What Makes Residential Different from Non-Residential REITs? Evidence from Multi-Factor Asset Pricing Models*

Daniele Bianchi[†], Massimo Guidolin[‡], and Francesco Ravazzolo[§]

This version: December 2012

Abstract

We use Bayesian methods to estimate a multi-factor linear asset pricing model characterized by structural instability in factor loadings, idiosyncratic variances, and factor risk premia. We use such a framework to investigate the key differences in the pricing mechanism that applies to residential vs. non-residential (such as office space, industrial buildings, retail property) REITs. Under the assumption that the subprime crisis has had its epicentre in the housing/residential sector, we interpret any differential dynamics as indicative of the propagation mechanism of the crisis towards business-oriented segments of the US real estate market. We find important differences in the structure as well as the dynamic evolution of risk factor exposures across residential vs. non-residential REITs. An analysis of cross-sectional mispricings reveals that only retail, residential, and mortgage-specialized REITs have been over-priced over the initial part of our sample, i.e., 1999-2006. However, the strongest mispricings occurred and may be still persisting in the office and regional mall-specialized REIT subsectors.

Key words: Bayesian estimation, Latent jumps, Linear factor models, Residential REITs. JEL codes: G11, C53.

1. Introduction

What are they key differences between residential and business-related—such as industrial buildings, offices, shops and malls—(publicly traded) real estate? Do such differences matter to investors and policy-makers, and how? Our paper takes a distinctive asset pricing perspective and exploits the massive movements in valuations and risk exposures that allegedly took place before and during the recent, Great Financial Crisis of 2007-2009 to find answers to such key questions. Because our asset pricing methods and tests need to rely on liquid and actively traded assets, we resort to total return index data concerning REITs and leverage on the ability to form homogeneous portfolios that oppose residential-specialized to non-residential REITs. Although some issues concerning the link between REITs and (subsequent) real estate valuations remain under scrutiny, an excellent case can be made that REITs are among the best available short-term proxies for real estate prices, even residential real estate, see e.g., Cotter and Roll (2011).

^{*}The views expressed in this paper are our own and do not necessarily reflect those of Norges Bank.

[†]Bocconi University, Milan. E-mail: andrea.tortora@phd.unibocconi.it.

[‡]Bocconi University, IGIER, and CAIR, Manchester Business School. E-mail: massimo.guidolin@unibocconi.it.

[§]Norges Bank and BI Norwegian Business School. E-mail: francesco.ravazzolo@Norges-Bank.no.

During 2009 and 2010, a number of commentators have stressed that while a mounting wave of delinquencies and foreclosures have since 2007 led to a massive devaluation and illiquidity in large segments (subprime and Alt-A sectors, especially for loans securitized during 2006 and 2007 by financial institutions and hence not enjoying of quasi-federal guarantees) of the mortgage markets (see Hendershott, Hendershott, and Shilling, 2010; Simon and Ng, 2009) and caused problems for both home owners and institutional lenders, the non-residential real estate market has suffered from a fall-out from the subprime crisis, but it has not bled as deeply as the residential/housing segment. One of our goals is to investigate whether this differential dynamics derives from a heterogeneous evolution of risk exposures of business-related vs. residential REITs, or whether this corresponds to a correction of pervasive mispricings previously existing in the residential sector that had not occurred in the non-residential sector. Our key assumption is that—assuming the literature has correctly identified the subprime housing sector as the origin of the recent real estate bust (see e.g., Cecchetti, 2009; Gorton, 2010; Mian and Sufi, 2009)—residential REITs were affected by the subprime crisis earlier and more strongly than other categories were.² The second panel of Figure 1 supports this reasonable assumption: the valuations of residential and mortgage REITs led other REIT sectors between early 2007 and the Summer of 2008; yet, they also recovered before and faster than most other subsectors after the Spring of 2009. Obviously, a related and even more interesting question is whether standard asset pricing models (see e.g., Plazzi, Torous and Valkanov, 2010) may be used to forecast whether commercial REITs are eventually bound to record similar and massive losses such as those recorded between 2007 and 2009 by house prices or mortgage-backed securities.

In our paper, we extend the methodologies and results in the literature on multi-factor, ICAPM-style models (see e.g., Ling and Naranjo, 1997; Karolyi and Sanders, 1998) to investigate whether there is any evidence of a systematic mispricing of various categories of REITs—with particular emphasis on the heterogeneous features of residential vs. non-residential REITs—in an asset pricing framework that is simultaneously estimated to price a wide range of equity and bond portfolios. Our extension is based on a Bayesian Monte Carlo approach that allows us to jointly estimate (the posterior distribution of) time-varying risk exposures and of risk premia in a single-step. This method preserves consistency and avoids the well-known statistical limitations of the standard, two-step Fama-MacBeth approach, as in Guidolin, Ravazzolo and Tortora (2012, henceforth GRT). When the multi-factor framework is extended to include a range of standard macroeconomic factors (the excess return on the market portfolio; the credit risk premium; the riskless term premium; the unexpected inflation rate; the rate of growth of industrial production, IP; the rate of growth of real personal consumption; the 1-month

 $^{^{1}}$ The commentators that have stressed that commercial and business property values usually rise more slowly and fall more slowly than residential home values do (see e.g., P. Grant, "Malls, Offices May Slump Less Steeply Than Homes," The Wall Street Journal, 3/10/2008) may imply that the exposures to pro-cyclical risk factors are different across residential vs. non-residential real estate.

²This means that while residential (in particular, apartment-investing) REITs are often classified themselves as commercial property, the key distinction in this paper is between real estate assets that are directly related to (supporting) business activities, such as industrial buildings, offices, regional and shopping malls, and free-standing shops, residential equity REITs that invest in and manage manufactured homes and apartments, and mortgage REITs that are mostly involved with purchasing housing-related loans and residential mortgage-backed securities.

real T-bill rate; surprises to the CPI inflation rate) that are assumed to drive the stochastic discount factor in a linear fashion, we find evidence that the model is not misspecified, in the sense that for most portfolios of equities and bonds there is no evidence of structural and persistent mispricing. In fact, a number of the assumed macroeconomic factors appear to be precisely priced in the cross-section of excess returns, with sensibly sized premia.

As far as our key research questions are concerned, we report two results that to the best of our knowledge have not been discussed before. First, we find differences in the structure as well the dynamic evolution of risk factor exposures across residential vs. industrial, office, and retail REITs. Residential REITs are characterized by a negative but rapidly increasing exposure to general stock market risk, by positive but also quickly retreating exposures to term premium and real interest rate risks, and by massive but falling beta towards inflation risk. Industrial, office, and retail REITs carry instead positive, equity-like, and increasing equity beta and positive, significant, and large exposures to unexpected inflation that have slowly increased over time. This comparison among residential and non-residential REITs sheds light on one potential cause of their differential behavior in the aftermath of the 2007-2009 financial crisis: by the end of our sample, the residential sector no longer carries any exposure to general market dynamics and its upward swing during 2010 and 2011 is mostly explained by increasing risks of unexpected inflation and real rate variations. Interestingly, mortgage REITs is the only securitized real estate sector that features non-zero and significant betas to typical macroeconomic factors, such as IP and real consumption growth; similarly to equity REITs, also mortgage REITs are exposed to inflation risk.

Second, an analysis of cross-sectional mispricings reveals that all the Jensen's alpha implied by REITs are positive and large, although they persistently decline between 2006 and 2008. However, when the uncertainty on the estimated alphas is taken into account, we find that only retail, residential, and mortgage-specialized REITs have been over-priced over the initial part of our sample, i.e., 1999-2006.³ Moreover, the claim that the great real estate bubble would have been a debt/mortgage-fueled one (see Hendershott, Hendershott, and Shilling, 2010) is consistent with our evidence that between 1999 and 2006 mortgage REITs were characterized by the largest, positive median alphas. Yet, there is no sign of a larger, residential real estate bubble because the alphas of manufactured homes-investing REITs turn out to be negative and declining throughout our sample; the mispricing of apartment-investing REITs did turn positive and large (reaching 1 percent per month) between 2004 and 2006, but such alphas were quickly corrected down after 2007.⁴ The more acute real estate over-pricing occurred instead—and in the perspective of our model this is potentially still under way—in the business-related sector: in particular, the estimated posterior median alphas of office and regional mall-specialized REITs remain obstinately at levels in the order of 2 percent per month over our entire sample, including the 2009-2011 alleged post-crisis period.

³Real estate turns out to be quite special among all asset classes as only 10- and 5-year Treasuries have also been significantly mis- (under-) priced; although exceptions may occasionally be found (e.g., health, energy, and technology stocks), none of the equity portfolios appears to have been persistently mispriced.

⁴These results imply the absence of any systematic overpricing of the REIT asset class as a whole over the 2003-2006 period. As shown in GRT (2012), the overpricing of REITs has been stronger and more persistent in the 1990s than over recent years.

Although there is now an ever expanding literature on rational asset pricing models for (commercial) real estate and REITs in particular, our contribution appears to be closely related to three earlier papers. Wheaton (1999) is a seminal paper on the heterogeneous cyclical properties—which may also be interpreted as different types and intensities of exposure to business cycle factors—of different types of real estate assets. To answer this question, he builds a (rather specific, tightly parameterized) microeconomic stock-flow model in which expectations, development lags, the degree of durability, and market elasticities all contribute to explain differential dynamics in alternative real estate sectors. He shows that fully rational structural models may be capable of producing price oscillations if they incorporate some institutional features that characterize real estate markets, such as long-term leases and the use of credit to finance development. Some subsectors (e.g., apartments and industrial space) are then structurally expected to produce deeper cycles and more variability in prices than other subsectors are (e.g., office and retail properties). In our paper, we refrain from building a microeconomic model of price determination and we resort instead to standard empirical asset pricing methods that estimate the time-varying risk exposures of different real estate subsectors to common, aggregate shocks. Payne and Waters (2007) have applied univariate time series methods to examine the existence of periodically (rational, i.e., consistent with the efficient market hypothesis) collapsing bubbles in the equity REIT market. They perform their analysis using similar NAREIT subsector decompositions as we use in our paper and find mixed results, in the sense that bubbles may be detected only in rather peculiar subsectors. In spite of these mixed findings, their paper draws the attention on the fact that different categories of REITs would be subject to heterogeneous pricing dynamics. However, there are also clear differences with respect to our paper: we use a multivariate, multi-factor model for excess returns instead of testing for the comovement properties of prices and dividends; our focus is not specifically devoted to the existence of speculative bubbles in the housing market (see e.g., Bjorklund and Soderberg, 1999) even though our linear multifactor asset pricing model does not rule out rational bubbles in which investors are compensated for the risk of sudden price surges. Finally, GRT (2012) employ econometric tools and specifications, and pursue goals that are similar to our paper, but their focus is generically on the cross-sectional differences between REITs vs. stocks and bonds as asset classes, and more specifically on whether or not real estate would have been prone to bubble conditions—rational or not—during the period preceding the great bust and ensuing financial crisis of 2007-2009. Obviously, our paper differs from theirs because we ask a different research question, as we aim at isolating any differences across different types of REIT portfolios.

The paper is structured as follows. Section 2 describes our methodology, with emphasis on the multi-factor model and its econometric implementation. Section 3 presents our data and summary statistics. Section 4 presents empirical findings concerning our Bayesian estimates of time-varying factor exposures as well as the unit risk premia estimates these imply. Section 5 represents the heart of the paper and contains our findings for heterogeneous mispricings across different segments of the REIT universe, with special emphasis on the dichotomy residential vs. business REIT subsectors. Section 6 performs a range of robustness checks. Section 7 concludes and emphasizes a few policy implications of our findings.

2. Research Design and Methodology

2.1. The Asset Pricing Framework

Our research design is based on an extension of the multi-factor models first introduced by Ferson and Harvey (1991) in the asset pricing literature and applied to a variety of empirical research questions (see e.g., Karolyi and Sanders, 1998 in the case of securitized real estate). A multi-factor asset pricing model (MFAPM) posits a linear relationship between asset returns and a set of macroeconomic factors that are assumed to capture business cycle effects on beliefs and/or preferences, as summarized by a pricing kernel factor with time-varying properties. These macroeconomic factors are typically identified with the market portfolio (i.e., aggregate wealth) returns, the credit quality spread on corporate bond yields, the term spread in the riskless (Treasury) yield curve, and changes in the rate of growth of industrial production (see Chen, Roll and Ross, 1986, or Liu and Mei, 1992). If we define the (shocks to) macroeconomic risk factors as $F_{j,t}$ (j = 1, ..., K) and $r_{i,t}$ to be the excess return on asset or portfolio i = 1, ..., N, then a typical MFAPM can be written as

$$r_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \epsilon_{i,t},$$
 (1)

where it is customary to assume $E[\epsilon_{i,t}] = E[F_{j,t}] = E[\epsilon_{i,t}F_{j,t}] = 0$ for all i = 1, ..., N and j = 1, ..., K. The $r_{i,t}$ are returns in excess of the risk-free rate proxied by the 1-month T-bill. The advantage of MFAPMs such as (1) consists of the fact that a number of systematic factors well below the number of test assets, $K \ll N$, may prove useful to capture relatively large portions of the variability in asset returns. Importantly, even though the notation $\beta_{ij,t}$ implies that the factor loadings are allowed to be time-varying, such patterns of time variation are in general left unspecified.

One problem with (1) is the difficulty with interpreting $\beta_{i0,t}$ (often called "Jensen's alphas") when some (or all) the risk factors are not traded portfolios. Although some analyses that use (1) to either understand realized excess returns or to decompose them may still be implemented, unless all the factors are themselves tradable portfolios, it is impossible to interpret any non-zero $\beta_{i0,t}$ as an abnormal return on asset i "left on the table" after all risks $(f_{j,t}, j = 1, ..., K)$ and risk exposures $(\beta_{ij,t}, j = 1, ..., K)$ have been taken into account. If some of the factors are not replicated by traded portfolios, there may be an important difference between the theoretical alpha that the model uncovers, and the actual alpha that an investor may achieve by trading assets on the basis of the MFAPM.

To eliminate such a possibility, we follow the literature (see e.g., Ferson and Korajczyk, 1995) and proceed as follows. When an economic risk factor is measured in the form of an excess return, such as the U.S. market portfolio, real T-bill rates, term structure spreads, and default spread variables, we use the excess return directly as a mimicking portfolio; Shanken (1992) has argued that such an approach delivers the most efficient estimate of the risk premiums. When a factor is not an excess return, such as industrial production growth, unexpected inflation, and real consumption growth, we construct mimicking portfolios by estimating time-series regressions of individual portfolio returns on M economic variables and lagged instruments (see Section 3 for additional details). Using the residuals of such regressions to form an estimate of the $N \times N$ (conditional) idiosyncratic covariance matrix,

 V_t , we then form in each month of our sample the factor-mimicking portfolios for each of the $K' \leq K$ factors for which these are needed by finding a vector of weights $\mathbf{w}_{j,t}$ (j = 1, ..., K') that solves

$$\min_{\mathbf{w}_{j,t}} \mathbf{w}_{j,t}' \mathbf{V}_t \mathbf{w}_{j,t} \quad \text{s.t. (i) } \mathbf{w}_{j,t}' \mathbf{B}_{[j],t} = \mathbf{0}; \text{ (ii) } \mathbf{w}_{j,t}' \mathbf{1}_N = 1,$$

where $\mathbf{B}_{[j],t}$ is the $N \times (M-1)$ matrix that excludes the jth row from the $N \times M$ matrix of slope coefficient estimates \mathbf{B}_t obtained by regressing returns data on the N portfolios on the M factors and instruments. The jth mimicking portfolio is formed from the individual stocks, using the portfolio weight $\mathbf{w}_{j,t}$.

In the conditional version of Merton's (1973) intertemporal CAPM (ICAPM), the expected excess return (risk premium) on asset i over the interval [t-1,t] may then be related to its "betas", i.e., factor loadings measuring the exposure of asset i to each of the systematic risk factors and the associated unit risk premia (i.e., average compensations for unit risk exposure)

$$E[r_{i,t}|\mathbf{Z}_{t-1}] = \lambda_0(\mathbf{Z}_{t-1}) + \sum_{i=1}^{K} \beta_{ij,t-1} \lambda_j(\mathbf{Z}_{t-1}),$$
(2)

where both the betas and the risk premia are conditional on the information publicly available at time t, here summarized by the $M \times 1$ vector of "instruments" \mathbf{Z}_t .

2.2. A Bayesian Mixture Estimation Approach

Stochastic, time-varying betas have been recently found to be crucial ingredients of conditional asset pricing, in the sense that there is a growing evidence that careful modelling of the dynamics in factor exposures may provide a decisive contribution to solve the typical anomalies associated with unconditional implementations of multifactor models. For instance, Jostova and Philipov (2005) find that in a Fama and MacBeth's style exercise (see Section 6.2), the CAPM is rejected when using rolling OLS beta estimates while the opposite verdict emerges when they allow for stochastic variation (in the form of a simple AR(1) process) in the conditional CAPM betas. In practice, we specify the relationship between excess returns and factors and the time-varying dynamics in factor loadings and idiosyncratic volatility in the following state-space (Bayesian time-varying stochastic volatility-with breaks, BTVSVB) form

$$r_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \sigma_{it} \epsilon_{i,t}$$

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{1ij,t} \eta_{ij,t} \qquad j = 0, ..., K,$$

$$\ln(\sigma_{i,t}^{2}) = \ln(\sigma_{i,t-1}^{2}) + \kappa_{2i,t} v_{i,t} \qquad i = 1, ..., N,$$
(3)

where $\epsilon_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{N,t})' \sim N(0, diag\{\sigma_{1,t}^2, \sigma_{2,t}^2, ..., \sigma_{N,t}^2\}), \boldsymbol{\eta}_{i,t} \equiv (\eta_{i0,t}, \eta_{i1,t}, ..., \eta_{iK,t}, \upsilon_{i,t})' \sim N(0, \mathbf{Q}_i)$ with \mathbf{Q}_i a diagonal matrix characterized by the parameters $q_{i0}^2, q_{i1}^2, ..., q_{iK}^2, q_{iv}^2$. Stochastic variations

⁵The conditional beta of the jth mimicking portfolio on the jth economic factor may change as \mathbf{B}_t and \mathbf{V}_t change over time. However, such mimicking portfolios are adjusted to have constant factor betas by combining them with T-bills so that the combined portfolio has a beta equal to the time-series average of the betas that are produced by the constrained optimizations. While in frequentist applications, this procedure is relatively straightforward and applied to simple expanding windows of data to have maximum power, we shall provide additional details on our Bayesian implementation in Section 2.3.

(breaks) in the level of both the beta coefficients and of the idiosyncratic variance σ_{it}^2 are introduced and modelled through a mixture innovation approach as in Ravazzolo, Paap, van Dijk and Franses (2007) and Giordani and Kohn (2008). The latent binary random variables $\kappa_{ij,t}$ and $\kappa_{iv,t}$ are used to capture the presence of random shifts in betas and/or idiosyncratic variance and, for the sake of simplicity, these are assumed to be independent of one another (i.e., across assets and factors) and over time. Note that even though we allow breaks to occur independently across assets, empirically we are not restraining breaks (such as big macroeconomic shocks) from occurring contemporaneously across assets and/or factor exposures.

This specification is very flexible as it allows for both constant and time-varying parameters. When $\kappa_{ij,\tau} = \kappa_{iv,\tau} = 0$ for some $t = \tau$, then (3) reduces to (1) when the factor loadings and the quantity of idiosyncratic risk are assumed to be constant, as $\beta_{ij,\tau} = \beta_{ij,\tau-1}$ and $\ln \sigma_{i,\tau}^2 = \ln \sigma_{i,\tau-1}^2$. However, when $\kappa_{ij,\tau} = 1$ and/or $\kappa_{iv,\tau} = 1$, then a break hits either beta or idiosyncratic variance or both, according to the random walk dynamics $\beta_{ij,\tau} = \beta_{ij,\tau-1} + \eta_{ij,\tau}$ and $\ln(\sigma_{i,\tau}^2) = \ln(\sigma_{i,\tau-1}^2) + v_{i,t}$ (or $\sigma_{i,\tau}^2 = \sigma_{i,\tau-1}^2 \exp(v_{i,\tau})$). Note that because when a break affects the betas and/or variances, the random shift is measured by variables collected in $\eta_{i,t}$, we can also interpret \mathbf{Q}_i not only as a standard, "cold" measure of the covariance matrix of the random breaks in $\eta_{i,t}$, but also of the "size" of such breaks: a large q_j^2 means for instance that whenever $\beta_{ij,t}$ is hit by a break, such a shift is more likely to be large (in absolute value). The same applies to the interpretation of q_{iv}^2 as the size of breaks in idiosyncratic variance. Moreover, (3) generalizes the idea of conditional heteroskedasticity in excess asset returns, to the possibility that such a process may be also subject to breaks, in the form of sudden change points that take $\ln(\sigma_{i,t}^2)$ away from its constant path.

We estimate (3) using a Bayesian approach, which is possibly the only numerically feasible method for a model with the features of the BTVSVB framework.⁶ Realistic values for the different prior distributions obviously depend on the problem at hand. In general, we use weak priors, excluding the size of the breaks \mathbf{Q}_i and the probabilities $\Pr(\kappa_{ij,\tau}=1)$ and $\Pr(\kappa_{iv,\tau}=1)$ for which our priors are quite informative (see the Appendix). We set the prior hyperparameters to imply, on average, breaks in $\beta_{ij,t}$ and $\sigma_{i,t}^2$ approximately 5% and 2% of the time. Priors are instead uninformative for breaks with prior mean for the size of the break smaller than 0.3. All other priors imply that the posteriors tend to be centered around their maximum likelihood estimates which eases comparison with the existing literature.⁷

Once estimates of the posterior density for the unknown coefficients are obtained, we also implement a second-stage estimation pass by estimating, for each month, the following cross-sectional multivariate regression:

$$r_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} + e_{i,t} \qquad i = 1, ..., N,$$
 (4)

where $e_{i,t} \sim N(0, \sigma_t^2)$ and $\beta_{ij,t|t-1}$ measures the expected time t sensitivity of asset i to factor j, based

⁶In a frequentist framework it would be hard to separately identify the stochastic shifts represented by the variables $\kappa_{ij,t}$ and $\kappa_{iv,t}$ from the continuous shocks in $\eta_{ij,t}$ and $v_{i,t}$ without specifying a parametric process for $\kappa_{ij,t}$ and $\kappa_{iv,t}$.

⁷These priors are commonly referred to as uninformative or "flat". However, Section 6.3 reports results obtained using tighter, more informative priors and show that these have a negligible impact on our qualitative findings.

on all information available on and upon to time t-1. $\beta_{ij,t|t-1}$ is carefully constructed for the purposes of our investigation: it is obtained by taking the lagged value from the updating step of the Kalman filter (see the Appendix for details) and simulating the occurrence of future breaks and the shock magnitude from the appropriate posteriors. This Bayesian approach provides an elegant way to take into account estimation uncertainty by averaging out over parameter draws. In particular, we consider the full posterior distribution of the expected factor sensitivities $\beta_{ij,t|t-1}$: for each draw of the betas at a given time t, corresponding values for the risk premia are "drawn" from the relevant posterior distribution; then for each time t we obtain the entire empirical distribution of a given large number of draws of $\lambda_{j,t}$ on which to base our multivariate inferences on.⁸

2.3. Decomposition Tests

Independently of the estimation methods employed, we use the estimated time series of factor loadings and risk premia from Sections 2.1-2.2 to perform a number of "economic" tests. We use (4) to decompose excess asset returns in each time period in a component related to risk, represented by the term $\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1}$ plus a residual $\lambda_{0,t} + e_{i,t}$. In principle, a multi-factor model is as good as the implied percentage of total variation in excess returns explained by the first component, $\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1}$. However, here we should recall that even though (4) refers to excess returns, these are simply statistical implementations of the asset pricing framework in (1). This implies that in practice it may be excessive to expect that $\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1}$ be able to explain most (or even much) of the variability in excess returns. A more sensible goal seems to be that $\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1}$ ought to at least explain the predictable variation in excess returns.

We therefore follow earlier literature, such as Karolyi and Sanders (1998), and adopt the following approach. First, the excess return on each asset is regressed onto a set of instrumental variables that proxy for available information at time t-1, \mathbf{Z}_{t-1} ,

$$r_{i,t} = \theta_{i0} + \sum_{m=1}^{M} \theta_{im} Z_{m,t-1} + \xi_{i,t},$$
(5)

to compute the sample variance of the resulting fitted values,

$$Var[P(r_{it}|\mathbf{Z}_{t-1})] \equiv Var\left[\hat{\theta}_{i0} + \sum_{m=1}^{M} \hat{\theta}_{im} Z_{m,t-1}\right],\tag{6}$$

where the notation $P(r_{it}|\mathbf{Z}_{t-1})$ means "linear projection" of r_{it} on a set of instruments, \mathbf{Z}_{t-1} . Second, for each asset i=1,...,N, a time series of fitted risk compensations, $\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1}$, is derived and regressed onto the instrumental variables,

$$\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} = \theta'_{i0} + \sum_{m=1}^{M} \theta'_{im} Z_{m,t-1} + \xi'_{i,t}$$
(7)

⁸To avoid generated regressor problems in the most resolute way, for each time t we avoid collapsing the posterior density of the factor loadings $\beta_{ij,t|t-1}$ to a single value (e.g., their mean or median) and use instead the entire posterior for the betas. In practice, we draw a large number of times from such a posterior across all N assets and for each draw we estimate a multivariate cross-sectional regression to obtain a corresponding (implicit) draw for the risk premia.

to compute the sample variance of fitted risk compensations:

$$Var\left[P\left(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right] \equiv Var\left[\hat{\boldsymbol{\theta}}'_{i0} + \sum_{m=1}^{M}\hat{\boldsymbol{\theta}}'_{im}Z_{m,t-1}\right].$$
 (8)

The predictable component of excess returns in (5) not captured by the model is then the sample variance of the fitted values from the regression of the residuals $\xi_{i,t}$ on the instruments, $Var[P(r_{i,t} \sum_{i=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1} \rangle$. At this point, it is informative to compute and report two variance ratios, commonly called VR1 and VR2, after Ferson and Harvey (1991):

$$VR1 \equiv \frac{Var\left[P\left(\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1}\right)\right]}{Var[P(r_{it}|\mathbf{Z}_{t-1})]} > 0$$
(9)

$$VR2 \equiv \frac{Var\left[P\left(r_{i,t} - \sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1}\right)\right]}{Var[P(r_{it}|\mathbf{Z}_{t-1})]} > 0.$$
(10)

VR1 should be equal to 1 if the multi-factor model is correctly specified, which means that all the predictable variation in excess returns ought to be captured by variation in risk compensations; at the same time, VR2 should be equal to zero if the multi-factor model is correctly specified.⁹

When these tests are implemented using the estimation outputs obtained from the BTVSVB framework, we preserve complete consistency with our Bayesian framework: drawing from the joint posterior densities of the factor loadings $\beta_{ij,t|t-1}$ and the implied risk premia $\lambda_{j,t}$, i=1,...,N, j=1,...,K, and t = 1, ..., T, and holding the instruments fixed over time, it becomes possible to actually compute VR1 and VR2 in correspondence to each of such draws. This means that any large set S of draws from the (matching) posterior distributions for the $\{\beta_{ij,t|t-1}\}$ and $\{\lambda_{j,t}\}$ generates a posterior distribution for the statistics VR1 and VR2 as well. This makes it possible to conduct standard Bayesian "inferences" concerning the properties of VR1 and VR2 in our sample.

Finally, the predictable variation of returns due to the MFAPM may be decomposed into components imputed to each of the individual systematic risk factors, by factoring as in

$$Var[P(\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1})] = \sum_{j=1}^{K} Var\left[P\left(\lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1}\right)\right] + \sum_{j=1}^{K} \sum_{k=1}^{K} Cov[P\left(\lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1}\right), P\left(\lambda_{k,t} \beta_{ik,t|t-1} | \mathbf{Z}_{t-1}\right)]$$
(11)

and tabulating and reporting $Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]$ for j=1,...,K as well as the residual factor $\sum_{j=1}^{K} \sum_{k=1}^{K} Cov[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right), P\left(\lambda_{k,t}\beta_{ik,t|t-1}|\mathbf{Z}_{t-1}\right)] \text{ to pick up any interaction terms. Note}$ that because of the existence of the latter term, the equality

$$\sum_{j=1}^{K} \frac{Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var\left[P\left(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]} = 1$$
(12)

fails to hold, i.e., the sum of the K risk compensations should not equal the total predictable variation from the asset pricing model because of the covariance among individual risk compensations. 10

⁹Ferson and Harvey (1991) note that this is too strong a condition to be met by any model and so it cannot be interpreted strictly. Moreover VR1=1 does not imply that VR2=0 and viceversa, because $Var[P(r_{it}|\mathbf{Z}_{t-1})]$ is not simply $Var[P(\sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1})] + Var[P(r_{i,t} - \sum_{j=1}^{K} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1})]$ as it also reflects a covariance effect.

The fact that in (1) the risk factors are assumed to be orthogonal does not imply that their time-varying total

3. Data and Summary Statistics

Our paper is based on a large number of monthly time series (45) sampled over the period 1994:01-2011:12. The 1994:01 starting date derives from the availability of monthly return series for all the subsector REIT total return indices used in this paper. The initial five year worth of observations are used to set priors and the analysis is implemented over the remaining 156 observations, per each series, over the interval 1999:01-2011:12. The series belong to three main categories. The first group, "Portfolio Returns", includes several asset classes like stocks, bonds and real estate, organized in portfolios, a procedure that is useful to tame the contribution of non-diversifiable risk. The stocks are publicly traded firms listed on the NYSE, AMEX and Nasdaq (from CRSP) and sorted according to two criteria. First, we form 10 industry portfolios by sorting firms according to their four-digit SIC code. Second, we form 10 additional portfolios by sorting (at the end of every year, and recursively updating this sorting at an annual frequency) NYSE, AMEX and Nasdaq stocks according to their size, as measured by the aggregate market value of the company's equity. Using industry and size-sorted criteria to form spread portfolios of stocks to trade-off "spread" and reduction of idiosyncratic risk due to portfolio formation, is typical in the empirical finance literature (see e.g., Dittmar, 2002). Moreover, industry- and size-sorting criteria are sufficiently unrelated to make it plausible that industry- and size-sorted equity portfolios may contain different and non-overlapping information on the underlying factors and risk premia.

Data on long- (10-year) and medium-term (5-year) government bond returns are from Ibbotson and available from CRSP. Data on 1-month T-bill and 10-year government bond yields are from FREDII at the Federal Reserve Bank of St. Louis and from CRSP. Data on junk bond returns are approximated from Moody's (10-to-20 year maturity) Baa average corporate bond yields and converted into returns using Shiller's (1979) approximation formula. The data on sector and subsector tax-qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association and consists of data downloaded at two different levels of disaggregation. The analysis is initially performed at a rather aggregate level that only distinguishes between Industrial and Office, Retail, Residential, and Mortgage REITs, for a total of 4 sectors. However, our analysis is also performed afresh with reference to a finer disaggregation that distinguishes among 7 different subsectors derived from the previously listed 3 equity REIT portfolios, i.e., Industrial, Office (these obviously amount to the aggregate "Industrial and Office" sector), Shopping Centers, Regional Malls, and Free Standing shops (these amount to the aggregate "Retail" sector), Apartments and Manufactured Homes (from the "Residential" sector). These are breakdowns common in the literature (see e.g., Wang, Hsing, Chen, Wu, and Chang-Chien, 2011; Payne and Waters, 2007; Stevenson, 2002). All excess return

risk compensations $(\lambda_{j,t}\beta_{ij,t|t-1} \text{ for } j=1,...,K)$ should be orthogonal. For instance, unit risk premia on many factors are well-known to co-move during the business cycle and very precise counter-balancing moves in risk exposures will be required for the covariance term in (11) to equal zero.

¹¹Approximated returns from this formula are correlated with actual, Baa rating bracket returns (from Bloomberg) over recent years (2005-2011), with a correlation of 0.87.

¹²REITs, estabilished in the United States in 1960, constitute a way to invest into large scale, income producing real estate, enjoying a privileged tax treatment. They have a unique structure: among the requirements they meet, at least 75% of their assets has to be invested in real estate and at least 90% of their taxable earnings have to be paid out as

series are computed as the difference between total returns and 1-month T-bill returns, as usual.

Finally, we use a range of macroeconomic factors as standard proxies for the systematic, economywide risk factors potentially priced in asset returns. Lagged values of these risk factors (or simple transformation of the factors) are also used as "instruments" when relevant in our methodology, our logic being that all these variables belonged to the information set of the investors when they had made their portfolio decisions. In practice, we employ seven factors (as in Ling and Naranjo, 1997): the excess return on a wide, value-weighted market portfolio (r_t^M) that includes all stocks traded on the NYSE, AMEX, and Nasdaq; the trailing, 12-month dividend yield on all stocks traded on the NYSE, AMEX, and Nasdaq; the default risk premium (def_t) measured as the difference between Baa Moody's yields and yields on 10-year Treasuries; the change in the term premium $(\Delta term_t)$, the difference between 10-year and 1-month Treasury yields; the unexpected inflation rate $(UInfl_t)$, computed as the residual of a simple ARIMA(0,1,1) model applied to (seasonally adjusted) CPI inflation; the rate of growth of (seasonally adjusted) industrial production (IP_t) ; the rate of growth of (seasonally adjusted) real personal consumption growth (PC_t) ; the 1-month real T-bill rate of return computed as the difference between the 1-month T-bill nominal return and realized CPI inflation rate (not seasonally adjusted).

Table 1 presents summary statistics for the time series under investigation over our overall 1994-2011 sample. In particular, the table reports sample means, medians, standard deviations, and the resulting Sharpe ratios (computed with reference to 1-month T-bill returns). The summary statistics in Table 1 show no big surprises. Starting with the four REIT sectors, the three equity groups imply largely similar sample means, medians, and standard deviations of returns; these yield comparable monthly Sharpe ratios that fall between 0.12 and 0.15 (here residential REITs display the highest Sharpe ratio of 0.148, as a result of a sample standard deviation that is slightly smaller than in the case of other sectors). As one would expect, mortgage REITs are characterized by lower mean and median returns; because their volatility is however similar to that of equity REITs, their realized sample Sharpe ratio is relatively low, only 0.06 per month.

The REIT subsector panel of Table 1 reveals little differences between Industrial and Office REITs (but the former are more volatile than the latter are). On the contrary, the realized risk-return performance of Retail REITs appears to be driven by Free Standing REITs with a monthly Sharpe ratio of 0.18 vs. the comparably poor performance of Shopping Center-related REITs, implying a sobering 0.11. Finally, and in spite of the recent housing bust, the Residential subsector reveals a good risk-reward trade-off, mostly driven by the Apartment-specialized subsector, as it is characterized by a strong mean realized return (1.2% per month), in spite of its high volatility (6% per month); Manufactured Home REIT returns give instead more stable, but lower returns.

Most industry portfolios and all cap-sorted portfolios have mean returns between 0.7 and 1.1% per month. Moreover, for all stock portfolios (but one, energy stocks) median returns are substantially higher than mean returns, a clear indication of asymmetric return distributions. Volatilities tend to be between 4.5 and 7.5 percent in monthly terms, which corresponds to annualized volatilities ranging from 16 to 26 percent per year, which is the range one expects; small stock portfolios are

dividends.

more volatile than large stocks, while the most volatile industries are high tech and durable goods. As a result, most Sharpe ratios are in the 0.10-0.15 range (on a monthly basis), with very few outliers such as telecommunication, durable goods (with ratios below 0.05) and non-durable goods with a high Sharpe ratio of 0.17. There is nothing abnormal to report with reference to returns on 5- and 10-year government bonds, apart from their high Sharpe ratios in excess of most stock portfolios, due to the fact that our sample contains the massive flight-to-quality into Treasuries that has occurred during the financial crisis and to the effects of quantitative easing measures implemented by the Federal Reserve since 2009. Interestingly, while equity REITs are characterized by means (around 13% per year), volatility (in excess of 20%), and Sharpe ratios (0.12-0.13) directly comparable to those of stocks (for instance, the value-weighted CRSP portfolio has a mean return of 9%, volatility of 16%, and a Sharpe ratio of 0.10), mortgage REITs have produced much lower mean returns (around 6-7% per year) but display volatilities in excess of long-term bonds, with resulting Sharpe ratios that are disappointing, as we have noted.

Figure 1 provides a visual summary of the movements of the REIT total return indices under investigation. As a benchmark, we also plot the total return index for the value-weighted market portfolio. To favor comparability across different sectors and subsectors, all total return indices are standardized to equal 100 in correspondence to the end of January 2007. This date is chosen because most the literature (see e.g., Aït-Sahalia et al., 2009) has dated the onset of the subprime crisis to early to mid-2007. The top panel of Figure 1 provides motivation for our analysis because it shows that the residential sector exactly peaks in correspondence to the end of 2006 and leads the remaining two equity REITs sectors during all of 2007 and 2008. In fact, the mortgage REIT sector had already boomed between 2003 and 2005, but had also reached a new, local peak just in early 2007 and consistently with most anecdotal accounts of the onset of the subprime crisis (see e.g., Mian and Sufi, 2009; Wheelock, 2010)—subsequently tumbles from the late Spring of 2007. Interestingly however, from the Fall of 2008—approximately after the demise of Lehmann Brothers—the industrial & office (henceforth, I&O) and retail sectors start leading (and fall at higher rate than) residential and mortgage REITs. This is consistent with the policy debate and the financial press accounts of the time (see e.g., Greenlee, 2009). Starting in the Spring of 2009 all four sectors recover somewhat, with their total return indices approximately returning to the levels of late 2003, but the residential REIT segment displays a steep "V-shaped" bounce-back that has no equivalent in the case of the other three sectors. In fact, a simple calculation for the period January 2007 - December 2011 reveals that residential REITs are the only portfolio plotted in Figure 1 for which average returns are positive, albeit small. Our goal in this paper is to explain this differential dynamics in the valuations of I&O and retail REITs vs. residential and, at least to some extent, mortgage REITs.

The bottom panel of Figure 1 presents similar information with reference to 8 REIT subsectors for which monthly data are published by NAREIT. On the one hand, the picture that emerges is qualitatively similar to the one commented already. For instance, both apartments and manufactured homes follow the lead-lag-lead pattern observed for the most aggregated data, even though the recovery of apartment-investing REITs appears to be slower than for manufactured homes. On the other hand,

a few additional patterns are visible. For instance, REITs specialized in free-standing retail unit investments seem to have been hardly affected by the financial crisis, while REITs specialized in industrial buildings seem to have suffered the most, arguably as a result of the deep recession and of the structural over-capacity accumulated between 2005 and 2007, with the result that gross valuations as of the end of 2011 still lag behind the levels last observed in 1999.

4. Empirical Results

4.1. Factor Loadings

Figures 2-8 report plots of the posterior medians of the factor loadings $\beta_{ij,t}$ for each of the four REIT sectors at the heart of our empirical investigation, along with the loadings for other four portfolios—two equity portfolios for deciles 10 (the largest stocks covered by CRSP) and 1 (the smallest stocks), and two bond portfolios (10-year Treasuries and long term corporate bonds below the investment grade threshold)—to be taken as representative of the range of results we have obtained for stocks and bonds.¹³ We have 7 sets of figures, one set in correspondence to each of the risk factors assumed in our analysis. In each set, we include 8 plots (one for each of the portfolios listed above), in which besides the posterior medians estimated over time, we also show the associated 90 percent posterior confidence bands.¹⁴ A time t, the 90% credibility interval is characterized by the 5th and 95th percentiles of the posterior density of $\beta_{ij,t}$. In what follows we limit ourselves to two types of comments: a general comparison of the real estate asset class to stocks and bonds; a comparison within the real estate asset class among different types of portfolios to answer our key questions concerning the differences among residential- and commercial/industrial-driven REITs.

Figures 2-8 show that the real estate asset class has rather specific exposures to risk factors that differ from those typical of stocks and bonds. For instance, in Figure 2 it is clear that over our 1999-2011 sample, (most) REIT portfolios have a market beta that is lower (often close to zero) than stocks do, similarly to what Cotter and Roll (2011) and Lee, Lee, and Chiang (2008) have recently reported. Moreover, for most REIT portfolios, such an exposure to market shocks appears to have increased over time, while in the case of equity portfolios (both reported and unreported), any oscillations tend to occur around a stationary level over time. However, REITs also display market exposures that on average exceed those of riskless bonds (although there are deep similarities to corporate bonds). Similarly, in Figure 4, concerning the betas to changes in term premia, while REIT portfolios tend to mostly show non-zero and often significant exposure to changes in the slope of the riskless yield curve, stocks and corporate bonds are instead characterized by negligible exposures; ¹⁵ in this respect.

¹³Our plots never specifically focus on results for the industry portfolios, even though these have been used in estimating the MFAPM and especially its implied risk premia. Complete results are available upon request.

¹⁴Pinning down the "statistical significance" of coefficients (betas or lambdas) on the basis of 90% credibility intervals represents a rather stringent criterion because the Bayesian posterior density will reflect not only the uncertainty on the individual coefficient but also the overall uncertainty on the entire model (e.g., the uncertainty on structural instability of all the coefficients), see e.g., the discussion in Uno et al. (2005).

 $^{^{15}}$ Although this is not a terminology that is completely be fitting our Bayesian econometric framework, in what follows we shall often state that a given time series $\beta_{ij,t}$ of risk factor exposures is "significant" when the associated posterior 90 percent confidence bands fail to include zero, which means that the absence of any exposures is unlikely in the light of

securitized real estate appears to be similar to Treasuries, that have a rather obvious negative exposure to the term premium factor (see GRT, 2012, for related interpretations and references). A similar comment applies to Figure 5, concerning the loadings on IP growth risk, even though in this case corporate bonds behave like stocks and hence show a modest exposure to IP risk, while Treasuries have a positive but declining beta on macroeconomic growth risk. However, the major differences between REITs and stocks and bonds appear with reference to one traditional and here important macroeconomic factor, unexpected inflation. In Figure 8, all sector REIT portfolios display a strongly time-varying and statistically significant positive exposure to unexpected inflation, once more in a way similar to Treasuries. On the contrary, equities and corporate bonds generally show small, negative and anyway rarely "significant" beta on unexpected inflation. This finding is consistent with two traditional views often discussed in the real estate finance literature. First, that real estate would represent a "composite" asset class that inherits mixed features (here, factor exposures) from both stocks and bonds, see e.g., Ibbotson and Siegel (1984), Simpson, Ramchander and Webb (2007). Figures 2-8 emphasize that indeed REITs sometimes share their risk exposures with corporate bonds and even more often with Treasuries which contrasts with the recent view of real estate as an investment vehicle potentially as risky as stocks. The second view is that real estate may represent a strong hedge against inflationary shocks (i.e., unexpected inflation, see e.g., Simpson, Ramchander and Webb, 2007; Hoesli, Lizieri and MacGregor, 2008), even though in our results this relationship appears to be a complex one as—after unexpected inflation has been replaced by the corresponding, tradable mimicking portfolio a positive REIT beta implies that long positions expose an investor to inflationary shocks, so that short positions would be required in a hedging perspective. 16,17

All bond portfolios carry a negative exposure to market risk once the six additional macroeconomic factors are controlled for but display positive betas on the credit risk factor. It is interesting that 10-year government bond risk premia may increase when the credit risk premium increases, although this may relate more to using this factor as a business cycle indicator than to the credit quality of the U.S. government; consistently with this intuition, we notice that the beta of 5-year Treasuries is small. All bond risk premia have negative exposures to the slope of the yield curve factor; these betas are large and with a posterior distribution clearly tilted away from zero especially in the case of 10-year government bonds, which may capture flight-to-quality effects, in the sense that Treasuries would command high prices and low risk premia exactly when the riskless yield curve is flat or inverted, as typical of the early stages of economic contractions. Treasury bonds, especially long-term ones, have a positive and precisely estimated exposure to unexpected inflation, which is sensible because government securities

the evidence in the likelihood of the data.

¹⁶Differences across asset classes appar instead to be weaker in the case of the credit risk premium (Figure 3), real consumption growth (Figure 6), and the real T-bill rate (Figure 7). This also occurs because the $\beta_{ij,t}$ s are mostly small and their 90% confidence bands often include a zero exposure.

¹⁷Many (but not all) industries are significantly exposed to market beta risk and load considerably more on changes in the riskless term structure factor than they do on the credit risk factor. With reference to industry betas concerning unexpected inflation and short-term rate factors, their posterior beta densities yield credibility intervals that often fail to include zero but the sign of the median posteriors are heterogeneous. In general, industry portfolios yield small betas on IP and real consumption growth, with posterior densities that tend to attach a substantial probability on coefficients close to zero.

are notoriously exposed to inflationary shocks. Treasury returns have weak exposures to IP growth and real consumption growth factors. Finally, the BTVSVB model allows us to infer considerable instability in the betas of all Treasuries vs. market, IP growth, and real consumption growth risks, with rather heterogeneous trends.

Figures 2-8 also show that for most portfolios and factors, including real estate assets, our flexible BTVSVB model yield rich and interesting time-variation in betas. However, we immediately emphasize that such time variation is not forced upon the data, in the sense that a casual look at the plots reveals that combinations of test assets and factors can be found for which the $\beta_{ij,t}$ s reveal little or no time variation. For instance, in the top left corner of Figure 3, concerning the exposure of I&O REITs to the credit risk factor, the plot reveals a posterior median of $\beta_{I\&O,credit,t}$ that is flat at approximately -0.25 throughout our entire sample period. Interestingly, for most factors the four REIT sectors tend to share a common dynamics in their exposures, even when such betas imply mean levels that are quite different. For instance, in Figure 5, the $\beta_{ij,t}$ s with respect to IP growth all generally increase over our sample, peaking between late 2008 and early 2009, but a simple comparison between the I&O and mortgage sectors reveal that while the former climbs from -0.7 in 1999 to -0.4 at the end of 2011, in the case of the latter the increase is from -3.2 in 1999 to basically zero at the end of the sample. ¹⁸

The third feature of the factor exposures in Figure 2-8 that deserves commenting is that different sectorial REITs portfolios are characterized by a substantially different dynamics of their estimated beta posteriors over time, at least as captured by their medians and 90% credibility regions. Moreover, residential REITs are clearly different from "commercial" and industrial REITs; additionally, mortgage REITs obviously have a risk factor structure that is very specific and that diverges from equity REITs. In this case it is useful to express comments on factor exposures across plots and for each portfolio. Residential housing-driven REITs—according to most of the literature, the area from which the subprime crisis would have originated, see Simon and Ng (2009)—are characterized by a negative but rapidly increasing exposure to market risk, by positive but also quickly retreating exposures to term premium and real interest rate risks (in the former case, with confidence bands that start including a zero exposure after 2001), and by massive although also falling beta for unexpected inflation.¹⁹ Interestingly, if one disregards median posteriors of betas the confidence region of which includes a zero exposure, by 2007 residential REITs came to mostly carry only unexpected inflation and real riskless rate risk, both with positive betas. REITs that specialize in I&O investments carry instead positive, equity-like, and rapidly increasing market beta that exceeds one after the Spring of 2007, and positive, significant, and large exposure to unexpected inflation that has also slowly increased over time; until 2005, this REIT portfolio also had some negative beta toward IP growth risk. Retail unit-specialized

¹⁸However, in Figure 4 ($\beta_{ij,t}$ s concerning the term premium factor), while the exposures of the I&O and mortgage REIT sectors are essentially constant, in the case of residential and retail REITs the betas are strongly declining over time.

¹⁹The apperance of negative betas towards the market portfolio should not be surprising on two accounts: first, we have to recall that the non-traded factors in this paper are surrogated by factor mimicking portfolios that—in spite of our efforts—may still contain themselves non-zero exposures to the market; second, while an expectation of positive market beta is typical of the CAPM where the market is the only factor picking up business cycle risk, in our extended MFAPM there are in principle other six factors that may represent exposures to business cycles.

REITs have a negative, significant and relatively stable exposure to market risk and positive and large exposures to real interest rate and unexpected inflation risks (declining in the former case and strongly time-varying with a trough in 2000 in the latter case).²⁰ A comparison among residential on the one hand, and I&O and retail REITs on the other hand, sheds light on one potential cause of the differential behavior in the aftermath of the 2007-2009 financial crisis: the residential sector no longer has any exposure to general market dynamics and its upward swing must then be explained by increasing risks of unexpected inflation and real rate variations, that represent a sensible story in the presence of massive quantitative easing interventions that have increased the quantity of high-powered money manyfold in the attempt to revive growth in the US.

Finally, REITs that specialize in mortgage investments are estimated to have a large, negative but progressively increasing exposure to IP growth risk (this tames to zero by early January 2009), a large positive but declining exposure to real consumption growth (which also disappears towards the end of our sample), and a positive and strongly time-varying exposure to inflation risk. Interestingly, mortgage REITs is the only sector that manifests non-zero betas to the typical macroeconomic factors, IP and real consumption growth risks. In this sense, mortgage REITs appear to be different from the equity REIT sectors analyzed above. However, similarly to all the equity REIT portfolios, also mortgage REITs are exposed to unexpected inflation risk, that is obviously one of the characterizing features of real estate as an asset class.

4.2. Evidence from REIT Subsectors

Figures 9-11 extend the evidence in Section 4.1 to the case in which the three aggregate equity REIT indices (I&O, retail, and residential) are further disaggregated in 7 REIT portfolios represented by industrial, office, shopping centers, regional malls, free-standing shops, apartments, and manufactured homes specialized REITs. The eighth index/portfolio collects also in this case mortgage REITs. To save space, Figures 9-11 only present plots concerning the risk factor exposures of the 8 REIT subsector portfolios and for three factors only—the market, the real T-bill rate, and unexpected inflation—which turned out to be the three most important factors (i.e., those that were significant for most of the REIT portfolios and most of the time) in Section 4.1. However, completely tabulated results remain available upon request. Crucially, results on estimated $\beta_{ij,t}$ s for the remaining portfolios, i.e., equities and bonds, remain unchanged relative to those in Figures 2-8.

Figure 9, concerning market betas, reveals that the posterior distributions resulting from subsector-based estimation are remarkably more accurate than those plotted in Figure 2 for the more aggregate sectorial REITs. Moreover, while the aggregate portfolios in Figure 2 displayed a tendency for the market factor to imply significant loadings for both residential and non-residential REITs, this is not the case in Figure 9, where for most subsectors the betas are small (in absolute value), strongly time varying and generally characterized by 90% confidence bands that include zero most of the time. This is to be expected from the fact that it is well known that precise factor loading estimation tends to be difficult

 $^{^{20}}$ Until 2003-2005, retail REITs also displayed negative and significant betas toward the credit risk and IP growth factors, and positive and significant betas toward the term premium factor.

because dominated by noise in the case of excessively disaggregated portfolios, as these 8 subsector REIT indices appear to be.²¹ For instance, while the market beta of I&O was positive, increasing and precisely estimated in Figure 2, in Figure 9 the separate betas for industrial property and office REITs tend indeed to be positive but they remain imprecisely estimated and highly variable, even though it is visible that for both portfolios these increase towards one after 2009. Moreover, while shopping center and regional mall-investing REITs in Figure 9 have mildly negative and imprecisely estimated market betas, in Figure 2 the exposures were larger in absolute value and precisely estimated. However, once more in Figure 9, the residential-specialized REITs (manufactured homes and apartments) appear different from the other portfolios because their market betas are smaller, essentially fluctuate around zero and decline to zero between 2009 and 2010. Interestingly, free standing retail property-specialized REITs that have performed very strongly after 2009, are characterized by a market exposure dynamics that is very similar to manufactured homes.

In Figure 10, concerning the betas computed with respect to real short rate risk, when the REIT portfolios are disaggregated, we find that none of the posterior $\beta_{i,Tbill,t}$ s are either large in absolute value or significant. Interestingly, the shapes of the posterior medians are very similar within each of the sectors from Figure 7.²² Figure 11 shows instead how it can be obviously informative to perform estimation on disaggregated data, because the plots considerably strengthen our earlier conclusion that all equity REITs carry a significant exposure to inflation risk, that has further grown to exposure levels well in excess of 1.5-2 after 2009-2010 in the case of residential REITs.²³

4.3. Risk Premia Estimates

The top panel of Table 2 shows results on the posterior densities for the time series of risk premia estimates $\{\hat{\lambda}_{j,t}\}$ (j=1,...,K). The table reports both summary statistics for the full sample as well as for the most recent, financial crisis-dominated sub-sample, 2007-2011. The risk premia are computed with reference to the most disaggregated asset menu composed of the seven equity REIT subsectors discussed in Sections 3 and 4.2.25 The table shows results that have to be interpreted with great caution. If one applies standard (but frequentist) statistical inference to the time series of mean posterior estimates of the risk premia $\{\hat{\lambda}_{j,t}\}$ —which however assumes normality of the resulting posterior distributions—and computes standard t-tests of the null of zero risk premia, then we have interesting evidence in favor of as many as four priced risk factors in the cross-section of excess asset

²¹Nonetheless, Ang, Liu, and Schwarz (2010) have shown that an approach based on larger portfolios results in large efficiency losses in cross-sectional tests of asset pricing models. In particular, while creating portfolios reduces estimation error in betas, the standard errors of risk premia estimates are larger due to the smaller spread in betas.

²²Here the free standing property-specialized REITs are the exception, both because their real T-bill positive beta has been precisely estimated between 1999 and 2001, and also because their posterior medians describe a path that differs from shopping centers and regional malls, as well as from the plot for retail REITs in Figure 7.

²³In the case of industrial property and regional malls, the exposure to unexpected inflation also increases between 2006 and 2009, but it then declines back to pre-2005 levels. Also in Figure 11, the beta for free-standing shops follows a dynamics similar to manufactured homes, although the beta of the latter is visibly larger.

²⁴Plots are available from the Authors upon request. However, risk premia are sufficiently variable over time that in this case plots were not particularly revealing, especially because the size of the 90% confidence bands is rather volatile.

²⁵We have also performed the same analysis using only the I&O, retail, and residential equity REIT portfolio data (besides mortgage REITs) obtaining qualitatively similar results.

returns used in this paper: the market, IP growth, real consumption growth and inflation risks appear to be priced, in the full as well as in the final sub-sample. Interestingly, while for the market and real consumption growth factors, this result is similar to the evidence reported in GRT (2012), using a shorter but more disaggregated (in terms of number of REIT portfolios) data set yields evidence of widespread significance of the risk (premia on the) factors assumed in our analysis. In particular, market risk carries a mean posterior price of 0.45% per month with a "frequentist-type" p-value of 0.017; IP growth risk implies a risk premium of 0.72% per month with a p-value of 0.016; consumption growth risk carries a mean posterior price of 0.45% per month, again with a p-value of 0.010; finally, inflation risk commands a premium of 0.35% per month with a p-value of 0.014. While the finding of a significantly priced market factor may be not surprising, the result that also typical macroeconomic risks be priced is consistent with earlier evidence centered on real estate data (see e.g., Ling and Naranjo, 1997, with reference to real consumption growth and inflation). Importantly, the time series mean of the intercept $\lambda_{0,t}$ —which should be zero if the assumed MFAPM held—is relatively small (0.54 percent) and not precisely estimated.

This result is robust across sub-samples, and it is remarkable that with only 60 observations, the most recent time interval that includes the financial crisis leads to precisely estimated risk premia for the same four factors already listed above.²⁶ All averages for the posterior estimates of the premia turn out to be larger than what we have found in the left panel of Table 2: these are 2.6, 1, 2, and 2.3 percent per month for the market, IP growth, real consumption growth, and unexpected inflation risks, respectively. The corresponding p-values are in fact all very small, essentially nil. However, over the most recent sub-sample, the time series mean of the intercept $\lambda_{0,t}$ is large and highly statistically significant, at least in a frequentist interpretation that disregards the empirical distribution of the posterior of $\lambda_{0,t}$.

The evidence turns inconclusive in the full sample if one tries to use (averages over time of) 90% Bayesian credibility intervals built using posterior densities from the model. The reason is that without any exceptions, all these densities attach a non-negligible probability to zero or small risk premia on the different factors. For instance, the 90% interval for the market risk premium ranges from -0.89% per month to 1.62% per month. Although the median of the posterior is a sensible 0.45% per month (i.e., a rather typical 5.4% annual equity risk premium), an (unreported) density plot reveals that almost 20% of the probability mass goes to zero and negative risk premia.²⁷ However, it is comforting to see that this does not occur through probability mass being shifted out of the risk factors and into the posterior density of the mispricing indicator $\hat{\lambda}_{0,t}$, as also the 90% credibility interval for the intercept is wide (e.g., spanning -1.36% to 2.51% in the full sample). Moreover, results are again strong and supportive of the assumed MFAPM when our attention turns to 90 percent Bayesian credibility intervals for the shorter, recent 2007-2011 sub-sample. In this case, three factors command credibility regions for their

²⁶In this case, also credit risk appears to be priced, even though the corresponding p-value is only 0.081 and the historical average of the posterior median premium is negative.

²⁷These different results derived from means vs. quantiles of the posterior densities of the risk premia coefficients are possible because the posterior densities have a highly-non normal, non-symmetric shape characterized by a number of outliers.

risk premia that fail to include negative values: specifically, this occurs for the market (the lower 5% bound of the posterior median of risk premia is 0.82%), real consumption growth (1.02%), and inflation (0.76%) risk premia.

These estimated risk premia provide an easy explanation for any differential behaviors of residential vs. strictly commercial REITs both at the height of the financial crisis as well as in recent periods, such as 2010 and 2011 (see Figure 1, middle panel). For instance, with reference to late 2008 and only focusing on precisely estimated exposures, Figures 2-8 reveal that residential REITs were then characterized by a large and negative market beta (approximately -1.5), large and positive betas on the real short term rate and unexpected inflation (approximately 2.5 and 3, respectively); because full sample means of posterior estimates indicate risk premia for these factors of 0.45, -0.13, and 0.35 percent per month, a quick back-of-the-envelope calculation yields an expected return of 0.05\% only, which is not inconsistent with a rapidly declining realized residential REIT index. At the same time, Figures 2-8 show that at the end of 2008, I&O REITs were characterized by a unit market beta, and by a large and positive beta on inflation risks (approximately 5); based on the same full sample risk premia provided above, this returns an expected return of 2\%, which may be taken to explain why in late 2008 industrial, office, and retail REIT prices were not declining, or at least they were not declining as fast as residential REITs did. If one performs similar back-of-the-envelope calculations with reference to 2011 and using the estimated risk premia for the our second sub-sample, one obtains an expected return of 10.1 percent per month in the case of residential REITs vs. 16.2 percent for I&O REITs, which is consistent with them growing at a similar rate in the last part of Figure 1.

4.4. Economic Tests

So far our discussion has focussed on the statistical performance of the models with emphasis on whether there was evidence of either the $\lambda_{0,t}$ s coefficients being different from zero and whether there was any evidence that the assumed risk factors had been priced in the cross-section of risky portfolios used in our tests. We have indeed uncovered encouraging evidence that the BTVSVB model may be consistent with the data. Therefore, with reference to (3), we compute the VR1 and VR2 ratios and proceed to factor $Var[P(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$ as the sum of the contributions given by each of the factors, following standard practice in the multifactor literature.²⁸

The first two columns of Table 3 present the posterior medians of VR1 and VR2 obtained from (3) for each of the 31 portfolios under examination. Variance ratio results are encouraging, although there is some difference between the VR1 vs. the VR2 perspectives. Under a VR1 perspective, we can claim that approximately 75% of all predictable variation in excess returns is indeed captured by our MFAPM. Such percentages are very high and fully satisfactory for the subsector REIT data (82% on average), still high (79% on average) but more heterogeneous (ranging from 53 to 90 percent) for the industry equity portfolios, and just a tad lower for size-sorted and bond portfolios (between 72

²⁸In what follows, the information at time t-1 (\mathbf{Z}_{t-1}) is proxied by the instrumental variables listed in Section 3, plus a dummy variable to account for the "January effect" in the cross-section of stock returns, a calendar anomaly that investors are likely to take into account.

and 74 percent). What is more interesting in our perspective is that the ability of the MFAPM to explain the (predictable) variation in REIT returns is excellent for most of the subsector portfolios, with peaks in excess of 85% for office, free-standing shop, and apartment REITs. This finding fits well our earlier ability to explain simple trends in REIT total return indices using average beta and risk premium estimates from Sections 4.1-4.3.

However, because VR1 + VR2 = 1 does not hold, the finding of good VR1 ratios fails to automatically imply that the VR2 ratios are always as close to zero as much as we would want. Even though VR2 is generally at or below 20% only, in Table 3 we occasionally notice portfolios for which as much as a quarter of total predictable variation cannot be explained by the macroeconomic risk factors assumed in our MFAPM, so that it is time-varying idiosyncratic variances that have to pick up the slack. These relatively high VR2 ratios characterize only free-standing and manufactured homes-specialized REITs, as far as our main objects of analysis are concerned, with VR2 ratios of 27 and 26 percent, respectively. However, and consistently with the empirical findings in GRT (2012), some equity portfolios (e.g., energy and health stocks) as well as junk corporate bonds occasionally spike up, signalling an unsatisfactory fit offered by the BTVSVB implementation of the multi-factor framework.

Table 3 also shows that the predictable variation in excess REIT returns is mostly explained by exposure to the IP growth risk factor (the contribution to explaining the variance of predictable variation is 0.52 on average), followed by the unexpected inflation (0.39) and real consumption growth (0.29) factors. Interestingly, the typical business cycle risk factor represented by IP growth plays a key role even though the associated exposures as well as cross-sectional risk premia failed to be precisely estimated in Sections 4.1-4.3 and confirms the idea that public real estate portfolios are mostly priced off pure aggregate real activity and inflation surprise shocks, more than off typical financial factors such as market or credit default risks. We also find a clear difference between the percentage contribution of the unexpected inflation factor to explain residential REIT excess returns vs. non-residential REITs: in the former case, the inflation risk factor plays a dominant role, with contributions of 0.80 (manufactured homes) or even 1.00 (apartments): in the case of business-oriented (I&O or retail-investing) REITs, we estimate inflation risk contributions of 0.33 at most (as small as 0.10 in the case of industrial REITs). This confirms a view of residential public real estate as an asset class dominated by inflation concerns, not only in terms of (posterior median) exposures and risk premia, but also in terms of the contribution of these concerns to explain the predictable variation of realized excess returns. Even though this occurs on a smaller scale, a similar difference between residential and non-residential REITs may also be noticed with respect to the contribution offered by the real consumption growth factor: its weight is large for residential portfolios (0.46 and 0.69) and relatively small in the case of commercial ones (it is practically nil in the case of industrial, shopping centers, and free standing-specialized REITs).²⁹

Let's also ask one more time what makes (if anything) REITs different from other asset classes.

²⁹Because non-residential REITs are explained "less" by inflationary and real consumption growth concerns than residential REITs are, one may wonder what makes up for the difference. Leaving aside the problematic (for the purpose of interpretation) interaction effects, we notice that office and most of the retail REITs tend to be explained by the IP growth factor with above-average contributions.

Table 3 reveals a very simple and yet powerful answer: in the case of stocks (and to some extent, bonds) the leading priced risk factor that explains the dynamics of asset prices is represented by the market, with an average contribution to $Var[P(\sum_{j=1}^{7} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1})]$ of 0.64 and several peaks well in excess of 1 in the case of a few industrial portfolios and the largest capitalization ones. Conversely, stocks are affected by pure macroeconomic IP growth risk on a scale that is inferior to what we have observed for REITs. One may say that while in the case of equities, general aggregate risk is mostly represented by the value-weighted market portfolio, on the contrary macroeconomic factors play this role in the case of all types of REITs. As far as stocks are concerned, the next most important factor contributions come from unexpected inflation risks, although the heterogeneity across portfolios is large.

5. Heterogeneous Mispricings in REIT Sectors

Figures 12 and 13 report (posterior median) estimates of $\beta_{i0,t}$. In an ICAPM interpretation of (1) and under the null of correct specification, $\beta_{i0,t} \neq 0$ represents evidence of non-zero excess returns for a portfolio i with zero exposures to the K risk factors, which implies the existence of an arbitrage opportunity and it is inconsistent with first principles (e.g., non-satiation). In the finance literature, estimates of quantities like $\beta_{i0,t}$ are often named Jensen's alphas (see Jensen, 1968) and interpreted as measures of abnormal (excess) returns.

Figure 12 starts by presenting medians of $\beta_{i0,t}$ posteriors as well as 90 percent confidence intervals computed in the usual way, with reference to the 4 aggregate REIT sectors as well as a few other stock and bond portfolios representative of the overall universe of 27 portfolios used in estimation. Therefore Figure 12 completes the set of Figures 2-8 commented in Section 4.1. If one ignores the considerable uncertainty in the data (both objective and related to parameter estimation), Figure 12 offers a rather stark view of a number of asset pricing trends that have involved real estate over the past decade: all the Jensen's alpha related to REITs are indeed positive and relatively large, although these are either structurally declining or at least they do so between 2006 and 2008. In particular, the retail REIT alpha is always in excess of 1 percent per month, which points to a considerable over-pricing of REITs—in the sense that their value would have grown over time generating excess returns that were not justified by their exposures to priced macroeconomic risks; also the alphas of residential and mortgage REITs average at approximately 1.5% per month over our sample, although these have declined somewhat during the subprime crisis (although they remain positive). Only the I&O's alpha is smaller, although it never declines below 50 basis points per month. These structurally high and positive alphas for REITs of all kinds differ from the typical alphas estimated for stock and bond portfolios, when the posteriors for the $\beta_{i0,t}$ s tend to yield medians that are generally small, often negative, and the sign of which changes several times between 1999 and 2011. For instance, even in

³⁰ As explained in Section 2.4, these ratios may exceed 100% because $Var[P(\sum_{j=1}^{7} \lambda_{j,t} \beta_{ij,t|t-1} | \mathbf{Z}_{t-1})]$ will also reflect the contribution of covariance terms between factor terms. In fact, in Table 3 the only 12 (out of 217) contributions grossly exceeding 100% are obtained in the presence of sizably negative covariance contributions.

³¹Interestingly, bond portfolios occupy an intermediate position, as in this case both market and IP growth risks are equally important in explaining predictable variations.

the case of small capitalization stocks—that the empirical finance literature has long debated as an obvious case of mispricing by standard factor models—their alpha starts out at 1% in the late 1990s to then slowly decline to essentially zero by early 2008. Another example is 10-year Treasuries, for which the estimated alpha has a posterior median that is systematically negative over our sample, possibly an indication of under-pricing.

Yet, when the uncertainty surrounding these posterior median estimates is added back into the picture, the story changes somewhat: only retail, residential, and mortgage-specialized REITs have been mispriced (arguably, over-priced) over the initial part of our sample, i.e., 1999-2006 (2005 in the case of mortgage REITs), in the sense that even the lower 5% bound of the 90% credibility region fails to include zero over this interval. Over the same portion of the sample, 10- and 5-year Treasuries would have been under-priced as their alpha is estimated to have been "significantly" negative. Although exceptions may occasionally be found (e.g., health and energy have long been over-priced, while the technology sector underwent a well known bubbly period in 1999-2000), none of the equity portfolios appear to have been persistently mispriced over our sample. These results are consistent with those obtained by GRT (2012) with reference to a longer sample that also includes the 1980s and 1990-1998.

Figure 13 repeats this exercise, when the equity REIT sectors are split into seven subsectors, as already commented in Section 4.2. Therefore, Figure 13 may be taken to complete the evidence on the performance of the MFAPM in Figures 9-11. Figure 13 tells us a story that does not completely match the popular press accounts of the unfolding of the great financial crisis and much of the previous literature. On the one hand, and ignoring confidence regions for the time being, the claim that the great real estate bubble would have been a debt/mortgage-fueled one (see e.g., Brueckner, Calem and Nakamura, 2012; Coleman, LaCour-Little and Vandell, 2008; Hendershott, Hendershott, and Shilling, 2010) is consistent with the fact that between 1999 and 2006 mortgage REITs imply the largest, positive median alphas among the 8 plots that appear in the figure. On the other hand, there is no evidence of a larger, residential real estate bubble because the alpha of manufactured homes-investing REITs is actually estimated to be negative and declining throughout our entire sample; the mispricing of apartment-investing REITs did turn out positive and large (reaching 1 percent per month) between 2004 and 2006, but such alphas were quickly corrected down after 2007. The actual real estate overpricing occurred instead—and in the perspective of our model it is indeed potentially still under way—in the industrial, office, and retail commercial real estate subsectors: in particular, the estimated posterior median alphas for office and regional mall-specialized REITs are flat and persistently at levels in the order of 2 percent per month over our entire sample, including the 2009-2011 alleged post-crisis period.³² Notice that the finding of positive alphas on these portfolios does not imply that they would have yielded high or positive returns during our sample, as from Figure 1, we know this was not the case between 2007 and 2009, while the recovery in O&I and regional malls valuations over 2010-2011 has also been muted: a positive alphas simply means that realized excess returns on these REITs on average should have been even lower than what the data reveal, based on their exposures to priced risk

³²On the opposite, industrial and shopping center REIT portfolios were never massively mispriced, in the sense that their alphas is persistently small as well as not precisely estimated. Moreover, there is some evidence of free-standing shops having been structurally underpriced, with the mispricing turning statistically significant between 1999 and 2003.

factors. Equivalently, the valuations of office and regional malls should have dropped even more than they actually did. This ongoing poor pricing of portions of the I&O and retail real estate universe held by REITs would therefore concern mostly office space and large-scale regional mall properties that have been affected by the deep 2008-2009 recession more severely than other asset types, and with effects that may still linger to depress their valuations.³³

6. Robustness Checks

6.1. A Bayesian Time-Varying Parameter Model

The BTVSVB model is characterized by a large number of parameters, in particular to model any breaks, so that problems of over-parameterization could arise. This is undesirable, because our conclusions could be driven by the details of the parameterization of the change point process. To investigate these issues, we have also estimated two restricted versions of the general framework in (3). The first version is a time-varying parameter model (TVPM) à la Jostova and Philipov (2005),

$$r_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \sigma_{it} \epsilon_{i,t}$$

$$\beta_{ij,t} = \beta_{ij,t-1} + \eta_{ij,t} \qquad j = 0, ..., K,$$

$$\ln(\sigma_{i,t}^{2}) = \ln(\sigma_{i,t-1}^{2}) + v_{i,t} \qquad i = 1, ..., N,$$
(13)

under the same distributional assumptions as in (3). This model is obtained from the BTVSVB when $\kappa_{ij,t} = \kappa_{iv,t} = 1 \ \forall t$, so that both the risk exposures and the idiosyncratic variances follow a random walk process. Clearly, the TVPM implies no need to estimate any breakpoints. A second restricted version (covered in Section 6.2) consists of the case of $\kappa_{ij,t} = \kappa_{iv,t} = 0 \ \forall t$, which implies that $\beta_{ij,t} = \beta_{ij,t-1} = \beta_{ij}$ and $\ln(\sigma_{i,t}^2) = \ln(\sigma_{i,t-1}^2) = \ln(\sigma_i^2)$ and consists of the two-step Fama-MacBeth model with constant betas and idiosyncratic variances.

The leftmost column of plots in Figures 14-16 and the second panel in Table 2 report selected estimation results from the TVPM estimated using Bayesian methods adapted from the BTVSVB model to this special case. The results are "selected" because they refer only to four REIT subsectors—the ones for which the most interesting differences had emerged in Section 5—i.e., office, regional mall, apartment, and mortgage-specialized REITs. A comparison with Figures 9-11 and the qualitative comments expressed in Section 4.2 reveals important differences in the shape of the posteriors for the factor loadings produced by the BTVSVB vs. the TVPM. Moreover, confidence regions around median posteriors tend now to be narrower than in the case in which discrete breakpoint in factor loadings were allowed. The downside is that, because variation in the parameters is a built-in feature deriving from the assumed random walk process $\beta_{ij,t} = \beta_{ij,t-1} + \eta_{ij,t}$, all the examples in Figures 14 and 15 show pervasive instability in the loadings, although their variation tends to occur smoothly over

³³When in Figure 13 we also take note of the 90 percent confidence regions, results remain qualitatively intact, in the sense that the alphas of office and regional mall REITs have remained statistically significant at least until 2006 and their posterior densities still attach at most a 15% probability to their mispricing being zero or negative. On the contrary, manufactured homes would have been mispriced up to until as recently as 2010 and at the end of our sample there was only less than 10% probability of their alpha being zero or positive.

time. For instance, in Figure 14, regional malls and apartments were significantly negatively exposed to IP growth risk between 1999 and 2005, to then swing to a positive, significant exposure between 2007/2008 and 2010. Such wild swings, implying posterior median $\beta_{ij,t}$ s that are often of large absolute magnitude but unstable signs do not appear to be completely realistic.³⁴

Interestingly, a TVPM implies very precisely estimated but also strongly time-varying posterior densities for the factor loadings accompanied by rather constant, and spread out posterior estimates for the Jensen's alphas, the $\beta_{i0,t}$ s in Figure 16. Under a TVPM, there is no story to be told about mispricing in REITs and the recent subprime bust too: all REIT categories, both those plotted in Figure 16 and those that are available on request, imply flat alphas over time and the corresponding 90 percent confidence regions systematically include zero for most of our sample (always, after 2001). Although the finding of imprecisely estimated $\beta_{i0,t}$ s tends to be associated with the idea that a MFAPM should not be rejected, in this cases a number of doubts remain. These are strengthened by the fact that in the middle panel of Table 2 and with reference to the full sample period, most of the risk premia posterior means are not precisely estimated, so that only the market and IP growth factors turn out to be significant when a frequentist approach is employed, but with relatively large p-values between 0.05 and 0.1.

6.2. Traditional Two-Stage Fama-MacBeth Estimation

The framework in (1)-(2) just describes a general conditional pricing framework that is actually known to hold under rather general conditions. Apart from the BTVSVB approach, a variety of alternative methodologies have been proposed to estimate the factor loadings and the risk premia. A number of papers have indeed pursued a rather simple, one would say "seminonparametric" rolling window approach, by now considered classical, that consists of the two-stage CAPM testing procedure à la Fama and Mac Beth (1973) first applied by Ferson and Harvey (1991) to the estimation of linear multi-factor models.³⁵ In the first stage, for each of the assets, the factor betas are estimated using time series regressions from historical excess returns on the assets and economic factors. That is, for month t, we estimate equation (1) using the previous sixty months (ranging from t - 61 to t - 1) in order to obtain estimates for the betas, $\hat{\beta}_{ij,t-1}^{60}$. This time-series regression is updated each month. The first-step residuals have zero mean and may be modelled to have a time-varying $\sigma_{i,t}^2$ variance. In this paper—also to preserve asymmetry with the heteroskedastic modelling choices made in Section

 $^{^{34}}$ More generally, the finding of $\beta_{ij,t}$ s (for instance, with respect to inflation risk in Figure 15) with posterior medians that average between 2 and 4 and hence levels that are easily between 50 and 200 percent higher than what found in Section 4.2 is also unrealistic because in the presence of the type of estimated risk premia reported in Table 2, these would translate in large expected monthly returns (in absolute value) that often exceed the very variance of realized excess returns.

 $^{^{35}}$ GRT (2012) explain that, even though it is widely used in the applied finance literature, the classical two-stage Fama-MacBeth approach has a number of obvious statistical drawbacks. To name only two, first, the second stage multivariate regression used to estimate risk premia suffers from obvious generated regressor (error-in-measurement) problems as the estimated first-stage, rolling window beta estimates $\hat{\beta}^{60}_{ij,t-1}$ are used as regressors on the right-hand side. Second, the need to perform the estimation of (1)-(2) in two distinct stages that use rolling windows to capture parameter instability is not only ad hoc but also largely inefficient. GRT point out that Bayesian estimation methods applied to the BTVSVB framework remedy to these drawbacks.

2.2—we model each of the asset specific variances as following a univariate EGARCH(1,1) process.³⁶ In the second stage, we estimate a cross-sectional regression, for each month, using ex-post realized excess returns

$$r_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t} \hat{eta}_{ij,t-1}^{60} + \zeta_{i,t} \quad i = 1,...,N,$$

(and for each t = 60, ..., T). In (4) $\lambda_{0,t}$ is the zero-beta (abnormal) excess return and the $\lambda_{j,t}$ s (j = 1, ..., K) are proxies for the factor risk premia on month t.

The rightmost column of plots in Figures 14-16 and the bottom panel in Table 2 report selected estimation results from a Fama-MacBeth two-step estimation strategy. Also in this case, results are selected because they refer only to office, regional mall, apartment, and mortgage-specialized REITs. A comparison with earlier findings reveal important differences in the estimates of the factor loadings produced by this classical procedure, with reference to both BTVSVB and the TVPM. Also in this case, the results are quite unrealistic. First, in a number of cases (combinations of test portfolios and factors), Figures 14 and 15 produce jagged shapes characterized by pervasive time variation and wide 90% confidence intervals (these assume conditional normality of the shocks) that not only differ from those reported in Section 4.2, but also from the TVPM ones that have just appeared in Section 6.1 and that are plotted in the leftmost column. However, a different dynamics in factor loadings is by itself hardly problematic, because not knowing exactly the structure of the data generating process, it is feasible for alternative estimation strategies to return different results in small samples, even though these are all valid in a statistical sense.

Also in this case, what is more of a concern is the fact that in Figure 16 and in Table 2 the results of the Fama-MacBeth strategy are rather implausible. In Figure 16, all the alphas that have been plotted are strongly gyrating, they repeatedly change sign even though there is little evidence of any statistical significance and—which may be more troublesome—assume relatively large values that are difficult to justify in economic terms, similarly to what has been observed already by GRT (2012) in a similar application. According to Figure 16 none of the REIT portfolios (residential or non-residential, plotted or unreported to save space) would have been systematically over- or under-priced during our sample period. For instance, the plot of the estimated alphas for office REITs exceeds the rather incredible level of 5% per month (i.e., 60 percent in annualized terms) on a few occasions between 2003 and 2004. Interestingly, such a level is approached again around the end of our sample. However, the same estimated alpha persistently drops below -3% per month over long periods in 2000-2001, and again for a few months at the end of 2008. Results get even worse in the bottom panel of Table 2. Over the full sample, only the market factor appears to be significantly priced (the p-value is 0.036), while there is evidence of a highly significant and absurdly large cross-sectional mean intercept $\bar{\lambda}_0$, 0.64% per month

³⁶As a result, estimation of both the multifactor model for the conditional mean and of the variance parameters is performed using quasi-maximum likelihood. Ferson and Harvey (1991) have explored a range of alternative beta estimation techniques, including conditional betas estimated from regressions on past information variables, sixty-month rolling betas regressed on economic risk variables and past information variables, and ARCH-style conditional betas à la Bollerslev, Engle and Wooldridge (1988). The results in their paper are unaffected by selecting simple, Fama-MacBeth style five-year rolling OLS regression betas. GRT (2012) document that the specifics of the conditional variance model and estimation methods hardly affect the results in the classical, rolling window Fama-MacBeth implementation.

with a p-value that is essentially nil. Finally, in the case of the Fama-MacBeth method, we also record considerable instability across the two subsamples covered by Table 2.

6.3. Informative Priors

Within the BTVSVB approach, we have also experimented with an informative prior in the second pass in order to put some structure (constraints) on the distribution and moments of the risk premia. These are postulated to be normally distributed with zero mean and variance such that there is 95% probability that annualized premia are *smaller in absolute value* than the maximum cumulative annual return observed in the sample. We record a striking reduction in the variability of the estimated posterior distributions (as well as their medians) for the risk premia relative to the baseline case. In essence, using informative priors on the premia to constrain their variability, we find both less variable premia (which is a direct result of priors used) and economic implications that strengthen the first and second panels of Table 2: inflation shocks, real consumption growth and market risks are important drivers of U.S. REIT returns. However, all the remarks concerning the differences between residential and business-driven real estate expressed in Sections 4.1-4.2 and 5 apply intact.

7. Conclusions

In this paper we have asked a simple question: can a standard, rational multi-factor asset pricing model in which typical macroeconomic factors measure risk, shed any light on the actual or alleged differences in the pricing mechanism applied to residential vs. non-residential real estate? To provide an answer to this question, we have made two critical choices. First, we have written and estimated using Bayesian methods a rich multi-factor stochastic volatility model with time-varying factor loadings and discrete breakpoints. Such a choice is intended to deal with the widespread evidence that—especially when these are restricted to be linear and to involve standard macroeconomic variables—asset pricing relationships may be deeply unstable, in the sense that the exposures of different portfolios to risk variables often change over time, and that the price of such exposures may be unstable too. Second, we have addressed our research question using not only commonly used data for portfolios of stocks and bonds, but also resorting to abundant and detailed data on publicly traded REIT total return indices. This occurs with reference to two alternative sets that increasingly disaggregate overall REIT returns in subsector portfolios to distinguish between purely or mostly residential investments, commercial (i.e., industrial, office, and retail) investments, and mortgage specializations.

We uncover two key results. First, there are differences in the structure as well the dynamic evolution of risk factor exposures across residential and non-residential REITs. Residential REITs—according to most of the literature, the area from which the subprime crisis would have originated—are characterized by a negative but rapidly increasing exposure to market risk, by positive but also quickly retreating exposures to term premium and real interest rate risks, and by massive but falling beta towards inflation risk. Commercial REITs (both I&O and retail) carry instead positive, equity-like, and increasing market beta and positive, significant, and large exposures to unexpected inflation that has slowly increased over time. A comparison among residential and non-residential REITs sheds light

on one potential cause of their differential behavior in the aftermath of the 2007-2009 financial crisis: the residential sector no longer has any exposure to general market dynamics and its upward swing in Figure 1 is explained by increasing risks of unexpected inflation and real rate variations. Interestingly, mortgage REITs is the only sector that manifests non-zero betas to the typical macroeconomic factors, IP and real consumption growth; similarly to equity REITs, also mortgage REITs are exposed to unexpected inflation risk.

Second, an analysis of cross-sectional mispricings reveals that all the Jensen's alpha implied by REITs are positive and large, although they persistently decline between 2006 and 2008. However, when the uncertainty of the estimated alphas is taken into account, we find that only retail, residential, and mortgage-specialized REITs have been over-priced over the initial part of our sample, i.e., 1999-2006. Real estate appears to be quite special among all asset classes as only 10- and 5-year Treasuries have been significantly mis (under)-priced; although exceptions may occasionally be found, none of the equity portfolios appears to have been persistently mispriced. Moreover, the claim that the great real estate bubble would have been a debt/mortgage-fueled one is consistent with our result that between 1999 and 2006 mortgage REITs imply the largest, positive median alphas. Yet, there is no evidence of a larger, residential real estate bubble because the alphas of manufactured homes-investing REITs are actually negative and declining throughout our sample. The actual real estate over-pricing occurred instead—and in the perspective of our model, it is potentially still under way—in the businessrelated real estate sector: in particular, the estimated posterior median alphas of office and regional mall-specialized REITs remain obstinately at levels in the order of 2 percent per month over our entire sample, including the 2009-2011 post-crisis period. This ongoing poor pricing of portions of the commercial real estate universe held by REITs would therefore concern mostly office space and largescale regional mall properties that have been affected by the deep 2008-2009 recession more severely than other asset types, and with effects that may still linger to depress their valuations.

This finding of a deeply rooted and persistent overpricing of specific types of commercial real estate properties, may have important policy repercussions. On the one hand, should the current regime of low rates of growth of the US economy and of low inflation risks persist, it is possible that the progressive removal of such mispricings may translate in future, low, potentially negative realized commercial real estate returns. As job losses have accelerated during 2009, demand for commercial property has declined and vacancy rates have increased. The higher vacancy levels and significant decline in the value of existing properties have placed particularly heavy pressure on construction and development projects that do not generate income until after completion.³⁷ As a result, starting in late 2008, a sharp deterioration in the credit performance of loans in banks' balance sheets and loans in commercial mortgage-backed securities (CMBS) has been reported. This means that four years after the alleged end of the great financial crisis of 2007-2009, a second, equally catastrophic dip into a commercial real estate bust cannot be completely ruled out in the light of our estimates. This undesirable development echoes the unfolding of the banking crisis that engulfed the US system in the

³⁷Developers typically depend on the sales of completed projects to repay their outstanding loans, and with prices depressed amid sluggish sales, many developers have struggled servicing existing loans.

early 1990s (see e.g., Berger, Kashyap, Scalise, Gertler, Friedman, 1995). On the other hand, in the measure in which—as sometimes discussed in policy circles (see e.g., Bernanke, 2012; Greenlee, 2009; Parkinson, 2011)—office and regional mall property investments sit on large amounts in the balance sheets of nationally- and regionally-relevant US banks, their exposure to macroeconomic and inflation risks may end up hindering the correct transmission of monetary policy impulses.³⁸

These conjectures concerning the wider scale implications of persistent underpricing in commercial real estate relative to residential real estate would of course deserve further analysis.³⁹ Moreover, it must be emphasized that in this paper we have analyzed the key question of the relative dynamics of mispricing of different categories of real estate through the lenses of an econometric approach that is based on a formal modelling of the latent process followed by risk exposures capable to capture structural shifts in parameters. Our application to monthly, 1999-2011 US data for stock, bond, and publicly traded real estate returns shows that the classical, two-stage approach that relies on a rolling window modelling of time-varying betas yields results that are unreasonable. On the contrary, the empirical implications of our Bayesian estimation of (3) are plausible and there are indications that the model may be consistent with the data. For instance, most stock and bond portfolios do not appear to have been grossly mispriced and a few risk premia are precisely estimated with a plausible sign, which makes the results for REITs even more striking. However, we cannot claim to have achieved complete success. It would be interesting both to further fine-tune the standard, more traditional part of the model—such as the number of macroeconomic factors specified, their identity and definition—and at the same time to work on the specific structure and assumptions appearing in (3) to test whether its empirical performance may be improved.

References

- [1] Aït-Sahalia J., Andritzky J., Jobst A., Nowak S., and Tamirisa, N., "How to Stop a Herd of Running Bears? Market Response to Policy Initiatives during the Global Financial Crisis." IMF working paper 2009/204.
- [2] Ang A., Liu J., and Schwarz K., "Using Stocks or Portfolios in Tests of Factor Models", working paper, Columbia University, 2010.
- [3] Berger A., Kashyap A. K., Scalise J. M., Gertler M., and Friedman B.M., "The Transformation of the U. S. Banking Industry: What a Long Strange Trip It's Been", *Brookings Papers on Economic Activity*, 1995, 2, 55-218.

³⁸At the end of 2010, approximately \$3.2 trillion of outstanding debt was linked to commercial real estate (CRE). Of this, \$1.6 trillion was held on the books of banks and thrifts, and an additional \$700 billion represented collateral for CMBS. At the same time, about 10 percent of CRE loans in bank portfolios were delinquent, a three-fold increase since the end of 2007.

³⁹High CRE concentrations have been the dominant factor in recent bank failures. Of the more than 300 commercial banks and thrifts that have failed since early 2008, more than three-fourths had high CRE concentrations at year-end 2007. In fact, the Federal Reserve has taken a number of actions directly affecting the exposure of the financial sector to the business-oriented segments of the CRE market. One program active during 2008-2010 has bee the Term Asset-Backed Securities Loan Facility (TALF). Under the TALF, eligible investors may borrow to finance purchases of the AAA-rated tranches of various classes of asset-backed securities, including both existing and newly issued CMBS.

- [4] Bernanke B., "Banks and Bank Lending: The State of Play", Remarks to the 48th Annual Conference on Bank Structure and Competition Sponsored by the Federal Reserve Bank of Chicago, May 10, 2012.
- [5] Bjorklund K. and Soderberg B., "Property Cycles, Speculative Bubbles and the Gross Income Multiplier", *Journal of Real Estate Research*, 1999, 18, 151-174.
- [6] Bollerslev T., Engle R. F., and Wooldridge J.M., "A Capital Asset Pricing Model with Time-Varying Covariances", *Journal of Political Economy*, 1988, 96, 116-131.
- [7] Brueckner J. K., Calem P. S., and Nakamura L.I., "Subprime Mortgages and the Housing Bubble", *Journal of Urban Economics*, 2012, 71, 230-243.
- [8] Carter C. and Kohn R., "On the Gibbs Sampling for State-Space Models", *Biometrika*, 1994, 81, 541-553.
- [9] Cecchetti S., "Crisis and Responses: The Federal Reserve in the Early Stages of the Financial Crisis." *Journal of Economic Perspectives*, 2009, 23, 51-75.
- [10] Chen N.-F., Roll R., and Ross S., "Economic Forces and the Stock Market", *Journal of Business*, 1986, 59, 383-403.
- [11] Coleman M., LaCour-Little M., and Vandell K. D, "Subprime Lending and the Housing Bubble: Tail Wags Dog?", *Journal of Housing Economics*, 2008, 17, 272-290.
- [12] Cotter J. and Roll R., "A Comparative Anatomy of REITs and Residential Real Estate Indexes: Returns, Risks and Distributional Characteristics", working paper, UCLA, 2011.
- [13] Dittmar R. F., "Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns", *Journal of Finance*, 57, 369-403.
- [14] Fama E. and MacBeth J., "Risk, Return and Equilibrium: Empirical Tests", Journal of Political Economy, 1973, 81, 607-636.
- [15] Ferson W. and Harvey C. R., "The Variation of Economic Risk Premiums", Journal of Political Economy, 1991, 99, 385-415.
- [16] Gerlach R., Carter C., and Kohn R., "Efficient Bayesian Inference for Dynamic Mixture Models", Journal of the American Statistical Association, 2000, 95, 819-828.
- [17] Giordani P. and Kohn R., "Efficient Bayesian Inference for Multiple Change-Point and Mixture Innovation Models", *Journal of Business and Economic Statistics*, 2008, 26, 66-77.
- [18] Gorton G., "The Panic of 2007," European Financial Management, 2010, 15, 10-46.
- [19] Greenlee, J., 2009, Testimony on Residential and Commercial Real Estate before the Subcommittee on Domestic Policy, Committee on Oversight and Government Reform, U.S. House of Representatives, Atlanta, GA, November 2, 2009.
- [20] Groen J.J.J., Paap R., and Ravazzolo F., "Real-Time Inflation Forecasting in a Changing World", Journal of Business and Economic Statistics, forthcoming.
- [21] Guidolin M., Ravazzolo F. and Tortora A. ""Myths and Facts about the Alleged Over-Pricing of U.S. Real Estate: Evidence from Multi-Factor Asset Pricing Models of REIT Returns", *Journal of Real Estate Finance and Economics*, forthcoming.

- [22] Hendershott P., Hendershott R., and Shilling J., "The Mortgage Finance Bubble: Causes and Corrections", *Journal of Housing Research*, 2010, 19, 1-16.
- [23] Hoesli M., Lizieri C. M., and MacGregor B. D., "The Inflation Hedging Characteristics of U.S. and U.K. Investments: A Multi-Factor Error Correction Approach", Journal of Real Estate Finance and Economics, 2008, 38, 183-206.
- [24] Ibbotson R. G. and Siegel L. B., "Real Estate Returns: A Comparison with Other Investments", Real Estate Economics, 1984, 12, 1540-6229.
- [25] Jensen M.C., "The Performance of Mutual Funds in the Period 1945-1964", Journal of Finance, 1968, 23, 389-416.
- [26] Jostova G. and Philipov A., "Bayesian Analysis of Stochastic Betas", *Journal of Financial and Quantitative Analysis*, 2005, 40, 4, 747-778.
- [27] Karolyi G.A. and Sanders A., "The Variation of Economic Risk Premiums in Real Estate Returns", Journal of Real Estate Finance and Economics, 1998, 17, 3, 245-262.
- [28] Kim S., Shepard N., and Chib S., "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models", *Review of Economic Studies*, 1998, 65, 361-93.
- [29] Lee M., Lee M., and Chiang K., "Real Estate Risk Exposure of Equity Real Estate Investment Trusts", Journal of Real Estate Finance and Economics, 2008, 36, 165-181.
- [30] Ling, D.C. and Naranjo A., "Economic Risk Factors and Commercial Real Estate Returns", Journal of Real Estate Finance and Economics, 1997, 15, 283-307.
- [31] Liu C. H. and Mei J., "The Predictability of Returns on Equity REITs and their Co-Movement with Other Assets", *Journal of Real Estate Finance and Economics*, 1992, 5, 401-418.
- [32] Merton R., "An Intertemporal Capital Asset Pricing Model", Econometrica, 1973, 41, 867-887.
- [33] Mian A. and Sufi A., "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis", Quarterly Journal of Economics, 124, 1449-1496.
- [34] Payne J. and Waters G., "Have Equity REITs Experienced Periodically Collapsing Bubbles?", The Journal of Real Estate Finance and Economics, 2007, 34, 207-224.
- [35] Parkinson P. M., Statement before the Congressional Oversight Panel Washington, D.C., February 4, 2011.
- [36] Plazzi, A., Torous, W., and Valkanov R., "Expected Returns and Expected Growth in Rents of Commercial Real Estate", Review of Financial Studies, 2010, 23, 3469-3519.
- [37] Primiceri G., "Time Varying Structural Vector Autoregressions and Monetary Policy", Review of Economic Studies, 2005, 72, 821-852.
- [38] Raftery A.E. and Lewis S.M., "How Many Iterations in the Gibbs Sampler?", Bernardo J.M (eds), Bayesian Statistics 4, Oxford University Press, 763-773.
- [39] Ravazzolo F., Paap R., van Dijk D. and Franses P.H., "Bayesian Model Averaging in the Presence of Sructural Breaks", Rapach D. and Wohar M. (eds.), "Forecasting in the Presence of Structural Breaks and Uncertainty", Frontiers of Economics and Globalization, Elsevier, 2007.
- [40] Shanken, J., "On the Estimation of Beta Pricing Models", Review of Financial Studies, 1992, 5, 1-34.

- [41] Shiller R.J., "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure", *Journal of Political Economy*, 1979, 87, 1190-1219.
- [42] Simon S. and Ng W.-L., "The Effect of the Real Estate Downturn on the Link Between REITs and the Stock Market", Journal of Real Estate Portfolio Management, 2009, 15, 211-219.
- [43] Simpson M. W., Ramchander S., and Webb J. R., "The Asymmetric Response of Equity REIT Returns to Inflation", *Journal of Real Estate Finance and Economics*, 2007, 34, 513-529.
- [44] Stevenson S., "Ex-Ante and Ex-Post Performance of Optimal REIT Portfolios", Journal of Real Estate Portfolio Management, 2002, 8, 199-207.
- [45] Tanner M.A. and Wong W.H., "The Calculation of Posterior Distributions by Data Augmentation", Journal of the American Statistical Association, 1987, 82, 528-550.
- [46] Wang H-C., Chen C-C., Wu M-C. and Chang-Chien I-I., "Forecasting the REITs' Returns in US: New Evidence from Structural Time Series Model", 2011 IEEE International Summer Conference of Asia Pacific, 2011, 205-209.
- [47] Wheaton W., "Real Estate 'Cycles': Some Fundamentals", Real Estate Economics, 1999, 27, 209-230.
- [48] Wheelock, D. "Lessons Learned? Comparing the Federal Reserve's Responses to the Crises of 1929-1933 and 2007-2009." Federal Reserve Bank of St. Louis *Review*, March/April 2010.

Appendix

We separately present each of the steps of our Bayesian implementation of the two-step Fama-MacBeth (1973) approach.

First pass:

For each asset i = 1, ..., N, the model in (3) is

$$\begin{array}{rcl} r_{i,t} & = & \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \sigma_{i,t} \epsilon_{i,t} \\ \\ \beta_{ij,t} & = & \beta_{ij,t-1} + k_{ij,t} \eta_{ij,t} & j = 0, ..., K \\ \ln(\sigma_{i,t}^2) & = & \ln(\sigma_{i,t-1}^2) + k_{2i,t} v_{i,t} & i = 1, ..., N, \end{array}$$

where $\epsilon_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{N,t})' \sim N(0, \mathbf{I}_N)$, $\boldsymbol{\eta}_{i,t} \equiv (\eta_{i0,t}, \eta_{i1,t}, ..., \eta_{iK,t}, v_{i,t})' \sim N(0, \mathbf{Q}_i)$ with \mathbf{Q}_i a diagonal matrix characterized by the parameters q_{i0}^2 , q_{i1}^2 , ..., q_{iK}^2 , q_{iv}^2 , and $\boldsymbol{\kappa}_{it} \equiv (\kappa_{i0,t}, ..., \kappa_{iK,t}, \kappa_{iv,t})'$ is a $((K+2)\times 1)$ vector of unobserved uncorrelated 0/1 processes with $\Pr[\kappa_{ij,t}=1]=\pi_{ij}$ for j=0,...,K+1 and $\Pr[\kappa_{iv,t}=1]=\pi_{iv}$. The model parameters are the structural break probabilities $\boldsymbol{\pi}_{i} \equiv (\pi_{i0},...,\pi_{iK},\pi_{iv})'$ and the vector of variances of the break magnitude $\mathbf{q}_i^2 \equiv (q_{i0}^2,q_{i1}^2,...,q_{iK}^2,q_{iv}^2)$. They are collected in a $(2(K+1)\times 1)$ vector $\boldsymbol{\theta}_i \equiv (\boldsymbol{\pi}_i',(\mathbf{q}_i^2)')'$. For the sake of brevity, we suppress the index i in the remaining part of this section.

Independent conjugate priors are used to ease posterior simulation. For the break probability we assume simple Beta distributions,

$$\pi_j \sim Beta(a_j, b_j) \qquad \pi_j^{\kappa_v} \sim Beta(a_j^{\kappa_v}, b_j^{\kappa_v}),$$
 (14)

where the hyperparameters a_j and b_j (j = 0, ...K + 1) reflect prior beliefs about the occurrence of breaks. For the variance parameters the inverted Gamma-2 prior is chosen,

$$q_j^2 \sim IG(\nu_j, \delta_j) \qquad q_v^2 \sim IG(\nu_v, \delta_v),$$
 (15)

where ν_j (ν_v) expresses the belief on the strength of the prior mean, which is proportional to δ_j (δ_v). We set the prior hyperparameters to imply, on average, breaks in $\beta_{ij,t}$ and $\sigma_{i,t}^2$ approximately 5% and 2% of the time; given our sample size, this means we expect 19 and 7 breaks in each of the $\beta_{ij,t}$ and $\sigma_{i,t}^2$, respectively, when $\delta_j = \delta_j = 0.3 \ \forall j$. Although an ex-ante belief of breaks occurring on average every 20-53 months are plausible, Groen, Paap, and Ravazzolo (2012) have investigated the importance of these priors to get insights on their effects. Priors are instead uninformative for breaks with prior mean for the size of the break smaller than 0.3.

For posterior simulation we run the Gibbs sampler in combination with the data augmentation technique by Tanner and Wong (1987). The latent variables $B = \{\beta_t\}_{t=1}^T$, $R = \{\sigma_t^2\}_{t=1}^T$, and $\mathcal{K} = \{\kappa_t\}_{t=1}^T$ are simulated alongside the model parameters, $\boldsymbol{\theta}$. The complete data likelihood function is given by

$$p(r, B, \mathcal{K}, R | \boldsymbol{\theta}, F) = \prod_{t=1}^{T} p(r_t | F_t, \beta_t, \sigma_t^2) \prod_{j=0}^{m} p(\beta_{j,t} | \beta_{j,t-1}, \kappa_{j,t}, q_j^2) \times$$

$$p(\sigma_t^2 | \sigma_{t-1}^2, \kappa_{v,t}, q_v^2) \prod_{j=0}^{k} \pi_j^{\kappa_{j,t}} (1 - \pi_j)^{1 - \kappa_{j,t}} \pi_v^{\kappa_{v,t}} (1 - \pi_v)^{1 - \kappa_{v,t}}.$$

Combining the prior and the data likelihood, we obtain the posterior density

$$p(\theta, B, \mathcal{K}, R|r, F) \propto p(\theta)p(r, B, \mathcal{K}, R|\theta, F).$$
 (16)

Defining $\mathcal{K}_{\beta} = \{\kappa_{0,t}, ..., \kappa_{K,t}\}_{t=1}^{T}$ and $\mathcal{K}_{\sigma} = \{\kappa_{v,t}\}_{t=1}^{T}$, the sampling scheme consists of the following iterative steps:

- 1. Draw \mathcal{K}_{β} conditional on $R, \mathcal{K}_{\sigma}, \boldsymbol{\theta}$, and r.
- 2. Draw B conditional on $R, \mathcal{K}, \boldsymbol{\theta}$ and r.
- 3. Draw \mathcal{K}_{σ} conditional on $B, \mathcal{K}_{\beta}, \boldsymbol{\theta}$, and r.
- 4. Draw R conditional on $B, \mathcal{K}, \boldsymbol{\theta}$ and r.
- 5. Draw $\boldsymbol{\theta}$ conditional on B, \mathcal{K} and r.

The first step applies the efficient sampling algorithm of Gerlach, Carter and Kohn (2000), the main advantage being drawing $\kappa_{j,t}$ without conditioning on the states $\beta_{j,t}$, as Carter and Kohn (1994) instead do. The conditional posterior density for $\kappa_{\beta,t}$, t = 1, ..., T unconditional on B is:

$$p(\kappa_{\beta,t}|\mathcal{K}_{\beta,-t},\mathcal{K}_{\sigma},R,\boldsymbol{\theta},r) \propto p(r|\mathcal{K}_{\beta},\mathcal{K}_{\sigma},R,\boldsymbol{\theta})p(\kappa_{t}|\mathcal{K}_{\beta,-t},\boldsymbol{\theta})$$

$$\propto p(r^{t+1,T}|r^{1,t},\mathcal{K},R,\boldsymbol{\theta})p(r_{t}|r_{1,t-1},\kappa_{\beta,1,t-1},R,\boldsymbol{\theta},x)p(\kappa_{\beta,t}|\mathcal{K}_{\beta,-t},\boldsymbol{\theta}). (17)$$

Gerlach, Carter and Kohn (2000) show how to evaluate the first two terms while the last one is obtained from the prior. When $\mathcal{K}_{\beta,t}$ and $\beta_{j,t}$ are highly dependent, the sampler of Carter and Kohn (1994) breaks down completely: the higher the correlation (dependence), the bigger the efficiency gain. The latent

process for the betas is estimated by means of the forward-backward algorithm of Carter and Kohn (1994).

 \mathcal{K}_{σ} and R are drawn in the same way as \mathcal{K}_{β} and B. To do so we follow Kim, Shepard and Chib (1998) and approximate the log of a $\chi^2(1)$ distribution by means of a mixture of seven normals. In each iteration of the Gibbs sampler we simulate a component of the mixture distribution in order to get a conditional linear state space model for $\ln(\sigma_t^2)$. Finally, the vector of parameters $\boldsymbol{\theta}$ is easily sampled as we use conjugate priors.

We use a burn-in period of 1,000 and draw 5,000 observations storing every other of them to simulate the posterior distributions of parameters and latent variables. The resulting autocorrelations of the draws are very low.⁴⁰

Second pass:

To estimate the cross section in (4) at each time t we first draw values for each $\beta_{ij,t|t-1}$, i=1,...,N and j=1,...,K. The parameters $\beta_{ij,t|t-1}$ measures the sensitivity of asset i to factor j. We take the lagged value from the updating step of the Kalman filter, simulate $K_{ij,t}$ and q_{ij}^2 from their posteriors derived above and obtain $\beta_{ij,t|t-1}$.

Then, we use natural conjugate priors. In particular,

$$\begin{array}{rcl} p(\lambda,\sigma^2) & = & p(\lambda|\sigma^2) \times p(\sigma^2) \\ \text{where } (\lambda|\sigma^2) & \sim & N(\underline{\lambda},\sigma^2\underline{V}) \text{ and } (\sigma^2) \sim IG(\frac{\underline{\nu}}{2}\underline{s}^2,\frac{1}{2}\underline{\nu}). \end{array}$$

Combining them with the data likelihood we obtain a joint posterior density with convenient analytical form. The resulting marginal posterior distributions are

$$(\lambda|r) \sim t(\overline{\lambda}, \overline{s}^2 \overline{V}, \overline{\nu}) \qquad (\sigma^2|r) \sim IG(\frac{\overline{\nu}}{2} \overline{s}^2, \frac{1}{2} \overline{\nu})$$

with

$$E(\lambda|r) = \overline{\lambda} \quad var(\lambda|r) = \frac{\overline{\nu}s^2}{\overline{\nu} - 2} \overline{V}$$

$$E(\sigma^2|r) = \frac{\overline{\nu}s^2}{\overline{\nu} - 2} \quad var(\sigma^2|r) = \frac{(\overline{\nu}s^2)^2}{(\overline{\nu} - 2)^2(\overline{\nu} - 2)}$$

where

$$\overline{V} = (\underline{V}^{-1} + (X'^{-1})^{-1} \qquad \overline{\lambda} = (\underline{V}^{-1} + (X'^{-1})^{-1}(\underline{V}^{-1}\underline{\lambda} + (X'^{-1}\hat{\lambda}))^{-1}(\underline{V}^{-1}\underline{\lambda} + (X'^{-1}\hat{\lambda}))^{-1}(\underline{V}^{-1}\underline{\lambda})^{-1}(\underline{V}^{-1$$

 $\overline{\nu} = \underline{\nu} + N$, and $\hat{\lambda}$ is the OLS estimate.

Results are presented with two different sets of priors. In the former case we are noninformative $(\underline{\nu}=0 \text{ and } \underline{V}^{-1}=0)$ and use the well known Jeffreys' prior while in the latter case we impose some prior information. In more detail, we opted for a small amount of strength $(\underline{\nu}=5)$ supporting a prior view for premiums with zero mean and variance equal to a twelfth of the maximum absolute cumulative annual return observed in the sample. Finally, the prior residual variance is centered at about 10, a value that appeared in the higher range of the maximum likelihood estimates.

⁴⁰In order to gain a rough idea of how well the chain mixes in our algorithm we follow Primiceri (2005) in looking at the autocorrelation function of the draws. Primiceri (2005) plots the 20th order sample autocorrelation for some of the parameters. He bases his assessment on two further and more elaborate indicators, such as the inefficiency factor (IF) and the Raftery and Lewis (1992) diagnostic. As for the stochastic variances, we obtain satisfactorily low levels of autocorrelation that in most cases essentially vanishes after a handful of lags. However, for the long term government bonds we observe a persistent autocorrelation which reaches values close to 0.4 after 20 lags, especially at the end of the sample.

Table 1
Summary Statistics for Financial and Macroeconomic Time Series Used in the Paper

| Mean | Median | Std. Dev | Sharpe Ratio | | | | |
|--|---|---------------------|---|--|--|--|--|
| | | | | | | | |
| 1.061 | 1.446 | 6.912 | 0.116 | | | | |
| 1.115 | 1.388 | 6.820 | 0.125 | | | | |
| 1.134 | 1.309 | 5.895 | 0.148 | | | | |
| 0.638 | 1.743 | 6.407 | 0.059 | | | | |
| Real Estate Returns - Subsectors, Value-weighted | | | | | | | |
| 1.083 | 1.365 | 9.428 | 0.087 | | | | |
| 1.109 | 1.584 | 6.590 | 0.129 | | | | |
| 0.955 | 1.304 | 6.642 | 0.105 | | | | |
| 1.335 | 1.584 | 7.970 | 0.135 | | | | |
| 1.188 | 1.607 | 5.115 | 0.181 | | | | |
| 1.153 | 1.331 | 6.013 | 0.148 | | | | |
| 0.883 | 0.987 | 5.317 | 0.117 | | | | |
| | 10 Industry Port | folios, Value-wei | ghted | | | | |
| 0.901 | 1.220 | 3.773 | 0.170 | | | | |
| 0.557 | 0.940 | 7.520 | 0.039 | | | | |
| 0.983 | 1.420 | 5.150 | 0.140 | | | | |
| 1.144 | 0.890 | 5.723 | 0.154 | | | | |
| 1.052 | 1.420 | 7.916 | 0.100 | | | | |
| 0.529 | 1.040 | 5.651 | 0.047 | | | | |
| 0.782 | 1.110 | 4.693 | 0.111 | | | | |
| 0.909 | 1.140 | 4.351 | 0.149 | | | | |
| | | | 0.124 | | | | |
| | | | 0.070 | | | | |
| | | | | | | | |
| 1.002 | 1.360 | 6.567 | 0.113 | | | | |
| 1.006 | 1.270 | 6.864 | 0.109 | | | | |
| 0.973 | 1.590 | 6.354 | 0.112 | | | | |
| 0.891 | 1.560 | 6.067 | 0.104 | | | | |
| 0.924 | 1.650 | 5.991 | 0.111 | | | | |
| 0.924 | 1.560 | 5.426 | 0.122 | | | | |
| 0.991 | 1.540 | 5.323 | 0.137 | | | | |
| 0.899 | 1.390 | 5.330 | 0.120 | | | | |
| 0.889 | 1.660 | 4.808 | 0.131 | | | | |
| 0.702 | 1.160 | 4.510 | 0.098 | | | | |
| | Bon | d Returns | | | | | |
| 0.554 | 0.630 | 2.090 | 0.141 | | | | |
| 0.493 | 0.556 | 1.300 | 0.178 | | | | |
| 0.746 | 0.854 | 2.717 | 0.179 | | | | |
| 0.261 | 0.310 | 0.171 | 0.000 | | | | |
| | | | | | | | |
| 0.489 | 1.070 | 4.704 | 0.104 | | | | |
| 2.373 | 2.210 | 0.871 | _ | | | | |
| -0.004 | -0.030 | 0.254 | _ | | | | |
| 0.001 | 0.004 | 0.254 | _ | | | | |
| 0.176 | 0.218 | 0.701 | _ | | | | |
| 0.154 | 0.165 | 0.247 | _ | | | | |
| 0.057 | 0.061 | 0.317 | _ | | | | |
| Instrumental Variables | | | | | | | |
| 1.234 | 1.265 | 1.041 | _ | | | | |
| 0.968 | 0.860 | 0.464 | _ | | | | |
| 1.839 | 1.808 | 0.470 | _ | | | | |
| | 1.115 1.134 0.638 1.083 1.109 0.955 1.335 1.188 1.153 0.883 0.901 0.557 0.983 1.144 1.052 0.529 0.782 0.909 0.793 0.644 1.002 1.006 0.973 0.891 0.924 0.991 0.899 0.793 0.891 0.924 0.991 0.899 0.702 0.554 0.493 0.746 0.261 0.489 2.373 -0.004 0.001 0.176 0.154 0.057 | Real Estate Returns | Real Estate Returns - Sectors, Value- 1.061 | | | | |

 $Table\ 2$ Summary Statistics for Second-Pass Bayesian Posterior Median Estimates of Risk Premia

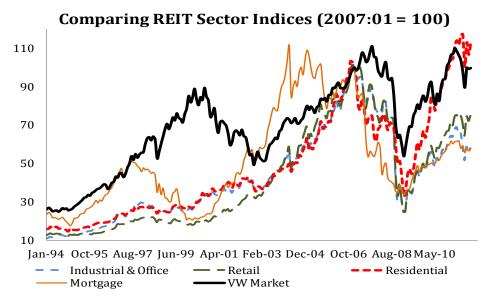
| | Full sample (Jan 1999 - Dec 2011) | | | | | Sub-sample (Jan 2007 - Dec 2011) | | | | | | |
|------------------------|---|------------|---------|---------|-------------|----------------------------------|-----------|------------|---------|---------|---------|--------|
| | Average | Std. Error | t-stat | p-value | 5% CI | 95% CI | Average | Std. Error | t-stat | p-value | 5% CI | 95% CI |
| | Bayesian model with stochastic breakpoints in loadings and in idiosyncratic variances | | | | | | | | | | | |
| Intercept | 0.5418 | 1.4571 | 0.4133 | 0.1850 | -1.3622 | 2.5119 | 2.7276 | 1.0255 | 2.4812 | 0.0046 | 1.4437 | 4.4674 |
| Market | 0.4449 | 1.0605 | 0.1977 | 0.0162 | -0.8851 | 1.6176 | 2.5671 | 1.4723 | 0.7915 | 0.0023 | 0.8191 | 4.3236 |
| Credit spread | -0.0754 | 0.7650 | -0.1502 | 0.1488 | -0.9654 | 0.8143 | -0.7942 | 1.4988 | -0.4892 | 0.0806 | -2.6740 | 1.0852 |
| Delta term spread | -0.1020 | 0.3835 | -0.2262 | 0.1621 | -0.5707 | 0.3265 | 0.5563 | 0.5217 | 0.7999 | 0.1895 | -0.2392 | 1.3100 |
| IP Growth | 0.7209 | 1.0770 | 0.4168 | 0.0159 | -0.8598 | 2.1149 | 1.0006 | 1.4092 | 0.9997 | 0.0011 | -0.8982 | 2.9456 |
| Real Consuption Growth | 0.4465 | 0.9757 | 0.2835 | 0.0101 | -0.8956 | 1.7185 | 1.9905 | 1.0547 | 1.2961 | 0.0014 | 1.0159 | 3.1681 |
| Real T-bill rate | -0.1281 | 0.2638 | -0.4403 | 0.1123 | -0.4522 | 0.1832 | 0.3899 | 0.3700 | 0.8273 | 0.1760 | -0.0706 | 0.8305 |
| Unexp Inflation | 0.3455 | 0.9348 | 0.2170 | 0.0143 | -0.8769 | 1.2578 | 2.3177 | 1.2394 | 0.7350 | 0.0036 | 0.7614 | 3.8786 |
| | | | | E | Bayesian ti | ime-varyin | g paramet | er model | | | | |
| Intercept | 1.0596 | 1.0804 | 0.9899 | 0.1675 | -0.3532 | 2.5809 | 0.8902 | 0.9747 | 0.9769 | 0.0266 | -0.2259 | 2.2390 |
| Market | 0.3739 | 0.9626 | 0.5451 | 0.0957 | -1.5835 | 0.9375 | 0.3873 | 0.8878 | 0.2812 | 0.0111 | -0.8914 | 1.1840 |
| Credit spread | -0.1337 | 0.5554 | -0.2852 | 0.1704 | -0.7078 | 0.5074 | 0.1497 | 0.6011 | -0.0025 | 0.0605 | -0.3223 | 0.6086 |
| Delta term spread | 0.0421 | 0.3840 | 0.1853 | 0.2081 | -0.4207 | 0.5322 | 0.2028 | 0.3452 | 0.2651 | 0.0803 | -0.2088 | 0.6835 |
| IP Growth | 0.4206 | 0.8645 | 0.6006 | 0.0552 | -1.5430 | 0.9311 | 0.1875 | 0.8301 | -0.1318 | 0.0255 | -0.6867 | 1.2485 |
| Real Consuption Growth | -0.2831 | 0.8387 | -0.5847 | 0.1160 | -1.2923 | 0.6206 | -0.0531 | 0.7056 | -0.2637 | 0.0438 | -0.6425 | 0.9411 |
| Real T-bill rate | -0.0244 | 0.2715 | -0.1843 | 0.2770 | -0.3389 | 0.2477 | -0.0375 | 0.1707 | -0.1993 | 0.2370 | -0.2925 | 0.1697 |
| Unexp Inflation | -0.2595 | 0.7890 | -0.5581 | 0.1047 | -1.3298 | 0.7081 | -0.0798 | 0.6982 | -0.4896 | 0.0062 | -0.8897 | 0.8432 |
| | | | | Cla | assical two | o-step Fam | a-MacBetl | n strategy | | | | |
| Intercept | 0.6390 | 0.1965 | 3.2515 | 0.0006 | -3.5797 | 4.5936 | 0.6757 | 0.3232 | 2.0906 | 0.0183 | -4.0474 | 5.4885 |
| Market | 0.3686 | 0.2054 | 1.7946 | 0.0364 | -4.6209 | 4.2686 | -0.3330 | 0.3691 | -0.9023 | 0.8166 | -6.9761 | 5.8719 |
| Credit spread | 0.1631 | 0.1480 | 1.1023 | 0.1352 | -2.2243 | 2.1278 | 0.1645 | 0.0967 | 1.7006 | 0.0445 | -2.1754 | 2.4387 |
| Delta term spread | 0.0461 | 0.0514 | 0.8976 | 0.1847 | -0.8571 | 0.9498 | 0.1767 | 0.0413 | 4.2789 | 0.0000 | -1.0214 | 1.3357 |
| IP Growth | 0.0368 | 0.4694 | 0.0783 | 0.4688 | -5.2618 | 4.8800 | -0.2284 | 0.3907 | -0.5848 | 0.7206 | -7.1313 | 5.6751 |
| Real Consuption Growth | 0.0799 | 0.4033 | 0.1981 | 0.4215 | -4.2102 | 4.1052 | -0.4202 | 0.3644 | -1.1532 | 0.8756 | -5.9887 | 4.9386 |
| Real T-bill rate | -0.0221 | 0.0584 | -0.3792 | 0.6477 | -0.9163 | 0.8071 | 0.0194 | 0.0188 | 1.0325 | 0.1509 | -0.6849 | 0.7051 |
| Unexp Inflation | 0.0596 | 0.3912 | 0.1522 | 0.4395 | -3.8481 | 3.8299 | -0.3442 | 0.3397 | -1.0133 | 0.8445 | -5.6958 | 4.6901 |

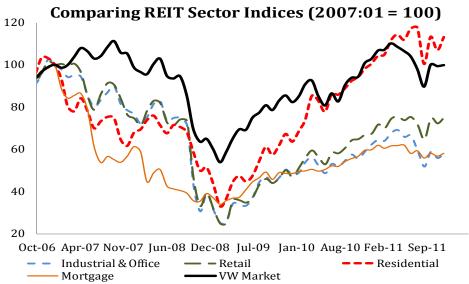
Table 3

Variance Ratio Coefficients and Predictable Variation Decompositions from BTVSVB Model

| | VR1 | VR2 | Market | Credit Risk Premiur | n Term Spread | IP Growth | Real Cons. Growth | Real T-Bill | Unexp. Inflation | Interaction Eff. |
|-----------------------------|---|--------|--------|---------------------|---------------|------------|-------------------|-------------|------------------|------------------|
| | 10 Industry Portfolios, Value weighted | | | | | | | | | |
| Non Durable Goods | 0.9006 | 0.2039 | 0.1552 | 0.0079 | 0.1015 | 0.0167 | 1.0308 | 0.0290 | 2.5531 | -2.8942 |
| Durable Goods | 0.7189 | 0.1172 | 0.9351 | 0.0049 | 0.0043 | 0.0390 | 2.1222 | 0.0446 | 0.2375 | -2.3876 |
| Manufacturing | 0.9176 | 0.1520 | 0.0409 | 0.0387 | 0.0072 | 0.0292 | 0.4297 | 0.0005 | 0.2695 | 0.1843 |
| Energy | 0.5258 | 0.2989 | 1.1132 | 0.1570 | 0.0499 | 0.2659 | 0.6252 | 0.5801 | 1.3518 | -3.1432 |
| High Tech | 0.6723 | 0.0875 | 2.8155 | 0.0185 | 0.1049 | 0.0242 | 0.0667 | 0.0034 | 0.9893 | -3.0226 |
| Telecom | 0.7523 | 0.1884 | 1.4007 | 0.1131 | 0.0125 | 0.0983 | 0.1832 | 0.0364 | 0.0279 | -0.8722 |
| Shops and Retails | 0.8922 | 0.2132 | 0.1392 | 0.0660 | 0.0983 | 0.0317 | 0.0715 | 0.0243 | 0.9669 | -0.3980 |
| Health | 0.8092 | 0.3671 | 2.5913 | 0.0035 | 0.0416 | 0.0450 | 0.4764 | 0.0089 | 0.1909 | -2.3577 |
| Utilities | 0.8317 | 0.2699 | 0.3812 | 0.0215 | 0.2616 | 0.3064 | 0.0645 | 0.0205 | 0.4463 | -0.5020 |
| Other | 0.8841 | 0.1950 | 0.1644 | 0.0306 | 0.0110 | 0.0110 | 0.0390 | 0.0073 | 0.3120 | 0.4247 |
| | 10 Size-sorted Portfolios, Value weighted | | | | | | | | | |
| Decile 1 | 0.7497 | 0.2023 | 0.4810 | 0.0012 | 0.0150 | 0.1822 | 0.0061 | 0.0011 | 0.1445 | 0.1689 |
| Decile 2 | 0.7221 | 0.1651 | 0.3656 | 0.0260 | 0.0123 | 0.1335 | 0.0349 | 0.0030 | 0.7257 | -0.3010 |
| Decile 3 | 0.6710 | 0.1463 | 0.0451 | 0.0197 | 0.0017 | 0.1266 | 0.0136 | 0.0003 | 1.1137 | -0.3206 |
| Decile 4 | 0.7177 | 0.1578 | 0.0384 | 0.0148 | 0.0174 | 0.0814 | 0.0313 | 0.0014 | 1.5912 | -0.7759 |
| Decile 5 | 0.8295 | 0.1252 | 0.0709 | 0.0040 | 0.0156 | 0.1053 | 0.0126 | 0.0010 | 1.0313 | -0.2407 |
| Decile 6 | 0.7736 | 0.1469 | 0.1289 | 0.0025 | 0.0161 | 0.0520 | 0.0134 | 0.0066 | 1.7472 | -0.9669 |
| Decile 7 | 0.8034 | 0.0994 | 0.1314 | 0.0022 | 0.0062 | 0.0436 | 0.1282 | 0.0022 | 0.7740 | -0.0878 |
| Decile 8 | 0.7109 | 0.1310 | 0.0716 | 0.0012 | 0.0046 | 0.0522 | 0.0220 | 0.0056 | 1.3909 | -0.5480 |
| Decile 9 | 0.7200 | 0.1371 | 0.0646 | 0.0009 | 0.0019 | 0.0721 | 0.0748 | 0.0050 | 0.2873 | 0.4934 |
| Decile 10 | 0.7069 | 0.2417 | 1.6873 | 0.0033 | 0.0007 | 0.0426 | 0.1400 | 0.0053 | 0.0633 | -0.9424 |
| | | | | | Вс | nd Returns | | | | |
| 10 - Yrs Treasury | 0.9319 | 0.2080 | 0.3968 | 0.0616 | 0.1119 | 0.4014 | 0.3972 | 0.0013 | 0.7969 | -1.1671 |
| 5 - Yrs Treasury | 0.7173 | 0.2335 | 0.4646 | 0.0726 | 0.0127 | 0.5550 | 0.3468 | 0.0037 | 0.9193 | -1.3748 |
| Baa Corporate Bonds (10-20) | 0.5249 | 0.2604 | 0.2261 | 0.0300 | 0.0453 | 0.7749 | 0.6899 | 0.0126 | 0.1697 | -0.9486 |
| | Real Estate Returns | | | | | | | | | |
| NAREIT - Industrial | 0.6957 | 0.1595 | 0.0137 | 0.0114 | 0.0135 | 0.2360 | 0.0122 | 0.0034 | 0.1009 | 0.6089 |
| NAREIT - Office | 0.8611 | 0.1111 | 0.0062 | 0.0641 | 0.0093 | 0.7697 | 0.4738 | 0.0031 | 0.3320 | -0.6582 |
| NAREIT - Shopping Centers | 0.8162 | 0.1859 | 0.0029 | 0.0019 | 0.0005 | 0.2746 | 0.0146 | 0.0002 | 0.2671 | 0.4383 |
| NAREIT - Regional Malls | 0.6960 | 0.1174 | 0.0031 | 0.0328 | 0.0010 | 0.6705 | 0.2384 | 0.0095 | 0.3190 | -0.2742 |
| NAREIT - Free Standing | 0.9795 | 0.2682 | 0.0033 | 0.0225 | 0.0019 | 0.6723 | 0.0094 | 0.0189 | 0.1482 | 0.1235 |
| NAREIT - Apartments | 0.8859 | 0.2255 | 0.0150 | 0.0176 | 0.0045 | 0.6477 | 0.4621 | 0.0012 | 1.0028 | -1.1509 |
| NAREIT - Manufactured Homes | 0.8134 | 0.2588 | 0.0230 | 0.1474 | 0.0442 | 0.3185 | 0.6937 | 0.0367 | 0.8025 | -1.0660 |
| NAREIT - Mortgage TR | 0.8345 | 0.1541 | 0.0110 | 0.0159 | 0.0026 | 0.5446 | 0.3891 | 0.0011 | 0.1564 | -0.1208 |

Figure 1
Comparing the Dynamics of Sector and Subsector REIT Indices Over Time





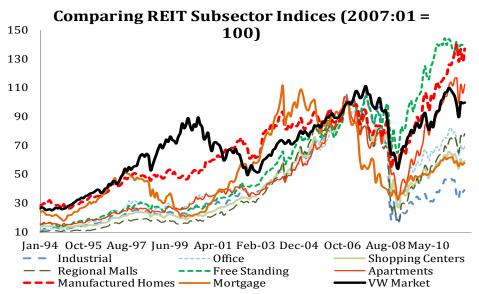


Figure 2
REIT and Other Portfolios Loadings on Macroeconomic Factors: VW Market Portfolio

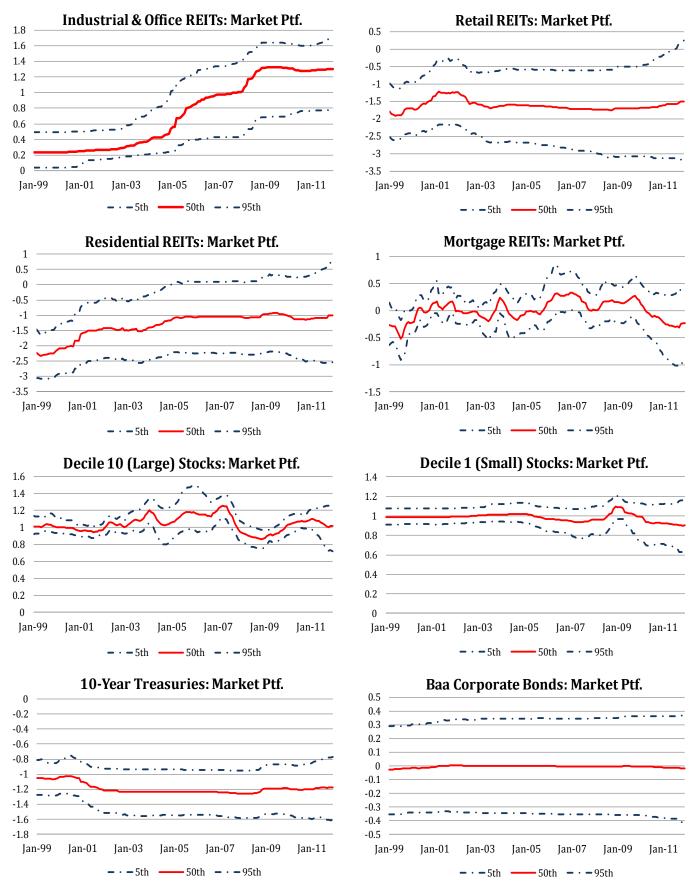


Figure 3

REIT and Other Portfolios Loadings on Macroeconomic Factors: Credit Risk Premium

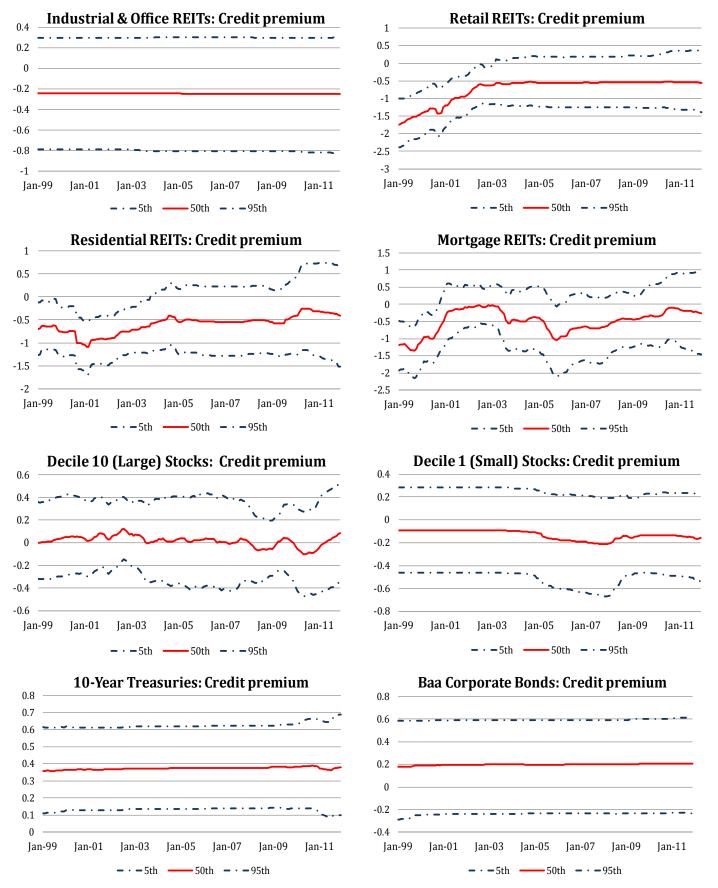


Figure 4

REIT and Other Portfolios Loadings on Macroeconomic Factors: Term Premium

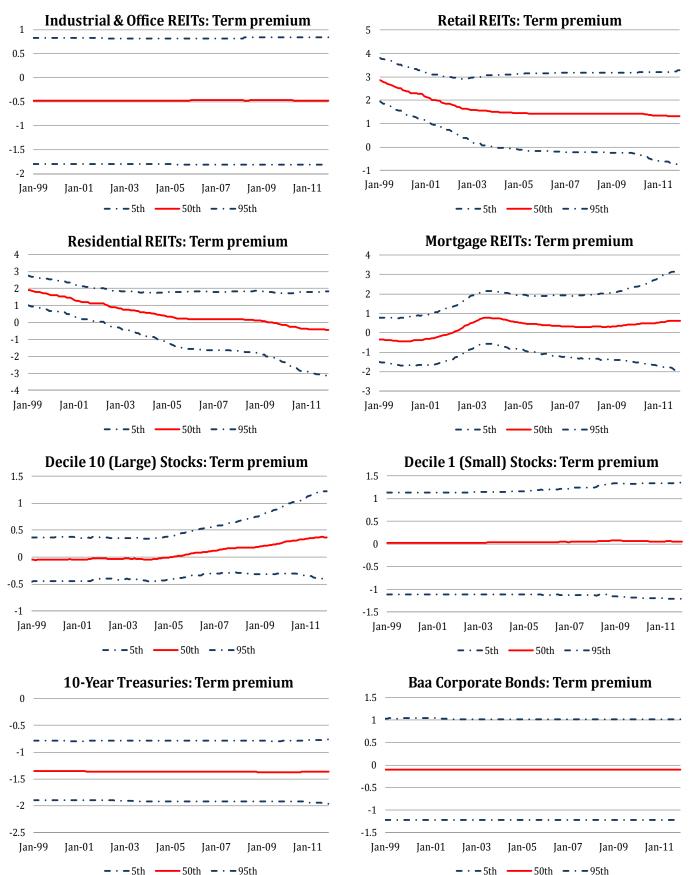


Figure 5
REIT and Other Portfolios Loadings on Macroeconomic Factors: IP Growth

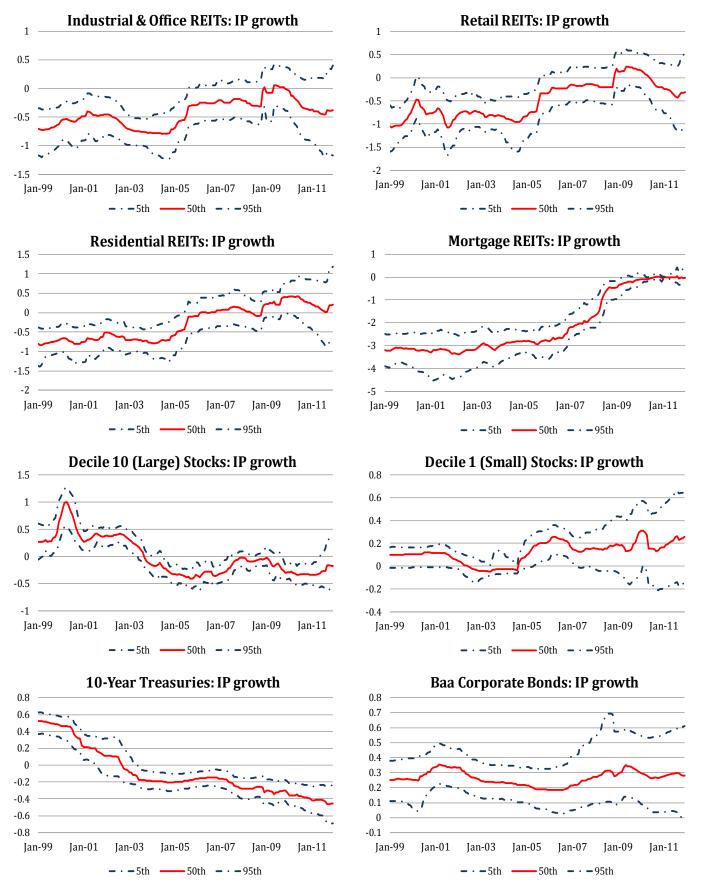


Figure 6
REIT and Other Portfolios Loadings on Macroeconomic Factors: Real Consumption Growth

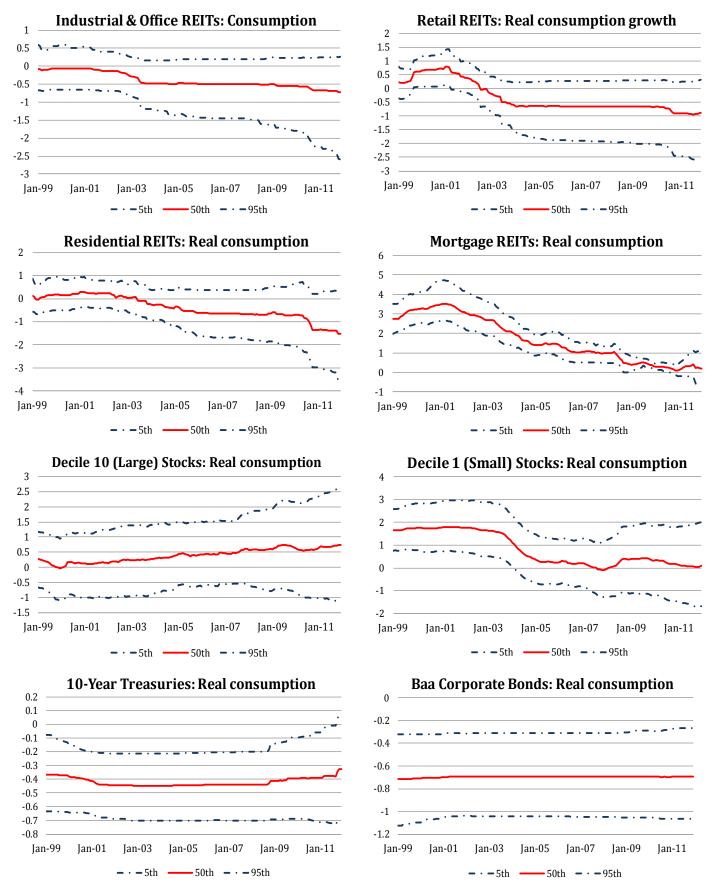


Figure 7
REIT and Other Portfolios Loadings on Macroeconomic Factors: Real T-Bill Rate



Figure 8

REIT and Other Portfolios Loadings on Macroeconomic Factors: Unexpected Inflation

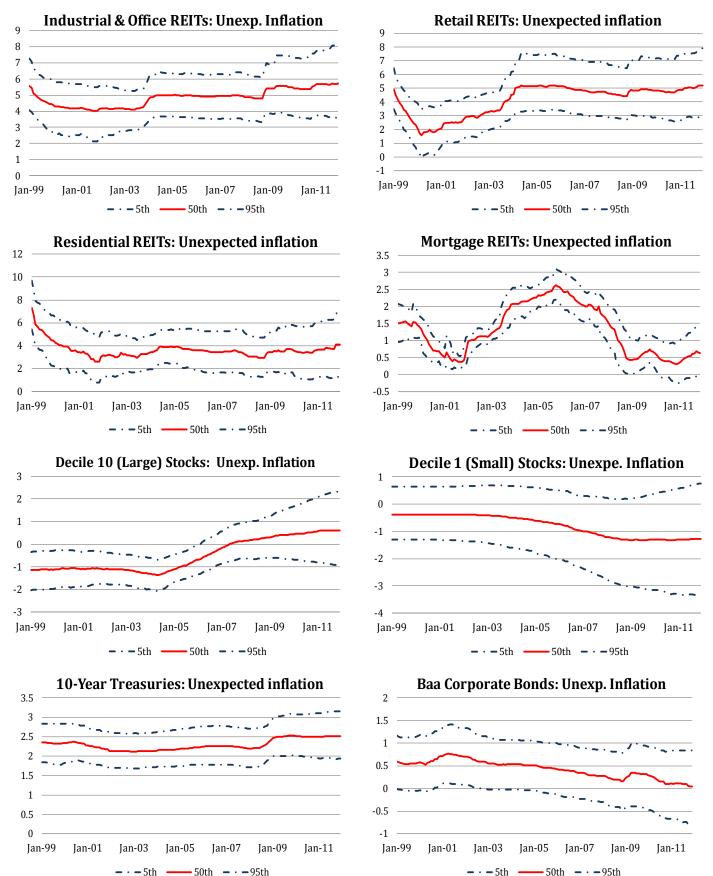


Figure 9
Subsector REIT Portfolios Loadings on Macroeconomic Factors: VW Market Portfolio

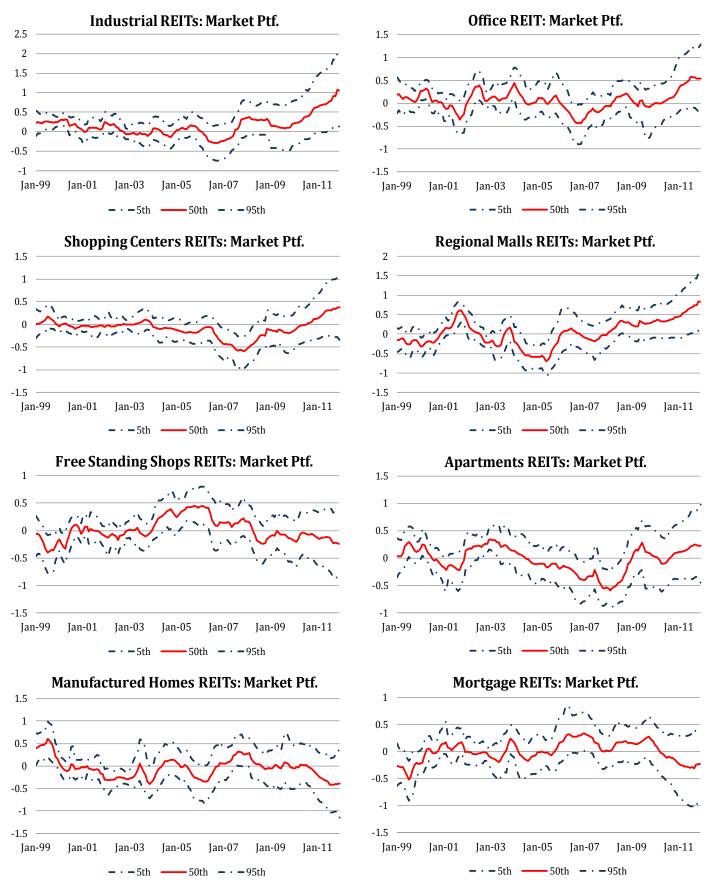


Figure 10
Subsector REIT Portfolios Loadings on Macroeconomic Factors: Real T-Bill Rate

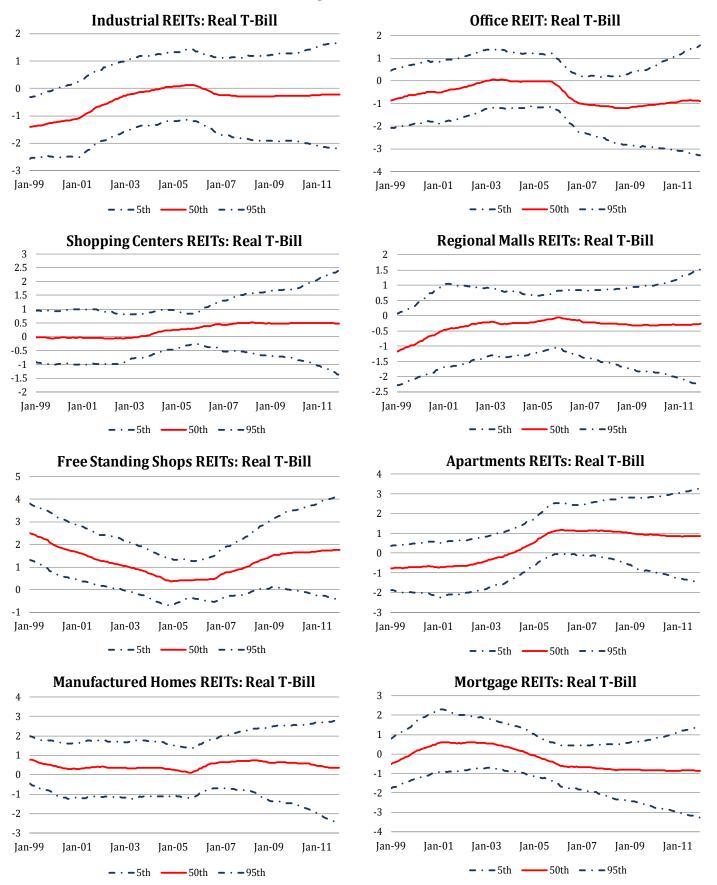


Figure 11
Subsector REIT Portfolios Loadings on Macroeconomic Factors: Unexpected Inflation

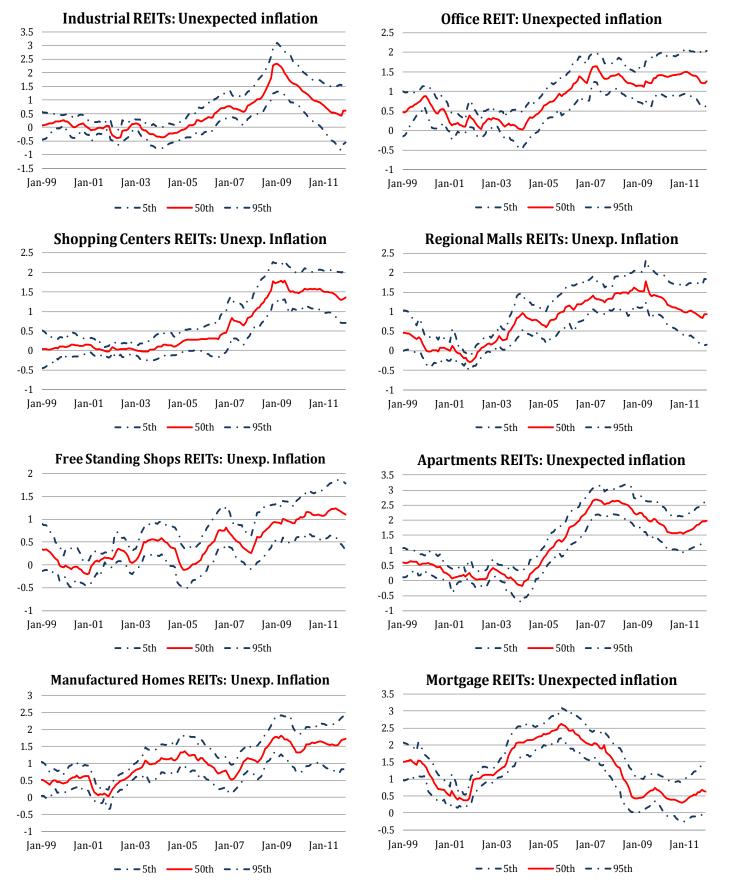


Figure 12
Bayesian (BTVSVB) Posterior Medians of the Jensen's Alpha: Sector REIT Asset Menu

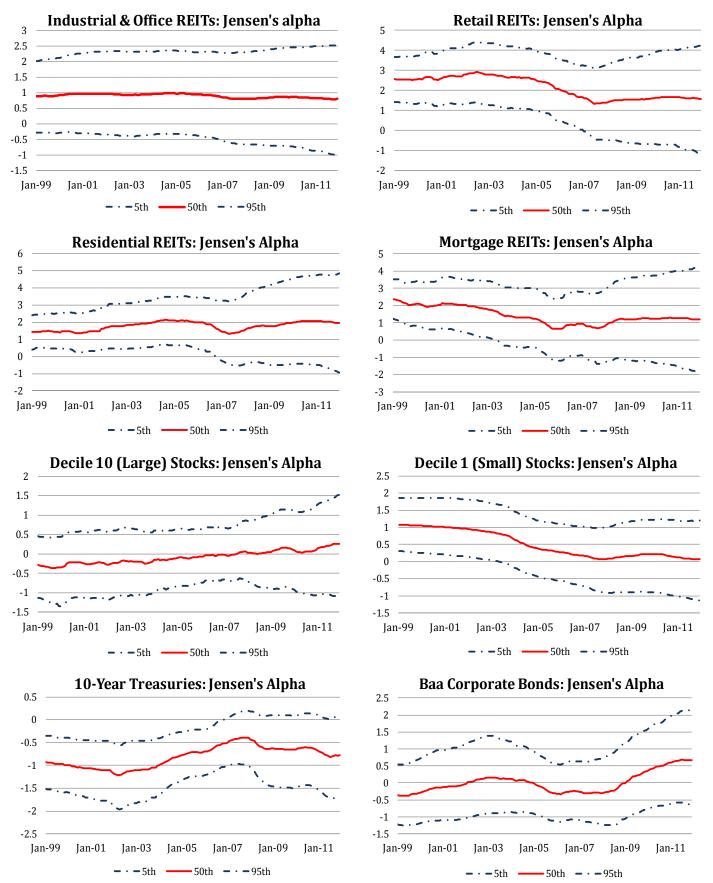


Figure 13
Bayesian (BTVSVB) Posterior Medians of the Jensen's Alpha: Subsector REIT Asset Menu

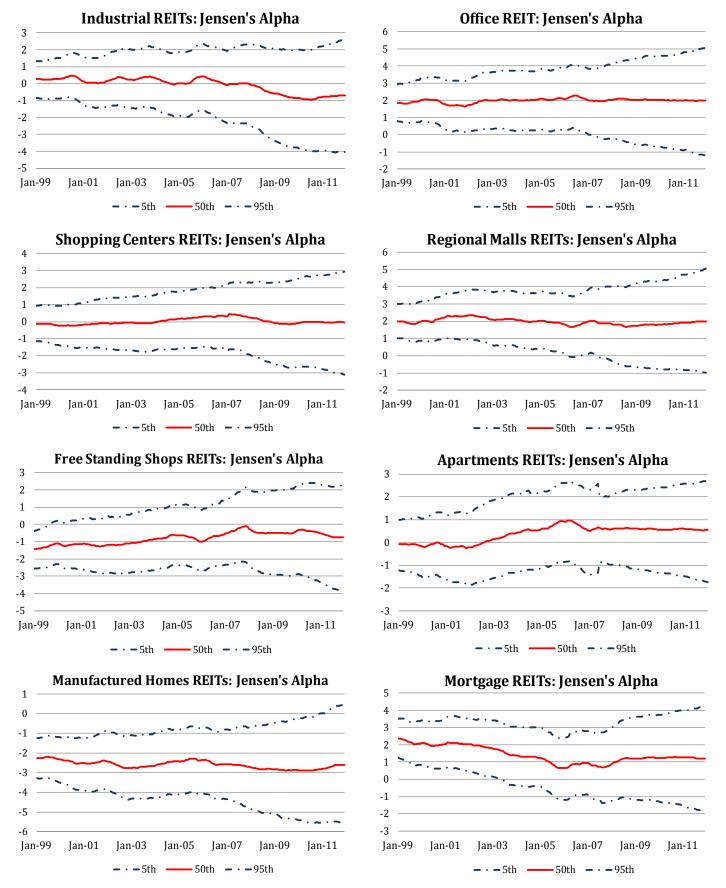


Figure 14
Selected Subsector REIT Loadings under Alternative Specifications: IP Growth
Bayesian Time-Varying Parameter Model
Fama- MacBeth 5-year Rolling Window

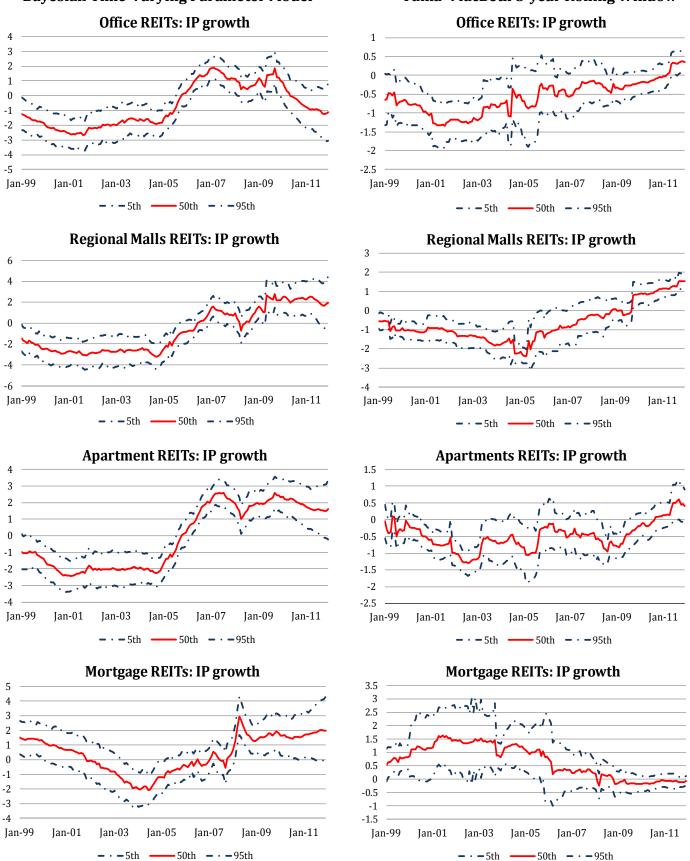


Figure 15
Selected Subsector REIT Loadings under Alternative Specifications: Unexpected Inflation
Bayesian Time-Varying Parameter Model
Fama- MacBeth 5-year Rolling Window

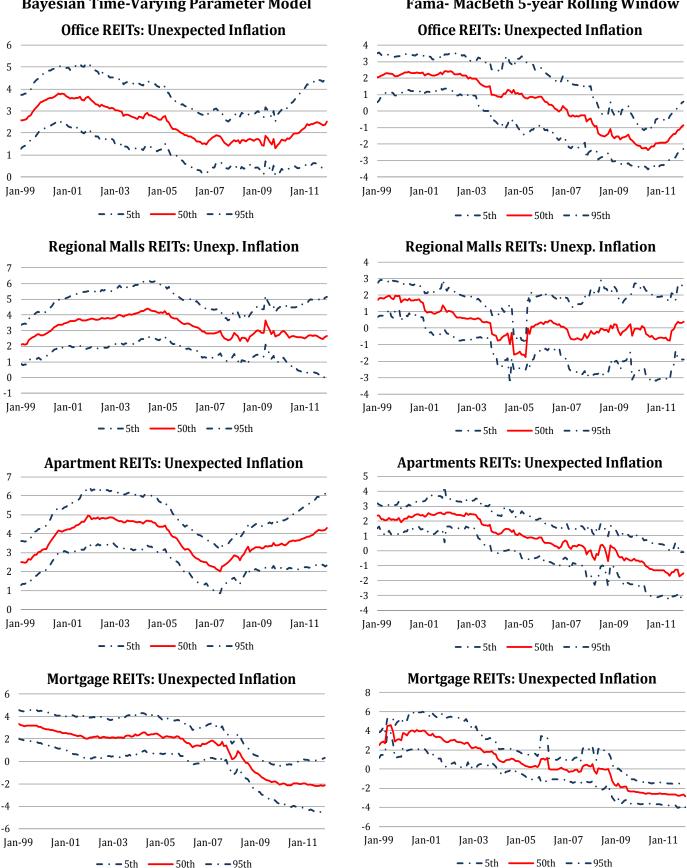


Figure 16
Subsector REIT Posterior Medians of the Jensen's Alpha under Alternative Specifications
Bayesian Time-Varying Parameter Model
Fama-MacBeth 5-year Rolling Window

