Incomplete Markets, Idiosyncratic Shocks and Optimal Monetary Policy *

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Abstract

A widespread result in monetary policy literature is that the price level should be stabilized and, as corollary, the nominal interest rate should vary with the *Wicksellian* determinants of the real interest rate. The present paper studies how this result is altered when the representative agent assumption is abandoned and financial wealth heterogeneity across households is introduced. I derive a welfare-based loss function for the policy maker which includes an additional *target* related to the cross-sectional distribution of household debt. My results differ from standard ones in two respects. First, thanks to its ability to affect interest payments volatility, monetary policy has real effects even in a flexible-price cashless-limit environment. Second, in a setup with nominal rigidities, price stability is no longer optimal. The extent of deviation from price stability depends on the initial level of debt dispersion. I use US micro data to calibrate the model and I find that the departure from price stability is still relatively small under the baseline calibration. Finally, the paper also studies the design of an optimal simple implementable rule. I find that *superinertial* rules

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that also include a separate target on debt dispersion outperforms standard Taylor rules.
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1 Introduction

Since the end of '80s many countries have experienced a sharp increase in households' debt, a phenomenon which has drawn the attention of policy makers and economists. This phenomenon is even more dramatic at a disaggregate level: aggregate data on the indebtedness of the household sector conceal substantial variation in the distribution of the debt across individual households. For example, in the United States, in 2001, around 45% of households had mortgage debt, while around one quarter of households held no debt at all.

In such an environment, monetary policy is likely to have stronger effects on the real sector.[1] In particular it may play a substantial redistributive role on households' wealth affecting their balance sheet.

In the present paper we assess whether households financial imbalances should be a (quantitatively) relevant source of concern for the monetary authority and ask how, in this scenario, monetary policy should be optimally designed.

Despite the relevance of this issue, the economic literature, so far, has not provided a clear-cut answer. A strand of the literature has studied the macroeconomic implications of household debt by introducing heterogeneous agents. Many works, however, lack welfare analysis and thus cannot provide any normative guidance. Barnes and Young (2003), for example, find that interest rate shocks contribute importantly to changes in household debt. Iacoviello (2005) shows that, in presence of borrowing constraints, a rise in income inequality could lead to an increase in debt and debt dispersion. Den Haan (1997) asks whether the cross-sectional distribution of asset holdings has a quantitative role in the determination of the real interest rate. In a recent work, Doepke and Schneider (2005) show that, a moderate inflation episode can lead to a high redistribution of wealth because of changes in the value of nominal assets.

Other papers, instead, do perform welfare analysis but lack business cycle considerations. Albanesi (2005), for example, studies optimal monetary and fiscal policy with heterogeneous holdings of money balances. In this case

[1]see Debelle (2004) for example.

distributional considerations may determine a departure from the Friedman rule. On similar lines, Akyol (2003) finds that, in a model with a liquid and illiquid asset, a positive inflation can improve risk sharing, and therefore, welfare.

Hence, the above mentioned literature misses to put together welfare analysis and business cycle fluctuations. Moreover, there is no role for monetary policy coming from nominal rigidities as in the recent monetary business cycle literature.[2] On the contrary, this second strand of literature, assuming a representative agent, has been widely focused on normative issues regarding the role of monetary policy in stabilizing the economic cycle - e.g. King, Khan and Wolman (2003) and Rotemberg and Woodford (1997). A distinctive conclusion, recurrent in this framework, may be illustrated by the recent work of Schmitt-Grohe Uribe (2005). They show that, even in a rich medium-scale model with a large variety of frictions, price stability remains quantitatively a central goal for monetary policy.

However to address questions regarding households financial imbalances it seems crucial to depart from a complete market/representative agent hypothesis.

This paper tries to link the two strands of literatures: I introduce heterogenous households in a tractable sticky price model - e.g. Gali (2001). In particular I relax the complete market assumption - only nominal riskless bonds are available - and I assume that households may differ in their asset holdings. This tantamount to a model where agents hold heterogeneous portfolios with different exposure to interest rate risk.

I show that, for this setup, the welfare-based loss function for the policy maker includes an extra *target variable* in addition to the ones typically found in the literature (inflation and output gap). In other words, the introduction of heterogenous nominal bond holdings entails that the central bank tries to minimize also a measure of consumption dispersion across households - which, in turn, is strictly related to the cross-sectional distribution of household-debt.

This implies a departure from standard results of the literature in two aspects. First, thanks to its ability to affect interest payments volatility,

[2]Exceptions can be found in Mendicino and Pescatori (2004)

monetary policy has real effects even in a flexible-price cashless-limit environment. Second, in a setup with nominal rigidities, price stability - the standard goal of monetary policy in that case - is no longer optimal.

In other words, the introduction of debt-burdened households creates a trade-off between interest rate reactions meant to stabilize prices and the ones that stabilize the debt service volatility. In fact, the volatility of interest payments introduces a source of idiosyncratic uncertainty at household level - which, in turn, is welfare reducing.

Finally, we also show that a measure of debt dispersion would be an important separate target for an optimally designed simple implementable rule. More precisely, rules that also include a separate target on debt dispersion outperforms standard rules which only target inflation and output gap.

The extent of deviation from price stability depends on the economy's *initial* level of debt dispersion. In order to calibrate the initial debt dispersion I use micro data from the US Board of Governors' Survey of Consumers Finances for the year 2001. Under the baseline calibration our model suggests that its representative agent counterpart (i.e. the equivalent model with symmetric asset positions) may constitute a reasonable approximation: the magnitude of deviation from zero inflation we get is small. However, unlike in the representative agent model, the initial response of nominal interest rate to disturbances is much smaller.

As last remark, we observe that a high dispersion in the initial net-debt positions does call into question the price stability goal. In this case, aggregate shocks affecting the *natural* rate of the economy would imply a large and persistent deviation from zero inflation.

2 The Model

The baseline model is a cashless limit dynamic sticky price model with common factor markets and no capital accumulation (Clarida et al., 1999, Gali, 2001; Rotemberg and Woodford, 1997, 1999). I depart from the baseline model in two aspects: markets are incomplete and the initial distribution of nominal debt across households is not degenerate.[3]

There are two sources of aggregate uncertainty: the level of total factor productivity, A, and the level of real government purchases, G, which are assumed to be financed with lump-sum taxes. Aggregate shocks may have an idiosyncratic impact on households budget constraint.

The government can finance the exogenous stream of public consumption with lump sum taxes T^G. In period-0 the government is also able to implement a redistributive transfers scheme, $\bar{\tau}$, to favor wealth equality. However it is not allowed to change it thereafter.

The monetary authority controls the short term nominal interest, R, takes the fiscal redistributive scheme as given and can commit to a state-dependent rule. This last one allows the monetary authority to respond to all of the relevant state variables of the economy.

In this section, I describe a recursive equilibrium, with households and firms solving dynamic optimization problems for given fiscal and monetary policy rule.

2.1 Households

I assume a continuum of households indexed by $h \in [0,1]$ maximizing the following utility

$$U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t^h) - v(N_t^h) \Big]$$

E_0 denotes the expectation operator conditional on the information set at date-0 and β is the inter-temporal discount factor, with $0 < \beta < 1$. Households get utility from consumption and disutility from working. Both func-

[3] Using the US Board of Governors Survey of Consumer Finances for the year 2001 I find that the net nominal credit position substantially differ across households (see calibration section for further details). The first 10% of of the distribution holds a stock a net-debt higher than 120,000USD; while the last 10% (the 90th percentile) holds a stock of net-credit of about 880,000USD. The median is approximately zero.
From a modeling point of view we could generate a non-degenerate distribution of assets across agents introducing idiosyncratic income or preference shocks at household level. However, for tractability reasons and because they are irrelevant for the exposition of the main arguments, we do not need introduce them.

tions are strictly increasing and twice differentiable, however $v(.) : [0, \bar{N}^{+}) \rightarrow \mathbb{R}$ is strictly convex while $u(.) : \mathbb{R}_{+} \rightarrow \mathbb{R}$ is strictly concave in the consumption index C. This is defined as a Dixit-Stiglitz aggregator of different goods produced in the economy with constant elasticity $\theta > 1$:[4]

$$C_t^h = \Big(\int_0^1 c^h(z)^{\frac{\theta-1}{\theta}} dz\Big)^{\frac{\theta}{\theta-1}}$$

Let P_t represent the aggregate price index such that

$$P_t^{1-\theta} = \int_0^1 P_t(z)^{1-\theta} dz$$

where $P_t(z)$ denotes the price of good-z. Then, for each household, the optimal allocation of a given amount of expenditures among the different goods generates the good-z demand schedules

$$c_t^h(z) = \Big(\frac{P_t(z)}{P_t}\Big)^{-\theta} C_t^h \tag{1}$$

Each household-h earns a nominal wage W_t per hour worked and can buy or issue a nominal riskless bond B_t^h (IOUs) - its market price $1/R_t$ is taken as given. The variable X_t^h collects terms which are rebated to households in lump sum fashion: it summarizes a lump sum government tax (transfer) T_t^h and lump sum profits from firms F_t^h. So the budget constraint takes the following form:

$$P_t C_t^h + B_t^h / R_t = B_{t-1}^h + W_t N_t^h + P_t X_t^h \tag{2}$$

where

$$X_t^h = T_t^h + F_t^h \tag{3}$$

In period-0 firms shares are equally split across households and are not subsequently traded.[5] In other words we can write $F_t^h = F_t$ where F_t is the

[4] In a representative agent economy having no upper bound for hours worked do not represent a serious concern. However, when there is a continuum of heterogenous agents, the possibility of supplying an unbound amount of hours, having the wage unaffected, is not realistic and would pose no lower bound for the natural debt limit.

[5] The trading restriction imposed here on stocks may not be innocuous given the absence of complete financial markets. However more than one concern has prevented us

total amount of profits made in the economy. The government tax (transfer) can be divided into an aggregate tax T_t^G - needed to finance current government spending G_t - and a household specific constant transfer $\bar{\tau}^h$. So the additive component X_t^h of the budget constraint can be written as $X_t^h = \bar{\tau}^h - T_t^G + F_t$.

I now turn to households necessary conditions for optimality. For each household-h the intra-temporal consumption-leisure choice reads (I write the real wage as $W_t^r \equiv W_t/P_t$)

$$W_t^r = v_n(N_t^h)/u_c(C_t^h) \tag{4}$$

while the inter-temporal optimality condition is given by the Euler equation

$$\beta R_t E_t \frac{u_c(C_{t+1}^h)}{P_{t+1}} = \frac{u_c(C_t^h)}{P_t} \tag{5}$$

Savers will purchase debt issued by borrowers only if they know that they can be repaid *almost surely*, I thus introduce a *natural debt limit*

$$B_t^h/P_t \geq -\phi_b^h \tag{6}$$

The value of ϕ_b^h is the maximum level of debt a household is able to repay satisfying the consumption plan $\{C_t^h\}_{t=0}^{\infty}$ to be a non-negative random sequence (for a derivation of the natural debt limit in this economy see Appendix section G).

2.2 Firms

I assume a continuum of firms, each producing a differentiated good with a technology

$$y_t(z) = A_t N_t(z) \tag{7}$$

where (log) productivity $a_t = \log(A_t)$ follows a Markov-stationary exogenous stochastic process.

to introduce this additional feature. Mainly I believe that a sticky price model is not well suited to describe firms' profits behavior over the business cycle - see for example Christiano et.al. 1997.

I will also assume that employment is subsidized at a constant subsidy rate $1-\tau_\mu$. Hence, all firms face a common real marginal cost, which in equilibrium is given by

$$mc_t = \frac{W_t^r}{A_t}\tau_\mu \tag{8}$$

The government has the same consumption aggregator as the private sector and it demands the same fraction, τ_t^G, of the output of each produced good $g_t(z) = \tau_t^G y_t(z)$.

Recalling the private sector static-optimality condition - equation (1) - I define the aggregate private sector demand for a good-z by summing up individual households' demands: $c_t(z) \equiv \int_0^1 c_t^h(z)dh$.[6]

Hence the total demand function for each differentiated good is

$$y_t^d(z) \equiv c_t(z) + g_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta} Y_t \tag{9}$$

where

$$Y_t = \left(\int_0^1 y(z)^{\frac{\theta-1}{\theta}}dz\right)^{\frac{\theta}{\theta-1}} \text{ and } G_t = \tau_t^G Y_t$$

denotes the aggregate (demanded) output and the aggregate government spending, respectively.

Firms are monopolistic competitors and are allowed to change prices with a Calvo probability $1-\psi$. Each household-shareholder h would like to have firms maximize discounted profits using its own stochastic discount factor $\Lambda_{t,t+k}^h$. The pricing-policy that a shareholder-h would like to see implemented in firm-z is:[7]

$$\sum_{k=0}^{\infty}(\psi\beta)^k E_t \Lambda_{t,t+k}^h P_{t+k}^\theta Y_{t+k}[P_t^{h,\star}(z)/P_{t+k} - \frac{\theta}{\theta-1}mc_{t+k}] = 0 \tag{10}$$

If managers have delegated a linear rule then, under the assumption of zero steady state inflation, shareholder-h would like to see implemented

[6]For notational convenience I will introduce the distribution of agents over variables only when strictly necessary.

[7]For a derivation and interpretation of the firms' optimality condition see Woodford 2003 or Gali 2001, among others

$$\log P_t^{h,\star}(z) = \frac{\theta}{\theta - 1} + (1 - \psi\beta) \sum_{k=0}^{\infty} (\psi\beta)^k E_t \Big[\log(mc_{t+k} P_{t+k}) \Big] \qquad (11)$$

From above expression we see that the equilibrium choice of the relative price $P_t^{h,\star}(z)$ is the same for all resetting firms and across all shareholders. In other words, whatever is the distribution of voting rights across households, in a first order approximation, they would like to see implemented the same pricing rule - which has a simple interpretation: firms set prices at a level such that a (suitable) weighted average of anticipated future markups matches the optimal frictionless markup $\theta/(\theta - 1)$.

2.3 The Government

The government lump sum tax/subsidy, T_t^h is household specific. However it can be split into two components: an aggregate component, T_t^G, and a constant redistributive component, $\bar{\tau}^h$. The latter is a constant redistribution scheme chosen at time-0 before any shock realization. I assume it has zero mean: $\int_0^1 \bar{\tau}^h dh = 0$. The former component, T_t^G, is instead the same for each household and is aimed to finance current government expenditure G_t such that the government runs a balanced budget deficit in each period. At all times it must hold

$$T_t = -\int_0^1 T_t^h = T_t^G - \int_0^1 \bar{\tau}^h = T_t^G = G_t \qquad (12)$$

The availability of lump sum taxes, and the absence of transaction frictions, renders the way government finances its current deficit irrelevant also in an heterogenous agent model - which is not necessarily true when money balances are not a dominated asset. [8] At the same time the availability of

[8] Akyiol (2004) studies a heterogenous-agents endowment economy with lump sum taxes and open-market operations. An open market operation involves a transfer to agents holding government debt. Given that there is a non-degenerate distribution of agents with respect to bond holdings, there is a different level of transfer to each agent which is not the case when the government makes a lump sum transfer to each agent. This does not happen in a representative agent models where lump sum transfers of money (i.e. "helicopter drop") and retiring existing debt through open market operations are equivalent.

lump sum taxes imply that there is no need of using inflation as absorber of unexpected adverse fiscal shocks - as often studied in the public finance literature.[9] Thus, the structure imposed to the government behavior allows us to focus the analysis on the household liabilities only.

I define a fixed redistributive transfers scheme to be a measure (or the cumulative marginal distribution) of households, Φ^{τ}, over transfers $\bar{\tau}$, satisfying $\int_{-\infty}^{\infty} \tau d\Phi^{\tau}(\tau) = 0$. The government has to choose Φ^{τ} once and for all at time-0. Let Φ_t be the distribution of households over the beginning of period bond holdings. I assume that the choice of Φ^{τ} must be made prior to any shock realization. This entails that the government information set of time-0 is simply given by the initial measure of households Φ_{-1} over bond holdings.[10]

For the role and interpretation of the transfer scheme and also for an alternative setup without transfers see section (4).

2.4 Monetary Authority

I abstract from monetary frictions and I assume that the central bank can control the riskless short-term gross nominal interest rate R_t.[11]

The zero lower bound on nominal interest rate is assumed to be never binding under the optimal policy regime. Finally, I also assume that the central bank has full information in setting its instrument.

The time-t available information is captured by the all relevant time-t state of the economy. In particular, as it will be clear shortly, I allow monetary authority to respond to an exogenous state vector Z_t, to an endogenous aggregate state vector S_t and to a third set of co-states denoted $\mathcal{L}_t$. As a matter of notation I write $\omega_t = (Z_t, S_t, \mathcal{L}_t)$ and $R_t = R(\omega_t)$.

[9] See, among others, Chari, Christiano and Kehoe (1994)

[10] In fact the government has more detailed information, it know the asset position of each household.

[11] See Woodford 2003 Ch2 for a discussion about a "cashless" limit economy.

2.5 Recursive Equilibrium

Let $Z_t = (A_t, \tau_t^G)$ be the vector of exogenous economy-wide stochastic processes and Φ_t be the measure (cumulative distribution) of households over asset holdings at time-t. The law of motion concerning Φ_t is described by the function $f(.)$ such that $\Phi_t = f(\Phi_{t-1}, Z_t)$.

Let also

$$\Delta_{p,t} = \int_0^1 \Big(\frac{P_t(z)}{P_t}\Big)^{-\theta} dz \tag{13}$$

represent the price dispersion in the economy. In the case of unfrequent possibilities of readjusting prices $\Delta_{p,t-1}$ becomes a state for our economy.

I can now introduce the aggregate state vector for this economy $\omega_t = (Z_t, \Phi_t, \Delta_{p,t-1}, \mathcal{L}_t)$ and the individual state vector $s_t^h = (b_{t-1}^h, X_t^h, \omega_t)$. The role of the aggregate state is to allow agents to predict future prices and monetary authority actions. The household's problem can be recast in the following recursive form

$$V(s,\omega) = \max\Big[u(C) - v(N) + \beta E\, V(s',\omega')\Big] \tag{14}$$
$$s.t.$$
$$c + b'/R(\omega) = b/\Pi(\omega) + w(\omega)N + X(s,\omega)$$
$$\Phi' = f(\Phi, Z, Z')$$
$$b \geq -\phi_b$$

The policy function for asset investment is $b' = b(s)$.

For given monetary policy and transfer scheme $\Big(R(\omega), \Phi^\tau\Big)$ and an initial condition ω_0 a *recursive imperfectly competitive equilibrium* is a law of motion $f(.)$, value and policy functions V and b, pricing functions $\Big(w(\omega), \Pi(\omega), (p(z))(\omega)_{z\in[0,1]})\Big)$ such that i)V and b solve (14). ii) The pricing functions, together with a law of motion for the price level, solve the resetting firm problem. iii) There is consistency between aggregate variables and summing up of agents optimal choices - i.e. Φ generates bond market clearing $\int_0^1 b' d\Phi = 0$ and labor market clears.[12]

[12] A formal proof of the existence of an equilibrium for an economy very similar ours can be found in Miao 2005.

3 Idiosyncratic Interest Payments Risk

This section is preliminary to the welfare analysis. Here I describe how portfolios heterogeneity coupled with incomplete markets may affect the aggregate equilibrium allocation.

For the rest of the paper I will use the following utility functional form: $u(.)$ is in the CRRA class such that $-u_{cc}C/u_c \equiv \sigma$ is a constant, while $v(.)$ is such that, given some $\delta > 0$, $\varphi \equiv v_{nn}N/v_n$ is at least approximately constant for $N \in I(\bar{N}, \delta)$ - where φ is the inverse of the Frisch labor elasticity.

3.1 Effects on Aggregate Labor Supply

Using the consumption-leisure relation we observe that the individual labor supplies are shifted by the different levels of consumption - which in turn are related to individual wealth. For example, a relatively "poor" household has its labor supply shifted downward: it will work relatively more, given the wage.

Incomplete Markets I now want to see the impact of this shift on the *aggregate* labor supply schedule. We can write the consumption-leisure choice as

$$C_t^h = W_t^{r\,1/\sigma} N_t^{h\,-\varphi/\sigma} \tag{15}$$

Integrating up the above equation with respect to households we recover a relation between aggregate consumption C_t and aggregate labor N_t:[13]

$$C_t = (W_t^r)^{1/\sigma} N_t^{-\varphi/\sigma} \int_0^1 (N_t^h/N_t)^{-\varphi/\sigma} \tag{16}$$

Let

$$\Delta_{n,t} \equiv \int_0^1 (N_t^h/N_t)^{-\varphi/\sigma} \tag{17}$$

[13] I have simply defined aggregate consumption as $C_t \equiv \int_0^1 C_t^h dh$ and aggregate labor supply as $N_t \equiv \int_0^1 N_t^h dh$

denote the labor supply distortion - ultimately linked to wealth dispersion. Taking a log-transformation and using *hats* for *logs* we can reformulate the above expression:

$$\hat{W}_t^r = \varphi \hat{N}_t + \sigma \hat{C}_t - \sigma \hat{\Delta}_{n,t} \tag{18}$$

By Jensen inequality we realize that $\forall \sigma > 0$ and $\varphi > 0$ we have $\log \Delta_{n,t} > 0$.[14] This means that, for a given aggregate consumption, the aggregate labor supply is pushed rightward by an amount proportional to a *measure* eventually related to the economy wide debt dispersion.[15] This creates a time-varying wedge, at aggregate level, between the factor price of labor and the "aggregate" marginal rate of substitution. To understand whether this wedge or its volatility involve an inefficiency we have to introduce a concept of efficiency.

Complete Markets Under the assumption of complete markets households can perfectly insure against interest rate risk.[16] Changes in the prevailing interest rate and inflation would affect each households' budget constraint differently depending on their nominal bonds asset position. However if a full set of state-contingent claims on consumption is available at time-0 then - regardless of the initial asset position - the consumption of each household is perfectly correlated with the one of every other household. This also means that each household will consume as much as the average consumtion times a constant of proportionality

$$C_t^h = \delta(h) C_t; \ \ \forall h \in [0,1] \tag{19}$$

The function $\delta : [0,1] \to \mathbb{R}^+$, satisfying $\int_0^1 \delta(h) dh = 1$, is time invariant and reflects wealth differences across households. It is possible to determine $\delta(h)$ from Φ_{-1} and $\{\tau^h\}_{h\in[0,1]}$.

[14] We can think of $X \equiv N^h/N$ as a positive random variable with mean equal to one. While $f(u) = u^{-\varphi/\sigma}$ is a strictly convex function $\forall \sigma > 0$ and $\varphi > 0$. This means that $E_h[f(X)] > f(E_h(X)) = f(1) = 1$

[15] If we set $\hat{C}_t = \hat{A}_t - \hat{g}_t + \hat{N}_t$ - as it will be clear later - we can write a proper labor supply schedule $\hat{W}_t^r = \sigma \hat{A}_t - \sigma \hat{g}_t + (\sigma + \varphi) \hat{N}_t - \sigma \hat{\Delta}_{n,t}$.

[16] This is true under our specified functional form for the households utility.

As before we can write the following equation

$$\hat{W}_t^r = \varphi \hat{N}_t + \sigma \hat{C}_t - \sigma \hat{\Delta}_{n,t} \tag{20}$$

However now

$$\Delta_{n,t} = \left(\int_0^1 \delta(h)^{-\sigma/\varphi} \right)^{\varphi/\sigma} \tag{21}$$

This means that $\Delta_{n,t}$ is a constant across time. We will refer to it simply as Δ_n.

3.2 Efficient allocation vs Flexible Price Equilibrium

To stress the role played by incomplete markets, I first shut off the distortion stemming from price stickiness, I will reintroduce it at the end of this section.

In a environment without nominal rigidities the price decision rule reduces to a constant mark-up μ over the real marginal cost regardless of household sector. Let the employment subsidy exactly offset the monopolistic distortion, i.e. $\mu\tau_\mu = 1$, thus a symmetric equilibrium implies that the real wage $\hat{W}_t^r = \hat{A}_t$ and $\hat{N}_t = \hat{Y}_t - \hat{A}_t$. Using the resource constraint, $\hat{C}_t = \hat{Y}_t - \hat{g}_t$ together with production function I substitute out aggregate consumption and aggregate labor from the previous equation.[17] Hence I am able to write the flexible price (*natural*) level of output Y_t^f as:

$$\hat{Y}_t^f \equiv \frac{\sigma}{\sigma + \varphi} \hat{g}_t + \frac{1 + \varphi}{\sigma + \varphi} \hat{A}_t + \frac{\sigma}{\sigma + \varphi} \hat{\Delta}_{n,t} \tag{22}$$

In the case of complete markets we have shown that Δ_n is a constant. I call the associated level of output as *efficient* output Y^e.[18][19] Using logs we have

$$\hat{Y}_t^e = \frac{\sigma}{\sigma + \varphi} \hat{g}_t + \frac{1 + \varphi}{\sigma + \varphi} \hat{A}_t + \frac{\sigma}{\sigma + \varphi} \hat{\Delta}_n \tag{23}$$

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[17] I have also made use of $\hat{g}_t \equiv -\log(1 - \tau_t^G)$

[18] When $\Delta_n = 1$ the flexible price allocation is equivalent to the one usually found in the literature.

[19] Aggregating individual Euler equations we can also define the *efficient rate of interest* as $r_t^e \equiv \sigma E_t \Delta \hat{C}_{t+1}^e = \sigma E_t \Delta \hat{Y}_{t+1}^e - \sigma E_t \Delta \hat{g}_{t+1}$.

This is the equilibrium allocation that would be obtained under flexible prices, perfect competition, no distortionary taxation plus complete markets.

So from the previous equation we can find an exact relation between the output prevailing in the flexible-prices environment and the *efficient* level of output

$$\hat{Y}_t^f - \hat{Y}_t^e = \frac{\sigma}{\sigma + \varphi}(\hat{\Delta}_{n,t} - \hat{\Delta}_n) \tag{24}$$

Thus deviations of $\Delta_{n,t}$ from Δ_n introduce a real imperfection in the economy that creates a wedge between the *natural* and the *efficient* level of output. [20] [21]

We also notice that not only Y_t^f does not deliver the efficient allocation but it is also not independent of monetary policy, to the extent that the latter can affect $\Delta_{n,t}$.

We now turn to the sticky price model. Using equation (9) we write the total hours demanded by firms

$$N_t = \int_0^1 N_t(z)dz = \frac{Y_t}{A_t}\Delta_{p,t} \tag{25}$$

where

$$\Delta_{p,t} = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\theta} dz \tag{26}$$

is the usual measure of price dispersion - which, in turn, is the source of welfare losses from inflation or deflation.

We now establish an exact relation between the sticky price output Y_t and its efficient level Y_t^e, which allow us to define an output gap measure $x_t \equiv \hat{Y}_t - \hat{Y}_t^e$ and to disclose the role played by the two sources of distortion.

Using the household first order conditions we can find an exact relation that expresses the marginal costs as function of the deviation of output from the *efficient* level of output and we write (see Appendix-B for details)

$$\hat{mc}_t = (\sigma + \varphi)x_t + \varphi\hat{\Delta}_{p,t} - \sigma(\hat{\Delta}_{n,t} - \hat{\Delta}_n) \tag{27}$$

[20] It is also worth noting that a higher dispersion of hours worked, shifting the labor supply downward, generates *overproduction* pushing aggregate output over its efficient level.

[21] For a related concept, although introduced in a different environment, see also Blanchard and Gali 2005

price distortion $\Delta_{p,t}$ and the labor supply distortion $\Delta_{n,t}$ affects the output gap.

Hence, we have determined an additional source of deviation from the efficient level of output which a benevolent policy maker would like to offset. Generally speaking we can identify two different dimensions at which we could analyze how heterogeneity may generate welfare concerns.

In a *static* dimension the level around which $\Delta_{n,t}$ oscillates - which will be Δ_n - may reduce social welfare given that it represents the long-run differences in consumption and leisure across households. However any policy action meant to change Δ_n would not be a pareto improvement but would depend on the way we express social preferences and we care about wealth inequality.

In a *dynamic* dimension instead, taken as given the level Δ_n, reducing the volatility of $\Delta_{n,t}$ represents a strictly pareto improvement. The volatility is in facts a consequence of households' impossibility to hedge perfectly against aggregate shocks that can affect interest rate and inflation an so interest rate payments.

In the next section I will analyze in further details the role played by fiscal and monetary policy. Prominently, I will show that the monetary authority does *not* have necessarily to deal with inequality - the static dimension of the problem which should be more a fiscal policy concern. However, even in this case, it will be clear that a central bank still plays a crucial role in offsetting the redistributive impact that *aggregate* shocks have on household budget constraint - the dynamic dimension of the problem. Moreover, I will clarify why the stock of debt/assets accumulated by households becomes a source of idiosyncratic uncertainty at household level - which, in turn, is the source of volatility for our distortion $\Delta_{n,t}$.

4 Welfare Analysis

In this section we lay out the problem of a benevolent policy maker reacting to aggregate exogenous disturbances when the economy is populated by a continuum of households which show a non-degenerate distribution over nominal asset holdings. The standard stabilization prescription of replicating the flexible price equilibrium allocation is challenged. With an incomplete market structure and portfolio heterogeneity featuring in the economy we must now face also the *redistributive* character of standard policy recommendations and the implied distortion.

The policy objective of a benevolent policy maker is maximizing a welfare function W which aggregates agents' utilities $\mathrm{W} : \mathcal{U} \rightarrow \mathbb{R}$.[22]

$$\mathrm{W}_t = E_t \sum_{k=0}^{\infty} \beta^k \int_0^1 \eta(h)[u(C_t^h) - v(N_t^h)]dh \tag{28}$$

where $\eta(h) : [0, 1] \rightarrow \mathbb{R}^+$ represents a time-invariant weighting function.

Transfers Scheme Approach When transfers are optimally chosen (see next section) our economy oscillates around the *efficient* and *socially desirable allocation* - for any arbitrary initial asset distribution.[23] This is a necessary condition for the derivation of a quadratic welfare-based loss function.

For the case $\eta(h) = 1$ every household is weighted the same: the above welfare criterion, given strictly concave utility functions, strictly prefers consumption (wealth) equality. In this case transfers would be chosen in order to restore - in absence of any shock realization - wealth equality.

Unequal Pareto Weights Approach Without any transfer scheme ($\tau^h = 0 \; \forall h \in [0, 1]$) wealth would be unequally distributed. Creditors would be rich and debtors poor. However we can always find a positive weighting function $\eta(h)$ such that - in absence of any shock realization - the welfare criterion is maximized. This is to say that the welfare criterion would relatively overweight rich households. It turns out that such a weighting function would be the one that makes a social planner recover the complete markets solution discussed in paragraph 3.1. The weights would be given by the inverse of each initial households marginal utility. They can be normalized such that

[22]Qualitatively, our results do not depend from the welfare criterion chosen, in fact the less utilitarian is the welfare function the stronger are our results.

[23]For a definition of efficient allocation in our economy see section 3

we can use the steady state consumption - i.e. $\eta(h) = 1/u'(\bar{C}^h)$.[24]

Both approaches would make the central bank accept the initial (and long run) wealth inequality. Loosely speaking this is equivalent to a monetary authority that accepts the wealth distribution *in statu quo nunc*.

In the appendix (D) I will show that the two approaches give the same results. In what follows I will take equal weights $\eta(h) = 1$ and transfers chosen to deliver a socially desirable steady state from which the monetary authority does not have incentive to deviate - i.e. wealth equality.

4.1 Optimal Policy

We assume that the optimal policy honors commitments made in the past. This form of policy commitment has been referred to as optimal from a *timeless perspective* (see Woodford). The difference with respect to a standard Ramsey problem is that we will be looking for policy functions that are time invariant. In other words the monetary or fiscal authority cannot exploit any advantage at time-0.[25]

The optimal fiscal and monetary policy is a rule for $\{R_t\}_{t\geq 0}$ and a feasible fixed transfer system $(\bar{\tau}^h)_{h\in[0,1]}$ which are consistent with the imperfectly competitive equilibrium (CE) defined in section (2.5) and maximize the welfare function as defined in equation (28) given exogenous processes Z_t, initial conditions S_{-1} and s_{-1}, and values for a set of Lagrange multipliers $\mathcal{L}$ associated with the constraints introduced for satisfying CE-conditions dated $t < 0$.

Thus we have now determined the extra state variables $\mathcal{L}_t$ to which the monetary authority was viewed as responding to in section (2.4.).

In carrying out our analysis we will not have to determine the value

[24]Let $\tilde{\eta}(h) = 1/u'(C_0^h)$. We use the following normalization:

$$\eta(h) \equiv \tilde{\eta}(h)\frac{u'(C_0^h)}{u'(\bar{C}^h)} = \tilde{\eta}(h)\frac{u'(C_0)u'(\delta(h))}{u'(\bar{C})u'(\delta(h))} = \tilde{\eta}(h)\frac{u'(C_0)}{u'(\bar{C})}$$

[25]In a closely related setup Khan et al. (2003) introduce in the standard (unconstrained) Ramsey problem lagged Lagrange multiplier corresponding to the forward-looking constraints in the initial period making the problem stationary. The initial values are chosen to be the steady state values. For a discussion see also Benigno-Woodford 2005

functions for the private sector behavior but we will simply focus on the first order conditions. In order to do it we will take a local approximation of the model which means we need to find a reasonable point around which to perform the approximation (for a discussion on the approximation procedure see appendix). A natural candidate is the steady state of the deterministic version of our model where aggregate shocks have been shut off. However even in this case fiscal and monetary authority can affect the steady state values of the endogenous variables of the system. Because we want to keep staying close to those values - when starting from initial conditions close enough to them and for small enough exogenous disturbances - then we have to characterize the optimal long run steady state.

In other words the presumption is that the optimal policy which guides the economy through business cycle fluctuations will be oscillating around the optimal *long-run* policy - and not about a generic steady state. In the next paragraph we specify what we mean for optimal long run policy.

4.1.1 Optimal Steady State

We characterize the solution to our policy problem only for initial conditions near certain steady-state values, allowing us to use local approximations when we characterize optimal policy.[26] Hence our local characterization describes policy that is optimal from a timeless perspective in the event of small disturbances.

For any given initial distribution of debt across households Φ_{-1} we wish to find an initial degree of price dispersion $\Delta_{b,-1}$ and a transfer system $\Phi^{\tau} = H(\Phi_{-1}, \Delta_{p,-1})$ - implemented before any shock realizations - such that the solution of the deterministic problem involves a constant policy in each period and where $\bar{\Delta} = \Delta_{p,-1}$ and $\bar{\Phi} = \Phi_{-1}$.

We state the following proposition (for a proof see Appendix-C)

[26] In a representative agent model it can be usually shown that these steady-state values have the property that if one starts from initial conditions close enough to the steady state, and exogenous disturbances thereafter are small enough, the optimal policy subject to the initial commitments remains forever near the steady state (see Benigno and Woodford (2005)). This does not necessarily entail the stronger result of convergence. Indeed, in many fiscal policy setup the deterministic model shows a unit root - if the stochastic version has a quasi-random walk - in government bonds.

Proposition 1 *In the deterministic equivalent model where all sources of uncertainty are shut off but for the Calvo signal, the long run optimal monetary policy entails no price dispersion*

$$\bar{\Pi} = \bar{\Delta}_p = 1 \tag{29}$$

and any given initial distribution of households over debt, Φ_{-1}, induces an optimal constant transfer system $\Phi^{\tau}(\bar{\tau}) \propto \Phi_{-1}$ such that for each household we have

$$\bar{\tau}^h = -\bar{b}^h(1/\bar{\Pi} - 1/\bar{R}) = -\bar{b}^h(1-\beta) \tag{30}$$

We next characterize the optimal steady state. By proposition (1) the steady state inflation rate is zero hence the steady state nominal gross interest rate is equal to the inverse of the subjective discount factor $R = 1/\beta$. We can write the steady state budget constraint for a generic household-h as:

$$\bar{C}^h = \bar{b}^h(1-\beta) + \bar{W}\bar{N}^h + \bar{F} - \bar{G} + \bar{\tau}^h \tag{31}$$

where $\bar{b}^h$ is the initial period bond holdings of household-h. Because the government sets a constant transfer $\bar{\tau}^h = -\bar{b}^h(1-\beta)$ for each household-h in steady state we have

$$\bar{C}^h = \bar{C} \text{ and } \bar{N}^h = \bar{N} \; \forall h \in [0,1] \tag{32}$$

Firms optimal price decision rule implies that the constant markup over marginal costs must be equal to one. Hence we can finally write

$$\bar{N} = \bar{A}^{\frac{1-\sigma}{\sigma+\varphi}}(1-\bar{\tau}^g)^{-\frac{\sigma}{\sigma+\varphi}} \; \forall h \in [0,1] \tag{33}$$

and

$$\bar{C}^h = \bar{C} = \bar{Y} - \bar{G} = \bar{A}^{\frac{1+\varphi}{\sigma+\varphi}}(1-\bar{\tau}^g)^{\frac{\varphi}{\sigma+\varphi}} \; \forall h \in [0,1] \tag{34}$$

The most important consideration to be made is that, thanks to the transfers scheme, even in presence of debt dispersion the steady state found is *non-distorted* and *socially desirable*: marginal rate of substitutions equal marginal rate of transformations and the consumption allocation maximizes the welfare criterion chosen.[27] The fact that I will analyze the economy oscillating about its efficient level of output is crucial for the derivation of a purely quadratic objective function for the policy maker.

4.2 The Economy under the Optimal Transfers Scheme

I define $\tilde{b}_t^h \equiv b_t^h - \bar{b}^h$ and, exploiting that $\bar{\tau}^h = -\bar{b}^h(1-\beta)$, I re-formulate the agent-h budget constraint:

$$C_t^h + \tilde{b}_t^h / R_t = \tilde{b}_{t-1}^h / \Pi_t + W_t^r N_t^h + F_t - G_t + \bar{b}^h \left(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t} \right) \quad (35)$$

From this expression we see that heterogenous debt holdings introduce a source of idiosyncratic uncertainty at household level, which is captured by $\bar{b}^h(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t})$.

If the economy oscillates close enough to its efficient level then the value of $\tilde{b}_t^h$ is relatively small compared to $\bar{b}^h$. This means that $\bar{b}^h(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t})$ represents the main component of the impact on households balance sheet of fluctuations in interest payments (or interest income).

Intuitively, anticipating results, a monetary authority who is willing to shut off the debt servicing volatility should set the above term to zero:

$$\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t} = 0 \quad (36)$$

which implies

$$R_t^o = \frac{1 + \pi_t}{\beta - (1 - \beta)\pi_t} \quad (37)$$

The economic intuition is that, ceteris paribus, if we want to leave households asset position unchanged $\tilde{b}_t^h = 0$ than the nominal interest rate must mildly react to inflationary (deflationary) pressures. On the other hand any reaction of the nominal rate different from R_t^o leads to what we call an *arbitrary redistribution*: it generates a wealth effects that adds consumption volatility.

[27] I recall that we have chosen $\eta(h) = 1$.

4.2.1 A Log-Linear Representation

We develop further the previous idea recasting the system in deviations from average quantities (for a discussion on the approximation method see Appendix 6.6). We first define the consumption and employment cross-sectional gap

$$\tilde{C}_t^h \equiv \log C_t^h / C_t = \hat{C}_t^h - \hat{C}_t$$
$$\tilde{N}_t^h \equiv \log N_t^h / N_t = \hat{N}_t^h - \hat{N}_t$$

The resource constraint for this economy without capital accumulation - which is simply $C_t = (1 - \tau_t^G) Y_t$ - can be written as

$$C_t = W_t^r N_t + F_t - G_t \tag{38}$$

So we can re-write the household-h budget constraint of equation (35) subtracting it the above resource constraint (38)

$$C_t^h - C_t + \tilde{b}_{t-1}^h / R_t = \tilde{b}_{t-1}^h / \Pi_t + W_t^r (N_t^h - N_t) + \bar{b}^h \left(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t}\right) \tag{39}$$

Taking a linear expansion of this equation around the steady state of the deterministic model we have

$$\tilde{C}_t^h + \beta \tilde{b}_t^h = \tilde{b}_{t-1}^h + \bar{W}^r \bar{N} \tilde{N}_t^h + \bar{b}^h (\beta \hat{R}_t - \pi_t) \tag{40}$$

Recalling the result of section (3)

$$C_t = W_t^{r\,1/\sigma} N_t^{-\varphi/\sigma} \Delta_{n,t} \tag{41}$$

we can find how $\tilde{N}_t^h$ is related to $\tilde{C}_t^h$:

$$\frac{\varphi}{\sigma} \tilde{N}_t^h = -\tilde{C}_t^h + \hat{\Delta}_{n,t} \tag{42}$$

In case of small enough exogenous disturbance the term $\Delta_{n,t}$ will be either small or constant for raking welfare. In fact we have $\Delta_{n,t} \simeq .5\frac{\varphi}{\sigma}Var_h\tilde{N}_t^h$. So we can substitute $\tilde{C}^h$ for $\tilde{N}^h$ in equation (40) using expression (42) [28] to get

$$\kappa_c\tilde{C}_t^h = \tilde{b}_{t-1}^h - \beta\tilde{b}_t^h + \bar{b}^h(\beta\hat{R}_t - \pi_t) \tag{43}$$

where $\kappa_c = 1 + \frac{\sigma}{\varphi}$.

This equation deserves attention. We have rewritten the saving decision of each household, $\tilde{b}_t^h$, as choosing how much to deviate from the steady state level of savings $\bar{b}^h$. This means that, in a local approximation and at any degree, the impact of interest rate and inflation in the budget constraint *through* $\tilde{b}_t^h$ is negligible. The all impact is instead captured by $\bar{b}^h(\beta\hat{R}_t - \pi_t)$ which represents debt servicing deviations from their steady state level for a household that enters the world with a stock of debt equal to $-\bar{b}^h$.

We conclude the description of the system at individual-level by taking a log-linear expansion of the households Euler equation in deviation from aggregate levels.

$$E_t\Delta\tilde{C}_{t+1}^h = -\varphi_b\tilde{b}_t^h \tag{44}$$

We have introduced the term $\varphi_b\tilde{b}_t^h$. If $\varphi_b = 0$ the approximated system would have an exact unit root in individual asset holdings. However, this is not the behavior of the non-linear model - which show a quasi-random walk given the existence of a natural borrowing limit - but is the result of our approximation. In our analysis we choose to capture the quasi-random walk behavior by assuming a very small but strictly positive value for φ_b.[29]

[28] We recall that in steady state we have offset the monopolistic distortion such that $\bar{W}^r\bar{N} = \bar{A}\frac{\tau_\mu}{\mu}\bar{Y}/\bar{A} = \bar{Y}$. We further normalize the output to one $\bar{Y} = 1$.

[29] This can be micro-funded by introducing small quadratic adjustment costs on debt transactions. For a related discussion see also Schimd-Grohe Uribe (2003). See also Kim Kim Kollman 2005 on barrier methods to convert an *optimization problem with borrowing constraints as inequalities into a problem with equality constraints* and then solving the converted model using a local approximation

4.3 A Linear Quadratic Approach - Loss Function

We derive a second order approximation of the policy objective, equation (28), about the deterministic Ramsey steady state derived above. Details of the derivation can be found in the appendix here we simply claim the result:

$$\mathrm{W}_t \simeq E_t \sum_{k=0}^{\infty} \beta^k L_t \tag{45}$$

where

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 (\tilde{C}_t^h)^2 + o(\|S_{t-1}\|^2) \tag{46}$$

The approximation error is strictly related to the deviations of our variables from their steady state values and $\|S_{t-1}\|$ represents a bound on the amplitude to exogenous shocks and to the deviations of the time-t state.[30]

The presence of staggered prices brings in gains from minimizing relative price fluctuations. The relative weight between inflation and output gap, is standard in the literature: $\lambda_x \equiv \frac{\kappa}{\theta}$, where κ is the Phillips curve parameter.[31]

However in our case an additional term is affecting the country welfare: the cross-sectional consumption dispersion. It enters with the following relative weight:

$$\lambda_c \equiv \frac{(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi} \frac{\sigma}{\theta} (1 - \bar{\tau}^G + \sigma/\varphi) \tag{47}$$

The crucial parameters for understanding the conflicts between price stability and debt dispersion are λ_x and the relative risk aversion σ. The higher

[30]In fact it is not always the case that imposing a bound on the amplitude of exogenous disturbance is enough to guarantee that the system will oscillate about the steady state. An explosive system is clearly a counter example, but even stable systems with important amplification mechanism are likely to spend many periods very far away from the point about which the model is approximated. In our case the only variable that is likely to deviate persistently from the steady state is b_t^h. A second problem is that the approximation is taken around the deterministic steady state. The system is more likely to oscillate about the mean of its stationary distributions (i.e. about the stochastic steady state). Aggregate shocks affect the moments of the household wealth distribution which in turn affects prices and hence quantities. Here, again, we have implicitly disregarded this contribution to the oscillation of aggregate and individual variables

[31]In this case is given by $\kappa \equiv \frac{(\sigma+\varphi)(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi}$

the distortion generated by nominal rigidities (i.e. the higher the stickiness ψ or the CES elasticity θ) the lower will be λ_x. On the other hand a higher curvature of the households' utility function would clearly imply - for any given consumption dispersion - a relatively higher social benefit from consumption equality.

The steady state level of government consumption lowers the weight simply because it reduces the steady state level of private consumption. However any $\bar{\tau}_t^G < 1$ gives still a strictly positive weight. The term σ/φ at first sight could be misleading. It is true that the higher the labor elasticity the higher the weight but at the same time the lower will be the consumption dispersion (see also the definition of κ_c in equation (43)).[32]

From the reformulated household budget constraint (equation (43)) we see that our new target $var_h(\tilde{C}_t^h)$ is mainly associated to the source of idiosyncratic uncertainty: the debt servicing (interest income) $\bar{b}^h(\beta\hat{R}_t - \pi_t)$. As we will see in the next section a central bank has enough instruments to soften the impact of aggregate shocks on the households' balance sheet stemming from debt servicing volatility.

4.4 Calibration

As common in the business cycle literature we let the relative risk aversion and the inverse of the Frisch elasticity parameters take values in the following range: $\sigma \in [1, 5]$ and $\varphi \in [0, 3]$.

The time is meant to be a quarter and we assign a value of 0.9902 to the subjective discount factor $\beta = .99$, which is consistent with an annual real rate of interest of 4 percent (see Prescott 1986)

We set the steady state share of government purchases $\bar{\tau}^G = 20\%$ matching the US historical experience in postwar period. Following Sbordone

[32] I will clarify it with an example. In the extreme case $\varphi = 0$ (labor supply perfectly elastic) $\sigma/\varphi \rightarrow \infty$. However it is also true, see equation (..), that $C_t^h = C_t \ \forall h \in [0, 1]$ such that consumption dispersion is zero and the loss function is no longer well defined. For such a case it would be useful to rewrite the loss in term of labor dispersion. After some algebra we can find a relation between the consumptions dispersion and the measure of hours worked dispersion: $\int_0^1 \tilde{C}_t^{h^2} \simeq 2\frac{\varphi}{\sigma}\hat{\Delta}_{n,t}$. If we substitute it in the loss we can define the weight on $\hat{\Delta}_{n,t}$ as$\lambda_\Delta = 2\lambda_x(1 - \frac{\sigma\tau^G}{\sigma+\varphi})$, if there is no government consumption this boils down to $\lambda_\Delta = 2\lambda_x$

(2002) and Gali and Gertler (1999), we assign a value of 2/3 to ψ, the fraction of firms that cannot change their price in any given quarter. This value implies that on average firms change prices every 3 quarters. The price elasticity of the demand θ is set to 11 such that the steady state markup is 10%.

For the driving processes I follow Schmitt-Grohe Uribe 2005 and I set the persistence parameters $\rho_a = .86$ and $\rho_g = .87$, for the productivity and government spending respectively. While the standard deviations of the correspondent innovations are $\sigma_a = .0064$ and $\sigma_a = .0160$. The two processes are assumed to be uncorrelated.

We calibrate the debt dispersion parameter $\zeta_b^2 \equiv \int_0^1 \bar{b}_{-1}^{h^2} dh$ [33] using micro data from the Federal Reserve Board's Survey of Consumers Finances (SCF) for the year 2001. We calculate the net debt position for each household in the survey. We calculate a gross credit position by summing up the following variables: saving accounts, money market account, investment in money market funds, CDs, total bonds.[34] On the other hand we proxy a debit position by summing up: mortgage debt, other lines of credit, residential debt, checking account debt, installation loans and other debt. The net debt is given by the algebraic difference between the credit and debit gross positions.

Because in our model in steady state everybody earns the same wage and financial income we divide the net-debit position by the total household income, then we calculate the variance of the sample. The value we find for the year 2001, calibrated for our quarterly model, is $\zeta_b = 2.96$. In table.1 we give a summary of the all parameters just described.

Tab.1

[33] Also defined by the distribution $\zeta_b^2 = \int_{-\infty}^{\infty} u^2 d\Phi_{-1}(u)$

[34] By that we mean: US saving bonds, Federal government bonds other than U.S. saving bonds, bonds issued by state and local governments, corporate bonds, mortgage-backed bonds and other types of bonds.

Parameter	Value	Description
β	.9902	Subjective discount factor (quarterly)
σ	2	Relative risk aversion
φ	.1	Frisch elasticity
θ	11	Price-elasticity of demand for a specific good variety
μ	.10	Firms markup
ψ	.75	Fraction of non-resetter firms
$\bar{\tau}^G$	.25	Steady state value of government consumption over GDP
ζ_b	2.96	Fixed-Income asset dispersion
ρ_A	.86	Serial correlation of (log) of technology process
ρ_G	.87	Serial correlation of (log) of government spending process
σ_A	.0064	Std. dev. innovation to (log) of technology
σ_G	.0160	Std. dev. innovation to (log) of government consumption

5 Optimal Monetary Policy

In this section we examine, in a formal manner, the design of optimal monetary policy when debt servicing stabilization (i.e. consumption dispersion) is a policy goal.

We first show the relations that must hold among aggregate variables. Once we have linearized the households' Euler equations aggregation is straightforward and delivers the same aggregate system usually found in the sticky price literature.

Using households' Euler equations and the output gap definition:

$$\sigma E_t \Delta x_{t+1} = \hat{R}_t - E_t \pi_{t+1} - r_t^e \tag{48}$$

where we have defined the *efficient interest rate* r_t^e as the real interest rate prevailing in the flexible price model without wealth dispersion.

From firms' optimal condition and exploiting the marginal cost relation with the output gap, we also can state the New Philips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{49}$$

where $\kappa \equiv (\sigma+\varphi)(1-\psi)(1-\psi\beta)/\psi$.

In order to close the system we only need to know the monetary policy behavior. In the next paragraphs we will find the optimal interest rate rule that a benevolent central bank would be willing to implement.

5.1 Flexible-Price Environment

In order to have a better understanding of the channel through which the redistributive effect of monetary policy has an impact on social welfare, we first analyze the case of fully flexible prices - for which we have an analytical solution.

If prices are perfectly flexible the Phillips curve (49) is no longer well defined and simply tells us that marginal costs are constant.[35] We recall the output gap in the flexible price case: $x_t = \hat{Y}_t^F - \hat{Y}_t^N = \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_{n,t}$.[36] In first order approximation this is driven only by exogenous aggregate shocks.[37] Hence we can rewrite the IS relation previously found (48) replacing the output gaps with the natural interest rate r_t^e [38]

$$\hat{R}_t - E_t\pi_{t+1} = r_t^e \tag{50}$$

Inflation creates no distortion, in fact we have $\Delta_{p,t} = 0$ at all times and x_t^2 is of order higher than the second - it drops out from the loss function. Hence the period by period loss function simplifies to

$$L_t = \lambda_c \int_0^1 \tilde{C}_t^{h^2} \tag{51}$$

Monetary authority seeks the state-contingent path for inflation $\{\pi_t\}_{t=0}^{\infty}$ that minimizes expected discounted sum of losses conditioning upon the initial state of the world in period-0 and subject to the constraint that this evolution represents a possible rational-expectations equilibrium.

The monetary authority faces the following problem:

[35] This is the limiting case of our model in which $\psi = 0$ and $1/\kappa = 0$.

[36] This is always true in log-deviations from the steady state, not only when $\hat{\Delta}_n = 0$.

[37] We have that $\frac{\sigma}{\sigma+\varphi}\hat{\Delta}_{n,t} = \frac{\varphi}{\sigma+\varphi}\int_0^1 \tilde{N}_t^{h^2}$

[38] This is a linear function of the exogenous aggregate stochastic processes $\hat{A}_t$ and $\hat{g}_t$ as standard in the literature. See Gali 2001

$$\begin{aligned}&\min E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \int_0^1 \tilde{C}_t^{h^2}\\ &s.t.\ \ (43),(44),(50),\\ &\ (\tilde{b}_{t-1}^h)_{h\in[0,1]}\ \text{ given}\end{aligned}$$

Together with the above constraints the optimality conditions take the form (see appendix for further details):

$$\tilde{C}^h: \quad \lambda_c \tilde{C}_t^h = \kappa_c \lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1}\lambda_{2,t-1}^h \quad \forall h \in [0,1] \tag{52}$$

$$\tilde{b}^h: \quad \beta(E_t \lambda_{1,t+1}^h - \lambda_{1,t}^h) = \varphi_b \lambda_{2,t}^h \qquad \forall h \in [0,1] \tag{53}$$

$$\pi: \quad \int_0^1 \bar{b}^h \lambda_{1,t}^h = \lambda_{3,t-1}/\beta \tag{54}$$

$$\hat{R}: \quad \beta \int_0^1 \bar{b}^h \lambda_{1,t}^h = \lambda_{3,t} \tag{55}$$

We have a collection of lagrange multipliers associated to the previous constraints $\mathcal{L} = \{\mathcal{L}_t\}_{t\geq 0}$ where $\mathcal{L}_t = ((\lambda_{1,t}^h)_{h\in[0,1]}, (\lambda_{2,t}^h)_{h\in[0,1]}, \lambda_{3,t})$ and new set of initial conditions $\mathcal{L}_{-1}$. The above system involves a continuum of equations difficult to handle directly. In order to find the optimal state-contingent path for inflation we introduce new aggregate variables. Recalling that $\bar{b}^h$ is the steady state household-h bond position, we define the covariances (or dispersions) among some key variables:

- the multipliers-debt covariance

$$\lambda_{i,t} \equiv \int_0^1 \bar{b}^h \lambda_{i,t}^h dh \tag{56}$$

- the consumption-debt covariance

$$w_t \equiv \int_0^1 \bar{b}^h \tilde{C}_t^h dh \tag{57}$$

- and the debt dispersion[39]

$$z_t \equiv \int_0^1 \bar{b}^h \tilde{b}_t^h dh \qquad (58)$$

We will make use of the already defined steady state debt-dispersion parameter $\zeta_b^2 \equiv \int_0^1 \bar{b}^{h^2}$.

First of all we notice that if the steady state household distribution over debt is uncorrelated with the household joint distribution over consumption and debt then we have $\lambda_{i,t} = z_t = w_t = 0$. In this case, because of a clear lack of instruments, monetary policy has nothing to say about how to reduce wealth dispersion even if its implied distortion $\Delta_{n,t}$ is not necessarily zero.

However this does not mean that monetary policy has no welfare impact. Even in a flexible price environment, the way the central bank reacts to inflation can be welfare reducing. We now state the following proposition (a formal proof is given in the appendix):

Proposition 2 *In a flexible price environment, with the only distortion created by wealth dispersion, optimal monetary policy is given by a state-contingent path for inflation*

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} \beta^j r_{t+j}^n + \frac{z_{t-1}}{\zeta_b^2}, \quad \forall\, t \geq 0 \qquad (59)$$

which implies a targeting rule [40]

$$\hat{R}_t = \frac{\pi_t}{\beta} + \frac{z_t - z_{t-1}/\beta}{\zeta_b^2}, \quad \forall\, t \geq 0 \qquad (60)$$

The optimal interest rate reaction is a function of inflation, π_t, and the debt dispersion, z_t. The coefficient on inflation, being of order 1.01, satisfies

[39] Notice that, from our definitions, we can also write $z_t/\zeta_b^2 = \int_0^1 \bar{b}^h b_t^h / \int_0^1 \bar{b}^{h^2} - 1$. In a simple 2-agents economy example say agent-1 has an initial debt of 10£, $\bar{b}^1 = -10£$, in this case $\zeta_b^2 = 50£^2$. If in the next periods he increases the debt of 10£ up to $-11£$ then we have $z/\zeta_b^2 = 1.1 - 1 = 10\% > 0$

[40] For a definition of targeting rules see Svensson 2003, Giannoni-Woodford, or Woodford Ch7

the Taylor principle but is much smaller than the standard Taylor prescription. The reason is that a stronger reaction to inflation (deflation) would entail a higher (lower) real cost (revenue) for debt servicing with respect to some long run average that hurts households who have accumulated a big stock of debt (credit).

The second term in the central bank rule reflects a measure of aggregate households financial imbalances. Whenever z_t is different from zero the central bank has something to do about the distortion created by the existence of a non-degenerate distribution of households over nominal asset holdings. A positive value for z_{t-1} says that there is a positive correlation between those households who are worst off (lower utility than the average) and households who are over-accumulating debt (i.e. that have accumulated a stock of debt higher than their long run average). In this case, ceteris paribus, the central bank should have a looser monetary policy and restore the second best optimum (i.e. $w_t = z_t = 0$).[41]

We can see that, in absence of any nominal distortion, the central bank, under the optimal policy, can always achieve such a goal. Rewriting the households budget constraint (43) in term of covariances and using the optimal policy we have that

$$\kappa_c w_t = -\beta z_t + z_{t-1} + \zeta_b^2(\beta \hat{R}_t - \pi_t) = 0, \ \forall t \geq 0 \tag{61}$$

which in turn implies $z_t = 0 \ \forall t \geq 0$. In other words, in equilibrium, for any given initial debt dispersion z_{-1}, the optimal monetary policy rule reads $\hat{R}_t = \pi_t/\beta \ \forall t \geq 1$ delivering what we can call a second best allocation.

This result tells us also, as corollary, that monetary policy has nothing to exploit from time-0 absence of commitment, and timeless perspective and standard Ramsey deliver the same problem. In fact, as shown in the appendix, if consumption-debt covariance w_t and debt dispersion z_t are zero for all $t \geq 0$ then it must be the case that

[41]Recalling the definition of z_t the steady state dispersion ζ_b^2 in the policy rule can be interpreted as a scaling parameter. An important check for the accuracy of our approximation can be indeed found in the ratio z_t/ζ_b^2. Whenever this ratio is higher than 1 the steady state debt dispersion is lower than the current deviation-from-steady-state debt dispersion; which means that the model is surely drifting away

$$\lambda_{1,t} = \lambda_{1,t-1} = \lambda_{1,-1} = 0 \tag{62}$$

which entails $\lambda_{2,-1} = \lambda_{3,-1} = 0$. However, this does not imply that for each multiplier-h we have $\lambda_{i,-1}^h = 0$[42] but it only says that there is no need of any inflation surprise at time zero given that the central bank is unable to target each individual household.

This means that a central bank is reacting to aggregate imbalances which are favoring either the group creditors or the group of debtors. What monetary can do for social welfare is realizing if, because of changes in the real interest rate, some groups in the economy are more affected than others.

5.2 Sticky-Price Environment

We now characterize the optimal responses to shocks in the case that prices are sticky ($\psi > 0$).

The central bank problem is to choose processes $\{\pi_t, \hat{R}_t, (\tilde{C}_t^h)_{h\in[0,1]}, (\tilde{b}_t^h)_{h\in[0,1]}\}_{t\geq 0}$ to minimize (45) subject to the constraint (43), (44), (48), (49) for every $t \geq 0$, [43] given initial conditions $(\tilde{b}_{-1}^h)_{h\in[0,1]}$ and the evolution of the exogenous shocks $\{A_t, \hat{g}_t\}_{t\geq 0}$.

Hence we have:

$$\begin{aligned}
\min \quad & E_0 \sum_{t=0}^{\infty} \beta^t \Big(\pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 \tilde{C}_t^{h^2}\Big) \\
s.t. \quad & \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \\
& \sigma E_t \Delta x_{t+1} = \hat{R}_t - E_t \pi_{t+1} - r_t^e \\
& \kappa_c \tilde{C}_t^h = -\beta \tilde{b}_t^h + \tilde{b}_{t-1}^h + \bar{b}^h(\beta \hat{R}_t - \pi_t), \quad \forall h \in [0,1] \\
& E_t \Delta \tilde{C}_{t+1}^h = -\varphi_b \tilde{b}_t^h, \quad \forall h \in [0,1] \\
& (\tilde{b}_{-1}^h)_{h\in[0,1]} \ \text{ given}
\end{aligned} \tag{63}$$

[42]in general it will not be the case if $b_{-1}^h \neq \bar{b}^h$ for some $h \in [0,1]$

[43]Together also with initial constraints of the form $\pi_0 = \bar{\pi}_0$ $x_0 = \bar{x}_0$ and $(\tilde{C}_0^h = \bar{C}_0^h)_{h\in[0,1]}$ which ensure the commitment from a timeless perspective.

Necessary conditions read (lagrange multipliers associated to the constraints are μ_1, μ_2 λ_1^h λ_2^h):[44]

$$\lambda_x x_t + \kappa\mu_{1,t} + \sigma\mu_{2,t} - \sigma\beta^{-1}\mu_{2,t-1} = 0 \tag{64}$$
$$\pi_t + \mu_{1,t-1} - \mu_{1,t} - \beta^{-1}\mu_{2,t-1} - \lambda_{1,t} = 0 \tag{65}$$
$$\mu_{2,t} + \beta\lambda_{1,t} = 0 \tag{66}$$
$$\lambda_c \tilde{C}_t^h = \kappa_c\lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1}\lambda_{2,t-1}^h \tag{67}$$
$$\beta(E_t\lambda_{1,t+1}^h - \lambda_{1,t}^h) = \tilde{\varphi}_b\lambda_{2,t}^h \tag{68}$$

Where again we use previous section definitions. The final system is:

$$\lambda_x x_t + \kappa\mu_{1,t} + \sigma\mu_{2,t} - \sigma\beta^{-1}\mu_{2,t-1} = 0 \tag{69}$$
$$\pi_t + \mu_{1,t-1} - \mu_{1,t} - \beta^{-1}\mu_{2,t-1} - \lambda_{1,t} = 0 \tag{70}$$
$$\mu_{2,t} + \beta\lambda_{1,t} = 0 \tag{71}$$
$$\lambda_c w_t = \kappa_c\lambda_{1,t} - \lambda_{2,t} + \beta^{-1}\lambda_{2,t-1} \tag{72}$$
$$\beta(E_t\lambda_{1,t+1} - \lambda_{1,t}) = \tilde{\varphi}_b\lambda_{2,t} \tag{73}$$
$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{74}$$
$$\sigma E_t\Delta x_{t+1} = R_t - E_t\pi_{t+1} - r_t^e \tag{75}$$
$$\kappa_c w_t = -\beta z_t + z_{t-1} + \zeta_b^2(\beta\hat{R}_t - \pi_t) \tag{76}$$
$$E_t w_{t+1} = w_t - \tilde{\varphi}_b z_t \tag{77}$$

We now analyze the optimal response of inflation and output gap to a transitory disturbance to the level of productivity and government spending and to what we have called a financial shock.

We perform the exercise for different values of the debt dispersion parameter. We show the results for our calibrated value, $\zeta_b = 2.96$, but also for a lower and higher value, respectively 2 and 3.87 (see fig.1).

I analyze the case when a (transitory) productivity and/or government spending shocks give rise to an increase in the *efficient* rate of interest, r_t^e. In this case, *ceteris paribus*, households who are net creditors in the economy

[44] see appendix for details on necessary conditions

would enjoy higher returns on their bond holdings. This can be seen also from the evolution of the consumption-debt covariance w_t which is positive: higher returns on assets allow those households to have, in average, a higher consumption than the aggregate per capita consumption. At the same time indebted households must decrease their consumption and, to service their debt, they are accumulating an even higher stock of debt $z_t < 0$.

Hence monetary policy faces a clear trade-off between dwindling the redistributive impact of an increase in the natural rate and shutting off nominal distortion pursuing price stability. In the baseline model without asset dispersion the stabilization policy would be straightforward: tracking the natural rate and closing all the gaps. However, as we can see in fig1 - the higher the debt dispersion ζ_b the bigger the deviation from price stability. At the time of the impact of the shock the nominal interest rate does not move together with the natural rate (what we would have found in the baseline model). On the contrary the reaction is much smaller and for $\zeta = 3.87$ we even have an inversion of sign: the nominal rate decreases at the time of the shock.

Repeating the analysis for higher values of ζ_b does not alter the main conclusion. However, from figure 3, we can see that the same shock now implies a stronger deviation from price stability: the higher the economy is indebted the higher will be the deviation from zero inflation.[45].

Summarizing the results for the two calibrated exercises, the deviation from price stability is still relatively small. Deviations are of the order of .02% for a 5% in the natural rate.

As the steady state debt dispersion increases, the deviation of the nominal interest rate from the natural rate increases further. In our calibrated example, for a $\zeta_b = 10.36$, a negative productivity shock (for example) which gives the efficient rate a deviation of about .4%, now implies an inflation rate stimulus of about .08% (5 times more than before).

We want to find a simple rule for our model that approximate the global optimal monetary policy. Hence we simulate our model for 9000 periods (we

[45] A crucial preference parameter for the persistence of the shocks is given by the intertemporal rate of substitution $1/\sigma$. The lower it is the higher the persistence. Our results are for $\sigma = 2$, for $\sigma = 10$ the persistence of deviation from price stability has is much more hump-shaped and long lasting.

discard the first 1000 observations) and we estimates the following simple rule (ERS):

$$\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_{z_0} z_t + \eta_{z_1} z_{t-1} \tag{78}$$

We find that under our calibration [46] the weights on current and past debt-dispersion, η_{z_0} and η_{z_1} respectively, are significantly non-zero (see Table1 appendix). Recalling the flexible price solution we see that, also for the sticky price model, an approximated optimal policy has a negative reaction on the beginning of period debt dispersion. The rule is also showing *super-inertia*: the coefficient on lagged interest rate is in fact greater than one. The coefficient on inflation is still very high meaning that the welfare loss stemming from price dispersion is still dominating. However, the higher the steady state parameter ζ_b the lower the weight given to inflation. In figure 4 I perform the same exercise of figure 1 but for using the ERS (rule) instead of the optimal one. The responses are quite similar to the optimal, however is worth noting how the inflation deviations, relatively small, are quite persistent.

Finally, having derived the central bank loss function we can easily compare different policy rules in terms of their welfare score. I rank alternative rules on the basis of the *unconditional* expected welfare. To compute it, I simulate 1200 paths for the endogenous variables over 600 quarters and then compute the average loss per period across all simulations. For the initial distribution of the state variables I run the simulation for 200 quarters prior to the evaluation of welfare. In order to calculate the consumption dispersion I use 100 households: I draw from the US CFS 100 net-bond positions which stand for $\bar{b}^h$. Table 2 gives a definition of the rule used and table 3 ranks all rules according to their welfare score.

Under the baseline calibration the optimal rule (GOMP) and the estimated simple rule (ERS) gives almost the same result: the percentage loss of the estimated rule with respect to the optimal is only 0.065%. This means that a simple rule can still be a very good approximation of the optimal one.

In the baseline framework without debt dispersion targeting zero inflation

[46] For this exercise we set $\sigma = 5$. Because under the baseline calibration we still have quantitatively small deviations from price stability, the estimates for η_p are not very precise.

(IT) would be optimal. However, once debt dispersion is introduce, the IT rule results in a 7.6% welfare loss with respect to GOMP. The sub-optimality of the IT rule can also be seen from figure 5 in which we compare it with the ESR (which is, as we said, a good approximation of the GOMP rule). As we can see, inflation and output gap are stabilized at the cost a much larger variation in w_t and z_t which in turn represent the $var_h(\tilde{C}_t)$ and so the welfare loss from consumption dispersion.

To see the importance of the debt-covariance, z_t and z_{t-1}, as a separate target we also estimate a simple rule only with inflation and interest rate as targets (ERSbis). As we can see from table 2 it delivers a welfare loss very close to the pure IT rule and, again, it turns out in almost an 8% loss with respect to ERS.

Finally we also analyze a quite standard Taylor rule (TR) and a Taylor rule augmented with the debt covariance z_t and z_{t-1} as targets. We observe a huge loss of order 10 times higher than the GOMP rule. This is not surprising given that it comes mainly from inflation losses, however we can see how the ATR outperforms the standard TR by more than 9 times. Moreover, targeting z_t and z_{t-1} not only reduces the losses stemming from consumption dispersion but reduces also the output gap and inflation volatility.

6 Conclusion

Most of the results in the recent monetary policy literature have been derived under the assumption of a representative household.[47] The present paper relaxes that assumption. We have introduce an effect on households' balance sheet stemming from variations in servicing (in the returns from) the stock of net-debt (credit) - variations that, in turn, are related to economy-wide aggregate disturbances. Those variations imply an *arbitrary* redistributive pattern in the economy and thus, potentially, a greater dispersion of consumption and hours worked.

To determine what a central bank could do, we first introduce a transfers scheme. This leaves aside problems related to "long run" wealth inequality.[48]

[47] Or assuming complete markets which, after all, is the same thing.

[48] For which a central bank does not have enough policy instruments, especially if does

The first result is that even in a flexible-price environment monetary policy has real effects through its ability to affect interest payment (income) volatility.

The second important result is found when price stickiness is introduced. In the baseline sticky price model this entail that price stability is the prominent policy goal. The direct corollary is that the nominal rate should track closely the *natural* rate.[49] However this is in clear contrast with the objective of stabilizing interest rate income (repayments) which would imply the inflation rate (and not the interest) to track the *natural* real rate.

Quantitatively, in our calibrated exercise, we find that the magnitude of deviation from zero inflation is still relatively small.

However, patterns of household debt dispersion should be monitored by any monetary authority who is willing to keep the price stability goal as credibly central.

Finally, designing a simple implementable rule, we find that *superinertial* rules, that incorporates also a measure of debt dispersion as separate target, outperforms standard Taylor rules.

not want to exploit surprise inflation.

[49]which, being exogenous, could be very high volatile.

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Appendix

A Appendix: Some Results

Some results used.

In a second order approximation, for any variable $x \in \mathbb{R}_+$ and $\bar{x} \in \mathbb{R}_+$

$$\frac{x-\bar{x}}{\bar{x}} \simeq \hat{x} + .5\hat{x}^2 \tag{A-1}$$

$$(\frac{x-\bar{x}}{\bar{x}})^2 \simeq \hat{x}^2 \tag{A-2}$$

Where $\hat{x} = \log(x/\bar{x})$.

Given a function of the following kind $f(x,y) = xg(y)$, with $y \in \mathbb{R}_+$, $g(.)$ twice differentiable and $\bar{x} = 0$, we have that

$$f_y(\bar{x}, y) = \bar{x}g'(y) = 0,$$
$$f_{yy}(\bar{x}, y) = \bar{x}g''(y) = 0,$$

This means that if take the 2-order approximation of f about $(\bar{x}, \bar{y})$ $\forall \bar{y}$ we find that

$$f(x,y) \simeq g(\bar{y})x + g'(\bar{y})x(y-\bar{y}) = g(\bar{y})x + \bar{y}g'(\bar{y})x\hat{y} \tag{A-3}$$

In order to calculate the effect of price and output dispersion on overall output we use the following result for a household or firms variable, say $x(h)$, in deviation from its average value, $x \equiv \int_0^1 x(h);dh$:

$$\int_0^1 \log(x_t(h)/x_t) \simeq -0.5 \int_0^1 \Big(\frac{x_t(h) - x_t}{x_t}\Big)^2 \tag{A-4}$$

This also means that

$$\int_0^1 \hat{x}_t(h) - \hat{x}_t \simeq -0.5 \int_0^1 (\hat{x}_t(h) - \hat{x}_t)^2 \tag{A-5}$$

We note that the first order effect is zero.

In relation with the previous result, if we let $x_t(h) = X_t(h)/X_t$ and we have $\bar{x} = 1$ and $\int_0^1 x_t(h) = 1$ then $\Delta_{x,t} = \int_0^1 x_t^\alpha(h)$ can be approximated as

$$\hat{\Delta}_{x,t} = \log \Delta_{x,t} \simeq -0.5\alpha \int_0^1 \hat{x}_t^2(h) \tag{A-6}$$

B Appendix: Output Gap

We have defined the efficient rate of output Y^e as the one prevailing with complete markets equal initial wealth distribution and flexible prices. In this case it is easy to show that

$$\hat{Y}_t^e = \frac{\sigma}{\sigma+\varphi}\hat{g}_t + \frac{1+\varphi}{\sigma+\varphi}\hat{A}_t + \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_n \tag{A-7}$$

The introduction of nominal rigidities does not alter any fundamental relation but for the markup determination. So we still have that $\hat{mc}_t = \hat{W}_t^r - \hat{A}_t$, from the consumption/leisure choice $\hat{W}_t^r = \sigma\hat{C}_t + \varphi\hat{N}_t$, from the resource constraint $\hat{Y}_t - \hat{g}_t = \hat{C}_t$. However it does alter output aggregation of the production functions $Y_t = A_t N_t/\Delta_{p,t}$ such that consumption (in *logs*) is given by

$$\hat{C}_t = \hat{A}_t + \hat{N}_t + \hat{g}_t - \hat{\Delta}_{p,t} \tag{A-8}$$

So we can write

$$\hat{mc}_t = (\sigma+\varphi)x_t + \varphi\hat{\Delta}_{p,t} \tag{A-9}$$

For our market structure we cannot exploit the aggregate consumption/leisure relation directly. However, even in the sticky price case, the aggregate consumption-labor relation found in section (3) must hold:

$$\hat{W}_t^r = \varphi\hat{N}_t + \sigma\hat{C}_t - \sigma\hat{\Delta}_{n,t} \tag{A-10}$$

Moreover it is always true that $\hat{W}_t^r = \hat{mc}_t + \hat{A}_t$ and that aggregate consumption is related to output as above in equation (A-8). Combining those two relations with (A-10) and using the output gap definition $x_t \equiv \hat{Y}_t - \hat{Y}_t^e$ we get

$$\hat{mc}_t = (\sigma+\varphi)x_t + \varphi\hat{\Delta}_{p,t} - \sigma(\hat{\Delta}_{n,t} - \hat{\Delta}_n) \tag{A-11}$$

as in equation (27) of the text.

C Appendix: The Optimal Deterministic Steady State

Here we show the existence of an optimal steady state, i.e., of a solution to the recursive policy problem defined in section (2.5), that involves (under appropriate initial conditions) constant values for all variables, in the case of no stochastic disturbances: $A_t \equiv \bar{A}$ and (without loss of generality) $G_t \equiv \bar{G} = 0$.

To prove the result we split the problem in two stages. In the first stage the government sets and commits to a redistributive policy Φ^τ taking as given inflation, price dispersion and total production - i.e. $R_t = R^\star$, $\Pi_t = \Pi^\star$, $Y_t = Y^\star$ $w_t = w^\star$. Using the consumption-leisure condition we can write $N_t^h = v_n^{-1}(w^\star / u_c(C_t^h))$. We accordingly redefine the momentary utility

$$u(C_t^h) - v(N_t^h) = \tilde{u}(C_t^h) \tag{A-12}$$

and the wage income

$$w_t N_t^h = g(C_t^h) \tag{A-13}$$

We can now formulate the deterministic version of the Ramsey problem for a given (and at the moment arbitrarily) initial distribution of households over debt Φ_{-1}

$$\max \textstyle\sum_{t=0}^{\infty} \beta^t \int_0^1 \tilde{u}(C_t^h) dh \tag{A-14}$$

$$\begin{gathered} s.t. \\ C_t^h + b_t^h / R^\star = b_{t-1}^h / \Pi^\star + \bar{\tau}^h + g(C_t^h) \ \forall h \in [0,1] \\ \textstyle\int_0^1 \bar{\tau}^h dh = 0 \\ \textstyle\int_0^1 C_t^h dh = Y^\star \end{gathered} \tag{A-15}$$

We denote the associate set of lagrange multiplier $\{(\varphi_t^h)_{h\in[0,1]}, \varphi_{1,t}, \varphi_{2,t}\}$. The FOC for optimal consumption allocation reads

$$\tilde{u}_c(C_t^h) = \varphi_t^h g'(C_t^h) + \varphi_{2,t} \ \forall h \in [0,1] \tag{A-16}$$

On the other hand we have the relation $\varphi_t^h = \varphi_{1,t}$. Putting together the two equations we realize that individual consumption must be equalized

$$C_t^h = \bar{C}_t \; \forall h \in [0,1] \tag{A-17}$$

The intuition is straightforward, for a given amount of available resources and (strictly) concave utilities the previous solution is a necessary and sufficient conditions which tells us that a social planner will (strictly) prefer to equate marginal utilities of consumption across agents.

The induced transfer system - denoted $\Phi^{\tau^\star}(\bar{\tau})$ - can be recovered from the intertemporal households budget constraint and will be proportional to the initial debt dispersion $\Phi^{\tau^\star}(\bar{\tau}) \propto \Phi_{-1}$ with the constant of proportionality function of $R^\star$ and $\Pi^\star$. In fact for each household we have:

$$\bar{\tau}^h = \bar{b}_{-1}^h (1/\Pi^\star - 1/R^\star) \; \forall h \in [0,1] \tag{A-18}$$

In the second stage we take $\Phi^{\tau^\star}(\bar{\tau})$ as given and we wish to find an initial degree of price dispersion Δ_{-1} such that the recursive problem involves a constant policy.

However, under the optimal transfer scheme we have shown that households consumes and work the same, this means that our second stage boils down to the same problem solved in Benigno-Woodford 2005 which show that zero price dispersion (i.e. zero inflation) is the optimal long-run monetary policy. Given no price dispersion $\bar{\Delta}_p = 0$ we have

$$1 = \bar{p}(z) = \mu \bar{mc} = \mu \bar{w}/\bar{A} \tag{A-19}$$

Because the employment is subsidized at a rate τ_μ which exactly offset the monopolistic distortion we have

$$\bar{W}^r = \frac{\bar{A}}{\mu \tau_\mu} = \bar{A} \tag{A-20}$$

So output is at its efficient level:

$$\bar{Y} = \bar{A}^{\frac{1+\varphi}{\sigma+\varphi}} (1 - \bar{\tau}^g)^{-\frac{\sigma}{\sigma+\varphi}} \tag{A-21}$$

D Appendix: Loss Function

We recall that the resource constraint implies at all time that $C_t = Y_t - G_t = Y_t(1-\tau_t^G)$. We start from the utility coming from consumption (we recall that we name $\widehat{C_t^h/C_t} \equiv \tilde{C}_t^h$)

$$u(C_t^h) = u(\frac{C_t^h}{C_t}(Y_t - G_t)) \simeq \tag{A-22}$$
$$\simeq \bar{u} + u_c(\bar{Y}-\bar{G})(\tilde{C}_t^h + .5\tilde{C}_t^{h^2}) + u_c\bar{Y}(\hat{Y}_t + .5\hat{Y}_t^2) + .5u_{cc}\bar{Y}^2[(1-\bar{\tau})^2\tilde{C}_t^{h^2} + \hat{Y}_t^2]$$
$$+(u_c\bar{Y} + u_{cc}\bar{Y}(\bar{Y}-\bar{G}))\tilde{C}_t^h\hat{Y}_t - u_{cc}\bar{Y}\bar{G}\hat{Y}_t\hat{G}_t - [u_c + (\bar{Y}-\bar{G})u_{cc}]\bar{G}\hat{G}_t\tilde{C}_t^h + t.i.p$$

Rearranging and integrating with respect households and using the fact that $\int_0^1 \tilde{C}_t^h = -.5\int_0^1 \tilde{C}_t^{h^2} + h.s.o.$ we get:[50]

$$\int_0^1 u(C_t^h) = \tag{A-23}$$
$$= t.i.p. + u_c\bar{Y}\hat{Y}_t - .5u_c\bar{Y}[\sigma(1-\bar{\tau}^G)\int_0^1 \tilde{C}_t^{h^2} - (1-\sigma)\hat{Y}_t^2] + u_c\bar{Y}\sigma\bar{\tau}^G\hat{Y}_t\hat{G}_t =$$
$$= t.i.p. + u_c\bar{Y}[\hat{Y}_t + (1-\sigma)\hat{Y}_t^2 + \sigma\bar{\tau}^G\hat{Y}_t\hat{G}_t - .5\sigma(1-\bar{\tau}^G)\int_0^1 \tilde{C}_t^{h^2}]$$

We turn to the disutility from working. We define $\tilde{N}_t^h \equiv \hat{N}_t^h - \hat{N}_t$ and we will make use of the two following facts: $\int_0^1 \tilde{N}_t^h \simeq -.5\int_0^1 \tilde{N}_t^{h^2}$ and from the labor supply conditions we realize that in a second order approximation it must be that $\tilde{N}_t^{h^2} \simeq \frac{\sigma^2}{\varphi^2}\tilde{C}_t^{h^2}$.

We now turn to the quadratic approximation of the disutility of labor.

$$\int_0^1 v(N_t^h)dh = \tag{A-24}$$
$$= \int_0^1 v(\frac{N_t^h}{N_t}N_t)dh = \int_0^1 v(\frac{N_t^h}{N_t}\frac{1}{A_t}\int_0^1 y(z)dz)dh \simeq t.i.p+$$
$$+v_n\frac{\bar{Y}}{\bar{A}}[\int_0^1 \hat{y}_t(z)dz + .5(1+\varphi)\int_0^1 \hat{y}_t^2(z)dz - (1+\varphi)\hat{A}_t\int_0^1 \hat{y}_t(z)dz + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}dh] = t.i.p+$$
$$+v_n\frac{\bar{Y}}{\bar{A}}[E_z\hat{y}_t(z) + .5(1+\varphi)[(E_z\hat{y}_t(z))^2 + V_z\hat{y}_t(z)] - (1+\varphi)\hat{A}_tE_z\hat{y}_t(z) + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}]$$

[50] In the text we have made use labor supply dispersion $\Delta_{n,t}$. However in the derivation of the loss function we will prefer to work with $\tilde{C}_t^h$. It is nonetheless not difficult to see how $\Delta_{n,t}$ would enter in the loss function derivation: just note that we can write $C_t^h = (Y_t - G_t)(N_t^h/N_t)^{-\varphi/\sigma}/\Delta_{n,t}$

Having defined $\varphi \equiv \varphi/\bar{A}$ and used $E_z[\hat{y}_t(z)^2] = (E_z\hat{y}_t(z))^2 + V_h\hat{y}_t(z)$. We will make use of the fact that $\hat{Y}_t = E_z\hat{y}_t(z) + .5(1 - 1/\theta)V_z\hat{y}_t(z)$ and $(E_z\hat{y}_t(z))^2 = \hat{Y}_t^2$ and also that $\hat{A}_t E_z\hat{y}_t(z) = \hat{A}_t\hat{Y}_t$ (being the other terms of order higher than the second).

We can write:

$$\int_0^1 v(N_t^h)dh \simeq$$
$$t.i.p + v_n\frac{\bar{Y}}{\bar{A}}[\hat{Y}_t + .5(1+\varphi)\hat{Y}_t^2 + .5(1/\theta+\varphi)V_z\hat{y}_t(z) - (1+\varphi)\hat{A}_t\hat{Y}_t + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}]$$

Using the steady state relation $u_c = v_n/\bar{A}$ we can put together both expressions we have found (up to a multiplicative constant) to define the loss function we were looking for:

$$L_t = \tag{A-25}$$
$$= (\sigma+\varphi)\hat{Y}_t^2 - 2(\sigma+\varphi)\hat{Y}_t\hat{Y}_t^N + (1/\theta+\varphi)V_z\hat{y}_t(z) + \sigma(1-\bar{\tau}^G+\sigma/\varphi)\int_0^1 \tilde{C}_t^{h^2} =$$
$$= (\sigma+\varphi)x_t^2 + (1/\theta+\varphi)V_h\hat{y}_t(h) + \sigma(1-\bar{\tau}^G+\sigma/\varphi)\int_0^1 \tilde{C}_t^{h^2}$$

We have made use of the fact that $(\sigma+\varphi)\hat{Y}_t^N = (1+\varphi)\hat{A}_t + \sigma\bar{\tau}^G\hat{G}_t$ and of the output gap definition $x_t \equiv \hat{Y}_t - \hat{Y}_t^N$. Then knowing that $V_z\hat{y}_t(z) = \theta^2 V_z\hat{p}_t(z)$ and following Woodford (Ch3) we write

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_c\int_0^1 \tilde{C}_t^{h^2} \tag{A-26}$$

where $\kappa \equiv (1-\psi)(1-\beta\psi)(\sigma+\varphi)/\psi/(1+\varphi\theta)$ is the Phillips Curve parameter, while $\lambda_x \equiv \frac{\kappa}{\theta}$ and $\lambda_c \equiv \frac{(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi}\sigma(1-\bar{\tau}^G+\sigma/\varphi)$.

If we had to use $\eta(h)$ we simply observe that when $\eta(h) = 1/u_c(C^h)$ we have that

$$\eta(h)u_c(C^h) = 1$$

and

$$\eta(h)v_n(N^h)/A = \eta(h)u_c(C^h) = 1$$

Hence all the results would hold (up to a multiplicative constant).

E Appendix: Optimal Monetary policy. Flex Case

For convenience we restate proposition (2).

In a flexible price environment with the only distortion created by wealth dispersion globally optimal monetary policy is given by a state-contingent path for inflation

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} \beta^j r^n_{t+j} + \frac{z_{t-1}}{\zeta_b}, \quad \forall\, t \geq 0 \tag{A-27}$$

which implies

$$\hat{R}_t = \frac{\pi_t}{\beta} + \frac{z_t - z_{t-1}/\beta}{\zeta_b}, \quad \forall\, t \geq 0 \tag{A-28}$$

Proof.

We write the Lagrangian for the policy problem

$$\mathcal{LG} = E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \lambda_c \int_0^1 \tilde{C}_t^{h^2} + \int_0^1 \lambda^h_{1,t} \Big(\kappa_c \tilde{C}^h_t - \tilde{b}^h_{t-1} + \beta \tilde{b}^h_t - \bar{b}^h (\beta \hat{R}_t - \pi_t) \Big) +$$
$$+ \textstyle\int_0^1 \lambda^h_{2,t} \Big(\Delta \tilde{C}^h_{t+1} + \varphi_b \tilde{b}^h_t \Big) + \lambda_{3,t} \Big(\hat{R}_t - \pi_{t+1} - r^n_t \Big) + \int_0^1 \lambda^h_{2,-1} \tilde{C}^h_0 / \beta - \lambda_{3,-1} \pi_0 / \beta$$

Necessary conditions read (we substitute out the interest rate $\hat{R}_t$):

$$\kappa_c \tilde{C}^h_t - \tilde{b}^h_{t-1} + \beta \tilde{b}^h_t - \bar{b}^h (\beta r^n_t + \beta E_t \pi_{t+1} - \pi_t) = 0 \forall h \in [0,1] \tag{A-29}$$
$$\Delta \tilde{C}^h_{t+1} + \varphi_b \tilde{b}^h_t = 0 \forall h \in [0,1] \tag{A-30}$$
$$\lambda_c \tilde{C}^h_t = \kappa_c \lambda^h_{1,t} - \lambda^h_{2,t} + \beta^{-1} \lambda^h_{2,t-1} \quad \forall h \in [0,1] \tag{A-31}$$
$$\beta (E_t \lambda^h_{1,t+1} - \lambda^h_{1,t}) = \varphi_b \lambda^h_{2,t} \qquad \forall h \in [0,1] \tag{A-32}$$
$$\textstyle\int_0^1 \bar{b}^h \lambda^h_{1,t} = \lambda_{3,t-1} / \beta \tag{A-33}$$
$$\beta \textstyle\int_0^1 \bar{b}^h \lambda^h_{1,t} = \lambda_{3,t} \tag{A-34}$$

We multiply the first four equations by $\bar{b}^h$ and we integrate up with respect to agent-h (we use the definitions given in the text).

$$\kappa_c w_t - z_{t-1}^h + \beta z_t^h - \bar{b}^h(\beta \hat{R}_t - \pi_t) = 0 \tag{A-35}$$

$$E_t \Delta w_{t+1} + \varphi_b z_t = 0 \tag{A-36}$$

$$\lambda_c w_t = \kappa_c \lambda_{1,t} - \lambda_{2,t} + \beta^{-1}\lambda_{2,t-1} \tag{A-37}$$

$$E_t \lambda_{1,t+1} - \lambda_{1,t} = \varphi_b \lambda_{2,t}/\beta \tag{A-38}$$

$$\lambda_{1,t} = \lambda_{1,t-1} \tag{A-39}$$

If our solution is right then $\zeta_b(\beta \hat{R}_t - \pi_t) = \beta z_t - z_{t-1}$. So equation A-35 can be written (for every $t \geq 0$) as $\kappa_c w_t = -\beta z_t + z_{t-1} + \beta z_t - z_{t-1} = 0$. Given that $w_t = 0$ $\forall t \geq 0$ then from equation (A-36) we also have $z_t = 0$ $\forall t \geq 0$.

From equation (A-39) we have that the first multiplier must be constant $\lambda_{1,t} = \lambda_{1,-1}$ and $\lambda_{3,t} = \beta \lambda_{1,-1}$. Using equation (A-38) this means that $\lambda_{2,t} = 0$ $\forall t \geq 0$.

Hence, by the last unused equation (A-37) we must also have that for all $t \geq 1$

$$0 = \lambda_c w_t = \kappa_c \lambda_{1,-1} - \lambda_{2,t} + \lambda_{2,t-1}/\beta = \kappa_c \lambda_{1,-1}$$

which implies $\lambda_{1,-1} = 0$. Now it is straightforward to see that also $\lambda_{2,-1} = 0$. So the system is satisfied and the initial values of the cross-lagrange multiplier consistent with our solution are exactly $\lambda_{i,-1} = 0$. $\square$

For clearness we write the system under optimal policy

$$\hat{R}_t - E_t \pi_{t+1} = r_t^n \tag{A-40}$$

$$\hat{R}_t = \pi_t/\beta + \frac{z_t - z_{t-1}/\beta}{\zeta_b} \tag{A-41}$$

$$z_t = 0 \ \forall t \geq 0, \quad z_{-1} \ \text{ given} \tag{A-42}$$

which also can be written as

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} r_{t+j}^n + z_{t-1}/\zeta_b, \quad z_{t-1} \ \text{ given} \tag{A-43}$$

F Appendix: Discussion on Aggregation

Krusell and Smith (1998) shows that in an economy with incomplete market, idiosyncratic income shocks and an asset (capital) available for partial self-insurance an *approximate* aggregation result holds. In their words "...all aggregate variables - consumption, the capital stock and relative prices - can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution and the aggregate productivity shock". Moreover, the marginal propensity to save out of current wealth is almost completely independent of the levels of wealth and labor income (even with leisure choice).

Den Haan (1997), in a setup similar to ours, shows that, without tight borrowing constraints, policy functions are almost-linear and the effects of changes of asset distribution on prices are much smaller than the ones implied by aggregate shocks. For example even if the stationary level of the interest is shifted by wealth heterogeneity (as also shown in Hugget 1993 in relation with the low-riskfree puzzle), the percentage changes during business cycle fluctuations are mainly driven by aggregate shocks.

The previous results suggested my conjecture that variations in the cross-sectional distribution of assets do not affect are of minor order with respect to variations in the other endogenous state variables. In this model, in fact, the first moment of the asset distribution - which is a "sufficient statistics" in Krusell and Smith - is constant by construction. Second and higher moments do affect endogenous variable but mainly their stationary levels (as shown by Hugget for example) rather than their oscillations around those levels - that is what I care for my welfare analysis.

G Appendix: The Natural Debt Limit

Imposing $C_t^h \geq 0$ and $N_t^h \leq \bar{N}^+$ implies the emergence of what Aiyagari, in a slightly simpler context, calls a *natural debt limit*. We want to solve the budget constraint forward imposing the "worst possible scenario" for repaying a contracted debt. Let, over all possible realization, $\underline{\beta} = \min R_{t+k}$, $\underline{T}^G = \min T_t^G$ and $\underline{w} = \min W_t^r$. We also set $\Pi_t = 1 \ \forall t \geq 0$. Call $\underline{y} = \underline{w}\bar{N}^- \underline{T}^G$.

The budget constraint can now be written as

$$-b_{t-1}^{h}(1-\underline{\beta}L^{-1}) = \underline{y} + \bar{\tau}^{h} \tag{A-44}$$

Let $\phi_b \equiv \underline{y}/(1-\underline{\beta})$ and recall that $\bar{\tau}^h = -\bar{b}^h(1-\beta)$ and $\tilde{b}_t^h = b_t^h - \bar{b}^h$. We can write

$$\tilde{b}_{t-1}^{h} \geq -\phi_b \geq -\phi_b - \bar{b}^h\Big(\frac{\beta-\underline{\beta}}{1-\underline{\beta}}\Big) \tag{A-45}$$

So taking $-\phi_b$ as natural borrowing limit entails that everybody has the same limit when the problem is formulated in deviation from the steady state and that the mass of agents hitting the limit in the stationary equilibrium is zero.

H Appendix: The Complete Markets Case

I assume a continuum of households indexed by $h \in [0,1]$ maximizing the following utility

$$U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t^h) - v(N_t^h)\Big]$$

the budget constraint takes the following form:

$$P_t C_t^h + E_t B_t^h Q_{t,t+1} = B_{t-1}^h + W_t N_t^h + P_t X_t^h \tag{A-46}$$

Where now B_t is a set of state-contingent securities that pays 1 dollar. While $Q_{t,t+1}$ is the pricing kernel.

From the Euler equations we have that

$$\frac{C_{t+1}^h}{C_t^h} = \frac{C_{t+1}^{h^o}}{C_{t+1}^{h^o}},\ \forall (h, h^o) \in [0,1]^2 \tag{A-47}$$

In the next proposition we claim that there exists an average household.

Proposition 3 *For any continuous initial distribution of wealth $\exists h^o \in [0,1]$ such that $C_t^{h^o} = C_t\ \forall t \geq 0$*

Proof. Given any continuous initial distribution of wealth $\exists h^o \in [0,1]$ $C_0^{h^o} = \int_0^1 C_0^h dh$.

From the Euler then we have that

$$C_t^{h^o} = \frac{C_0^{h^o}}{C_0^h} C_t^h = \frac{C_0}{C_0^h} C_t^h \tag{A-48}$$

So

$$C_t^{h^o} \int_0^1 C_0^h dh = C_0 \int_0^1 C_t^h dh \tag{A-49}$$

which shows the above proposition. ■

So we can now introduce a metric for the deviations of consumptions from the average consumption:

$$C_t^h = \frac{C_0^h}{C_0} C_t = \delta(h) C_t \tag{A-50}$$

$$\Delta_{n,t}^{CM} = \left(\int_0^1 \delta(h)^{-\varphi/\sigma} \right)^{\varphi/\sigma} \tag{A-51}$$

So under complete markets $\Delta_{n,t}$ is constant.

To determine the value of this constant we have to specify the initial wealth - so the transfer scheme.

We can always find a transfer scheme such that $\delta(h) = 1$ $\forall h \in [0,1]$.

This would also be the optimal scheme that a benevolent government would implement weighting households the same.

To find this transfer scheme we write the inter-temporal budget constraint and we impose that $C_t^h = C_t$ $\forall h \in [0,1], t \geq 0$. Then inter-temporal budget constraint is:

$$B_{-1}^h = \sum_{t=0}^{\infty} E_0 Q_{0,t} [W_t N_t^h + P_t \bar{\tau}^h - P_t C_t^h] \tag{A-52}$$

Given that $C_t^h = C_t$ then it must also be that $N_t^h = N_t$ so that (considering that the profits equals the taxes for subsidies[51]) we have that $C_t = W_n N_t$. So the budget constraint reduces to

[51] subsidy rate is constant but total subsidies are not and are always equal to profits

$$B_{-1}^{h} = \bar{\tau}^{h} \sum_{t=0}^{\infty} E_0 Q_{0,t} P_t \tag{A-53}$$

or

$$\bar{\tau}^{h} = -\frac{b_{-1}^{h}/\Pi_0}{\sum_{t=0}^{\infty} E_0 Q_{0,t} P_t / P_0} = -\frac{b_{-1}^{h}/\Pi_0}{\sum_{t=0}^{\infty} \beta^t E_0 u_{c,t}/u_{c,0}} \tag{A-54}$$

If we call $1 - \beta^* = \sum_{t=0}^{\infty} \beta^t E_0 u_{c,t}/u_{c,0}$ we can write

$$\bar{\tau}^{h} = -\frac{b_{-1}^{h}}{\Pi_0}(1 - \beta^*) \tag{A-55}$$

Given that $\Pi_0 = 1$ is optimal in case of no initial relative price distortion we set:

$$\bar{\tau}^{h} = -b_{-1}^{h}(1 - \beta^*) \tag{A-56}$$

Figures and Tables

Estimated Simple Rule

Tab.1

Parameters	mean	standard deviation
η_r	1.0325	0.0076
η_p	64.1613	6.5743
η_{z_0}	1.4852	0.1832
η_{z_1}	-0.8381	0.0809

We simulate our model for 9000 periods (we discard the first 1000 observations) and we estimates the following simple rule(ERS): $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_{z_0} z_t + \eta_{z_1} z_{t-1}$.

Monetary Policy Rules

Tab.2

Rule Code	η_r	η_p	η_x	η_{z0}	η_{z1}	
GOMP	-	-	-	-	-	Optimal Policy Rule
ESR	1.033	64.16	0	1.485	-0.838	
ESRbis	1.033	64.16	0	0	0	
ATR	0	3	.5	1.485	-0.838	
TR	0	3	.5	0	0	
IT	-	∞	-	-	-	$\pi_t = 0$

Rules used for welfare comparison. For ESR, ESRbis, ATR and TR the functional form is: $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_x x_t + \eta_{z_0} z_t + \eta_{z_1} z_{t-1}$.

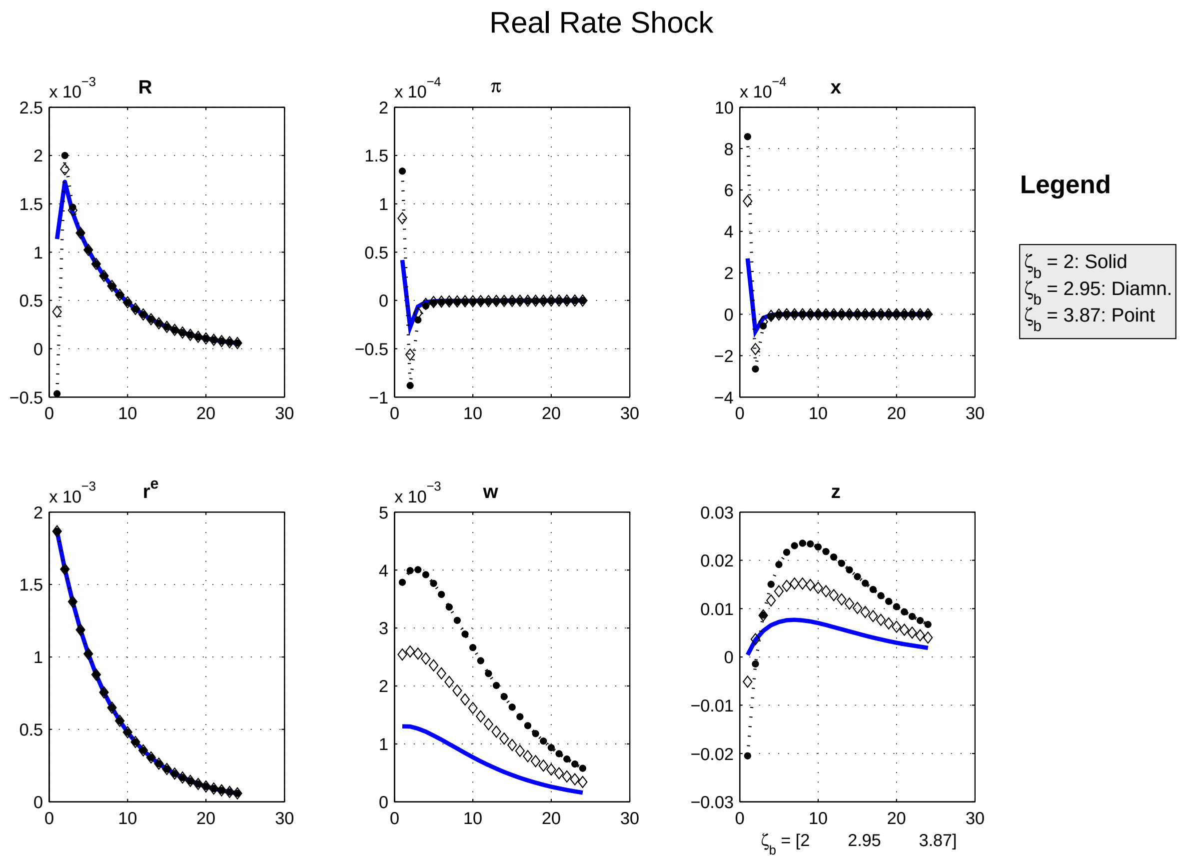
Welfare Comparison

Tab.3

Losses	GOMP	ESR	ESRbis	ATR	TR	IT
Deviations	1	1.00065	1.07596	6.17589	55.92921	1.07643
Levels	6.28e-5	6.29e-5	6.76e-5	3.88e-4	3.51e-3	6.76e-5
Inflation	1.17e-5	1.18e-7	5.58e-6	2.46e-4	3.45e-3	0
Output gap	1.12e-5	3.94e-6	2.27e-6	1.12e-4	2.48e-5	0
Cons. Disp.	3.99e-5	5.88e-5	5.98e-5	3.05e-5	3.81e-5	6.76e-5

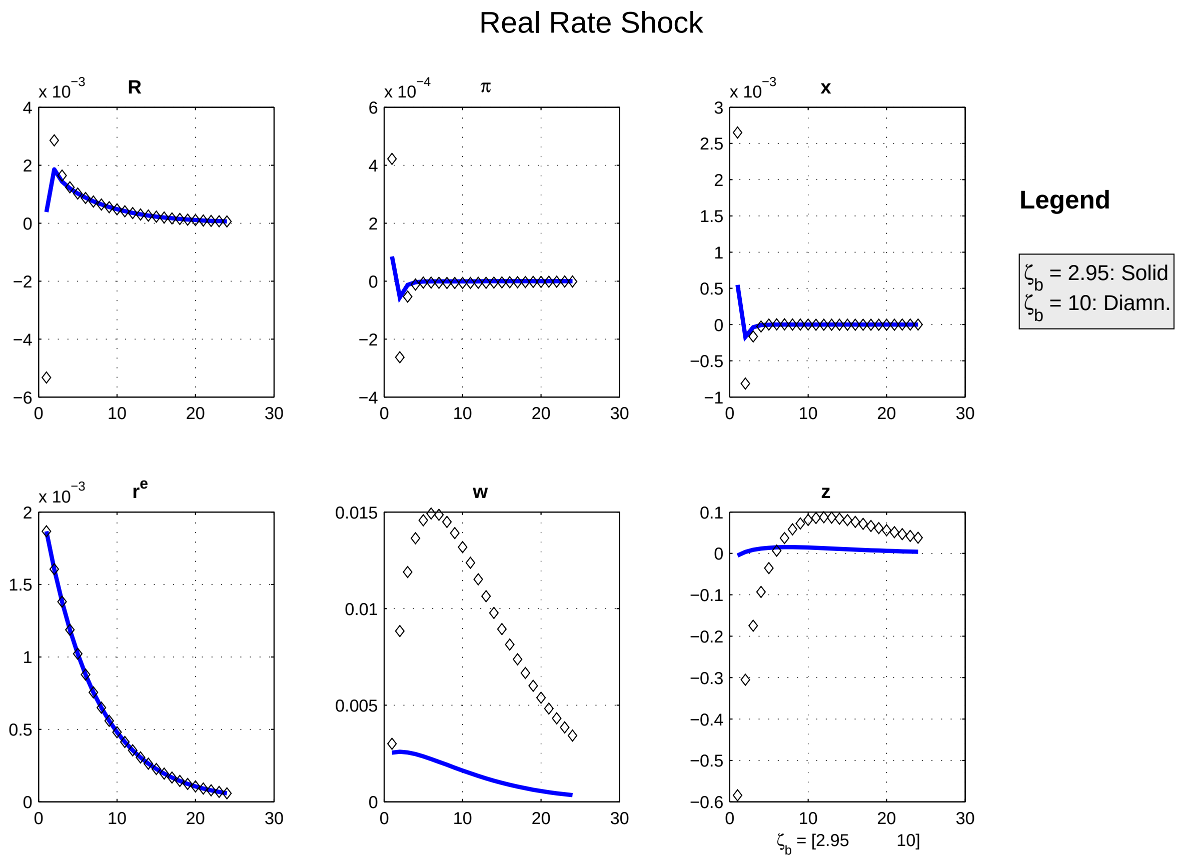
The welfare loss is expressed in: percentage terms with respect to the optimal rule (Deviations) in steady state consumption (Levels). This last one is also split by target (losses stemming from: inflation, output gap, consumption dispersion)

Figure 1: Low-Medium Steady State Debt Dispersion



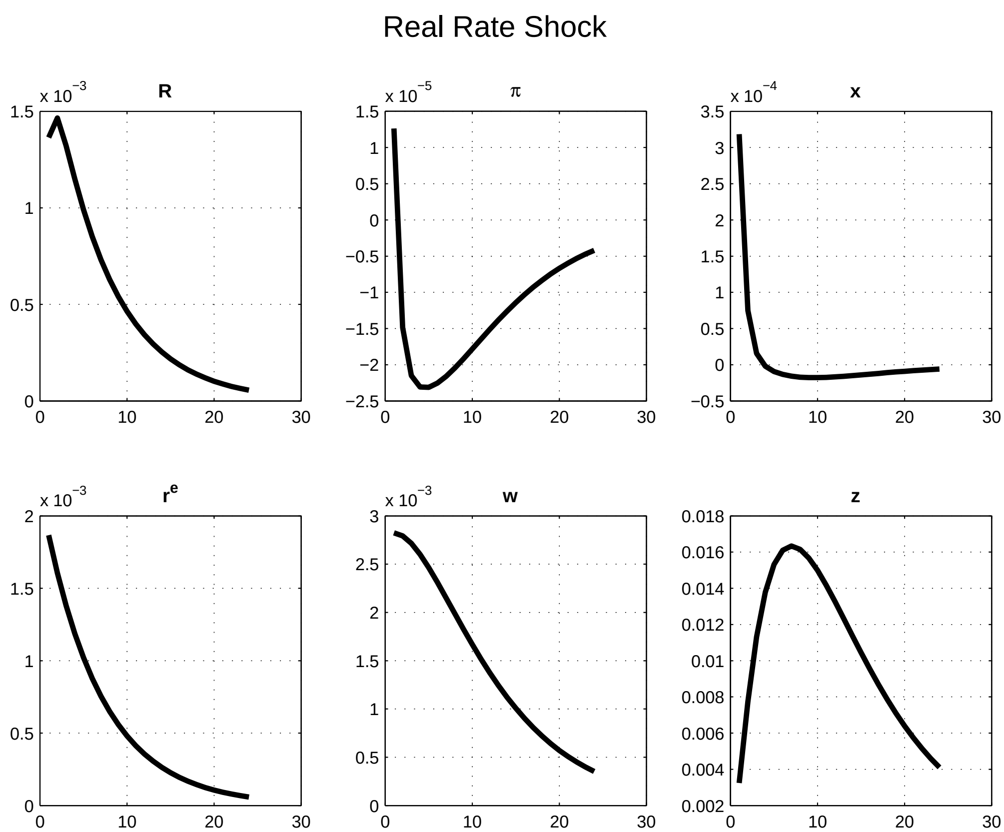
Impulse response functions to a positive shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). The debt standard deviation across households, ζ_b, takes values: 2.00, 2.95 and 3.87 represented by a solid line, diamonds and points, respectively.

Figure 2: Medium-High Steady State Debt Dispersion



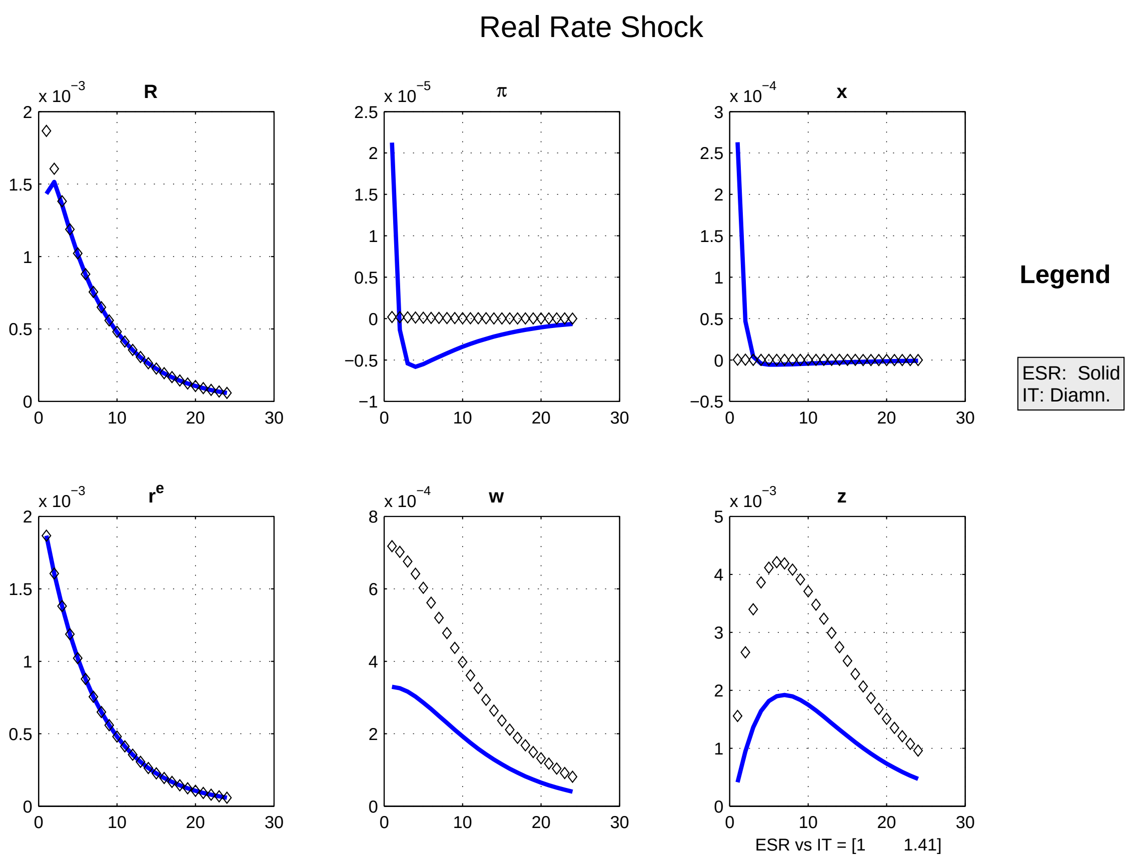
Impulse response functions to a positive shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). The debt standard deviation across households, ζ_b, takes values: 2.95 and 10.0 represented by a solid line and diamonds, respectively.

Figure 3: Estimated Policy Rule



Impulse response functions to a positive shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). Policy Rule adopted: $\hat{R}_t = 1.033\hat{R}_{t-1} + 64.16\pi_t + 1.485z_t - .838z_{t-1}$ (ESR) - see Table 2

Figure 4: Estimated Policy Rule vs Targeting Zero Inflation



Impulse response functions to a positive shock to the *efficient* real rate of interest r_t^e (the one prevailing in a flex-complete market environment). Comparing Policy Rules: $\hat{R}_t = 1.033\hat{R}_{t-1} + 64.16\pi_t + 1.485z_t - .838z_{t-1}$ (ESR) vs $\pi_t = 0$ (IT) - see Table 2. ESR solid line, IT diamonds.