# Beyond Tax Smoothing

By

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and

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Abstract: Analyses of optimal government capital structure generally follow Bohn (1990) and Barro (1995) in assuming risk neutrality or an exogenous risk premium. These analyses usually conclude that the optimal government capital structure stabilizes tax rates over time and states of nature to the greatest extent possible, something known as "tax smoothing." In this paper, we show that when an endogenous risk premium is introduced, the optimal government capital structure will no longer smooth tax rates. Under likely conditions, the optimal structure requires a larger short position in risky assets than that implied by tax smoothing.

Key Words: tax smoothing; capital structure;

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#### 1. Introduction

If taxes are non-distortionary, the result is Ricardian equivalence - there is no unique level of government debt that is optimal and the timing of tax burdens has no impact on social welfare. If there are no transaction costs associated with securities transactions, then what is commonly called the "Miller Modigliani Theorem of Public Finance" holds as well - the optimal government capital structure is indeterminate. Papers that articulate these results include Barro (1974), Wallace (1981), Chan (1983), and Stiglitz (1983).

Should, however, taxation engender deadweight losses, the timing of taxation will generally matter. Barro (1979) shows that in a deterministic setting, if the marginal welfare loss associated with taxation is an increasing function of tax rates, then the optimal fiscal strategy is to hold tax rates constant over time – something known as "tax smoothing" or the "inter-temporal Ramsey Rule." Papers such as Lucas and Stokey (1983) and Aiyagari et.al. (2002) show that Barro's result holds even in a stochastic setting, with the caveat being that governments no longer stabilize tax rates but rather expected tax rates, which then evolve in a random walk as unexpected events alter economic conditions and spending requirements. A textbook exposition of these ideas is offered in Romer (2000). <sup>1</sup>

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<sup>&</sup>lt;sup>1</sup> Anderson and Dogonowski (2004) show that stabilization of expected tax rates will generally not be optimal when output, rather than government consumption, is stochastic.

Following this logic, Bohn (1990) and Barro (1995) demonstrate that in a stochastic setting, the optimal government capital structure is also no longer indeterminate – social welfare can be enhanced if governments establish a capital structure whose risk/return characteristics eliminate or at least minimize the need to alter tax rates over time and states of nature. Over the years, an extensive literature has developed surrounding the issue of tax smoothing and optimal government capital structure, including papers such as Kingston (1991), Zhu (1992), Chari, Christiano, and Kehoe (1994), Barro (1999), Missale (1997), Judd (1999), Angeletos (2002), Buera and Nicolini (2004), Fisher and Kingston (2004), Nosbusch (2008), Angyridis (2009), Marcet and Scott (2009) and Niemann and Pichler (forthcoming). These studies either follow Bohn (1990) in assuming the public to be risk neutral or Barro (1995) in assuming that the risk premium is exogenous.

In this paper we consider the government's choice of capital structure when the risk premium is modeled as an endogenous variable. We find that the optimal government capital structure strategy no longer stabilizes tax rates. Introduction of an endogenous risk premium alters the government's decision problem in two ways. First, it gives the government an interest in manipulating the risk premium in order to enhance the profitability of its chosen capital market position. Depending on whether the government is long or short risky assets, this may encourage the government to choose a capital structure consistent with either a lower or higher risk premium than that of a tax smoothing strategy. Second, introduction of an endogenous risk premium allows the government to attach greater weight to the minimization of deadweight losses associated with "bad" states of nature and less weight towards the reduction of deadweight losses

associated with "good" states of nature. All else equal, this encourages the government to choose a capital structure with a lower risk premium than that consistent with a tax smoothing strategy.

This paper is divided into four sections. In Section Two, we sketch out a simple two period model of the government's capital structure problem. In our model, the public is risk averse and makes expected utility maximizing portfolio decisions, taking the amount of government debt as exogenous. Symmetrically, the government chooses its asset position to maximize the public's expected utility, but treats the risk premium as exogenous. The resulting policy rule replicates the results of other analyses of the government's capital structure problem - the best policy is to choose a capital structure that stabilizes taxation to the greatest extent possible. In Section Three, we re-evaluate the government's capital structure problem, but drop the assumption that the government is a price taker as far as the risk premium is concerned. The resultant policy rule no longer attempts to stabilize tax rates. Section Four concludes the paper and evaluates the implications of our findings for government policy.

## 2. The Model with an Exogenous Risk Premium

Assume that there are many identical citizens whose objective is to maximize their expected utility, EU(C), where C is consumption and U' is positive while U" is negative. Each citizen, i, has an initial endowment of wealth, W, in the form of shares of a risky asset. In period one, citizens allocate that wealth to portfolios made up of risk

free government bonds, B, and risky assets, A. Hence, for citizen i,  $W = B_i + A_i$ . Let A and B equal the average values of  $A_i$  and  $B_i$ . A need not equal  $A_i$  since individual citizens, in addition to being able to trade risky assets and bonds with the government, are also able to trade risky assets and bonds with each other. We assume that the values of B and A are not constrained to be positive, although W must be positive. Hence, both the government as well as private citizens are able to take long or short positions in risk free bonds or in risky assets.

During the second period, citizens receive labor income, L. Their net wage is (1-t)L - D(tL), where t is the rate of taxation and D(tL) is the deadweight welfare loss associated with this taxation. Following Barro (1979), we assume that D' and D" are positive. In addition to wage income, citizens also earn interest, r, on their bond portfolio, and they earn a stochastic return, R, on their risky asset portfolio. To keep the analysis as simple as possible, we assume that no taxes are levied on r or R.

Following receipt of net wages and realization of the returns on their investments, citizens consume all of their income and wealth, hence

(1) 
$$C_i = (1-t)L - D(tL) + (1+r)B_i + (1+R)A_i.$$

The taxes raised from labor income are used to finance government spending. Let G represent government spending per capita. The government holds a portfolio of risky assets worth W-A per capita. As a result, the government's per capita bond position equals A–W. The government may choose to issue bonds and allocate the resultant funds to the purchase of a portfolio of risky assets (W-A > 0), or alternatively, the government

could sell short risky assets and buy risk free bonds issued by the private citizens (W-A < 0).

The government's budget constraint, in per capita terms, is given by:

(2) 
$$G = tL + R (W-A) - r(W-A),$$

which can be solved for t:

(3) 
$$t = [G - (W-A)(R-r)]/L.$$

The tax rate, t, is stochastic because R and G are stochastic. Using (2) and (3) and substituting W- $A_i$  for  $B_i$ , we can now express (1) as

(4) 
$$C_i = L - G + W (1+R) + (R-r)(A_i-A) - LD[G/L - (W-A)(R-r)/L].$$

We assume that the government's objective is to maximize the welfare of citizens. Hence, the government will choose the level of risky assets to hold (and risk free bonds to issue) that maximizes EU(C). Citizens seek to maximize  $EU(C_i)$  and choose the mix of risky and risk free assets that will do so.

In addition to the first order conditions for these maximizations, the equilibrium conditions for the economy are provided by the symmetry of the citizens (which assures that  $A=A_i$ ) and by conservation of assets (total holdings of assets are W and total holdings of bonds are 0). For now, we will also assume that the government as well as private citizens act as price takers.

In order to identify the values of A and  $A_i$  that maximize EU(C), we expand EU(C) around the means of R and G, which are its only stochastic elements

$$EU \approx U(C(ER,EG)) + U_{RR}\sigma_{RR}/2 + U_{GG}\sigma_{GG}/2 + U_{RG}\sigma_{RG}$$

where  $\sigma_{RR}$  and  $\sigma_{GG}$  are the variances of R and G, while  $\sigma_{RG}$  is the covariance between R and G. The terms in this approximation are given by:

(5) 
$$U_{RR} = U'' \Big[ \Big( W + A_i - A \Big) + D'(W - A) \Big]^2 + U'(-LD'' \Big( \frac{W - A}{L} \Big)^2 \Big),$$

(6) 
$$U_{GG} = U''(1+D')^2 - \frac{U'D''}{L},$$

and

(7) 
$$U_{RG} = -U''(1+D')[W+A_i-A+D'(W-A)]+U'D''\frac{W-A}{I}.$$

In order to write the first order conditions, the derivatives evaluated at the mean values of G and R are also needed. These are given by:

(8) 
$$\frac{\partial U(EG, ER)}{\partial A} = U' \left[ -(ER - r) - LD' \frac{(ER - r)}{L} \right]$$

and

(9) 
$$\frac{\partial U(EG, ER)}{\partial A_i} = U'(ER - r)$$

Now we find the FOC by taking the derivatives of (5) with respect to A<sub>i</sub> and A:

(10) 
$$\frac{\partial EU}{\partial A_{i}} = U'(ER - r) + U''[W + A_{i} - A + D'(W - A)]\sigma_{RR} - U''(1 + D')\sigma_{RG} = 0$$

And

$$\frac{\partial EU}{\partial A} = U'(-1)(1+D')(ER-r) + U''(W+A_i - A+D'(W-A))(1+D')(-1)\sigma_{RR} + U'D''(\frac{(W-A)}{L})\sigma_{RR} + \left[U''(1+D')^2 - \frac{U'D''}{L}\right]\sigma_{RG} = 0$$

By solving for ER-r in both equations, setting them equal to each other, and noting that  $A=A_i$ , we find that:

$$(12) W - A^* = \frac{\sigma_{RG}}{\sigma_{RR}}$$

The expression in (12) is the familiar minimum variance hedge, and implies that the government maximizes social welfare by adopting a capital structure that minimizes the variance in tax rates required to finance government spending. In other words, a welfare maximizing government should engage in tax smoothing – the stabilization of tax rates over different future states of nature - through the use of financial instruments. This result is identical to those offered in Bohn (1990) and Barro (1995).

Now, let us continue the analysis. To solve for the equilibrium risk premium, we use (12) to substitute out W-A\* in the FOC for  $A_i$ :

(13) 
$$ER - r = -\frac{U''}{U'} \left( W + D' \frac{\sigma_{RG}}{\sigma_{RR}} \right) \sigma_{RR}$$

If the government seeks to smooth taxation, the equilibrium risk premium is larger than if it did nothing when the covariance of R and G is positive. Similarly, the equilibrium risk premium will be lower if the covariance of R and G is negative. If the later is the case, then government efforts to stabilize tax rates involve the shorting of risky assets.

It is interesting that the equilibrium risk premium declines (rises) when private citizens increase (reduce) their aggregate holdings of risky assets. The reason for this is that the increase (decrease) in citizens' direct exposure to portfolio risk is more than offset by the opposite change in the indirect exposure to portfolio risk that citizens face as taxpayers responsible for the financing of government activities.

# 3. The Model with an Endogenous Risk Premium

Now, let us relax the assumption that the government is a price taker. Instead, consider the situation when the government explicitly takes into account the effect of its choice of capital structure on the equilibrium risk premium. This means that instead of

finding the value of A that maximizes EU(C), the government must now identify the value of A that maximizes EU(C) subject to (10):

(13) 
$$\frac{dEU}{dA} = \frac{\partial EU}{\partial A} \bigg|_{A=A_i} + \frac{\partial EU}{\partial A_i} \bigg|_{A=A_i} + \frac{\partial EU}{\partial r} \frac{dr}{dA} .$$

There are three terms on the right hand side of the expression. The first term is simply (11), while the second term reflects the fact that  $A_i$  is no longer exogenous but is now itself determined by the government since  $dA_i/dA = 1$ . The second term is equal to zero because it is the private asset pricing constraint given by (10).

In order to evaluate the third term, we derive the tradeoff between A and r that faces the government. To find the slope of the change in r with respect to a change in A, we impose  $A = A_i$ , and take the derivatives of (10):

$$\frac{\partial}{\partial A} \frac{\partial U}{\partial A_i} = -U "(ER - r)^2 D' - U " \sigma_{RR} D'$$

and

$$\frac{\partial}{\partial r}\frac{\partial U}{\partial A_{i}} = -U "D'(W - A)(ER - r) - U'.$$

Using these expressions, we find that

(14) 
$$\frac{dr}{dA} = -\frac{U''(ER - r)^2 D' + U'' \sigma_{RR} D'}{U'' D'(W - A)(ER - r) + U'}$$

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This expression is positive as long as W-A is negative - when the government takes a short position in risky assets. In this case C also increases in r. So the third term in (14) is positive. As a result, (14) will only equal zero if its first term is negative. Since the derivative of (11) with respect to A is negative, this requires A to be larger than required to maximize A when A<sub>i</sub> is taken as given. In other words, when the government takes a short position and treats the risk premium as endogenous, citizens will hold more risky assets than when the government is treating the risk premium as exogenous. Since the policy rule with an exogenous risk premium is to stabilize tax rates, the government's capital structure will be "shorter" than that implied by tax smoothing.

A more complicated situation arises when W-A is positive and the government has taken a long position in risky assets. In that case, the sign of dr/dA is ambiguous. If W-A is sufficiently large, dr/dA will be positive. As a result, the value of A required for (14) to equal zero will be smaller than that required to stabilize tax rates and the government's capital structure will be "longer" than that implied by tax smoothing. For sufficiently small values of W-A, however, dr/dA will be negative. In that case, the value of A required is larger and the government's capital market position is "less long" than that implied by tax smoothing.

These results illustrate the interplay of two separate effects. The first stems from the government's desire to enhance the profitability of its capital market position. By increasing the profitability of its investments, the government can lower taxation, and hence lower the deadweight losses associated with that taxation.

If the government is planning to sell short risky assets, it can increase its profits by lowering the risk premium. This raises the valuation of the risky assets that the

government is selling. If, on the other hand, the government is planning to purchase risky assets, profits are increased by raising the risk premium. This lowers the valuation of the assets that the government is buying. Since the risk premium declines when the government takes a short position in risky assets and rises when the government takes a long position in risky assets, this effect leads a government that is short selling to take a larger short position than it otherwise would, while it leads a government that is taking a long position in risky assets to take a larger long position than it otherwise would.

The second effect relates to the impact of deadweight losses over different future states of nature. As a result of risk aversion, the deadweight losses stemming from taxation will cause more damage to social welfare should future realized states of nature turn out to be "bad." As a result, the optimal government capital structure attaches greater weight to the minimization of deadweight losses associated with bad states of nature and less weight towards the reduction of deadweight losses associated with good states of nature. This leads the government to always take a position that is "shorter" than that consistent with the stabilization of tax rates, and the government's capital structure leads to tax reductions in the event of bad states of nature while requiring tax hikes in the event of good states of nature.

When the government is taking a short position, both of the effects lead it to choose a short position even larger than that implied by tax stabilization. Hence, dr/dA is unambiguously negative when W-A is negative. When the government is taking a long position, however, the desire to maximize profits leads to a longer capital market position, while the desire to minimize deadweight losses in bad states of nature lead to a shorter position. Overall, the result is that the sign of dr/dA is ambiguous and depends

critically on the value of W-A. If W-A is sufficiently large, the importance of increasing the profitability of the government capital market position outweighs the importance of minimizing deadweight losses in bad states of nature – hence, dr/dA is positive. If the government's choice of W-A is small, however, profits on its capital market position will play a relatively small role in determining the optimal capital structure, and dr/dA is negative.

### 4. Conclusion

In this paper, we demonstrated that in an economy with an endogenous risk premium, a government seeking to enhance social welfare will not choose a capital structure that stabilizes tax rates over time and states of nature. Rather, the government will take a larger short position than that implied by tax smoothing when the return on risky assets is negatively correlated with fluctuations in government spending. When the correlation between risky asset returns and spending is positive, however, the socially optimal capital structure may require a greater or smaller long position in risky assets when compared to the position consistent with tax smoothing.

Papers such as Bohn (1990), Buera and Nicolini (2004), Missale (1997), and Faraglia et. al. (2006), estimate the capital structure required to stabilize tax rates and generally find that tax smoothing requires short positions in risky assets. This makes sense, given that events likely to cause positive shocks in government spending include cyclical downturns, natural disasters, and wars. Given our analysis, tax stabilizing government capital structures will not be "short enough" to maximize expected social

welfare. In other words, a government seeking to maximize social welfare should go "beyond tax smoothing" – hence the title of this paper.

In practice, does this matter? As long as governments are limited to the use of traditional instruments such as nominal debt of various maturities, the answer is probably not. Government capital structures required to achieve tax stabilization involve short positions in nominal debt *several hundred times* the size of GDP. Since a position that large is clearly not feasible, any suggestion that the optimal short position is even larger than that seems to be of no consequence.

There are, however, some policies that may help a government achieve the capital structure consistent with social welfare optimization. One possibility is to aggressively privatize state owned firms.<sup>2</sup> A second alternative is to issue GDP linked debt. Papers such as Borensztein et. al. (2004) and Kamstra and Shiller (2009) have argued that issuance of reasonable quantities of such securities could greatly expedite efforts to achieve "short" government capital structures.

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<sup>&</sup>lt;sup>2</sup> See Berck et. al. (2006) for an analysis of the role that state owned firms play in contributing to a government capital structure that is at cross-purposes with a strategy of tax stabilization.

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