A Model of Discovery

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Abstract: Empirical research has reached the puzzling conclusion that stronger patents do little or nothing to encourage innovation. We show that the facts that have led to the assumption of fixed cost in the discovery process can be equally well explained by a standard model of diminishing returns. This may explain much of the misunderstanding of the (supposedly positive) role of monopoly in innovation and growth, thereby accounting for the empirical puzzle.

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1. Introduction

There is an emerging body of evidence (Bessen and Meurer [2008], Lerner [2008] and Mokyr [2008], for example) that patents do not have much impact on innovation. It is true that standard models of capital ladders such as Scotchmer [1991], Boldrin and Levine [2004], and Llanes and Trento [2008] allow for the possibility of patents discouraging innovation. However this can only be a long-run consequence: innovation is discouraged when so many patents have been created that additional innovation becomes heavily dependent on them. The evidence in Lerner [2008], among other, shows that even in the short-run there is no increase in innovation from strengthening patent protection. From the perspective of existing theory this is a puzzle.

The standard theory of innovation is essentially a static theory: incurring a fixed cost creates a unit of knowledge that is then available to all for free. So-called Schumpeterian models of knowledge creation, such as those of Aghion and Howitt [1992], while ostensibly dynamic, simply stack a sequence of these static models end-to-end.

This standard approach rests on two technological assumptions: fixed cost of creation and free availability of newly created knowledge. In earlier work (Boldrin and Levine [2002, 2008b]) we challenged the latter assumption, and we discussed the evidence against it extensively in Boldrin and Levine [2008a]. However, like the rest of the profession, until recently we accepted the conventional wisdom that, following the expense of a fixed cost, knowledge springs full-grown in the form of an "eureka moment". The goal of this paper is to challenge such wisdom and to argue that diminishing returns in knowledge creation is at least as plausible – and has profoundly different implications for the impact of monopoly on innovation. The implications for the facts that motivated the standard fixed cost theory are the same as for that theory. However, from the perspective of a theory of diminishing returns in knowledge creation the failure of government grants of monopoly to increase innovations is not a puzzle at all – it is one of the main predictions of the theory.

What is the evidence that knowledge arises in "eureka moments?" It is true that innovators work on new ideas for some period of time before marketing them or implementing them in new products. It is also true that, after ideas are brought to market,

the process of creating original knowledge is replaced by the process of making cheap copies. Superficially this suggests that ideas have little value until they reach "fruition", after which they are revealed and cheaply copied. Our goal is to show that these same facts are consistent with a diminishing returns technology for the creation of new ideas. The key intuition is that even with diminishing returns and perfect divisibility, the first few shards of new knowledge – the unfinished notes, the dead-ends that have been encountered, the computer program with many bugs – have enormous value in the process of further knowledge creation. This means that even if incomplete or imperfectly polished products are valued positively by consumers, it is optimal to keep them off the market for an initial period of time.

Our story is this. Knowledge is encapsulated in perfectly divisible blueprints. In general, original knowledge creation generates new blueprints from existing blueprints and labor. Initially, though, labor alone is used as no (copies of) original knowledge actually exist; implying that the technology for producing original knowledge must allow for positive output from labor alone. It is also natural to think that initially the first few bits of knowledge are much more useful in the production of additional new knowledge than in consumption or in being spread around by making copies of themselves. In such a world, the social optimum is to use labor and all the knowledge acquired so far to increase the amount of new knowledge for a time. Eventually diminishing returns to the production of new knowledge sets in: it becomes optimal to produce cheap copies and consume them; at this point the original knowledge creation process is phased out as the idea is "complete" and it comes to "fruition". If we treated new knowledge creation as a black box and thought of a static model in which "usable" knowledge jumped full-grown out of the box, this would look like a "fixed cost plus cheap reproduction" model. Yet probing the black box, we see that, in fact, this is a standard diminishing returns economy and has no increasing returns to scale.

In the fixed cost plus cheap reproduction story, there is no cumulative process of knowledge creation: there are "eureka moments" in which a fully usable piece of knowledge appears. However, knowledge creation does not generally proceed in that way. We do not generally write a finished first chapter first: we write an outline of the whole book, a sketch of each chapter, and so forth. Then we begin the process of revision and polishing until we get to the complete product. In doing this we go back to what we

did before and change it, producing additional bits of usable knowledge in an almost seamless path. So it is far from clear that two copies of an incomplete discovery – the notes containing the intuition, the first few experiments, and so forth – would be worth less to consumers than one copy of the complete one. But regardless, if we take the extreme view that there is an indivisibility – that the "discovery" is worthless until it is available as an entire unit – its indivisibility may not matter simply because the "idea" was not going to be used anyway until that point.

In terms of patents: the implication is that for those products and industries for which diminishing returns is the relevant model, government awarded monopolies, although they will be valued by those getting the monopolies, will strictly reduce welfare. Worse, while monopolists may bring their product to market earlier, they will do so by skimping on research and development. In this setting individual patents will reduce, rather than increase, innovation.

2. A Model of Creation and Copying

We take time $0 \leq t < \infty$ to be continuous. We consider the market for a new product that did not previously exist. Consumers derive utility from consuming x_t units of the product, and provide labor , ℓ_t, according to the discounted present value

$$\int_0^\infty e^{-\rho t}[u(x_t) - w\ell_t]dt\,.$$

We assume that u is strictly increasing, finite, strictly concave, and twice continuously differentiable for $u(x_t) < \sup u(\cdot)$. Since the good is not aggregate consumption, but rather a new product, we think it is natural to assume that $u'(0) < \infty$ with $\lim_{x_t \to \infty} u'(x_t) = 0$. Labor is in limited supply with $\ell_t \leq L$.

In the model, knowledge is encapsulated in blueprints $k_t \geq 0$. Initially there are no blueprints, so $k_0 = 0$. However, there is an original knowledge creation technology for creating blueprints from labor and blueprints. The simplest and most traditional method of combining capital and labor might seem the Cobb-Douglas production function. However, the Cobb-Douglas cannot produce any output if the capital input is zero, so we use a perturbed Cobb-Douglas technology to express the fact that new knowledge can be created from labor alone

$$\dot{k}_{ot} = A({k_{ot}}^{\alpha} + \eta){\ell_t}^{\beta}$$

where $A > 0, \alpha, \beta > 0, \alpha + \beta < 1, \eta > 0$, and $k_{ot} \geq 0, \ell_t \geq 0$ are the blueprints and labor used in creating original knowledge. Notice that once labor is allocated to the original creation process, the marginal product of blueprints is very high (infinite at $k_o = 0$). This is the opposite of the usual assumption. In standard theory it is assumed that nothing of value is produced until a threshold is reached. By contrast we assume that the very few first bits of knowledge – incomplete sketches, intuitions, and so forth – are extremely valuable in the production of additional original knowledge. The apple contributed great value to Newton's understanding of gravitation.

We assume there is also a technology for the inexpensive copying, or imitation, of blueprints. This is given by $\dot{k}_{ct} = Bk_{ct}$, where $B > \rho$ and k_{ct} are the blueprints used for making copies. This is like a copying machine, or the competitive market: put in an original or copy, and some number B of new copies are produced. As it is often the case, original knowledge and its imitations are perfect substitutes.

Lastly, blueprints can be used to produce a flow of consumption. It is convenient to assume that the units are chosen such that $x_t = k_{xt}$ where k_{xt} are the blueprints used to produce consumption.

We can summarize the accumulation technology by the equation of motion for the total amount of blueprints available

$$\dot{k}_t = A({k_{ot}}^{\alpha} + \eta){\ell_t}^{\beta} + B(k_t - x_t - k_{ot})$$

along with the constraints $k_{ot}, \ell_t, x_t \geq 0, k_t - x_t - k_{ot} \geq 0$. Notice that the rate of increase of knowledge capital is bounded by

$$\dot{k}_t \leq \max\{A({k_t}^{\alpha} + \eta)L^{\beta}, Bk_t\},$$

hence there is a function $K(t)$ such that $k(t) \leq K(t)$. We assume that utility is finite in the sense that[2]

$$\int_0^{\infty} e^{-\rho t} u(K_t) dt < \infty .$$

An example of a function satisfying our assumptions is $u(x_t) = 1 - e^{-x_t}$.

3. Characterizing the Optimal Plan

We first give a technical characterization of the optimal plan.

[2] Note that this forces $\lim_{x_t \to \infty} u'(x_t) = 0$ so that assumption is redundant.

Proposition 1: *A unique continuous optimal plan* $\{k_t, k_{ot}, \ell_t, x_t\}$ *exists and is characterized by Lagrange multipliers* $\lambda_t \geq 0$ *that evolve according to*

$$\dot{\lambda}_t = \lambda_t \left(\rho - \left[A\alpha {k_{ot}}^{\alpha-1} {\ell_t}^{\beta} \right] \right)$$

and satisfy the transversality condition $e^{-\rho t}\lambda_t k_t \to 0$*, and by first order conditions*

$$\lambda_t A\beta({k_{ot}}^{\alpha} + \eta){\ell_t}^{\beta-1} \geq w \text{, with equality if } \ell_t < L$$

$$u'(x_t) \leq \lambda_t \left[A\alpha {k_{ot}}^{\alpha-1} {\ell_t}^{\beta} \right] \text{, with equality if } x_t > 0$$

$$B \leq \left[A\alpha {k_{ot}}^{\alpha-1} {\ell_t}^{\beta} \right] \text{, with equality if } k_t - x_t - k_{ot} > 0$$

Proof: See the Appendix.

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Our primary goal is to understand how the technology for original creation interacts with the copying technology and with consumption. Our key result is that, initially, only the original creation technology is used and, for a while, blueprints are not used either for copying or consumption.

Proposition 2*: There is a time* $T > 0$ *such that if* $0 \leq t \leq T$ *then in the optimal plan has* $k_{ct}, k_{xt} = 0$.

Proof: From the first order conditions and $u'(x_t) \leq u'(0)$, if

(*) $$\lambda_t A\alpha {k_{ot}}^{\alpha-1} {\ell_t}^{\beta} > u'(0), A\alpha {k_{ot}}^{\alpha-1} {\ell_t}^{\beta} > B$$

then the optimal plan is $k_{ct}, k_{xt} = 0$. In particular, since the optimal plan is continuous, it suffices to prove that these inequalities hold as $k_{ot} \to 0$. Notice that, by continuity, $\lambda_t \to \lambda_0 > 0$. If $\ell_0 = L$ the result follows immediately. Otherwise we may solve the first order condition for the optimal use of labor,

$$\ell_t = \left[\frac{\lambda_t A\beta({k_{ot}}^{\alpha} + \eta)}{w} \right]^{\frac{1}{1-\beta}}$$

and substitute it into (*) to find the conditions

$$\lambda_t^{\frac{1}{1-\beta}} A\alpha {k_{ot}}^{\alpha-1} \left[\frac{A\beta({k_{ot}}^{\alpha} + \eta)}{w} \right]^{\frac{\beta}{1-\beta}} > u'(0)$$

$$\lambda_t^{\frac{\beta}{1-\beta}} A\alpha k_{ot}^{\alpha-1}\left[\frac{A\beta(k_{ot}^{\alpha}+\eta)}{w}\right]^{\frac{\beta}{1-\beta}} > B,$$

from which the result again follows as $k_{ot} \to 0$ and $\lambda_t \to \lambda_0 > 0$.

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Our second result shows that the use of the original creation technology is temporary in the sense that asymptotically it is not used at all, and after a point in time, some capital is always used for consumption.

Proposition 3: *In the optimal plan:* λ_t *is decreasing,* $\lim_{t\to\infty}\lambda_t = 0$, x_t *is increasing,* $\lim_{t\to\infty} u'(x_t) = 0$, *and* $\lim_{t\to\infty} k_{ot}, \ell_t \to 0$.

Proof: Observe that $\dot{\lambda}_t = \lambda_t\left(\rho - \left[A\alpha k_{ot}^{\alpha-1}\ell_t^{\beta}\right]\right)$ and $B \le \left[A\alpha k_{ot}^{\alpha-1}\ell_t^{\beta}\right]$ imply $\dot{\lambda}_t / \lambda_t \le \rho - B < 0$. Notice that $\lambda_\tau < \infty$ for $\tau > 0$. Hence not only is λ_t decreasing, but by integrating both sides of the inequality, it satisfies a bound of the form $\lambda_t \le \lambda_\tau e^{-(B-\rho)(t-\tau)}$, so certainly $\lim_{t\to\infty}\lambda_t = 0$.

Next suppose for $s > t$ that $x_s < x_t$. The labor supply can be solved from the first order condition as

$$\ell(\lambda_t, k_{ot}) = \min\left\{L, \left[\frac{\lambda_t A\beta(k_{ot}^{\alpha}+\eta)}{w}\right]^{\frac{1}{1-\beta}}\right\},$$

from which

$$A\alpha k_{ot}^{\alpha-1}\ell(\lambda_t, k_{ot})^{\beta} =$$

$$A\alpha \min\left\{k_{ot}^{\alpha-1}L^{\beta}, \left[\frac{A\beta\left(k_{ot}^{\frac{\alpha+\beta-1}{\beta}} + \eta k_{ot}^{(1-\beta)(\alpha-1)}\right)}{w}\right]^{\frac{\beta}{1-\beta}} \lambda^{\frac{\beta}{1-\beta}}\right\}$$

This is increasing in λ_t and decreasing in k_{ot}. Since $x_t > 0$ from the first order conditions $u'(x_t) = \lambda_t\left[A\alpha k_{ot}^{\alpha-1}\ell(\lambda_t, k_{ot})^{\beta}\right]$. Since $x_s < x_t$ and since λ_t is decreasing it follows from $u'(x_s) \le \lambda_s\left[A\alpha k_{os}^{\alpha-1}\ell(\lambda_s, k_{os})^{\beta}\right]$ that

$$A\alpha k_{os}^{\alpha-1}\ell(\lambda_s, k_{os})^{\beta} > A\alpha k_{ot}^{\alpha-1}\ell(\lambda_t, k_{ot})^{\beta}$$

and so $k_{cs} > k_{ct} \ge 0$. This implies

$$B = A\alpha k_{os}{}^{\alpha-1}\ell(\lambda_s, k_{os})^{\beta} > A\alpha k_{ot}{}^{\alpha-1}\ell(\lambda_t, k_{ot})^{\beta} \geq B\,,$$

a contradiction.

Next suppose that $u'(x_t)$ is bounded away from zero, say by $\overline{u}'$. From $\dot{\lambda}_t = \lambda_t\left(\rho - \left[A\alpha k_{ot}{}^{\alpha-1}\ell_t{}^{\beta}\right]\right)$ and $u'(x_t) \leq \lambda_t\left[A\alpha k_{ot}{}^{\alpha-1}\ell_t{}^{\beta}\right]$ we have $\dot{\lambda}_t \leq \rho\lambda_t - \overline{u}'$. Since $\lim_{t\to\infty}\lambda_t = 0$, eventually $\lambda_t < \overline{u}'/\rho$ implying that $\lambda_t < 0$ after a certain time, which is impossible.

Next observe that $B \leq A\alpha k_{ot}{}^{\alpha-1}L^{\beta}$ implies an upper bound on k_{ot}. Since $\lambda_t \to 0$ and k_t is bounded above we see that this implies $\ell_t \to 0$. Then the first order condition $B \leq A\alpha k_{ot}{}^{\alpha-1}\ell_t{}^{\beta}$ also implies $k_{ot} \to 0$.

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The role of the copying technology is less straightforward, in accordance with available evidence. If marginal utility, hence demand, falls rapidly to zero, it may be that the copying technology is never used and knowledge goes directly from the original creator to the consumers, without imitators stepping in to copy. We would expect copying to be relevant when there is strong demand for a large number of copies of the consumption good. Specifically, let us define strong asymptotic demand.

Definition 4: *We say that demand is strong asymptotically if* $u'(x_t) \geq we^{-wx_t}$.

An example of a utility function with asymptotically strong demand is $u(x_t) = 1 - e^{-x_t}$. We can then show that, while asymptotically the original creation technology is abandoned, the copying technology is used – that is, when demand is strong, after a time copying takes over the discovery process and imitators enter the market.

Proposition 5: *If demand is asymptotically strong, then in the optimal plan* $\limsup k_{ct} > 0$.

Proof: As before, we observe from the first order conditions that $\dot{\lambda}_t \geq (\rho - B)\lambda_t$ and that, for $\gamma = B - \rho$ and $G = \lambda_\tau e^{\tau\gamma}$, this implies that $\lambda_t \leq Ge^{-\gamma t}$.

Next observe that since $x_t > 0$ for large t we have, for all sufficiently large t, that $\dot{\lambda}_t = \rho\lambda_t - u'(x_t)$. Define

$$U_t \equiv \lambda_t - \int_0^{\infty} e^{-\rho(s-t)}u'(x_{t+s})ds\,.$$

Since

$$0 \le \int_0^\infty e^{-\rho(s-t)} u'(x_{t+s}) ds \le u'(0)/\rho$$

and $\lambda_t \to 0$, we see that U_t is bounded asymptotically. Since $\dot{U}_t = \rho U_t$, it follows that $U_t = 0$.

Combining these two steps, we see that for t large enough we can write $\lambda_t \le Ge^{-\gamma t}$ as

$$\int_0^\infty e^{-\rho(s-t)} u'(x_{t+s}) ds \le Ge^{-\gamma t} .$$

Consequently, for any $T > 0$

$$\int_0^\infty e^{-\rho s} u'(x_{t+s}) ds \le Ge^{-\gamma t}$$

$$\int_0^T e^{-\rho s} u'(x_{t+s}) ds \le Ge^{-\gamma t}$$

$$e^{-\rho T} \int_0^T u'(x_{t+s}) ds \le Ge^{-\gamma t}$$

$$\int_0^T u'(x_{t+s}) ds \le Ge^{\rho T - \gamma t}$$

$$\min_{0 \le s \le T} u'(x_{t+s}) \le \frac{Ge^{\rho T - \gamma t}}{T}$$

$$u'(\max_{0 \le s \le T} x_{t+s}) \le \frac{Ge^{\rho T - \gamma t}}{T}$$

$$u'(k_{t+T}) \le \frac{Ge^{\rho T - \gamma t}}{T}$$

By assumption $u'(k_{t+T}) \ge we^{-wk_{t+T}}$, so

$$wTe^{-wk_{t+T}} \le Ge^{\rho T - \gamma t} .$$

Taking logs

$$k_{t+T} \ge -\frac{\log(G/wT) + \rho T}{w} + \frac{\gamma}{w} t .$$

Since $k_{ot}, \ell_t \to 0$ we must have $\dot{k}_{ot} \to 0$, hence $\limsup \dot{k}_{ct} > 0$, implying $\limsup k_{ct} > 0$.

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4. Impact of Monopoly on Innovation

Suppose that a single monopolist controls the market for this product, and that she maximizes the present value of her profits. This amounts to replacing $u(x_t)$ with $v^R(x_t) = u'(x_t)x_t$ in the optimization problem. Notice that revenue $v^R(x_t)$ may actually be decreasing for large x_t and may fail to be concave. Assuming that it is concave and that $u(x_t)$ is three times continuously differentiable, we may define $v(x_t) = \max_{x_s \leq x_t} v^R(x_s)$. Then $v(x_t)$ satisfies the properties we have assumed of a utility function, and since x_t will never be chosen so large that $v(x_t) \neq v^R(x_t)$ yields the solution to the monopoly problem. The key difference with the original problem is that $v(x_t)$ has a lower marginal utility of consumption than $u(x_t)$ at all levels of $x_t > 0$, because $v'(x_t) = u'(x_t) + u''(x_t)x_t$.

We can develop some simple intuition about the impact of introducing a monopoly on the timing and nature of innovation. Use superscripts u and v to denote, respectively, the solution to the competitive and the monopolistic problem. Let T be the time for which $x_t^u > 0$ for $t > T$ and $x_t^u = 0$ for $t \leq T$. Notice that

$$\lambda_T^v = \int_0^\infty e^{-\rho(t-T)} v'(x_{T+t}^v) dt .$$

If this remains unchanged the monopoly solution, x_t^v, must involve less final product sales than under competition, x_t^u for $t \geq T$. Therefore, there must be less capital under monopoly than competition, $k_T^v < k_T^u$. There are two ways to produce less capital and the optimum will require both be used. First, less labor should be used in the original creation process $\ell^v < \ell^u$ for $t \leq T$. Second, x^v should become positive earlier than x^u.

One could say that, in this sense, "innovation takes place more quickly" under monopoly than competition because the new consumption good is brought to the consumers earlier. However, a smaller quantity is sold and at a higher price; further, less labor is used in original creation, $\ell^v < \ell^u$, meaning that the monopolist also produces less original knowledge and does less "R&D" than under competition. In this, more relevant, sense the monopolist innovates less than under competition.

One way to think of this is in terms of the "public-private partnership" under which universities are encouraged to patent ideas developed using government funding. By awarding a monopoly we would expect less actual research to be done at universities, but the results of the research that did take place would be made available to industry

sooner. It is claimed that the "public-private partnership" has been a great success because of the latter. In this model, that is unambiguously bad, as scientific resources (k) are misallocated to industrial applications when it would be better, from a social point of view, to use them in producing more original research that would, optimally, be brought to industrial fruition somewhat later.

5. Conclusion

The standard theory of innovation based on the "fixed cost of discovery plus cheap copying" predicts that strengthening patents should increase innovation. Overwhelming empirical evidence shows that strengthening patents does little, or nothing, positive to innovation, which is most puzzling for standard theory. We argue that, even if it "looks like" there is a fixed cost of creation, in fact there is none. We argue instead that the discovery activity is best represented by a decreasing returns technology in which the first few units of new knowledge are so valuable that, for a while, they are optimally invested in producing further knowledge instead of making copies of themselves or producing consumption.

We develop a model that captures this intuition and is consistent with a set of widely held facts about innovative activity. This is a model of competitive discovery with copying under decreasing returns to scale; in this world, introducing a patent is damaging to welfare: it does not increase the rate of innovation and may even reduce it.

The widely discussed puzzle, according to which stronger patents do not increase innovative activity, is no longer a puzzle.

Appendix: Proposition 1

The result is relatively standard, and can be derived by defining $V(k_t)$ to the sup of utility achievable with continuous paths of the controls starting at the initial capital k_t. This is strictly increasing and strictly concave, so quite well behaved from a differential point of view. We can derive the optimal conditions by solving

$$\frac{(1-e^{-\rho\tau})V(k_t)}{\tau} = \lim_{\tau\to 0}\frac{\max_{x_t,k_{ot},k_{ct},\ell_t} u(x) + e^{-\rho\tau}\left[V(k_t+\tau\dot{k}_t) - V(k_t)\right]}{\tau}$$

subject to $\dot{k}_t = A({k_{ot}}^{\alpha} + \eta){\ell_t}^{\beta} + B(k_t - x_t - k_{ot})$. The Lagrange multipliers (or costate variables) are just $\lambda_t = V'(k_t)$.

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