

Collateral, Financial Intermediation, and the Distribution of Debt Capacity*

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Abstract

To seize investment opportunities due to temporarily low asset prices, borrowers may optimally conserve debt capacity. However, debt capacity is limited when financing is subject to collateral constraints due to limited enforcement. Borrowers may choose to exhaust their debt capacity and thus may be unable to take advantage of such opportunities, even if they can arrange for loan commitments or hedge financing needs. The cost of conserving debt capacity is the opportunity cost of foregone investment, which is higher for more productive and less well capitalized borrowers. Such borrowers may hence exhaust their debt capacity and may be forced to downsize when investment opportunities arise but cash flows are low. Thus, capital may be less productively deployed then. Higher collateralizability may make the contraction more severe. Financial intermediaries, who are better able to collateralize claims, require capital and, in such times, intermediary capital may be scarce and spreads between intermediated and direct finance high, forcing borrowers who exhaust their debt capacity to downsize by even more.

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1 Introduction

When asset prices are temporarily low, investment opportunities arise. In order to be able to take advantage of these, borrowers must either have funds available or be able to raise financing. We study whether borrowers optimally conserve debt capacity to take advantage of such opportunities when financing is subject to collateral constraints due to limited enforcement. We find that borrowers may exhaust their debt capacity and hence be unable to exploit opportunities that arise, even if they can arrange for loan commitments or hedge financing needs and contracting is constrained efficient. Conserving debt capacity has a cost: it reduces earlier investment. Our first main finding regards the distribution of debt capacity: borrowers who are more productive may exhaust their debt capacity, since the opportunity cost of conserving debt capacity is too high for them, while less productive borrowers conserve debt capacity. Our second main finding regards the dynamics of debt capacity: more productive borrowers are likely more constrained and may downsize when asset prices and cash flows are low. In contrast, less productive borrowers are able to use their free debt capacity in such times to expand. This implies that capital may be less productively deployed on average in such times. In addition, the availability of internal funds affects the distribution of debt capacity: borrowers with less internal funds exhaust their debt capacity, rendering them unable to seize investment opportunities due to low asset prices, while borrowers with more internal funds conserve some of their debt capacity, allowing them to seize opportunities. The third main result regards corporate risk management: our model implies that the more constrained firms hedge less, because the financing needs for investment override hedging concerns. This is consistent with the evidence that smaller firms, which are likely more financially constrained, hedge less. This fact is considered a puzzle in the literature, since models, which take up front investment as given, predict that more financially constrained firms are effectively more risk averse and hedge more (see, for example, Froot, Scharfstein, and Stein (1993)).

We also consider the role of financial intermediaries, which are modeled as lenders who are better able to collateralize claims but have limited capital. In the model, borrowers are able to obtain collateralized loans from both lenders directly as well as through the financial intermediaries. When financial intermediary capital is scarce, intermediated finance is more expensive than direct finance, that is, the spread between intermediated finance and direct finance is positive. The cross-sectional capital structure implication is that the more productive and more constrained borrowers borrow from intermediaries. Our model allows the analysis of the dynamics of intermediary capital and the spread

between intermediated finance and direct finance.¹ Our fourth main result regards the effect of this spread on borrowers: if spreads are high when asset prices are temporarily low, which is the case if loan demand is sufficiently high in such times, then borrowers which have exhausted their debt capacity may be forced to downsize by more than they otherwise would. Indeed, they downsize for two reasons: first, because cash flows are low, and second, because intermediated finance becomes more expensive. Importantly in our model both borrowers and financial intermediaries are able to enter into contracts contingent on all states, that is contracting is complete. The only friction in our model is limited enforcement. Hence, we do not make an assumption that aggregate states are not contractible, in contrast to most of the literature.

The model has several additional implications that are worth noting: First, the model with limited enforcement implies that borrowers can borrow in a state-contingent way and that borrowing against each state is limited by the collateral value in that state. This allows us to be precise about the meaning of *debt capacity* and to show that debt capacity is endogenous and jointly determined with investment. Second, we show that attention can be restricted to one period state-contingent debt in our model, and there is no additional role for long term debt. Third, we show that borrowers can conserve debt capacity in a state-contingent way by taking out loan commitments. Thus, loan commitments are a practical implementation of the contracts predicted by our model. Fourth, we show that when the collateralizability increases, which we interpret as financial innovation, the effects analyzed here may become more important, that is, the contraction of borrowers who exhaust their debt capacity may be more severe. Finally, the minimum down payment requirements, or “lending standards,” in our model vary endogenously with expected capital appreciation. When the price of capital is expected to rise, down payment requirements are low, and vice versa when the price of capital is expected to decline. This prediction is empirically plausible and consistent with anecdotal evidence.

The paper provides two main theoretical results. First, we endogenize collateral constraints similar to the ones in Kiyotaki and Moore (1997) in an economy with limited contract enforcement in the spirit of Kehoe and Levine (1993, 2001, 2006). We assume that borrowers have limited commitment and can default on their promises to pay and abscond with all cash flows and a fraction of capital. We assume that borrowers who default can be excluded from neither the market for capital nor from borrowing and lending. Kehoe and Levine and most of the subsequent literature assume instead that borrowers who

¹See Holmström and Tirole (1997) for a related model of financial intermediation in a static environment in which there is a spread between the cost of intermediated and direct finance since intermediaries have limited capital.

default are excluded from intertemporal trade. A notable exception is Lustig (2007) who considers limited enforcement similar to the one in our model in an endowment economy. Deriving collateral constraints from a dynamic environment with limited commitment, as we do, allows the explicit analysis of their dynamic effects without requiring “ad hoc” extensions of constraints motivated by a static contracting problem to a dynamic setting. Second, we provide a new model of financial intermediaries as collateralization specialists which allows us to study the role and dynamics of intermediary capital.

The paper proceeds as follows: Section 2 discusses the related literature. Section 3 provides the model of collateral constraints due to limited enforcement and discusses the role of long term debt and loan commitments. Section 4 studies the distribution of debt capacity and conditions under which borrowers who exhaust their debt capacity are forced to downsize. Section 5 considers financial intermediation and Section 6 concludes. Proofs are relegated to the Appendix.

2 Related Literature

Dynamic models with limited commitment are used extensively in the literature to study optimal risk sharing² and asset pricing with heterogeneity,³ for example. Albuquerque and Hopenhayn (2004) and Hopenhayn and Werning (2007) analyze the implications for dynamic firm financing and Cooley, Marimon, and Quadrini (2004) and Jermann and Quadrini (2008) consider the aggregate implications of firm financing with limited commitment.

The collateral constraints we derive are similar to the ones in Kiyotaki and Moore (1997), albeit in our model they are state contingent. This is important because in our model borrowers can arrange additional financing contingent on states in which they require funding and would otherwise be constrained, which is the case in practice but is typically ruled out in theoretical models. Kiyotaki and Moore motivate their collateral constraints with an incomplete contracting model based on Hart and Moore (1994) and do not consider state-contingent borrowing. Several authors study models with collateral constraints with a similar motivation as in Kiyotaki and Moore. For example, Krishnamurthy (2003) studies a model in which both borrowers and lenders have to collateralize their promises and considers situations where lenders’ collateral is scarce.⁴ In contrast,

²See, e.g., Kocherlakota (1996), Ligon, Thomas, and Worrall (1997), Kehoe and Perri (2002, 2004), and Krueger and Uhlig (2006).

³See, e.g., Alvarez and Jermann (2000, 2001), Lustig (2007), and Lustig and van Nieuwerburgh (2007).

⁴See also, Iacoviello (2005) who studies a business cycle model with collateral constraints; and Eisfeldt and Rampini (2007, 2008) who study firm financing subject to collateral constraints.

we focus on borrowers incentives to arrange contingent financing when lenders have abundant funds and collateral. Most closely related to our model are Lorenzoni and Walentin (2007) who study a model with similar collateral constraints. Their focus is on the relation between investment, Tobin's q , and cash flow, and they do not consider aggregate shocks. Moreover, they restrict attention to the case in which borrowers always exhaust their debt capacity, whereas we analyze the incentives to conserve debt capacity and the implications for the cross-sectional distribution of debt capacity.

Shleifer and Vishny (1992) study debt capacity and the choice of optimal leverage in a model with aggregate states. They argue that debt may result in forced liquidations in bad times which in turn may limit the leverage that firms choose. They do not consider contingent financing, which is the focus here.

The role of intermediary capital is studied by Holmström and Tirole (1997). Intermediary capital in their model provides intermediaries with incentives to monitor and the amount of intermediary capital affects the availability of financing. They do not consider the dynamics of intermediary capital as we do here.⁵ The role of financial intermediaries during times where financing is constrained has been studied by Allen and Gale (1998, 2004), Gorton and Huang (2004), and Acharya, Shin, and Yorulmazer (2007), among others.⁶

This paper is also related to the emerging literature on contracting models of dynamic firm financing, see Bolton and Scharfstein (1990), Gromb (1994), and, more recently, Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007a, 2007b), Biais, Mariotti, Plantin, Rochet (2007), DeMarzo, Fishman, He, and Wang (2007), and Atkeson and Cole (2008) in addition to the papers mentioned above. These papers consider dynamic financing in the presence of private information or moral hazard, whereas we, and the literature discussed above, consider dynamic financing with limited commitment.

Finally, several other roles of collateral have been considered in the literature. When cash flows are private information, collateral may be used to induce borrowers to repay loans (see Diamond (1984), Lacker (2001), and Rampini (2005)). It has also been argued that collateral affects the interest rate that borrowers pay (see Barro (1976)), alleviates credit rationing due to adverse selection (see Bester (1985))⁷, reduces underinvestment

⁵Bolton and Freixas (2000) and Cantillo (2004) provide theories of financial intermediaries as specialized lenders. Diamond and Rajan (2000) provide a model of bank capital which trades off liquidity creation and costs of distress.

⁶Gromb and Vayanos (2002) study the dynamics of a model with financially constrained arbitrageurs.

⁷See also Chan and Kanatas (1985), Besanko and Thakor (1987a, b), and Chan and Thakor (1987), who study the role of collateral in models with adverse selection, and Berger and Udell (1995) and Boot,

problems (see Stulz and Johnson (1992)), provides lenders with an incentive to monitor (see Rajan and Winton (1995)), and renders markets more complete (see Dubey, Geanakoplos, and Shubik (2005) and Geanakoplos (1997)).

3 Modeling collateralized borrowing

We propose a dynamic model of collateralized borrowing. We consider an economy with limited enforcement which constrains borrowers' ability to make credible promises. We show that this economy is equivalent to an economy in which lending is subject to collateral constraints. Our model allows us to analyze the role of long term debt and show how the optimal lending contract can be implemented with loan commitments. Moreover, we define *debt capacity* explicitly in the context of the model. Finally, we study the dynamics of minimum down payment requirements.

3.1 Environment

There are 3 dates, 0, 1, and 2. There is a continuum of agents of measure 1. We index agents by their types $n \in \mathcal{N}$ and denote the density of agents of type n by $\psi(n)$ (and the cumulative distribution by $\Psi(n)$). We suppress agents' types for now and whenever possible, but do make the dependence on type explicit when it is useful to do so. Agents are risk neutral, subject to limited liability, and have preferences over (non-negative) dividends given by

$$E \left[\sum_{t=0}^2 d_t \right].$$

There are two goods in the economy, output goods and capital. Each agent is endowed with w_0 units of the output good at time 0 and no capital. Agents also have access to a production technology described below. These agents can be interpreted as entrepreneurs, for example, and typically have a financing need and hence we refer to them throughout as "borrowers."

The entrepreneurs' production technology is as follows. An amount of capital k_0 invested at time 0 returns $A_1(s)f(k_0)$ in output goods at time 1 in state s , where $s \in \mathcal{S}$, as well as the depreciated capital $(1-\delta)k_0$. Entrepreneurs also have access to a production technology at time 1 which, for an investment of $k_1(s)$, returns $A_2(s)f(k_1(s))$ in output goods at time 2 as well as the depreciated capital $(1-\delta)k_1(s)$.

Thakor, and Udell (1991), who study the role of collateral in models with moral hazard.

In addition to the borrowers described above, there is also a continuum of measure 1 of lenders in the economy which are unconstrained and risk neutral and discount the future at a rate $\beta < 1$. Lenders have a large endowment of funds in all dates and states. Lenders cannot run the production technology. Lenders have a large amount of collateral and hence are not subject to enforcement problems but rather are able to commit to deliver on their promises. Lenders are thus willing to provide any state-contingent loan at an expected rate of return $R = 1/\beta$ subject to borrowers' enforcement constraints.

We assume that markets are complete but there is limited enforcement; borrowers can abscond with the cash flows from the production technology and with fraction $1 - \theta$ of capital. Importantly we assume that entrepreneurs cannot be excluded from future borrowing or the market for capital. We show below that this is equivalent to assuming the following specification of financing constraints: borrowers can borrow in a state-contingent way, at time t , up to $\theta \in (0, 1)$ times the resale value of capital against each state at time $t + 1$.

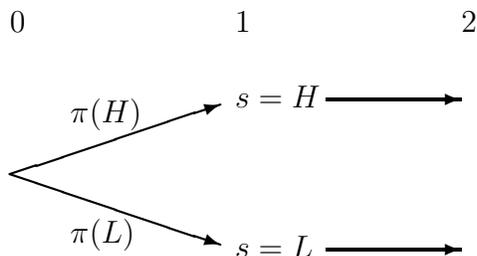
Finally, we assume that output goods can be transformed into capital goods (and vice versa) at a rate ϕ_0 at time 0 and at a rate of $\phi_t(s)$ at time $t \in \mathcal{T} \equiv \{1, 2\}$ in state $s \in \mathcal{S} \equiv \{H, L\}$, where state s has probability $\pi(s)$. Thus, for simplicity, we assume a very simple stochastic structure with two states at time 1 and no further uncertainty as illustrated in Figure 1. We assume that $\phi_1(H) > \phi_1(L)$ and that $A_1(H) > A_1(L)$, that is, we assume that in state L capital is relatively cheap, but cash flows are low at the same time. This is meant to capture the idea that state L is an economy wide downturn. The assumption that the “price” of capital is exogenously determined by a technological rate of transformation allows us to focus on the corporate finance implications of our model, whereas much of the literature has focused on the endogenous determination of this price (see, most notably, Kiyotaki and Moore (1997)).⁸ Moreover, our assumption effectively reduces our model to a one good economy which suggests that the allocation is constrained efficient.

3.2 Limited enforcement

Suppose that enforcement of contracts is limited as follows: borrowers can default on their promises, that is walk away from their debt obligations and abscond with all cash flows and fraction $1 - \theta$ of capital, and that lenders can seize only fraction θ of the capital and do not have access to any other enforcement mechanism. In particular, borrowers cannot be excluded from further borrowing or from purchasing capital goods. Thus, enforcement

⁸Endogenizing the price would not change our main conclusions, however.

Figure 1: Time Line



is limited as in Kehoe and Levine (1993) but unlike in their model, borrowers cannot be excluded from intertemporal markets here.⁹

The borrower chooses dividends $\{d_0, d_t(s)\}$, capital levels $\{k_0, k_t(s)\}$, loan amounts $\{l_0, l_1(s)\}$ and state-contingent repayments by $\{b_{t-1}(s)\}$, $\forall s \in \mathcal{S}, t \in \mathcal{T}$, to maximize the expected value of dividends,

$$d_0 + \sum_{s \in \mathcal{S}} \pi(s) \left\{ \sum_{t \in \mathcal{T}} d_t(s) \right\} \quad (1)$$

subject to the budget constraints at time 0, 1, and 2,

$$w_0 + l_0 \geq d_0 + \phi_0 k_0 \quad (2)$$

$$A_1(s)f(k_0) + \phi_1(s)k_0(1 - \delta) + l_1(s) \geq d_1(s) + \phi_1(s)k_1(s) + Rb_0(s), \quad \forall s \in \mathcal{S}, \quad (3)$$

$$A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1 - \delta) \geq d_2(s) + Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (4)$$

the lender's ex ante participation constraint at time 0,

$$\sum_{s \in \mathcal{S}} \pi(s) \left\{ \sum_{t \in \mathcal{T}} R^{-(t-1)} b_{t-1}(s) \right\} \geq l_0 + \sum_{s \in \mathcal{S}} \pi(s) R^{-1} l_1(s), \quad (5)$$

the enforcement constraints at time 1 and 2,

$$d_1(s) + d_2(s) \geq \hat{d}_1(s) + \hat{d}_2(s), \quad \forall s \in \mathcal{S}, \quad (6)$$

$$d_2(s) \geq A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1 - \theta)(1 - \delta), \quad \forall s \in \mathcal{S}, \quad (7)$$

⁹If θ were equal to 0, that is, if the borrower could abscond with all cash flows and all capital and would not be excluded from future lending, borrowers could not borrow at all (see Bulow and Rogoff (1989)).

limited liability constraints, and non-negativity constraints on capital,

$$d_0 \geq 0, \quad d_t(s) \geq 0, \quad k_0 \geq 0, \quad k_1(s) \geq 0, \quad \forall s \in \mathcal{S} \text{ and } t \in \mathcal{T}, \quad (8)$$

where $\{\hat{d}_t(s)\}_{t \in \mathcal{T}}$ are the dividends that the borrower could achieve after absconding, that is, $\{\hat{d}_t(s), \hat{k}_1(s), \hat{b}_1(s)\}_{t \in \mathcal{T}}$ maximize

$$\sum_{t \in \mathcal{T}} d_t(s) \quad (9)$$

subject to

$$A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta) + b_1(s) \geq d_1(s) + \phi_1(s)k_1(s), \quad (10)$$

the time 2 budget constraint (4), the time 2 enforcement constraint (7), and the non-negativity constraints (8). The borrower's problem after absconding at time 1 in state s is identical to the continuation problem at time 1 in state s , when he does not default, except that the borrower has net worth $A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta)$ after default, as opposed to net worth $A_1(s)f(k_0) + \phi_1(s)k_0(1 - \delta) - Rb_0(s)$, when he does not default.

3.3 Irrelevance of long term debt

We show that long term debt cannot add value.¹⁰ Intuitively, the enforcement constraints imply that the borrower can only credibly promise payment streams with present value less than or equal to the value of capital the borrower cannot abscond with. Any long term debt contract which satisfies this restriction can be implemented with a sequence of one period debt contracts. Hence, long term debt is irrelevant.

Lemma 1 *Considering state-contingent one period debt is sufficient, that is, without loss of generality, $l_0 = \sum_{s \in \mathcal{S}} \pi(s)b_0(s)$ and $l_1(s) = b_1(s)$, $\forall s \in \mathcal{S}$.*

In contrast, when borrowers can be excluded from intertemporal trade, long term contracts are not irrelevant in general.

3.4 Collateral constraints due to limited enforcement

We now show that the model with limited enforcement is equivalent to a model with state-contingent collateral constraints.

¹⁰We do not to write the problem recursively above, since, in principle, long term contracts could add value.

Lemma 2 *Enforcement constraints (6) and (7) are equivalent to collateral constraints*

$$\phi_1(s)\theta k_0(1 - \delta) \geq Rb_0(s), \quad \forall s \in \mathcal{S}, \quad (11)$$

$$\phi_2(s)\theta k_1(s)(1 - \delta) \geq Rb_1(s), \quad \forall s \in \mathcal{S}. \quad (12)$$

Lustig (2007) considers a similar outside option in an endowment economy and Lorenzoni and Walentin (2007) consider collateral constraints with a similar motivation in an economy with constant returns to scale. The original formulation of the enforcement constraints is in the same spirit as the one used to endogenize debt constraints in Kehoe and Levine (1993), although the limits on enforcement are different here. Kehoe and Levine assume that borrowers who default are excluded from intertemporal markets whereas we assume that borrowers cannot be excluded. Lemma 2 shows that, given our assumptions about the limits on enforcement, the constraints can equivalently be formulated as collateral constraints in the spirit of Kiyotaki and Moore (1997), but, importantly, are aggregate state contingent.

The equivalent formulation has the important advantage that the implementation of the optimal dynamic lending contract is rather simple: borrowers have access to state-contingent secured loans only.¹¹ Such lending arrangements are hence decentralized relatively easily by defining an equilibrium with collateral constraints with trade in state-contingent one-period loans which are subject to a state-contingent collateral constraint equal to fraction θ times the resale value of capital.¹²

3.5 Collateral constraints

To summarize, we now restate the borrower's problem restricting attention to state-contingent one period debt and replacing the enforcement constraints (6) and (7) with the collateral constraints (11) and (12). The borrower chooses $\{d_0, d_t(s)\}$, capital levels $\{k_0, k_1(s)\}$, and state-contingent one period borrowing $\{b_{t-1}(s)\}$ for all $(s, t) \in \mathcal{S} \times \mathcal{T}$ to maximize (1) subject to the budget constraints,

$$w_0 + \sum_{s \in \mathcal{S}} \pi(s)b_0(s) \geq d_0 + \phi_0 k_0 \quad (13)$$

$$A_1(s)f(k_0) + \phi_1(s)k_0(1 - \delta) + b_1(s) \geq d_1(s) + \phi_1(s)k_1(s) + Rb_0(s), \quad \forall s \in \mathcal{S}, \quad (14)$$

$$A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1 - \delta) \geq d_2(s) + Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (15)$$

¹¹Another advantage of this equivalent formulation is that the constraint set (2)-(4), (8), and (11)-(12) is convex. We study this problem henceforth.

¹²Similarly, Alvarez and Jermann (2000) define an equilibrium with solvency constraints to decentralize optimal allocations in an environment with limited commitment as in Kehoe and Levine (1993). The solvency constraints in their model are agent and state specific in contrast to the simple collateral constraints here.

the collateral constraints (11) and (12), and the limited liability and non-negativity constraints (8). Note that if the borrower promises to pay $Rb_0(s)$ in state s at time 1, he receives an amount of funds $\pi(s)b_0(s)$ at time 0. This guarantees the lender an expected return of R on the loan. Moreover, note that the amount that the borrower can credibly promise to repay at time t in state s is limited to a fraction θ of the resale value of capital in that state.

3.6 Thinking about debt capacity

Our model allows us to be precise about the meaning of *debt capacity*. At time 0, one unit of capital has state s debt capacity equal to a fraction θ of the present value of the resale value of capital, $R^{-1}\phi_1(s)\theta(1 - \delta)$. One unit of capital has (overall) debt capacity equal to a fraction θ of the present value of the expected resale value of capital, $R^{-1}\sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)\theta(1 - \delta)$. The overall debt capacity of a firm, of course, depends on the amount of capital the firm acquires and hence is endogenous. A firm exhausts its state s debt capacity if $R^{-1}\phi_1(s)\theta(1 - \delta) \geq b_0(s)$ holds with equality and has free state s debt capacity otherwise, and analogously for the firm's overall debt capacity.

Debt capacity is a property of the capital that a firm acquires. The amount of capital that a firm is able to acquire is jointly determined by the firm's net worth and the debt capacity of the capital that the firm is investing in. The overall debt capacity of a firm is endogenous; for example, keeping free debt capacity implies lower investment which in turn reduces the amount of capital that the firm can borrow against, that is, the debt capacity. In contrast, discussions in the literature often seem to imply that the debt capacity is an exogenous, pre-determined characteristic of the firm itself. In our dynamic model, the extent to which the firm uses its debt capacity for state s , say, determines the firm's net worth in that state. The firm's net worth, together with the debt capacity of the capital that the firm is considering, in turn determine the feasible investment in state s . Thus, in each state, the firm's net worth is pre-determined, while the debt capacity is endogenous and determined by the type and amount of capital that the firm acquires.

3.7 The role of loan commitments

In practice, borrowers conserve debt capacity in a state-contingent way by taking out loan commitments. We show that loan commitments are a practical implementation of the state-contingent loans determined by the model.

Define a *loan commitment* as a binding agreement to provide a loan of a particular size at some future date for a fee paid up front. So far, we have set $l_1(s) = b_1(s)$, $\forall s \in \mathcal{S}$, which

is without loss of generality given Lemma 1. Clearly this implies that the net present value of the loan from the lender's vantage point $NPV_1(s) \equiv -l_1(s) + R^{-1}Rb_1(s) = 0$, $\forall s \in \mathcal{S}$, that is, all loans have zero net present value to the lender when extended. Such loans do not of course require any ex ante commitment or up front fees.

Now consider a loan commitment $\{c_0(s), l_1(s), b_1(s)\}$ in which for an up front fee $c_0(s)$ to be paid at time 0, the lender agrees to provide a loan $l_1(s) > b_1(s)$ in state s at time 1 such that

$$c_0(s) + \pi(s)R^{-1}\{-l_1(s) + R^{-1}Rb_1(s)\} = 0,$$

which means that the loan commitment has zero net present value at time 0 due to competition in the market for loan commitments. In contrast, the net present value to the lender of a loan commitment in state s at time 1 is $NPV_1(s) = -l_1(s) + R^{-1}Rb_1(s) < 0$, that is, negative, which is why it is in fact a commitment.

Suppose the borrower chooses $\{b_{t-1}(s)\}_{s \in \mathcal{S}, t \in \mathcal{T}}$ and conserves debt capacity for state s , that is, $b_0(s) < R^{-1}\phi_1(s)\theta k_0(1 - \delta)$. To implement this with a loan commitment, suppose the borrower instead promises a repayment in state s of $\hat{b}_0(s) \equiv R^{-1}\phi_1(s)\theta k_0(1 - \delta)$ and arranges a commitment for a loan of $l_1(s) \equiv b_1(s) + R(\hat{b}_0(s) - b_0(s))$. The loan extended at time 1 in state s now has negative net present value, $NPV_1(s) = -l_1(s) + R^{-1}Rb_1(s) = -R(\hat{b}_0(s) - b_0(s)) < 0$, and thus requires a commitment from the lender and up front fees paid to the lender in the amount of $c_0(s) = -\pi(s)R^{-1}NPV_1(s) = \pi(s)(\hat{b}_0(s) - b_0(s))$ given competitive pricing of loan commitments. The borrower can finance these up front fees using the extra amount being borrowed against state s , which equals $\pi(s)(\hat{b}_0(s) - b_0(s))$. Thus, loan commitments are a way to implement the saving of contingent debt capacity.¹³ With this implementation, the key insight is that lining up loan commitments requires internal funds up front and thus has a cost in terms of reduced investment up front. Arranging for loan commitments or contingent financing is akin to conserving contingent debt capacity. Borrowers who choose to exhaust their debt capacity thus do not arrange for loan commitments either.

3.8 Dynamics of minimum down payments

This model of collateralized borrowing has the property that the minimum down payment is lower when the price of capital is expected to rise. This property seems empirically plausible and is consistent with anecdotal evidence that down payment requirements (or "lending standards") vary inversely with expected capital appreciation. To see this, define

¹³Indeed, here we could alternatively implement the optimal contract with loan commitments and non-state contingent debt only.

the minimum down payment \wp_0 and $\wp_1(s)$ as

$$\wp_0 \equiv \phi_0 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s) \phi_1(s) \theta (1 - \delta) \quad \text{and} \quad \wp_1(s) \equiv \phi_1(s) - R^{-1} \phi_2(s) \theta (1 - \delta).$$

The minimum amount that a borrower needs to pay down per unit of the asset is the price of the asset minus the collateralizable fraction of the discounted expected resale value, that is, minus the maximum amount that the borrower can borrow against the asset. The minimum down payment as a fraction of the price of capital at time 0, for example, is $\wp_0/\phi_0 \equiv 1 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s) \phi_1(s)/\phi_0 \theta (1 - \delta)$ and thus is decreasing in the expected capital appreciation $\sum_{s \in \mathcal{S}} \pi(s) \phi_1(s)/\phi_0$. Thus, expectations about future asset prices have an important effect on current down payment requirements. We are not aware of other models that predict such variation in down payment requirements.

4 The Distribution of Debt Capacity

In this section, we study the distribution of debt capacity and the dynamics of investment by different firms. We also analyze the effect of collateralizability and asset prices on the extent to which constrained firms might downsize, that is, scale down their investment. Furthermore, we consider the role of borrower net worth and the implications for risk management. We obtain three main results. First, more productive borrowers may exhaust their debt capacity since the opportunity cost of conserving debt capacity, which is foregone investment earlier on, is higher for them. Second, in states where asset prices and cash flows are low, capital may hence be less productively deployed on average, since more productive borrowers, who have exhausted their debt capacity, downsize relative to less productive borrowers. Third, in terms of risk management, the most constrained firms in our model choose not to hedge, as the financing needs for investment override the hedging concerns.

4.1 Conserve or exhaust debt capacity?

Define the return $R_1(k_0, s)$ as

$$R_1(k_0, s) \equiv \frac{A_1(s) f'(k_0) + \phi_1(s) (1 - \theta) (1 - \delta)}{\wp_0} \tag{16}$$

and define $R_2(k_1(s), s)$ analogously, which are the returns on the borrower's internal funds when he invests by making the minimum down payment (that is, by choosing maximal leverage). In order to abstract from net worth effects for now, we assume that investment

exhibits constant returns to scale, that is, $f(k) = k$ and hence $f'(k) = 1$. With constant returns to scale, $R_1(k_0, s)$ and $R_2(k_1(s), s)$ do not depend on k_0 or $k_1(s)$ and we hence simplify the notation to $R_1(s)$ and $R_2(s)$.

Moreover, we assume that investment at time 1 is sufficiently productive, namely that

Assumption 1 $R_2(s) > R, \forall s \in \mathcal{S}$.

This simplifies the analysis by implying that borrowers are constrained at time 1 and do not pay dividends before time 2, which in turn enables us to solve the borrower's problem at time 1 in state s explicitly. Define the net worth at time 1 in state s as

$$w_1(s) \equiv A_1(s)k_0 + \phi_1(s)k_0(1 - \delta) - Rb_0(s)$$

and the value attained by a borrower at time 1 in state s with that net worth as $V_1(w_1(s), s)$.

Lemma 3 *Given Assumption 1, borrowers are constrained at time 1, that is, the collateral constraints (12) bind, and dividends at time 0 and time 1 are zero, that is, $d_0 = d_1(s) = 0, \forall s \in \mathcal{S}$. Moreover, borrowers invest their entire net worth at time 1, that is, $k_1(s) = w_1(s)/\phi_1(s)$ and $V_1(w_1(s), s) = R_2(s)w_1(s), \forall s \in \mathcal{S}$.*

Having solved the time 1 problem, we can now solve the borrower's time 0 problem. This leads to our first main result. Depending on how productive investment is in the first period, that is at time 0, borrowers either invest as much as they can and exhaust their debt capacity with respect to all states at time 1 or conserve all their net worth and debt capacity for state s' at time 1, at which point they invest the maximal amount. The state s' is the state where the return is the highest, that is, $s' \in \arg \max_{s \in \mathcal{S}} R_2(s)$.

Proposition 1 *Productive borrowers exhaust their debt capacity, that is, if*

$$\sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s) > \max_s \{RR_2(s)\},$$

then $k_0 = w_0/\phi_0$ and $V_0(w_0) = \sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s)w_0$. Less productive borrowers conserve their net worth, that is, if the condition is not met, $k_0 = 0, w_1(s') = R/\pi(s')w_0$, and $V_0(w_0) = RR_2(s')w_0$, where s' such that $R_2(s') = \max_s \{R_2(s)\}$.

The condition for investment is $\sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s) > \max_s \{RR_2(s)\}$ and thus borrowers with higher productivity in the first period, say higher $\sum_{s \in \mathcal{S}} \pi(s)R_1(s)$, are more likely to invest and exhaust their debt capacity, all else equal. Moreover, the correlation between returns in the first period and returns in the second period, that is, investment opportunities, of course also matters. Higher autocorrelation of returns makes investment more likely. Hence, borrowers are more likely to exhaust their debt capacity when returns are more persistent.

4.2 Downsizing of productive firms

Now consider a borrower who invests at time 0 and who exhausts his debt capacity. Such a borrower may not be able to deploy as much capital at time 1 as he deploys at time 0, thus, he may be “forced to” downsize. This occurs in a state s in which cash flows $A_1(s)$ are sufficiently low. Importantly, this occurs despite the fact that the borrower could arrange for contingent financing. The borrower chooses not to do so because the opportunity cost is too high.

Proposition 2 *Borrowers are “forced to” downsize for $A_1(s)$ sufficiently low, that is, $k_1(s) < k_0$.*

Proposition 2 is our second main result and implies that productive borrowers may downsize when less productive borrowers, who did not previously invest, expand. If borrowers’ productivity is persistent, average productivity may hence decline in such states.

4.3 Effect of collateralizability on contraction

When the collateralizability θ increases, borrowers who invest at time 0 may downsize by more. Thus, financial innovation, which increases the collateralizability, may result in more severe contractions of borrowers who exhaust their debt capacity. This means that the effects we stress in this paper may become even more important over time as the ability to collateralize increases, consistent with recent events in financial markets.

Proposition 3 *With higher collateralizability, borrowers who exhaust their debt capacity may be forced to downsize by more. Suppose the parameters are as in Proposition 2 such that $k_1(s)/k_0 < 1$. Then $\frac{\partial}{\partial \theta} (k_1(s)/k_0) < 0$ as long as $\phi_1(s)/\phi_2(s) > k_1(s)/(Rk_0)$.*

This condition is satisfied for example when $\phi_1(s) = \phi_2(s)$. A higher θ has two effects. First, the borrower is able to pledge more funds at time 0 and hence has less “free net worth” left at time 1. Second, the borrower has a greater ability to borrow at time 1 going forward and hence requires a smaller “down payment requirement” in terms of net worth then. The two effects go in opposite directions, but as long as the price of capital is not too much higher at time 2, the first effect dominates: higher leverage due to higher pledgeability leads to a more severe contraction in capital.

4.4 Effect of asset prices on contraction

How does the extent of the contraction vary with the price of capital $\phi_1(s)$? That is, if the price drops by less in state s at time 1, do borrowers who exhausted their debt capacity downsize by more or by less?

Proposition 4 *Borrowers, who exhaust their debt capacity, downsize by more when asset prices fall by less, that is, $\frac{\partial}{\partial \phi_1(s)} (k_1(s)/k_0) < 0$.*

A higher price of capital at time 1 in state s has again two effects, raising the “free net worth,” since the borrower retains fraction $1 - \theta$ of the resale value of capital, while at the same time raising the “down payment requirement” $\wp_1(s)$. The second effect dominates the first. The higher the price of capital, the more capital downsizes as more net worth is required to purchase capital.

4.5 Role of borrower net worth

To study the effect of borrower net worth, we drop the assumption of constant returns to scale and instead assume that $f(k)$ is strictly concave and satisfies $\lim_{k \rightarrow 0} f'(k) = +\infty$. Then $k_0 > 0$ and $k_1(s) > 0$. Moreover, we again assume that productivity at time 1 is sufficiently high such that

Assumption 2 $R_2(k_1(s), s) > R, \forall s \in \mathcal{S}$.

With these assumptions, borrowers are again constrained at time 1 in state s and dividends at time 0 and 1 are zero. Defining net worth at time 1 in state s as $w_1(s) \equiv A_1(s)f(k_0) + \phi_1(s)k_0(1 - \delta) - Rb_0(s)$, the solution to the time 1 problem is characterized as before.¹⁴

Lemma 4 *Given Assumption 2, borrowers are constrained at time 1 and dividend payouts before time 2 are zero, that is, $d_0 = d_1(s) = 0, \forall s \in \mathcal{S}$. Moreover, borrowers invest their entire net worth at time 1, that is, $\forall s \in \mathcal{S}, k_1(s) = w_1(s)/\wp_1(s)$ and $V_1(w_1(s), s) = (A_2(s)f'(w_1(s)/\wp_1(s)) + \phi_2(s)(1 - \theta)(1 - \delta) - Rb_0(s)) \times w_1(s)/\wp_1(s)$.*

Suppose that the parameters satisfy the following assumption:

Assumption 3 **(i)** $R_2(k, H) < R_2(k, L)$, for k in the relevant range; and **(ii)** $k_1(H) > k_1(L)$, where $k_1(s) \equiv (A_1(s)f(w_0/\wp_0) + \phi_1(s)w_0/\wp_0(1 - \theta)(1 - \delta))/\wp_1(s)$ for w_0 in the relevant range.

¹⁴The proof of Lemma 4 is analogous to the proof of Lemma 3 and is hence omitted.

This assumption is satisfied, for example, when $A_2(H) = A_2(L)$ and $\phi_2(H) = \phi_2(L)$ and $A_1(H) \gg A_1(L)$. Intuitively, the assumption requires that the return on investment is higher in the low state at time 1, but that cash flows are sufficiently higher in the high state so that a borrower, who invests his entire net worth in the technology, has more capital in the high state than the low state. Given this assumption, the borrower exhausts his total debt capacity when net worth is very low, conserves debt capacity for the low state only when net worth is in an intermediate range, and is unconstrained in terms of first period investment when net worth is high enough. Thus, whether or not a borrower conserves debt capacity for state L now depends on the borrower's net worth. In particular, borrowers with low net worth do not conserve debt capacity and hence are more sensitive to aggregate conditions, consistent with empirical evidence. The following proposition summarizes this result:

Proposition 5 *Borrowers conserve debt capacity for the low state only if they are not too constrained. Under Assumption 3, there exist $\underline{w}_0 < \bar{w}_0$ such that (i) for $w_0 \leq \underline{w}_0$, $\lambda_0(s) > 0, \forall s \in \mathcal{S}$, $k_0 = w_0/\varphi_0$, and $k_1(s) = (A_1(s)f(k_0) + \phi_1(s)k_0(1-\theta)(1-\delta))/\varphi_1(s)$; (ii) for $\underline{w}_0 < w_0 < \bar{w}_0$, $\lambda_0(H) > 0$ and $\lambda_0(L) = 0$; and (iii) for $\bar{w}_0 \leq w_0$, $\lambda_0(s) = 0, \forall s \in \mathcal{S}$, $R_2(k_1(H), H) = R_2(k_1(L), L)$, and $R = \sum_{s \in \mathcal{S}} \pi(s)R_1(k_0, s)$.*

4.6 Reconsidering risk management

The state contingent loans in our model allow firms to engage in “corporate risk management.” Conserving state s contingent debt capacity amounts to buying state s Arrow claims, that is, partially hedging the amount of net worth in that state. The main theory of risk management, formalized by Froot, Scharfstein, and Stein (1993), is based on the effective risk aversion of firms subject to financial constraints. The rationale for hedging in this theory is that when firms are subject to financial constraints, hedging ensures that firms have sufficient internal funds to take advantage of investment opportunities. Importantly, this intuition suggests that more constrained firms should hedge more as they are effectively more risk averse. In practice, however, large firms, which are arguably less financially constrained, hedge more. Thus, this fact presents an important puzzle from the vantage point of received theory. Our theory resolves this “risk management puzzle,” since it predicts that the more constrained firms, that is, the more productive or less well capitalized firms, exhaust their debt capacity and hence do not hedge. In our model, firms’ ability to credibly promise to pay is limited, and firms have an incentive to hedge net worth in the low state for the usual reasons. However, investment up front is endogenous in our model and the overriding concern may be to finance up front investment.

Indeed, the more constrained the firm, the more likely it is that investment financing needs override the hedging concerns. This is the main implication of our model for risk management. Thus, we expect that smaller firms, which are likely more financially constrained, hedge less and, as a result, are more sensitive to aggregate fluctuations than larger firms, consistent with empirical evidence. In contrast, Froot, Scharfstein, and Stein (1993) take up front investment as exogenously given in their model, in effect making risk management the only concern, and thus reach the opposite conclusion.

Reinterpreting our model in terms of household finance, the prediction is that less well-off, and hence likely more constrained, households insure less and are more vulnerable to economic downturns. While we are not aware of systematic evidence to this effect, this prediction seems consistent with anecdotal evidence at least. Received theory, by contrast, would again have the prediction that less well-off households insure more, which we think is counterfactual.

5 Financial intermediation

In this section we study how financial intermediaries affect the distribution of debt capacity as well as how collateralized borrowing in turn affects the dynamics of intermediary capital. In addition to the lenders considered above, which we henceforth refer to as providing *direct finance*, we introduce *financial intermediaries*. We model financial intermediaries as lenders which are able to collateralize a larger fraction of capital, that is, are able to enforce their claims better, but have limited internal funds. Thus, intermediaries in our model are “collateralization specialists.” Relatedly, Holmström and Tirole (1997) and Diamond (2007) model intermediaries as lenders which are better able to monitor borrowers. We provide conditions for intermediary capital to be scarce when asset prices and cash flows are low, implying higher spreads between the cost of intermediated finance and direct finance. Moreover, we show that in that case borrowers who exhaust their debt capacity may downsize for two reasons: They have low cash flow and hence low net worth, and the cost of intermediated funds is higher.

5.1 A model of financial intermediaries

Suppose a representative financial intermediary with capital w_0^i is able to collateralize up to fraction $\theta^i > \theta$ of the resale value of capital.¹⁵ In other words, a borrower who borrows

¹⁵We consider a representative financial intermediary since intermediaries have constant returns to scale in our model and hence aggregation in the intermediation sector is straightforward. The distribution of intermediaries’ net worth is hence irrelevant and only the aggregate capital of the intermediation sector

from a financial intermediary can abscond with only $1 - \theta^i$ of capital that is pledged to an intermediary as well as all cash flows. The greater ability to enforce claims can be interpreted as an ability to monitor collateral more closely and limit diversion by the borrower, similar to Diamond (2007).

To simplify the exposition, we start by considering a one period problem and study the capital structure implications for the cross section of borrowers. The intermediary lends at a state-contingent interest rate $R_0^i(s)$, $\forall s \in \mathcal{S}$, to be determined in equilibrium. The intermediary solves

$$\max_{\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}} d_0^i + \sum_{s \in \mathcal{S}} \pi(s) R^{-1} d_1^i(s)$$

subject to

$$w_0^i \geq d_0^i + \sum_{s \in \mathcal{S}} \pi(s) l_0^i(s)$$

and

$$R_0^i(s) l_0^i(s) \geq d_1^i(s), \quad \forall s \in \mathcal{S},$$

as well as $d_0^i \geq 0$, $d_1^i(s) \geq 0$, $l_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$, where $l_0^i(s)$ is the amount that the intermediary lends against state s . This statement of the intermediary's problem does not explicitly involve collateral constraints since the intermediary is in fact lending, and collateral constraints are instead imposed on the borrowers for both direct as well as intermediated finance. Moreover, $R_0^i(s) \geq R$, $\forall s \in \mathcal{S}$, since the intermediary could always lend to the direct lenders at an expected return of R .

Importantly, we state the problem as if lenders provide finance to the borrowers directly, rather than explicitly keeping track of lenders' funds provided to intermediaries and passed on to the borrowers. This simplifies the notation and analysis, without affecting the results. Nevertheless, the interpretation should be clear. Of 1 unit of capital, intermediaries can seize θ^i . In turn, direct lenders can seize θ of the collateral backing an intermediated loan. This means that the intermediary can finance an amount θ from the lenders at an expected rate R and pass this amount on to the borrower. The additional amount, $\theta^i - \theta$, which the intermediary can finance due to the better ability to collateralize, however, has to be financed with the intermediary's internal funds. Since direct lenders cannot seize any of the additional capital, which the intermediary is able to seize, they cannot provide financing for it. Thus, our model provides a rationale for intermediary capital due to limited commitment.¹⁶ Note that this rationale for intermediary capital would not disappear even if the intermediary were to diversify by financing (w_0^i) matters.

¹⁶The intermediary's problem above can be derived from the problem subject to enforcement constraints as before. The intermediary's enforcement constraint requires that the dividend that the inter-

many borrowers with independent projects. In contrast, diversification does eliminate the need for intermediary capital in Holmström and Tirole (1997) in the limit and they hence assume that all projects are perfectly correlated.

We have suppressed borrowers' types thus far, and continue to do so whenever possible, but, to define an equilibrium, it is useful to make the dependence on type explicit. Recall that we index borrowers by their types $n \in \mathcal{N}$ and denote the density of borrowers of type n by $\psi(n)$ (and the cumulative distribution by $\Psi(n)$). For example, we assume that both borrowers' initial endowment $w_0(n)$ and productivity $A_t(s|n)$ may depend on n . An *equilibrium* consists of state-contingent interest rates on intermediated funds $R_0^i(s)$, $\forall s \in \mathcal{S}$, and an allocation such that $\{d_0(n), d_1(s|n), k_0(n), b_0(s|n), b_0^i(s|n)\}_{s \in \mathcal{S}}$ solves borrower n 's problem, $\forall n \in \mathcal{N}$, and $\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}$ solves the representative intermediary's problem, and such that the market for intermediated finance clears, that is,

$$\int_{\mathcal{N}} b_0^i(s|n) d\Psi(n) \leq l_0^i(s), \quad \forall s \in \mathcal{S},$$

with equality if $R_0^i(s) > R$.¹⁷

In the one period problem, intermediaries charge the same interest rate on intermediated loans for both states, since the value of net worth to the intermediary at time 1 is the same in the two states as such net worth is simply paid out as dividends:

Lemma 5 *The interest rates on state-contingent loans are the same for the two states, that is, $R_0^i(H) = R_0^i(L) \equiv R_0^i$ without loss of generality.*

Thus, the borrower can borrow using direct finance at an expected rate of R as before and from financial intermediaries at a rate R_0^i as determined above, stated formally:

$$\max_{\{d_0, d_1(s), k_0, b_0(s), b_0^i(s)\}_{s \in \mathcal{S}}} d_0 + \sum_{s \in \mathcal{S}} \pi(s) d_1(s)$$

subject to the budget constraints,

$$\begin{aligned} w_0 + \sum_{s \in \mathcal{S}} \pi(s) \{b_0(s) + b_0^i(s)\} &\geq d_0 + \phi_0 k_0 \\ A_1(s) f(k_0) + \phi_1(s) k_0 (1 - \delta) &\geq d_1(s) + R b_0(s) + R_0^i b_0^i(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

mediary receives exceeds the amount the intermediary can abscond with, which is the payments that the intermediary receives minus the capital that the direct lenders can seize.

¹⁷The markets for output goods, capital goods, and direct finance do not impose additional restrictions due to Walras' law, the fact that capital goods can be transformed into output goods with a linear and reversible technology, and the fact that direct lenders are risk neutral and have plenty of funds at all dates and in all states.

two sets of collateral constraints,

$$\begin{aligned}\phi_1(s)\theta k_0(1-\delta) &\geq Rb_0(s), & \forall s \in \mathcal{S}, \\ \phi_1(s)\theta^i k_0(1-\delta) &\geq Rb_0(s) + R_0^i b_0^i(s), & \forall s \in \mathcal{S},\end{aligned}$$

and $d_0 \geq 0$, $d_1(s) \geq 0$, $k_0 \geq 0$, $b_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$ and $t \in \mathcal{T}$. There are now two collateral constraints for each state: the first constraint restricts direct finance and is as before; the second constraint restricts the total promises the borrower makes against state s , which cannot exceed the amount that the intermediary can collateralize.

5.2 Capital structure: intermediated vs. direct finance

In the cross section, the capital structure of firms varies as follows: the least productive firms do not invest; more productive firms invest and exhaust the direct financing capacity; and the most productive firms exhaust both their direct financing as well as their intermediated financing capacity. The next proposition states this formally:

Proposition 6 *Suppose $R_0^i > R$. The most productive (and hence most constrained) borrowers borrow from intermediaries. If $R \geq \sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1-\delta))/\phi_0$, then $k_0 = 0$ and $V(w_0) = Rw_0$; otherwise, if $R_0^i \geq \mu_0^* \equiv \sum_{s \in \mathcal{S}} \pi(s)R_1(s)$, then $k_0 = (1/\varphi_0)w_0$ and $V(w_0) = \mu_0^*w_0$, and if $R_0^i < \mu_0^*$, then $k_0 = (1/\bar{\varphi}_0)w_0$ and $V(w_0) = \bar{\mu}_0^*w_0$ where $\bar{\varphi}_0 = \phi_0 - \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(R^{-1}\theta(1-\delta) + (R_0^i)^{-1}(\theta^i - \theta)(1-\delta))$ is the minimum down payment requirement in the presence of intermediaries and $\bar{\mu}_0^*$ is defined in the proof.*

The proof is in the appendix. The value of internal funds is $\mu_0 \geq R$ and thus exceeds the value of external funds when the borrower is constrained. Moreover, the more productive the borrower is, the higher the value of internal funds is, and the more constrained the borrower is. Thus, it is the more constrained borrowers which borrow from the financial intermediary in our model.

Similarly, if investment is subject to decreasing returns to scale and all borrowers have the same productivity but differ in their initial endowment, then the borrowers with less internal funds are more constrained and borrow from the financial intermediary. The static cross sectional capital structure implications are hence similar to that in Holmström and Tirole (1997). Next, we consider the dynamics of financial intermediation explicitly, thus going beyond Holmström and Tirole. They study the comparative statics with respect to intermediary capital in a static model, and indeed argue in the conclusion (see p. 688-689) that an explicitly dynamic model would be required for a proper investigation of the effects of intermediary capital.

5.3 Dynamics of intermediary capital

We analyze the dynamics of intermediary capital in the environment introduced in Section 3. Consider the intermediary's dynamic problem. We start with the intermediary's problem at time 1 in state s , given intermediary capital $w_1^i(s)$:

$$\max_{\{d_1^i(s), d_2^i(s), l_1^i(s)\}} d_1^i(s) + R^{-1}d_2^i(s)$$

subject to the budget constraints

$$\begin{aligned} w_1^i(s) &\geq d_1^i(s) + l_1^i(s), \\ R_1^i(s)l_1^i(s) &\geq d_2^i(s), \end{aligned}$$

as well as $d_1^i(s) \geq 0$, $d_2^i(s) \geq 0$, $l_1^i(s) \geq 0$, where $l_1^i(s)$ is the amount that the intermediary lends against time 2. Assuming that $R_1^i(s) \geq R$, we have $d_1^i(s) = 0$, $l_1^i(s) = w_1^i(s)$, $d_2^i(s) = R_1^i(s)l_1^i(s)$, and hence $V_1^i(w_1^i(s)) = R^{-1}R_1^i(s)w_1^i(s)$.

At time 0 the intermediary solves

$$\max_{\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}} d_0^i + \sum_{s \in \mathcal{S}} \pi(s)R^{-2}R_1^i(s)w_1^i(s)$$

subject to the budget constraints

$$\begin{aligned} w_0^i &\geq d_0^i + \sum_{s \in \mathcal{S}} \pi(s)l_0^i(s) \\ R_0^i(s)l_0^i(s) &\geq w_1^i(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

as well as $d_0^i \geq 0$, $w_1^i(s) \geq 0$, $l_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$, where $l_0^i(s)$ is the amount that the intermediary lends against state s .¹⁸ The first order conditions are $\mu_0 = 1 + \nu_0^d$, $\mu_1(s) = R^{-2}R_1^i(s) + \nu_1^w(s)$, and $\mu_0 = R_0^i(s)\mu_1(s)$. As long as $R_1^i(s) > R$ for some s, t , $d_0^i = 0$. Moreover, we have

$$R^{-2}R_0^i(s)R_1^i(s) + R_0^i(s)\nu_1^w(s) = R^{-2}R_0^i(s')R_1^i(s') + R_0^i(s')\nu_1^w(s').$$

If the intermediary has positive net wealth in both states at time 1 we have

$$R_0^i(s)R_1^i(s) = R_0^i(s')R_1^i(s').$$

To characterize the dynamics of intermediary capital and the spread between intermediated finance and direct finance, we first study the case in which intermediaries have plenty of capital and then consider the case in which intermediaries have limited capital.

¹⁸The intermediary may choose to lend some funds to the direct lenders as well, in order to conserve debt capacity for state s , for example, but there is no need to keep track of such lending separately from lending to the borrowers. The reason is that whenever the intermediary lends funds to the direct lenders, the interest rate on intermediated funds for that state equals R .

5.4 Well capitalized intermediaries

Suppose that the representative intermediary is well capitalized, that is, w_0^i is sufficiently large such that the intermediary has excess funds at time 0 and at time 1 in all states and $R_0^i(s) = R = R_1^i(s)$, $\forall s \in \mathcal{S}$. The borrower's problem is then equivalent to the problem without intermediation studied in Sections 3 and 4 except that θ is replaced by θ^i since borrowers are able to borrow up to fraction θ^i of capital in total. Here we determine the cutoff level of intermediary capital such that the intermediary is well capitalized. This facilitates the analysis of the case where intermediary capital is limited, which we consider in the next section.

Assume that there are two types of borrowers, more productive, "good" borrowers with measure $\psi(g)$, and less productive, "bad" borrowers with measure $\psi(b)$ ($= 1 - \psi(g)$), that is, $\mathcal{N} = \{g, b\}$. Assume that the more productive entrepreneurs (type g) optimally choose positive investment at time 0, $k_0(g) > 0$, which means that parameters are such that $\sum_{s \in \mathcal{S}} \pi(s) R_1(s|g) R_2(s|g) > \max_{s \in \mathcal{S}} R R_2(s|g)$ evaluated at θ^i instead of θ . We furthermore assume that here there is a maximum scale \bar{k} at which the technology can be operated. When \bar{k} is sufficiently high, the model is as before. But we consider the case where \bar{k} binds in state H , which implies that borrowers use their high cash flows in that state to partially pay down their loans from the intermediaries.¹⁹ For the less productive entrepreneurs (type b) assume that the inequality is reversed and that $L = \arg \max_{s \in \mathcal{S}} R R_1(s|b)$ again evaluating all expressions at θ^i , so that the less productive entrepreneurs conserve their net worth at time 0 and invest at time 1 in state L only.²⁰ From the solution to the equivalent problem we can determine the minimum amount of financing that intermediaries must provide to implement the solution. The intermediary extends loans in the amount of

$$\underline{l}_1^i(s) = R^{-1} \phi_2(s) (\theta^i - \theta) \left(\sum_{n \in \{g, b\}} \psi(n) k_1(s|n) \right) (1 - \delta)$$

at time 1 in state s , where $k_1(s|n) = w_1(s|n) / \varphi_1^i(s)$, $\forall n \in \{g, b\}$, and $\varphi_1^i(s)$ is $\varphi_1(s)$ with θ replaced by θ^i , if investment is less than maximum scale, and extends loans of $\max\{\bar{k}(\phi_1(s) - R^{-1} \phi_2(s) \theta (1 - \delta)) - w_1(s|n), 0\}$ to borrowers of type n otherwise. Borrowers' net worth in turn is $w_1(s|g) = (A_1(s|g) + \phi_1(s)(1 - \theta^i)(1 - \delta))k_0(g)$, $w_1(H|b) = 0$, and

¹⁹This requires a slight modification of the condition for investment at time 0 to be optimal, namely that $\pi(H)R_1(H|g)R + \pi(L)R_1(L|g)R_2(L|g) > \max_{s \in \mathcal{S}} R R_2(s|g)$. Note that the return in state H in the second period is now R since additional funds are simply used to pay down debt.

²⁰This is the case as long as investment by type b is sufficiently unproductive at time 0 and $(A_2(L|b) + \phi_2(L)(1 - \theta_i)(1 - \delta)) / \varphi_1^i(L) > (A_2(H|b) + \phi_2(H)(1 - \theta_i)(1 - \delta)) / \varphi_1^i(H)$ where $\varphi_1^i(s)$ is defined as in the text.

$w_1(L|b) = R/\pi(L)w_0(b)$. The total loan repayments to the intermediary at time 1 in state s are

$$RL_0^i(s) = \phi_1(s)(\theta^i - \theta)\psi(g)k_0(g)(1 - \delta),$$

where $k_0(g) = w_0(g)/\wp_0^i$ and \wp_0^i is \wp_0 with θ replaced by θ^i . Thus, the net lending of the financial intermediary at time 1 in state s is

$$\begin{aligned} \underline{nl}_1^i(s) &\equiv \underline{l}_1^i(s) - RL_0^i(s) \\ &= \left(R^{-1}\phi_2(s) \left(\frac{A_1(s) + \phi_1(s)(1 - \theta^i)(1 - \delta)}{\wp_1^i(s)} \right) - \phi_1(s) \right) (\theta^i - \theta)(1 - \delta)\psi(g)k_0(g) \\ &\quad + R^{-1}\phi_2(s)(\theta^i - \theta)(1 - \delta)\psi(b)\frac{1}{\wp_1^i(s)}w_1(s|b), \end{aligned} \quad (17)$$

as long as investment is below maximum scale. Net lending is thus higher in state s at time 1 when cash flows of productive borrowers are high which allows them to expand. It is also higher when there are more less productive borrowers entering. If in state H the productive borrowers have sufficient net worth to reach maximum scale, then net lending is

$$\underline{nl}_1^i(H) = \max\{\bar{k}(\phi_1(H) - R^{-1}\phi_2(H)\theta(1 - \delta)) - w_1(H|g), 0\} - \phi_1(H)(\theta^i - \theta)(1 - \delta)\psi(g)k_0(g).$$

In this case, net lending is lower when cash flows of productive borrowers are high since they repay loans rather than expand capital further.

When the aggregate net worth of financial intermediaries is sufficiently high, intermediaries are well capitalized and the spreads between intermediated finance and direct finance are zero, as the next results shows.

Proposition 7 *If $w_0^i \geq \underline{w}_0^i \equiv \sum_{s \in \mathcal{S}} \pi(s) (\underline{l}_0^i + R^{-1} \max\{\underline{nl}_1^i(s), 0\})$, banks are well capitalized and $R_0^i(s) = R = R_1^i(s)$, $\forall s \in \mathcal{S}$.*

5.5 Limited intermediary capital

Suppose instead that the intermediary is not well capitalized, that is, that $w_0^i < \underline{w}_0^i$. Clearly, $R_0^i(s) = R = R_1^i(s)$, $\forall s \in \mathcal{S}$ is then not an equilibrium, and there is a spread between intermediated funds in some dates and states. Consider the case in which the intermediary requires additional capital in the low state only, that is, the case in which loan repayments fall short of the net lending demand by borrowers in state L at a cost of intermediated loans of R . This will be the case as long as the demand for intermediated loans from the less productive borrowers who are investing is sufficiently high. Formally, we assume that

Assumption 4 *Net lending is positive in state L only, that is, $\underline{nl}_1^i(L) > 0 > \underline{nl}_1^i(H)$.*

We discuss the conditions under which this is the case more explicitly below. The next result characterizes the dynamics of the cost of intermediated financing:

Proposition 8 *Suppose Assumption 4 holds. Then $\exists \varepsilon > 0$ such that $\forall w_0^i < \underline{w}_0^i$ and $\varepsilon > \underline{w}_0^i - w_0^i$, there is a premium for intermediated loans at time 0 for state H contingent loans and at time 1 in state L , that is, $R^i \equiv R_0^i(H) = R_1^i(L) > R$, and $R_0^i(L) = R_1^i(H) = R$.*

Denoting the time 0 spread on a loan requiring the payment of 1 unit in all states at time 1 by $\varsigma_0 \equiv \sum_{s \in \mathcal{S}} \pi(s) R_0^i(s) - R$ and the time 1 spread in state s on a loan requiring the payment of 1 unit at time 2 by $\varsigma_1(s) \equiv R_1^i(s) - R$, we have the following immediate corollary of this proposition which characterizes the dynamics of the spread between intermediated and direct financing:

Corollary 1 *Under the conditions of Proposition 8, the spread between intermediated and direct finance is highest in state L at time 1 and positive at time 0, that is, $\varsigma_1(L) > \varsigma_0 > \varsigma_1(H) = 0$.*

Proposition 8 and Corollary 1 say that there is a positive spread between intermediated finance both at time 0 as well as in state L at time 1. The spread is highest in state L at time 1, however. The fact that intermediary capital is expected to be scarce in some future dates and states, implies that it is scarce at time 0 as well, and that spreads are positive then, too. Moreover, spreads are positive at time 0 even if the intermediaries are able to fund all current loans, because intermediaries optimally conserve some of their funds for future states with positive net loan demand.

The intermediary responds to the positive net loan demand in state L by conserving net worth for state L , but not to the point where spreads between intermediated finance and direct finance are zero. Intermediary capital is scarce and hence earns a higher return.

When is Assumption 4 satisfied? Consider first state H . Since we assume that the less productive borrowers do not invest in this state, the demand for loans is determined by the more productive borrowers. Given the high cash flows, their net worth increases in this state and hence their investment expands. This in turn raises the loan demand. Thus, it is possible for net loan demand to be positive in this state. However, if the productive borrowers reach maximum scale, then they use their net worth to pay down intermediated debt and net loan demand is negative. Second, in state L the less productive borrowers enter. The larger their aggregate net worth, $\psi(b)w_0(b)$, the higher is net loan demand. Moreover, net loan demand of the more productive borrowers depends again on their cash

flows. If cash flows are sufficiently high, net loan demand by these borrowers may still be positive, but when cash flows are low enough, such that the more productive borrowers are forced to downsize, net loan demand by these borrowers is negative. Aggregate net loan demand in state L is still positive, as long as the demand for intermediated loans from the less productive borrowers who are investing is sufficiently high.

5.6 Impact of limited intermediary capital on borrowers

When financial intermediary capital is scarce, then, the scarcer intermediary capital, the more borrowers downsize (or the less they expand) in the state where intermediary capital is scarce.

Proposition 9 *Suppose w_0^i is as in Proposition 8. If s such that $\underline{nl}_1^i(s) > 0 > \underline{nl}_1^i(s')$, $s' \neq s$, then $\frac{d}{dw_0^i}(k_1^g(s)/k_0^g) > 0$.*

Thus, productive borrowers may now downsize for two reasons: first, because they have low cash flow and hence low net worth in state L , and second, because the cost of intermediated funds increases in state L . Moreover, scarce intermediary capital increases the down payment requirement, $\bar{\varphi}_1(s) \equiv \phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta) - R_1^i(s)^{-1}\phi_2(s)(\theta^i - \theta)(1 - \delta)$, and, as a fraction of total debt, intermediated finance becomes less important.

5.7 An example with limited intermediary capital

To illustrate the dynamics of the spread between intermediated funds and direct finance in our model, we provide an example. The parameters of the example are provided in Panel A of Table 1. The parameters satisfy the assumptions in this section. In Panel B we consider the case of a well capitalized intermediary. The more productive borrowers invest at time 0 and the less productive borrowers invest at time 1 in state L only. Moreover, the more productive borrowers downsize at time 1 in state L , that is, $k_1(L|g) < k_0(g)$. Since intermediaries are well capitalized the spreads are zero and hence not reported. In Panel C we consider the case in which intermediaries have 2.5% less capital than they would require to be well capitalized. Since intermediary capital is scarce, investment is reduced. Note that the more productive borrowers now downsize by more at time 1 in state L . Moreover, spreads between intermediated finance and direct finance are now positive. Indeed, the spread at time 0 is almost 1% and the spread at time 1 in state L almost 2%. Of course, this example is illustrative only and is not calibrated. Nevertheless, this suggests that a relatively modest reduction in intermediary capital might have a substantial impact on spreads.

6 Conclusion

We provide a dynamic model of collateralized lending, allowing for both direct lending as well as lending by financial intermediaries. We endogenously derive the collateral constraints based on limited enforcement. We show that considering one period state-contingent debt is sufficient, and that long term debt is redundant, that is, does not increase debt capacity. We show that taking out loan commitments is equivalent to conserving debt capacity. Thus, loan commitments are a plausible way in which the state contingent loans predicted by our model are implemented in practice. The cross-sectional distribution of debt capacity in our model is endogenous. In particular, we show that more productive borrowers may be more constrained when asset prices and cash flows are low, and may hence not be able to seize investment opportunities that arise due to low asset prices. Similarly, borrowers with less internal funds may exhaust their debt capacity as well, while borrowers with more internal funds conserve some debt capacity to take advantage of such investment opportunities. More productive borrowers may be forced to scale down investment in such times, and they may be forced to scale down investment by more, the more collateralizable the assets. The reason is that higher collateralizability allows them to borrow more ex ante, but leaves them with less net worth ex post when cash flows are low. This implies that capital may be less productively deployed in such times. Moreover, if collateralizability increases over time, as arguably it has recently, the effects stressed in this paper become even more important. For risk management, our model predicts that more constrained firms hedge less, since financing needs override hedging concerns, consistent with the empirical evidence. In contrast, this evidence is considered a puzzle from the vantage point of the standard theory of risk management, which takes investment as given.

We model financial intermediaries as lenders which are able to collateralize a larger fraction of capital but have limited funds. Such financial intermediaries finance borrowers with higher leverage. We study the dynamics of intermediation capital and spreads between intermediated finance and direct finance. Spreads on intermediated finance are high when the demand for intermediated finance is high. In states where there are investment opportunities due to low asset prices, spreads are high when the demand from borrowers trying to take advantage of the investment opportunities is high. These higher spreads may force borrowers who previously invested to downsize by more in such states, consistent with anecdotal evidence.

Appendix

Proof of Lemma 1. Note that $Rb_1(s)$ is the total payment from the borrower to the lender at time 2, and there is no need to distinguish payments due to funds lent at time 0 (l_0) and at time 1 in state s ($l_1(s)$). Moreover, the program only determines the net payment $Rb_0(s) - l_1(s)$, $\forall s \in \mathcal{S}$, and thus we are free to set $l_1(s) = b_1(s)$, $\forall s \in \mathcal{S}$. Equation (5) then simplifies to $\sum_{s \in \mathcal{S}} \pi(s)b_0(s) \geq l_0$ and using the fact that this equation holds with equality we can substitute for l_0 . \square

Proof of Lemma 2. Notice that (4) holds with equality due to non-satiation. Substituting for $d_2(s)$ in (7) using (4) and canceling terms implies (12). Conversely, (12) together with (4) at equality implies (7).

To obtain (11), assume that $Rb_0(s) > \phi_1(s)\theta k_0(1-\delta)$. Let $X(s) \equiv \{d_t(s), k_1(s), b_1(s)\}_{t \in \mathcal{T}}$ be the allocation from time 1 onward in state s . Consider default at time 1 to an allocation $X'(s) = X(s)$. Note that (4) implies

$$\begin{aligned} A_1(s)f(k_0) + \phi_1(s)k_0(1-\theta)(1-\delta) + b_1(s) &> A_1(s)f(k_0) + \phi_1(s)k_0(1-\delta) - Rb_0(s) + b_1(s) \\ &\geq d_1(s) + \phi_1(s)k_1(s), \end{aligned}$$

and hence $X'(s)$ is feasible. Moreover $d'_1(s)$ can be increased which violates (6), a contradiction. Conversely, (11) implies that the optimal allocation after default, $\hat{X}(s)$ say, is a feasible allocation and hence the contractual allocation $X(s)$ must attain at least that value, implying that (6) is satisfied. \square

Proof of Lemma 3. The first order conditions of the problem of maximizing (1) subject to (8) and (11)-(15), which are necessary and sufficient, are

$$\mu_0 = 1 + \nu_0^d, \quad (18)$$

$$\mu_t(s) = 1 + \nu_t^d(s), \quad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \quad (19)$$

$$\mu_0 = R\mu_1(s) + R\lambda_0(s), \quad \forall s \in \mathcal{S}, \quad (20)$$

$$\mu_1(s) = R\mu_2(s) + R\lambda_1(s), \quad \forall s \in \mathcal{S}, \quad (21)$$

$$\phi_0\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) \{ (A_1(s)f'(k_0) + \phi_1(s)(1-\delta))\mu_1(s) + \phi_1(s)\theta(1-\delta)\lambda_0(s) \} + \nu_0^k \quad (22)$$

$$\phi_1(s)\mu_1(s) = (A_2(s)f'(k_1(s)) + \phi_2(s)(1-\delta))\mu_2(s) + \phi_2(s)\theta(1-\delta)\lambda_1(s) + \nu_1^k(s), \quad \forall s, \quad (23)$$

where $\lambda_0(s)$, $\lambda_1(s)$, μ_0 , $\mu_1(s)$, and $\mu_2(s)$ are the multipliers on constraints (11)-(15), and ν_0^d , $\nu_t^d(s)$, ν_0^k , and $\nu_1^k(s)$ are the multipliers on the constraints in (8).

Using the return definitions (16) and equations (20) and (21), (22) and (23) can be written as

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s)R_1(k_0, s)\mu_1(s) + \frac{1}{\wp_0}\nu_0^k \quad (24)$$

$$\mu_1(s) = R_2(k_1(s), s)\mu_2(s) + \frac{1}{\wp_1(s)}\nu_1^k(s). \quad (25)$$

Using (19), (21), (25), and Assumption 1, $R\mu_2(s) + R\lambda_1(s) = \mu_1(s) \geq R_2(s)\mu_2(s) > R\mu_2(s)$ and thus $\lambda_1(s) > 0, \forall s \in \mathcal{S}$. Moreover, $\mu_0 \geq \mu_1(s) \geq \mu_2(s) + \lambda_2(s) > \mu_2(s) \geq 1$. Then (18) and (19) imply $\nu_0^d > 0$ and $\nu_1^d(s) > 0, \forall s \in \mathcal{S}$.

Since $d_1(s) = 0$ and using (3) and (12) at equality we have $k_1(s) = w_1(s)/\wp_1(s)$. Moreover, (4) and (12) at equality imply that $d_2(s) = (A_2(s) + \phi_2(s)(1 - \theta)(1 - \delta))k_1(s)$ and hence $V_1(w_1(s), s) = d_1(s) + d_2(s) = R_2(s)w_1(s), \forall s \in \mathcal{S}$. \square

Proof of Proposition 1. Suppose $k_0 = 0$. Then $w_1(s) = -Rb_0(s)$ and using Lemma 3 we have

$$V_0(w_0) \equiv \max_{\{b_0(s)\}_{s \in \mathcal{S}}} \sum_{s \in \mathcal{S}} \pi(s)(-RR_2(s)b_0(s))$$

subject to $w_0 \geq -\sum_{s \in \mathcal{S}} \pi(s)b_0(s)$ and $-Rb_0(s) \geq 0, \forall s \in \mathcal{S}$. If s' such that $R_2(s') = \max_s \{R_2(s)\}$, then $b_0(s') = -w_0/\pi(s')$ and $V_0(w_0) = RR_2(s')w_0$.

Suppose $k_0 > 0$. Then $w_1(s) \geq (A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta))k_0 > 0$, which implies that $k_1(s) > 0$ (and $\nu_1^k(s) = 0$) and $d_2(s) > 0$ (and $\mu_2(s) = 1$). From (25), $\mu_1(s) = R_2(s)$, and (20) and (24) can be written as

$$\mu_0 = RR_2(s) + R\lambda_0(s), \quad \forall s \in \mathcal{S}, \quad (26)$$

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s). \quad (27)$$

Note that this is only possible if $\sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s) \geq \max_s \{RR_2(s)\}$. Moreover, the case where the inequality is an equality is not generic and hence generically $\lambda_0(s) > 0, \forall s \in \mathcal{S}$. But then (11) implies $b_0(s) = R^{-1}\phi_1(s)\theta k_0(1 - \delta)$ and (2) implies $k_0 = w_0/\wp_0$. Using Lemma 3 we get $V_0(w_0) = \sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s)w_0$.

Thus, if $\sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s) > \max_s \{RR_2(s)\}$, $k_0 > 0$ attains a higher value and the optimal k_0 and value attained are as stated in the proposition. Otherwise, $k_0 = 0$ attains a higher value and is hence optimal. \square

Proof of Proposition 2. Suppose $\sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s) > \max_s \{RR_2(s)\}$. Then, by Proposition 1, $k_0 = w_0/\wp_0 > 0$ and $w_1(s) = (A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta))k_0$. Moreover, $k_1(s) = w_1(s)/\wp_1(s)$ by Lemma 3. Thus, $k_1(s)/k_0 = (A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta))/\wp_1(s)$, which is less than 1 as long as $A_1(s) < \phi_1(s)\delta + (\phi_1(s) - R^{-1}\phi_2(s))\theta(1 - \delta)$. Any $A_1(s) < \min\{\phi_1(s)\delta, \phi_1(s) - R^{-1}\phi_2(s)(1 - \delta)\}$ satisfies this condition. Moreover, the condition is satisfied for some $A_1(s) \geq 0$ as long as $\phi_1(s) - R^{-1}\phi_2(s)(1 - \delta) > 0$. \square

Proof of Proposition 3. Note that $\frac{\partial}{\partial \theta} (k_1(s)/k_0) \propto ((\phi_2(s)/\phi_1(s))k_1(s)/(Rk_0) - 1) < 0$ as long as the condition in the statement of the proposition is satisfied. \square

Proof of Proposition 4. Differentiating $k_1(s)/k_0$ with respect to $\phi_1(s)$ gives

$$\frac{\partial}{\partial \phi_1(s)} \left(\frac{k_1(s)}{k_0} \right) = \frac{(1 - \theta)(1 - \delta)}{\wp_1(s)} \left(1 - \frac{\frac{A_1(s)}{(1 - \theta)(1 - \delta)} + \phi_1(s)}{\phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta)} \right) < 0 \quad \square$$

Proof of Proposition 5. Since $k_0 > 0$ and $k_1(s) > 0$, $s \in \mathcal{S}$, (24) and (25) simplify to

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) \mu_1(s) \quad (28)$$

$$\mu_1(s) = R_2(k_1(s), s) \mu_2(s) \quad (29)$$

and $d_2(s) \geq A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1 - \theta)(1 - \delta) > 0$, which implies $\mu_2(s) = 1$. Therefore by (29) $\mu_1(s) = R_2(k_1(s), s)$ and (20) and (28) simplify to

$$\mu_0 = RR_2(k_1(s), s) + R\lambda_0(s), \quad \forall s \in \mathcal{S}, \quad (30)$$

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) R_2(k_1(s), s). \quad (31)$$

Assumption 3 together with equation (30) imply that there are three cases to consider ($\lambda_0(s)$ positive for both states, for the high state only, and for neither state) since $R_2(k_1(H), H) + \lambda_0(H) = R_2(k_1(L), L) + \lambda_0(L)$. When $\lambda_0(s) > 0$, $\forall s \in \mathcal{S}$, $k_0 = w_0/\wp_0$, $k_1(s) = (A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta))/\wp_1(s)$. When moreover $\lambda_0(L) = 0$, then $\mu_0 = RR_2(k_1(L), L)$. Thus, there exists \underline{w}_0 such that the collateral constraint for state L is just satisfied, and

$$\sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s) R_2(\underline{k}_1(s), s) = RR_2(\underline{k}_1(L), L)$$

where $\underline{k}_0 = \underline{w}_0/\wp_0$ and $\underline{k}_1(s) = (A_1(s)f(\underline{k}_0) + \phi_1(s)\underline{k}_0(1 - \theta)(1 - \delta))/\wp_1(s)$. Furthermore, $RR_2(\underline{k}_1(L), L) = \sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s) R_2(\underline{k}_1(s), s) < (\sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s)) R_2(\underline{k}_1(L), L)$ and thus $R < \sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s)$.

When $\lambda_0(s) = 0$, $\forall s \in \mathcal{S}$, $R_2(k_1(H), H) = R_2(k_1(L), L)$, and $\sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) = R$. Thus, there exists \bar{w}_0 such that the collateral constraint against the high state is just satisfied, $\bar{k}_1(H) = (A_1(H)f(\bar{w}_0/\wp_0) + \phi_1(H)\bar{w}_0/\wp_0(1 - \theta)(1 - \delta))/\wp_1(H)$, $R_2(\bar{k}_1(H), H) = R_2(\bar{k}_1(L), L)$ and $\sum_{s \in \mathcal{S}} \pi(s) R_1(\bar{k}_0, s) = R$, where $\bar{k}_1(L)$ is determined using the budget constraints for time 0 and state L at time 1. Finally, $\bar{w}_0 \geq \bar{k}_0\wp_0 > \underline{k}_0\wp_0 = \underline{w}_0$. \square

Proof of Lemma 5. First, $l_0^i(s) \geq 0$ is implied by $R_0^i(s)l_0^i(s) \geq d_1^i(s) \geq 0$ and hence redundant. The first order conditions of the intermediary's problem are $\mu_0 = 1 + \nu_0^d$, $\mu_1(s) = R^{-1} + \nu_1^d(s)$, and $\mu_0 = R_0^i(s)\mu_1(s)$, $\forall s \in \mathcal{S}$. Thus, $R_0^i(H)(R^{-1} + \nu_1^d(H)) = R_0^i(L)(R^{-1} + \nu_1^d(L))$. Since $R_0^i(s) \geq R$ we can set $d_0^i = 0$ w.l.o.g., and hence at most one of $\nu_1^d(s)$ can be strictly positive. Now suppose $R_0^i(s) > R_0^i(s')$, $s \neq s'$. Then $\nu_1^d(s') > 0$ and hence $l_0^i(s') = 0$, that is, there is no intermediated lending against state s' . But for the intermediary to be willing to lend against state s' , he would require an expected return of $R_0^i(s)$, so we can set $R_0^i(H) = R_0^i(L) \equiv R_0^i$. \square

Proof of Proposition 6. The first order conditions are $\mu_0 = 1 + \nu_0^d$, $\mu_1(s) = 1 + \nu_1^d(s)$,

and

$$\mu_0 = R(\mu_1(s) + \lambda_0^i(s)) + R\lambda_0(s) \quad (32)$$

$$\mu_0 = R_0^i(\mu_1(s) + \lambda_0^i(s)) - \nu_0^i(s) \quad (33)$$

$$\begin{aligned} \phi_0\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) \{ & A_1(s)f'(k_0) + \phi_1(s)(1 - \delta)\mu_1(s) + \phi_1(s)\theta(1 - \delta)\lambda_0(s) \\ & + \phi_1(s)\theta^i(1 - \delta)\lambda_0^i(s) \} + \nu_0^k, \end{aligned} \quad (34)$$

where (32), (33), and (34) are the first order conditions with respect to direct finance, intermediated finance, and capital, respectively, and $\lambda_0(s)$ and $\lambda_0^i(s)$ are the Kuhn-Tucker multipliers on the collateral constraints for direct finance and total promises, respectively.

Suppose $\nu_0^k > 0$ and hence $k_0 = 0$. Then $b_0^i(s) = 0, \forall s \in \mathcal{S}$, and $V(w_0) = Rw_0$. Thus, henceforth assume that $\nu_0^k = 0$ and $k_0 > 0$. When $k_0 > 0$, the time 1 budget constraints together with the collateral constraints imply that $d_1(s) > 0$ and hence $\mu_1(s) = 1, \forall s \in \mathcal{S}$.

Suppose $\lambda_0(s) = 0$, for some s . Then (32) and (33) imply that $\nu_0^i(s) > 0$ and hence $\lambda_0^i(s) = 0$. Using (32) for s and s' we have $\mu_0 = R = R + R(\lambda_0^i(s') + \lambda_0(s'))$ and thus $\lambda_0^i(s') = 0 = \lambda_0(s')$. Substituting into (34) we conclude that $R = \sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1 - \delta))/\phi_0$ which is not generically true. Hence, $\lambda_0(s) > 0$ for some s . Indeed, since $R + R\lambda_0^i(s) + R\lambda_0(s) = R + R\lambda_0^i(s') + R\lambda_0(s')$, $\lambda_0(s') > 0$ as well, since otherwise the right hand side would equal R (due to the fact that $\lambda_0(s') = 0$ implies $\lambda_0^i(s') = 0$), a contradiction. Hence, $\lambda_0(s) > 0, \forall s \in \mathcal{S}$.

There are three cases to consider. First, consider the case where $\nu_0^i(s) > 0$ and $\lambda_0^i(s) = 0, \forall s \in \mathcal{S}$. Then $\lambda_0(s) = R^{-1}\mu_0 - 1$ and (34) implies

$$\mu_0^* \equiv \frac{\sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta))}{\phi_0 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)\theta(1 - \delta)}$$

and $V(w_0) = \mu_0^*w_0$. Suppose instead that $\nu_0^i(s) = 0, \forall s \in \mathcal{S}$. Then $\lambda_0^i(s) = (R_0^i)^{-1}\mu_0 - 1$ and $\lambda_0(s) = (R^{-1} - (R_0^i)^{-1})\mu_0, \forall s \in \mathcal{S}$. Substituting into (34) implies

$$\bar{\mu}_0^* \equiv \frac{\sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1 - \theta^i)(1 - \delta))}{\phi_0 - \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(R^{-1}\theta(1 - \delta) + (R_0^i)^{-1}(\theta^i - \theta)(1 - \delta))}$$

and $V(w_0) = \bar{\mu}_0^*w_0$. Also, let $\bar{\varphi}_0$ denote the denominator in $\bar{\mu}_0^*$. Let C denote the numerator in μ_0^* such that $\mu_0^* = C/\varphi_0$ and note that

$$\bar{\mu}_0^* = \frac{C - \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}{\bar{\varphi}_0 - (R_0^i)^{-1} \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}.$$

Hence, $\bar{\mu}_0^* > \mu_0^*$ iff $R_0^i < \mu_0^*$.

Finally, suppose $\nu_0^i(s) = 0$ and $\nu_0^i(s') > 0$. Proceeding analogously we obtain

$$\bar{\mu}_0^*(s) = \frac{C - \pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}{\bar{\varphi}_0 - (R_0^i)^{-1}\pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}.$$

Thus, $\bar{\mu}_0^*(s) > \mu_0^*$ iff $R_0^i < \mu_0^*$. Let $\bar{\mu}_0^* = \bar{C}/\bar{\phi}_0$ and note that

$$\bar{\mu}_0^*(s) = \frac{\bar{C} + \pi(s')\phi_1(s')(\theta^i - \theta)(1 - \delta)}{\bar{\phi}_0 + (R_0^i)^{-1}\pi(s')\phi_1(s')(\theta^i - \theta)(1 - \delta)}.$$

Now, $\bar{\mu}_0^* > \bar{\mu}_0^*(s)$ iff $R_0^i < \bar{\mu}_0^*$. But then, whenever $R_0^i < \mu_0^*$ then $\bar{\mu}_0^* > \bar{\mu}_0^*(s) > \mu_0^*$. \square

Proof of Proposition 7. If $w_0^i \geq \underline{w}_0^i$, the intermediary has sufficient net worth at time 0 to fund the loans that borrowers demand at time 0 at a cost of intermediary funds of R for all dates and states. Moreover, the intermediary has sufficient funds to fund the net lending borrowers require at time 1 in all states at this cost of intermediary funds (by lending to the direct lenders at rate R). Moreover, the lender is indifferent at the margin between consuming a dividend at time 0 and lending to the direct lenders at rate R . \square

Proof of Proposition 8. The borrowers' problem is the maximization of a concave function on a convex set defined by the constraints. By the theorem of the maximum the solution is hence continuous. Thus aggregate loan demand is continuous as well. This is also true for the lender's problem. Now, to be able to provide the required loans, a well capitalized intermediary needs to conserve a strictly positive amount of net worth for state L and, if $w_0^i = \underline{w}_0^i$, conserves no net worth for state H . By continuity then, for w_0^i less than but sufficiently close to \underline{w}_0^i , the financial intermediary continues to conserve net worth for state L . But then $R_0^i(L) = R$. Moreover, again by continuity, the financial intermediary continues to have excess funds in state H at time 1 implying $R_1^i(H) = R$. Since $R_0^i(H)R_1^i(H) = R_0^i(L)R_1^i(L)$, $R^i \equiv R_0^i(H) = R_1^i(L)$. Moreover, $R^i > R$ since otherwise there would be excess demand for intermediary loans. Thus, $\varsigma_1(L) = R^i - R > \varsigma_0 = \pi(L)(R^i - R) > \varsigma_1(H) = 0$. \square

Proof of Proposition 9. When the intermediary is almost well capitalized we have by continuity that

$$\frac{k_1^g(s)}{k_0^g} = \frac{A_1(s) + \phi_1(s)(1 - \theta^i)(1 - \delta)}{\phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta) - R_1^i(s)^{-1}\phi_2(s)(\theta^i - \theta)(1 - \delta)}$$

and thus $\frac{d}{dw_0^i}(k_1^g(s)/k_0^g) = \frac{\partial}{\partial R_1^i(s)}(k_1^g(s)/k_0^g) \frac{d}{dw_0^i}(R_1^i(s)) > 0$. \square

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Table 1: Limited Intermediary Capital: An Example

| Panel A: Parameters | | | |
|---|---------------------------|---------------------------|------------------------------|
| Type distribution | $\psi(g) = 0.50$ | $\psi(b) = 0.50$ | |
| Endowments | $w_0(g) = 1$ | $w_0(b) = 0.50$ | |
| Lenders' time preference | $R = 1/\beta = 1.05$ | | |
| Technology | $\delta = 0.10$ | $\theta = 0.80$ | $\theta^i = 0.90$ |
| Maximum scale | $\bar{k} = 7$ | | |
| Distribution of states | $\pi(H) = 0.5$ | $\pi(L) = 0.5$ | |
| Capital prices | $\phi_0 = 1$ | | |
| | $\phi_1(H) = 1$ | $\phi_1(L) = 0.965$ | $\phi_2(H) = \phi_2(L) = 1$ |
| Productivity type g | $A_1(H g) = 0.45$ | $A_1(L g) = 0.10$ | $A_2(H g) = A_2(L g) = 0.20$ |
| Productivity type b | $A_1(H b) = 0.35$ | $A_1(L b) = 0.00$ | $A_2(H b) = A_2(L b) = 0.15$ |
| Panel B: Well Capitalized Intermediary ($\underline{w}_0^i = 0.281$) | | | |
| Net worth type g | $w_1(H g) = 2.231$ | $w_1(L g) = 0.772$ | |
| Net worth type b | $w_1(H b) = 0.000$ | $w_1(L b) = 1.050$ | |
| Capital type g | $k_0(g) = 4.131$ | | |
| | $k_1(H g) = 7.000$ | $k_1(L g) = 3.988$ | |
| Capital type b | $k_0(b) = 0.000$ | | |
| | $k_1(H b) = 0.000$ | $k_1(L b) = 5.424$ | |
| Panel C: Intermediary with Limited Capital (w_0^i 2.5% less than \underline{w}_0^i) | | | |
| Net worth type g | $w_1(H g) = 2.224$ | $w_1(L g) = 0.770$ | |
| Net worth type b | $w_1(H b) = 0.000$ | $w_1(L b) = 1.050$ | |
| Capital type g | $k_0(g) = 4.118$ | | |
| | $k_1(H g) = 7.000$ | $k_1(L g) = 3.944$ | |
| Capital type b | $k_0(b) = 0.000$ | | |
| | $k_1(H b) = 0.000$ | $k_1(L b) = 5.382$ | |
| Spreads | $\varsigma_0 = 0.95\%$ | | |
| | $\varsigma_1(H) = 0.00\%$ | $\varsigma_1(L) = 1.90\%$ | |