THE EVOLUTION OF INTERTEMPORAL PREFERENCES

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The Evolution of Intertemporal Preferences*

Arthur Robson and *Larry Samuelson*

Where do preferences come from? What determines their properties? Though traditionally reluctant to ask such questions, economists have recently turned to evolutionary models for answers.

We focus on intertemporal preferences here, arising out of the evolutionary implications of different reproductive strategies or *life histories.* An agent's life history specifies the agent's number and timing (and, in a richer model, such aspects as quality) of offspring. Evolution will select the life history that maximizes the growth rate of the associated group of individuals.

We begin with the simplest possible biological life history—that of a *semelparous* agent that, if it survives a fixed number of years, reproduces and then dies. We can solve for the growth rate explicitly in this case. The evolutionary criterion for success entails hyperbolic time discounting of the log of the number of offspring produced. However, the rate of time preference is a function of age, not of time relative to the present, and hence there are no preference reversals in the sense of behavioral economics. At the same time, the optimal strategy maximizes the exponentially discounted number of offspring, provided we discount *at the maximal growth rate.* Conventional discounting thus suffices to induce optimal choices from the agent, with positive rates of population growth inducing impatience.

More generally, if the animal is *iteroparous*, and so may have a non-degenerate profile of offspring, the indifference curves are hyperplanes that are not parallel, but tilt to reflect greater impatience as the growth rate increases. This implies that the rate of time discounting of offspring falls with age, if there is a positive population growth rate to begin with. Evolution's preferences thus exhibit a present bias, though this again cannot imply preference reversals in the sense of behavioral economics.

There is no additively separable function of the age profile of expected offspring that is globally equivalent to the basic biological growth-rate criterion, even if arbitrary age dependence is allowed. Nevertheless, there is still a close local relationship between the biological growth rate criterion and the sum of expected offspring, exponentially discounted by the maximum growth rate.

An implication of these results is that the preferences evolution induces in an agent within a population need not match the preferences by which evolution chooses across populations. In particular, the preferences of an individual agent within a population can provide a guide as to which life histories could potentially invade that population, but not necessarily as to which of these would have the greatest ultimate advantage as an invader, and hence might be the most likely successful mutant. In addition, preferences that appear to be quite conventional in the environment in which they evolved may exhibit anomalies when pushed outside this environment.

We refer the reader to Arthur Robson and Larry Samuelson (2006) for a more complete treatment, including references to the literature.

1 Semelparous Life Histories

We begin with the simplest case, that of a *semelparous* life history, in which an organism reproduces at a fixed, single age (if it survives that long) and then dies.

Let time be discrete, given by $t = 0, 1, 2, \ldots$. An agent is characterized by a life pair (x, τ) identifying the agent's expected number of offspring x, to be produced at age τ, conditional on the agent's surviving until age τ.

In keeping with our focus on preferences over reproduction, we represent the strategy (x, τ) simply as a number of offspring and a time, though it presumably reflects a collection of activities, such as where to forage, what food so eat, when to mate, and so on, that determine reproduction. The strategy (x, τ) is heritable, so that offspring are characterized by the same strategy as their parents. We are interested in which such strategy will be favored by evolution.

While waiting to reproduce, the agent faces an instantaneous death risk of $\delta > 0$. An agent choosing (x, τ) thus survives for τ periods with probability $e^{-\delta\tau}$. If and only if the agent survives, the x offspring appear, at which point the agent dies. We work with a continuum of agents and without aggregate uncertainty.

Consider a population characterized by strategy (x, τ). Let $N^T(t)$ be a (row) vector $(N_1(t), N_2(t), \ldots, N_\tau(t))$ describing the measures of agents of each age in the population, at the beginning of time t. During time t, the τ-period-old agents produce an average of x offspring apiece, and then die. Proportion $e^{-\delta}$ of the remaining agents then survive until the next period, each becoming one period older in the process. The basic difference equation governing the population is

$$N^T(t) = N^T(t-1) \begin{bmatrix} 0 & e^{-\delta} & 0 & \ldots & 0 & 0 \\ 0 & 0 & e^{-\delta} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & e^{-\delta} \\ e^{-\delta}x & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}.$$

In this $\tau \times \tau$ matrix, each row $n \in \{1, \ldots, \tau - 1\}$ applies to the agents who enter period t at age n. The single term $e^{-\delta}$ that appears in row n identifies the proportion of these agents who survive until the beginning of the next period, at which point they are one period older. The final row corresponds agents who enter at age τ. These agents have x offspring each, with these offspring surviving to become next period's 1-period-olds and with the parents disappearing from the population. This is referred to as a *Leslie matrix.*

Asymptotically, the growth factor for this population is given by the leading eigenvalue of the Leslie matrix, the *Frobenius root* $\tilde{\lambda} > 0$. More precisely, the population vector $N(t)$

satisfies

$$\frac{N^T(t)}{\tilde{\lambda}^t} \quad \rightarrow \quad N^T(0)wv^T, \text{ as } t \rightarrow \infty,$$

where v^T and w are the strictly positive left (row) and right (column) eigenvectors of the Leslie matrix associated with $\tilde{\lambda} > 0$, and $v^T w = 1$. If $\sum\limits_k v_k = 1$, then v describes the limiting proportion of each age and w gives the reproductive value of an individual of each age in the initial population.

The growth factor $\tilde{\lambda}$ is the unique positive real root of the characteristic equation of the Leslie matrix. The characteristic equation is equivalent to

$$\lambda^\tau = e^{-\delta\tau}x,$$

so that the Frobenius root is $e^{-\delta}(x)^{\frac{1}{\tau}}$.

We now consider a population of agents characterized by a variety of life histories (x, τ). Evolution will select for the value (x, τ) that maximizes $e^{-\delta}(x)^{\frac{1}{\tau}}$ or, equivalently, that maximizes

$$\frac{\ln x}{\tau}.$$

Hence, evolution evaluates births according to the function $\ln(\cdot)$ and discounts them hyperbolically—evolution exhibits a present bias. The equilibrium population will grow geometrically at rate $\tilde{\lambda}$, or (equivalently) exponentially at rate $\ln\tilde{\lambda}$.

While evolution's preferences are characterized by a present bias, evolution can induce agents to behave optimally via conventional exponential discounting. Let (x_1, τ_1) be the optimal life history, giving rise to exponential growth at rate $r_1 = \ln\left(e^{-\delta}x_1^{\frac{1}{\tau_1}}\right) = -\delta + \frac{1}{\tau_1}\ln x_1$. Let the alternative strategy (x_2, τ_2) give growth rate $r_2 = \ln\left(e^{-\delta}x_2^{\frac{1}{\tau_2}}\right) = -\delta + \frac{1}{\tau_2}\ln x_2 < r_1$. Suppose that agents are designed to evaluate births linearly and discount exponentially at rate $-(\delta + r_1)$, so that (x, τ) is evaluated as $e^{-(\delta+r_1)\tau}x$. This choice of discount rates is intuitive. There are two costs of delaying reproduction. One of these is simply that death occurs at rate δ. The other is that a given number of offspring will comprise a smaller fraction of a population growing at rate r_1. The sum of these two is the rate at which delaying births causes an agent to fall behind the population.

The comparison between two reproductive strategies is then given by (using $e^{-(\delta+r_1)\tau_1}x_1 = 1 = e^{-(\delta+r_2)\tau_2}x_2$)

$$\begin{aligned} & e^{-(\delta+r_1)\tau_1}x_1 > e^{-(\delta+r_1)\tau_2}x_2 \\ \Leftrightarrow\quad & e^{-(\delta+r_1)\tau_1}x_1 > e^{-r_1\tau_2}e^{r_2\tau_2}e^{-(\delta+r_2)\tau_2}x_2 \\ \Leftrightarrow\quad & 1 > e^{-r_1\tau_2}e^{r_2\tau_2} \\ \Leftrightarrow\quad & r_1 > r_2. \end{aligned}$$

Hence, exponential discounting, at the sum of the death and optimal growth rates, identifies the optimal strategy.

2 Iteroparous Life Histories

We now generalize the analysis to *iteroparous* life histories, in which an individual may have offspring at more than one age.

2.1 Asymptotic Growth Rate

Suppose that agents live for ℓ periods, producing x_i offspring in each period $i = 1, \ldots, \ell$. A life history is then a collection $(x_1, x_2, \ldots, x_\ell)$, where some of these entries may be zero. The Leslie matrix is

$$\begin{bmatrix} e^{-\delta}x_1 & e^{-\delta} & 0 & \ldots & 0 & 0 \\ e^{-\delta}x_2 & 0 & e^{-\delta} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-\delta}x_{\ell-1} & 0 & 0 & \ldots & 0 & e^{-\delta} \\ e^{-\delta}x_\ell & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}.$$

The population's asymptotic growth factor is again the largest real root $\tilde{\lambda}$ of the characteristic equation of this matrix. This equation can be written as

$$1 = \frac{x_1}{e^\delta \lambda} + \frac{x_2}{(e^\delta \lambda)^2} + \ldots + \frac{x_\ell}{(e^\delta \lambda)^\ell}. \tag{1}$$

Evolution will select for the life history that maximizes this growth factor. Along any indifference surface for evolution, $e^\delta \tilde{\lambda}$ is constant and hence we have a linear function of the values $x_1, \ldots, x_\ell$. Evolution's indifference surfaces are thus hyperplanes in a space of ℓ dimensions. Figure 1 illustrates.

It is immediate from (1) that evolution is indifferent over two semelparous strategies (x_1, τ_1) and (x_2, τ_2) if and only if $x_1^{\frac{1}{\tau_1}} = x_2^{\frac{1}{\tau_2}}$. This confirms that the semelparous analysis is a special case of this more general model. Preferences over the remaining iteroparous strategies are captured by connecting indifferent semelparous strategies with linear indifference surfaces.

2.2 Induced Preferences Over Consumption

Economists are typically interested in preferences over consumption rather than births. The simplest transition from preferences over births to preferences over consumption is made by assuming that births are a function of consumption, and that preferences over consumption are those induced by the underlying preferences over births.

Consider for simplicity the case in which age-τ births depend only on age-τ consumption. It is then straightforward to obtain a description of the indifference curves over consumption. Let $f_\tau(c_\tau)$ give age-τ births as a function of period-τ consumption c_τ. Suppose that all the f_τ are strictly increasing and concave.

For any consumption vector c, an indifference curve is defined by (from (1), replacing $e^\delta \lambda$ with θ)

$$1 \;=\; \frac{f_1(c_1)}{\theta} + ... + \frac{f_\tau(c_\tau)}{\theta^\tau} + \ldots + \frac{f(c_{\ell-1})}{\theta^{\ell-1}} + \frac{f_\ell(c_\ell)}{\theta^\ell}, \tag{2}$$

where $\theta > 0$ is constant on a particular indifference surface. A higher value of θ indicates a higher indifference curve, so that consumption plan $(c'_1, \ldots, c'_\ell)$ is preferred to $(c_1, \ldots, c_\ell)$ if and only if

$$1 = \frac{f_1(c_1)}{\theta} + \ldots + \frac{f_\ell(c_\ell)}{\theta^\ell} < \frac{f_1(c'_1)}{\theta} + \ldots + \frac{f_\ell(c'_\ell)}{\theta^\ell}.$$

It follows that evolution's indifference surfaces over consumption bundles $(c_1, \ldots, c_\ell)$ have the usual shape, in the sense that evolution's preferences can be described by a utility function $\theta(c_1, ..., c_\ell)$ that is strictly increasing and quasi-concave.

2.3 Present Bias

As long as the population growth rate is positive, so that evolution has a preference for early births, evolution's biological criterion generates a decreasing rate of impatience with age. We illustrate this by considering a type that lives to age three with fertility/utility f_1, f_2 and f_3 in periods 1, 2 and 3. Consider the possibilities of augmenting fertility/utility by η_1 in the first period and augmenting it by η_2 in the second period. Suppose there is indifference between these alternatives, so that, for some $\theta > 1$ (cf. (2)),

$$1 = \frac{f_1 + \eta_1}{\theta} + \frac{f_2}{\theta^2} + \frac{f_3}{\theta^3} = \frac{f_1}{\theta} + \frac{f_2 + \eta_2}{\theta^2} + \frac{f_3}{\theta^3}$$

and hence $\theta\eta_1 = \eta_2$. Now consider the possibilities of augmenting fertility/utility by η_1 in the second period and augmenting it by η_2 in the third. Let the first option lie on an indifference surface characterized by θ', so that

$$1 = \frac{f_1}{\theta'} + \frac{f_2 + \eta_1}{(\theta')^2} + \frac{f_3}{(\theta')^3}.$$

As long as $\eta_1 > 0$, we have $\theta' < \theta$ (i.e., a pure postponement of births is disadvantageous when the population is growing). Hence, we have $\theta'\eta_1 < \eta_2$, implying

$$1 = \frac{f_1}{\theta'} + \frac{f_2 + \eta_1}{(\theta')^2} + \frac{f_3}{(\theta')^3} < \frac{f_1}{\theta'} + \frac{f_2}{(\theta')^2} + \frac{f_3 + \eta_2}{(\theta')^3}.$$

Evolution is thus indifferent when deferring an increment in fertility/utility from age 1 to 2, but strictly prefers to defer from period 2 to 3, i.e., evolution's preferences exhibit a present bias. In essence, deferring increments in fertility/utility lowers the associated growth rate, leading to less impatience.

Evolution's present bias does not lead to preference reversals here, in contrast to the models of present bias that form the heart of behavioral economics. Preferences are defined over age, rather than over time relative to the present. Suppose, for example, that

$$1 \;=\; \frac{f_1}{\theta'} + \frac{f_2+\eta_1}{(\theta')^2} + \frac{f_3}{(\theta')^3} \;<\; \frac{f_1}{\theta'} + \frac{f_2}{(\theta')^2} + \frac{f_3+\eta_2}{(\theta')^3}$$

as before, but we break the indifference in the first choice to give

$$1 \;=\; \frac{f_1+\eta_1}{\theta} + \frac{f_2}{\theta^2} + \frac{f_3}{\theta^3} \;>\; \frac{f_1}{\theta} + \frac{f_2+\eta_2}{\theta^2} + \frac{f_3}{\theta^3}.$$

Hence the individual would choose $(f_1, f_2, f_3+\eta_2)$ over $(f_1, f_2+\eta_1, f_3)$ if offered this choice at age 0. If again offered this choice at age 1, having already obtained utility/fertility f_1, the individual will reiterate her original choice. That is, the growth rate that is generated by $(f_1, f_2, f_3+\eta_2)$ exceeds that generated by $(f_1, f_2+\eta_1, f_3)$ even if this calculation is redone at age 1.

2.4 No Time Separable Representation

The basic biological criterion of maximizing the population growth rate is a complex function of the fertility profile. Let $\theta(x_1, x_2, ...)$ be the function implicitly defined by (1) (cf. (2)). Then the marginal rate of substitution between x_t and x_{t+1} is θ itself, which is a strictly increasing function of each x_s for $s \neq t, t+1$. It is then immediate that there then can be no additively separable representation of preferences, even if an arbitrary age dependence is permitted.

2.5 Exponential Discounting

Agents can once again be induced to make optimal choices via exponentially discounting offspring at the optimal growth rate. Letting $(x_1, \ldots, x_\ell)$ be the optimal life history and $\theta = e^\delta \tilde{\lambda}$, where $\tilde{\lambda}$ is the leading eigenvalue of the corresponding Leslie matrix, (1) gives

$$1 = \frac{x_1}{\theta} + \frac{x_2}{\theta^2} + \ldots + \frac{x_\ell}{\theta^\ell}.$$

Now suppose an alternative life history $(x_1', \ldots, x_\ell')$ gives a smaller growth rate. Then we must have

$$\frac{x_1}{\theta} + \frac{x_2}{\theta^2} + \ldots + \frac{x_\ell}{\theta^\ell} \quad = \quad 1 \quad > \quad \frac{x_1'}{\theta} + \frac{x_2'}{\theta^2} + \ldots + \frac{x_\ell'}{\theta^\ell}.$$

The agent can thus be induced to make the optimal decision by exponentially discounted preferences.

2.6 Whose Preferences?

"Evolution's" preferences provide a complete ranking of life histories in terms of their induced population growth rates. It is this ranking that exhibits the hyperbolic discounting of log births in the semelparous case, and that more generally exhibits a present bias. However, a complete ranking is unnecessary—evolution need only select the optimal life history. As we have seen, this is consistent with exponentially discounting births. In particular, there is nothing we could observe in evolutionarily optimal behavior that is inconsistent with exponential discounting. Then why do we care about evolution's preferences?

One answer is theoretical. The exponentially discounted preferences consistent with optimal behavior can provide a guide as to which life histories could invade that population, but cannot provide additional information such as which mutants are will ultimately be the most successful. Another answer is more practical. Preferences that would appear to be quite conventional in the environment in which they evolved may exhibit anomalies when pushed outside this environment by our current, evolutionarily novel circumstances.

References:

Arthur Robson and Larry Samuelson (2006), "The Evolution of Intertemporal Preferences," Simon Fraser University and University of Wisconsin.

Footnotes:

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Figure 1:

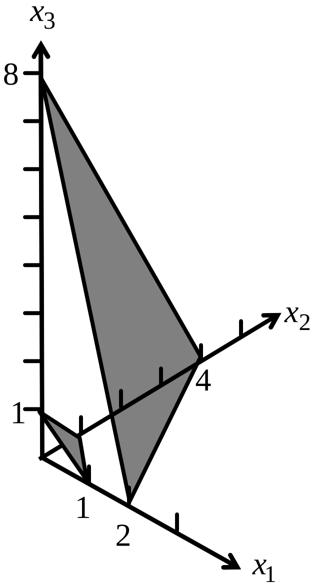

Figure 1 caption:

Illustration of evolution's indifference surface when $\ell = 3$. Evolution is indifferent over the three semelparous strategies $(x, 1)$, $(x^2, 2)$ and $(x^3, 3)$. Indifference surfaces are hyperplanes containing such equivalent semelparous strategies.