's operations in a way that affects the firm's cash flows, then the cash flows are instead endogenous and depend on market prices and on the trading process. Our modeling innovation is to introduce this effect into an analysis of information aggregation.

In the model, the government makes an intervention decision based on market information and on other information it has about the firm or the financial institution. Such information can come from the government's own supervision activities conducted by the Federal Reserve Banks, the Federal Deposit Insurance Corporation, etc. The government uses market signals because they contain information, but their informational content is endogenous and determined by the trading incentives of speculators, which in turn are affected by the government's policy and the extent to which it relies on the market price.

We identify two opposing effects of the government's reliance on stock prices on price informativeness. The first effect is the *Information Importance Effect*. When the government puts more weight on the price and less weight on its own information in the intervention decision, it makes speculators' information, conditional on the price, less important in predicting the government's action and hence the value of the security. This reduces speculators' incentives to trade on their information, and hence it reduces price informativeness. The second effect is the *Residual Risk Effect*. When the government puts more weight on the price and less weight on its own information in the intervention decision, it reduces the uncertainty that speculators are exposed to when they trade. Being risk averse, speculators then trade more aggressively on their information, and this leads to an increase in price informativeness.

Overall, which effect dominates depends on the parameters of the model. The residual risk effect is weakened when the risk for speculators is driven mostly by exogenous risk (i.e., risk from an unforecastable and exogenous cash flow shock) rather than by endogenous risk (i.e., risk due to the unknown government action), and so in this case price informativeness is decreasing in the extent to which the government relies on market prices. We show that this effect is strong enough to imply that the government follows the market too much, and, if possible, would gain from a commitment to marginally underweight market prices in its intervention decision.[4] Overall, our model delivers the somewhat paradoxical result that a government should marginally reduce its reliance on prices precisely when they are informative, because in this case prices forecast the government's action well, and so endogenous

[4]Note that this result, in common with other results in Section 3, is local.

risk is low. Similarly, and again paradoxically, a government should marginally increase its reliance on prices when its own information is relatively precise.

These results are in contrast to a common theme in the literature. While papers by Faure-Grimaud (2002), Rochet (2004), Hart and Zingales (2011) and others suggest that governments should commit to intervene in a pre-determined way based on publicly observable prices, our paper highlights another consideration to be taken into account when assessing the costs and benefits of these proposals: the effect that such proposals have on price informativeness. In particular, in the circumstances above, commitment to a market-based rule reduces the price-informativeness that the rule makes use of, and hence reduces the rule's value. In addition, in such circumstances, our model also implies that the government's own information is valuable beyond its direct effect on the efficiency of the government's decision. When the government has more precise information, it relies less on the market price, and this makes the market price more informative. Hence there are complementarities between the government's own information and the market's information, and so it is not advisable for the government to rely completely on market information.

Another important aspect of government policy is transparency. Should the government reveal its own information publicly? This issue has been hotly debated recently in relation to regulatory stress tests of financial institutions. There are various views on whether the results of such stress tests should be publicly disclosed (see Goldstein and Sapra (2013) for a survey). Our model sheds light on this debate from a new angle: is disclosure of information to the market desirable when the government is trying to learn from the market? In our framework, the answer to this question depends on the type of information being disclosed. If the government discloses information about a variable about which speculators have at least some additional information, then the government harms itself because the disclosed information reduces the incentives of speculators to trade on their information (due to the information importance effect) and reduces the government's ability to learn. If instead the government discloses information about a variable that speculators know less than the government (i.e., their information is a coarsening of the government's), it helps itself, because the disclosed information reduces the risk that speculators face (due to the residual risk effect), causing them to trade more and increasing the government's ability to learn from prices. This distinction is new to the literature on transparency.[5] In practice, it seems likely that individual bank conditions are an area in which speculators have substantial information not possessed by a government. At the opposite extreme, a government knows its own policy objectives, so there is no room for speculators to have useful information in this dimension. Consequently, transparency about policy objectives is useful for the government, but transparency about stress-test findings on individual bank conditions might be harmful.

Our paper adds to a growing literature on the informational feedback from asset prices to real decisions.[6] In particular, it complements papers such as Bernanke and Woodford (1997), Goldstein and Guembel (2008), Bond, Goldstein and Prescott (2010), Dow, Goldstein and Guembel (2010), and Lehar, Seppi and Strobl (2010), which analyze distinct mechanisms via which the use of price information in real decisions might reduce the informational content of the price. For a recent review of this literature, see Bond, Edmans, and Goldstein (2012). Relative to these papers, our focus is on the efficiency of aggregation of dispersed information by market prices. This topic, which has long been central in economics and finance (e.g., Hellwig (1980)), has not been analyzed in any of the related papers.

The remainder of the paper is organized as follows. Sections 1 and 2 first describe and then analyze the basic model. Section 3 analyzes how a government should optimally use market information. Section 4 looks at the importance of the government's own information. Section 5 analyzes the costs and benefits of transparency. Section 6 considers alternative

[5] There are recent papers showing that transparency might be welfare reducing, e.g., Morris and Shin (2002) and Angeletos and Pavan (2007). In these papers, the source for the result is the existence of coordination motives across economic agents. In contrast, such coordination motives do not exist in our model, where, conditional on the price (which is observed to all), speculators do not care about what other speculators do. Importantly, the above-mentioned papers do not explore the implications of transparency about different types of information, as we do here.

[6] See, for example, Fishman and Hagerty (1992), Leland (1992), Khanna, Slezak, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997), Subrahmanyam and Titman (1999), Fulghieri and Lukin (2001), Foucault and Gehrig (2008), and Bond and Eraslan (2010).

notions of price informativeness. Section 7 considers varying government subsidies to security holders. Section 8 concludes. The appendix contains most proofs.

1 The model

We focus on one firm (a financial institution, for example), whose shares trade in a financial market. At $t = 0$, speculators obtain private signals about a variable that affects the government's incentive to intervene in the firm, and trade on these signals. At $t = 1$, the government observes the firm's share price and an additional private signal, and makes a decision about its intervention. At $t = 2$, cash flows are realized and speculators are paid.

1.1 Cash flows and government intervention

The firm's cash flow is $X = \delta + T$. The component δ is exogenous, and unforecastable: neither speculators nor the government receive any signal about δ before its realization at $t = 2$. The distribution of δ is normal, with mean $\bar{\delta}$ and variance $var\left[\delta\right]$. The mean $\bar{\delta}$ can vary across firms, states of the world, and time, but it is publicly known as of $t = 0$. The precision of prior information about δ is $\tau_\delta \equiv var\left[\delta\right]^{-1}$.

The component T of the firm's cash flow is the result of endogenous government intervention. Positive values of T represent cash injections or other interventions that increase the firm's cash flow, while negative values represent penalties or interventions that reduce the firm's cash flow. As discussed in the introduction, a wide variety of government actions affect firm cash flows. We adopt a general formulation that accommodates many such examples. In particular, T is chosen by the government to maximize an objective function of the form

$$E\left[v\left(T-\theta\right)-\mu T | s_G, P\right]. \tag{1}$$

Here, v is a concave function that represents the benefits of intervention; θ is a state variable that is unobserved by the government but that affects these benefits; and μ is a scalar, which allows for an additional linear cost of intervention. The unobserved state variable θ is normally distributed with mean $\overline{\theta}$ and variance $var[\theta]$, and is independent of the cash flow shock δ. The precision of prior information about θ is $\tau_\theta \equiv var[\theta]^{-1}$.

The government makes its intervention decision after observing two pieces of information: a market price P, discussed in detail below, and a noisy signal $s_G \equiv \theta + \varepsilon_G$ of θ, where the noise term ε_G is normally distributed with mean 0 and variance $var[\varepsilon_G]$, and is independent of θ and δ. The precision of the government's signal is $\tau_G \equiv var[\varepsilon_G]^{-1}$.

Let us elaborate on the government's objective function, before proceeding to discuss the trading environment in the next subsection. As mentioned above, the specification of the government's objective function is general enough to cover a range of possible applications.

Many interventions in the recent crisis—such as TARP, TALF, and other related government programs—intended to provide resources to banks in the hope of increasing lending to non-financial firms in a period when government officials believed the credit market was impaired. Our framework can capture such motives, as follows. Consider the case in which the firm is a bank, and bank loans generate some social surplus. In particular, let $s(x-\theta_1)$ be the marginal social surplus created by the xth dollar loaned. The function s is decreasing, reflecting diminishing marginal social returns to lending, and θ_1 is a state variable that affects the social surplus of bank loans. Absent government intervention, the bank lends its available resources $L+\theta_2$, where L is a publicly observable quantity related to the bank's balance sheet, and θ_2 is a state variable that determines the bank's ability to access funds in external credit markets. The government's intervention provides additional resources T to the bank, so that its overall lending changes to $L+\theta_2+T$. Finally, write v for the anti-derivative of s. Consequently, if the government's cost of funds is μ, then conditional on θ_1 and θ_2 the social surplus associated with an intervention T is:[7]

$$\int_0^{L+\theta_2+T} s(x-\theta_1)\,dx - \mu T = v(L+T-(\theta_1-\theta_2)) - v(-\theta_1) - \mu T.$$

[7] Note that negative values of $L+\theta_2+T$ correspond to the bank absorbing funds at the expense of projects.

Since L is known and $v(-\theta_1)$ is outside the government's control, this is consistent with the objective function (1), with $\theta = \theta_1 - \theta_2$. To summarize, the government is concerned about the amount of credit banks provide, and sets T to influence it. When choosing T, the government faces uncertainty about both the desirability of bank lending (θ_1) and the amount of resources banks can get without government intervention (θ_2).

As noted in the introduction, another oft-invoked rationale for government intervention in financial institutions is the need to maintain financial-sector stability. The concern is that the failure of a systemically important financial institution (SIFI) might severely harm the whole financial system. A possible remedy is for the government to inject capital or provide loan guarantees to troubled institutions. However, such remedies have the cost of engendering future moral hazard problems, encouraging banks to take excessive risks. Events in the recent crisis reflect this dilemma. On the one hand, the bailout of Citigroup, Bear Stearns, AIG, and others, was driven by a concern that due to their systemic importance, the cost of their failure would be very large. On the other hand, Lehman Brothers was allowed to fail, probably because of concerns that a bailout would create a severe moral hazard issue. In our model, $v(T - \theta)$ reflects the benefit from injecting capital into a distressed financial institution, and is concave in the amount injected. The benefit depends on the state variable θ, which reflects the size of the reduction in negative externalities stemming from reducing a SIFI's failure probability, net of the cost of increased moral-hazard. The government does not have perfect information about the state θ, and may try to glean information from the stock market (see introduction). Note that the government may choose T to be negative, corresponding to reducing the size of the financial institution in an effort to promote stability.

While the above motivations involve the financial sector, which is a prime focus of government intervention, our framework also covers non-financial firms. For example, the three large US automakers also received significant government assistance in the recent crisis, and the justification for this assistance was again the mitigation of negative externalities associated with bankruptcy. There was concern that the failure of a large automaker would harm employees, dealers, and suppliers, in turn harming the aggregate economy. In our framework, $v(T-\theta)$ represents the social benefit from transfers to an automaker, where θ, which the government is not sure of, represents the size of negative externalities.

In summary, in each of these applications the government intervenes in a firm or financial institution to try to increase overall efficiency in cases where the firm does not internalize the externalities it generates. We do not take a stand on the source of externalities, but instead use a general formulation that encompasses multiple cases, as described above.[8] We focus on the interaction between the government and the financial market when the government is only partially informed about the state θ that determines its desired level of intervention, while financial market participants possess some information about this state.

We conclude this subsection by briefly highlighting and discussing the assumptions in our framework:

Remark 1: The restriction that the benefit function v is concave is standard, mild, and likely to be satisfied by many potential applications. The stronger assumptions imposed in our framework are that (a) the intervention T and state variable θ enter linearly, and (b) the cost of intervention is linear, i.e., μT, rather than strictly convex. With respect to (a), we note that this property arises naturally when T and θ have the same units. The application to impaired credit markets illustrates this well: both T and θ_2 are resources on the bank's balance sheet, and so are directly comparable. In other cases, (a) is better viewed as an approximation made to obtain analytic tractability. With respect to (b), we note first that linearity of the cost of intervention is the appropriate assumption when the intervention is small relative to the economy. This is the case for most single-institution interventions. Second, our framework also covers cases in which the benefit of intervention is linear, but the government is unsure of the cost, which takes a convex form $-v(T-\theta)$. That said, for extremely large government interventions (the Irish bank bailout may be a good example),

[8] In fact, our analysis remains valid if the government egoistically intervenes to maximize private benefits, e.g., campaign contributions or reelection prospects. Separately, our framework has other applications aside from government intervention—e.g., to managerial decisions. Details are available upon request.

the government's objective may be best modeled as $v(T-\theta)-\mu(T)$, where $\mu(\cdot)$ is a convex cost function, i.e., concave benefit and convex cost.[9] For these cases, analytic tractability requires stronger assumptions on the form of the functions v and μ: for example, if v takes the same form as speculator preferences, namely constant absolute risk aversion, and μ is likewise an exponential function, then these cases also fall within our framework.

Remark 2: A key ingredient in our analysis is that the intervention T affects the value of the traded security. The fact that T affects the cash flow one-for-one, i.e., $X=\delta+T$, is unimportant and is assumed here only for simplicity. Section 7 analyzes an extension in which a fraction of the injection T is taxed away, so that security holders do not benefit from the full injection. As discussed in the introduction, there is ample evidence that government interventions affect security values. For example, both O'Hara and Shaw (1990) and Gandhi and Lustig (2013) provide evidence that financial institutions' share prices reflect expectations of government bailouts. We do not take a stand on why governments do not design interventions to avoid windfall gains for shareholders,[10] but instead take this feature from the data and analyze the interaction between government intervention and market prices.

Remark 3: The main effects in our model all stem from the fact that speculators try to forecast government actions, and these forecasts affect market prices and their informativeness. As discussed in the introduction, we think this is important for many firms. To focus our analysis on this effect, in our model the only information speculators have is about a state variable, θ, that affects government actions. However, a prior draft of the paper (available on request) analyzes a variant of our model in which speculators also have information that is directly relevant for the cash flows of the firm even absent government intervention.

1.2 Trading in the financial market

We now complete the description of the model by describing the financial market and the price formation process. There is a continuum $[0,1]$ of speculators, each with constant

[9]The case $v(T)-\mu(T-\theta)$, where v is strictly concave and μ is strictly convex is handled similarly.

[10]Political considerations may certainly be a contributing factor.

absolute risk aversion (CARA) utility, $u(c) = -e^{-\alpha c}$, where c denotes consumption and α is the absolute risk aversion coefficient. Speculators trade shares in the firm. The shares pay out the firm cash flow X. Each speculator i receives a noisy signal $s_i \equiv \theta + \varepsilon_i$ of the state θ. The noise terms ε_i are independently and identically distributed across speculators, and each is normally distributed with mean 0 and variance $var[\varepsilon_i]$. The precision of each speculator's signal is $\tau_\varepsilon \equiv var[\varepsilon_i]^{-1}$. One interpretation of these private signals is that different speculators have different assessments of the extent to which a firm affects the rest of the economy, and the government can benefit from their combined knowledge. For example, as Bear Stearns and Lehman Brothers approached failure, it was unclear to everyone—including the government—how much their failures would damage the economy.

Each speculator chooses a trade size x_i to maximize his expected utility, conditional on the information in his private signal s_i and the (endogenously determined) share price P:

$$x_i(s_i, P) = \arg\max_{\tilde{x}} E\left[-e^{-\alpha\tilde{x}(\delta+T-P)}|s_i, P\right]. \tag{2}$$

Here, if a speculator trades x_i, his overall wealth is $x_i(\delta + T - P)$, where $\delta + T$ is the cash flow from the security after intervention, and P is the price paid for it.

In addition to informed trading by speculators, there is a noisy supply shock, $-Z$, which is normally distributed with mean 0 and variance $var[Z]$. We again use the notation $\tau_z \equiv var[Z]^{-1}$. Finally, the market-clearing condition is

$$\int x_i(s_i, P)\, di = -Z. \tag{3}$$

2 Equilibrium outcomes

In equilibrium, individual speculators' demands maximize utility given s_i and P (i.e., (2) holds), the market clearing condition (3) holds, and the government's choice of T maximizes its objective (1) given its signal s_G and the price P. As is standard in almost all the literature, we focus on linear equilibria in which the price P is a linear function of the average signal realization—which equals the realization of the state θ—and the supply shock $-Z$. The complication in our model relative to the existing literature is that the firm's cash flow is affected by the government's endogenous intervention T. Nonetheless, and as we next show, there is an equilibrium in which not only is price a linear function of the primitive random variables, but the intervention T is also linear in these same primitive random variables.

Let us conjecture that in equilibrium T is indeed a linear function of the primitive random variables. In the proof of Proposition 1 below, we show (by largely standard arguments) that this leads to a linear price function. Then, given that the government learns from the price P and its own signal s_G, and given that all primitive random variables in the model are normally distributed, the conditional distribution of θ given the government's information (P, s_G) is also normal.[11] Consequently, we can apply the following useful result, which confirms our conjecture that T is a linear function.

Lemma 1 *If the conditional distribution of the state variable θ given government information (P, s_G) is normal, then there exists a function g such that the intervention T that maximizes the government's objective (1) is*[12]

$$T = E\left[\theta|P, s_G\right] + g\left(\mu, var\left[\theta|P, s_G\right]\right). \tag{4}$$

The proof of Lemma 1 is short, and we give it here. The intervention T that maximizes the government's objective (1) satisfies the first-order condition

$$E\left[v'\left(T-\theta\right)|P, s_G\right] = \mu. \tag{5}$$

[11] A recent paper by Breon-Drish (2012) relaxes the normality assumptions in the canonical model. The key step in his generalization is to place enough structure on distributions so that the demand of an informed speculator is still linear in the informed speculator's signal. However, his model does not feature a feedback effect from the price of the security to the cash flows it generates.

[12] A slightly modified version of Lemma 1 holds when the government objective takes the form described in Remark 1 above, namely $-\exp\left(-\alpha_G\left(T-\theta\right)\right) - c_1\exp\left(c_2 T\right)$ for constants α_G, c_1, c_2, i.e., concave benefit *and* convex costs. The first-order condition (5) is $E\left[\alpha_G\exp\left(-\alpha_G\left(T-\theta\right)\right) - c_1 c_2\exp\left(c_2 T\right)|P, s_G\right] = 0$, which is equivalent to $E\left[\exp\left(-\left(\alpha_G + c_2\right)T + \alpha_G\theta\right)|P, s_G\right] = \frac{c_1 c_2}{\alpha_G}$. The same steps as in the main text then imply that the intervention T is a linear function of the government's expectation $E\left[\theta|P, s_G\right]$.

The government knows T, and so, by hypothesis, the conditional distribution of $T-\theta$ given (P, s_G) is normal; consequently, it is fully characterized by its first two moments. Hence, the expectation $E[v'(T-\theta)|P, s_G]$ can be written as a function of the first two moments of the conditional distribution of $T-\theta$, i.e., there exists some function $G : \Re^2 \to \Re$ such that

$$E[v'(T-\theta)|P, s_G] = G(E[T-\theta|P, s_G], var[T-\theta|P, s_G]). \qquad (6)$$

Substituting (6) into (5) and defining the inverse $g(y, x)$ of G by $G(g(y, x), x) = y$ delivers[13]

$$T - E[\theta|P, s_G] = g(\mu, var[\theta|P, s_G]),$$

completing the proof of Lemma 1.

Proposition 1 below uses Lemma 1 to establish the existence of a linear equilibrium, and to characterize the associated level of price informativeness. Before stating the formal result, we give an informal derivation of price informativeness.

In a linear equilibrium, the price can be written as

$$P = p_0 + p_Z(\rho\theta + Z), \qquad (7)$$

for some (endogenous) scalars p_0, p_Z and ρ. Here, $\rho^2\tau_Z$ measures price informativeness, since the informational content of the price is the same as the linear transformation $\frac{1}{\rho p_Z}(P - p_0) = \theta + \rho^{-1}Z$, which is an unbiased estimate of the state θ with precision $\rho^2\tau_Z$. Intuitively, the price of the security is affected by both changes in the state θ and changes in the noise variable Z; price informativeness is greater when ρ, the ratio of the effect of θ on the price relative to the effect of Z on the price, is greater. Because we typically take τ_Z as fixed in our comparative statics and policy analysis, we often refer to ρ, which is affected by the underlying parameters and government policy, as price informativeness.

[13] By the concavity of v, G is strictly decreasing in its first argument, and hence $g(y, x)$ is well-defined.

It is worth highlighting that the informativeness measure ρ relates to the state θ, and not the cash flow $T+\delta$. This is because the government is attempting to learn the state θ from the price, and so informativeness about θ is the relevant object for the government's maximization problem. We discuss this distinction in more detail in Section 6 below.

To characterize price informativeness, we first analyze the government's decision. Given normality of the state θ, the supply shock $-Z$, and the error term ε_G in the government's signal s_G, along with the linear form of the price function (7), the government's posterior of the state θ is normal, with the posterior mean taking the linear form

$$E\left[\theta|s_G,P\right]=w\left(\rho\right)s_G+K_P\left(\rho\right)\frac{1}{\rho p_Z}\left(P-p_0\right)+K_\theta\left(\rho\right)\bar{\theta}, \tag{8}$$

where $K_\theta\left(\rho\right)$, $K_P\left(\rho\right)$ and $w\left(\rho\right)$ are weights that sum to one and are derived by standard Bayesian updating, i.e.,

$$K_P\left(\rho\right) \equiv \frac{\rho^2\tau_z}{\tau_\theta+\rho^2\tau_z+\tau_G} \tag{9}$$

$$w\left(\rho\right) \equiv \frac{\tau_G}{\tau_\theta+\rho^2\tau_z+\tau_G}. \tag{10}$$

In particular, $w\left(\rho\right)$ is the weight the government puts on its own signal in estimating the state, which depends on the information available in the price. As one would expect, the government puts more weight on its own signal when it is precise (τ_G high) and less when the price is informative (ρ and/or τ_Z high). Given the policy rule (4), the intervention is

$$T\left(s_G,P\right)=w\left(\rho\right)s_G+K_P\left(\rho\right)\frac{1}{\rho p_Z}\left(P-p_0\right)+K_\theta\left(\rho\right)\bar{\theta}+g\left(\mu,var\left[\theta|P,s_G\right]\right). \tag{11}$$

Turning to the speculators, each speculator assigns a normal posterior (conditional on his own signal s_i and price P) to the state θ. Then, from (11), each speculator also assigns a normal posterior to the intervention T. Consequently, applying the well known expression for a CARA individual's demand for a normally distributed stock, speculator i trades

$$x_i(s_i, P) = \frac{1}{\alpha} \frac{E[T|s_i, P] + \overline{\delta} - P}{var[T|s_i, P] + var[\delta]}. \tag{12}$$

Hence, speculators trade more when there is a large gap between the expected security value $E[T|s_i, P] + \overline{\delta}$ and the security price P, but, due to risk aversion, this tendency is reduced by the conditional variance in security value $var[T|s_i, P] + var[\delta]$.

To characterize the equilibrium informativeness of the stock price, consider simultaneous small shocks of φ to the state θ and $-\varphi\rho$ to Z. By construction (see (7)), this shock leaves the price P unchanged. Moreover, the market clearing condition (3) must hold for all realizations of θ and Z. Consequently,

$$\varphi \frac{\partial}{\partial \theta} \int x_i(s_i, P)\, di = \varphi\rho.$$

Substituting in (11) and (12) yields equilibrium price informativeness:

$$\rho = \frac{1}{\alpha} \frac{\frac{\partial}{\partial s_i} E[T|s_i, P]}{var[T|s_i, P] + var[\delta]} = \frac{1}{\alpha} \frac{w(\rho) \frac{\partial}{\partial s_i} E[\theta|s_i, P]}{w(\rho)^2 (var[\theta|s_i, P] + var[\varepsilon_G]) + var[\delta]}. \tag{13}$$

The informativeness of the price is determined by how much speculators trade on their information about θ. This is determined by two factors: the relation between their information and asset's value (the numerator), and the asset's variance (the denominator).

Proposition 1 *A linear equilibrium exists. Equilibrium price informativeness ρ satisfies (13). For $var[\varepsilon_G]$ sufficiently small, there is a unique linear equilibrium, which is continuous in $var[\varepsilon_G]$.*

Our model nests the case of exogenous cash flows (the assumption of the prior literature). To obtain this special case, set $var[\varepsilon_G] = 0$, so that the government directly observes the state θ. Consequently, it ignores the price in choosing its intervention, and so speculators treat the firm's cash flow as exogenous. In this case, $w(\rho) \equiv 1$, and (13) reduces to

$$\rho = \frac{1}{\alpha} \frac{\frac{\partial}{\partial s_i} E[\theta | s_i, P]}{var[\theta | s_i, P] + var[\delta]}. \tag{14}$$

Since $var[\theta|s_i, P] = \frac{1}{\tau_\theta + \rho^2 \tau_z + \tau_\varepsilon}$ and $\frac{\partial}{\partial s_i} E[\theta|s_i, P] = \tau_\varepsilon var[\theta|s_i, P]$, it is easy to see that the right-hand side of (14) is decreasing in ρ, and so (14) has a unique solution in ρ. Consequently, there is a unique (linear) equilibrium.

Essentially by continuity, there is also a unique linear equilibrium when $var[\varepsilon_G]$ is strictly positive, but sufficiently small. However, when $var[\varepsilon_G]$ is large, multiple equilibria may exist. Economically, when price informativeness is low (high), the government puts a lot of (little) weight on its own signal, which causes speculators to face a lot of (little) residual risk, and hence trade cautiously (aggressively), generating low (high) price informativeness. All the results below are stated in a way that allows for the possibility of multiple equilibria.[14]

2.1 Empirical implications

We conclude this section with a few comparative-statics results that provide empirical implications of the basic model:

Corollary 1 *The equilibrium weight $K_P(\rho)$ that the government attaches to the price in decisions is decreasing in risk-aversion, α; the variance of the supply shock, $var[Z]$; the noise in speculator signals, $var[\varepsilon_i]$; and the unforecastable component of cash flows, $var[\delta]$. However, the noise in government signals, $var[\varepsilon_G]$, has an ambiguous effect on both price informativeness and the weight the government attaches to the price.*[15]

Testing these results requires empirical proxies. The weight that the government attaches to the price in its decision can be assessed by measuring the sensitivity of government

[14] Our paper is not the first to show that the uniqueness of equilibrium in Grossman and Stiglitz (1980) and Hellwig (1980) is not robust to extensions of the model. For example, Ganguli and Yang (2008) show that introducing private information about the aggregate liquidity shock may lead to multiplicity of equilibria.

[15] In cases of multiple equilibria, these statements should all be understood as applying to the equilibrium set.

intervention to price changes; see, for example, Chen, Goldstein, and Jiang (2007) for an implementation of this in the context of corporate investment. The important but hard-to-observe terms relating to the volume of noise trading, $var[Z]$, and the information of speculators, $var[\varepsilon_i]$, can be proxied using microstructure measures such as the probability of informed trading (PIN) or price non-synchronicity, which are often deployed for this purpose. Alternatively, one could proxy $var[Z]$ and $var[\varepsilon_i]$, along with risk-aversion α, by using characteristics of the base of investors who trade in the stock: who they are, how informed they are, how much they trade due to hedging and liquidity needs (i.e., noise), etc.

The results in Corollary 1 are mostly straightforward and have the sign one would expect (and for the reasons one would expect). This includes the results for the parameters α, $var[Z]$, $var[\varepsilon_i]$, $var[\delta]$. When traders trade less aggressively due to risk aversion, when there is more noise trading, when traders have less precise information, and when there is more unforecastable uncertainty regarding the value of the firm, then the price ends up being less informative and the government relies less on it. The one result that is more surprising is the comparative static with respect to $var[\varepsilon_G]$: one might instead have conjectured that more imprecise private information would always lead the government to pay more attention to the price. But as the noise in the government signal increases, the direct effect is that the government puts less weight on its own signal. Under some circumstances, this decreases price informativeness. (We discuss this point in much greater detail in the next section, when we consider exogenous perturbations of w.) When this effect is large enough, the weight the government puts on the price drops.

3 Does the government follow the market too much or too little?

In the equilibrium characterized in the prior section, the government makes ex-post optimal use of the information in market prices when making its intervention decision. In this sense, the government rationally follows the market. As discussed in the introduction, this behavior is consistent with empirical evidence and with comments from policymakers themselves indicating the use of market information in government decisions.

We next analyze whether the government follows the market *to the correct extent.* The issue we focus on is that when the government decides ex post how much weight to put on market prices, it does not internalize the effect that this decision has on equilibrium price informativeness. In essence, the government acts ex post as if price informativeness is fixed. But as should be clear from the analysis of Section 2, price informativeness is determined in part by the weight the government puts on market prices.

To characterize whether the government follows the market too much or too little, we consider whether, and in what direction, deviations from the ex-post optimal rule can help the government achieve a higher ex-ante expected value for its objective function (given the effect of such deviations on price informativeness). Recall that in the equilibrium characterized above, the government optimally follows a linear policy rule. We now consider a more general class of linear policy rules defined by weights $\tilde{w}$, $\tilde{K}_P$ and the constant $\bar{T}$:

$$\tilde{T}\left(s_G, P; \tilde{w}, \tilde{K}_P, \bar{T}\right) \equiv \tilde{w} s_G + \tilde{K}_P \frac{1}{\rho p_Z}\left(P - p_0\right) + \bar{T}. \tag{15}$$

This class of rules nests the behavior of the government in the equilibrium characterized above, i.e., for an equilibrium ρ^*: $\tilde{w} = w\left(\rho^*\right)$, $\tilde{K}_P = K_P\left(\rho^*\right)$, and $\bar{T} = K_\theta\left(\rho^*\right)\bar{\theta} + g\left(\mu, var\left[\theta | P, s_G\right]\right)$. We often refer to this particular set of weights as the government's *ex-post optimal rule.* We then analyze when this rule implies that the government follows the market too much or too little. Formally:

Definition 1 *The government follows the market too much (respectively, too little) in equilibrium ρ^* if it would be better off committing to a rule $\left(\tilde{w}, \tilde{K}_P, \bar{T}\right)$ that puts marginally more weight on its own information, i.e., $\tilde{w} > w(\rho^*)$ (respectively, less weight, i.e., $\tilde{w} < w(\rho^*)$).*

As a first step, we determine the equilibrium—and in particular, price informativeness—

under rules that differ from the ex-post optimal rule. A straightforward adaptation of the proof of Proposition 1 implies that, given a policy rule of the form (15), equilibrium price informativeness is given by the unique solution to[16]

$$\rho = \frac{1}{\alpha} \frac{\tilde{w} \frac{\partial}{\partial s_i} E\left[\theta | s_i, P\right]}{\tilde{w}^2 \left(var\left[\theta | s_i, P\right] + var\left[\varepsilon_G\right]\right) + var\left[\delta\right]}. \tag{16}$$

Note that uniqueness here follows from the fact that the government uses fixed weights rather than adjusting the weights in an ex-post optimal way based on the informativeness ρ.

The government's payoff is determined by a combination of the informativeness of the price P, and the effectiveness with which it then uses this information. Consequently, the government's objective is not just to maximize price informativeness. To give an extreme example, even if price informativeness were maximized by the government completely ignoring the price (i.e., $\tilde{K}_P = 0$), the government would certainly not adopt this rule, since the fact that it ignores the price means that it derives no value from price informativeness.

Nonetheless, for small departures from the ex post optimal rule, the government's payoff is directly determined by price informativeness: see Part (A) of Proposition 2 below. This is a straightforward application of the envelope theorem: a small perturbation of $\tilde{w}$ away from the ex-post optimal weight $w(\rho^*)$ has only a second-order direct effect on the government's payoff, but has a first-order impact via the informativeness ρ.

Part (B) of Proposition 2 characterizes when the government follows the market too much (i.e., when a small increase in $\tilde{w}$ away from the ex-post optimal weight $w(\rho^*)$ increases price informativeness). The condition boils down to comparing the size of the two risks a speculator is exposed to when trading. The first risk is *exogenous* cash flow risk stemming from the cash flow component δ, and is unaffected by speculative trading or government intervention. It is given simply by $var[\delta]$. The second risk is *endogenous* cash flow risk which stems from the government's intervention T, which is endogenous to the model, and in particular, is

[16] In the following expression, $var\left[\theta | s_i, P\right] = \left(\tau_\varepsilon + \tau_\theta + \rho^2 \tau_Z\right)^{-1}$ and $\frac{\partial}{\partial s_i} E\left[\theta | s_i, P\right] = \tau_\varepsilon var\left[\theta | s_i, P\right]$.

affected by speculative trading activity. The size of this risk is the variance of T conditional on a speculator's information s_i and P, which depends on both price informativeness ρ and the government's rule w, and we denote it by $N(w,\rho)$. For a policy rule (15),

$$N(w,\rho) \equiv var\left[T\left(s_G, P; w, K_P, \bar{T}\right)|s_i, P\right] = w^2\left(var\left[\theta|s_i, P\right] + var\left[\varepsilon_G\right]\right).$$

Proposition 2 *The government follows the market too much (respectively, too little) in equilibrium ρ^* if and only if one of the following equivalent conditions is satisfied:*
(A) A marginal increase in $\tilde{w}$ away from $w(\rho^*)$ increases (respectively, decreases) price informativeness.
(B) Exogenous risk exceeds (respectively, falls below) endogenous risk, i.e., $var[\delta] > N(w(\rho^*), \rho^*)$ (respectively, $var[\delta] < N(w(\rho^*), \rho^*)$).

To understand Part (B) of Proposition 2, consider (16) and note that the effect of exogenously increasing the weight $\tilde{w}$ on equilibrium price informativeness is determined by the following two opposing forces:

Information importance: This effect is captured by the numerator in the right-hand side of (16). Increasing $\tilde{w}$ increases the importance of a speculator's signal s_i in forecasting the cash flow. To see this, consider the extreme case in which the government puts no weight on its own information ($\tilde{w} = 0$). Then, its intervention is a function of prices only, and each speculator's signal contains no information about cash flows beyond that contained in the price. As the government increases the weight on its own information to positive levels ($\tilde{w} > 0$), each speculator's signal contains additional information about cash flows because it contains information about the component θ of the government's signal $s_G = \theta + \varepsilon_G$. This effect increases price informativeness, since, when their signals are more relevant, speculators trade more aggressively on their private signals.

Residual risk: This effect is captured by the denominator in the right-hand side of (16). The more weight the government puts on its own information s_G, the more residual risk speculators are exposed to. This risk is composed of both the uncertainty about the state variable θ and the noisy component ε_G of the government's signal. Because speculators are risk averse they then trade less aggressively, decreasing price informativeness.

Essentially, there is a risk-return tradeoff here. When the government bases its action more on its private information rather than on public information, it makes speculators' private information more important in predicting the government's action and so increases their return to trading on this information. But, on the other hand, this also increases the risk that speculators are exposed to when they trade on their information.

Part (B) of Proposition 2 gives a simple condition for when the information importance effect dominates—namely that the majority of risk be exogenous rather than endogenous. To gain intuition, note that without any exogenous risk, the residual risk effect always dominates the information importance effect. This is simply due to the weight $\tilde{w}$ affecting endogenous risk in the denominator via $\tilde{w}^2$, while having only a linear effect on information importance in the numerator. However, as exogenous risk increases, the weight $\tilde{w}$ has relatively less effect on the total residual risk that speculators are exposed to. So when exogenous risk is significant enough, the information importance effect dominates the residual risk effect.

We next use Part (B) of Proposition 2 to characterize which parameters of the model lead the government to follow the market too much (or too little). First, and perhaps paradoxically, the government follows the market too much when the price is highly informative (i.e., ρ is high and/or $var\left[Z\right]$ is low). This is because in this case endogenous risk $N\left(w\left(\rho^*\right), \rho^*\right)$ is low—directly, because $var\left[\theta|s_i, P\right]$ is low, and indirectly, because $w\left(\rho^*\right)$ is low when the price is informative. Hence, in cases in which the ex-post optimal rule leads to highly informative equilibrium prices, the government could actually obtain even more informative prices if it could commit to put a little more weight on its own signal s_G.

Second, and similarly, the government follows the market too little when its own signal s_G is accurate ($var\left[\varepsilon_G\right]$ is low). To see this, note that here there are two effects. The first effect is that a low $var\left[\varepsilon_G\right]$ reduces endogenous risk directly, while the second effect is that it increases the weight w the government puts on its own information, which increases the share of endogenous risk. The second effect dominates (see proof of Corollary 2). Hence, the government would increase both its own payoff, and price informativeness, if it could commit to put a little less weight on its information when its information is very precise.

These two results are summarized in the following corollary.

Corollary 2 *The government follows the market too much when either:*

(I) Risk-aversion α is low and/or the variance of supply $var\left[Z\right]$ is low (and consequently, when equilibrium price informativeness is high).

(II) The government's own information is imprecise, i.e., $var\left[\varepsilon_G\right]$ is high.

A popular idea in some policy circles is that the government should commit to intervene in a pre-determined way based on publicly observable prices: see, e.g., Rochet (2004) and Hart and Zingales (2011). This suggestion is motivated by a number of concerns, some of which are outside our model—in particular, a concern that, absent clear rules, the government acts too softly ex post. However, our analysis highlights another consideration to be taken into account when assessing the costs and benefits of these proposals: the effect that such proposals have on price informativeness. In particular, our analysis shows that absent commitment the government may follow the market too much rather than too little (see Corollary 2). Under these circumstances, the government would like to commit to place less rather than more weight on market prices.

While Corollary 2 deals with parameters that change endogenous risk $N\left(w, \rho\right)$, we now analyze the effect of exogenous risk $var\left[\delta\right]$:

Corollary 3 *The government follows the market too much if exogenous risk $var\left[\delta\right]$ is sufficiently high.*

Note that the result is not quite as straightforward as it may appear. When exogenous risk $var\left[\delta\right]$ is high, price informativeness ρ is low because speculators bear a lot of risk.

Consequently, when $var\left[\delta\right]$ is high, both exogenous and endogenous risk are high. However, because endogenous risk is bounded above, the direct exogenous risk effect dominates as $var\left[\delta\right]$ grows large (see proof). Note that for firms with high exogenous risk, the government's reliance on the price is low to begin with, since the price is a noisy indicator of what the government is trying to learn. However, Corollary 3 implies that the reliance on the price should decrease further, given the effect this reliance has on price informativeness.

4 The importance of a government's own information

The results above establish that in many circumstances the government follows the market too much. In this section, we explore some practical implications of this result.

An immediate practical implication is that in those cases in which the government follows the market too much, it would benefit from an ex-ante commitment to reduce its ex-post reliance on the price. Of course, such commitment is hard to achieve, and a government may be unable to credibly commit to underweight price information—especially since the government's signal may be unobservable even ex post, making it hard for outsiders to know how much weight the government puts on its own information relative to the price. One possible solution is to select policymakers who are overconfident about the precision of their information. This would achieve the desired commitment, though may also carry other costs.

A less controversial way for the government to commit to reduce its reliance on the price is to improve the quality of its own private information. As we show below, whenever an exogenous increase in the weight the government puts on its own information, $\tilde{w}$, increases price informativeness, then an increase in the quality of the government's information also increases price informativeness—even absent government commitment.

Formally, suppose that the precision of the government's information, τ_G, is a choice variable. What are the benefits of increasing τ_G? Because the price aggregates speculators' information only imperfectly, the government uses both the price and its private information

s_G in its intervention decisions. So an increase in the precision of the government's private signal has a direct positive effect on the quality of the government's overall information about the state θ. More interesting, however, is that an increase in τ_G may also have a positive indirect effect, in that more accurate government information leads to more informative prices. This follows from previous results on the effect of the government's use of market information on the quality of this information: An increase in τ_G increases the weight w that the government puts on its own information, which, in the cases characterized by Proposition 2 and Corollaries 2 and 3, increases equilibrium price informativeness. Hence, in these cases, the government should be willing to spend more on producing its own information than the direct contribution of this information to its decision making would imply.[17]

Proposition 3 *Suppose that in equilibrium ρ^* a small exogenous increase in $\tilde{w}$ away from $w(\rho^*)$ raises price informativeness. Then a small exogenous increase in the quality of the government's information, τ_G, results in an equilibrium with price informativeness strictly above ρ^* when the government acts ex post optimally.*

This result is interesting because it goes against a common belief that the availability of market information makes the government's own information unnecessary. Indeed, it is tempting to interpret policy proposals to use market information as implying that governments do not need to engage in costly collection of information on their own. For example, in the context of banking supervision, one might imagine that the government could substantially reduce the number of bank regulators when it can learn from prices. Our framework demonstrates that when the usefulness of market information is endogenous and affected by the government's use of this information, a decrease in the quality of the government's

[17] Bond, Goldstein, and Prescott (2010) also note that the government's own information helps the government make use of market information. However, in that model, the market price perfectly reveals the expected value of the firm, and the problem is that the expected value does not provide clear guidance as to the optimal intervention decision. Hence, the government's information can complement the market information in enabling the government to figure out the optimal intervention decision. Here, on the other hand, the fact that the government is more informed encourages speculators to trade more aggressively, and thus leads the price to reflect the expected value more precisely.

information might worsen price informativeness. Hence, the usual argument that market information can easily replace the government's own information is incorrect.

5 Transparency

Governments are often criticized for not being transparent enough about their information and policy goals. But is government transparency actually desirable when the government itself is trying to elicit information from prices? How does the release of government information affect speculators' incentives to trade on their information? Our model's implications for these questions hinge on the type of transparency in question—in particular, on whether the information in question concerns variables that the government can learn more about.

5.1 Transparency: variables the government can learn more about

In our model, the government directly observes a signal s_G about θ, but also tries to extract further information about θ from the market price. To analyze the effect of transparency, we consider what happens if the government is fully transparent about its own information, s_G. Specifically, suppose the government announces s_G before speculators trade.[18] Transparency of this type has an extreme effect in our model, as we now show.

Recall that the price is determined by the market-clearing condition

$$\frac{1}{\alpha}\frac{E\left[T\left(s_G,P\right)|s_i,s_G,P\right]+\overline{\delta}-P}{var\left[T\left(s_G,P\right)|s_i,s_G,P\right]+var\left[\delta\right]}+Z=0,$$

where note that, because of transparency, the government's signal s_G now enters speculators' information sets. Conditional on the price and s_G, there is now no uncertainty about the government's intervention T: hence $var\left[T\left(s_G,P\right)|s_i,s_G,P\right]=0$, and moreover,

[18]Throughout this section we assume that the government can credibly disclose information.

$E\left[T\left(s_G, P\right)|s_i, s_G, P\right] = T\left(s_G, P\right)$. Hence the market-clearing condition collapses to

$$\frac{1}{\alpha}\frac{T\left(s_G, P\right) + \overline{\delta} - P}{var\left[\delta\right]} + Z = 0. \tag{17}$$

From this identity, it is clear that the equilibrium price contains no information about θ beyond that available directly in s_G, and so transparency leads the government to ignore the price in making its intervention decision. Consequently:

Proposition 4 *If the government discloses its own information s_G, then the price ceases to be a useful source of information for the government, and the value of the government's objective function decreases.*

Rearrangement of (17) delivers an explicit expression for the price under full transparency:

$$P\left(s_G, Z\right) = T\left(s_G\right) + \overline{\delta} + \alpha var\left[\delta\right] Z. \tag{18}$$

It is worth highlighting that the price (18) *does* provide information about the fundamental θ to an *uninformed* outsider,[19] since it is still a noisy signal of s_G (which is a noisy signal of θ). But, as expressed in Proposition 4, the price contains no information that the government does not already have. In contrast, absent transparency the price is a noisy signal of the fundamental θ that is conditionally independent of the government's own noisy signal $s_G = \theta + \varepsilon_G$. It follows directly that the government is worse off by revealing its signal s_G.

Intuitively, when revealing its signal, the government eliminates the incentive of informed speculators to trade on their private information about the fundamental θ, since this private information no longer has any predictive power for the government's behavior beyond the publicly available information (P, s_G). Hence, the price reveals nothing of the private information available to speculators and becomes useless as a source of information for the government's intervention decision. As a result, the government then has to make the intervention decision with less precise information and is clearly worse off.

[19]Specifically, an uninformed outsider who somehow missed the government's announcement of s_G.

5.2 Transparency: variables the government cannot learn more about

We next consider the effects of transparency about variables the government cannot learn more about. To do so, suppose now that the government's benefit from intervention is

$$v\left(T-\psi-\theta\right),$$

where ψ is a normally distributed variable independent of θ. As before, both the government and speculators observe conditionally independent noisy signals (s_G and s_i, respectively) of θ. In contrast, the government observes a signal of ψ, $\sigma_G = \psi + \zeta_G$, but speculators observe only noisy signals of σ_G, $\sigma_i = \psi + \zeta_G + \zeta_i$. Consequently, the government is unable to learn anything from market prices about the realization of ψ, for the simple reason that speculators do not have any information about ψ beyond that which the government already has. Parallel to before, ζ_G and ζ_i are independently distributed normal random variables.

A leading interpretation of the state variable ψ is that it represents the government's policy objectives. In this case, it is natural to assume $\sigma_G \equiv \psi$, and so it is impossible for speculators to have information that the government does not already have. A second possible interpretation is that ψ is the aggregate state of the economy, while θ is a bank-specific state variable; and that a speculator's information about the aggregate state is weak coarsening of the government's own information.

Our main result is:

Proposition 5 *Disclosure of the government's information σ_G about ψ increases equilibrium price informativeness and hence the expected value of the government's objective function.*

The intuition behind Proposition 5 is clearest in the limit case in which speculators learn nothing about the government signal σ_G, i.e., $var\left[\zeta_i\right] = \infty$. In this case, there is a linear equilibrium that takes the same form as before, and equilibrium price informativeness again satisfies the first equality in (13). Moreover, by Lemma 1, the government's intervention is

$$T = E[\psi|\sigma_G] + E[\theta|s_G, P] + g(\mu, var[\psi|\sigma_G] + var[\theta|s_G, P]).$$

In this case, transparency about σ_G has no effect on the information importance term $\frac{\partial}{\partial s_i} E[T|s_i, P]$ in (13) while it unambiguously reduces the residual risk term $var[T|s_i, P]$. Consequently, price informativeness is increased.

The general case of $var[\zeta_i] < \infty$ is handled in full in the Appendix. The first complication is that the equilibrium price now depends on the government signal σ_G, which is known collectively by speculators. Consequently, the price is of the form $P = p_0 + \rho p_Z \theta + \xi \rho p_Z \sigma_G + p_Z Z$, for some scalars p_0, p_Z, ρ and ξ. The existence of an equilibrium of this type is proved in the Appendix. Because the government observes σ_G, the price conveys the same information to the government as does $\theta + \rho^{-1} Z$. Consequently, ρ remains the relevant measure of price informativeness. Moreover, the same argument as before implies that equilibrium price informativeness ρ still satisfies the first equality in (13).

The second complication is that a speculator i's signal s_i now has multiple effects his forecast of the intervention T. To see this, observe that the government's intervention T is

$$T = E[\theta|P, s_G, \sigma_G] + E[\psi|\sigma_G] + \text{constant}.$$

Consequently, speculator i's signal s_i affects his forecast of T not just via its effect on $E[s_G|P, s_i, \sigma_i]$ (the effect in the basic model); but also via its effect on $E[\sigma_G|P, s_i, \sigma_i]$. The proof of Proposition 5 establishes that disclosing σ_G increases the information importance of s_i. Loosely speaking, disclosure of σ_G makes it easier for a speculator to forecast the intervention T, and hence $E[T|P, s_i, \sigma_i]$ becomes more sensitive to s_i. Moreover, and as in the limit case, disclosing σ_G decreases residual risk, so that price informativeness is again unambiguously increased.

Proposition 5 captures what is perhaps the usual intuition about transparency and the reason why it is strongly advocated. The idea is that when the government reveals its information (e.g., about its policy goal), it reduces uncertainty for speculators. This encourages them to trade more aggressively on their information, resulting in higher price informativeness. The government is then better off as it can make more informed decisions.

5.3 Discussion

In general, when the government reveals information to speculators, there are two effects on speculator trading. On the one hand, making the government's information public may reduce speculators' incentive to trade because it reduces the informational advantage that brings them to the market in the first place. On the other hand, making the government's information public may increase speculators' incentive to trade because it reduces the overall uncertainty that speculators are exposed to, allowing them to trade more aggressively. In our analysis, if the government releases information about θ, about which speculators have information that the government does not, it destroys its ability to learn about θ from speculators. This is the first effect above. However, if the government releases information about ψ, about which the speculators have coarser information than the government, it enhances its ability to learn about θ from speculators. This is the second effect above.

Hence, our model provides justification for disclosing the government's policy goal, as this is a variable about which the government is unlikely to have anything to learn about from the speculators. But, the government should be more cautious when disclosing information about the state of an individual bank, as this is a variable the government may want to learn more about and speculators may be informed about. A little more speculatively, disclosure of information about economic aggregates may be desirable, as the government is less likely to be able to learn something about them from the public.

6 The appropriate measure of informativeness

In this paper we define price informativeness as the amount of information the price provides about the state variable θ. This is not the definition used in much of the finance literature, and in this section we discuss the difference.

In the context of the model, our definition of price informativeness is a natural one, since it is the state variable θ that is relevant for the government's intervention decision. Indeed, Proposition 2 shows that this notion of price informativeness is directly linked to the government's objective function. Hence, to the extent that we care about price informativeness because of its effect on the efficiency of the government's intervention, then the relevant notion of informativeness is one that captures the amount of information provided by the price for the intervention decision; this is the informativeness of the price about θ.

In contrast, the traditional definition of price informativeness in the literature is the accuracy with which the price forecasts future firm cash flows. Specifically, this is captured by the inverse of the variance of cash flows conditional on observing the price, but no other information (for example, Brunnermeier (2005) and Peress (2010)). In our model, this measure is given by

$$\varsigma \equiv \left(var\left[\delta\right] + w^2\left(var\left[\theta|P\right] + var\left[\varepsilon_G\right]\right)\right)^{-1}. \tag{19}$$

This is a measure of how well the market predicts future cash flows, i.e., how "efficient" the market is. In this section, we show that the measure ς is disconnected from real efficiency, which in our model is the efficiency of the government's intervention decision. The measure ς is built on the premise that the market is a side show that predicts future cash flows, rather than providing information that guides future cash flows. Hence, focusing on the measure ς might lead to very misleading answers if we care about the efficiency of the actions that are guided by market information. To make this point, we provide two examples:

Example 1, Pure price-based intervention: If the government makes intervention decisions based purely on the price, the weight w on its own information is 0. On the one hand, pure price-based intervention maximizes the informativeness measure ς: by (19), ς reaches its upper bound of $var[\delta]^{-1}$ (which is determined by the exogenous component of cash flow that is impossible to forecast). But on the other hand, pure price-based intervention minimizes our price informativeness measure ρ, since from (16) (and provided $var[\delta] > 0$), $\rho = 0$. Hence, for the purpose of achieving greater efficiency in government intervention, it would be a mistake to focus on ς instead of on ρ when deciding on the weight w that the government puts on its own information

Example 2, Transparency about s_G: From Section 5, if the government publicly announces s_G then the price ceases to be a useful source of information. However, at least when $var[\delta]$ is small, transparency increases the informativeness measure ς.[20] This can be seen from (18): when $var[\delta]$ is small, the price forecasts intervention T very well, so that ς approaches its upper bound $var[\delta]^{-1}$.[21] Again, if we care about the efficiency of the real action—the government intervention decision—then releasing the government signal s_G is a mistake even though it may increase the traditional measure of price informativeness.

As a specific application of Example 2, consider evaluating the success of government stress tests, which can be viewed as disclosures of government information. It may seem tempting to evaluate stress tests by asking whether they increase the traditional measure ς of market efficiency. But if the reason we care about price informativeness is that prices can guide decisions, this is the wrong metric to use.

Our overall point in this discussion is that it is hard, and probably impossible, to talk meaningfully about price informativeness in a completely theory-free way. Instead, one must specify why one cares about price informativeness in the first place, and let this inform the appropriate definition.[22] In our model, speculative trading is driven by the possibility of

[20] Here, ς captures the information about cash flow conditional on the price, but not conditional on the government signal.

[21] In contrast, when the government does not announce s_G (no transparency), then ς does not approach $var[\delta]^{-1}$ even as $var[\delta] \to 0$.

[22] See, for example, Paul (1992) and Bresnahan, Milgrom and Paul (1992) for related discussions about price informativeness. We ourselves discuss a closely related point in Bond, Edmans and Goldstein (2012); see also other references cited therein.

government intervention, and we care about the efficiency of this real decision—so it is the informativeness of the price about the relevant state variable θ that matters. Note that our efficiency criterion ignores the welfare of both noise traders and speculators; in this, we stop short of a full welfare analysis. Incorporating such considerations requires endogenizing the motives of noise traders, which is beyond the scope of our paper, and most likely beyond the scope of government policy. If these agents were included in welfare considerations, then one would use a weighted average of the government objective and trader welfare. Our analysis approximates the case in which the weight on the government objective is large.

7 Injection subsidies

So far, our model takes the extent to which government interventions affect shareholder payoffs as given. In particular, the full amount T injected by the government benefits shareholders. Of course, in reality this does not need to be the case. The government can recapture some of the funds injected to the firm from shareholders. For example, the government can structure cash infusions as loans, with a (gross) interest rate of R. The case $R = 0$ then corresponds to a pure gift to shareholders, as in our model so far; while when $R = 1$, shareholders do not benefit at all from the cash injection (which instead affects only the externalities that the government is concerned about). In this section, we extend our model to allow for interventions in which $R \neq 0$.

In practice, the government's choice of interest rate is likely to be affected by lots of different factors, many of them political, and a full analysis of the government's behavior in this dimension is beyond the scope of the current paper.[23] Instead, we want to make two points. First, our main results continue to hold for any value of $R \neq 1$. Second, the government potentially faces an unpleasant tradeoff between minimizing the subsidy to shareholders and maintaining price informativeness.

[23] As we mentioned before, empirical evidence suggests that subsidies do exist, as shareholders seem to benefit from the prospect of government bailouts of their financial institutions.

In the extension we analyze, the firm's cash flow is $X = \delta + (1-R)\,T$. Equilibrium price informativeness ρ is determined by the solution to

$$\rho = \frac{1}{\alpha} \frac{|1-R|\, w(\rho) \frac{\partial}{\partial s_i} E[\theta|s_i, P]}{(1-R)^2\, w(\rho)^2 \left(var[\theta|s_i, P] + var[\varepsilon_G]\right) + var[\delta]}. \tag{20}$$

Inspecting this expression, one can see that the main forces of our model are still at work, provided $R \neq 1$. The effects of changing the government's weight w on its own signal, for example, are the same as in our main model (i.e., $R = 0$). In particular, the assumption $R = 0$ is not an important driver of our results.

In addition to verifying robustness, one can use this extension to analyze how reducing subsidies (i.e., increasing R) would affect price informativeness ρ. The most striking implication is that the no-subsidy case of $R = 1$ minimizes price informativeness. Relative to this no-subsidy case, prices would be more informative if the government either added a subsidy (i.e., $R < 1$), or raised the interest rate so that interventions are strictly profitable (i.e., $R > 1$). If feasible, the latter approach is better for the government, since it both increases price informativeness and raises revenue. However, setting $R > 1$ means that government intervention makes shareholders worse off, which might be politically challenging. If only values of $R \leq 1$ are feasible, the government faces an unpleasant tradeoff between minimizing the subsidy to shareholders and maintaining price informativeness.

More generally, changes in the gross interest rate R affect informativeness in the opposite direction as exogenous changes in the government's weight w on its own signal (when $R < 1$). Economically, an increase in the rate R decreases the effect of the government's signal s_G on shareholders' payoffs in exactly the same way that a decrease in w does. Consequently:

Proposition 6 *If the government injection is subsidized, i.e., $R < 1$, then a marginal increase in the rate R increases price informativeness if and only if the government is following the market too little.*

This analysis suggests that, in setting R, the government may want to consider the effect on the informativeness of the price, in addition to other considerations. In some cases, heavily subsidizing government cash injections (reducing R) reduces price informativeness prices, and the government should cut subsidies. In other cases, the effect of subsidies on price informativeness is the reverse, and the government faces a more difficult tradeoff between reducing the cost of intervention and increasing the informativeness of market prices. In particular, this is the case in the neighborhood of no subsidies ($R = 1$).

8 Concluding remarks

We analyze how market-based government policy affects the trading incentives of risk-averse speculators in a rational-expectations model of financial markets. Increasing the reliance of government intervention on market information affects the risk-return tradeoff faced by speculators, and hence changes their incentive to trade. Our analysis shows that the use of market prices as an input for policy might not come for free and might damage the informational content of market prices themselves. We characterize cases in which the government would be better off limiting its reliance on market prices and increasing their informational content. However, the government always benefits from some reliance on market prices. Also, and counter to common belief, transparency by the government might be a bad idea in that it might reduce trading incentives and price informativeness, which hurts the government. While we focus in this paper on market-based government policy, our analysis and results apply more generally to other non-governmental actions based on the price. For example, our framework naturally covers the case of a manager or board of directors making decisions that affect firm cash flows. Hence, our paper contributes more generally to the understanding of the interaction between financial markets and corporate decisions, and in particular, the ways in which secondary financial market have real effects.

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Appendix

Appendix

We start by defining some notation, which we use throughout the appendix:

$$\upsilon_\varepsilon(\rho) \equiv \tau_\theta + \rho^2\tau_z + \tau_\varepsilon \tag{A-1}$$

$$\upsilon_G(\rho) \equiv \tau_\theta + \rho^2\tau_z + \tau_G \tag{A-2}$$

$$F(w,\rho) = \frac{w\tau_\varepsilon\upsilon_\varepsilon(\rho)^{-1}}{w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}}. \tag{A-3}$$

Proof of results other than Proposition 5

Proof of Proposition 1 [existence]: To establish existence, we show that there exist p_0, ρ and p_Z such that the price function (7), i.e., $P = p_0 + \rho p_Z\theta + p_Z Z$, is an equilibrium. Specifically, we show that there exist p_0, ρ and p_Z such that market clearing (3) holds.

First, and as discussed in the main text, when the price function is of the form (7), observing the price is equivalent to observing $\tilde{P} \equiv \frac{1}{\rho p_Z}(P - p_0) = \theta + \rho^{-1}Z$, which is an unbiased estimate of θ, is normally distributed, and has precision $\rho^2\tau_Z$. Hence by a standard application of Bayes' rule to normal distributions, the conditional distributions of θ given the information of, respectively, the government and a speculator i are normal, with conditional variances $var[\theta|s_G,P] = \upsilon_G(\rho)^{-1}$ and $var[\theta|s_i,P] = \upsilon_\varepsilon(\rho)^{-1}$, where $\upsilon_G(\rho)$ and $\upsilon_\varepsilon(\rho)$ are as defined in (A-1) and (A-2). Likewise, the conditional expectations of θ are

$$E[\theta|s_G,P] = \frac{\tau_\theta\bar{\theta} + \rho^2\tau_Z\tilde{P} + \tau_G s_G}{\upsilon_G(\rho)}$$

$$E[\theta|s_i,P] = \frac{\tau_\theta\bar{\theta} + \rho^2\tau_Z\tilde{P} + \tau_\varepsilon s_i}{\upsilon_\varepsilon(\rho)}.$$

Substituting $E[\theta|s_G,P]$ and $var[\theta|s_G,P]$ into (4), and using the definition (10) of $w(\rho)$, the government's intervention is

$$T = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P} + \tau_G s_G}{\upsilon_G(\rho)} + g\left(\mu, \upsilon_G(\rho)^{-1}\right) = w(\rho)\, s_G + \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}}{\upsilon_G(\rho)} + g\left(\mu, \upsilon_G(\rho)^{-1}\right).$$

So a speculator i's conditional expectation and conditional variance of T are

$$\begin{aligned} E[T|s_i, P] &= w(\rho)\, E[\theta|s_i, P] + \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}}{\upsilon_G(\rho)} + g\left(\mu, \upsilon_G(\rho)^{-1}\right) \\ var[T|s_i, P] &= w(\rho)^2 \left(var[\theta|s_i, P] + var[\varepsilon_G]\right). \end{aligned}$$

Substituting in (12) and $\int s_i di = \theta$, the market-clearing condition (3) is

$$\frac{1}{\alpha} \frac{w(\rho)\, E[\theta|s_i = \theta, P] + \bar{\delta} + \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}}{\upsilon_G(\rho)} + g\left(\mu, \upsilon_G(\rho)^{-1}\right) - P}{w(\rho)^2 \left(var[\theta|s_i, P] + var[\varepsilon_G]\right) + var[\delta]} + Z = 0.$$

This is a linear expression in the random variables θ and Z. Consequently, market clearing (3) is satisfied for all θ and Z if and only if the intercept term and the coefficients on θ and Z all equal zero. Written explicitly, these three conditions are

$$w(\rho) \frac{\tau_\theta \bar{\theta}}{\upsilon_\varepsilon(\rho)} + \bar{\delta} + \frac{\tau_\theta \bar{\theta}}{\upsilon_G(\rho)} + g\left(\mu, \upsilon_G(\rho)^{-1}\right) - p_0 = 0 \quad \text{(A-4)}$$

$$w(\rho) \frac{\rho^2 \tau_Z + \tau_\varepsilon}{\upsilon_\varepsilon(\rho)} + \frac{\rho^2 \tau_Z}{\upsilon_G(\rho)} - \rho p_Z = 0 \quad \text{(A-5)}$$

$$w(\rho) \frac{\rho^2 \tau_Z}{\upsilon_\varepsilon(\rho)} \rho^{-1} + \frac{\rho^2 \tau_Z}{\upsilon_G(\rho)} \rho^{-1} - p_Z + \alpha\left(w(\rho)^2 \left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}\right) = 0. \quad \text{(A-6)}$$

To complete the proof of existence, we must show that this system of three equations in p_0, ρ and p_Z has a solution. Observe that $\rho \times$(A-6)$-$(A-5) is

$$-w(\rho) \frac{\tau_\varepsilon}{\upsilon_\varepsilon(\rho)} + \alpha\rho\left(w(\rho)^2 \left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}\right) = 0,$$

which using the definition (A-3) of F can be rewritten as

$$\rho = \frac{1}{\alpha} \frac{w(\rho) \tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1}}{w(\rho)^2 \left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}} = \frac{1}{\alpha} F(w(\rho), \rho).$$

(Note that this equation coincides with the equilibrium condition (13) given in the main text.) Hence to show existence, we show that there exist p_0, ρ and p_Z with $\rho \neq 0$ that satisfy (A-4), (A-5) and $\alpha\rho = F(w(\rho), \rho)$. Since p_0 appears only once, in (A-4), and p_Z appears only once, in (A-5), existence is established if $\alpha\rho = F(w(\rho), \rho)$ has a non-zero solution. This is indeed the case: $F(w(\rho), \rho)$ is continuous in ρ, and by Lemma A-1 below, $F(w(0), 0) > 0$ and $\lim_{\rho\to\infty} F(w(\rho), \rho) < \infty$. This completes the proof of equilibrium existence.

Lemma A-1 *(I) $F(w(0), 0) > 0$ and $\lim_{\rho\to\infty} F(w(\rho), \rho) < \infty$. (II) For any $\tilde{w} > 0$, $F(\tilde{w}, 0) > 0$ and $\lim_{\rho\to\infty} F(\tilde{w}, \rho) < \infty$.*

Proof of Lemma A-1: We prove part (I); part (II) is similar, but more straightforward.

The fact that $F(w(0), 0) > 0$ follows from $w(0)\tau_\varepsilon > 0$ and $w(0)^2 \left(1 + \upsilon_\varepsilon(0)\tau_G^{-1}\right) + \upsilon_\varepsilon(0)\tau_\delta^{-1} < \infty$.

Both $\upsilon_\varepsilon(\rho)$ and $\upsilon_G(\rho)$ are strictly increasing in ρ. So certainly $\lim_{\rho\to\infty} w(\rho)\tau_\varepsilon\upsilon_\varepsilon(\rho)^{-1} < \infty$. If $\tau_\delta^{-1} > 0$, it is then immediate that $\lim_{\rho\to\infty} F(w(\rho), \rho) < \infty$. If instead $\tau_\delta^{-1} = 0$ and $\tau_G^{-1} > 0$, then note that

$$F(w(\rho), \rho) = \frac{\tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1}}{\tau_G \upsilon_G(\rho)^{-1} \left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right)},$$

and hence $\lim_{\rho\to\infty} F(w(\rho), \rho) < \infty$ because $\lim_{\rho\to\infty} \upsilon_G(\rho)/\upsilon_\varepsilon(\rho) = 1$. Finally, if $\tau_G^{-1} = \tau_\delta^{-1} = 0$, then $F(w(\rho), \rho) = \tau_\varepsilon$ for all ρ. **QED**

Proof of Proposition 1 [uniqueness]: The equilibrium is unique within the class of linear equilibria if $\alpha\rho = F(w(\rho), \rho)$ has a unique positive solution. Equilibrium uniqueness at $\tau_G^{-1} = 0$ follows from the fact that, in this case, $w(\rho) = 1$, and so $F(w(\rho), \rho)$ is decreasing in ρ, and so has a unique positive solution.

For $\tau_G^{-1} > 0$, we deal separately with the cases $\tau_\delta^{-1} > 0$ and $\tau_\delta^{-1} = 0$:

Case: $\tau_\delta^{-1} > 0$: Differentiation of $F(w(\rho), \rho)$ by τ_G^{-1} gives

$$\frac{\partial F(w(\rho), \rho)}{\partial\left(\tau_G^{-1}\right)} = \frac{\partial w(\rho)}{\partial\left(\tau_G^{-1}\right)} F_w(w, \rho) - \frac{w^3 \tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1}}{\left(w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}\right)^2},$$

where

$$\begin{aligned} F_w(w, \rho) &= \tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1} \frac{w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1} - 2w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right)}{\left(w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}\right)^2} \\ &= \tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1} \frac{\tau_\delta^{-1} - w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right)}{\left(w^2\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}\right)^2} \end{aligned} \quad \text{(A-7)}$$

and

$$\frac{\partial w(\rho)}{\partial\left(\tau_G^{-1}\right)} = \frac{\partial w(\rho)}{\partial \tau_G} \frac{\partial \tau_G}{\partial\left(\tau_G^{-1}\right)} = -\frac{\upsilon_G(\rho) - \tau_G}{\left(\upsilon_G(\rho)\right)^2} \tau_G^2 = -\frac{\tau_\theta + \rho^2 \tau_Z}{\left(\tau_G + \tau_\theta + \rho^2 \tau_Z\right)^2} \tau_G^2 = -\frac{\tau_\theta + \rho^2 \tau_Z}{\left(1 + \tau_\theta \tau_G^{-1} + \rho^2 \tau_Z \tau_G^{-1}\right)^2}.$$

For $\tau_G^{-1} > 0$, it follows straightforwardly that there exists some constant κ such that for all $\rho \in [0, \infty)$, $\left|\frac{\partial F(w(\rho),\rho)}{\partial\left(\tau_G^{-1}\right)}\right| \leq \kappa$. At $\tau_G^{-1} = 0$, note that $w(\rho) \equiv 1$, and hence

$$\left.\frac{\partial F(w(\rho), \rho)}{\partial\left(\tau_G^{-1}\right)}\right|_{\tau_G^{-1}=0} = -\left(\tau_\theta + \rho^2 \tau_Z\right) \frac{\tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1}\left(\tau_\delta^{-1} - \upsilon_\varepsilon(\rho)^{-1}\right)}{\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_\delta^{-1}\right)^2} - \frac{\tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1}}{\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_\delta^{-1}\right)^2},$$

and so again there exists some constant κ such that for all $\rho \in [0, \infty)$, $\left|\frac{\partial F(w(\rho),\rho)}{\partial\left(\tau_G^{-1}\right)}\right| \leq \kappa$. By standard arguments, the fact that $\alpha\rho = F(w(\rho), \rho)$ has a unique solution at $\tau_G^{-1} = 0$ then implies that it also has a unique solution for all τ_G^{-1} sufficiently small.

Case: $\tau_\delta^{-1} = 0$: In this case, $F(w(\rho), \rho)$ simplifies to

$$F(w(\rho), \rho) = \frac{\tau_\varepsilon \upsilon_\varepsilon(\rho)^{-1}}{w(\rho)\left(\upsilon_\varepsilon(\rho)^{-1} + \tau_G^{-1}\right)} = \tau_\varepsilon \frac{1 + \tau_G^{-1}\left(\tau_\theta + \rho^2 \tau_Z\right)}{1 + \tau_G^{-1}\left(\tau_\varepsilon + \tau_\theta + \rho^2 \tau_Z\right)},$$

which is decreasing in τ_G^{-1}. Hence for all $\tau_G^{-1} \geq 0$, we know the equilibrium lies in the compact set $\left[0, \frac{\tau_\varepsilon}{\alpha}\right]$. Because $F\left(w\left(\rho\right), \rho\right)$ is well-behaved over this compact set, equilibrium uniqueness at $\tau_G^{-1} = 0$ implies equilibrium uniqueness for all τ_G^{-1} sufficiently small. **QED**

Proof of Corollary 1: Substituting into (13), the equilibrium value of ρ is determined by the solution to

$$\rho = \frac{1}{\alpha} \frac{\tau_G \upsilon_G\left(\rho\right)^{-1} \tau_\varepsilon \upsilon_\varepsilon\left(\rho\right)^{-1}}{\tau_G^2 \upsilon_G\left(\rho\right)^{-2}\left(\upsilon_\varepsilon\left(\rho\right)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}}. \tag{A-8}$$

The statement about τ_G is established by a numerical example: a particularly simple example is that if $\tau_Z = \tau_\delta = \tau_\theta = \tau_\varepsilon = \alpha = 1$, then $K_P\left(\rho\right)$ is first increasing then decreasing in τ_G.

To establish the other comparative statics, we show that $\rho^2 \tau_Z$ is increasing in τ_δ, τ_ε, τ_Z and decreasing in α; given the expression for $K_P\left(\rho\right)$, the result then follows.

From the existence proof (Proposition 1), we know that at the least- and most-informative equilibria, the RHS of (A-8) has a slope below 1. The implication that the equilibrium value of ρ is increasing in τ_δ and decreasing in α is then immediate from the fact that the RHS of (A-8) is increasing in τ_δ and decreasing in α. The implication that the equilibrium value of ρ is increasing in τ_ε also follows from the fact the RHS of (A-8) is increasing in τ_ε; to see this, multiply both the numerator and denominator of the RHS by $\upsilon_\varepsilon\left(\rho\right)$. Finally, the comparative static with respect to τ_Z follows by a change of variables: defining $\varrho = \rho^2 \tau_Z$, the equilibrium equation is

$$\varrho^{1/2} = \frac{1}{\alpha} \tau_Z^{1/2} \frac{\tau_G^{-1}\left(\tau_\theta + \tau_G + \varrho\right)^{-1} \tau_\varepsilon \left(\tau_\theta + \tau_\varepsilon + \varrho\right)^{-1}}{\tau_G^2\left(\tau_\theta + \tau_G + \varrho\right)^{-2}\left(\left(\tau_\theta + \tau_\varepsilon + \varrho\right)^{-1} + \tau_G^{-1}\right) + \tau_\delta^{-1}},$$

and the same argument as above implies that the equilibrium value of ϱ is increasing in τ_Z. **QED**

Proof of Proposition 2: *Part (A):* By definition, for ρ fixed at ρ^*, setting $\tilde{w} = w\left(\rho^*\right)$

solves the government's maximization problem

$$\max_{\tilde{w},\tilde{K}_P,\bar{T}} E_{\theta,\varepsilon_G,Z}\left[v\left(\tilde{w}s_G+\tilde{K}_P\left(\theta+\rho^{-1}Z\right)+\bar{T}-\theta\right)-\mu\left(\tilde{w}s_G+\tilde{K}_P\left(\theta+\rho^{-1}Z\right)+\bar{T}\right)\right]. \tag{A-9}$$

By the envelope theorem, the derivative with respect to $\tilde{w}$ of the maximand in (A-9) equals

$$\frac{\partial\rho}{\partial\tilde{w}}\frac{\partial}{\partial\rho}E_{\theta,\varepsilon_G,Z}\left[v\left(\tilde{w}s_G+\tilde{K}_P\left(\theta+\rho^{-1}Z\right)+\bar{T}-\theta\right)-\mu\left(\tilde{w}s_G+\tilde{K}_P\left(\theta+\rho^{-1}Z\right)+\bar{T}\right)\right]. \tag{A-10}$$

Since v is concave, an increase in ρ increases the expected value of the government's objective function by reducing variance and so creating a second-order stochastically dominant gamble. Hence (A-10) is strictly positive if $\frac{\partial\rho}{\partial\tilde{w}}>0$ and strictly negative if $\frac{\partial\rho}{\partial\tilde{w}}<0$, i.e., a marginal increase in $\tilde{w}$ away from $w(\rho^*)$ affects price informativeness and the expected value of the government's objective in the same direction, completing the proof of Part (A).

Part (B): We show that a marginal increase in $\tilde{w}$ away from $w(\rho^*)$ increases price informativeness if and only if $var[\delta]>N(w(\rho^*),\rho^*)$.

First, fix any $\tilde{w}\neq w(\rho^*)$, and let $\tilde{\rho}$ denote equilibrium price informativeness when the government follows the rule $\tilde{w}$. Let F be as defined in (A-3) at the start of the appendix, which coincides with the RHS of (16). We know $\alpha\rho^*=F(w(\rho^*),\rho^*)$. Since $\upsilon_\varepsilon(\rho)$ is increasing in ρ, it follows that $F_\rho\leq 0$. The unique equilibrium $\tilde{\rho}$ when the government follows rule $\tilde{w}$ is given by the solution to $\alpha\tilde{\rho}=F(\tilde{w},\tilde{\rho})$. Hence (by Lemma A-1) if $F(\tilde{w},\rho^*)>F(w(\rho^*),\rho^*)$ then $\tilde{\rho}>\rho^*$, while if $F(\tilde{w},\rho^*)<F(w(\rho^*),\rho^*)$ then $\tilde{\rho}<\rho^*$.

Given this, to establish the result we sign the derivative $F_w(w,\rho)=\frac{\partial}{\partial w}\frac{w\tau_\varepsilon\upsilon_\varepsilon(\rho)^{-1}}{N(w,\rho)+var[\delta]}$, or equivalently, $N(w,\rho)+var[\delta]-w\frac{\partial N(w,\rho)}{\partial w}$. Using $\frac{\partial N(w,\rho)}{\partial w}=\frac{2N(w,\rho)}{w}$, this expression equals $var[\delta]-N(\tilde{w},\rho)$, completing the proof of Part (B). **QED**

Proof of Corollary 2: For both statements we show that endogenous risk $N(w(\rho^*),\rho^*)$ approaches 0 under the conditions stated, and then apply Part (B) of Proposition 2**.**

Part (I): We claim, and show below, that $\rho^{*2}\tau_Z\to\infty$ as either $\alpha\to 0$ or $\tau_Z\to\infty$. The result follows easily from the claim, since $\rho^{*2}\tau_Z \to \infty$ implies that both $\upsilon_\varepsilon(\rho^*) \to \infty$ and $\upsilon_G(\rho^*) \to \infty$, so that $w(\rho^*) \to 0$ and endogenous risk $N(w(\rho^*), \rho^*) \to 0$.

To prove the claim for $\alpha \to 0$, note first that $F(w(\rho), \rho) > 0$ for all ρ, and that $F(w(\rho), \rho)$ is independent of the risk-aversion parameter α. Hence as $\alpha \to 0$, the minimum solution to $\alpha\rho = F(w(\rho), \rho)$ grows unboundedly large.

For $\tau_Z \to \infty$, write $F(w(\cdot), \cdot; \tau_z)$ to emphasize the dependence on τ_Z, and observe that the equality $\alpha\rho = F(w(\rho), \rho; \tau_Z)$ is equivalent to the equality $\alpha\tau_Z^{-1/2}\left(\rho\tau_Z^{1/2}\right) = F\left(w\left(\rho\tau_Z^{1/2}\right), \rho\tau_Z^{1/2}; \tau_Z = 1\right)$. So by exactly the same argument as for $\alpha \to 0$, it follows that as $\tau_z \to \infty$, $\rho^2\tau_Z$ grows unboundedly large.

Part (II): Rewriting, endogenous risk $N(w(\rho^*), \rho^*)$ equals $w(\rho^*)(w(\rho^*)\, var[\theta|s_i, P] + w(\rho^*)\, var[\varepsilon_G])$. Observe that $w(\rho^*) \to 0$ as $var[\varepsilon_G] \to \infty$, regardless of how equilibrium informativeness ρ^* changes. Moreover, $w(\rho^*)\, var[\varepsilon_G] = 1/\upsilon_G(\rho^*)$, which is bounded above by $1/\tau_\theta$; likewise, $var[\theta|s_i, P] = 1/\upsilon_\varepsilon(\rho^*)$ is bounded above $1/\tau_\theta$. Hence $N(w(\rho^*), \rho^*) \to 0$ as $var[\varepsilon_G] \to \infty$.
QED

Proof of Corollary 3: Endogenous risk is bounded above by $var[\theta + \varepsilon_G]$. Consequently, for sufficiently large $var[\delta]$ exogenous risk exceeds endogenous risk, and the result follows from Part (B) of Proposition 2. **QED**

Proof of Proposition 3: Let F be defined as in (A-3) at the start of the appendix. A small exogenous change in τ_G affects $F(w(\rho^*), \rho^*)$ according to

$$\frac{\partial F(w(\rho^*), \rho^*)}{\partial \tau_G} + \frac{\partial w(\rho)}{\partial \tau_G} F_w(w(\rho^*), \rho^*).$$

By hypothesis, and from the proof of Part (B) of Proposition 2, we know $F_w(w(\rho^*), \rho^*) > 0$. Moreover, $\frac{\partial w(\rho)}{\partial \tau_G} > 0$ and $\frac{\partial F(w(\rho^*), \rho^*)}{\partial \tau_G} > 0$. Hence if τ_G is slightly increased, $F(w(\rho^*), \rho^*) > \rho^*$. By Lemma A-1, $\lim_{\rho\to\infty} F(w(\rho), \rho)$ is finite, so it follows that at the new τ_G, there exists an equilibrium strictly above ρ^*. **QED**

Proof of Proposition 5

Equilibrium existence

To establish existence, we show that there exist p_0, ρ, ξ, p_Z such that the price function $P = p_0 + \rho p_Z \theta + \xi \rho p_Z \sigma_G + p_Z Z$ is an equilibrium. Specifically, we show that there exist p_0, ρ, ξ, p_Z such that market clearing (3) holds.

First, when the price function is of this form, for the government observing the price is equivalent to observing $\tilde{P}_G \equiv \frac{1}{\rho p_Z}(P - p_0) - \xi \sigma_G = \theta + \rho^{-1} Z$, which is an unbiased estimate of θ, is normally distributed, and has precision $\rho^2 \tau_Z$. Hence by a standard application of Bayes' rule to normal distributions, the conditional distribution of θ given the information of the government is normal, with conditional variance $var\left[\theta | s_G, \tilde{P}_G\right] = \upsilon_G(\rho)^{-1}$ and conditional expectation

$$E\left[\theta | s_G, \tilde{P}_G\right] = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}_G + \tau_G s_G}{\upsilon_G(\rho)}.$$

Substituting $E\left[\theta | s_G, \tilde{P}_G\right]$ and $var\left[\theta | s_G, \tilde{P}_G\right]$ into (4), the government's intervention is

$$T\left(s_G, \sigma_G, \tilde{P}_G\right) = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}_G + \tau_G s_G}{\upsilon_G(\rho)} + E\left[\psi | \sigma_G\right] + g\left(\mu, \upsilon_G(\rho)^{-1} + var\left[\psi | \sigma_G\right]\right).$$

For speculator i, when σ_G remains hidden, observing the price P has the same information content as observing $\tilde{P}_H \equiv \frac{1}{p_Z}(P - p_0) = \rho\theta + \xi \rho \sigma_G + Z$. Observe that $\tilde{P}_G = \frac{1}{\rho}\tilde{P}_H - \xi \sigma_G$. So a speculator i's conditional expectation and conditional variance of T are

$$\begin{aligned}
E\left[T | s_i, \sigma_i, \tilde{P}_H\right] &= \frac{1}{\upsilon_G(\rho)}\left(\tau_\theta \bar{\theta} + \rho^2 \tau_Z E\left[\tilde{P}_G | s_i, \sigma_i, \tilde{P}_H\right] + \tau_G E\left[\theta | s_i, \sigma_i, \tilde{P}_H\right]\right) \\
&\quad + E\left[E\left[\psi | \sigma_G\right] | s_i, \sigma_i, \tilde{P}_H\right] + g\left(\mu, \upsilon_G(\rho)^{-1} + var\left[\psi | \sigma_G\right]\right) \\
var\left[T | s_i, \sigma_i, \tilde{P}_H\right] &= var\left[\frac{\rho^2 \tau_Z(-\xi \sigma_G) + \tau_G s_G}{\upsilon_G(\rho)} + E\left[\psi | \sigma_G\right] | s_i, \sigma_i, \tilde{P}_H\right].
\end{aligned}$$

Speculator i's demand is

$$x_i\left(s_i,\sigma_i,\tilde{P}_H\right)=\frac{1}{\alpha}\frac{E\left[T|s_i,\sigma_i,\tilde{P}_H\right]-\left(p_Z\tilde{P}_H+p_0\right)}{var\left[T|s_i,\sigma_i,\tilde{P}_H\right]+var\left[\delta\right]}.$$

Since all variables are normally distributed, $x_i\left(s_i,\sigma_i,\tilde{P}_H\right)$ is linear in $s_i,\sigma_i,\tilde{P}_H$. Hence $\int x_i\left(s_i,\sigma_i,\tilde{P}_H\right)di=x_i\left(s_i=\theta,\sigma_i=\sigma_G,\tilde{P}_H\right)$, and the market-clearing condition is simply $x_i\left(s_i=\theta,\sigma_i=\sigma_G,\tilde{P}_H\left(\theta,\sigma_G,Z\right)\right)+Z=0$. This is a linear function of θ, σ_G, Z. Hence we must show that there exist p_0, ρ, ξ, p_Z such the intercept and the coefficients on θ, σ_G, Z are all zero, i.e., show that the system of four equations in p_0, ρ, ξ, p_Z has a solution. Note that p_0 only enters the intercept term, so can be freely chosen to set the intercept to 0. The conditions that the coefficients on θ, σ_G, Z are zero can be written, respectively, as

$$\frac{\partial x_i}{\partial s_i}+\rho\frac{\partial x_i}{\partial\tilde{P}_H}=0 \tag{A-11}$$

$$\frac{\partial x_i}{\partial\sigma_i}+\xi\rho\frac{\partial x_i}{\partial\tilde{P}_H}=0 \tag{A-12}$$

$$1+\frac{\partial x_i}{\partial\tilde{P}_H}=0. \tag{A-13}$$

To show that this system of equations has a solution, we show that (A-12) together with

$$\xi=\frac{\partial x_i}{\partial\sigma_i}/\frac{\partial x_i}{\partial s_i} \tag{A-14}$$

$$\rho=\frac{\partial x_i}{\partial s_i} \tag{A-15}$$

has a solution. (Note that (A-14) and (A-12) imply (A-11), while (A-14), (A-12) and (A-15) together imply (A-13).)

Note that $\frac{\partial x_i}{\partial\sigma_i}$ and $\frac{\partial x_i}{\partial s_i}$ are independent of p_Z, while $\frac{\partial x_i}{\partial\tilde{P}_H}$ takes the form

$$\frac{\partial x_i}{\partial\tilde{P}_H}=\frac{\frac{\partial}{\partial\tilde{P}_H}E\left[T|s_i,\sigma_i,\tilde{P}_H\right]-p_Z}{\alpha var\left[T+\delta|s_i,\sigma_i,\tilde{P}_H\right]}.$$

Hence, given ξ and ρ satisfying (A-14) and (A-15), p_Z can be freely chosen to satisfy (A-12). Hence it remains to show that there exist ξ and ρ satisfying (A-14) and (A-15).

There exist functions $w(\rho)$ and $q(\xi,\rho)$ such that the derivatives $\frac{\partial x_i}{\partial s_i}$ and $\frac{\partial x_i}{\partial \sigma_i}$ have the form

$$\begin{aligned}
\frac{\partial x_i}{\partial s_i} &= \frac{\frac{\partial}{\partial s_i} E\left[w(\rho)\,\theta + q(\xi,\rho)\,\sigma_G | s_i, \sigma_i, \tilde{P}_H\right]}{\alpha var\left[T+\delta | s_i, \sigma_i, \tilde{P}_H\right]} \\
\frac{\partial x_i}{\partial \sigma_i} &= \frac{\frac{\partial}{\partial \sigma_i} E\left[w(\rho)\,\theta + q(\xi,\rho)\,\sigma_G | s_i, \sigma_i, \tilde{P}_H\right]}{\alpha var\left[T+\delta | s_i, \sigma_i, \tilde{P}_H\right]}.
\end{aligned}$$

In particular, $w(\rho) = \frac{\tau_G}{\upsilon_G(\rho)}$ as before, while there exist $\kappa \in [0,1]$ such that $q(\xi,\rho) = \kappa - \frac{\rho^2 \xi \tau_Z}{\upsilon_G(\rho)}$ A standard (though tedious) application of the properties of multivariate normal distributions yields the following result; the proof is at the end of the paper:

Lemma A-2

$$\begin{aligned}
E\left[w\theta + q\sigma_G | \tilde{P}_H, s_i, \sigma_i\right] &= w\left(\rho^2\xi^2 var\left[\zeta_i\right] var\left[\sigma_G\right] + var\left[Z\right] var\left[\sigma_i\right]\right) \frac{var\left[\theta\right]}{|\Sigma_{22}|} s_i \\
&\quad + q\left(-\rho\rho\xi var\left[\zeta_i\right] var\left[\sigma_G\right]\right) \frac{var\left[\theta\right]}{|\Sigma_{22}|} s_i \\
&\quad + w\left(-\rho\rho\xi var\left[\theta\right] var\left[\varepsilon_i\right]\right) \frac{var\left[\sigma_G\right]}{|\Sigma_{22}|} \sigma_i \\
&\quad + q\left(\rho^2 var\left[\theta\right] var\left[\varepsilon_i\right] + var\left[Z\right] var\left[s_i\right]\right) \frac{var\left[\sigma_G\right]}{|\Sigma_{22}|} \sigma_i \\
&\quad + terms\ independent\ of\ s_i, \sigma_i
\end{aligned}$$

where

$$|\Sigma_{22}| = var\left[Z\right] var\left[s_i\right] var\left[\sigma_i\right] + \rho^2 var\left[\theta\right] var\left[\varepsilon_i\right] var\left[\sigma_i\right] + \rho^2\xi^2 var\left[\zeta_i\right] var\left[\sigma_G\right] var\left[s_i\right].$$

Applying Lemma A-2 yields

$$\frac{\frac{\partial x_i}{\partial \sigma_i}}{\frac{\partial x_i}{\partial s_i}} = \frac{w(\rho)\left(-\rho^2 \xi var[\theta] var[\varepsilon_i]\right) + q(\xi,\rho)\left(\rho^2 var[\theta] var[\varepsilon_i] + var[Z] var[s_i]\right)}{w(\rho)\left(\rho^2 \xi^2 var[\zeta_i] var[\sigma_G] + var[Z] var[\sigma_i]\right) + q(\xi,\rho)\left(-\rho^2 \xi var[\zeta_i] var[\sigma_G]\right)} \frac{var[\sigma_G]}{var[\theta]}.$$

We first show that, for any given ρ, there exists ξ satisfying

$$\xi = \frac{w(\rho)\left(-\rho^2 \xi var[\theta] var[\varepsilon_i]\right) + q(\xi,\rho)\left(\rho^2 var[\theta] var[\varepsilon_i] + var[Z] var[s_i]\right)}{w(\rho)\left(\rho^2 \xi^2 var[\zeta_i] var[\sigma_G] + var[Z] var[\sigma_i]\right) + q(\xi,\rho)\left(-\rho^2 \xi var[\zeta_i] var[\sigma_G]\right)} \frac{var[\sigma_G]}{var[\theta]}.$$

To see this, note that rearrangement of this equality yields

$$\begin{aligned} & w(\rho)\,\xi var[\theta]\left(\rho^2 \xi^2 var[\zeta_i] var[\sigma_G] + var[Z] var[\sigma_i]\right) + w(\rho) var[\sigma_G] \rho^2 \xi var[\theta] var[\varepsilon_i] \\ = \;& q(\xi,\rho) var[\sigma_G]\left(\rho^2 var[\theta] var[\varepsilon_i] + var[Z] var[s_i]\right) + q(\xi,\rho) var[\theta] \rho^2 \xi^2 var[\zeta_i] var[\sigma_G], \end{aligned}$$

so that

$$\frac{q(\xi,\rho)}{\xi w(\rho)} = \frac{\rho^2 \xi^2 var[\theta] var[\zeta_i] var[\sigma_G] + var[\theta] var[Z] var[\sigma_i] + \rho^2 var[\sigma_G] var[\theta] var[\varepsilon_i]}{\rho^2 var[\sigma_G] var[\theta] var[\varepsilon_i] + var[\sigma_G] var[Z] var[s_i] + \rho^2 \xi^2 var[\theta] var[\zeta_i] var[\sigma_G]}. \tag{A-16}$$

The RHS is continuous as a function of ξ, is finite at $\xi = 0$, and converges to 1 as $\xi \to \infty$. Since $q(\xi,\rho)/\xi = \kappa/\xi - \frac{\rho^2 \tau_Z}{\upsilon_G(\rho)}$, continuity implies that for any ρ there exists $\xi > 0$ such that this equality is satisfied. Denote the smallest such solution by $\xi(\rho)$. Observe that $\xi(\rho)$ is continuous in ρ, and remains bounded away from both 0 and ∞ both as $\rho \to 0$ and as $\rho \to \infty$.

To complete the proof of existence, we must show there exists ρ satisfying $\rho = \frac{\partial x_i}{\partial s_i}$, where $\frac{\partial x_i}{\partial s_i}$ is evaluated using ρ and $\xi(\rho)$. Again, this follows from continuity, as follows. As $\rho \to 0$, the government ignores the price, and $\frac{\partial x_i}{\partial s_i}$ remains bounded away from 0 since the signal s_i gives speculator i information about the government's signal s_G. As $\rho \to \infty$, the government ignores its own signal s_G, and the price conveys the same information to speculator i as does $\theta + \xi \sigma_G$; consequently, a change in s_i changes the speculator's expectation about σ_G, but the effect is finite (and moreover, $var\left[T|s_i,\sigma_i,\tilde{P}_H\right]$ remains bounded away from 0).

Increased informativeness under transparency

Writing $\rho=\frac{\partial x_i}{\partial s_i}$ explicitly, price informativeness when the government does not disclose σ_G is given by the solution to

$$\alpha\rho=\frac{\left(w\left(\rho\right)\left(\rho^2\xi\left(\rho\right)^2 var\left[\zeta_i\right]var\left[\sigma_G\right]+var\left[Z\right]var\left[\sigma_i\right]\right)+q\left(\xi\left(\rho\right),\rho\right)\left(-\rho^2\xi\left(\rho\right)var\left[\zeta_i\right]var\left[\sigma_G\right]\right)\right)\frac{var[\theta]}{|\Sigma_{22}|}}{var\left[T|s_i,\sigma_i,\tilde{P}_H\right]+var\left[\delta\right]}. \tag{A-17}$$

If instead the government makes its signal σ_G public, i.e., transparency, price informativeness is determined by the solution to

$$\alpha\rho=\frac{w\left(\rho\right)\frac{\partial}{\partial s_i}E\left[\theta|s_i,P\right]}{var\left[T|s_i,P\right]+var\left[\delta\right]}. \tag{A-18}$$

To establish the result, we show that, for any ρ, the RHS of (A-17) is strictly less than the RHS of (A-18). It is immediate that, for any ρ, the denominator in (A-18) is less than the denominator in (A-17). The main step in the proof is to show that the the numerator in (A-17) is less than the numerator in (A-18).

Expanding, the numerator in (A-18) is

$$\frac{w\left(\rho\right)\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\rho^2\tau_Z}=\frac{w\left(\rho\right)var\left[\theta\right]var\left[Z\right]}{var\left[\varepsilon_i\right]var\left[Z\right]+var\left[\theta\right]var\left[Z\right]+\rho^2 var\left[\theta\right]var\left[\varepsilon_i\right]}.$$

Define

$$\begin{aligned}D\left(\rho\right) &= var\left[\varepsilon_i\right]var\left[Z\right]+var\left[\theta\right]var\left[Z\right]+\rho^2 var\left[\theta\right]var\left[\varepsilon_i\right]\\ &= var\left[s_i\right]var\left[Z\right]+\rho^2 var\left[\theta\right]var\left[\varepsilon_i\right].\end{aligned}$$

Using this definition,

$$|\Sigma_{22}| = D(\rho)\, var[\sigma_i] + \rho^2 \xi^2 var[\zeta_i]\, var[\sigma_G]\, var[s_i].$$

Hence we must show

$$\begin{aligned} & \frac{w(\rho)\, var[\theta]\, var[Z]}{D(\rho)} \\ > \quad & \frac{w(\rho)\left(\rho^2 \xi(\rho)^2 var[\zeta_i]\, var[\sigma_G] + var[Z]\, var[\sigma_i]\right) + q(\xi(\rho), \rho)\left(-\rho^2 \xi(\rho)\, var[\zeta_i]\, var[\sigma_G]\right)}{D(\rho)\, var[\sigma_i] + \rho^2 \xi(\rho)^2 var[\zeta_i]\, var[\sigma_G]\, var[s_i]} var[\theta], \end{aligned}$$

or equivalently,

$$\begin{aligned} & w(\rho)\, var[Z]\left(D(\rho)\, var[\sigma_i] + \rho^2 \xi(\rho)^2 var[\zeta_i]\, var[\sigma_G]\, var[s_i]\right) \\ > \quad & D(\rho)\left(w(\rho)\left(\rho^2 \xi(\rho)^2 var[\zeta_i]\, var[\sigma_G] + var[Z]\, var[\sigma_i]\right) - q(\xi, \rho)\, \rho^2 \xi(\rho)\, var[\zeta_i]\, var[\sigma_G]\right) \end{aligned}$$

or equivalently,

$$\begin{aligned} & w(\rho)\, var[Z]\, \rho^2 \xi(\rho)^2 var[\zeta_i]\, var[\sigma_G]\, var[s_i] \\ > \quad & D(\rho)\left(w(\rho)\, \rho^2 \xi(\rho)^2 var[\zeta_i]\, var[\sigma_G] - q(\xi(\rho), \rho)\, \rho^2 \xi(\rho)\, var[\zeta_i]\, var[\sigma_G]\right), \end{aligned}$$

or equivalently,

$$w(\rho)\, \rho^2 \xi(\rho)^2 var[Z]\, var[s_i] > D(\rho)\left(w(\rho)\, \rho^2 \xi(\rho)^2 - q(\xi(\rho), \rho)\, \rho^2 \xi(\rho)\right),$$

or equivalently,

$$w(\rho)\, \rho^2 \xi(\rho)^2 \left(var[Z]\, var[s_i] - \left(1 - \frac{q(\xi(\rho), \rho)}{w(\rho)\, \xi(\rho)}\right) D(\rho)\right) > 0,$$

or equivalently (using $w(\rho) > 0$),

$$\frac{q(\xi(\rho), \rho)}{w(\rho)\xi(\rho)} > 1 - \frac{var[Z]\,var[s_i]}{D(\rho)} = \frac{\rho^2 var[\theta]\,var[\varepsilon_i]}{D(\rho)}.$$

Substituting in (A-16), along with the definition of $D(\rho)$, this inequality is in turn equivalent to

$$\begin{aligned} &\frac{\rho^2\xi(\rho)^2 var[\theta]\,var[\zeta_i]\,var[\sigma_G] + var[\theta]\,var[Z]\,var[\sigma_i] + \rho^2 var[\sigma_G]\,var[\theta]\,var[\varepsilon_i]}{var[\sigma_G]\,D(\rho) + \rho^2\xi(\rho)^2 var[\theta]\,var[\zeta_i]\,var[\sigma_G]} \\ > \quad & \frac{\rho^2 var[\theta]\,var[\varepsilon_i]}{D(\rho)}, \end{aligned}$$

or equivalently,

$$\frac{\rho^2\xi(\rho)^2 var[\theta]\,var[\zeta_i]\,var[\sigma_G] + var[\theta]\,var[Z]\,var[\sigma_i]}{\rho^2\xi(\rho)^2 var[\theta]\,var[\zeta_i]\,var[\sigma_G]} > \frac{\rho^2 var[\theta]\,var[\varepsilon_i]}{D(\rho)}.$$

This is indeed the case since the LHS is above 1 and the RHS is below 1, completing the proof.

Standard analysis of multivariate normal

Proof of Lemma A-2: Define

$$\Sigma_{22} = \begin{pmatrix} var\left[\tilde{P}_H\right] & cov\left[\tilde{P}_H, s_i\right] & cov\left[\tilde{P}_H, \sigma_i\right] \\ cov\left[s_i, \tilde{P}_H\right] & var[s_i] & cov[s_i, \sigma_i] \\ cov\left[\sigma_i, \tilde{P}_H\right] & cov[\sigma_i, s_i] & var[\sigma_i] \end{pmatrix} = \begin{pmatrix} var\left[\tilde{P}_H\right] & \rho var[\theta] & \rho\xi var[\sigma_G] \\ \rho var[\theta] & var[s_i] & 0 \\ \rho\xi var[\sigma_G] & 0 & var[\sigma_i] \end{pmatrix}.$$

and

$$\Sigma_{12} = \begin{pmatrix} cov\left[\theta, \tilde{P}_H\right] & cov[\theta, s_i] & cov[\theta, \sigma_i] \\ cov\left[\sigma_G, \tilde{P}_H\right] & cov[\sigma_G, s_i] & var[\sigma_G] \end{pmatrix} = \begin{pmatrix} \rho var[\theta] & var[\theta] & 0 \\ \rho\xi var[\sigma_G] & 0 & var[\sigma_G] \end{pmatrix}.$$

Then

$$E\left[w\theta + q\sigma_G | \tilde{P}_H, s_i, \sigma_i\right] = \begin{pmatrix} w & q \end{pmatrix} \Sigma_{12}\Sigma_{22}^{-1} \begin{pmatrix} \tilde{P}_H \\ s_i \\ \sigma_i \end{pmatrix}.$$

Evaluating

$$\Sigma_{22}^{-1} = \frac{1}{|\Sigma_{22}|} \begin{pmatrix} var[s_i]\,var[\sigma_i] & -\rho var[\theta]\,var[\sigma_i] & -\rho\xi var[\sigma_G]\,var[s_i] \\ -\rho var[\theta]\,var[\sigma_i] & var\left[\tilde{P}_H\right] var[\sigma_i] - (\rho\xi var[\sigma_G])^2 & \rho var[\theta]\,\rho\xi var[\sigma_G] \\ -\rho\xi var[\sigma_G]\,var[s_i] & \rho var[\theta]\,\rho\xi var[\sigma_G] & var\left[\tilde{P}_H\right] var[s_i] - (\rho var[\theta])^2 \end{pmatrix},$$

where

$$\begin{aligned} |\Sigma_{22}| &= var\left[\tilde{P}_H\right] var[s_i]\,var[\sigma_i] - (\rho var[\theta])^2\,var[\sigma_i] - (\rho\xi var[\sigma_G])^2\,var[s_i] \\ &= var[Z]\,var[s_i]\,var[\sigma_i] + \rho^2 var[\theta]\,var[\varepsilon_i]\,var[\sigma_i] + \rho^2\xi^2 var[\zeta_i]\,var[\sigma_G]\,var[s_i]. \end{aligned}$$

So

$$\begin{aligned} \Sigma_{12}\Sigma_{22}^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} &= \frac{1}{|\Sigma_{22}|}\Sigma_{12} \begin{pmatrix} -\rho var[\theta]\,var[\sigma_i] \\ var\left[\tilde{P}_H\right] var[\sigma_i] - (\rho\xi var[\sigma_G])^2 \\ \rho var[\theta]\,\rho\xi var[\sigma_G] \end{pmatrix} \\ &= \frac{var[\theta]}{|\Sigma_{22}|} \begin{pmatrix} -\rho^2 var[\theta]\,var[\sigma_i] + var\left[\tilde{P}_H\right] var[\sigma_i] - (\rho\xi var[\sigma_G])^2 \\ -\rho\rho\xi var[\sigma_i]\,var[\sigma_G] + \rho\rho\xi var[\sigma_G]^2 \end{pmatrix} \\ &= \frac{var[\theta]}{|\Sigma_{22}|} \begin{pmatrix} \rho^2\xi^2 var[\zeta_i]\,var[\sigma_G] + var[Z]\,var[\sigma_i] \\ -\rho\rho\xi var[\zeta_i]\,var[\sigma_G] \end{pmatrix} \end{aligned}$$

where the final inequality follows from

$$\begin{aligned}
& -\rho^2 var\left[\theta\right] var\left[\sigma_i\right] + var\left[\tilde{P}_H\right] var\left[\sigma_i\right] - \left(\rho\xi var\left[\sigma_G\right]\right)^2 \\
= & -\rho^2 var\left[\theta\right] var\left[\sigma_i\right] + \left(\rho^2 var\left[\theta\right] + \rho^2\xi^2 var\left[\sigma_G\right] + var\left[Z\right]\right) var\left[\sigma_i\right] - \left(\rho\xi var\left[\sigma_G\right]\right)^2 \\
= & \left(\rho^2\xi^2 var\left[\sigma_G\right] + var\left[Z\right]\right) var\left[\sigma_i\right] - \left(\rho\xi var\left[\sigma_G\right]\right)^2 \\
= & \rho^2\xi^2 var\left[\zeta_i\right] var\left[\sigma_G\right] + var\left[Z\right] var\left[\sigma_i\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Sigma_{12}\Sigma_{22}^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} &= \frac{1}{|\Sigma_{22}|}\Sigma_{12}\begin{pmatrix} -\rho\xi var\left[\sigma_G\right] var\left[s_i\right] \\ \rho var\left[\theta\right]\rho\xi var\left[\sigma_G\right] \\ var\left[\tilde{P}_H\right] var\left[s_i\right] - \left(\rho var\left[\theta\right]\right)^2 \end{pmatrix} \\
&= \frac{var\left[\sigma_G\right]}{|\Sigma_{22}|}\begin{pmatrix} -\rho var\left[\theta\right]\rho\xi var\left[s_i\right] + var\left[\theta\right]\rho var\left[\theta\right]\rho\xi \\ -\rho\xi var\left[\sigma_G\right]\rho\xi var\left[s_i\right] + var\left[\tilde{P}_H\right] var\left[s_i\right] - \left(\rho var\left[\theta\right]\right)^2 \end{pmatrix} \\
&= \frac{var\left[\sigma_G\right]}{|\Sigma_{22}|}\begin{pmatrix} -\rho\rho\xi var\left[\theta\right] var\left[\varepsilon_i\right] \\ \rho^2 var\left[\theta\right] var\left[\varepsilon_i\right] + var\left[Z\right] var\left[s_i\right] \end{pmatrix}
\end{aligned}$$

where the final inequality follows from

$$\begin{aligned}
& -\rho\xi var\left[\sigma_G\right]\xi\rho var\left[s_i\right] + var\left[\tilde{P}_H\right] var\left[s_i\right] - \left(\rho var\left[\theta\right]\right)^2 \\
= & -\rho^2\xi^2 var\left[\sigma_G\right] var\left[s_i\right] + \left(\rho^2\xi^2 var\left[\sigma_G\right] + \rho^2 var\left[\theta\right] + var\left[Z\right]\right) var\left[s_i\right] - \rho^2 var\left[\theta\right]^2 \\
= & \left(\rho^2 var\left[\theta\right] + var\left[Z\right]\right) var\left[s_i\right] - \rho^2 var\left[\theta\right]^2 \\
= & \rho^2 var\left[\theta\right] var\left[\varepsilon_i\right] + var\left[Z\right] var\left[s_i\right].
\end{aligned}$$

The result follows. **QED**